Introduction to GAME THEORY

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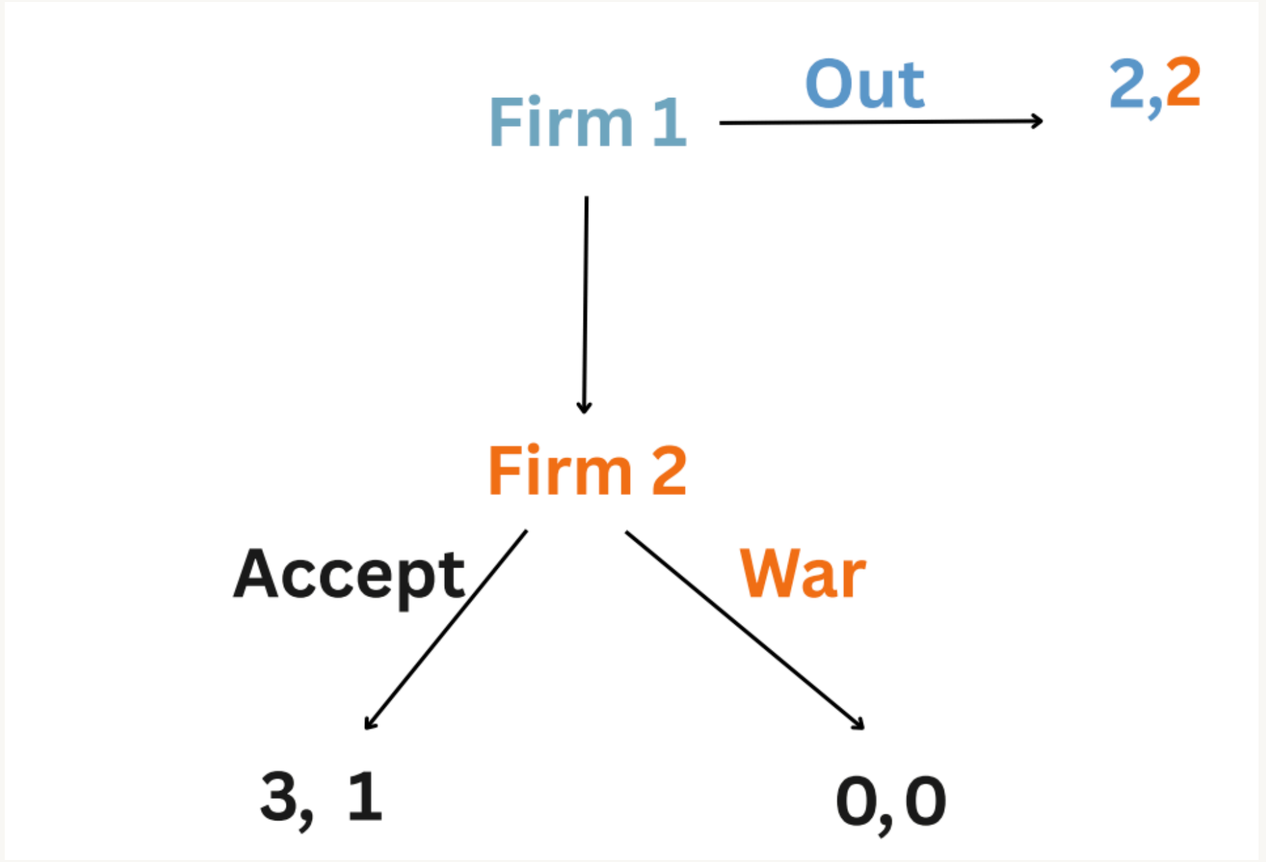
The Game

