

# **Introduction To Game Theory**

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**Stamatics**

# What is Game Theory ?

- **Definition:** the study of strategically interdependent behavior.
  - **Strategic interdependence:** what I do affects your outcomes and what you do affects my outcomes.
- Not just about winning and losing (though it could be).

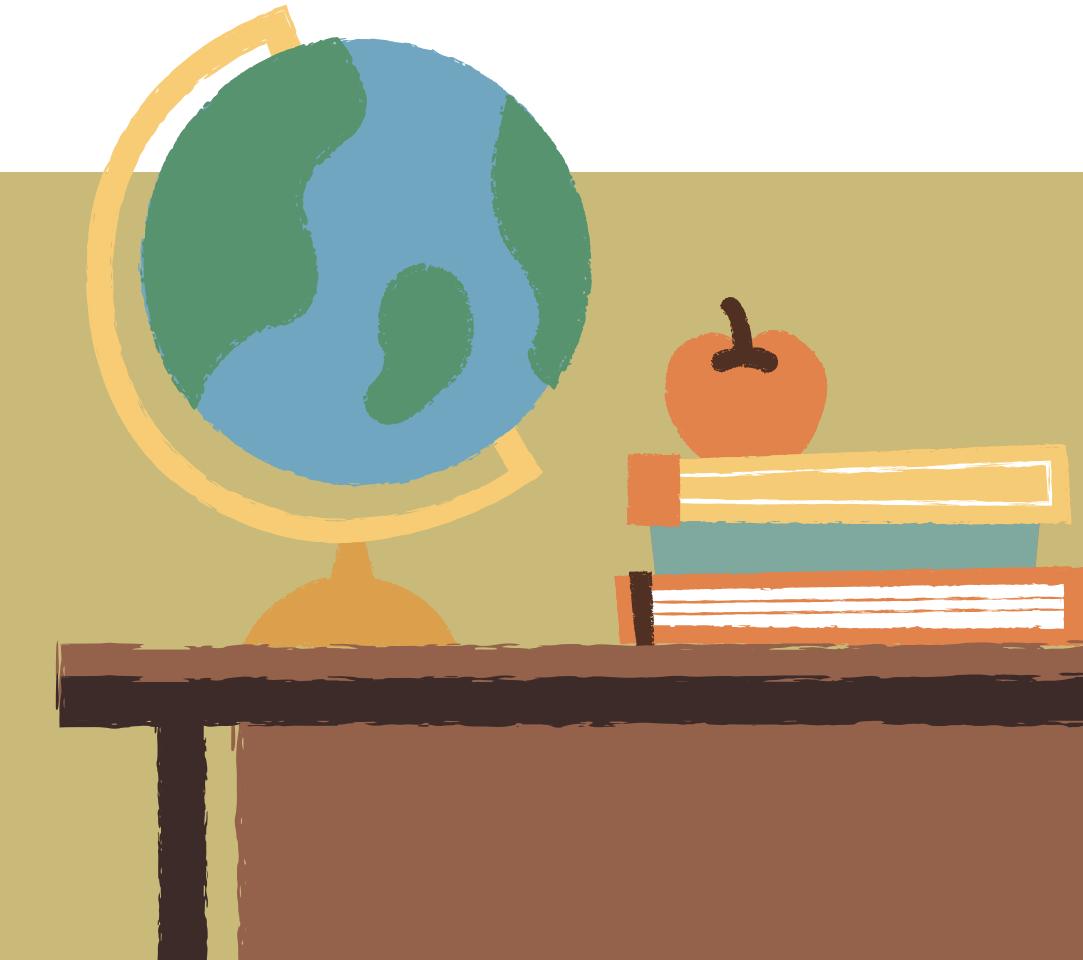
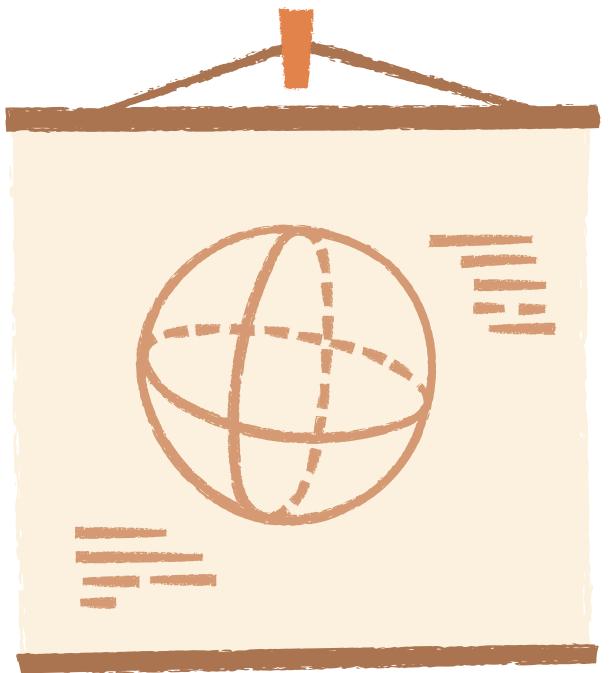
# Why Study Game Theory?

