

# Some Deep Learning for Neuroscientists

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MAIN 2019

(Slides ruthlessly stolen from Andrej Karpathy)

# Today's Goals:

## Theoretical

- Understand when to bother using deep learning.
- Understand the basic math behind training a deep learning neural network.

## Practical (Optional)

- See how this is done with a simple feedforward network in **Numpy**.

What's the deal with  
deep learning?

# Image Classification: a core task in Computer Vision



(assume given set of discrete labels)  
{dog, cat, truck, plane, ...}



cat

# The problem: semantic gap

Images are represented as  
3D arrays of numbers, with  
integers between [0, 255].

E.g.  
300 x 100 x 3

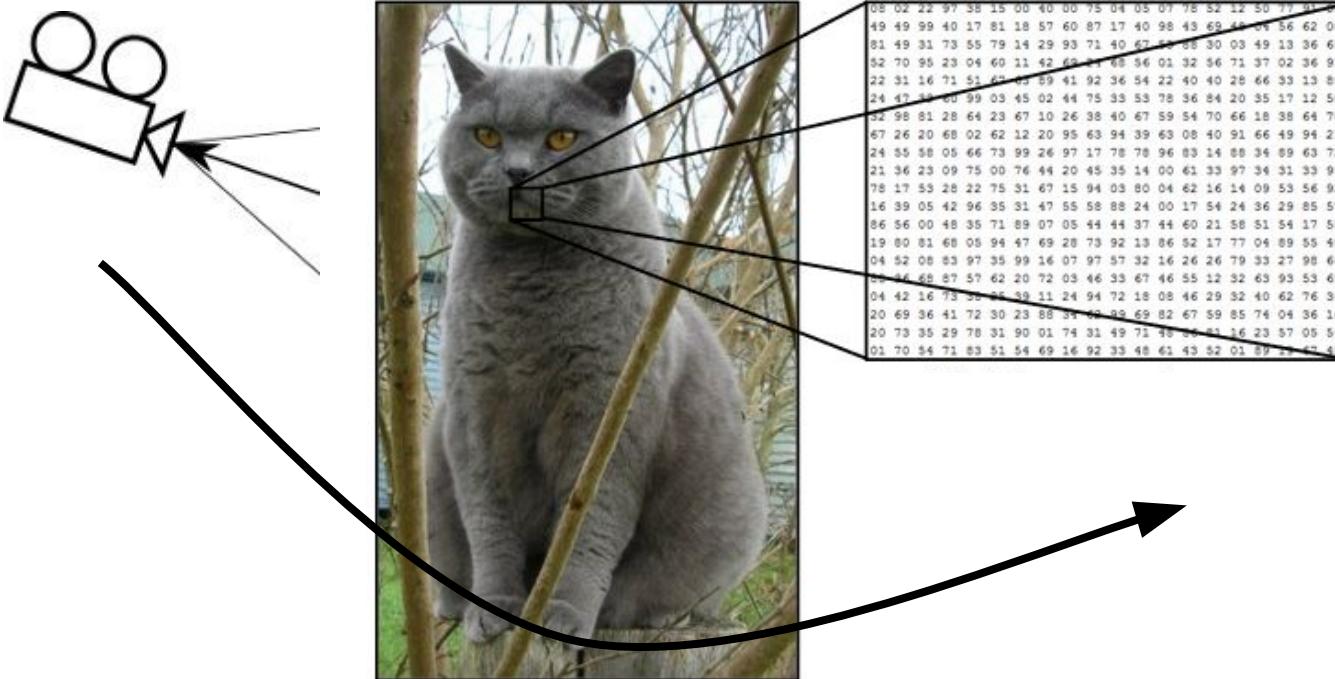
(3 for 3 color channels RGB)



98 02 22 97 38 15 00 40 00 75 04 05 07 78 52 12 50 77 91
49 49 99 40 17 81 18 57 60 87 17 40 98 43 69 44 06 56 62 00
81 49 31 73 55 79 14 29 93 71 40 67 53 85 30 03 49 13 36 65
52 70 95 23 04 60 11 42 63 1 68 56 01 32 56 71 37 02 36 91
22 31 16 71 51 62 63 59 41 92 36 54 22 40 40 28 66 33 13 80
24 17 39 60 99 03 45 02 44 75 33 53 78 36 84 20 35 17 12 50
32 98 81 28 64 23 67 10 26 38 40 67 59 54 70 66 18 38 64 70
67 26 20 68 02 62 12 20 95 63 94 39 63 08 40 91 66 49 94 21
24 55 58 05 66 73 99 26 97 17 78 78 96 03 14 08 34 89 63 72
21 36 23 09 75 00 76 44 20 45 35 14 00 61 33 97 34 31 33 95
78 17 53 28 22 75 31 67 15 94 03 80 04 62 16 14 09 53 56 92
16 39 05 42 96 35 31 47 55 58 88 24 00 17 54 24 36 29 85 57
86 56 00 48 35 71 89 07 05 44 44 37 44 60 21 58 51 54 17 58
19 80 81 61 05 94 47 69 28 73 92 13 86 52 17 77 04 89 55 40
04 52 08 83 97 35 99 16 07 97 57 32 16 26 26 79 33 27 98 66
55 66 68 87 57 62 20 72 03 46 33 67 46 55 12 32 63 93 53 69
04 42 16 73 35 85 39 11 24 94 72 18 08 46 29 32 40 62 76 36
20 69 36 41 72 30 23 88 34 02 88 69 82 67 59 85 74 04 36 16
20 73 35 29 78 31 90 01 74 31 49 71 45 84 87 16 23 57 05 54
01 70 54 71 83 51 54 69 16 92 33 48 61 43 52 01 89 37 47 48

What the computer sees

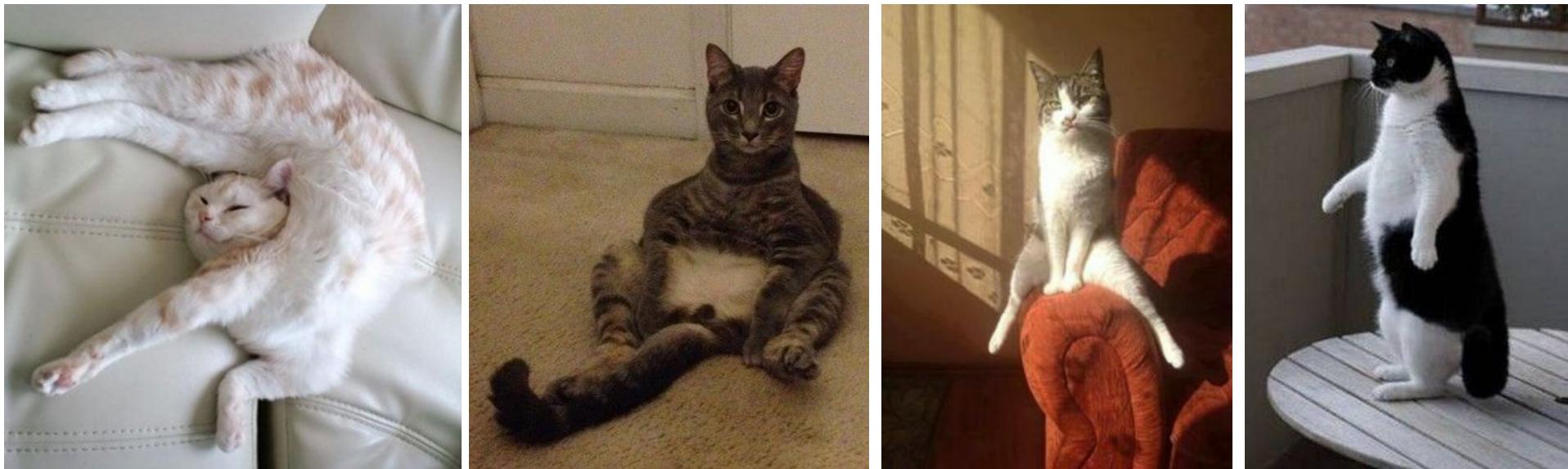
# Challenges: Viewpoint Variation



# Challenges: Illumination



# Challenges: Deformation



# Challenges: Occlusion



# Challenges: Background clutter



# Challenges: Intraclass variation



# An image classifier

```
def predict(image):  
    # ????  
    return class_label
```

Unlike e.g. sorting a list of numbers,

**no obvious way** to hard-code the algorithm for  
recognizing a cat, or other classes.

How do we compare the images? What is the **distance metric**?

**L1 distance:**

$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$

test image			
56	32	10	18
90	23	128	133
24	26	178	200
2	0	255	220

training image			
10	20	24	17
8	10	89	100
12	16	178	170
4	32	233	112

pixel-wise absolute value differences

46	12	14	1
82	13	39	33
12	10	0	30
2	32	22	108

add  
→ 456

```

import numpy as np

class NearestNeighbor:
    def __init__(self):
        pass

    def train(self, X, y):
        """ X is N x D where each row is an example. Y is 1-dimension of size N """
        # the nearest neighbor classifier simply remembers all the training data
        self.Xtr = X
        self.ytr = y

    def predict(self, X):
        """ X is N x D where each row is an example we wish to predict label for """
        num_test = X.shape[0]
        # lets make sure that the output type matches the input type
        Ypred = np.zeros(num_test, dtype = self.ytr.dtype)

        # loop over all test rows
        for i in xrange(num_test):
            # find the nearest training image to the i'th test image
            # using the L1 distance (sum of absolute value differences)
            distances = np.sum(np.abs(self.Xtr - X[i,:]), axis = 1)
            min_index = np.argmin(distances) # get the index with smallest distance
            Ypred[i] = self.ytr[min_index] # predict the label of the nearest example

        return Ypred

```

## Nearest Neighbor classifier

```

import numpy as np

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```

## Nearest Neighbor classifier

remember the training data

```

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        return Ypred

```

## Nearest Neighbor classifier

- for every test image:
- find nearest train image with L1 distance
  - predict the label of nearest training image

