Introduction to GAME THEORY

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Matching Pennies

- You and your friend simultaneously reveal a penny
- If both pennies show heads or both show tails, your friend has to pay you $1
- If one penny shows head and other tail, you have to pay your friend $1

Matching pennies

		Your friend: Head	Your friend: Tail
You	Head	1 -1	-1 1
	Tail	-1 1	1 -1

NASH THEOREM

- There must be at least one Nash equilibrium for all finite games

    -But here there are no equilibria in pure strategies

- But there is another type of equilibrium

Bar Scene

Question: Understand the situation and find out if the situation explained in this scene is a Nash equilibrium or not?

*jk:)

MIXED STRATEGY NASH EQUILIBRIUM

- If no equilibrium exists in pure strategies, one must exist in mixed strategy
- A mixed strategy is a probability distribution over two or more pure strategies
  - -That is players choose randomly among their options in equilibrium.
  - -If mixtures are mutual best responses, the set of strategies is a mixed Nash equilibrium.

Matching Pennies 2.0

- Suppose you are playing the game against Mind reader.
- How can you avoid losing?
- Answer: Flip the coin
- - At best, the mind reader could only win half the time.

Matching pennies

		Your friend	
		Head (.5)	Tail (.5)
You	Head(.5)	1 -1	-1 1
	Tail (.5)	-1 1	1 -1

Weighted Matching pennies

		Player 2	
		Left (Pl)	Right (Pr)
Player 1	Up (Pu)	3 -3	-2 2
	Down (Pd)	-1 1	0 0

Zero-Sum Mixed Strategy game

Solving for Player 1's Mixed Strategy

EU(l) = EU(r)
EU(l) = f(Pu)
EU(r) = f(Pu)
Three equations
three variables

	Left (Pl)	Right (Pr)
Up (Pu)	3 -3	-2 2
Down (Pd)	-1 1	0 0

Solving for Player 1's Mixed Strategy

- What is EU(l) = f(Pu)
- Some % of time, Player 2 gets -3
- The rest of the time she gets 1.

	Left (Pl)	Right (Pr)
Up (Pu)	3 -3	-2 2
Down (Pd)	-1 1	0 0

SOLVING FOR PLAYER 1'S MIXED STRATEGY

- What is EU(l) = f(Pu)
- Some % of time, Player 2 gets -3
- The rest of the time she gets 1.
- EU(l) = Pu(-3) + (1-Pu)(1)

	Left (Pl)	Right (Pr)
Up (Pu)	3 -3	-2 2
Down (Pd)	-1 1	0 0

Solving for Player 1's Mixed Strategy

- What is EU(r) = f(Pu)
- Some % of time, Player 2 gets 2.
- The rest of the time she gets 0.
- EU(r) = Pu(2) + (1-Pu)(0)

	Left (Pl)	Right (Pr)
Up (Pu)	3 -3	-2 2
Down (Pd)	-1 1	0 0

Solving for Player 1's Mixed Strategy

- EU(l) = EU(r)
- EU(l) = Pu(-3)+(1-Pu)(1)
- EU(r) = Pu(2) + (1-Pu)(0)
- Pu(-3)+(1-Pu)(1) = Pu(2) + (1-Pu)(0)

>> -3Pu + 1 - Pu = 2Pu

>> 6Pu = 1

>> Pu = 1/6

Solving for Player 2's Mixed Strategy

EU(u) = EU(d)

EU(u) = f(Pl)

EU(d) = f(Pl)

Three equations

three variables

	Left (Pl)	Right (Pr)
Up (Pu)	3 -3	-2 2
Down (Pd)	-1 1	0 0

SOLVING FOR PLAYER 2'S MIXED STRATEGY

- What is EU(u) = f(Pl)
- Some % of time, Player 2 gets 3.
- The rest of the time she gets -2.
- EU(u) = Pl(3) + (1-Pl)(-2)

	Left (Pl)	Right (Pr)
Up (Pu)	3 -3	-2 2
Down (Pd)	-1 1	0 0

SOLVING FOR PLAYER 2'S MIXED STRATEGY

- What is EU(d) = f(Pl)
- Some % of time, Player 2 gets -1
- The rest of the time she gets 0
- EU(d) = Pu(-1) + (1-Pu)(0)

	Left (Pl)	Right (Pr)
Up (Pu)	3 -3	-2 2
Down (Pd)	-1 1	0 0

SOLVING FOR PLAYER 1'S MIXED STRATEGY

- EU(u) = EU(d)
- EU(u) = Pl(3)+(1-Pl)(-2)
- EU(d) = Pl(-1) + (1-Pl)(0)
- Pl(3)+(1-Pl)(-2) = Pl(-1) + (1-Pl)(0)