- The logic of strategically interdependent situations gets extremely complicated extremely fast.
  - Game theory gives us *accounting* tools.
- Game theory allows us to quickly draw parallels from one situation to another.
  - This will allow you to think on your feet much better than you can today.

# Project Evaluation

There will be one Mid-Term Evaluation, possibly presentation-based as per Stamatics norms. Details will be shared in due course.

Additionally, attendance and timely assignment submissions will be key factors in final ratification. Please stay engaged throughout the project.



# The Situation

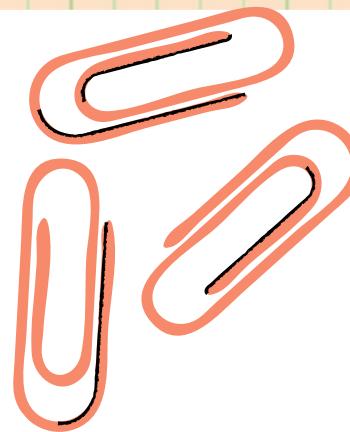
- Two suspects are arrested.
- The police think that they were trying to rob a store.
  - But the cops can only prove that the suspects were trespassing.
- Thus, the police need one of criminals to rat out the other.

# The Potential Deal

- If no one confesses to robbery, the police can only charge the prisoners for trespassing.
  - Punishment: 1 month in jail each.
- If one confesses and the other doesn't, the police will be lenient on the rat and severely punish the quiet one:
  - Punishment: 12 months in jail for the quiet one; 0 months for the rat.
- If both confess, the police punish both of them equally.
  - Punishment: 8 months in jail each.

# The Question

Suppose the thieves only want to minimize the number of months they spend in jail. Should they confess to the police?



Player 1

# The Prisoner's Dilemma

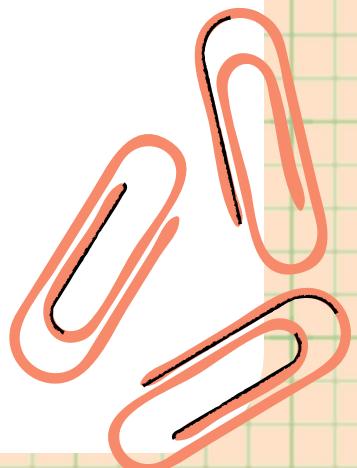
Player 2

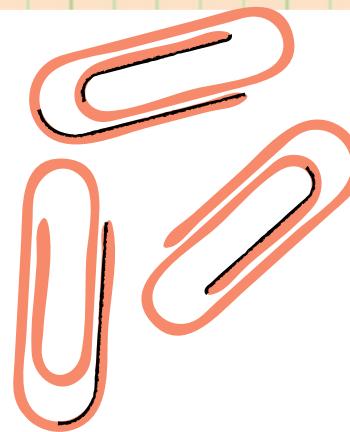
Keep Quiet   Confess

Keep  
Quiet

Confess

	-1, -1	-12, 0
	0, -12	-8, -8





Player 1

# The Prisoner's Dilemma

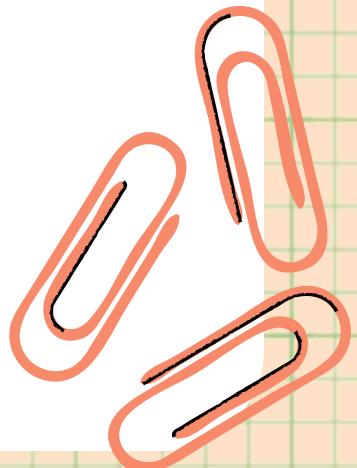
Player 2

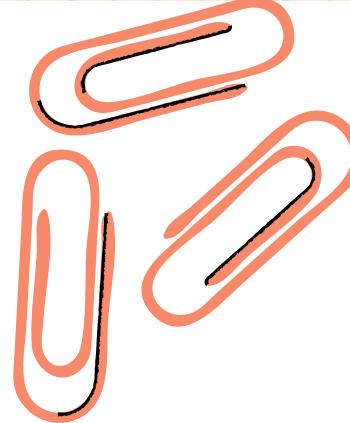
Keep Quiet

Keep  
Quiet

Confess

<b>-1, -1</b>
<b>0, -12</b>





Player 1

# The Prisoner's Dilemma

Player 2

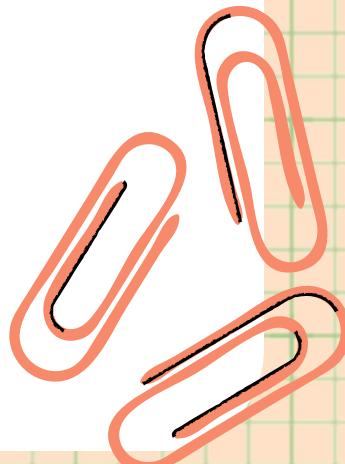
Confess

Keep  
Quiet

Confess

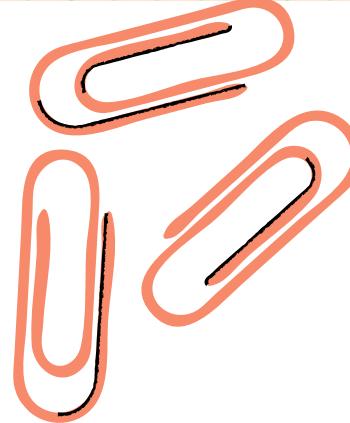
-12, 0

-8, -8



# Strict Dominance

- Strategy  $x$  strictly dominates strategy  $y$  for a player if  $x$  generates a greater payoff than  $y$  regardless of what the other players do.
- Rational players never play strictly dominated strategies.
  - Why play  $y$  when you can play  $x$  instead?