```
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```

## Nearest Neighbor classifier

**Q: how does the classification speed depend on the size of the training data?**

```

import numpy as np

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```

## Nearest Neighbor classifier

Q: how does the classification speed depend on the size of the training data?  
**linearly :(**

This is **backwards**:  

- test time performance is usually much more important in practice.
- CNNs flip this: expensive training, cheap test evaluation

The choice of distance is a **hyperparameter**  
common choices:

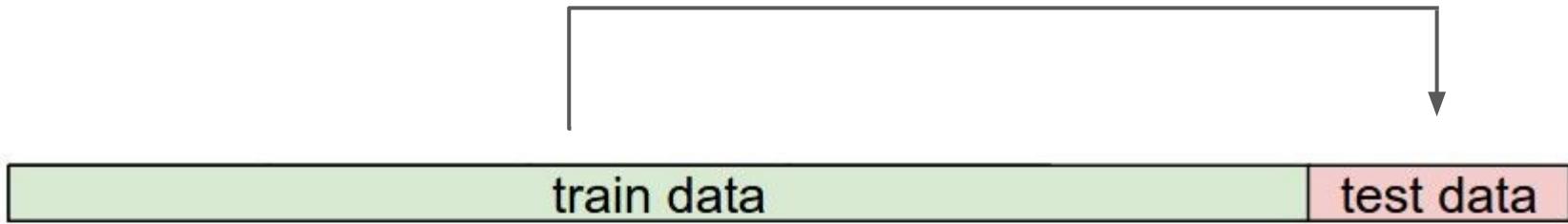
L1 (Manhattan) distance

$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$

L2 (Euclidean) distance

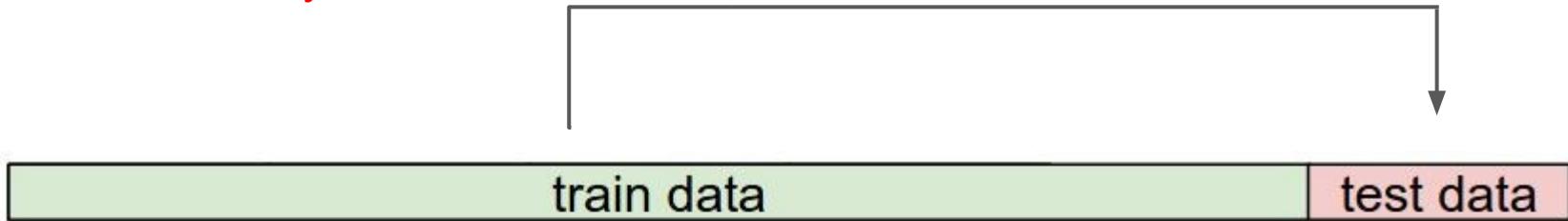
$$d_2(I_1, I_2) = \sqrt{\sum_p (I_1^p - I_2^p)^2}$$

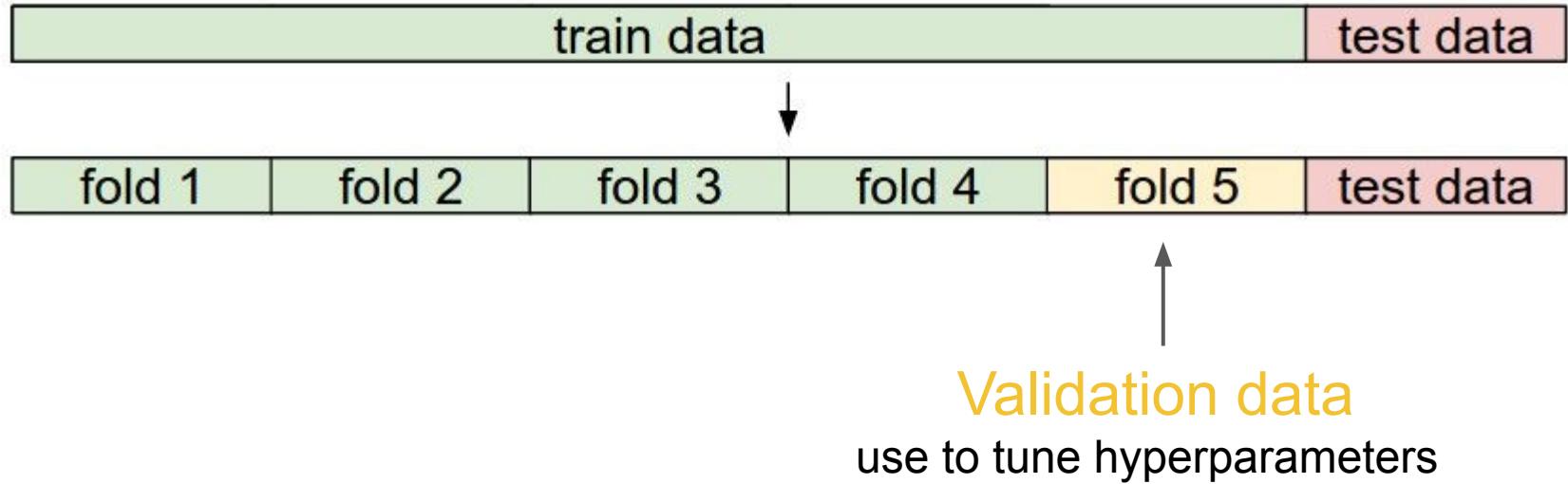
Try out what hyperparameters work best on test set.

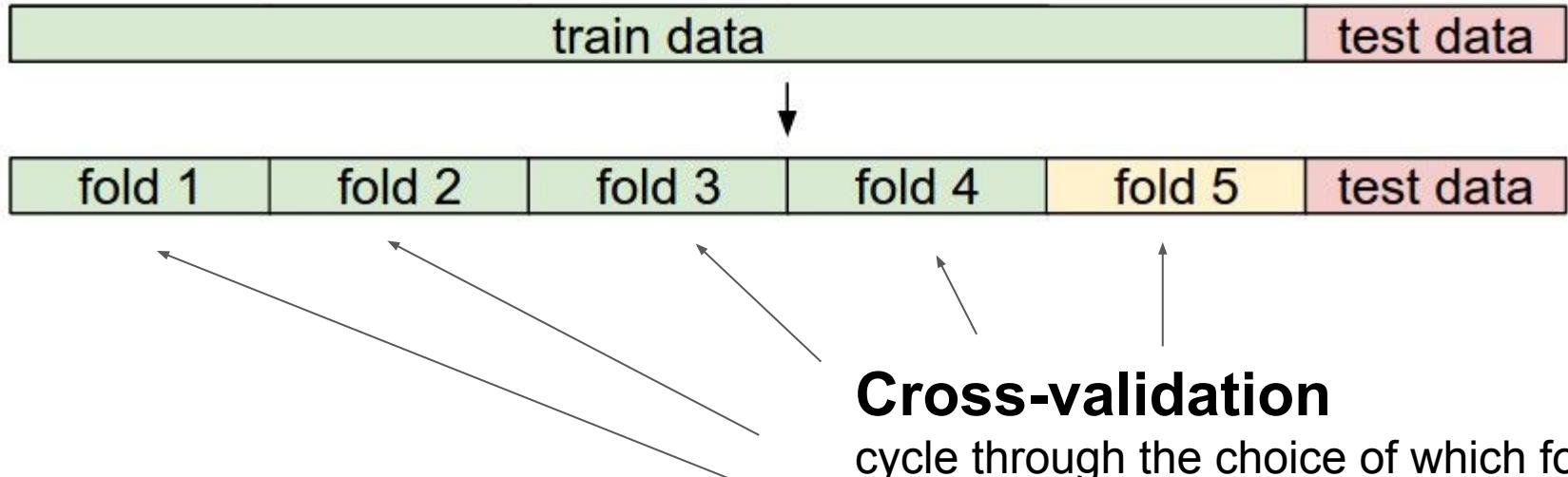


Trying out what hyperparameters work best on test set:

Very bad idea. The test set is a proxy for the generalization performance!  
Use only **VERY SPARINGLY**, at the end.



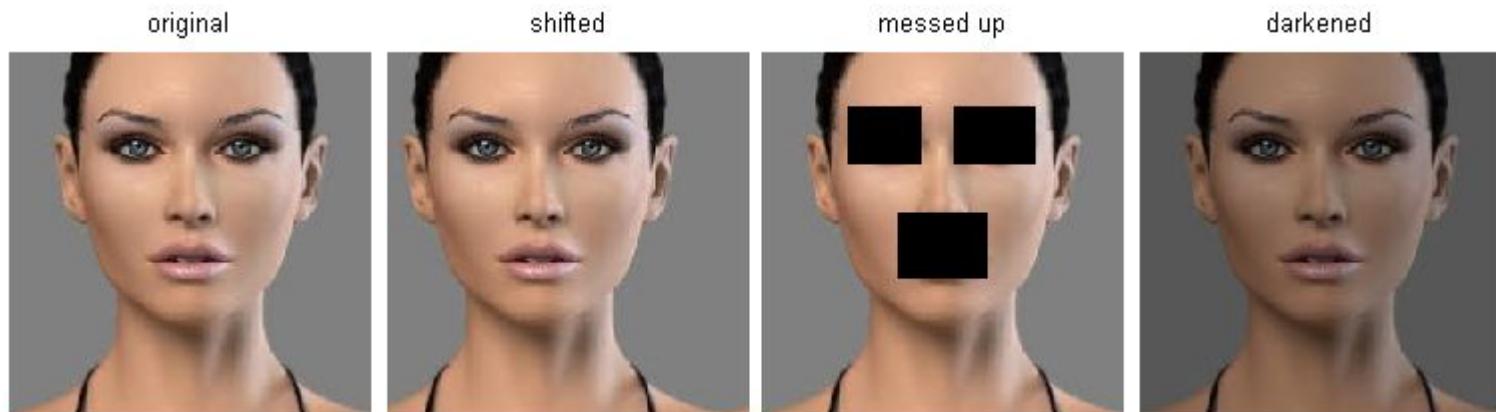




cycle through the choice of which fold  
is the validation fold, average results.

# k-Nearest Neighbor on images **never used**.

- terrible performance at test time
- distance metrics on level of whole images can be very unintuitive



(all 3 images have same L2 distance to the one on the left)

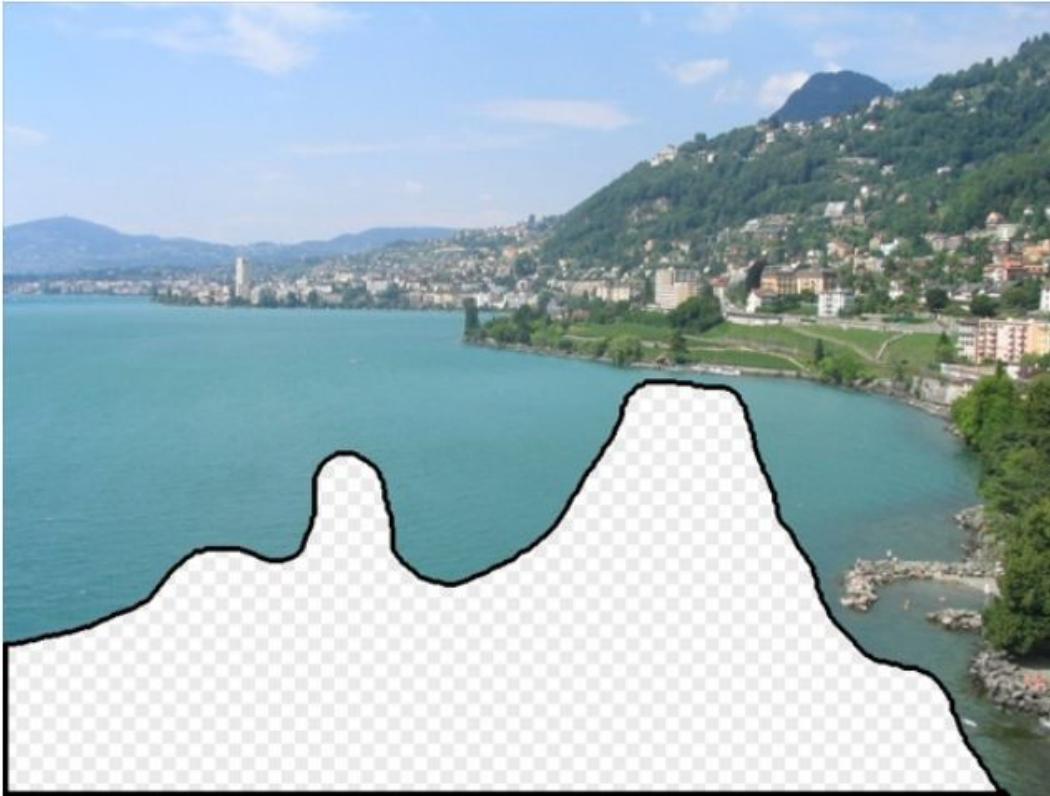
# Scene Completion

---



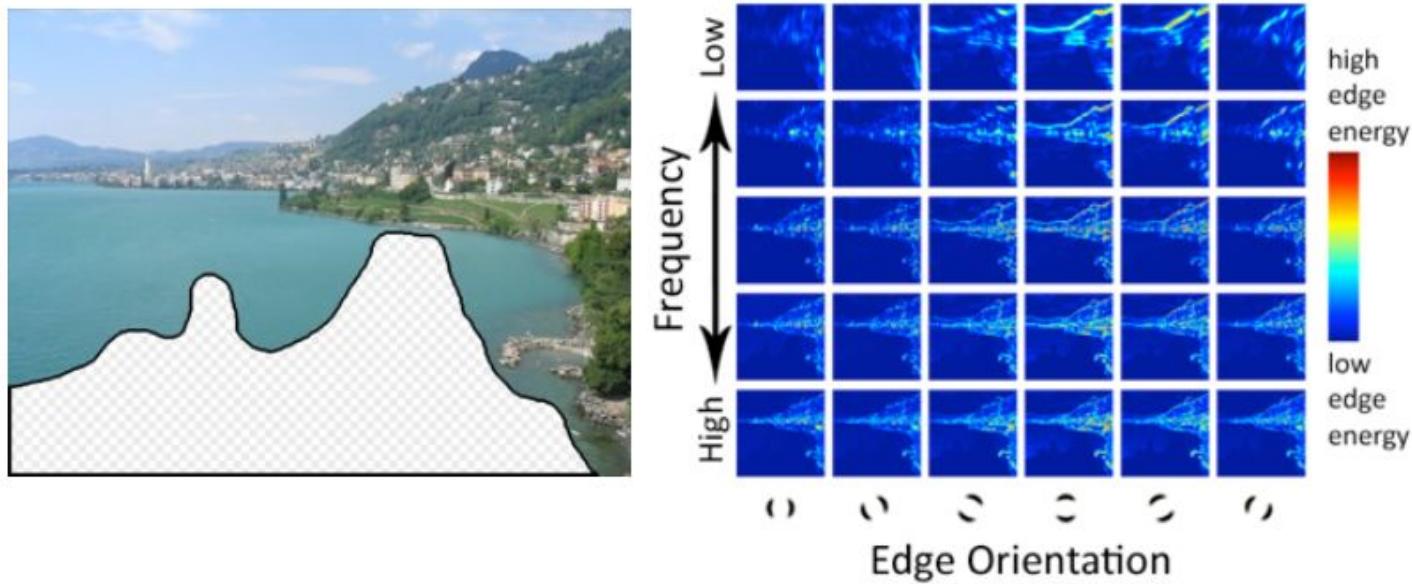
# Scene Matching

---



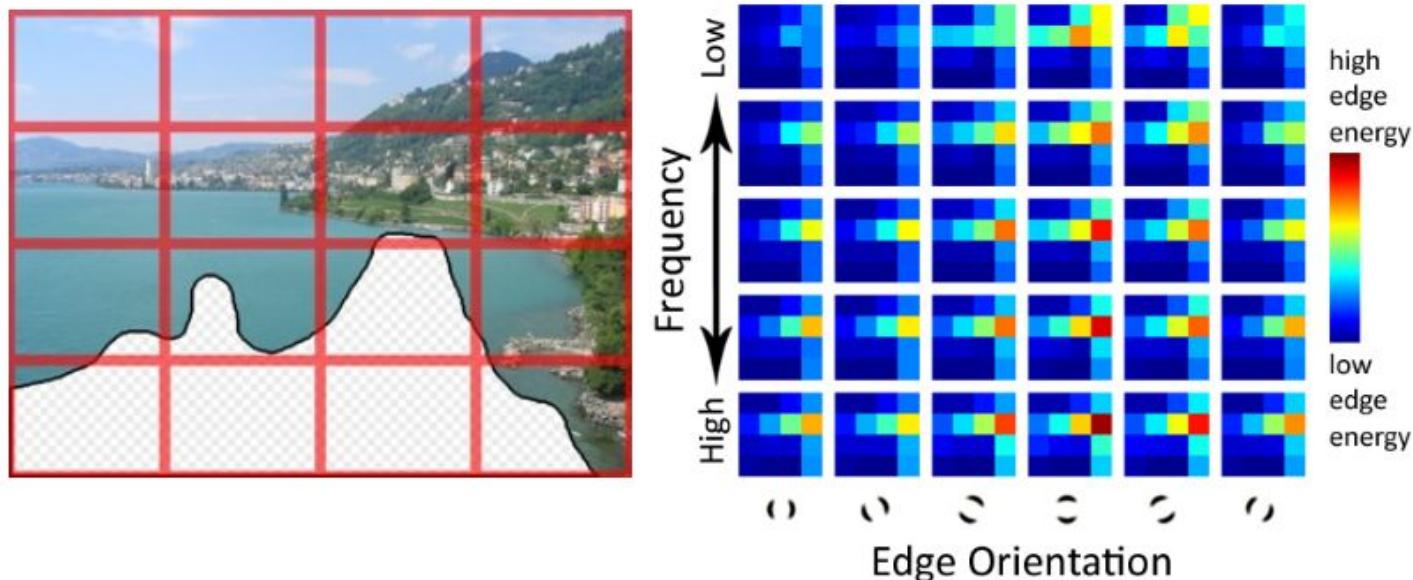
# Scene Descriptor

---



# Scene Descriptor

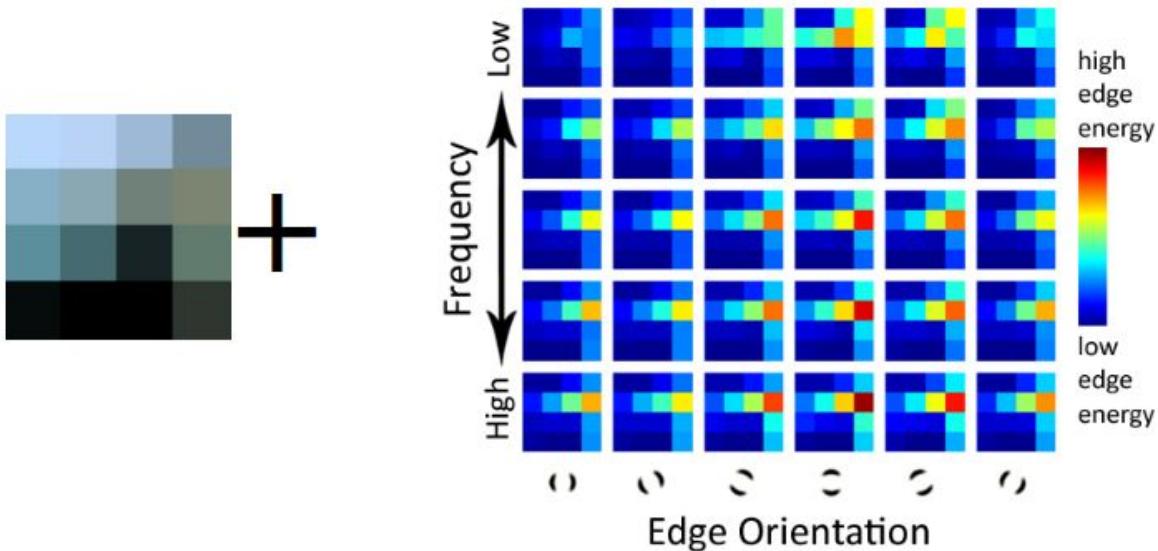
---



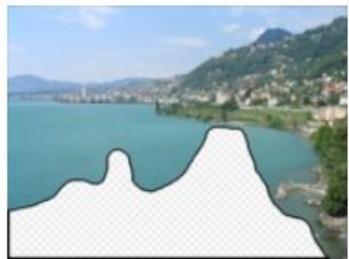
Scene Gist Descriptor  
(Oliva and Torralba 2001)

# Scene Descriptor

---

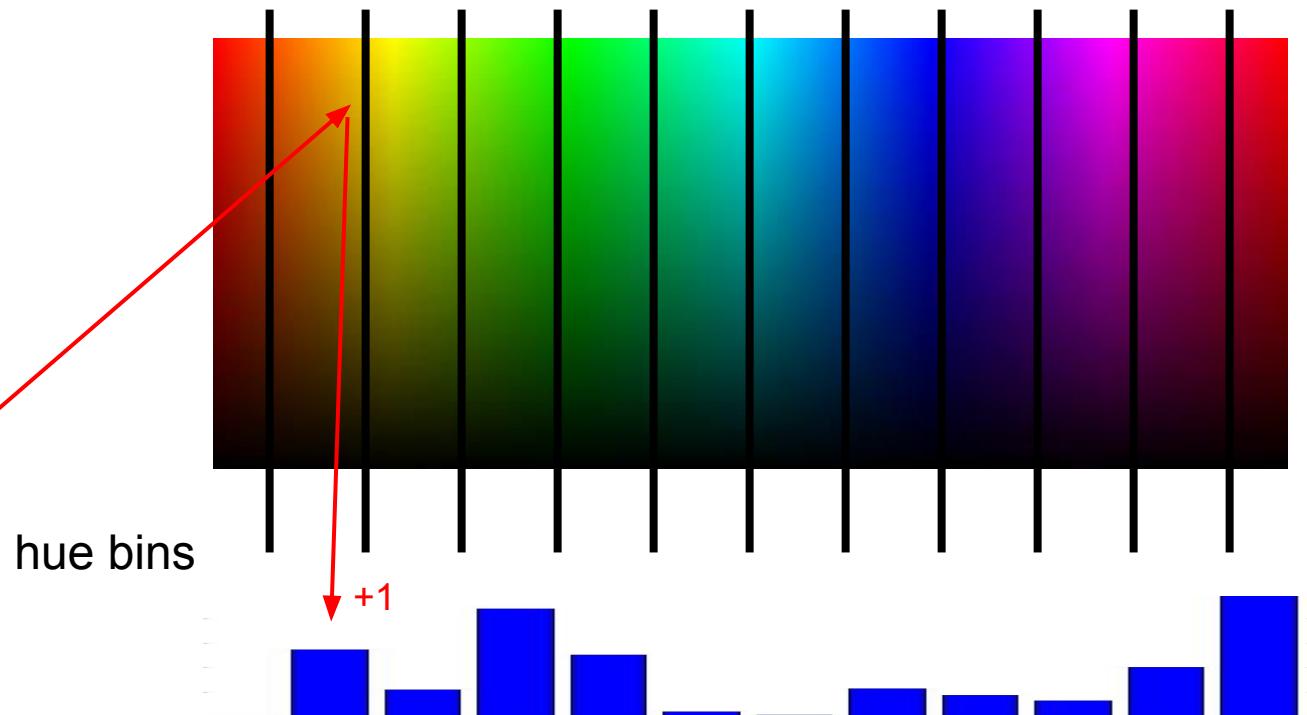
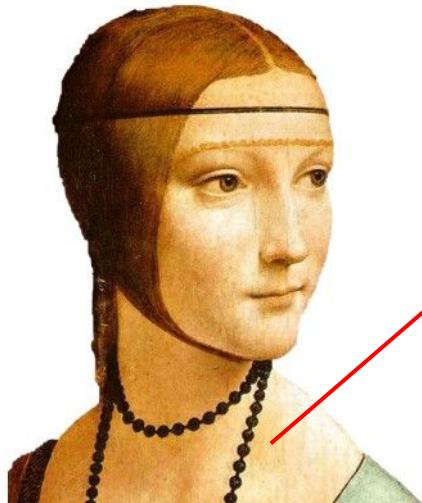


Scene Gist Descriptor  
(Oliva and Torralba 2001)



2 Million Flickr Images → 200 matches.

# Example: Color (Hue) Histogram



# Let's be lazy instead

- Instead of trying to enumerate all of the possible functions requires to decompose images, let's just use neural networks which can **learn on their own** what those functions are!
- Neural networks are “universal function approximators”.  
<http://neuralnetworksanddeeplearning.com/chap4.html>



## Feature Extraction

vector describing various  
image statistics



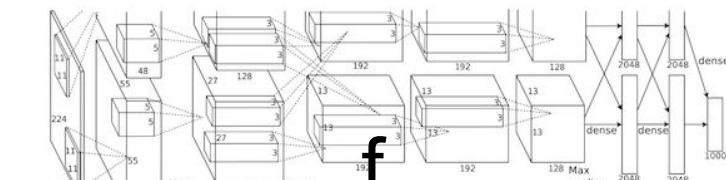
$$f \xrightarrow{\text{training}}$$

10 numbers, indicating  
class scores

[32x32x3]



**f**



training

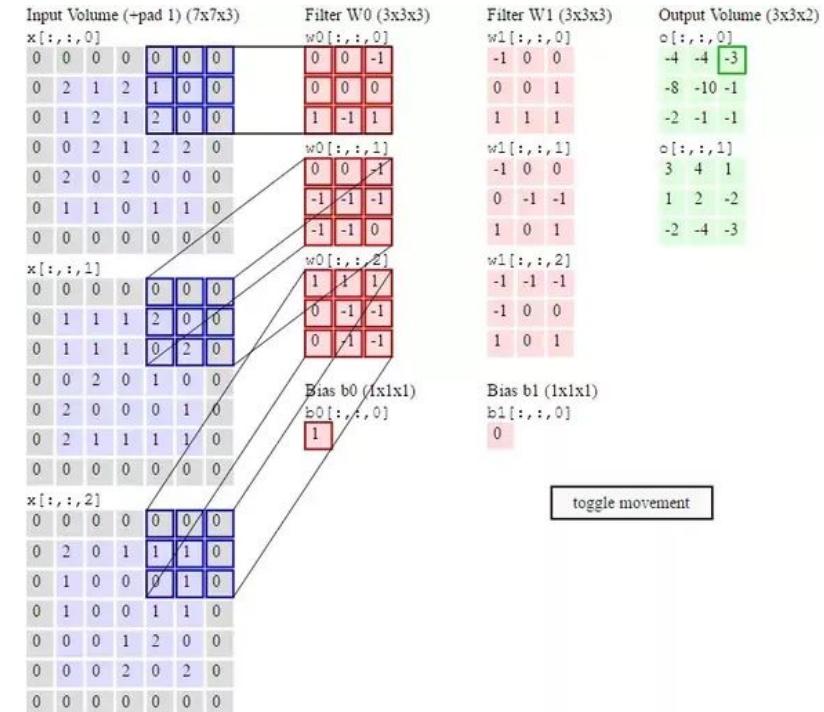
10 numbers, indicating  
class scores

[32x32x3]

# Enter Convolutional Neural Networks

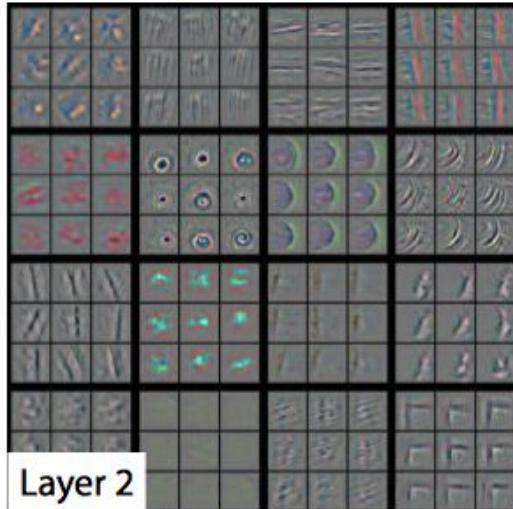
ONLY ASSUMPTION: Things occur close together in the inputs (like in images)!

Hyperparameter: Kernel size!

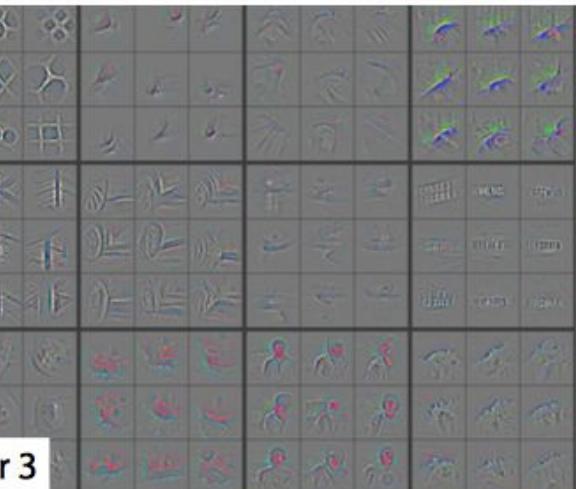


# Visualizing and Understanding Convolutional Networks

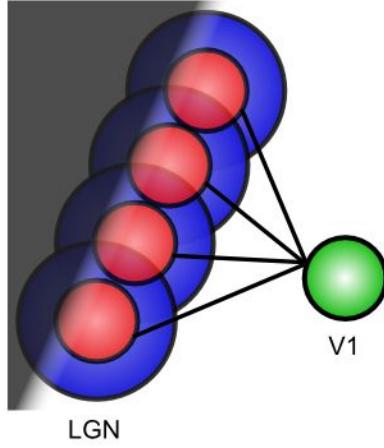
Layer 1



Layer 2



V1 simple cell edge detector



LGN

V1

# Why don't we use neural networks for everything?

- Neural networks are **too strong**: if you let them, they will just memorize the training data and not work on any new data.
  - **Regularization.**
  - **Lots of training data (the best regularizer).**
  - Specific model architectures.

# When should I consider a neural network?

- Neural networks are a **good** candidate when we have lots of:
  - Data.
  - Time (your time).
  - No idea what the functions generating the data might be.

# How to know if neural networks are appropriate?

- Establish a **baseline!**
  - **Do it now!**
- You will be surprised how well a linear regression / SVM / random forest model (3 lines of code in scikit-learn) will perform on your task if you have good feature engineering done already.

# Summary

- Deep learning does (some) feature engineering for us.
