

t-SNE

Giảm chiều dữ liệu

$$X = \{x_1, x_2, \dots, x_n \in \mathbb{R}^n\} \Rightarrow Y = \{y_1, y_2, \dots, y_n \in \mathbb{R}^d\}$$

$$\min_Y C(X, Y)$$

$$p_{j|i} = \frac{\exp(-\|x_i - x_j\|^2 / 2\sigma_i^2)}{\sum_{k \neq i} \exp(-\|x_i - x_k\|^2 / 2\sigma_i^2)}$$

$$q_{j|i} = \frac{\exp(-\|y_i - y_j\|^2)}{\sum_{k \neq i} \exp(-\|y_i - y_k\|^2)}$$

$$\left. \begin{array}{l} p_{i|j} : x_s \\ q_{i|j} : x_s \end{array} \right\} \begin{array}{l} x_i \Rightarrow x_j \\ y_i \Rightarrow y_j \end{array} \left\{ \begin{array}{l} \text{Muốn } x_s \text{ } p_{i|j} \text{ \& } q_{i|j} \text{ giống nhau} \end{array} \right.$$

$$P_i = \{p_{1|i}, p_{2|i}, \dots, p_{n|i}\} \Rightarrow \text{sim}(x_1, x_i)$$

$$\text{and } Q_i = \{q_{1|i}, q_{2|i}, \dots, q_{n|i}\} \Rightarrow \text{sim}(y_1, y_i)$$

Tính khoảng cách KL giữa $\text{sim}(x_1, x_i)$ và $\text{sim}(y_1, y_i)$

$$C = \sum_i KL(P_i \parallel Q_i) = \sum_i \sum_j p_{j|i} \log \frac{p_{j|i}}{q_{j|i}}$$

$$= \sum_{k \neq i} p_{i/k} \log p_{i/k} - p_{i/k} \log q_{i/k}$$

$$= \sum_{k \neq i} p_{i/k} \log p_{i/k} - p_{i/k} \log E_k + p_{i/k} \log Z_k$$

$$\Rightarrow \frac{\partial C}{\partial y_i} = \sum_{k \neq i} -p_{i/k} \log E_k + \sum_{k \neq i} p_{i/k} \log Z_k$$

$$\sum_{k \neq i} -p_{i/k} \log E_k = \sum_{j \neq i} -p_{j|i} \log E_{ij} - p_{i|j} \log E_{ji}$$

$$\omega' \log E_{ij} = E_{ij} (-2(y_i - y_j)) \text{ là } \omega'$$

$$\sum_{j \neq i} -p_{j|i} \frac{E_{ij}}{E_j} (-2(y_i - y_j)) - p_{i|j} \frac{E_{ji}}{E_i} (2(y_i - y_j))$$

$$= 2 \sum_{j \neq i} (p_{j|i} + p_{i|j}) (y_i - y_j)$$

b' $E_{i \neq j} p_{i|j} = 1$ và z_j phụ thuộc vào k .

$$\sum_{i, k \neq j} p_{k|j} \partial \log z_j = \sum_j \partial \log z_j.$$

Đạo hàm $\neq 0$ khi $k=i$ hoặc $j=i$.

$$= \sum_j \frac{1}{z_j} \sum_{k \neq j} \partial \bar{E}_{jk}$$

$$= \sum_{j \neq i} \frac{E_{ji}}{z_j} (2(y_i - y_j)) + \sum_i \frac{E_{ij}}{z_i} (-2(y_i - y_j))$$

$$= 2 \sum_{j \neq i} (-q_{j|i} - q_{i|j}) (y_i - y_j)$$

$$\Rightarrow \frac{\partial C}{\partial y_i} = 2 \sum_{j \neq i} (p_{j|i} - q_{j|i} + p_{i|j} - q_{i|j}) (y_i - y_j)$$

t-distribution .

$$q_{ji} = q_{ij} \frac{(1 + \|y_j - y_i\|^2)^{-1}}{\sum_{k, l \neq k} (1 + \|y_k - y_l\|^2)^{-1}} = \frac{E_{ij}^{-1}}{\sum_{k, l \neq k}} = \frac{E_{ij}^{-1}}{2}$$

Loss function :

$$\begin{aligned} C &= \sum_{k, l \neq k} p_{lk} \log \frac{p_{lk}}{q_{lk}} = \sum_{k, l \neq k} p_{lk} \log(p_{lk}) - p_{lk} \log q_{lk} \\ &= \sum_{k, l \neq k} p_{lk} \log p_{lk} - p_{lk} \log E_{kl}^{-1} + p_{lk} \log 2 \end{aligned}$$

Đạo hàm biến y_i :

$$\frac{\partial C}{\partial y_i} = \sum_{k, l \neq k} -p_{lk} \partial \log E_{kl}^{-1} + \sum_{k, l \neq k} p_{lk} \partial \log 2$$

$$\sum_{k, l \neq k} -p_{lk} \partial \log E_{kl}^{-1} = -2 \sum_{i \neq j} p_{ji} \partial \log E_{ij}^{-1}$$

$$\partial E_{ij}^{-1} = \frac{E_{ij}^{-2}}{E_{ij}^{-1}} (-2(y_i - y_j)) = 4 \sum_{i \neq j} p_{ji} E_{ij}^{-1} (y_i - y_j)$$

$\sum_{k, l \neq k} p_{kl} = 1$ và $2 =$ phụ thuộc k hoặc l.

$$\begin{aligned} \sum_{k, l \neq k} p_{lk} \partial \log 2 &= \frac{1}{2} \sum_{k, l \neq k} \partial E_{kl}^{-1} \\ &= 2 \sum_{i \neq j} \frac{E_{ji}^2}{2} (-2(y_j - y_i)) \\ &= 4 \sum_{i \neq j} q_{ij} E_{ji}^{-1} (y_i - y_j) \end{aligned}$$

$$\Rightarrow \frac{\partial C}{\partial y_i} = 4 \sum_{j \neq i} (p_{ji} - q_{ji}) E_{ji}^{-1} (y_i - y_j)$$

$$\frac{\partial C}{\partial y_i} = 4 \sum_{j \neq i} (p_{ji} - q_{ji}) (1 + \|y_i - y_j\|^2)^{-1} (y_i - y_j)$$