Assigmment-2

1) $\sum_{i=1}^{n} x_i = 11211$ $\qquad \sum_{i=1}^{n} y_i = 44520.80$

$\sum_{i=1}^{n} x_i y_i = 1996904.105$

$\sum_{i=1}^{n} x_i^2 = 543503$ $\qquad \sum_{i=1}^{n} y_i^2 = 8110405.02$

a) we know that,

$$\hat{w}_0 = \bar{y} - \hat{w}_1 \bar{x} \quad \text{and} \quad \hat{w}_1 = \frac{\sum_{n=1}^{N} y_n x_n - N\bar{x}\bar{y}}{\sum_{n=1}^{N} x_n^2 - \frac{(N\bar{x})^2}{N}}$$

$$\therefore \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{11211.00}{250} = 44.844$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{44520.80}{250} = 178.0832$$

$$\therefore w_1 = \frac{250 \times 1996904.105 - 11211.00 \times 44520.80}{250 \times 543503.00 - (11211.00)^2}$$

slope $= \dfrac{-3671.3}{1014229} \approx -0.00358$

$$\therefore w_0 = 178.0832 - (-.00358) \times 44.844$$
$$= 178.0832 + 0.1607 = 178.2439$$

Intercept

$\therefore$ Equation $= y = 178.24 - 0.00358x$.

b) The predicted weight that would be observed on average for a 25 year old man =

$\Rightarrow y = 178.24 - 0.00358x$.

$= 178.24 - 25 \times 0.00358$

$= 178.1505$ lbs

c) Residual part of observation =

$(170 - 178.1505)$ lbs

$= -8.15$ lbs.

d) The residual weight was negative, the predicted value was an therefore the predicted value was an overestimate.

⇒ Given.

$\sum y_i = 572$ $\quad \sum x_i = 43$ $\quad \sum x_i y_i = 1697.8$

$\sum y_i^2 = 23,530$ $\quad \sum x_i^2 = 157.42$

$\Rightarrow \bar{x} = \frac{43}{14} = 3.0714.$

$\Rightarrow \bar{y} = \frac{572}{14} = 40.8571.$

$\therefore \hat{w}_1 = \frac{14 \times 1697.8 - 43 \times 572}{14 \times 157.42 - (43)^2} = \frac{-926.8}{354.88}$

$= -2.612$

$\therefore \hat{w}_0 = \bar{y} - w_1 \cdot \bar{x} = 40.8571 - (-2.612) \times 3.0714.$

$= 40.8571 + 8.026 = 48.8831$

$\therefore$ Regression equation $\Rightarrow y = \underbrace{48.88}_{\text{intercept.}} - \underbrace{2.61}_{\text{slope}} x$

b) Predicted permeability for $x = \cancel{3.7}$ 4.3

$\Rightarrow y = \hat{w}_0 + \hat{w}_1 x.$

$= 48.88 - 2.61 \times 4.3 = 48.88 - 11.223$

$= 37.657.$

c) Estimate for $x = 3.7$

$\Rightarrow y = \hat{w}_0 + \hat{w}_1 x$

$= 48.88 - 2.61 \times 3.9 = 39.223.$

d) Residual for observed $y = 46.1$ and $x = 3.7$

$= 46.1 - 39.233 = 6.877$

(Underestimate)

3) Regression model,

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$$

putting $k=2$, in least square normal equations we get,

$$n\hat{w}_0 + (\Sigma x_{i1})\hat{w}_1 + (\Sigma x_{i2})\beta_2 = \Sigma y_i$$

$$(\Sigma x_{i1})\beta_0 + (\Sigma x_{i1}^2)\beta_1 + (\Sigma x_{i1} x_{i2})\beta_2 = \Sigma x_{i1} y_i$$

$$(\Sigma x_{i2})\beta_0 + (\Sigma x_{i1} x_{i2})\beta_1 + (\Sigma x_{i2}^2)\beta_2 = \Sigma x_{i2} y_i$$

Given,

$n = 10$ $\qquad \Sigma y_i = 1916$

$\Sigma x_{i1} = 223$ $\qquad \Sigma x_{i1}^2 = 5200.9$

$\Sigma x_{i2} = 553$ $\qquad \Sigma x_{i2}^2 = 31729$

$\Sigma x_{i1} y_1 = 43550.8$ $\qquad \Sigma x_{i1} x_{i2} = 12352$

$\Sigma x_{i2} y_2 = 104736.8$

So, the system is.