Player 1

# The Prisoner's Dilemma

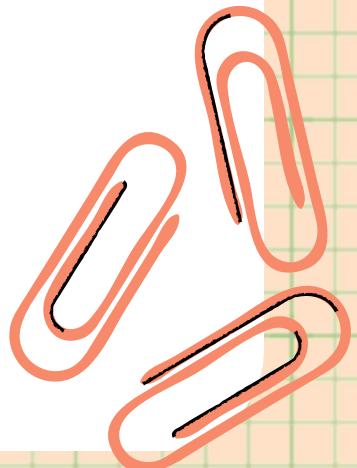
Player 2

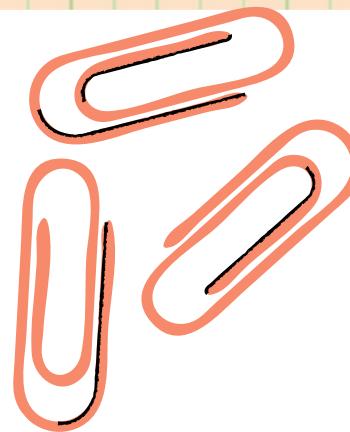
Keep Quiet   Confess

Keep  
Quiet

Confess

	-1, -1	-12, 0
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Player 1

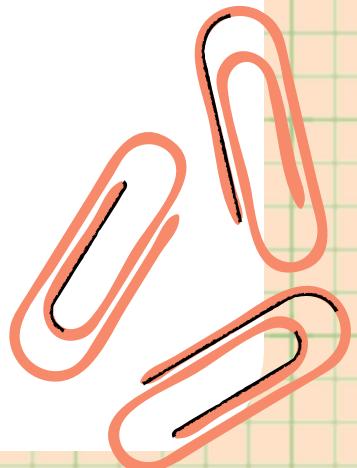
# The Prisoner's Dilemma

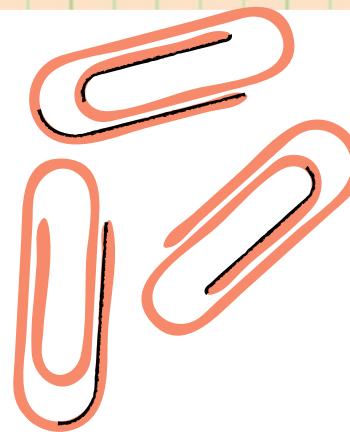
Player 2

Keep Quiet   Confess

Confess

0, -12	-8, -8
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Player 1

# The Prisoner's Dilemma

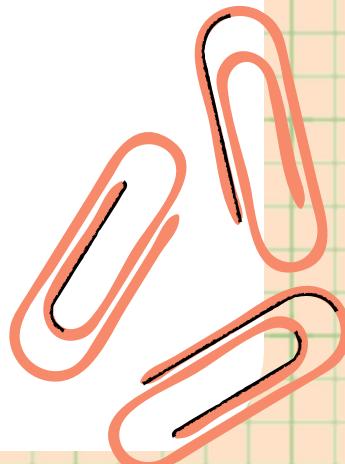
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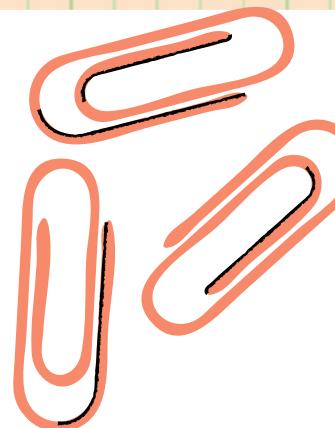
Keep Quiet   Confess

