Introduction To Game Theory

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Midterm Evaluation

- Date: 15th of next month
- Format: Group presentation (~20 minutes) based on all topics covered to date
- Evaluation: Judged by Stamatics
- Coordinators; questions may be asked during the presentation

Participation is mandatory for ratification

Topics to cover

- STRICTLY DOMINANT MIXED STRATEGIES
- WEAK DOMINANCE
- INFINITELY MANY EQUILIBRIA
- THE ODD RULE AND THE FREE MONEY GAME
- SUBGAME PERFECT EQUILIBRIUM

Strict Dominance and Mixed Strategies

- If you find a mixture between two strategies (or more) strictly dominates another strategy, eliminate that last strategy immediately.
- Strictly dominated strategies are irrational, whether pure strategies or mixed strategies dominate them.

The game

Strictly Dominant Mixed Strategies

		Player 2 Left	Player 2 Right
Player 1	Up	3, -1	-1, 1
	Middle	0, 0	0, 0
	Down	-1, 2	2, -1

The game

Strictly Dominant Mixed Strategies

		Player 2	
		Left	Right
Player 1	Up(1/2)	3 -1	-1 1
	Middle	0 0	0 0
	Down(1/2)	-1 2	2 -1

The game

Strictly Dominant Mixed Strategies

Player 1 \ Player 2	Left	Right
Up(1/2)	3, -1	-1, 1
Middle	0, 0	0, 0
Down(1/2)	-1, 2	2, -1

The game

		Player 2	
		Left	Right
Player 1	Up	3 -1	-1 1
	Down	-1 2	2 -1

Solving for Player 1's Mixed Strategy

What is $EU_L = f(\sigma_D)$?

Some percentage of the time, player two gets -1. The rest of the time, she gets 2.

$$EU_L = \sigma_U(-1) + (1 - \sigma_U)(2)$$

	Left	Right
Up	3, -1	-1, 1
Down	-1, 2	2, -1

Solving for Player 1's Mixed Strategy

What is $EU_R = f(\sigma_U)$?

Some percentage of the time,player two gets -1.The rest of the time, she gets 2.

$$EU_L = \sigma_U(1) + (1 - \sigma_U)(-1)$$

	Left	Right
Up	3 -1	-1 1
Down	-1 2	2 -1

Solving for Player 1's Mixed Strategy

$$EU_L = EU_R$$

$$EU_L = \sigma_U(-1) + (1-\sigma_U)(2)$$

$$EU_R = \sigma_U(1) + (1-\sigma_U)(-1)$$

$$\Rightarrow \sigma_U(-1) + (1-\sigma_U)(2) = \sigma_U(1) + (1-\sigma_U)(-1)$$

$$\Rightarrow -\sigma_U + 2 - 2\sigma_U = \sigma_U - 1 + \sigma_U$$

$$\Rightarrow -3\sigma_U + 2 = 2\sigma_U - 1$$

$$\Rightarrow 3 = 5\sigma_U$$

$$\Rightarrow \sigma_U = 3/5$$

Solving for Player 2's Mixed Strategy

What is $EU_U = f(\sigma_L)$

Some percentage of the time,player two gets -1.The rest of the time, she gets 2.

$$EU_D = \sigma_L(-1) + (1 - \sigma_L)(2)$$

	Left		Right	
Up	3	-1	-1	1
Down	-1	2	2	-1

Solving for Player 2's Mixed Strategy

What is $EU_U = f\left(\sigma_L\right)$

Some percentage of the time, player two gets 3. The rest of the time, she gets -1.