$$10\beta_0 + 223\beta_1 + 553\beta_2 = 1916$$

$$223\beta_0 + 5200.9\beta_1 + 12352\beta_2 = 43550.8$$

$$553\beta_0 + 12352\beta_1 + 31729\beta_2 = 104,736.8$$

b) For
need to

$$\begin{bmatrix} 10 \\ 22 \\ 55 \end{bmatrix}$$

Solving
$\beta_0 = 1$

So, the
$y =$

c) Putt
the a
$y = 17$
$=$
$=$

b) For estimating the parameters, we need to solve the equations.

$$\begin{bmatrix} 10 & 223 & 553 \\ 223 & 5200.9 & 12352 \\ 553 & 12352 & 31729 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 1916 \\ 43350.8 \\ 104736.8 \end{bmatrix}$$

Solving this system we get,

$\beta_0 = 171.06 \qquad \beta_1 = 3.71 \qquad \beta_2 = -1.13.$

So, the estimated regression equation is,

$y = 171.06 + 3.71x_1 - 1.13x_2.$

c) Putting $x_1 = 18$ and $x_2 = 43$ in the above equation we get,

$y = 171.06 + 3.71 \times 18 - 43 \times 1.13$

$= 171.06 + 66.78 - 48.59$

$= 189.25$

4) $(X'X)^{-1} = \begin{pmatrix} 2.9705 & -4.0042\times10^{-2} & -4.1679\times10 \\ -0.4004 & 6.0774\times10^{-4} & -7.3875\times10 \\ -0.00417 & -7.3875\times10^{-5} & 2.5766\times10 \end{pmatrix}$

$$(X'y) = \begin{pmatrix} 4757.9 \\ 834335.8 \\ 179706.8 \end{pmatrix}$$

we wish to find least square estimate

$$\hat{w} = (\hat{w}_1, \hat{w}_2, \hat{w}_3)^T$$

such that

$$y_i = \hat{w}_0 + \hat{w}_1 \cancel{x}_i + \beta_2 x_{2i} + \epsilon_i.$$

$$\hat{w} = (X'X^{-1})(X'y)$$

$$= \begin{pmatrix} -6744.1277 \\ -1715.1498 \\ 1.7637281 \end{pmatrix}$$

$\therefore$ % Body fat $= -6744.1277$

$+ (-1715.1498) \times$ (height in inches)

$+ (1.7637281) \times$ (waist in inches)

5) $f(x_1, x_2) = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_1 x_2 + w_4 x_1^2 + w_5 x_2^2$

we have N traing points i.e

$(x_1^{(i)}, x_2^{(i)}, y^{(i)})$, $i = 1(1)N$.

we wish to fit,

$$y^{(i)} = w_0 + w_1 x_1^{(i)} + w_2 x_2^{(i)} + w_3 x_1^{(i)} x_2^{(i)} + w_4 [x_1^{(i)}]^2 + w_5 [x_2^{(i)}]^2 + \varepsilon^{(i)}.$$

let $w = \begin{pmatrix} w_0 \\ w_1 \\ \vdots \\ w_6 \end{pmatrix}$ $y = \begin{pmatrix} y^{(1)} \\ \vdots \\ y^{(N)} \end{pmatrix}$

$$X = \begin{pmatrix} 1 & x_1^{(1)} & \cdots & (x_2^{(1)})^2 \\ \vdots & \vdots & & \vdots \\ 1 & x_1^{(N)} & \cdots & (x_2^{(N)})^2 \end{pmatrix}$$

$$X'X = \begin{bmatrix} \sum_{i=1}^{N} 1 \cdot 1 & \sum_{i=1}^{N} 1 \cdot x_1^{(i)} & \sum_{i=1}^{N} 1 \cdot x_2^{(i)} & \cdots & \sum_{i=1}^{N} 1 \cdot [x_2^{(i)}]^2 \\ \sum_{i=1}^{N} x_1^{(i)} \cdot 1 & & & & \vdots \\ \vdots & & & & \vdots \\ \sum_{i=1}^{N} [x_1^{(i)}]^2 \cdot 1 & \cdots & & \cdots & \sum_{i=1}^{N} [x_2^{(i)}]^4 \end{bmatrix}$$