  >> 3Pl -2 + 2Pl = -Pl

  >> 6Pl = 2

  >> Pl = 1/3

THE MIXED STRATEGY NASH EQUILIBRIUM

$Pl = 1/3, \ Pu = 1/6$

	Left (Pl)	Right (Pr)
Up (Pu)	3 -3	-2 2
Down (Pd)	-1 1	0 0

BATTLE OF THE SEXES

- A man and a woman want to get together for an evening of entertainment, but they have no means of communication.
- They can either go to the ballet or the fight.
  - the man prefers going to the fight
  - The woman prefers going to the ballet
  - But they prefer being together than being alone

Battle of Sexes

		Woman	
		Ballet	Fight
Man	Ballet	1 2	0 0
	Fight	0 0	2 1

Battle of Sexes

		Player 2	
		Left	Right
Player 1	Up	1 2	0 0
	Down	0 0	2 1

Solving for Player 1's Mixed Strategy

- What is EU(l) = f(Pu)
- Some % of time, Player 2 gets 2
- The rest of the time she gets 0
- EU(l) = Pu(2) + (1-Pu)(0)

	Left	Right
Up	1 2	0 0
Down	0 0	2 1

Solving for Player 1's Mixed Strategy

- What is EU(r) = f(Pu)
- Some % of time, Player 2 gets 0
- The rest of the time she gets 1
- EU(r) = Pu(0) + (1-Pu)(1)

	Left	Right
Up	1 2	0 0
Down	0 0	2 1

Solving for Player 1's Mixed Strategy

- EU(l) = EU(r)
- EU(l) = Pu(2)+(1-Pu)(0)
- EU(d) = Pu(0) + (1-Pu)(1)
- Pu(2)+(1-Pu)(0) = Pu(0) + (1-Pu)(1)

  >> 2Pu = 1- Pu

  >> 3Pu = 1

  >> Pu = 1/3

Solving for Player 2's Mixed Strategy

EU(u) = EU(d)
EU(u) = f(Pl)
EU(d) = f(Pl)
Three equations
three variables

	Left	Right
Up	1 2	0 0
Down	0 0	2 1

Solving for Player 2's Mixed Strategy

- What is EU(u) = f(Pl)
- Some % of time, Player 2 gets 1.
- The rest of the time she gets 0.
- EU(u) = Pl(1) + (1-Pl)(0)

	Left	Right
Up	1 2	0 0
Down	0 0	2 1

SOLVING FOR PLAYER 2'S MIXED STRATEGY

- What is EU(d) = f(Pl)
- Some % of time, Player 2 gets 0.
- The rest of the time she gets 2.
- EU(d) = Pl(0) + (1-Pl)(2)

	Left	Right
Up	1 2	0 0
Down	0 0	2 1

SOLVING FOR PLAYER 2'S MIXED STRATEGY

- EU(u) = EU(d)
- EU(u) = Pl(1)+(1-Pl)(0)
- EU(d) = Pl(0) + (1-Pl)(2)
- Pl(1)+(1-Pl)(0) = Pl(0) + (1-Pl)(2)

  >> Pl = 2 - 2Pl

  >> 3Pl = 2

  >> Pl = 2/3

Battle of Sexes

Player 2

Player 1		Ballet(2/3)	Fight(1/3)
	Ballet (1/3)	1 2	0 0
	Fight (2/3)	0 0	2 1

HOW TO CALCULATE PAYOFFS

1. Find the probability of each outcome occurs in equilibrium.
2. For each outcome, multiply that probability by a particular player's payoff.
3. Sum all of those numbers together.

How to Calculate Payoffs

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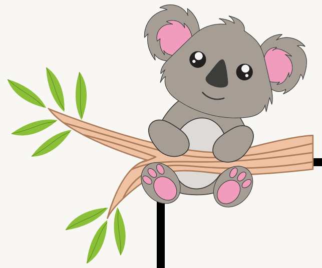

Battle of Sexes

Player 2

Player 1

	Ballet(2/3)	Fight(1/3)
Ballet (1/3)	1 2 2/9	0 0 1/9
Fignt (2/3)	0 0 4/9	2 1 2/9

How to Calculate Payoffs

1. Find the probability of each outcome occurs in equilibrium.
2. For each outcome, multiply that probability by a particular player’s payoff.
3. Sum all of those numbers together.

Battle of Sexes

		Player 2	
		Ballet(2/3)	Fight(1/3)
Player 1	Ballet (1/3)	1 * 2/9	0 * 1/9
	Fignt (2/3)	0 * 4/9	2 * 2/9

How to Calculate Payoffs

1. Find the probability of each outcome occurs in equilibrium.
2. For each outcome, multiply that probability by a particular player's payoff.
3. Sum all of those numbers together.

Battle of Sexes

Player 2

Player 1

	Ballet(2/3)	Fight(1/3)
Ballet (1/3)	1 * 2/9	0 * 1/9
Fignt (2/3)	0 * 4/9	2 * 2/9

Sum=6/9

=2/3

Battle of Sexes

Player 1 \ Player 2	Ballet(2/3)	Fight(1/3)
Ballet (1/3)	2 * 2/9	0 * 1/9
Fignt (2/3)	0 * 4/9	1 * 2/9

Sum=6/9
=2/3