$$EU_U = \sigma_L(3) + (1 - \sigma_L)(-1)$$

	Left	Right
Up	3 -1	-1 1
Down	-1 2	2 -1

Solving for Player 1's Mixed Strategy

$$EU_U = EU_D$$

$$EU_U = \sigma_L(3) + (1 - \sigma_L)(-1)$$

$$EU_D = \sigma_L(-1) + (1 - \sigma_L)(2)$$

$$\Rightarrow \sigma_L(3) + (1 - \sigma_L)(-1) = \sigma_L(-1) + (1 - \sigma_L)(2)$$

$$\Rightarrow 3\sigma_L - 1 + \sigma_L = -\sigma_L + 2 - 2\sigma_L$$

$$\Rightarrow 4\sigma_L - 1 = -3\sigma_L + 2$$

$$\Rightarrow 7\sigma_L = 3$$

$$\Rightarrow \sigma_L = 3/7$$

The game

Strictly Dominant Mixed Strategies

		Player 2	
		Left(3/7)	Right(4/7)
Player 1	Up(3/5)	3, -1	-1, 1
	Middle	0, 0	0, 0
	Down(2/5)	-1, 2	2, -1

Solving Simultaneous Move Games

- Goal: find all Nash equilibria.
- Iterated elimination of strictly dominated strategies never eliminates Nash equilibria.
- This is why we are free to use it, and you should eliminate any strictly dominated strategy immediately.

Weak Dominace

Player 2

Player 1

	Left	Right
Up	3 1	0 0
Down	2 2	2 2

Weak Dominace

		Player 2	
		Left	Right
Player 1	Up	3 1	0 0
	Down	2 2	2 2

Weak Dominace

		Player 2	
		Left	Right
Player 1	Up	3 1	0 0
	Down	2 2	2 2

Weak Dominance

- Left weakly dominates right for player two.
- That is, left is at least as good as right and sometimes better.

Weak Dominace

Player 2

Player 1		Left
	Up	3 1
	Down	2 2

Weak Dominace

Player 2

Left

Player 1

	Left
Up	3 1

Weak Dominace

		Player 2	
		Left	Right
Player 1	Up	3 1	0 0
	Down	2 2	2 2

Weak Dominance

- After using iterated elimination of weakly dominated strategies, any remaining Nash equilibrium must be a Nash equilibrium of the original game.
    - However, there may be more Nash equilibria.
    - You won’t know until you go back to the original game and check

Weak Dominace

		Player 2	
		Left	Right
Player 1	Up	3 1	0 0
	Down	2 2	2 2

Infinitely many equilibria

		Player 2 Left	Player 2 Right
Player 1	Up	3 1	12 0
	Down	3 -2	2 -5

Infinitely many equilibria

		Player 2	
		Left	Right
Player 1	Up	3 1	12 0
	Down	3 -2	2 -5

Infinitely many equilibria

Player 2

Player 1		Left	Right
	Up	3 1	12 0
	Down	3 -2	2 -5

Infinitely many equilibria

Player 2

Player 1

	Left
Up	3, 1
Down	3, -2

Infinitely many equilibria

Player 2

Player 1		Left
	Up	3 1
	Down	3 -2

Infinitely many equilibria

Player 2

Player 1

	Left
Up(p)	3 1
Down(1-p)	3 -2

The Odd Rule

- Almost all games have an odd number of equilibria.
  - Examples: Prisoner's dilemma (1), matching pennies (1), mixed strategy algorithm game (1), battle of the sexes (3), stag hunt (3), stoplight game (3).
- Games rarely have an infinite number or an even number of equilibria.
  - Weak dominance is usually to blame.

The Free Money Game

- Today, I am offering you and your best friend free money.
- All you have to do is unanimously vote to approve the resolution.
- Votes are simultaneous and blind.