$$X'y = \begin{pmatrix} \sum_{i=1}^{n} 1 \cdot y^{(i)} \\ \sum_{i=1}^{n} x_1^{(i)} y^{(i)} \\ \sum_{i=1}^{n} x_2^{(i)} y^{(i)} \\ \sum_{i=1}^{n} [x_1^{(i)} x_2^{(i)}] y^{(i)} \\ \sum_{i=1}^{n} [x_1^{(i)}]^2 y^{(i)} \\ \sum_{i=1}^{n} [x_2^{(i)}]^2 y^{(i)} \end{pmatrix}$$

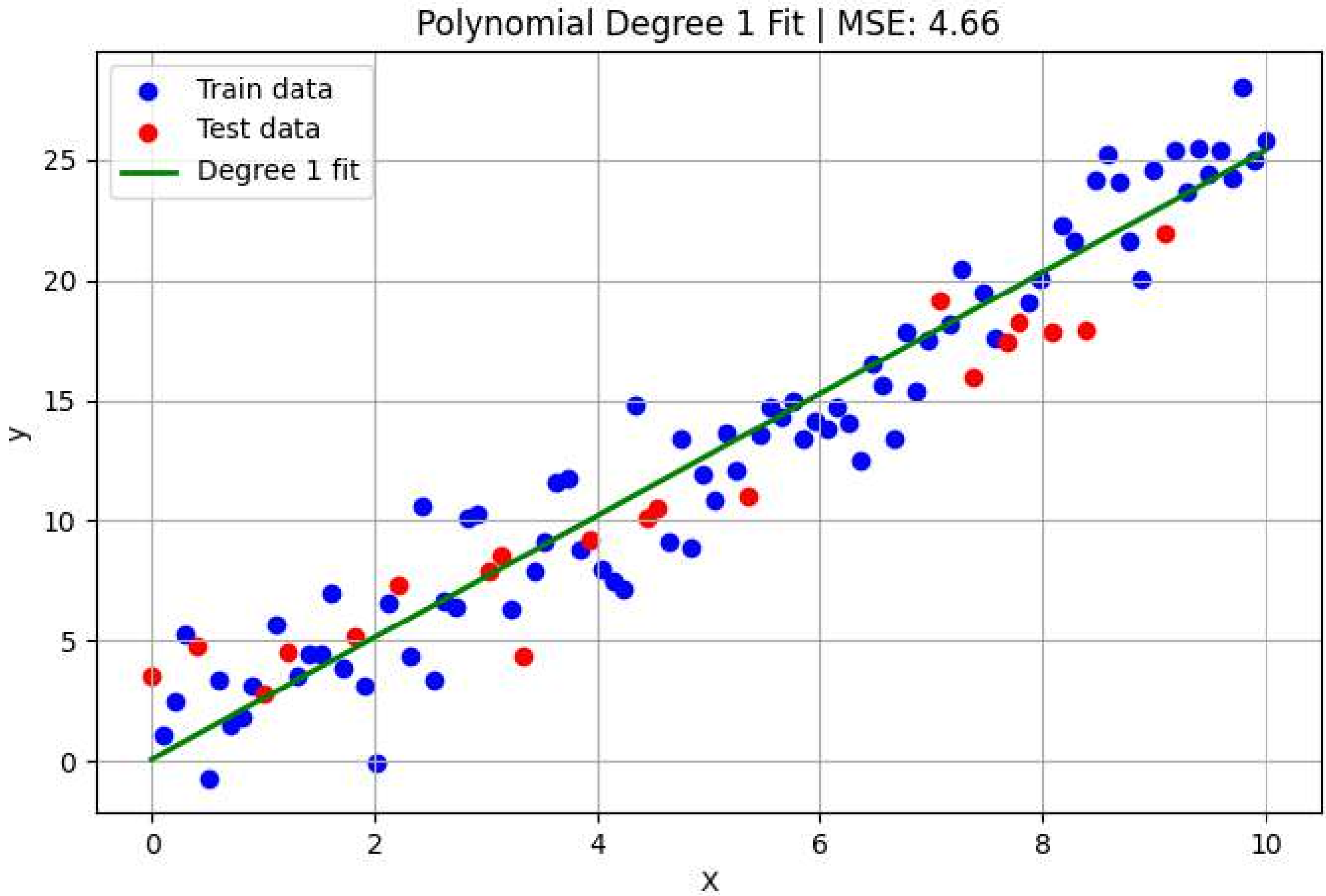
Hence the normal equations $X'X \hat{w} = X'y$ will become equation in unknowns, $w_0, w_1, \ldots, w_5$.