Keep  
Quiet

Confess

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	0, -12	-8, -8





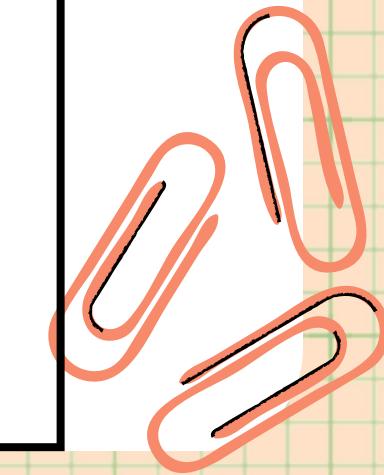
# IESDS

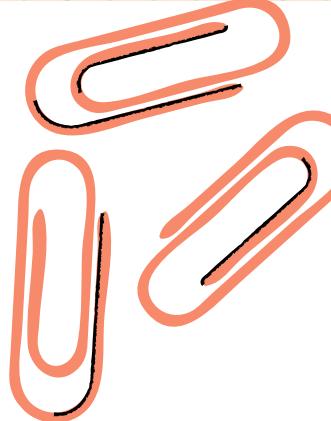
## Game

Up Middle Down

Left Center Right

13, 3	1, 4	7, 3
4, 1	3, 3	6, 2
-1, 9	2, 8	8, 1





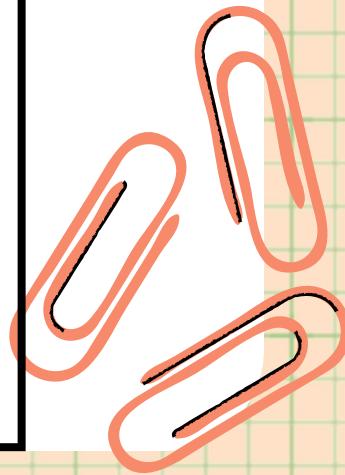
# IESDS

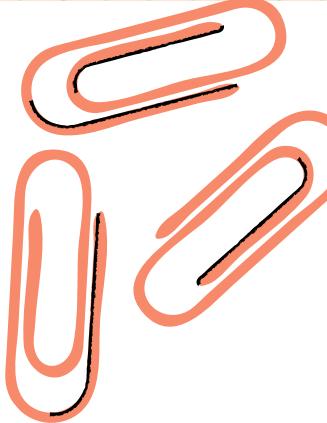
## Game

What would player  
1 play?

Left      Center      Right

13, 3	1, 4	7, 3
4, 1	3, 3	6, 2
-1, 9	2, 8	8, 1





# IESDS

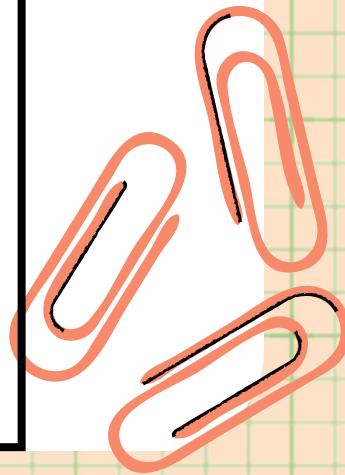
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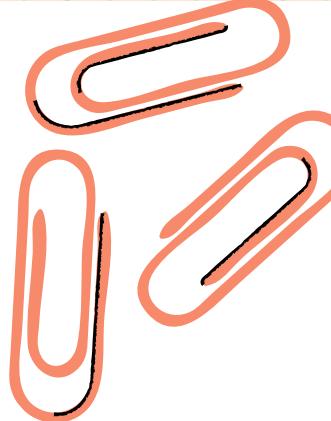
What would player

1 play?

Up  
Down Middle

	Left	Center	Right
Up	13, 3	1, 4	7, 3
Down Middle	4, 1	3, 3	6, 2
Down	-1, 9	2, 8	8, 1





# IESDS

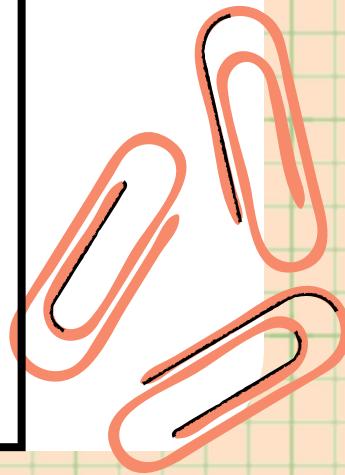
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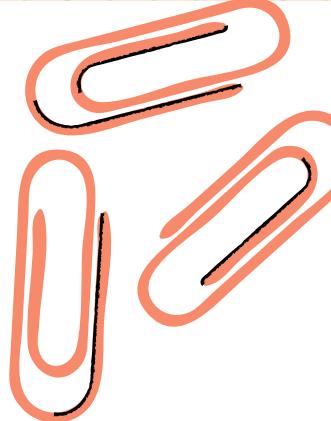
What would player  
1 play?

Up Middle Down

Left Center Right

13, 3	1, 4	7, 3
4, 1	3, 3	6, 2
-1, 9	2, 8	8, 1





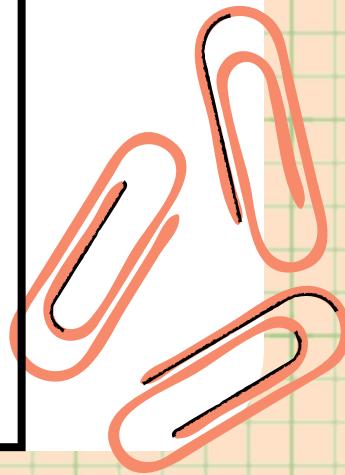
# IESDS

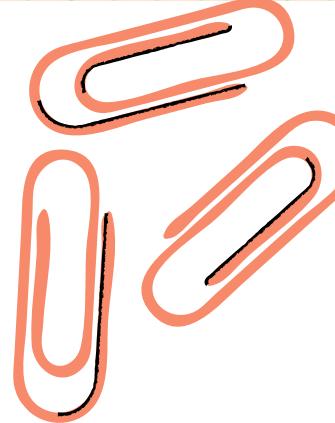
## Game

Will player 2 ever  
play right?

