

we have that

$$p(w|x, t, \alpha, \beta) = \frac{p(t|x, w, \beta)p(w|\alpha)}{p(x, t, \alpha, \beta)}$$

We are trying to maximize the posterior to find w , which means we have to maximize $p(t|x, w, \beta)p(w|\alpha)$. Suppose $p(w|\alpha)$ is a normal distribution, we have:

$$p(w|\alpha) = N(w|0, \alpha^{-1}I) = ((\frac{\alpha}{2\pi i})^{(M+1)/2}) \exp(-\frac{\alpha}{2} w^T w)$$

So

$$p(w|x, t, \alpha, \beta) \propto p(t|x, w, \beta) p(w|\alpha) \exp(-\frac{\beta}{2} \sum_{n=1}^N (y(x_n, w) - t_n)^2 - \frac{\alpha}{2} w^T w)$$

we find that the maximum of the posterior by the minimum of

$$\frac{\beta}{2} \sum_{n=1}^N (y(x_n, w) - t_n)^2 + \frac{\alpha}{2} w^T w$$

or minimize

$$Q = (\|Xw - t\|)^2 + \lambda w^T w$$

we have

$$\begin{aligned} \frac{dQ}{dw} &= \frac{d}{dw} (Xw - t)^T (Xw - t) + 2\lambda(Iw) = \frac{d}{dw} (w^T X^T X w - t^T X w + X^T X w - 2X^T t + 2\lambda(Iw)) = 0 \\ &< - > X^T X w = X^T t + \lambda(Iw) \\ &< - > w = (X^T X + \lambda(I))^{-1} X^T t \end{aligned}$$