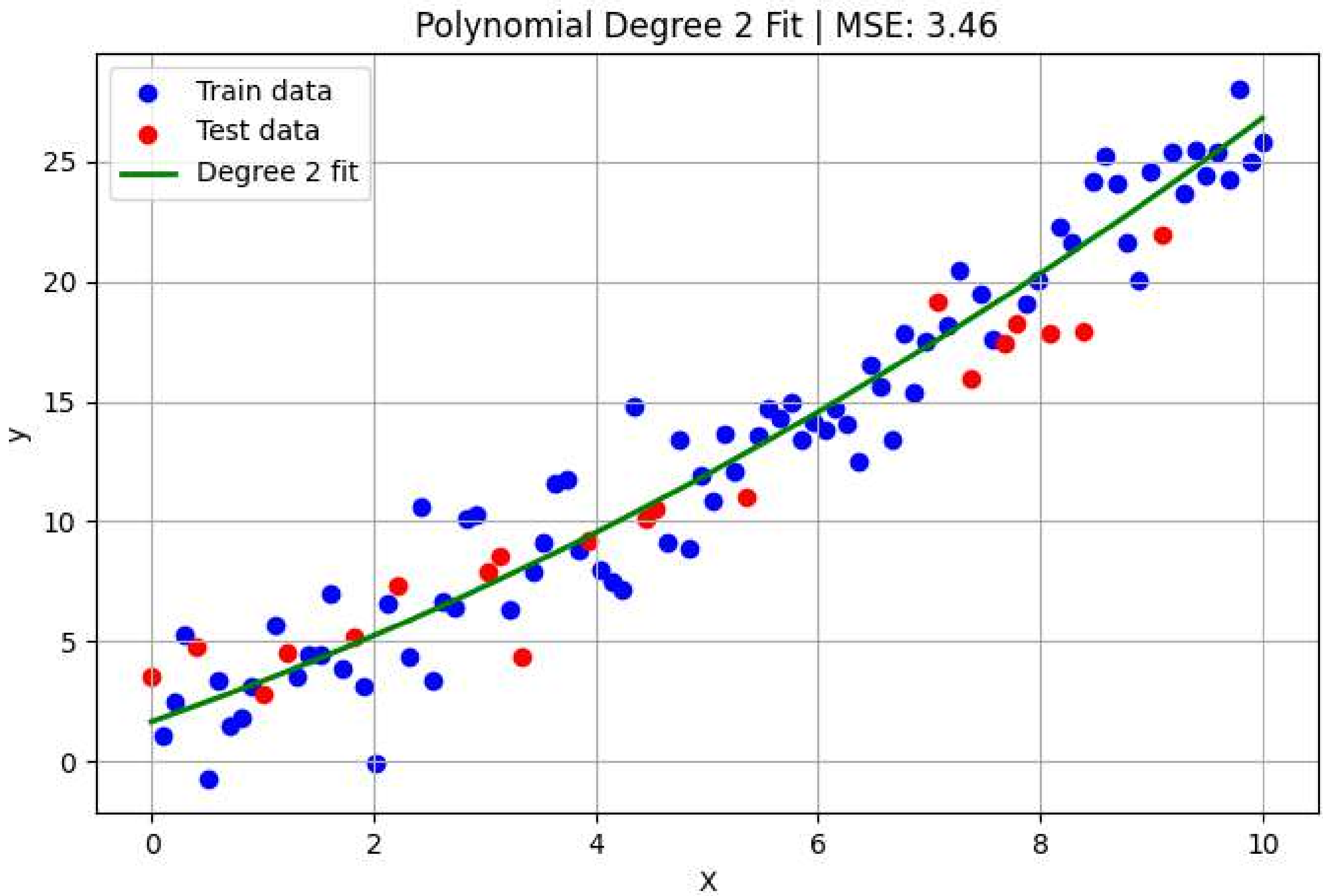
Now if we multiply $(XX')^{-1}$ with $X'y$ ~~will~~ we will get

$$w = (XX')^{-1}(X'y) = \begin{pmatrix} \hat{w}_1 \\ \hat{w}_2 \\ \vdots \\ \hat{w}_6 \end{pmatrix}$$