- Deep learning is slow to train but quick to predict!
- Still required to pick **hyperparameters** and evaluate those choices on **held-out data**.
- Otherwise we will **overfit** and our model will be useless.
  - **Generalize don't Memorize.**

How does deep  
learning work?

# Parametric approach



image parameters

$$f(\mathbf{x}, \mathbf{W})$$

10 numbers,  
indicating class  
scores

[32x32x3]

array of numbers 0...1  
(3072 numbers total)

# Parametric approach: Linear classifier

$$f(x, W) = Wx$$



**10** numbers,  
indicating class  
scores

**[32x32x3]**

array of numbers 0...1

# Parametric approach: Linear classifier



[32x32x3]

array of numbers 0...1

$$f(x, W) = \boxed{W} \boxed{x} \quad 3072 \times 1$$

**10x1**      **10x3072**

10 numbers,  
indicating class  
scores

parameters, or “weights”

# Parametric approach: Linear classifier



[32x32x3]

array of numbers 0...1

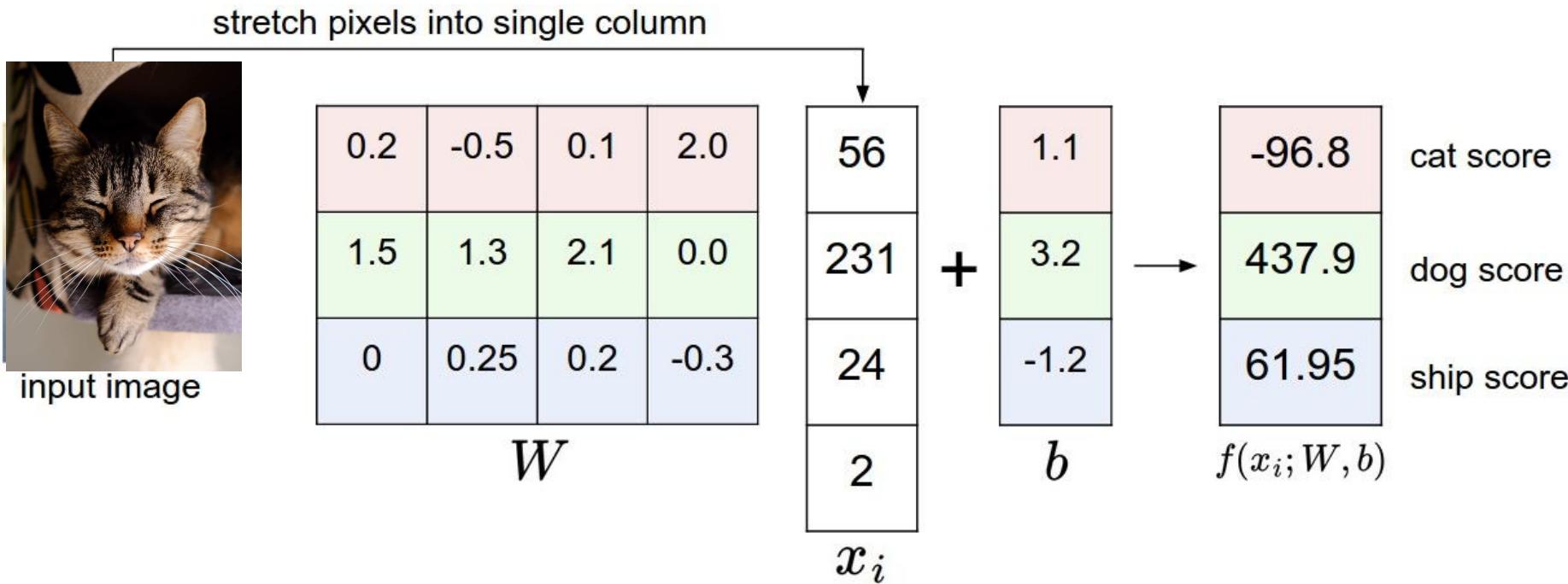
$$f(x, W) = Wx \quad \begin{matrix} 3072 \times 1 \\ 10 \times 3072 \end{matrix}$$

$$(+b) \quad \begin{matrix} 10 \times 1 \end{matrix}$$

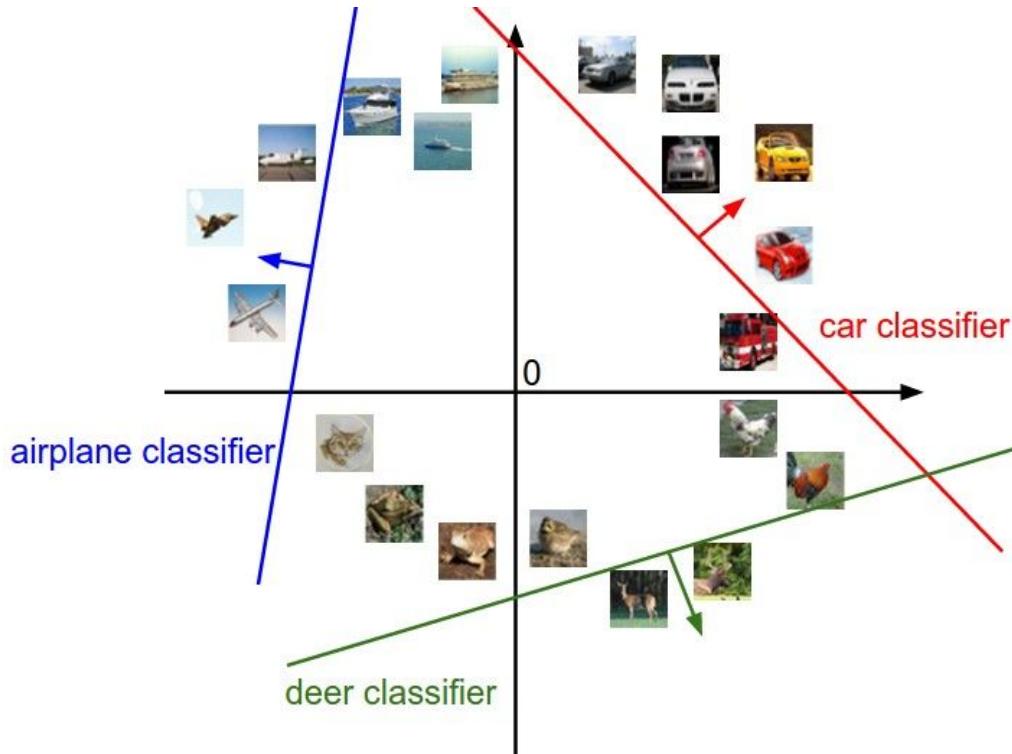
10 numbers,  
indicating class  
scores

parameters, or “weights”

# Example with an image with 4 pixels, and 3 classes (**cat**/dog/**ship**)



# Interpreting a Linear Classifier



$$f(x_i, W, b) = Wx_i + b$$



[32x32x3]  
array of numbers 0...1  
(3072 numbers total)

Suppose: 3 training examples, 3 classes.

With some  $W$  the scores  $f(x, W) = Wx$  are:



cat	<b>3.2</b>	1.3	2.2
car	5.1	<b>4.9</b>	2.5
frog	-1.7	2.0	<b>-3.1</b>

# Softmax Classifier (Multinomial Logistic Regression)



cat	<b>3.2</b>
car	5.1
frog	-1.7

# Softmax Classifier (Multinomial Logistic Regression)



**scores = unnormalized log probabilities of the classes.**

$$s = f(x_i; W)$$

cat	<b>3.2</b>
car	5.1
frog	-1.7

# Softmax Classifier (Multinomial Logistic Regression)



**scores = unnormalized log probabilities of the classes.**

$$P(Y = k | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}} \quad \text{where} \quad s = f(x_i; W)$$

cat	<b>3.2</b>
car	<b>5.1</b>
frog	<b>-1.7</b>

# Softmax Classifier (Multinomial Logistic Regression)



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$$P(Y = k | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$
 where  $s = f(x_i; W)$

cat	3.2	Softmax function
car	5.1	
frog	-1.7	

# Softmax Classifier (Multinomial Logistic Regression)



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cat	<b>3.2</b>
car	5.1
frog	-1.7

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$L_i = -\log P(Y = y_i | X = x_i)$$

# Softmax Classifier (Multinomial Logistic Regression)



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frog	-1.7

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$L_i = -\log P(Y = y_i | X = x_i)$$

---

in summary:  $L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$

# Softmax Classifier (Multinomial Logistic Regression)



$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

cat  
car  
frog

3.2
5.1
-1.7

unnormalized log probabilities

# Softmax Classifier (Multinomial Logistic Regression)



$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

cat  
car  
frog

3.2	
5.1	
-1.7	

exp →

24.5
164.0
0.18

unnormalized log probabilities

# Softmax Classifier (Multinomial Logistic Regression)



$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

cat  
car  
frog

3.2
5.1
-1.7

exp

24.5
164.0
0.18

normalize

0.13
0.87
0.00

unnormalized log probabilities

probabilities

# Softmax Classifier (Multinomial Logistic Regression)



$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

cat  
car  
frog

3.2
5.1
-1.7

exp

24.5
164.0
0.18

normalize

0.13
0.87
0.00

$$\begin{aligned} L_i &= -\log(0.13) \\ &= 0.89 \end{aligned}$$

unnormalized log probabilities

probabilities

Goal: optimize the weights  $\mathbf{W}$  to minimize the loss  $\mathbf{L}$ .

$$= \arg \min_{\theta} \frac{1}{n} \sum_{i=1}^n L(y^{(i)}, f(\mathbf{x}^{(i)}, \theta))$$

# Strategy #1: A first very bad idea solution: Random search

