

1. Likelihood function of logistic regression:

Consider the case has two classes. The posterior probability for  $C_1$  can be written:

$$p(C_1|x) = \frac{p(x|C_1)p(C_1)}{p(x|C_1)p(C_1)+p(x|C_2)p(C_2)}$$

where we have defined:

$$a = \log \frac{p(x|C_1)p(C_1)}{p(x|C_2)p(C_2)}$$

So the logistic sigmoid function defined by

$$\sigma(a) = \frac{1}{1+e^{-a}}$$

The model logistic regression:

$$p(C_1|\phi) = y(\phi) = \sigma(w^T \phi) \quad p(C_2|\phi) = 1 - p(C_1|\phi)$$

The likelihood function:

$$p(t|w) = \prod_{n=1}^N (y_n)^{t_n} (1 - y_n)^{1-t_n} \quad L = -\log(t|w) = -\sum_{n=1}^N (t_n \log y_n + (1 - t_n) \log(1 - y_n))$$

want to minimize L by taking derivative of it with respect to w, we have:

$$\frac{\delta L}{\delta w_0} = \frac{\delta L}{\delta y_n} \frac{\delta y_n}{\delta z} \frac{\delta z}{\delta w_0}$$

we have:  $\frac{\delta L}{\delta y_n} = -\frac{t}{y} - \frac{1-t}{1-y}$   
 $\frac{\delta y_n}{\delta z} = \sigma(z)(1 - \sigma(z))$   
 $\frac{\delta z}{\delta w_0} = 1$   
 so that:

$$\rightarrow \frac{\delta L}{\delta w_0} = y_n - t \quad \frac{\delta L}{\delta w_1} = (y_n - t)\phi_1 \rightarrow \frac{\delta L}{\delta w} = \sum_{n=1}^N (y_n - t_n)\phi_n$$

2. Find f(x) that f'(x) = f(x)(1-f(x))

$$f'(x) = f(x)(1 - f(x))$$

$$\rightarrow \int dx = \int \frac{df(x)}{f(x)-f^2(x)}$$

$$x = \int \frac{df(x)}{f(x)(1-f(x))}$$

$$x = \int \left( \frac{1}{f(x)} - \frac{1}{f(x)-1} \right) df(x) -$$

$$-x = \int \frac{df(x)}{f(x)} - \int \frac{df(x)}{f(x)-1}$$

$$-x = \ln|f(x)| - \ln|1-f(x)|$$

$$-x = \ln|f(x) \frac{f(x)}{f(x)-1}|$$

$$\rightarrow e^{-x} = \frac{f(x)}{f(x)-1}$$

$$\rightarrow \frac{1}{e^{-x}} = \frac{f(x)-1}{f(x)} = 1 - \frac{1}{f(x)}$$

$$\rightarrow \frac{1}{f(x)} = 1 - \frac{1}{e^{-x}} = \frac{e^{-x}-1}{e^{-x}}$$

$$f(x) = e^{-x} \frac{e^{-x}}{e^{-x}-1}$$