Up Middle Down

	Left	Center	Right
Up	13, 3	1, 4	7, 3
Middle	4, 1	3, 3	6, 2
Down	-1, 9	2, 8	8, 1





# IESDS

## Game

Up Middle Down

Left

Center

13, 3

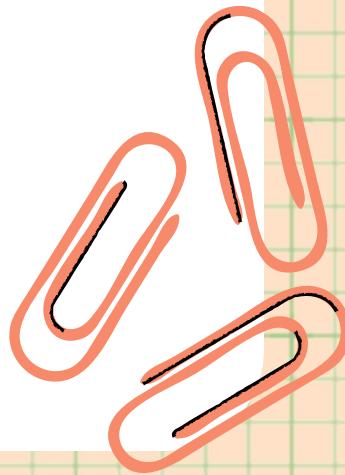
1, 4

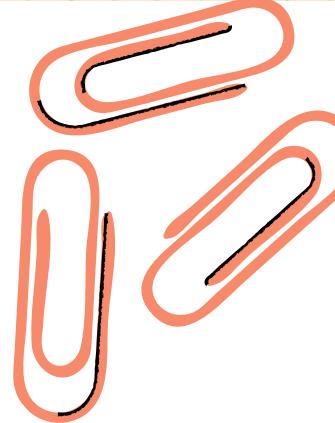
4, 1

3, 3

-1, 9

2, 8





# IESDS

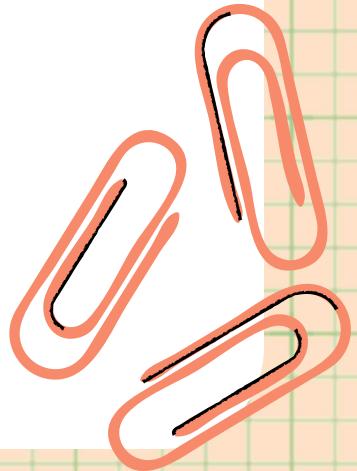
## Game

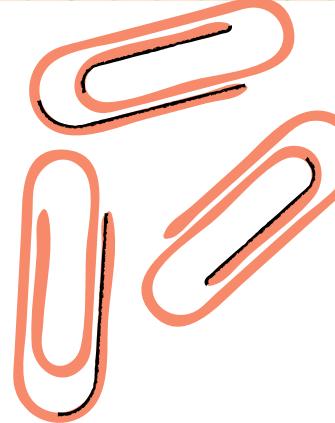
Up Middle Down

Left

Center

13, 3	1, 4
4, 1	3, 3



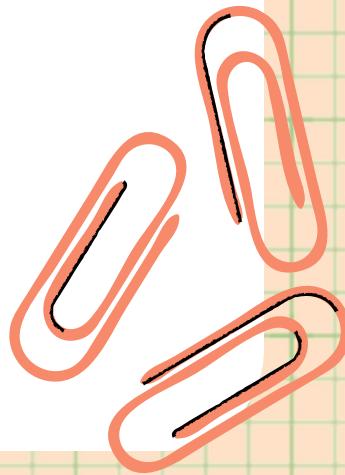


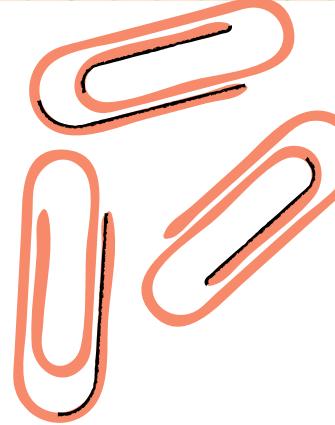
# IESDS

## Game

Middle up

	Left	Center
Up	13, 3	1, 4
Middle	4, 1	3, 3





# IESDS

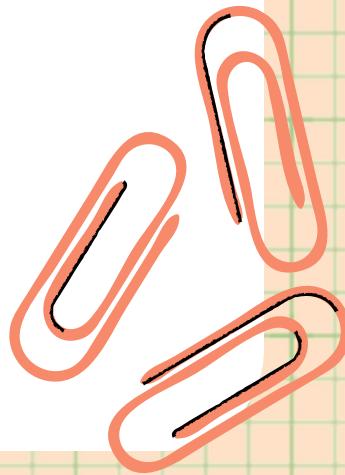
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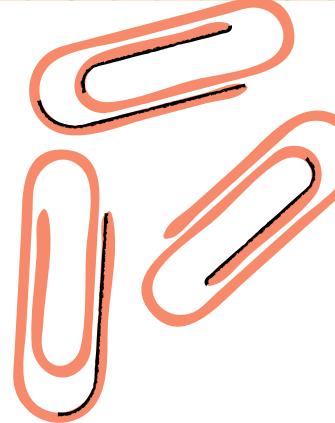
Middle up