```
# assume X_train is the data where each column is an example (e.g. 3073 x 50,000)
# assume Y_train are the labels (e.g. 1D array of 50,000)
# assume the function L evaluates the loss function

bestloss = float("inf") # Python assigns the highest possible float value
for num in xrange(1000):
    W = np.random.randn(10, 3073) * 0.0001 # generate random parameters
    loss = L(X_train, Y_train, W) # get the loss over the entire training set
    if loss < bestloss: # keep track of the best solution
        bestloss = loss
        bestW = W
    print 'in attempt %d the loss was %f, best %f' % (num, loss, bestloss)

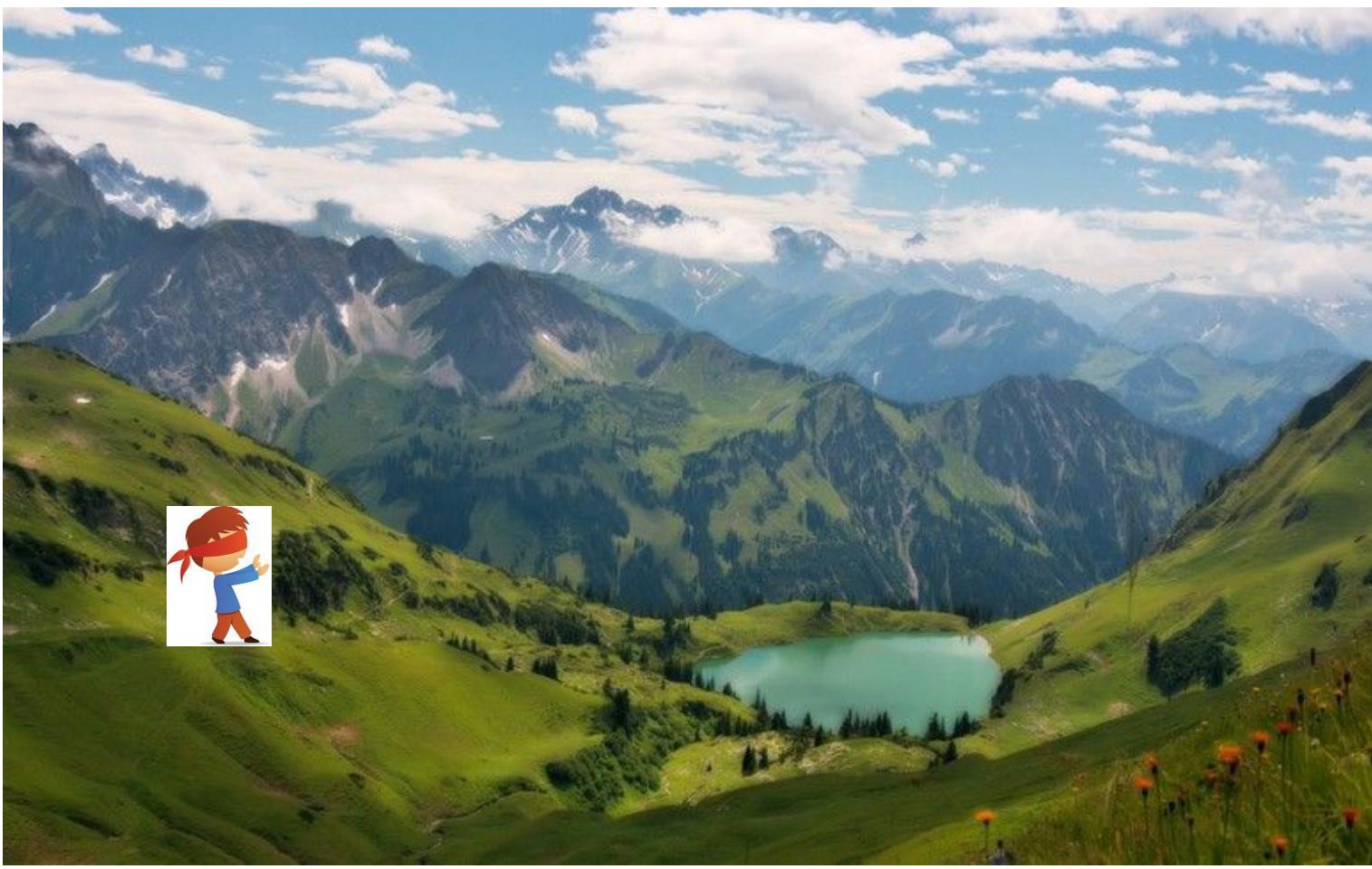
# prints:
# in attempt 0 the loss was 9.401632, best 9.401632
# in attempt 1 the loss was 8.959668, best 8.959668
# in attempt 2 the loss was 9.044034, best 8.959668
# in attempt 3 the loss was 9.278948, best 8.959668
# in attempt 4 the loss was 8.857370, best 8.857370
# in attempt 5 the loss was 8.943151, best 8.857370
# in attempt 6 the loss was 8.605604, best 8.605604
# ... (truncated: continues for 1000 lines)
```

# Lets see how well this works on the test set...

```
# Assume X_test is [3073 x 10000], Y_test [10000 x 1]
scores = Wbest.dot(Xte_cols) # 10 x 10000, the class scores for all test examples
# find the index with max score in each column (the predicted class)
Yte_predict = np.argmax(scores, axis = 0)
# and calculate accuracy (fraction of predictions that are correct)
np.mean(Yte_predict == Yte)
# returns 0.1555
```

15.5% accuracy! not bad!  
(SOTA is ~95%)





## Strategy #2: Follow the slope

In 1-dimension, the derivative of a function:

$$\frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

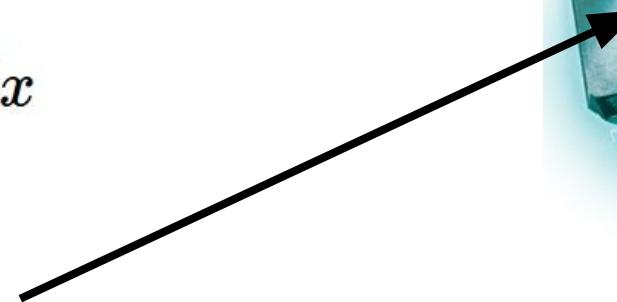
In multiple dimensions, the **gradient** is the vector of (partial derivatives).

The loss is just a function of W, so

$$s = f(x; W) = Wx$$

want  $\nabla_W L$

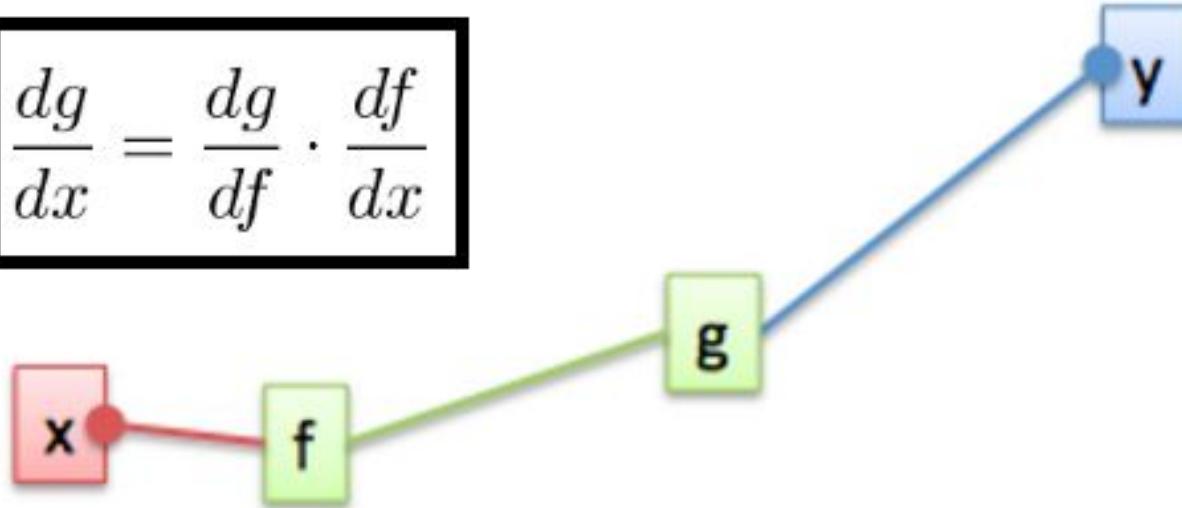
Calculus



# Chain Rule

$$y = g(f(x))$$

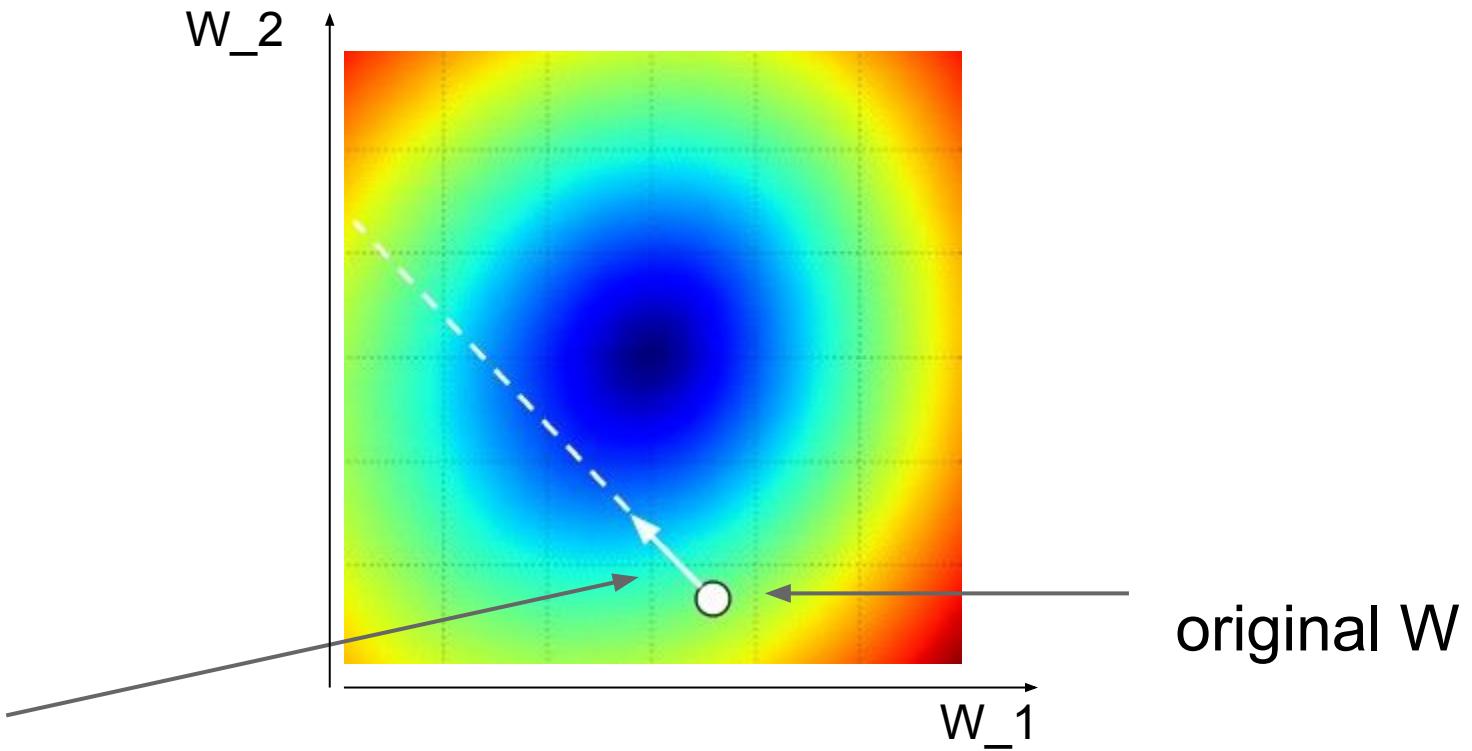
$$\frac{dg}{dx} = \frac{dg}{df} \cdot \frac{df}{dx}$$



# Gradient Descent

```
# Vanilla Gradient Descent

while True:
    weights_grad = evaluate_gradient(loss_fun, data, weights)
    weights += - step_size * weights_grad # perform parameter update
```



negative gradient direction

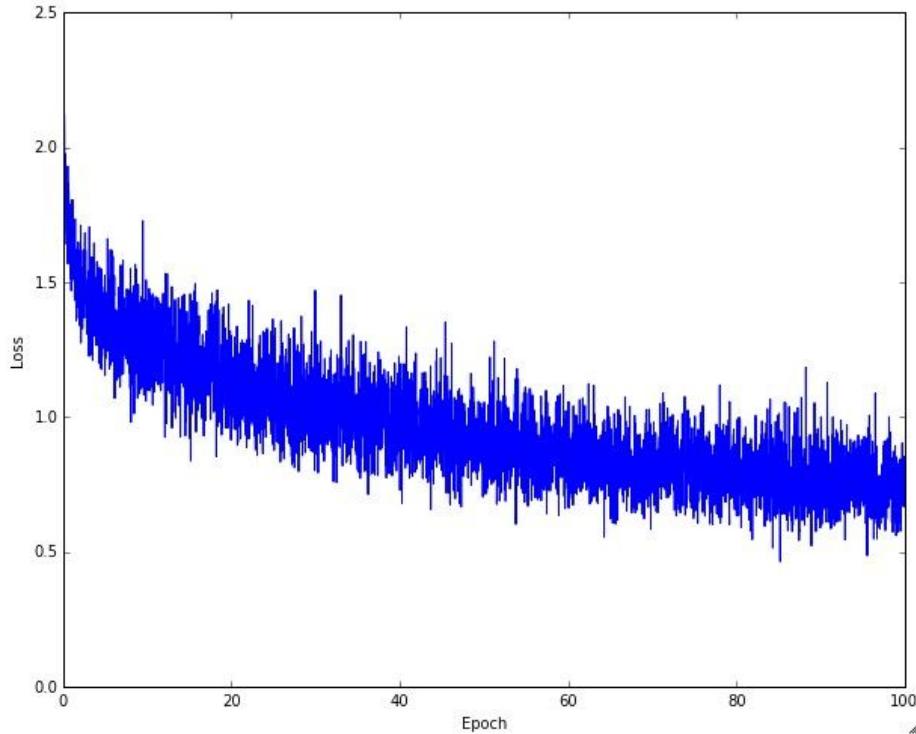
# Mini-batch Gradient Descent

- only use a small portion of the training set to compute the gradient.

