Module I : Units, Dimensions and Vectors

Mainak Pal

July 13, 2020

What is Physics ?

Physics is the study of all phenomenon around us ranging from the atomic scale to the galactic scale.

Figure: Milky Way, Diameter 105,700 light-years $\approxeq 1 \times 10^{21}$m

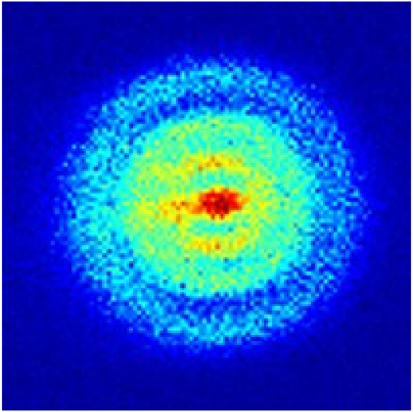

Figure: Hydrogen Atom Radius 53pm $\approxeq 10^{-10}$m

Figure: The Scientific Method

A Hypothesis should be Falsifiable

Experiments and the Conclusions should be Repeatable

Nature of Physical Law

- Universal, i.e. valid every where (Earth, Moon, Saturn or even the Anrdomeda galaxy or anywhere) and at all times (since the beginning of Universe)
- So far Physicists have found that all the Laws of Nature are expressed in terms of simple mathematical equations.
- Laws of nature we find are valid in certain regimes, we need to make certain assumptions to simplify the calculations, but if these conditions are satisfied we find the laws to be universal.
- In future we may find Laws which require less assumptions and that would be a more general law of nature.

Use of Physical Laws in Technology

Faraday demonstrated at Royal Institute in London that electricity can affect magnetism. When asked "What good is it?" Faraday replied: "What good is a newborn baby?"

- Thermodynamics, Mechanics $\rightarrow$ Steam Engines, Transport
- Electricity, Magnetism, Oscillations and Waves $\rightarrow$ Radio communication, Internet (invented at CERN - particle accelerator), WiFi
- Electron Transport in Materials $\rightarrow$ Semiconductor Devices and Digital Computers
- Aerodynamics $\rightarrow$ Airplanes
- Gravitation $\rightarrow$ Global Positioning System (GPS)
- Atomic and Nuclear Physics $\rightarrow$ Atomic Bomb, Nuclear Energy

Units: A standardized method to compare measurements of physical observables.

Examples : *kg* for mass, *light-year* for distance, *Ampere* for current.

C.G.S. (Centimeter - Gram - Second) and S.I.(System International) are the most common unit systems.

Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	kelvin	K
Luminous Intensity	candela	cd
Amount of Substance	mole	mol

Figure: SI unit system

Significant Digits:

$$53 \text{ pm} = \underbrace{0.53}_{2 \text{ significant digits}} \times 10^2 \text{ pm} = \underbrace{0.53}_{\text{still } 2 \text{ significant digits}} \times 10^{-10} \text{ m}$$

$$105,700 \text{ light-years} = \underbrace{0.105700}_{6 \text{ significant digits}} \times 10^5 \text{ light-years}$$

65040

650.40

0.065040

Significant, Insignificant

Any number you come across can be expressed as follows

$0.\#\#0..0\#..\#..$

Express them in this way and count the number of digits to the right of decimal point. All of them are significant and come from Experimental Accuracy

Dimension: Describes how a physical quantity has been constructed from fundamental building blocks

- Mass - M
- Length - L
- Time - T
- Electrical Charge - Q
- Temperature - θ
- ..

Example : $\dim(\text{velocity}) = LT^{-1}$, $\dim(\text{acceleration}) = LT^{-2}$
$\dim(\text{electrical current}) = QT^{-1}$, $\dim(\text{power}) = ML^2T^{-3}$

In general dimension $= M^x L^y T^z Q^u \theta^w ...$

A physical quantity can be expressed in various units but its dimension will always be the same.

Question: Can you make force out of power and velocity?

Principle of Homogeneity of Equations

Principle of Homogeneity of Equations

dim(force) $= MLT^{-2}$ assume force $\propto$ (power)^x(velocity)^y

Principle of Homogeneity of Equations

dim(force) $= MLT^{-2}$ assume force $\propto$ (power)^x(velocity)^y

Equating dimensions of both side

$$\begin{aligned} MLT^{-2} &= (ML^2T^{-3})^x \times (LT^{-1})^y \\ &= M^x L^{2x+y} T^{-3x-y} \end{aligned}$$

Principle of Homogeneity of Equations

dim(force) $= MLT^{-2}$ assume force $\propto$ (power)^x(velocity)^y

Equating dimensions of both side

$$\begin{aligned} MLT^{-2} &= (ML^2T^{-3})^x \times (LT^{-1})^y \\ &= M^x L^{2x+y} T^{-3x-y} \end{aligned}$$

