Suppose we have dataset $\{x_1, ...x_N\}$ consisting of N observations of a random D-dimensional Euclidean variable x. A set of D-dimensional vectors μk, where k = 1,...,K in which μk is a prototype associated with the k$^{th} cluster.$

$Function$:

$$J = \sum_{n=1}^{N} \sum_{K=1} K r_{nk} \parallel x_n - \mu_k \parallel^2 \, with \left\{ r_{n,k} \in \{0, 1\}, \forall n, ksum_{j=1} K r_{n,k} = 1 \forall n \right.$$

Goal: find values for r_{nk} and the μk => minimize J.

Consider the optimization of the μ_k with the r_{nk} held fixed. The J is a quadratic funcion of μ_k, and it can be minimized by setting its derivative with respect to μ_k to zero.

$$2 \sum_{n=1} N r_{nk}(x_n - \mu_k) = 0 \; \mu_k = \frac{sum_{n=1} r_{nk} x_n}{\sum_{n=1} r_{nk}}$$

So μ_k equal to the mean of all of the data points x_n assigned to cluter k.