Center

1, 4

3, 3



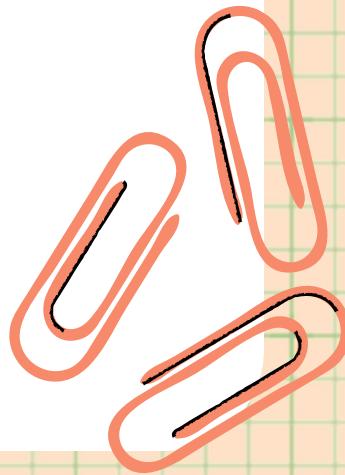


# IESDS Game

Middle

Center

3, 3



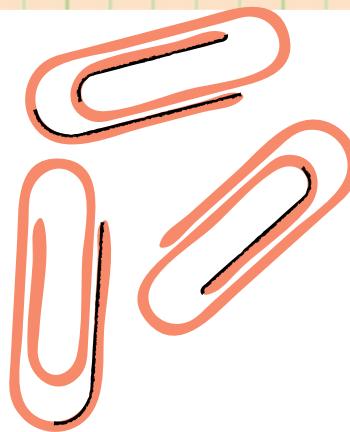
# Iterated Elimination of Strictly Dominated Strategies

- This process is called IESDS.
- If you ever see a strictly dominated strategy, eliminate it immediately.
  - Order does not matter.
  - If IESDS leads to a single outcome, you will arrive at that outcome whether you eliminate strategy #1 or strategy #2 first.

# Stag Hunt

- Two hunters go out to catch meat.
- There are two hares in the range and one stag. The hunters can each bring the equipment necessary to catch one type of animal.
- The stag has more meat than the hares combined, but both hunters must chase the stag to catch it.
- Hare hunters can catch all of their prey by themselves.

Player 1



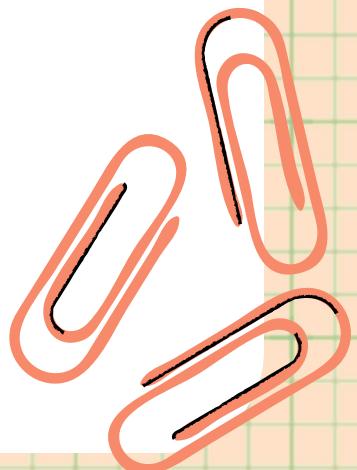
# Stag Hunt

## Player 2

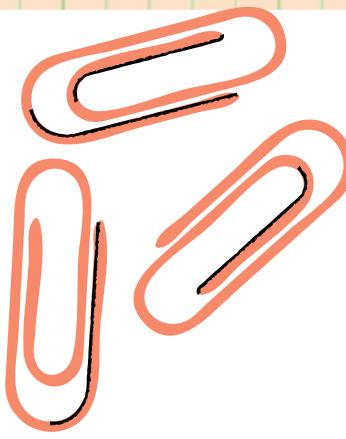
Stag      Hare

Stag  
Hare

	3 3	0 2
	2 0	1 1



Player 1



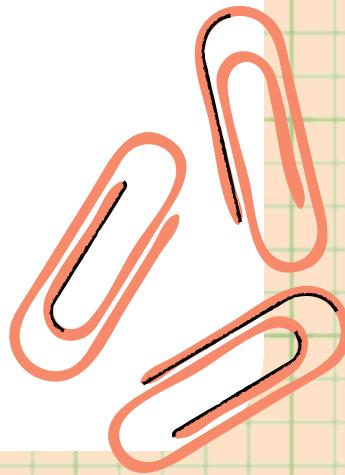
# Stag Hunt

## Player 2

Stag                      Hare

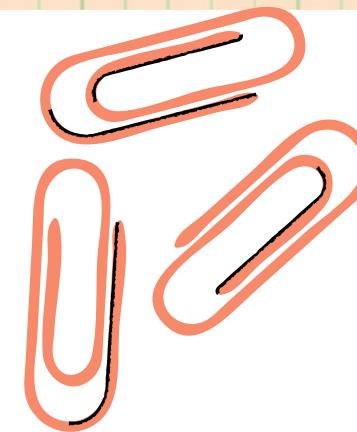
Stag  
Hare

	3 3
	2 0



Player 1

Stag  
Hare



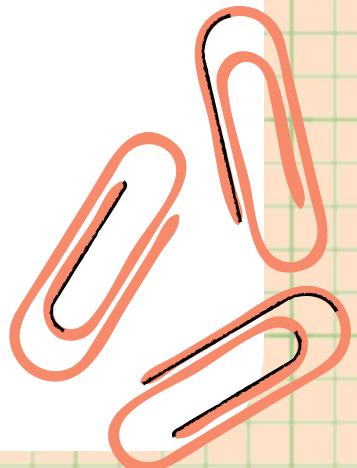
# Stag Hunt

## Player 2

Stag

Hare

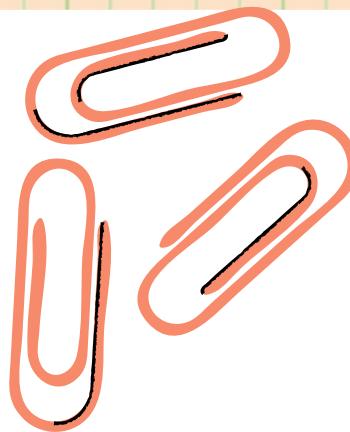
0	2
1	1



# Nash Equilibrium

- A Nash equilibrium is a set of strategies, one for each player, such that no player has incentive to change his or her strategy.
  - We only care about individual deviations, not group deviations.
  - Nash equilibria are inherently stable.
    - What you are doing is optimal given what I am doing and vice versa.
    - No regrets.