```
# Vanilla Minibatch Gradient Descent

while True:
    data_batch = sample_training_data(data, 256) # sample 256 examples
    weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
    weights += - step_size * weights_grad # perform parameter update
```

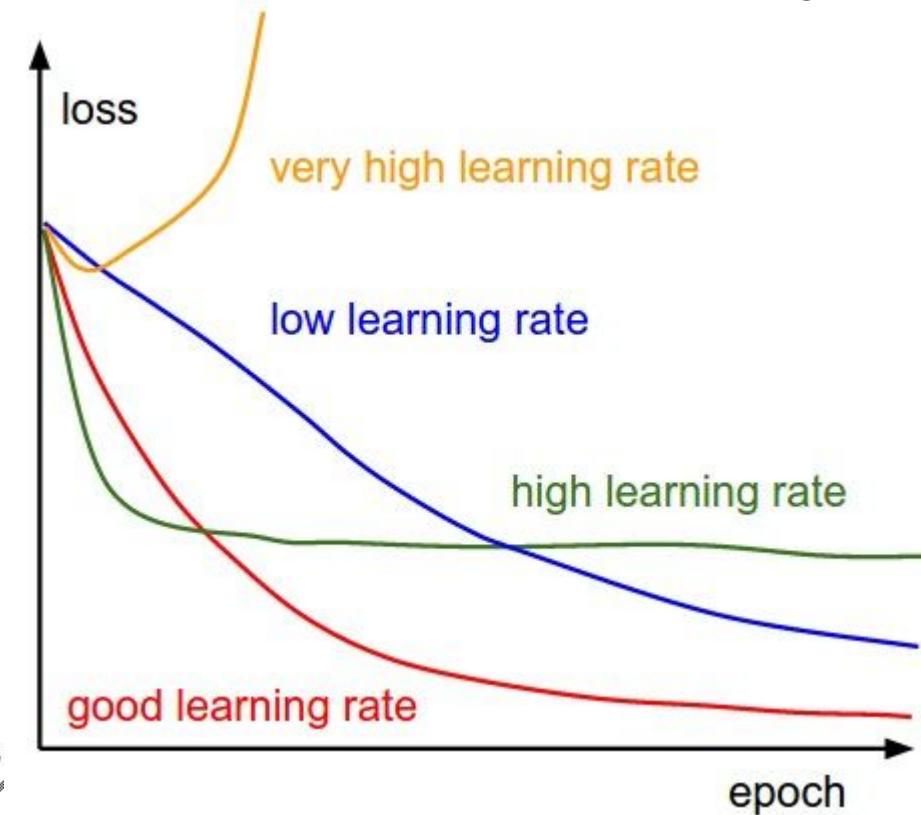
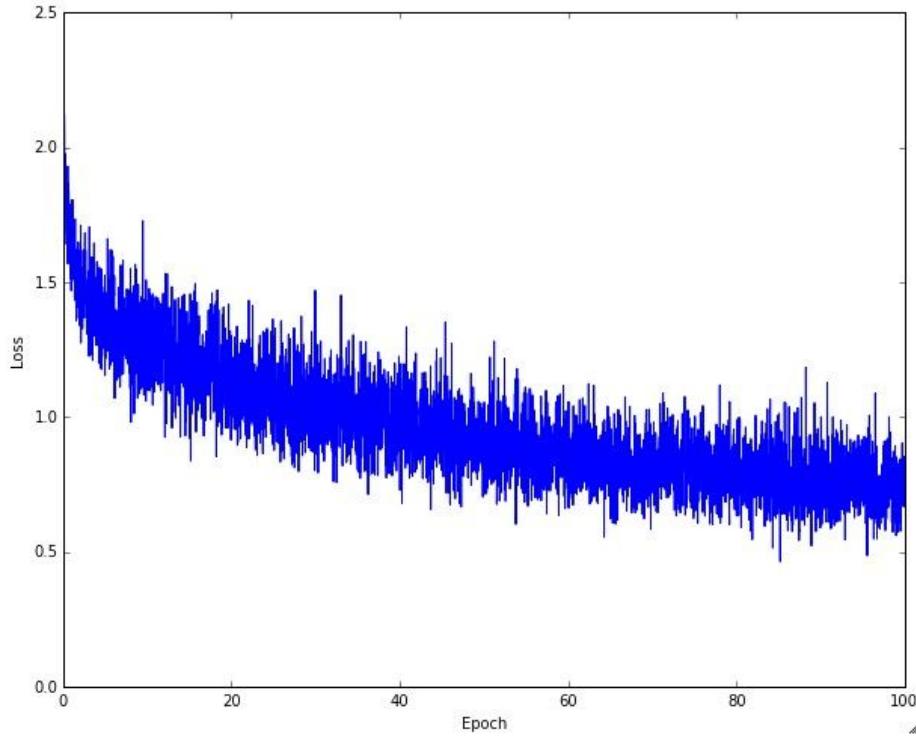
Common mini-batch sizes are 32/64/128 examples  
e.g. Krizhevsky ILSVRC ConvNet used 256 examples



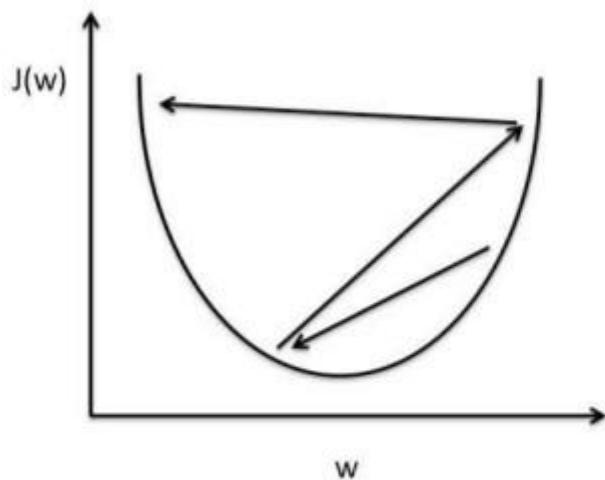
Example of optimization progress while training a neural network.

(Loss over mini-batches goes down over time.)

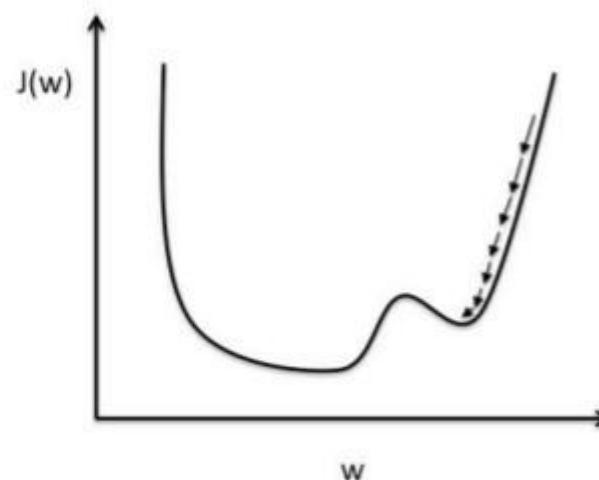
## The effects of step size (or “learning rate”)



# Learning rate



Overshooting



Learn too slow

# Neural Network: without the brain stuff

(Before) Linear score function:

$$f = Wx$$

# Neural Network: without the brain stuff

**(Before)** Linear score function:

$$f = Wx$$

**(Now)** 2-layer Neural Network

$$f = W_2 \max(0, W_1 x)$$

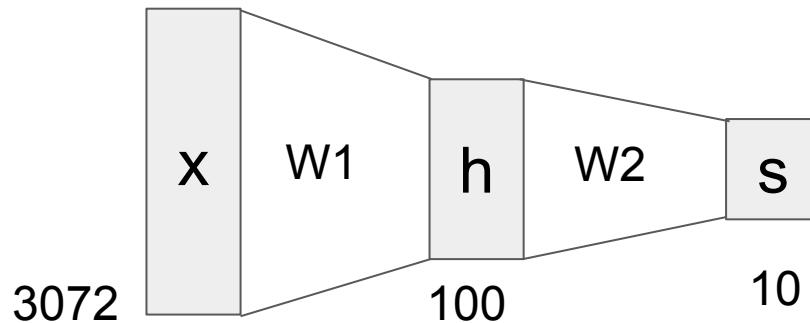
# Neural Network: without the brain stuff

(Before) Linear score function:

$$f = Wx$$

(Now) 2-layer Neural Network

$$f = W_2 \max(0, W_1 x)$$



# Neural Network: without the brain stuff

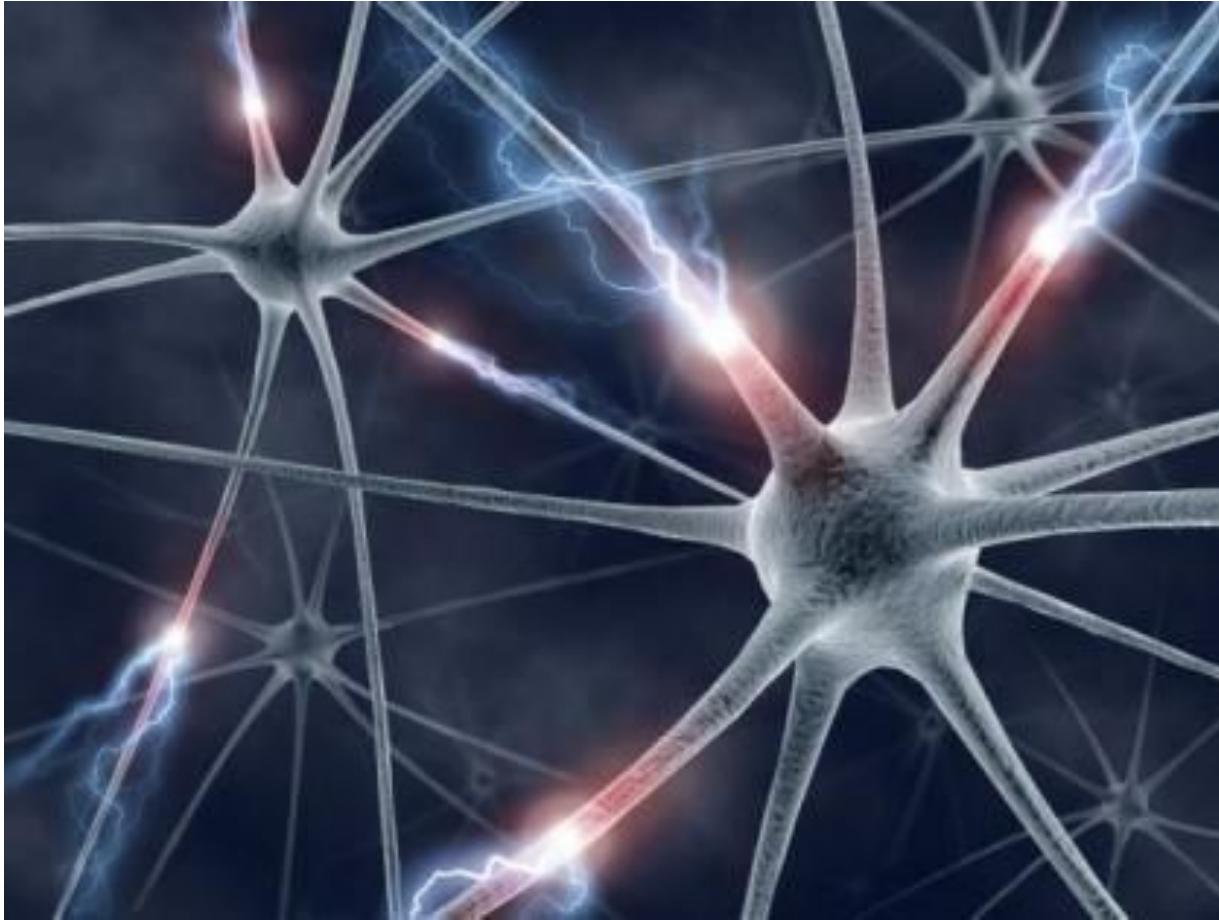
**(Before)** Linear score function:

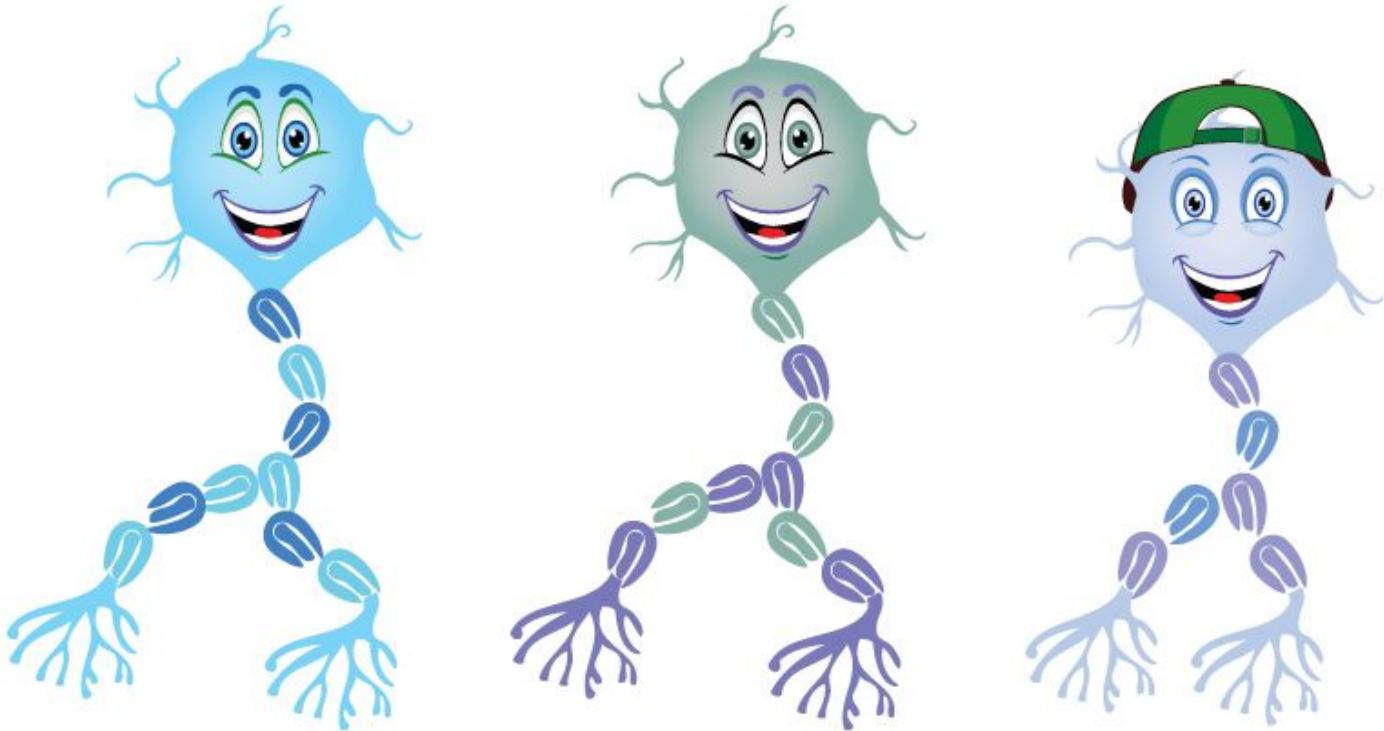
$$f = Wx$$

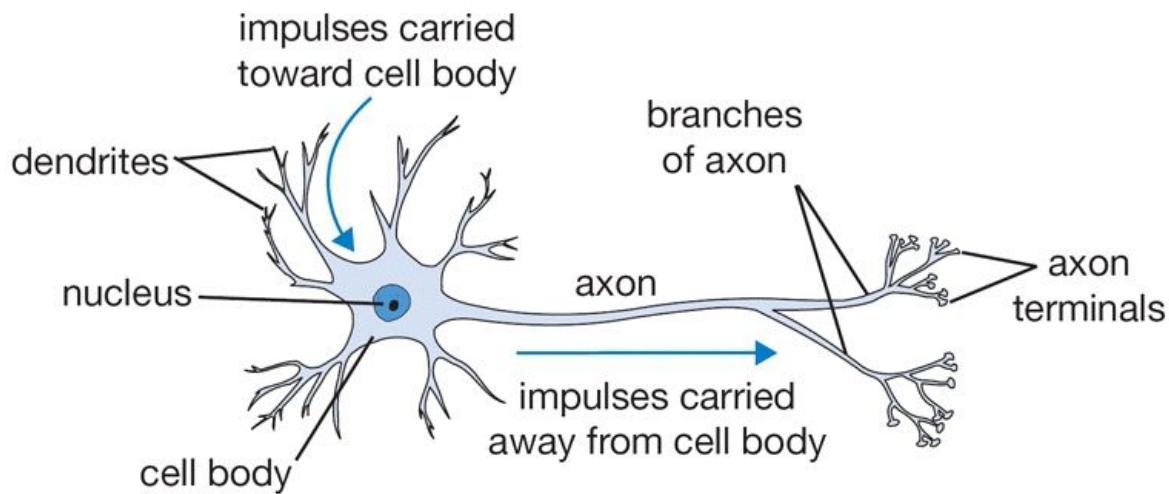
**(Now)** 2-layer Neural Network  
or 3-layer Neural Network

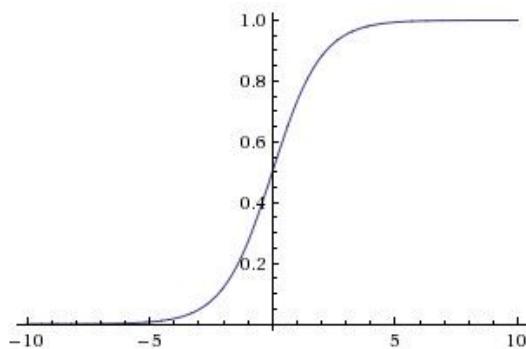
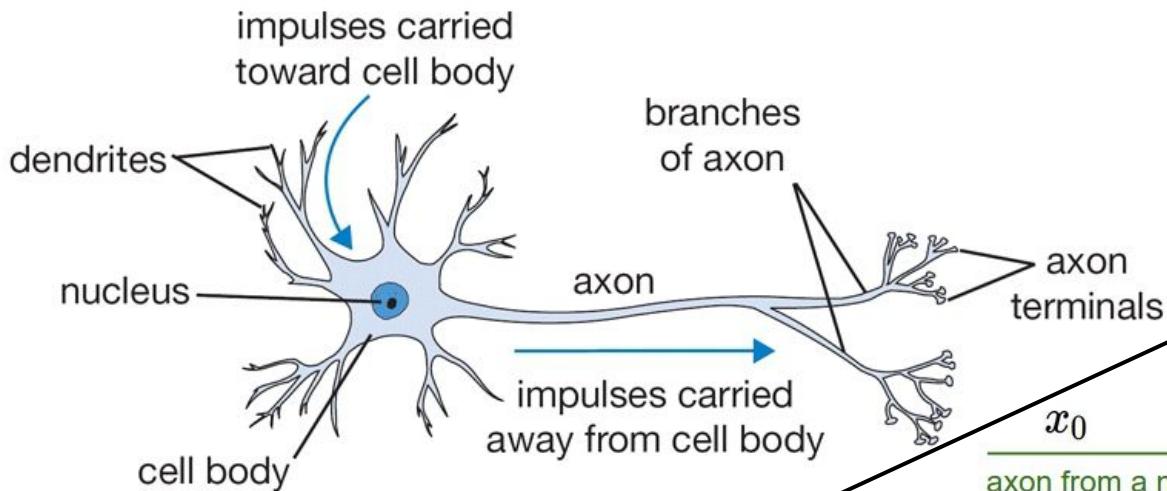
$$f = W_2 \max(0, W_1 x)$$

$$f = W_3 \max(0, W_2 \max(0, W_1 x))$$



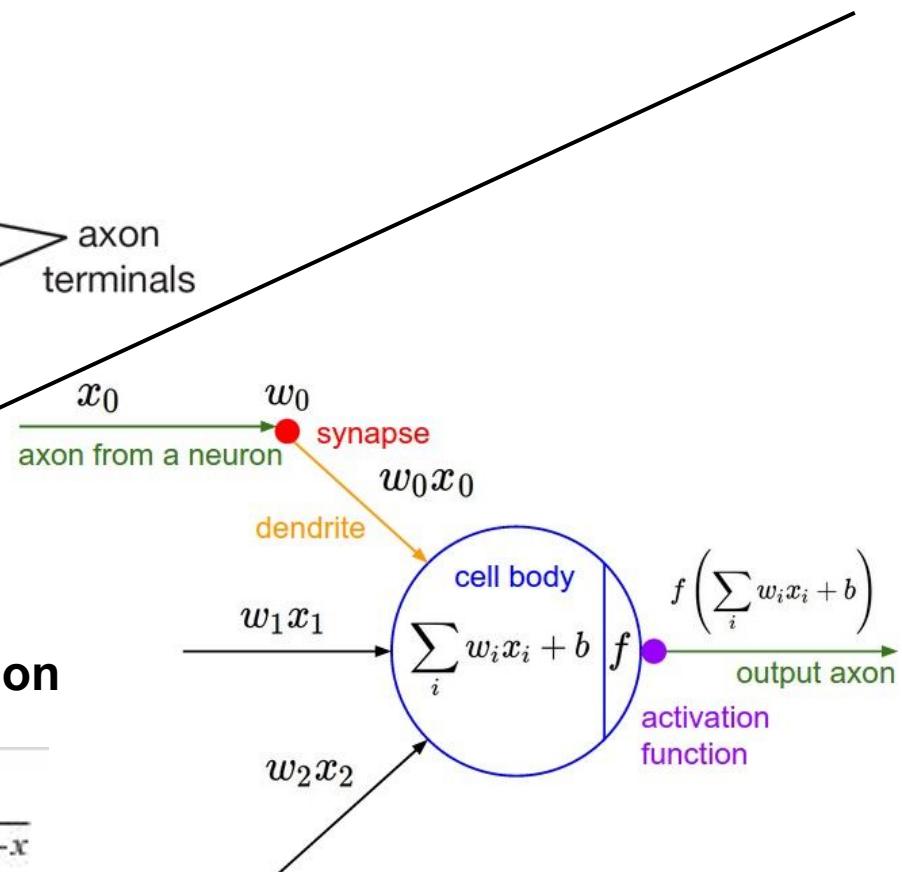






## sigmoid activation function

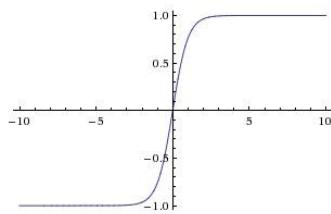
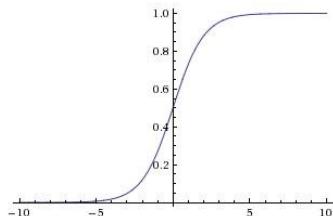
$$\frac{1}{1 + e^{-x}}$$