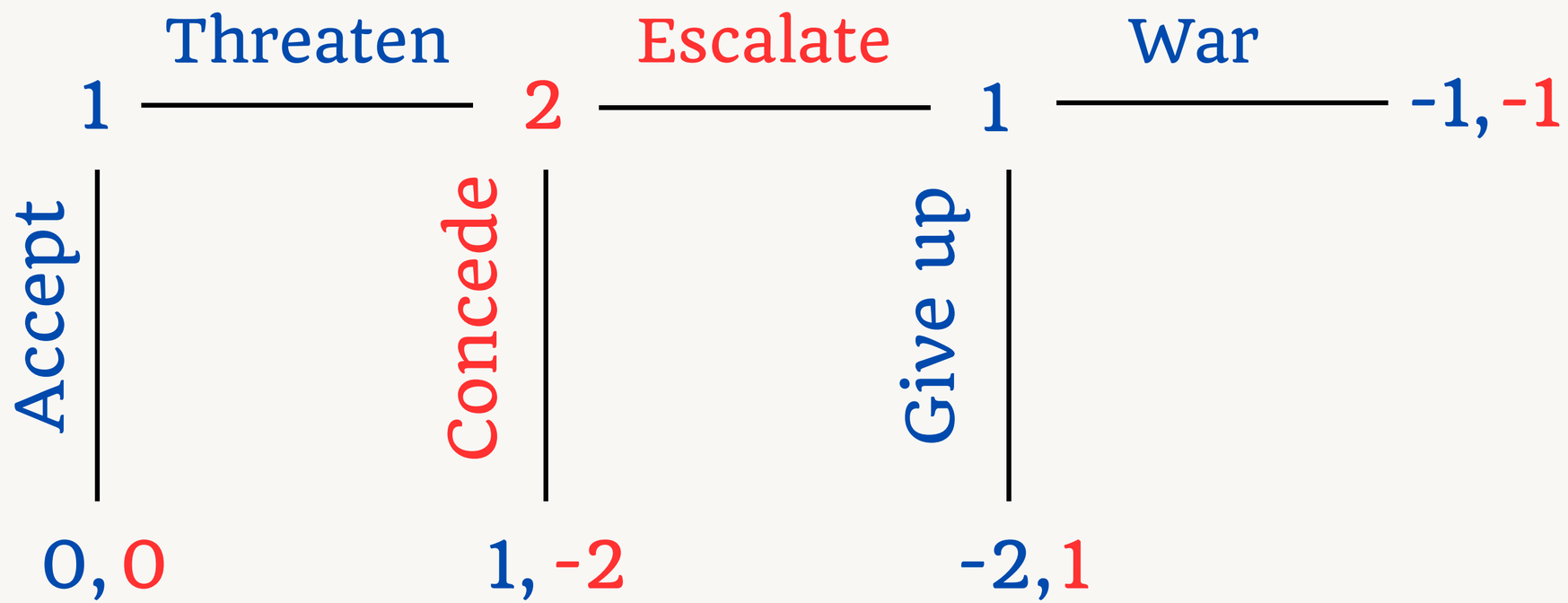
- A firm is deciding whether to enter the market, which another firm currently has a monopoly over.
- If the firm enters, the monopolist chooses whether to accept or declare a price war.
  - The firm only wants to enter if the monopolist won't engage in a price war.
  - A price war is unprofitable for the monopolist.

Remember this game!