Player 1



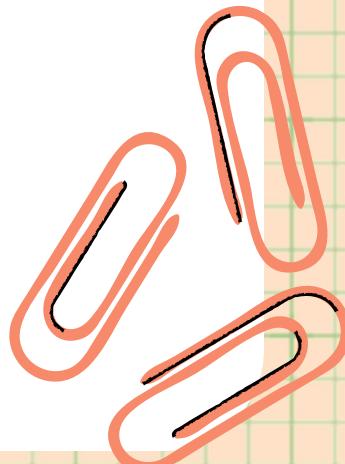
# Stag Hunt

## Player 2

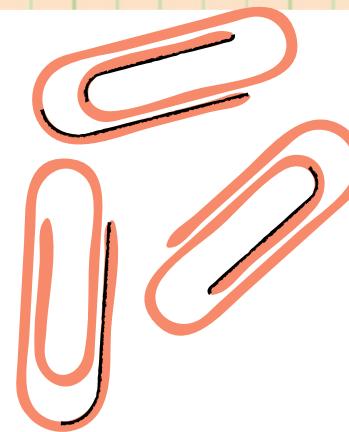
Stag      Hare

Stag  
Hare

	3 3	0 2
	2 0	1 1



Player 1



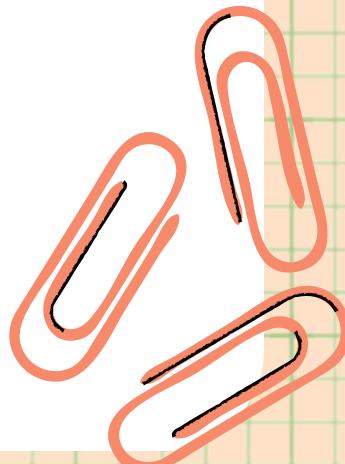
# Stag Hunt

## Player 2

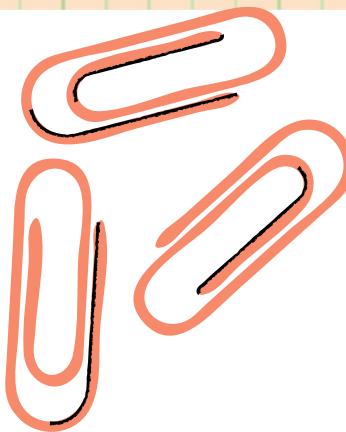
Stag      Hare

Stag  
Hare

	3 3	0 2
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Player 1

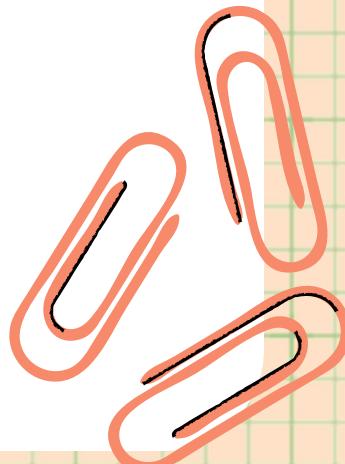


# Stag Hunt

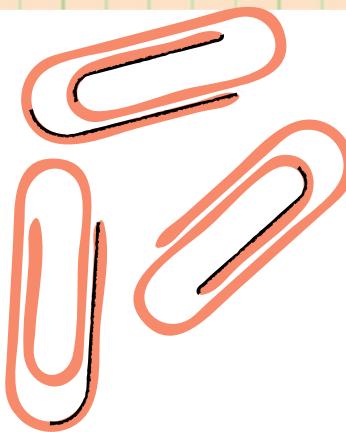
## Player 2

Stag                      Hare

	Stag	3 3	0 2
	Hare	2 0	1 1



Player 1



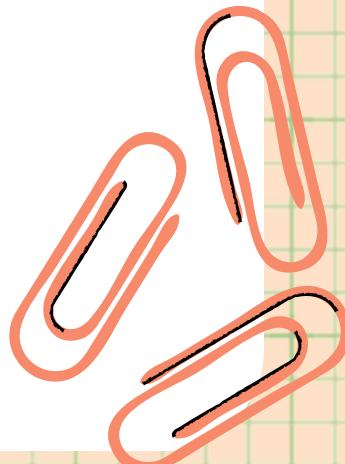
# Stag Hunt

## Player 2

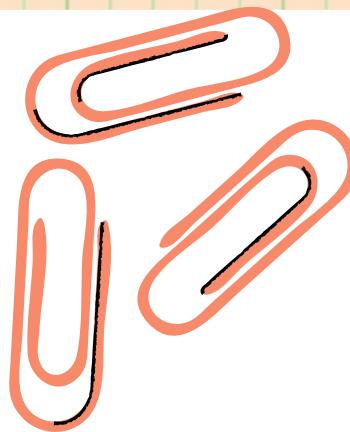
Stag      Hare

Stag  
Hare

	3 3	0 2
	2 0	1 1



Player 1



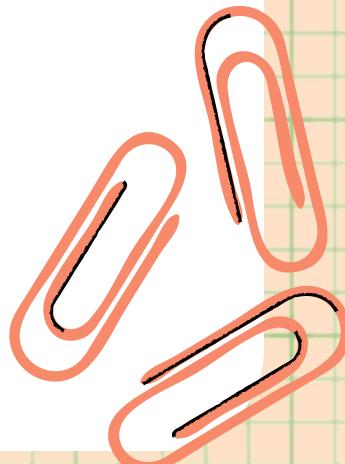
# Stag Hunt

## Player 2

Stag      Hare

Stag  
Hare

	3 3	0 2
	2 0	1 1



# Formal Definition

- A Nash equilibrium is a set of strategies, one for each player, such that no player has incentive to change his or her strategy given what the other players are doing.
  - A pure strategy Nash equilibrium is when players do not randomize between two or more strategies.

# But What Does that Mean?

- A Nash equilibrium is a law that no one would want to break even in the absence of an effective police force.
  - Pretend the police do not exist.
  - The government passes a law.
  - The law is a Nash equilibrium if everyone would want to follow it.

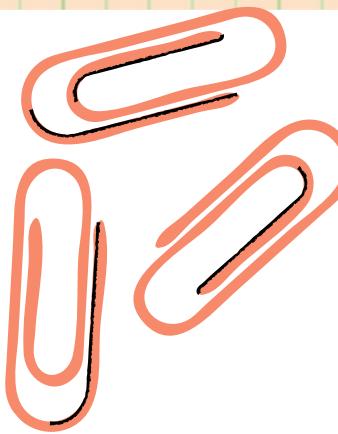
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# Think about Traffic

- In some situations, following stoplights is a Nash equilibrium.
  - Suppose two cars are driving at each other from perpendicular directions.