# Activation Functions

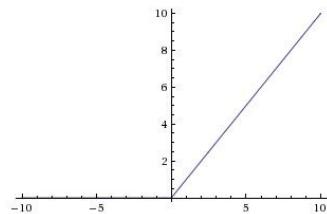
## Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$



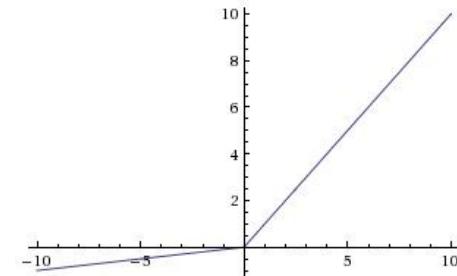
$$\tanh \quad \tanh(x)$$

$$\text{ReLU} \quad \max(0, x)$$



## Leaky ReLU

$$\max(0.1x, x)$$

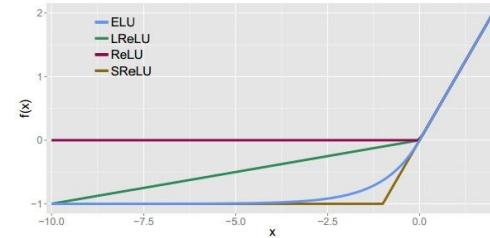


## Maxout

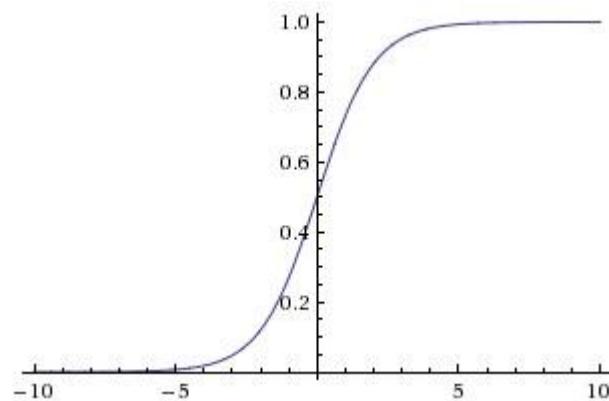
$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

## ELU

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha (\exp(x) - 1) & \text{if } x \leq 0 \end{cases}$$



# Activation Functions



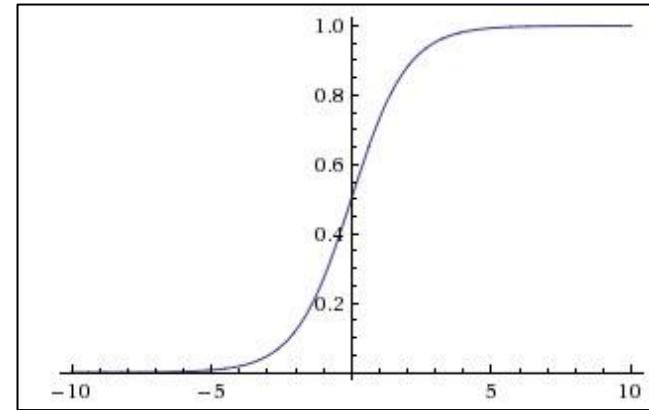
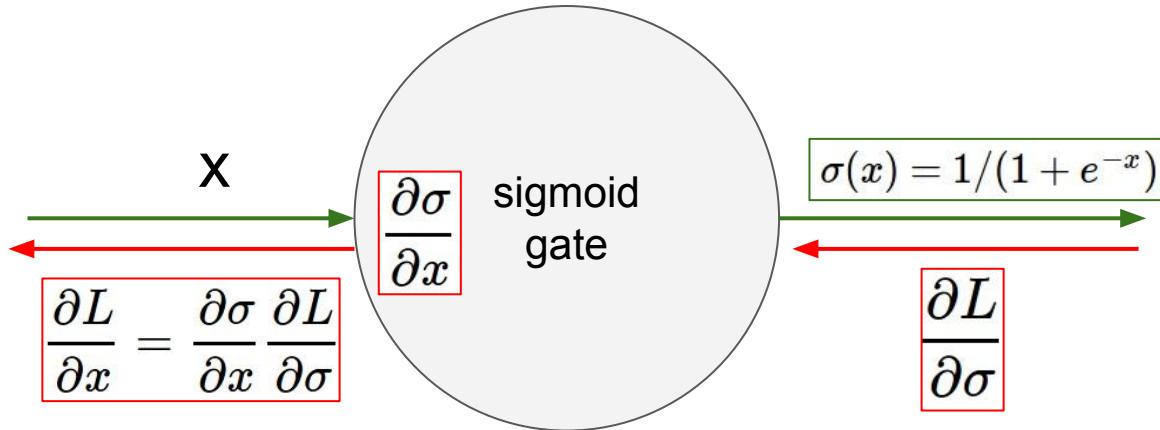
Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

3 problems:

1. Saturated neurons “kill” the gradients



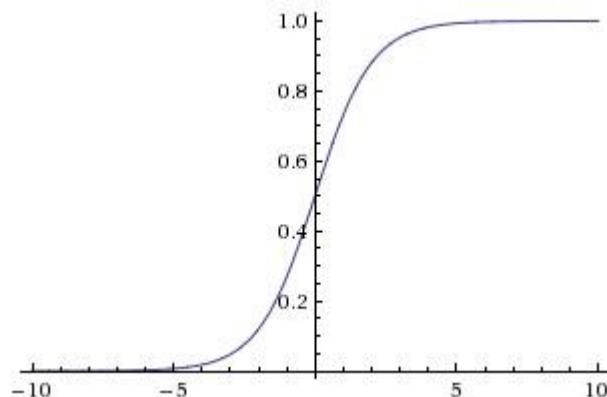
What happens when  $x = -10$ ?

What happens when  $x = 0$ ?

What happens when  $x = 10$ ?

# Activation Functions

$$\sigma(x) = 1/(1 + e^{-x})$$



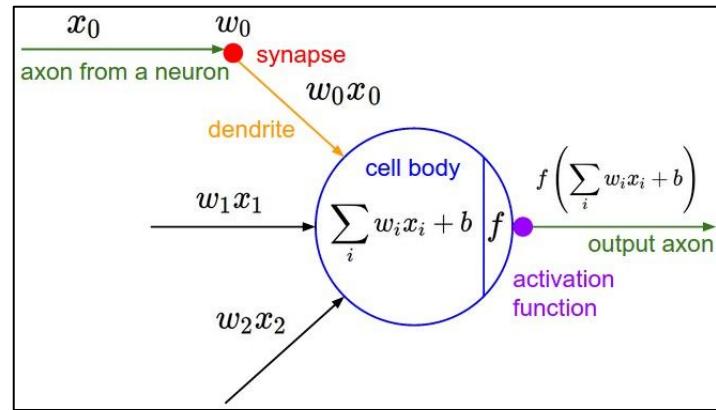
**Sigmoid**

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

3 problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered

Consider what happens when the input to a neuron ( $x$ ) is always positive:

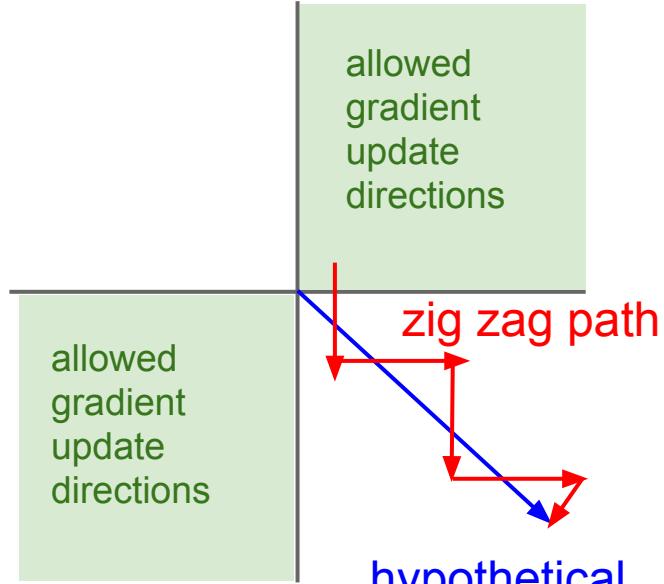


$$f \left( \sum_i w_i x_i + b \right)$$

What can we say about the gradients on  $w$ ?

Consider what happens when the input to a neuron is always positive...

$$f \left( \sum_i w_i x_i + b \right)$$



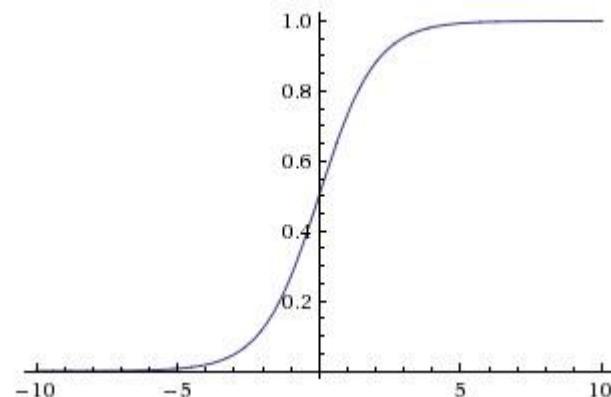
What can we say about the gradients on  $w$ ?

Always all positive or all negative :(

(this is also why you want zero-mean data!)

hypothetical  
optimal  $w$   
vector

# Activation Functions



Sigmoid

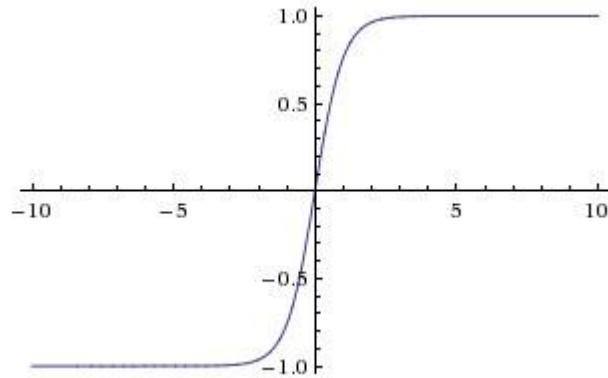
$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

3 problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered