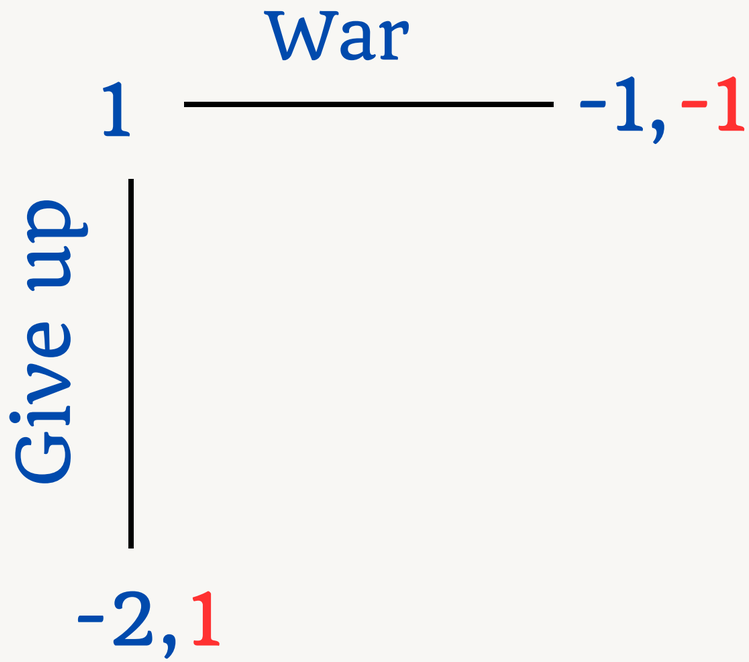

Firm 1
Out
2,2
Firm 2
Accept
War
3, 1
0,0

SUBGAME PERFECT EQUILIBRIUM

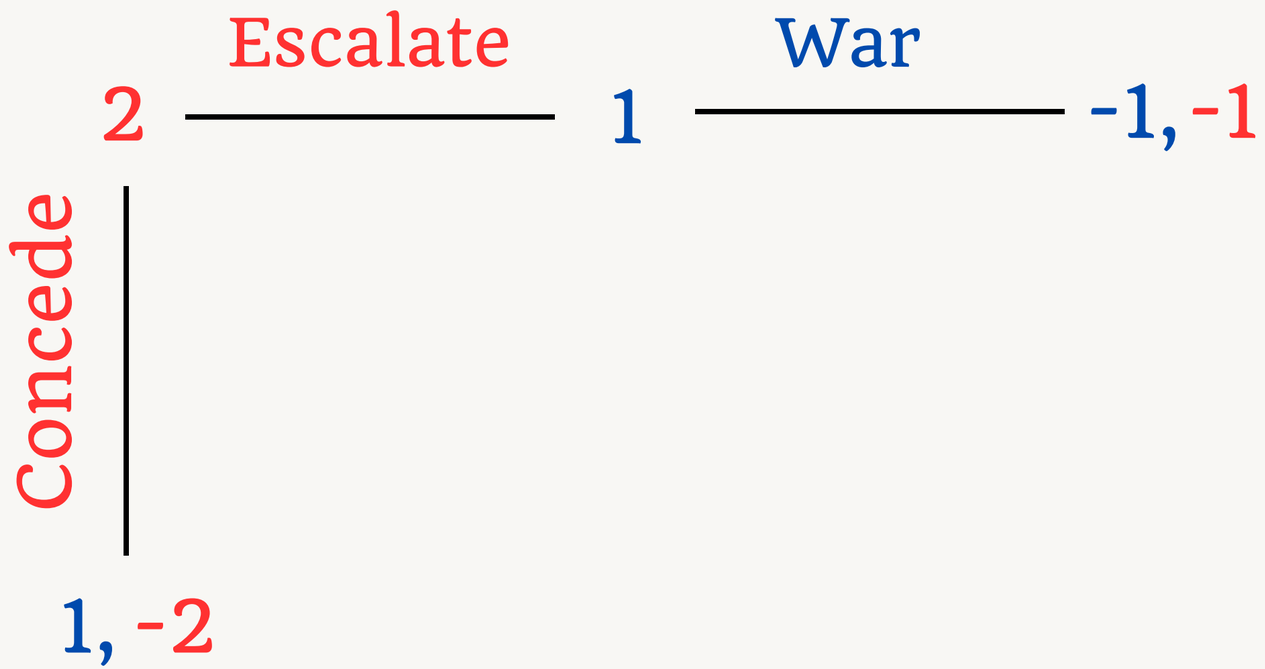
ESCALATION GAME