Free Money

Player 1 \ Player 2	Yes	No
Yes	1 1	0 0
No	0 0	0 0

Free Money

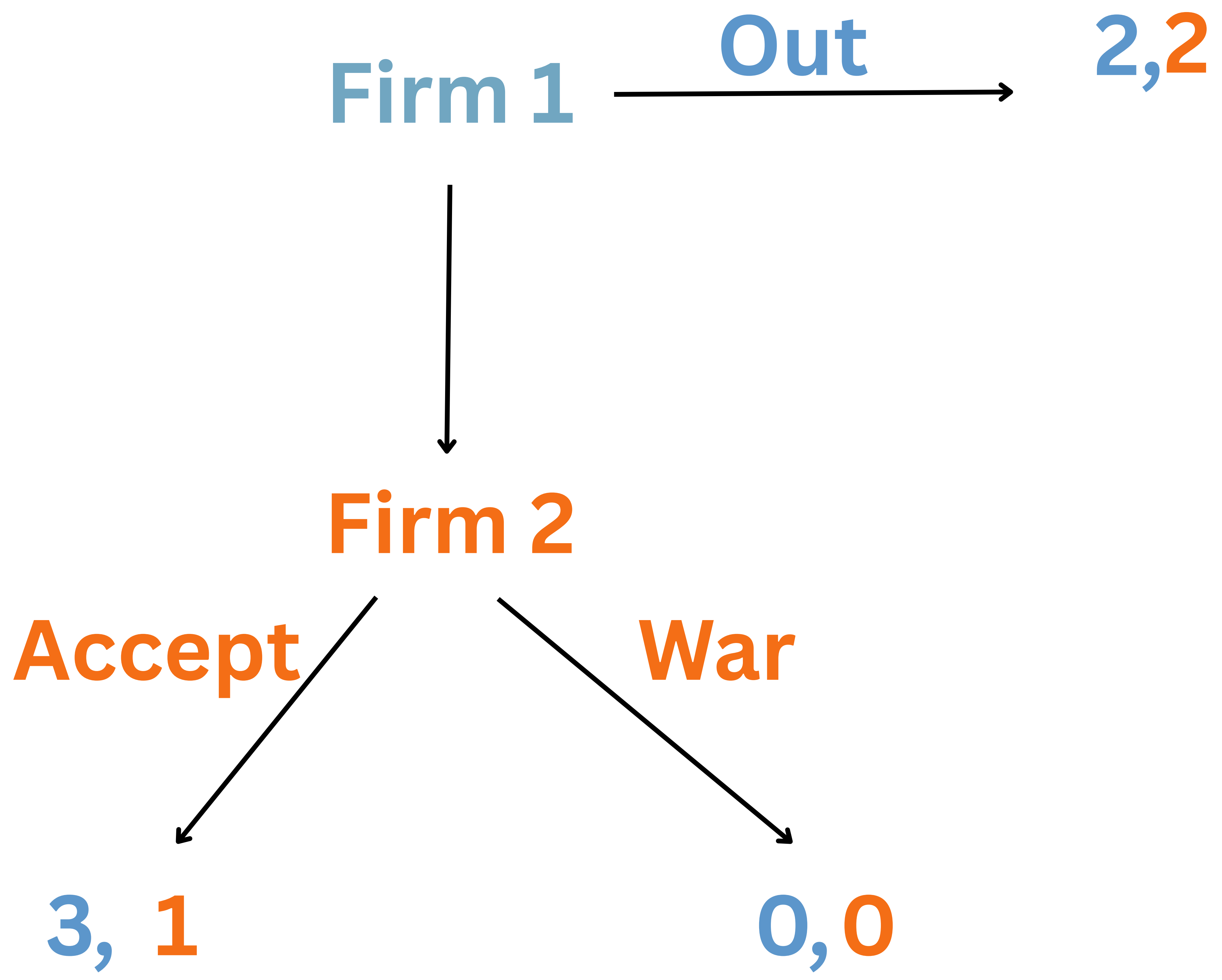
		Player 2: Yes	Player 2: No
Player 1	Yes	1 1	0 0
	No	0 0	0 0