3.  $\exp()$  is a bit compute expensive

# Activation Functions

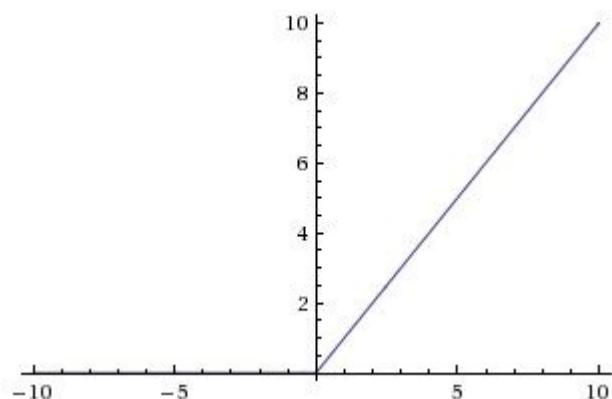


**tanh(x)**

- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated :(

[LeCun et al., 1991]

# Activation Functions

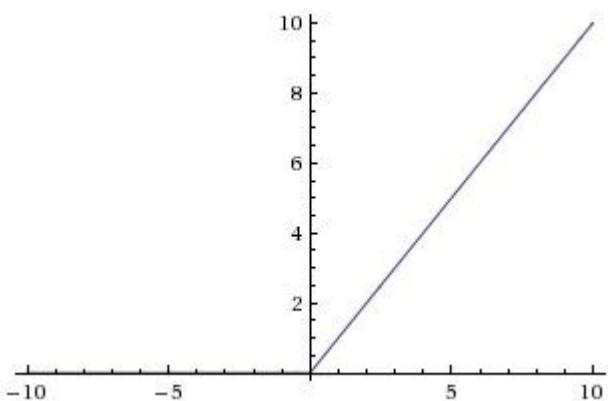


- Computes  $f(x) = \max(0, x)$
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

**ReLU**  
(Rectified Linear Unit)

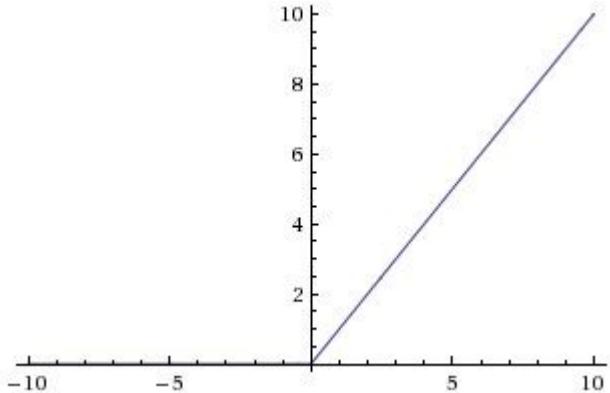
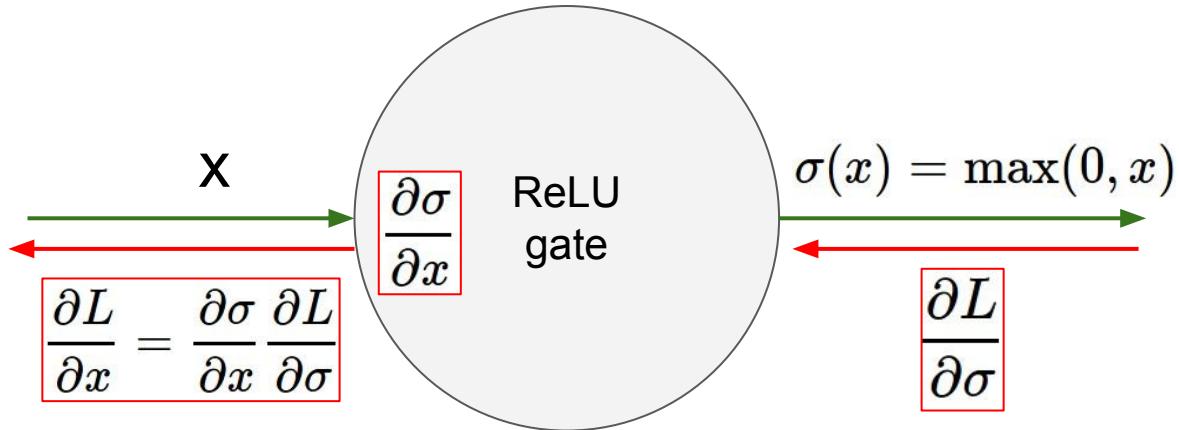
[Krizhevsky et al., 2012]

# Activation Functions



**ReLU**  
(Rectified Linear Unit)

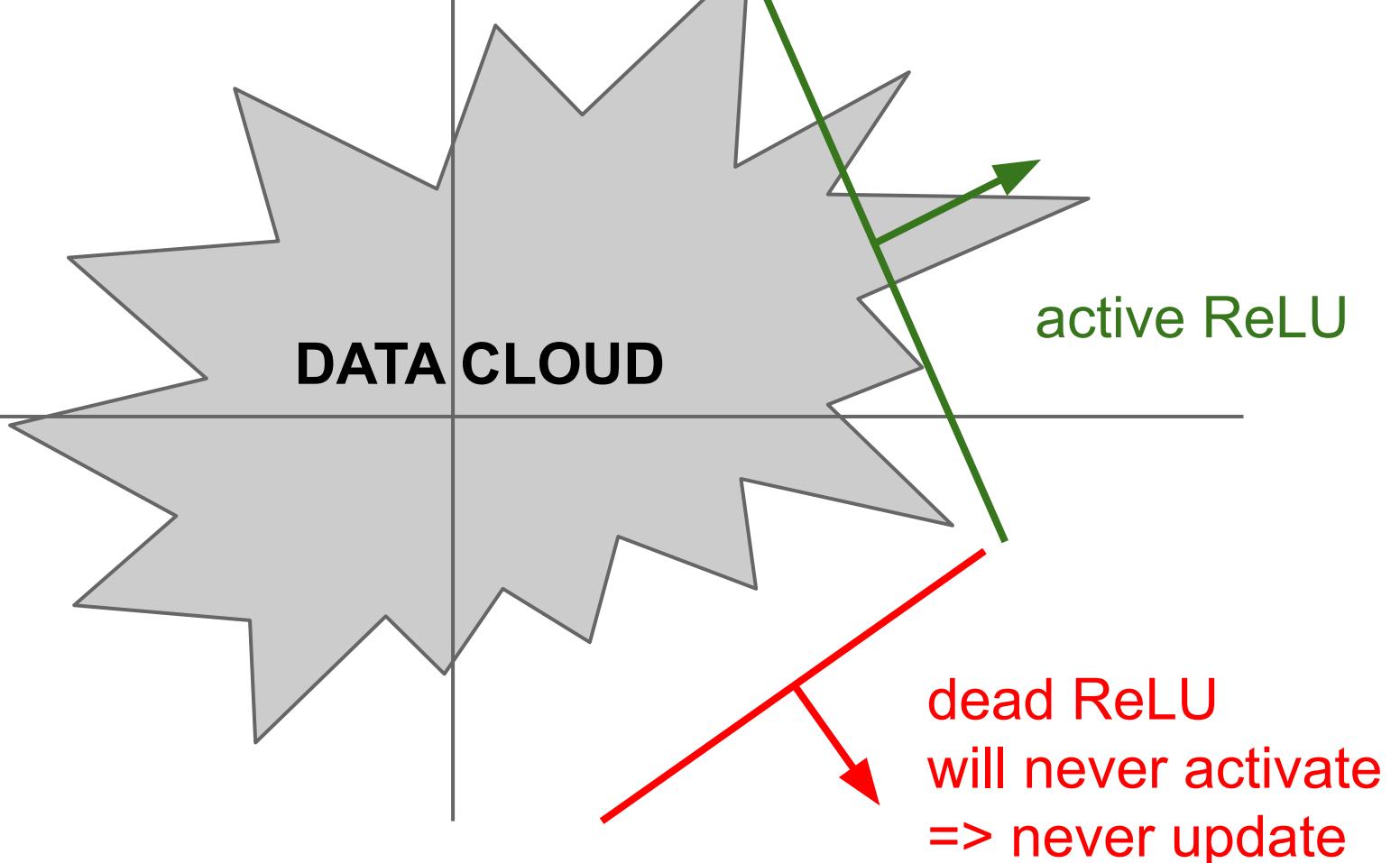
- Computes  $f(x) = \max(0, x)$
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- Not zero-centered output
- An annoyance:  
hint: what is the gradient when  $x < 0$ ?

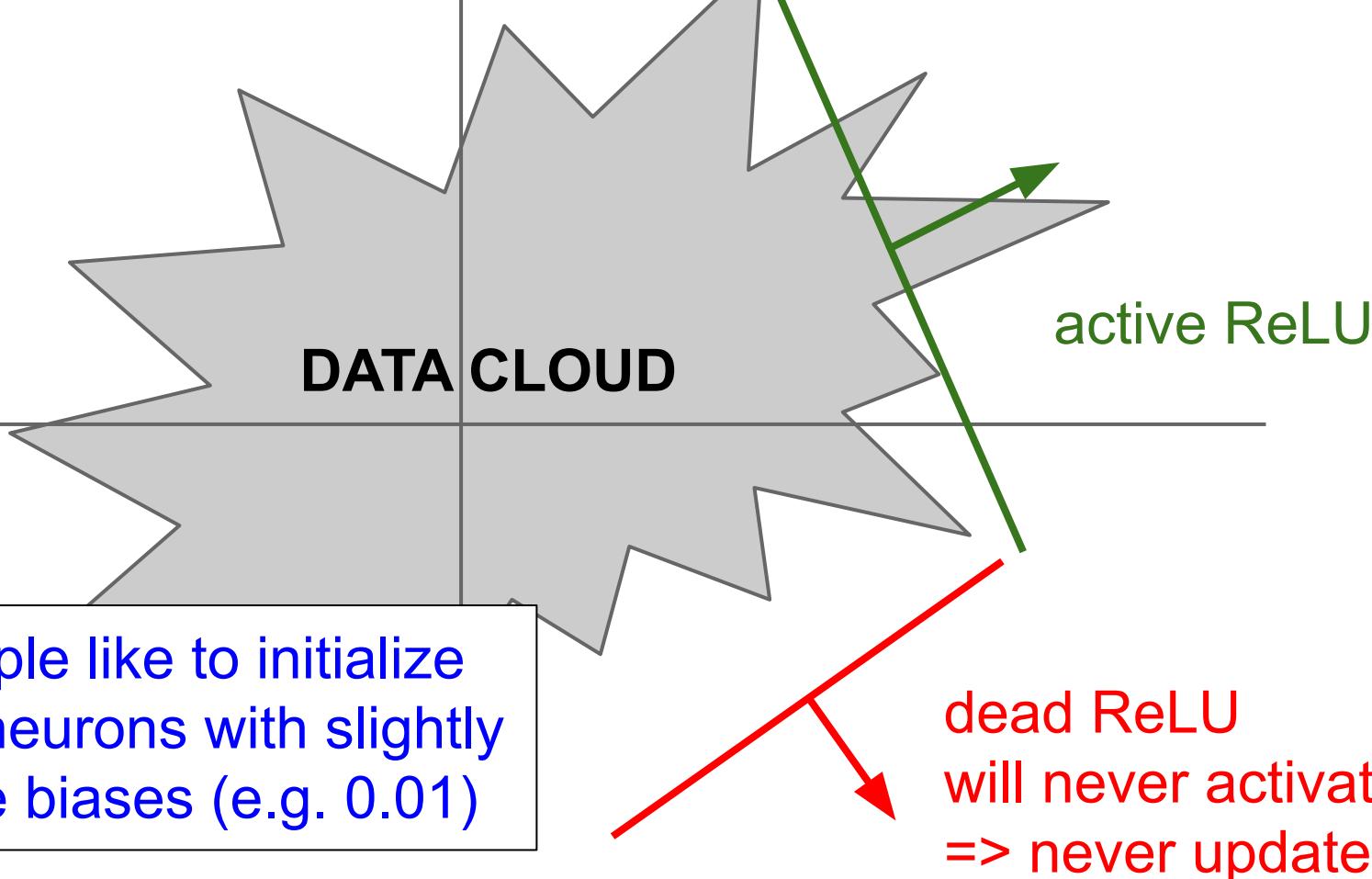


What happens when  $x = -10$ ?

What happens when  $x = 0$ ?

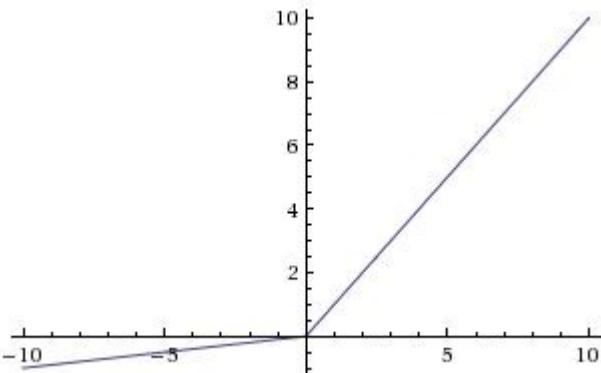
What happens when  $x = 10$ ?





# Activation Functions

[Mass et al., 2013]  
[He et al., 2015]



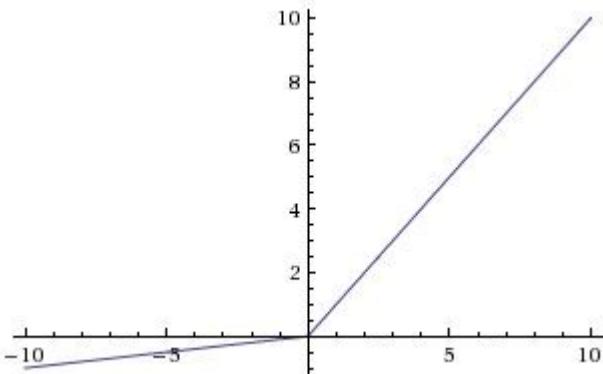
- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

## Leaky ReLU

$$f(x) = \max(0.01x, x)$$

# Activation Functions

[Mass et al., 2013]  
[He et al., 2015]



## Leaky ReLU

$$f(x) = \max(0.01x, x)$$

- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

## Parametric Rectifier (PReLU)

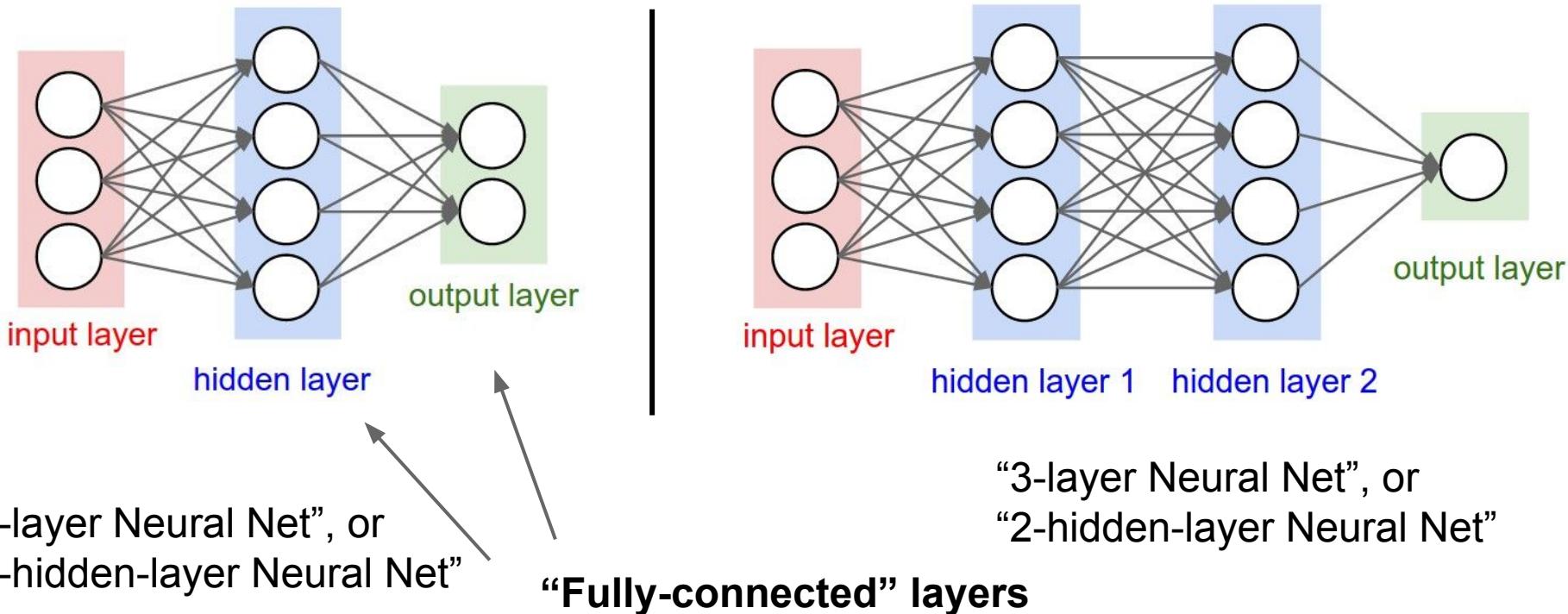
$$f(x) = \max(\alpha x, x)$$

backprop into  $\alpha$   
(parameter)

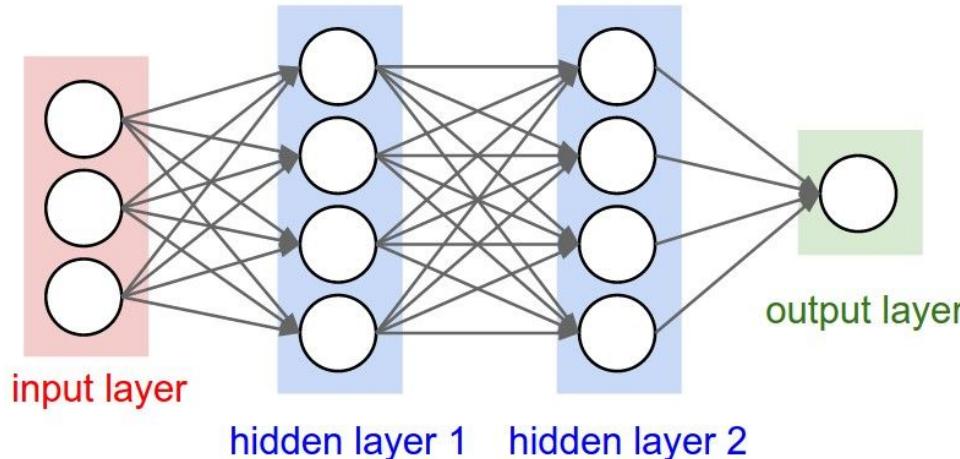
# TLDR: In practice:

- Use ReLU. Be careful with your learning rates
- Try out Leaky ReLU / Maxout / ELU
- Try out tanh but don't expect much
- Don't use sigmoid

# Neural Networks: Architectures

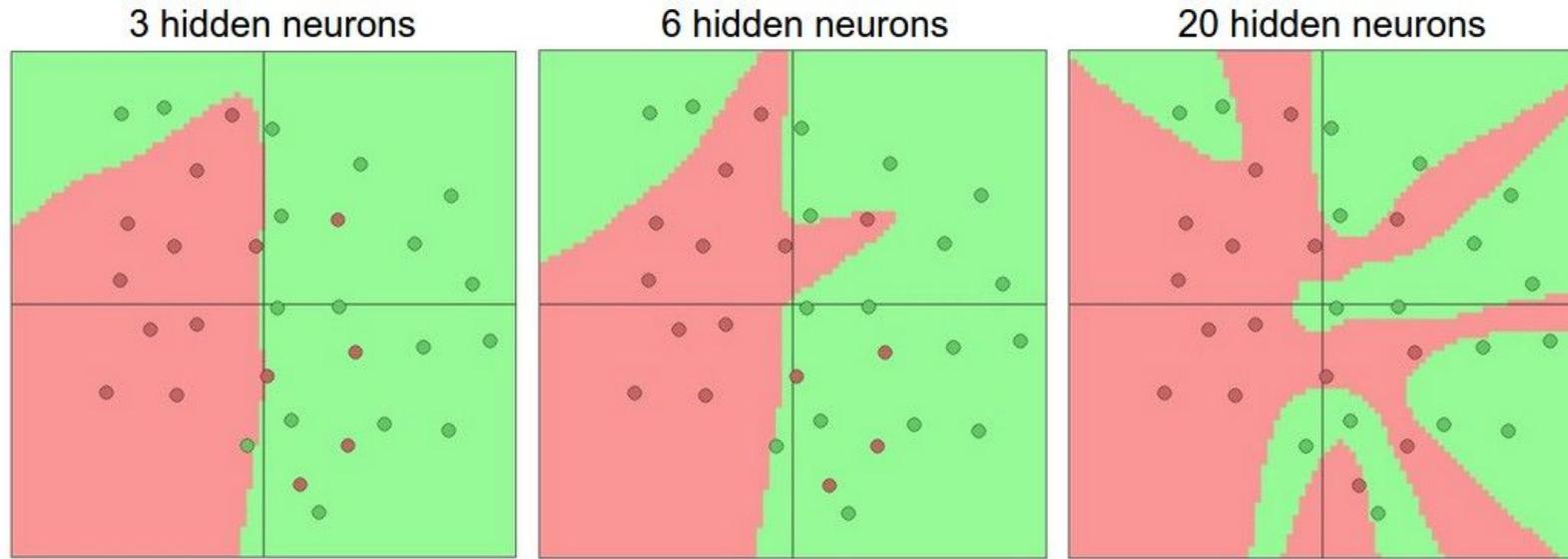


# Example Feed-forward computation of a Neural Network