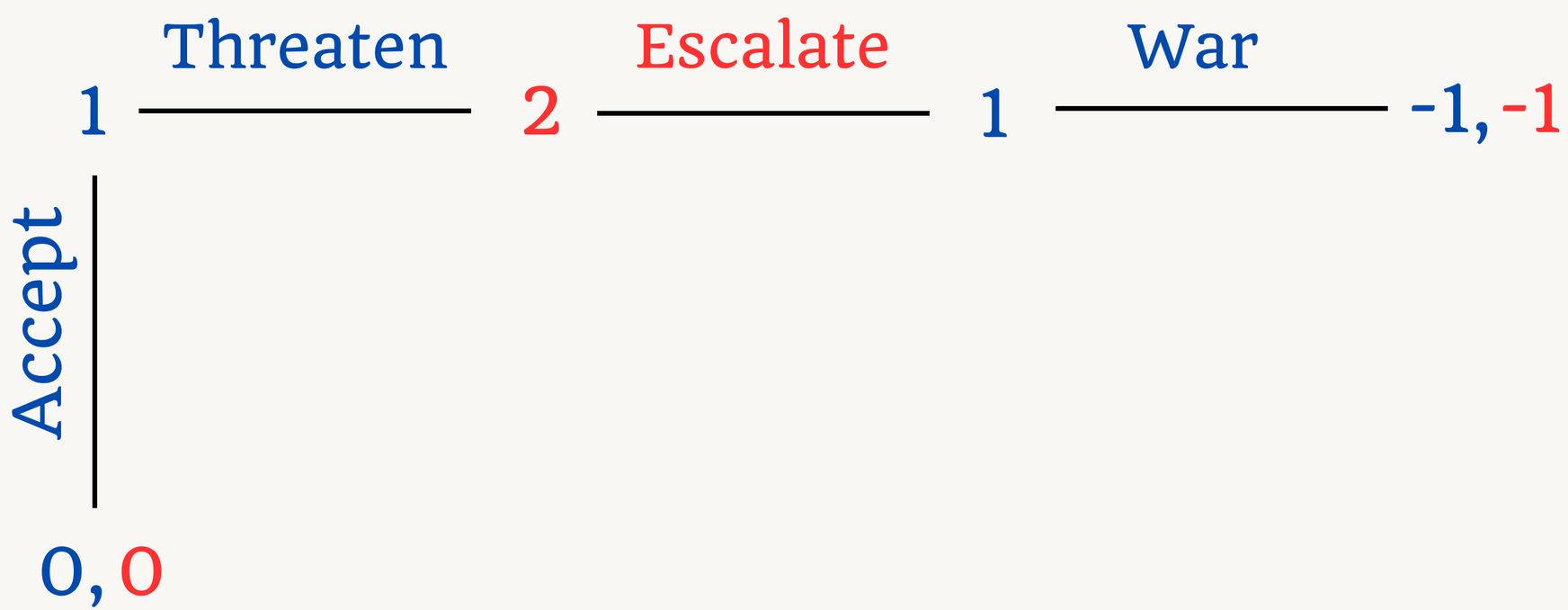
ESCALATION GAME



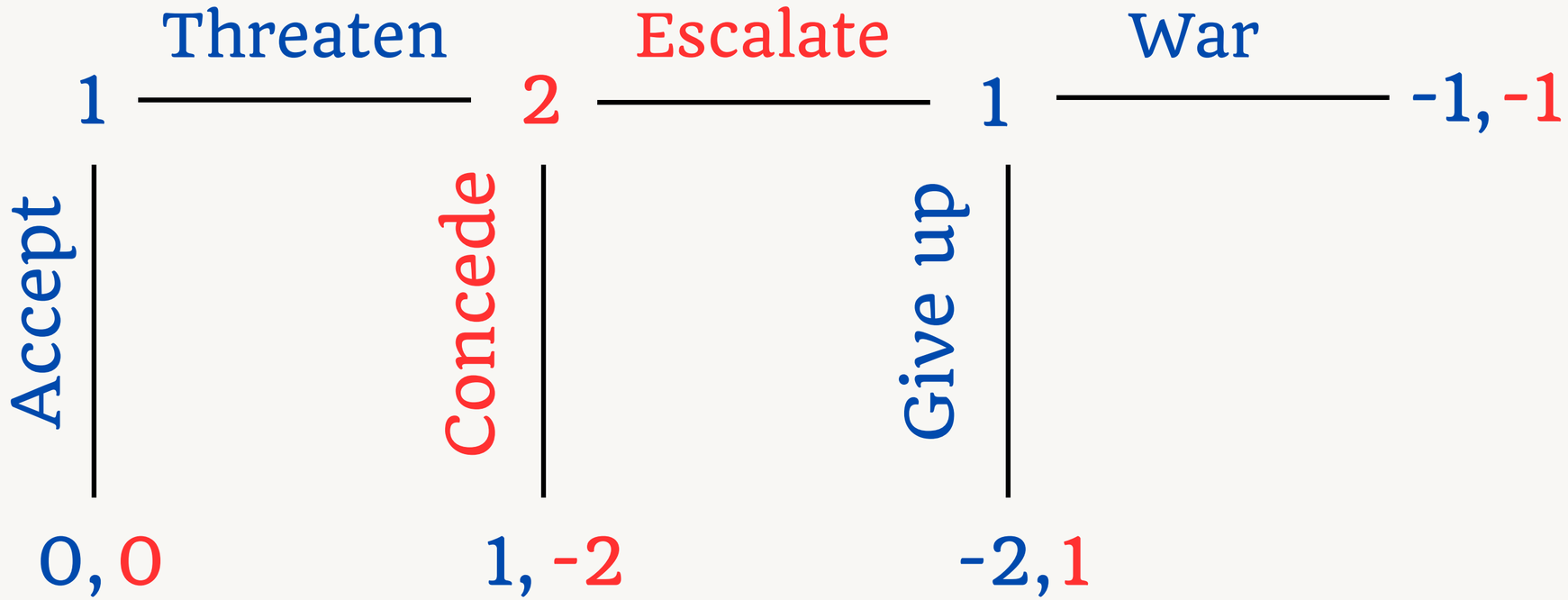
ESCALATION GAME



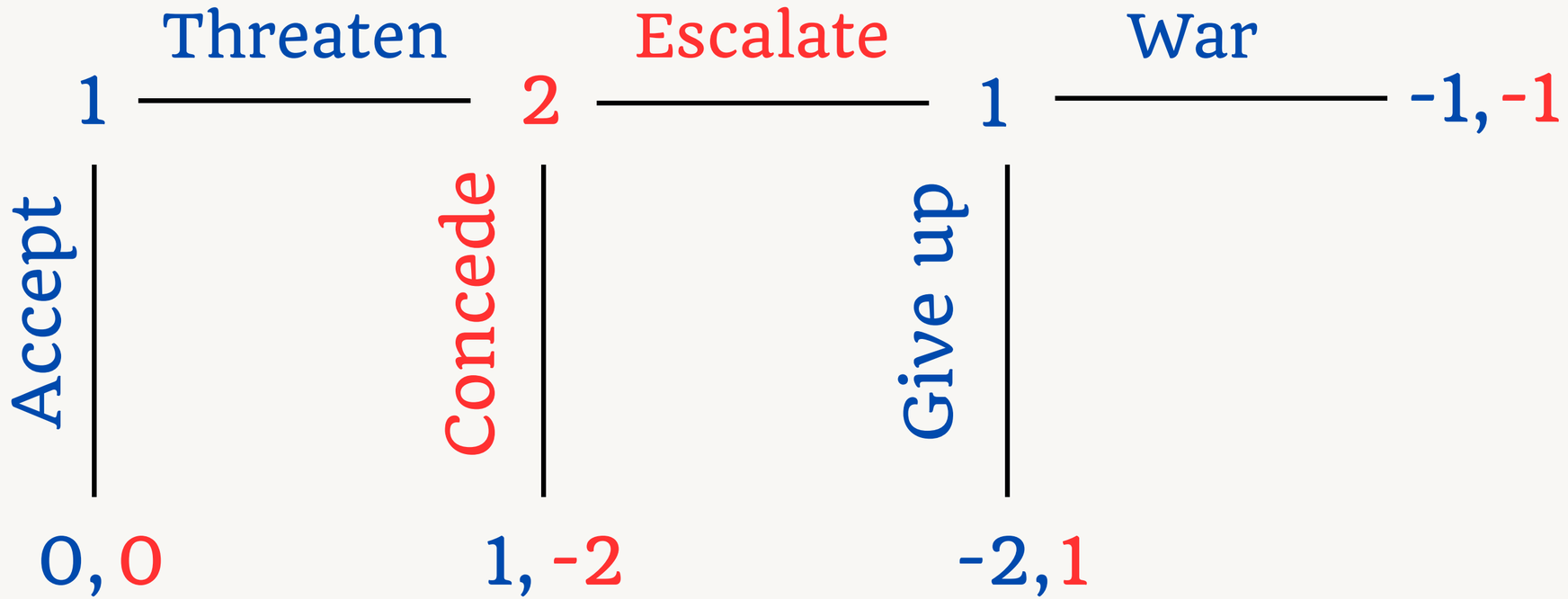
ESCALATION GAME