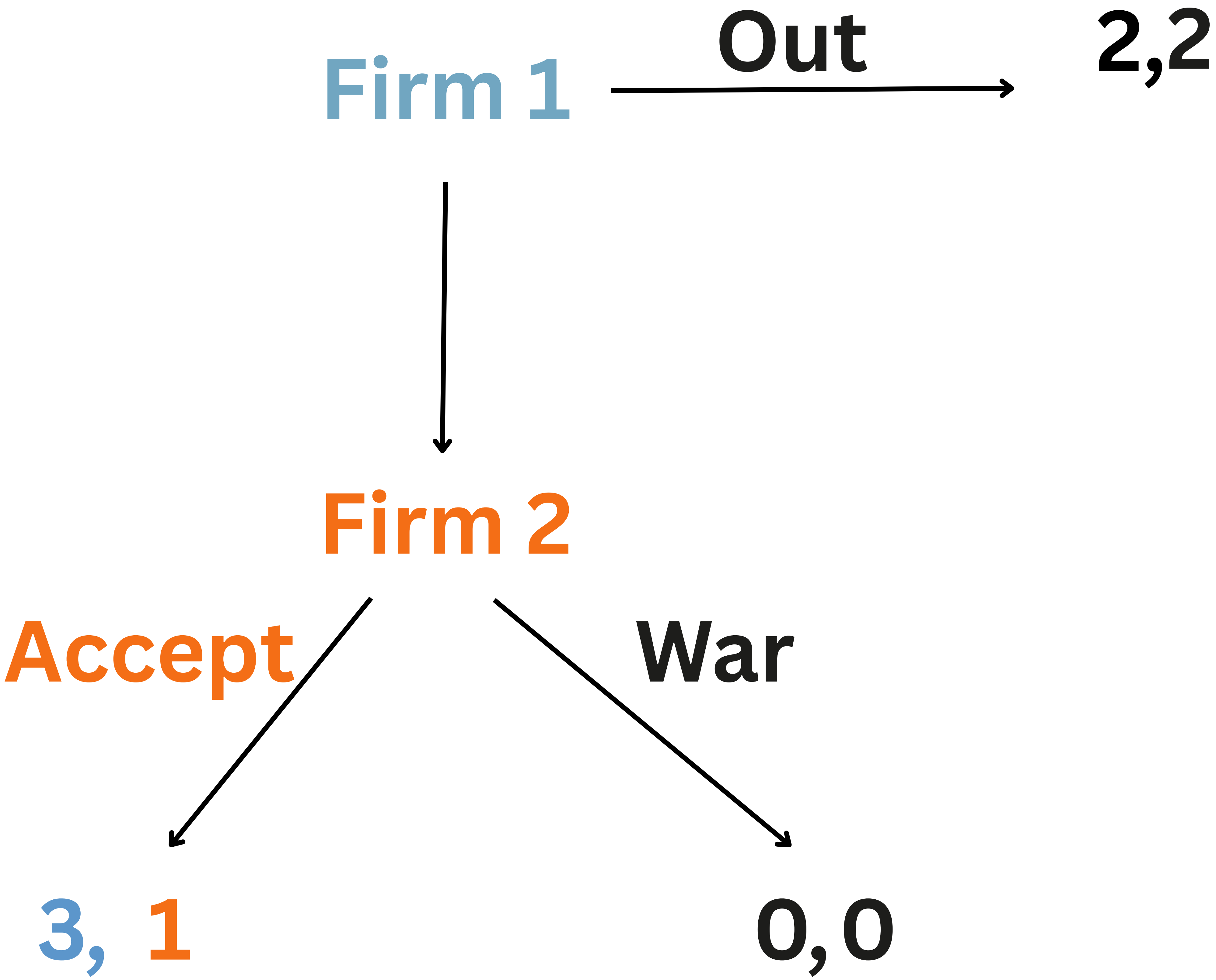
Free Money

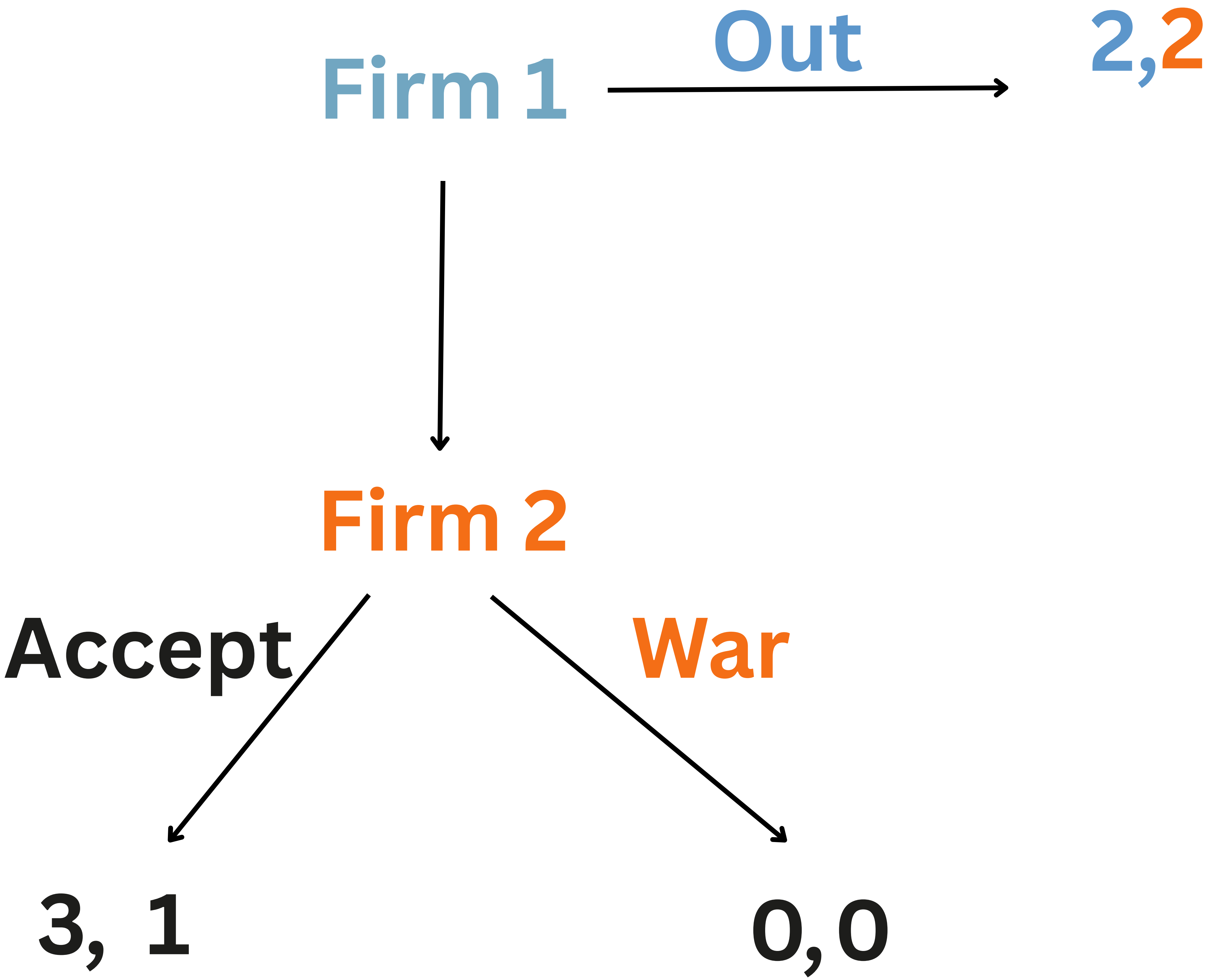
		Player 2	
		Yes	No
Player 1	Yes	1 1	0 0
	No	0 0	0 0

The Game

- A firm is deciding whether to enter the market, which another firm currently has a monopoly over.
- If the firm enters, the monopolist chooses whether to accept or declare a price war.
    - The firm only wants to enter if the monopolist won't engage in a price war.
    - A price war is unprofitable for the monopolist.


Firm 1
Out
2,2
Firm 2
Accept
War
3, 1
0, 0


Firm 1
Out
2,2
Firm 2
Accept
War
3, 1
0,0


Firm 1
Out
2,2
Firm 2
Accept
War
3, 1
0,0

Firm Entry Game

Firm 2

Firm 1

	Accept	War
In	3 1	0 0
Out	2 2	2 2

Thank You