ASSIGNMENT # 1

FORMAL METHODS

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Exercise 1.1

Question #①

Use $\neg, \rightarrow, \wedge, \vee$ to express the following.

b- Robert was jealous of Yvone ----

p: Robert was jealous of Yvone.
q: Robert was in a good mood.

Proposition: $p \vee \neg q$

d- If a request occurs, then either it will ----

p: A request occurs.
q: The request will be eventually acknowledged.
r: The requesting process will make progress.

Propositional Expression: $p \rightarrow (q \vee \neg r)$

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i- If interest rates go up, ...

p: Interest rate goes up
q: Share prices go down

Expression: $p \rightarrow q$

h- Today it will rain or shine, but not both.

p: It will rain today
q: It will shine today

Expr: $(p \vee q) \wedge \neg (p \wedge q)$

j- No shoes, no shirt, no service.

p: No shoes
q: No shirt
r: No service.

Expression: $p \vee q \rightarrow r$

Question #②

b- $(p \rightarrow q) \wedge \neg (r \vee p \rightarrow q)$

$(p \rightarrow q) \wedge (r \vee (p \rightarrow q))$

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d- $p \vee (\neg q \rightarrow p \wedge r)$.

$p \vee (\neg q \rightarrow (p \wedge r))$

f- $p \vee p \rightarrow \neg q$

$(p \vee p) \rightarrow \neg q$

Exercise 1.2

Prove the validity of following sequents.

b- $p \wedge q \vdash q \wedge p$

1	$p \wedge q$	premise
2	p	$\wedge e_2 1$
3	q	$\wedge e_1 1$
4	$p \wedge q$	$\wedge i 2,3$

d- $p \rightarrow (p \rightarrow q), p \vdash q$

1	$p \rightarrow (p \rightarrow q)$	premise
2	p	premise
3	$p \rightarrow q$	e 2, 1
4	q	e 2, 3

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f- $\vdash (p \wedge q) \rightarrow p$.

1	$p \wedge q$	assumption
2	p	e_1 1
3	$(p \wedge q) \rightarrow p$	i 1-2

h- $p \vdash (p \rightarrow q) \rightarrow q$

1	p	premise
2	$p \rightarrow q$	assumption
3	q	e 2, 1
4	$(p \rightarrow q) \rightarrow q$	i 2-3.

j- $q \rightarrow r \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$

1	$q \rightarrow r$	premise
2	$p \rightarrow q$	assumption
3	p	assumption
4	q	e 2, 3
5	r	e 1, 4.
6	$p \rightarrow r$	→i 3-5
7	$(p \rightarrow q) \rightarrow (p \rightarrow r)$	→i 2-6.

l- $p \rightarrow q, r \rightarrow s \vdash p \vee r \rightarrow q \vee s$

1	$p \rightarrow q$	premise
2	$r \rightarrow s$	premise
3	$p \vee r$	assumption
4	p	assumption

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5	q	$\to e$ 1,4
6	$q \vee s$	$\vee i$ 5
7	r	assumption
8	s	$\to e$ 2,7
9	$q \vee s$	$\vee i$ 8
10	$q \vee s$	$\vee e$ 3,4-6,7-9
11	$p \vee r \to q \vee s$	$\to i$ 3-10

n $(p \vee (q \to p)) \wedge q \vdash p$

1	$(p \vee (q \to p)) \wedge q$	premise
2	q	$\wedge e_2$ 1
3	$p \vee (q \to p)$	$\wedge e_1$ 1
4	p	assumption
5	$q \to p$	$\vee e_2$ 3
6	p	$\to e$ 5,2

o- $p \to q,\ r \to s \vdash p \wedge r \to q \wedge s$

1	$p \to q$	premise
2	$r \to s$	premise
3	$p \wedge r$	assumption
4	p	$\wedge e_1$ 3
5	q	$\to e$ 1,4
6	r	$\wedge e_2$ 3
7	s	$\to e$ 2,6
8	$q \wedge s$	$\wedge i$ 5,7
9	$(p \wedge r) \to (q \wedge s)$	$\to i$ 3-8

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q- $\vdash q \to (p \to (p \to (q \to p)))$

1	q	assumption
2	p	assumption
3	p	assumption
4	q	assumption
5	p	same as 2
6	$q \to p$	$\to$ i 4,5
7	$p \to (q \to p)$	$\to$ i 3-6
8	$p \to (p \to (q \to p))$	$\to$ i 2-7
9	$q \to (p \to (p \to (q \to p)))$	$\to$ i 1-8

s- $(p \to q) \wedge (p \to r) \vdash p \to q \wedge r$

1	$(p \to q) \wedge (p \to r)$	premise
2	$p \to q$	$\wedge$ e, 1
3	$p \to r$	$\wedge$ e, 1
4	p	assumption
5	q	$\to$ e 4, 2
6	r	$\to$ e 4, 3
7	$q \wedge r$	$\wedge$ i 5, 6
8	$p \to q \wedge r$	$\to$ i 4-7

u- $p \to q \vdash \neg q \to \neg p$

1	$p \to q$	premise
2	$\neg q$	assumption
3	$\neg p$	MT 1, 2
4	$\neg q \to \neg p$	$\to$ i 2-3