This gives us system of equations

$$\begin{aligned} x &= 1 \\ 2x + y &= 1 \\ -3x - y &= -2 \end{aligned}$$

Principle of Homogeneity of Equations

dim(force) $= MLT^{-2}$ assume force $\propto$ (power)^x(velocity)^y

Equating dimensions of both side

$$\begin{aligned} MLT^{-2} &= (ML^2T^{-3})^x \times (LT^{-1})^y \\ &= M^x L^{2x+y} T^{-3x-y} \end{aligned}$$

This gives us system of equations

$$\begin{aligned} x &= 1 \\ 2x + y &= 1 \\ -3x - y &= -2 \end{aligned}$$

<u>Solution</u>

$$\begin{aligned} x &= 1 \\ y &= -1 \end{aligned}$$

Principle of Homogeneity of Equations

dim(force) $= MLT^{-2}$ assume force $\propto$ (power)^x(velocity)^y

Equating dimensions of both side

$$\begin{aligned} MLT^{-2} &= (ML^2T^{-3})^x \times (LT^{-1})^y \\ &= M^x L^{2x+y} T^{-3x-y} \end{aligned}$$

This gives us system of equations

$$x = 1$$
$$2x + y = 1$$
$$-3x - y = -2$$

Solution

$$x = 1$$
$$y = -1$$

This is in fact very good, we will later learn that Power, $P = \vec{F} \cdot \vec{v}$

Dimensional Analysis

- You can only add two quantities if they have the same dimension.
- You can very simply construct relationship between different quantities, though you can miss some numerical factors, trigonometric factors etc. since numbers are dimensionless.
- From the dimension of a quantity you can very easily know what unit to use.

From dimensional analysis we can test if a relation could be correct or not and test if some relation is definitely wrong. We say that some relation is correct only using dimensional arguments

Problem-1

1. Experiments with a simple pendulum show that its time period depends on its length (l) and the acceleration due to gravity (g). Use dimensional analysis to obtain the dependence of the time period on l and g .

Solution

Solution

Objective : To find relationship between length of a pendulum l, its time period of oscillation T and the acceleration due to gravity g

Solution

Objective : To find relationship between length of a pendulum l, its time period of oscillation T and the acceleration due to gravity g

$$\text{dim}(l) = L,\ \ \text{dim}(T) = T,\ \ \text{dim}(g) = LT^{-2}$$

Solution

Objective : To find relationship between length of a pendulum l, its time period of oscillation T and the acceleration due to gravity g

$$\text{dim}(l) = L,\ \text{dim}(T) = T,\ \text{dim}(g) = LT^{-2}$$

$$T = l^x g^y$$

Solution

Objective : To find relationship between length of a pendulum l, its time period of oscillation T and the acceleration due to gravity g

$$\text{dim}(l) = L, \ \ \text{dim}(T) = T, \ \ \text{dim}(g) = LT^{-2}$$

$$T = l^x g^y \implies T = (L)^x \times (LT^{-2})^y$$

Solution

Objective : To find relationship between length of a pendulum l, its time period of oscillation T and the acceleration due to gravity g

$$\text{dim}(l) = L,\ \text{dim}(T) = T,\ \text{dim}(g) = LT^{-2}$$

$$T = l^x g^y \implies T = (L)^x \times (LT^{-2})^y$$

Comparing power of L and T on both sides of the equation (Principle of Homogeneity of Equations)

Solution

Objective : To find relationship between length of a pendulum l, its time period of oscillation T and the acceleration due to gravity g

$$\text{dim}(l) = L, \ \text{dim}(T) = T, \ \text{dim}(g) = LT^{-2}$$

$$T = l^x g^y \implies T = (L)^x \times (LT^{-2})^y$$

Comparing power of L and T on both sides of the equation (Principle of Homogeneity of Equations)

$$x + y = 0$$
$$-2y = 1$$

$$y = -1/2$$
$$-2y = 1$$

Solution

Objective : To find relationship between length of a pendulum l, its time period of oscillation T and the acceleration due to gravity g

$$\text{dim}(l) = L,\ \ \text{dim}(T) = T,\ \ \text{dim}(g) = LT^{-2}$$

$$T = l^x g^y \implies T = (L)^x \times (LT^{-2})^y$$

Comparing power of L and T on both sides of the equation (Principle of Homogeneity of Equations)

$$x + y = 0 \qquad\qquad y = -1/2$$

$$-2y = 1 \qquad\qquad -2y = 1$$

We find $T \propto \sqrt{\frac{l}{g}}$ The exact relation is $T = 2\pi\sqrt{\frac{l}{g}}$