Thus our final quadratic fit model will be,

$$\hat{f}(x_1 x_2) = \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2 + w_3$$


Polynomial Degree 1 Fit | MSE: 4.66
Train data
Test data
Degree 1 fit
y
X
0
5
10
15
20
25
0
2
4
6
8
10


Polynomial Degree 2 Fit | MSE: 3.46
Train data
Test data
Degree 2 fit
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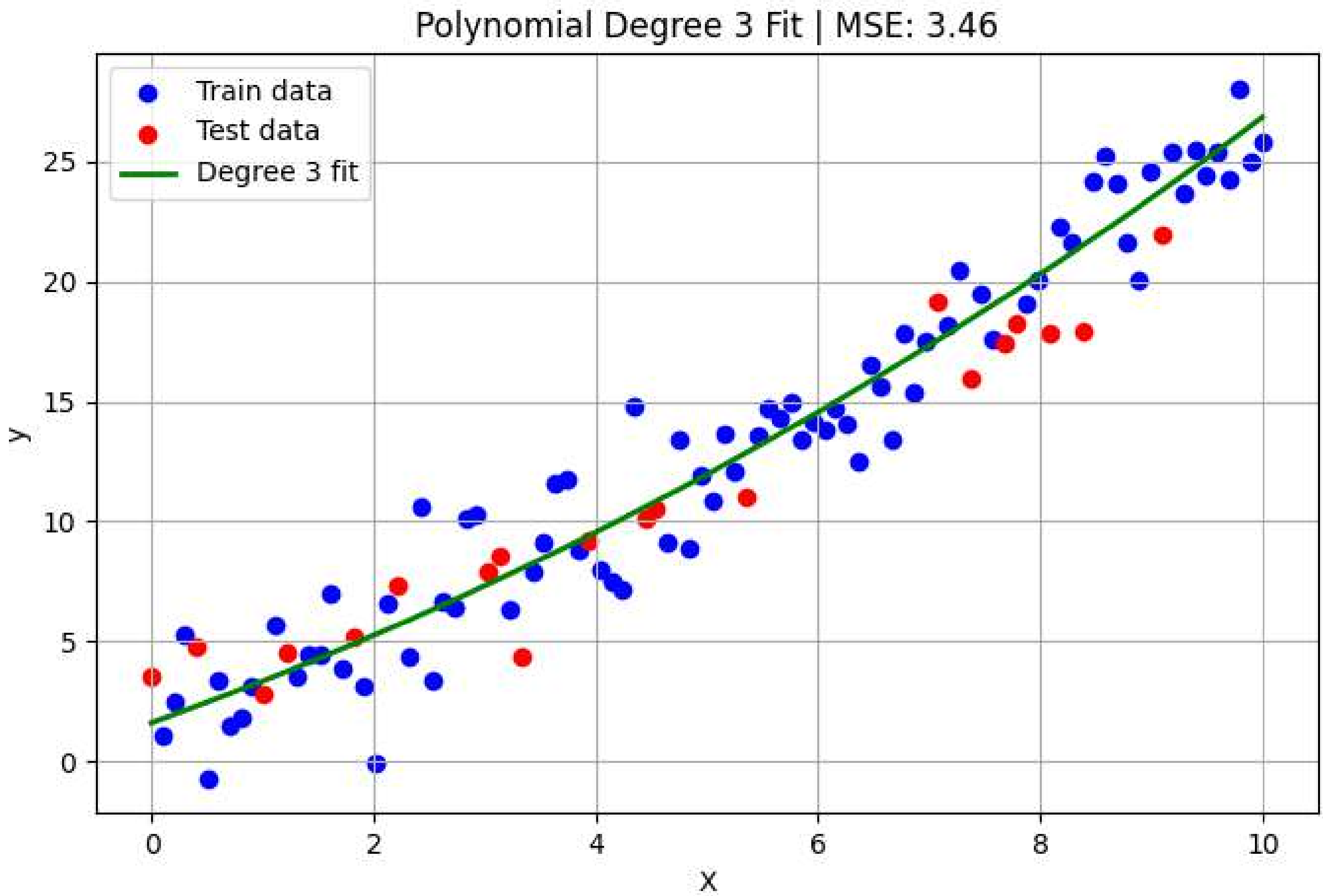
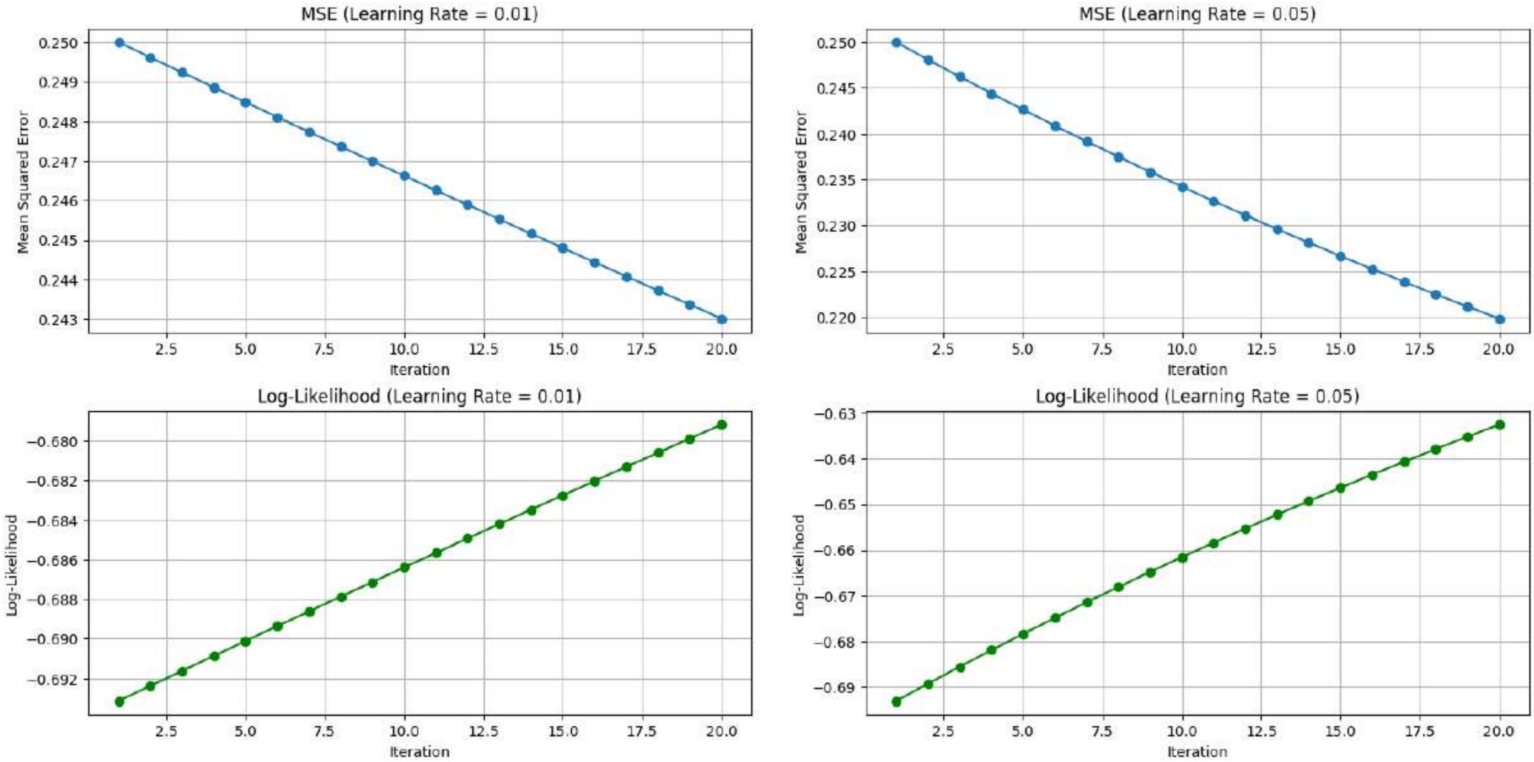
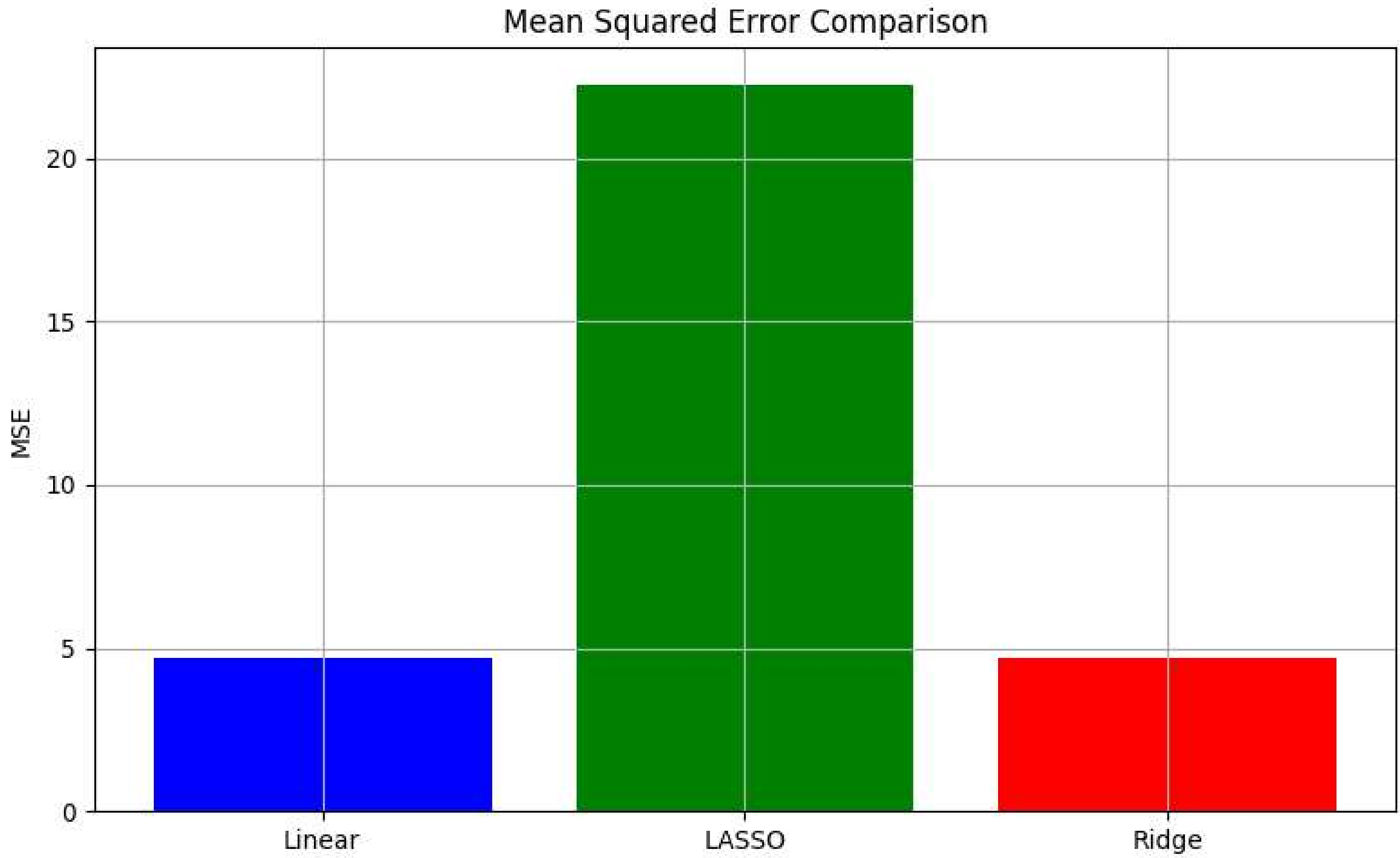
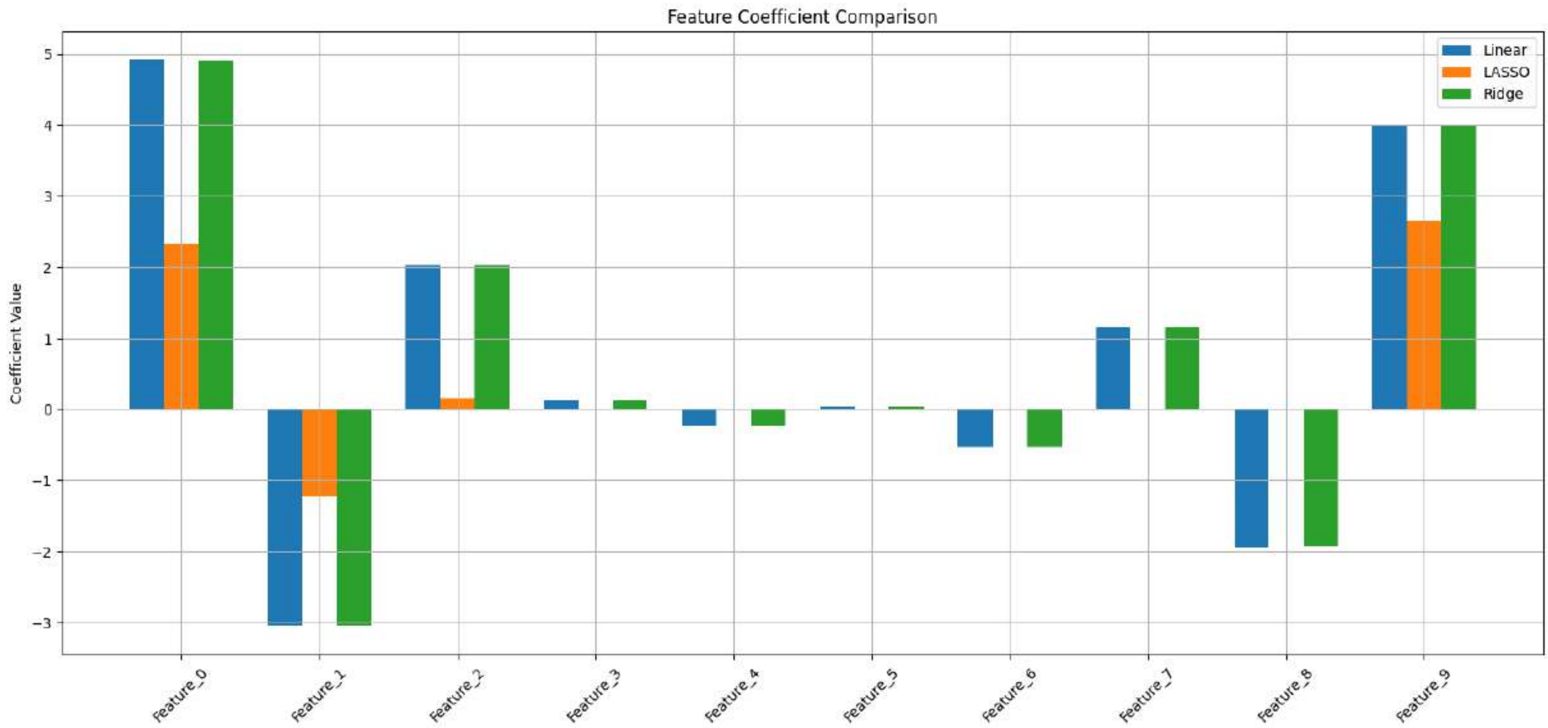
5
10
15
20
25
y
0
2
4
6
8
10
X


Polynomial Degree 3 Fit | MSE: 3.46
Train data
Test data
Degree 3 fit
y
X
0
5
10
15
20
25
0
2
4
6
8
10


MSE (Learning Rate = 0.01)
Mean Squared Error
Iteration
MSE (Learning Rate = 0.05)
Mean Squared Error
Iteration
Log-Likelihood (Learning Rate = 0.01)
Log-Likelihood
Iteration
Log-Likelihood (Learning Rate = 0.05)
Log-Likelihood
Iteration


Mean Squared Error Comparison
MSE
0
5
10
15
20
Linear
LASSO
Ridge


Feature Coefficient Comparison
Linear
LASSO
Ridge
Coefficient Value
5
4
3
2
1
0
−1
−2
−3
Feature_0
Feature_1
Feature_2
Feature_3
Feature_4
Feature_5
Feature_6
Feature_7
Feature_8
Feature_9