1. Likelihood function of logistic regression:
Consider the case has two classes. The posterior probability for C_1 can be written:

$$p(C_1|x) = \frac{p(x|C_1)p(C_1)}{p(x|C_1)p(C_1)+p(x|C_2)p(C_2)}$$

where we have defined:

$$a = log\frac{p(x|C_1)p(C_1}{p(x|C_2)p(C_2}$$

So the logistic sigmoid function defined by

$$\sigma(a) = \frac{1}{1+e^(-a)}$$

The model logistic regression:

$$p(C_1|\phi) = y(\phi) = \sigma(w^T\phi)\ p(C_2|\phi) = 1 - p(C_1|\phi)$$

The likelihood function:

$$p(t|w) = \prod_{n-1}^{N}(y_n)^{t_n}(1-y_n)^{1-t_n}\ L = -log(t|w) = -\sum_{n-1}^{N}(t_n log y_n + (1-t_n)log(1-y_n)$$

want to minimize L by taking derivative of it with respect to w, we have:

$$\frac{\delta L}{\delta w_0} = \frac{\delta L}{\delta y_n}\frac{\delta y_n}{\delta z}\frac{\delta z}{\delta w_0}$$

we have: $\frac{\delta L}{\delta y_n} = -\frac{t}{y} - \frac{1-t}{1-y}$
$\frac{\delta y_n}{\delta z} = \sigma(z)(1-\sigma(z))$
$\frac{\delta z}{\delta w_0} = 1$
so that:

$$\rightarrow \frac{\delta L}{\delta w_0} = y_n - t\ \frac{\delta L}{\delta w_1} = (y_n - t)\phi_1 \rightarrow \frac{\delta L}{\delta w} = \sum_{n=1}^{N}(y_n - t_n)\phi_n$$

2. Find f(x) that f'(x) = f(x)(1-f(x))

$$f'(x) = f(x)(1-f(x))$$

$$\rightarrow \frac{df(x)}{dx} = f(x) - f^2(x)$$

$$\text{dx} = \text{df(x)}\frac{}{f(x)-f^2(x)}$$

$$\rightarrow \int dx = \int \frac{df(x)}{f(x)-f^2(x)}$$

$$\text{x} = \int \frac{df(x)}{f(x)(1-f(x))}$$

$$\text{x} = \int (\frac{1}{f(x)} - \frac{1}{f(x)-1})df(x)$$

$$\text{x} = \int \frac{df(x)}{f(x)} - \int \frac{df(x)}{f(x)-1}$$

x = ln|f(x)| - ln|1-f(x)|

$$\text{x} = \text{ln|f(x)}\frac{}{f(x)-1|}$$

$$\rightarrow e^x = \frac{f(x)}{f(x)-1}$$

$$\rightarrow \frac{1}{e^x} = \frac{f(x)-1}{f(x)} = 1 - \frac{1}{f(x)}$$

$$\rightarrow \frac{1}{f(x)} = 1 - \frac{1}{e^x} = \frac{e^x - 1}{e^x}$$

$$\text{f(x)} = \text{e}^x\frac{}{e^x-1}$$