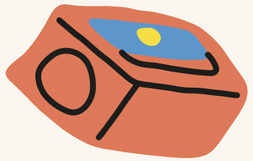

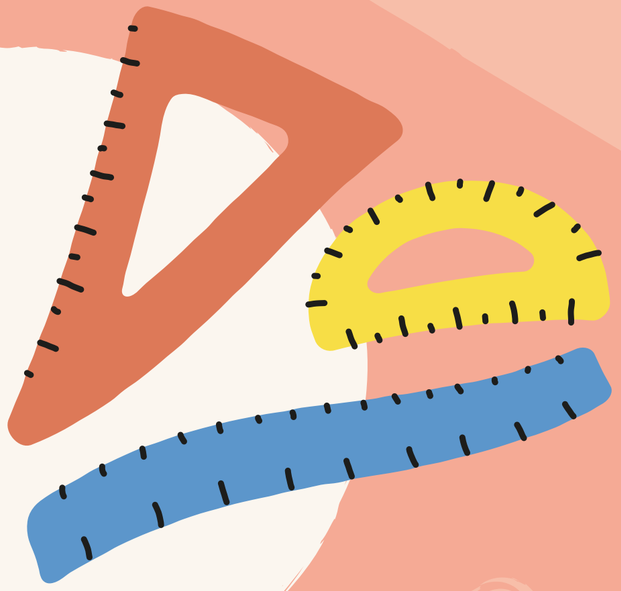

Introduction To Game Theory

MAINAK SARKAR

Stamatics

Topics to cover

- Soccer Penalty Games
- Establishing Causation
- Comparative Statics

Penalty Games

- Striker can kick left or kick right.
- Goalie can dive left or dive right. Because the kick is so fast, she guess at the same time the striker kicks.
- Assume:
  - Superhuman goalie.
  - Kicker has perfect accuracy to his left side but only hits the net with probability X on the right.

Penalty Game

Goalie

Striker	Dive Left	Dive Right
Left	0 0	X -X
Right	1 -1	0 0

Penalty Game

Solving for Striker's Mixed Strategy

What is $EU_{DL} = f(\sigma_{KL})$?

- Some percentage of the time, goalie gets 0.
- The rest of the time, she gets -1.

Striker \ Goalie	Dive Left	Dive Right
Left	0 0	X -X
Right	1 -1	0 0

$$EU_{DL} = \sigma_{KL}(0) + (1 - \sigma_{KL})(-1)$$

Penalty Game

Solving for Striker's Mixed Strategy

What is $EU_{DR} = f(\sigma_{KR})$?

- Some percentage of the time, goalie gets -X.
- The rest of the time, she gets 0.

Striker \ Goalie	Dive Left	Dive Right
Left	0 0	X -X
Right	1 -1	0 0

$$EU_{DR} = \sigma_{KL}(-X) + (1 - \sigma_{KL})(0)$$

Solving for Strikers Mixed Strategy

$$EU_{DL} = EU_{DR}$$

$$EU_{DL} = \sigma_{KL}(0) + (1 - \sigma_{KL})(-1)$$

$$EU_{DR} = \sigma_{KL}(-X) + (1 - \sigma_{KL})(0)$$

$$\Rightarrow \sigma_{KL}(0) + (1 - \sigma_{KL})(-1) = \sigma_{KL}(-X) + (1 - \sigma_{KL})(0)$$

$$\Rightarrow \sigma_{KL} = \frac{1}{(1 + X)}$$

Penalty Game

Solving for Goalie's Mixed Strategy

What is $EU_{KL} = f(\sigma_{DL})$?

- Some percentage of the time, striker gets 0.
- The rest of the time, he gets X.

Striker \ Goalie	Dive Left	Dive Right
Left	0 0	X -X
Right	1 -1	0 0

$$EU_{KL} = \sigma_{DL}(0) + (1 - \sigma_{DL})(X)$$

Penalty Game

Solving for Goalie's Mixed Strategy

What is $EU_{KR} = f(\sigma_{DL})$?

- Some percentage of the time, striker gets 1.
- The rest of the time, he gets 0.

Striker \ Goalie	Dive Left	Dive Right
Left	0 0	x -x
Right	1 -1	0 0

$$EU_{KR} = \sigma_{DL}(1) + (1 - \sigma_{KL})(0)$$

Solving for Goalie's Mixed Strategy

$$EU_{DL} = EU_{DR}$$

$$EU_{KL} = \sigma_{DL}(0) + (1 - \sigma_{DL})(X)$$

$$EU_{KR} = \sigma_{DL}(1) + (1 - \sigma_{KL})(0)$$

$$\Rightarrow \sigma_{DL}(0) + (1 - \sigma_{DL})(X) = \sigma_{DL}(1) + (1 - \sigma_{DL})(0)$$

$$\Rightarrow \sigma_{KL} = \frac{X}{(1 + X)}$$

Penalty Game

Goalie

Striker	Dive Left X/(1+X)	Dive Right 1/(1+X)
Left 1/(1+X)	0 0	X -X
Right X/(1+X)	1 -1	0 0

Establishing Causation

INPUTS

Players
Information
Actions
Payoffs

Game Theory

OUTPUTS

Strategies Played
Outcomes Realized
Payoffs

Establishing Causation

INPUTS

Players
Information
Actions
High Costs

Game Theory

OUTPUTS

Outputs
Strategies
Played
Peace
Realized
Payoffs

Establishing Causation

INPUTS

Players
Information
Actions
Low Costs

Game Theory

OUTPUTS

Outputs
Strategies Played
War
Realized Payoffs

Penalty Game

Goalie

Striker	Dive Left	Dive Right
Left	0 0	X -X
Right	1 -1	0 0

The Question

As the striker’s accuracy improves to his left side, does he aim to the left more or less frequently?

Comparative Statics

- We answer this type of question by using comparative statics.
- Definition: How a game’s equilibrium behaviors and outcomes change as a function of the game’s parameters.
- – How do the outputs change as a function of inputs?

Comparative Statics

- How to find them:
- Solve for the game’s equilibria.
- Calculate the element of interest.
- Take the derivative of that element of interest with respect to the given exogenous variable.
- Check whether it is increasing or decreasing as a function of that variable.

Penalty Game

Goalie

Striker	Dive Left X/(1+X)	Dive Right 1/(1+X)
Left 1/(1+X)	0 0	X -X
Right X/(1+X)	1 -1	0 0

Taking Derivatives

- Pr(kick left) = 1/(1 + X)
- [1/(1 + X)]' = [(1)'(1 + X) – (1)(1 + X)']/(1 + X)^2
- [(0)(1 + X) – (1)(1)]/(1 + X)^2
- -1/(1 + X)^2
- – Numerator is negative.
- – Since X > 0, denominator is always positive.
    - - Therefore, the derivative is always negative. The function is always decreasing. So the striker aims left less frequently as X increases.

What?

- Does the result seem strange?
  - – After all, the better the striker is at aiming left, the less frequently he aims that way.
- Intuition: the game is strategic.
  - – If the striker aimed right more frequently (his strong side), then the goalie could easily counteract him by diving right.
  - – The striker therefore compensates for his weakness by aiming left more often.
  -

Thank You