```
# forward-pass of a 3-layer neural network:  
f = lambda x: 1.0/(1.0 + np.exp(-x)) # activation function (use sigmoid)  
x = np.random.randn(3, 1) # random input vector of three numbers (3x1)  
h1 = f(np.dot(W1, x) + b1) # calculate first hidden layer activations (4x1)  
h2 = f(np.dot(W2, h1) + b2) # calculate second hidden layer activations (4x1)  
out = np.dot(W3, h2) + b3 # output neuron (1x1)
```

# Setting the number of layers and their sizes



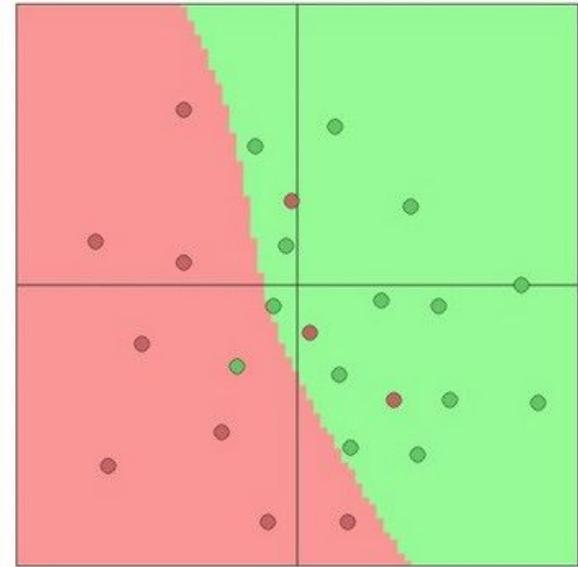
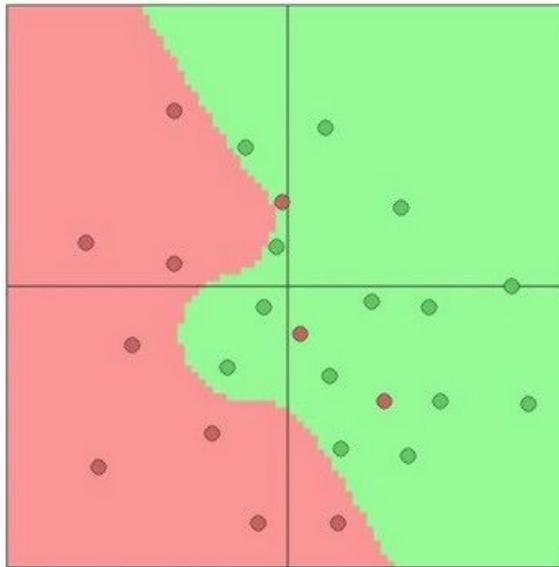
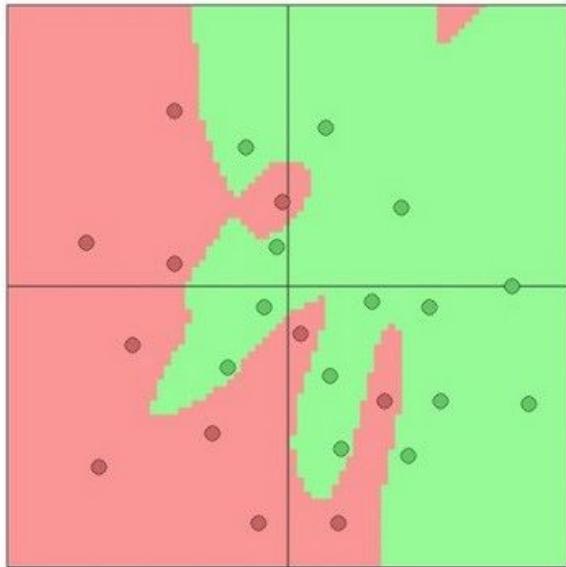
more neurons = more capacity

Do not use size of neural network as a regularizer. Use stronger regularization instead:

$\lambda = 0.001$

$\lambda = 0.01$

$\lambda = 0.1$



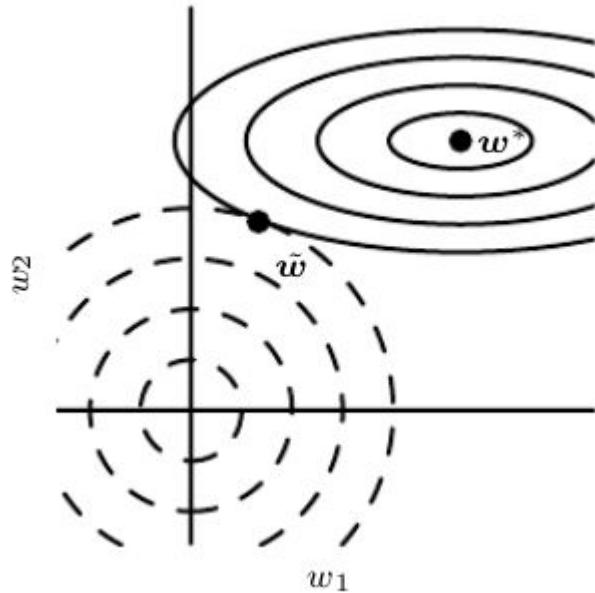
(you can play with this demo over at ConvNetJS:  
<http://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html>)

L1 regularization on least squares:

$$\mathbf{w}^* = \arg \min_{\mathbf{w}} \sum_j \left( t(\mathbf{x}_j) - \sum_i w_i h_i(\mathbf{x}_j) \right)^2 + \lambda \sum_{i=1}^k |w_i|$$

L2 regularization on least squares:

$$\mathbf{w}^* = \arg \min_{\mathbf{w}} \sum_j \left( t(\mathbf{x}_j) - \sum_i w_i h_i(\mathbf{x}_j) \right)^2 + \lambda \sum_{i=1}^k w_i^2$$



- L1: makes  $\mathbf{W}$  sparse!
- L2: reduces the number of large numbers in  $\mathbf{W}$ !

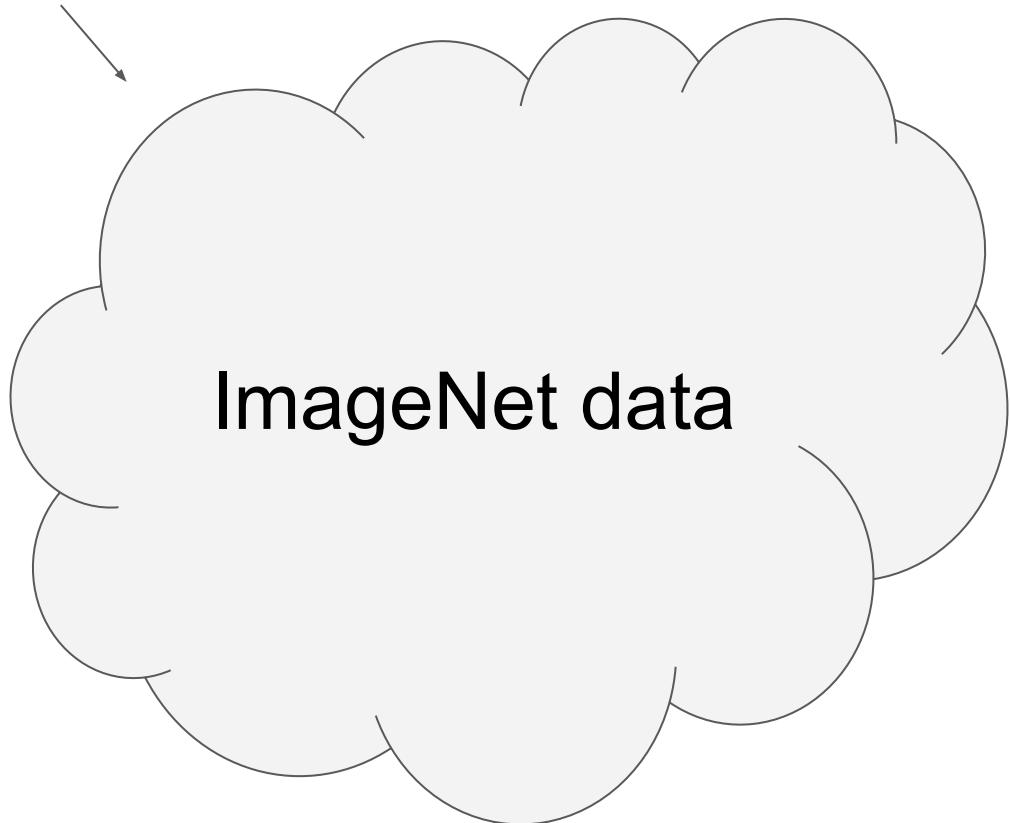
“ConvNets need a lot  
of data to train”

“ConvNets need a lot  
of data to train”

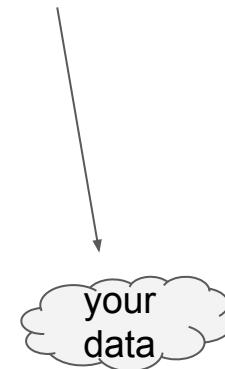


**finetuning!** we rarely ever  
train ConvNets from scratch.

1. Train on ImageNet



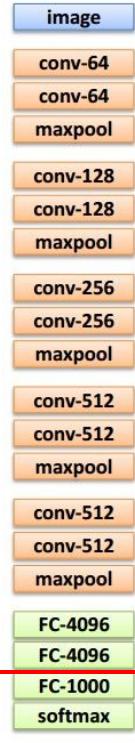
2. Finetune network on  
your own data



# Transfer Learning with CNNs



1. Train on ImageNet



2. If small dataset: fix all weights (treat CNN as fixed feature extractor), retrain only the classifier

i.e. swap the Softmax layer at the end



3. If you have medium sized dataset, “finetune” instead: use the old weights as initialization, train the full network or only some of the higher layers

retrain bigger portion of the network, or even all of it.