  - The stoplight is red for one of them and green for the other.
  - If the police could not ticket the drivers, would they want to break the law?

Player 1



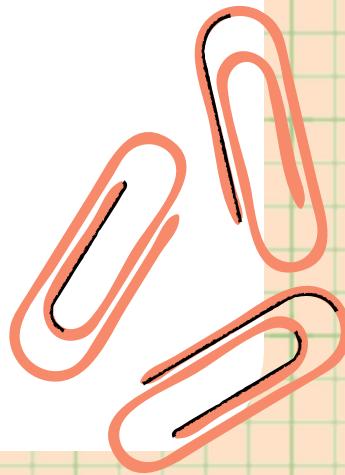
# Stoplight Game

Player 2

Go Stop

Go Stop

-5, -5	1, 0
0, 1	-1, -1

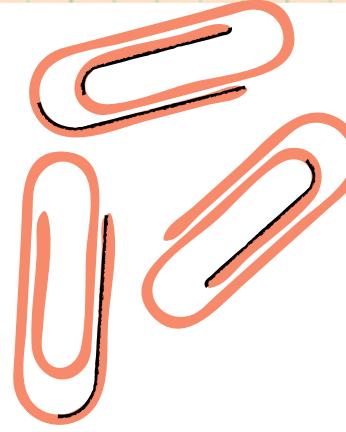
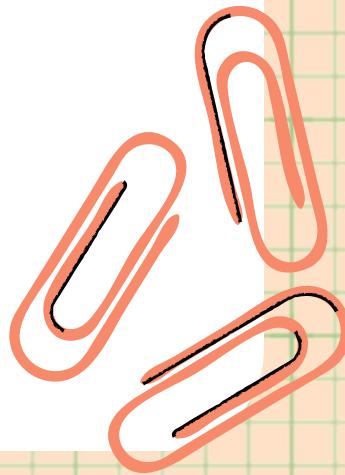


Player 1

# Stoplight Game

## Player 2

	Go	Stop
Go	-5, -5	1, 0
Stop	0, 1	-1, -1



# Nash Equilibrium, Intuitively

A Nash equilibrium is a law that no one would want to break even in the absence of an effective police force.

# The Game

- Two generals have three units.
- They simultaneously decide how many units they should allocate to an upcoming battle.
  - Or, either side can unilaterally opt out of the battle.
- The side with more units at the battle wins. If they have the same amount or at least one decides not to play, then the game ends in a draw

# The Game

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- They simultaneously decide how many units they should allocate to an upcoming battle.
  - Or, either side can unilaterally opt out of the battle.
- The side with more units at the battle wins. If they have the same amount or at least one decides not to play, then the game ends in a draw

Three Two One Pass

	Pass	One	Two	Three
Pass	0, 0	0, 0	0, 0	0, 0
One	0, 0	0, 0	-1, 1	-1, 1
Two	0, 0	1, -1	0, 0	-1, 1
Three	0, 0	1, -1	1, -1	0, 0

# Best Responses

- Given what all other players are doing, a strategy is a best response if and only if a player cannot gain more utility from switching to a different strategy.
- A game is in a Nash equilibrium if and only if all players are playing best responses to what the other players are doing



# Thank You