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w- $r, p \to (r \to q) \vdash p \to (q \wedge r)$

1	r	premise
2	$p \to (r \to q)$	premise
3	p	assumption
4	$r \to q$	$\to e_2$ 2
5	q	$\to e$ 4,1
6	$q \wedge r$	$\wedge i$ 5,1
7	$p \to (q \wedge r)$	$\to i$ 3-6

y- $(p \wedge q) \vee (p \wedge r) \vdash p \wedge (q \vee r)$

1	$(p \wedge q) \vee (p \wedge r)$	premise
2	$p \wedge q$	$\vee e_1$ 1
3	p	$\wedge e_1$ 2
4	q	$\wedge e_2$ 2
5	$p \wedge r$	$\vee e_2$ 1
6	r	$\wedge e_2$ 5
7	$q \vee r$	$\vee i$ 4,6
8	$p \wedge (q \vee r)$	$\wedge i$ 3,7.

Question #(2)
Prove which ones of following are valid and which not.

b- $\neg p \vee \neg q \vdash \neg(p \wedge q)$

1	$\neg p \vee \neg q$	premise
2	$\neg p$	$\vee e_1$ 1
3	$\neg q$	$\vee e_2$ 1
4	$\neg\neg p \equiv p$	$\neg i$ 2
5	$\neg\neg q \equiv q$	$\neg i$ 3
6	$p \wedge q$	$\wedge i$ 4,5

(7)

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7	$\neg(p \wedge q)$	$\neg i\ 6$

Hence this is valid

d $p \vee q, \neg q \vee r \vdash p \vee r$

1	$p \vee q$	premise
2	$\neg q \vee r$	premise
3	$\neg q$	assumption
4	$p \vee q$	same as 1
5	p	assumption
6	$p \vee q$	$\vee i_1\ 5$
7	q	assumption
8	$\perp$	$\neg e\ 7,3$
9	$p \vee r$	$\perp e\ 8$
10	$p \vee r$	$\vee e\ 4, 5\text{-}6, 7\text{-}9$
11	r	assumption
12	$p \vee r$	$\vee i_2\ 11$
13	$p \vee r$	$\vee e\ 2, 3\text{-}10, 11\text{-}12$

Hence, valid

f- $\neg p \wedge \neg q \vdash \neg(p \wedge q)$

1	$\neg p \wedge \neg q$	premise
2	$p \vee q$	assumption
3	p	assumption
4	$\neg p$	$\wedge e_1 1$
5	$\perp$	$\neg e\ 3,4$
6	q	assumption
7	$\neg q$	$\wedge e_2\ 1$
8	$\perp$	$\neg e\ 6,7$
9	$\perp$	$\vee e\ 2, 3\text{-}5, 6\text{-}8$
10	$\neg(p \vee q)$	$\neg i\ 2\text{-}9$

Hence, valid

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h- $p \to q, s \to t \vdash p \vee s \to q \wedge t$

1	$p \to q$	premise
2	$s \to t$	premise

Now, here we have the following contradiction.

p : False
q : False
s : True
t : True

i.e with these value $p \vee s \to q \wedge t$ becomes a contradiction So, this is not valid.

Question # (3)
Prove the validity of following.

b $\neg p \vdash p \to q$

1	$\neg p$	premise
2	$\neg p$	ii
3	p	assumption
3		$\neg$ e 1,2
4		$\to$ I 3-3

b $\neg p \vdash p \to q$

1	$\neg p$	premise
2	p	assumption
3	$\perp$	$\neg$ e 1,2
4	q	$\perp$ e 3
5	$p \to q$	$\to$ I 2-4

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d- $\vdash \neg p \to (p \to (p \to q))$

1	$\neg p$	assumption
2	p	assumption
3	$p \to q$	~~same as 2~~ assumption
4	$p \to (p \to q)$	$\to$i 2,3
5	$\neg p \to (p \to (p \to q))$	$\to$i 1-4

f- $p \to q \vdash \neg p \vee q$.

1	$p \to q$	premise
2	$\neg p$	assumption
3	p	assumption
4	$\bot$	$\neg$e 2,3
5	q	$\bot$ 4
6	p	assumption
7	q	$\to$e 1
8	q	$\vee$e 2, 3-5, 6-7
9	$\neg p \vee q$	$\vee$i 2-8

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