ESCALATION GAME



<Accept, War>; <Escalate>



Do not do this

<Accept>

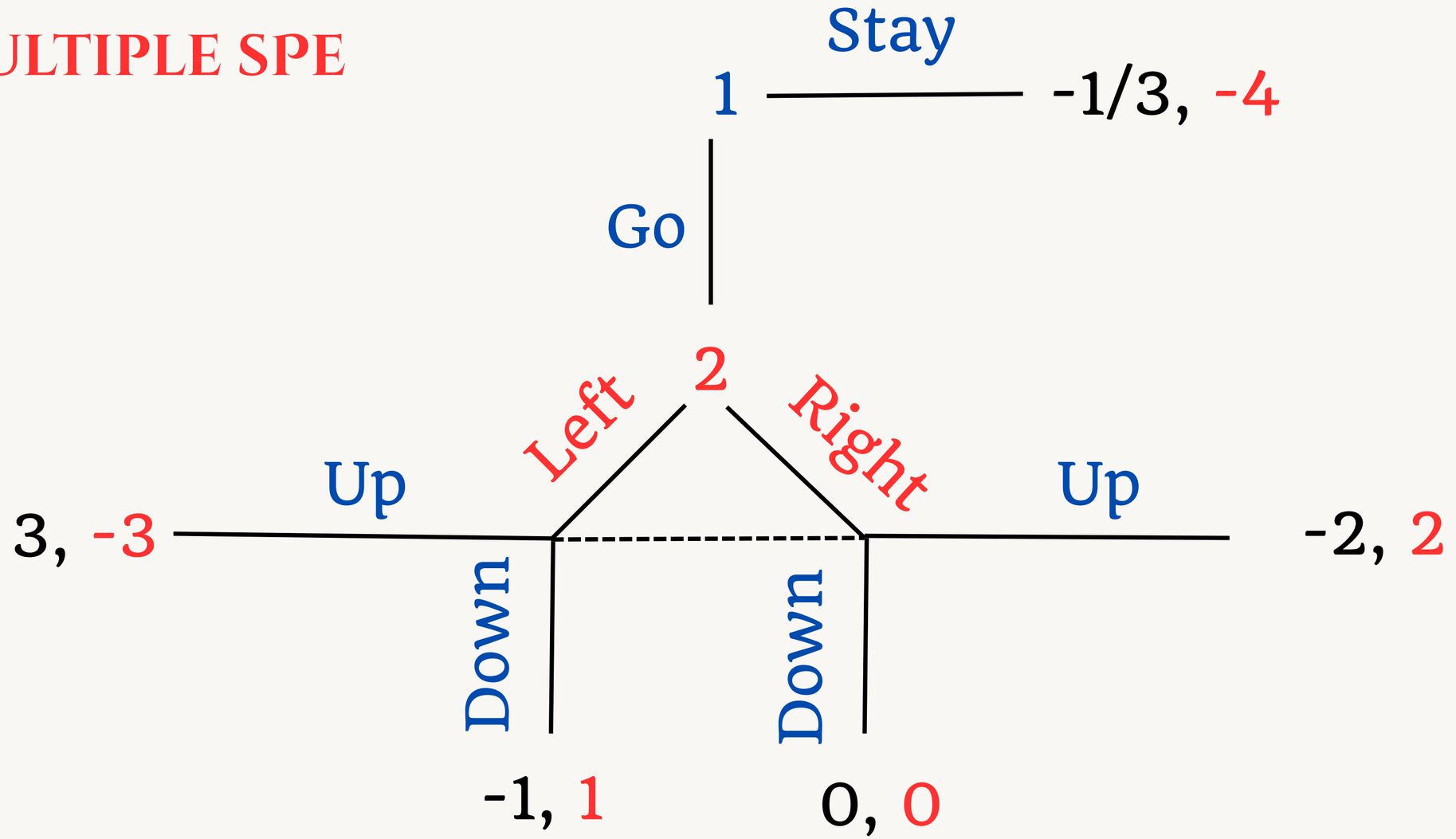
SUBGAME PERFECT EQULLIBRIUM

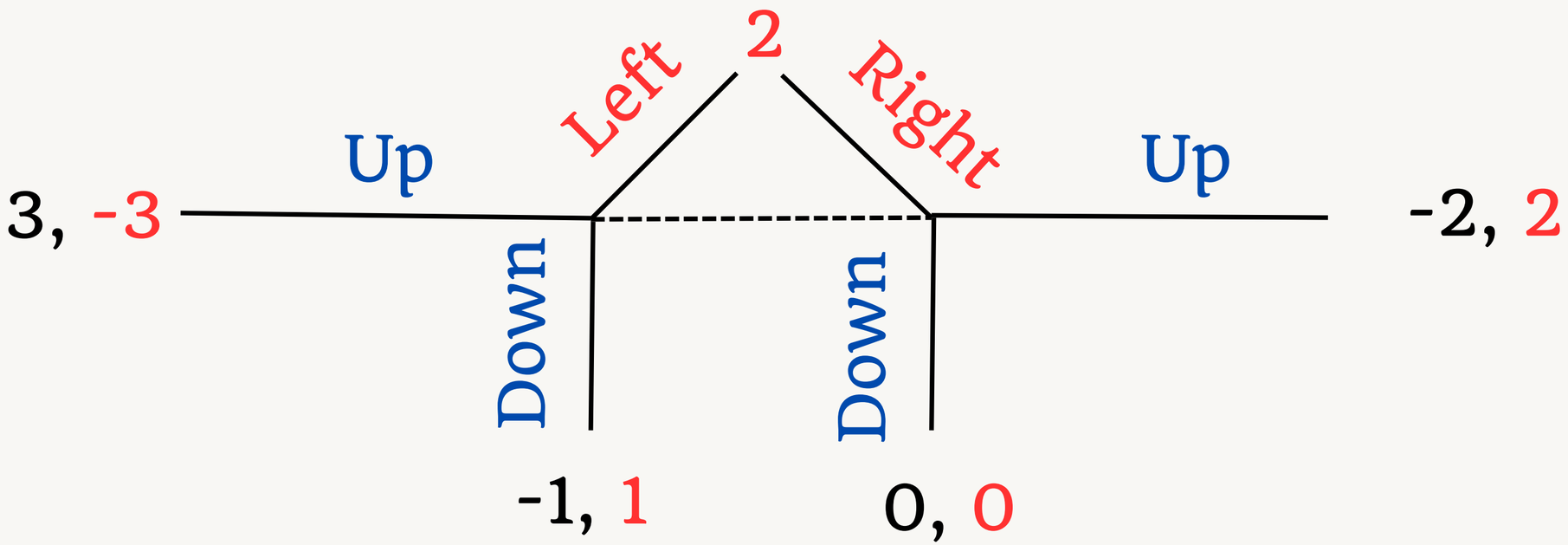
- A subgame perfect equilibrium is a complete and contingent plan of action.

  -It must state what happens on and off the equilibrium path of play

Multiple Subgame Perfect Equilibrium

MULTIPLE SPE




3, -3
Up
Left
2
Right
Up
-2, 2
Down
Down
-1, 1
0, 0

Player 2

Player 1	Left (Pl)	Right (Pr)
Up (Pu)	3 -3	-2 2
Down (Pd)	-1 1	0 0

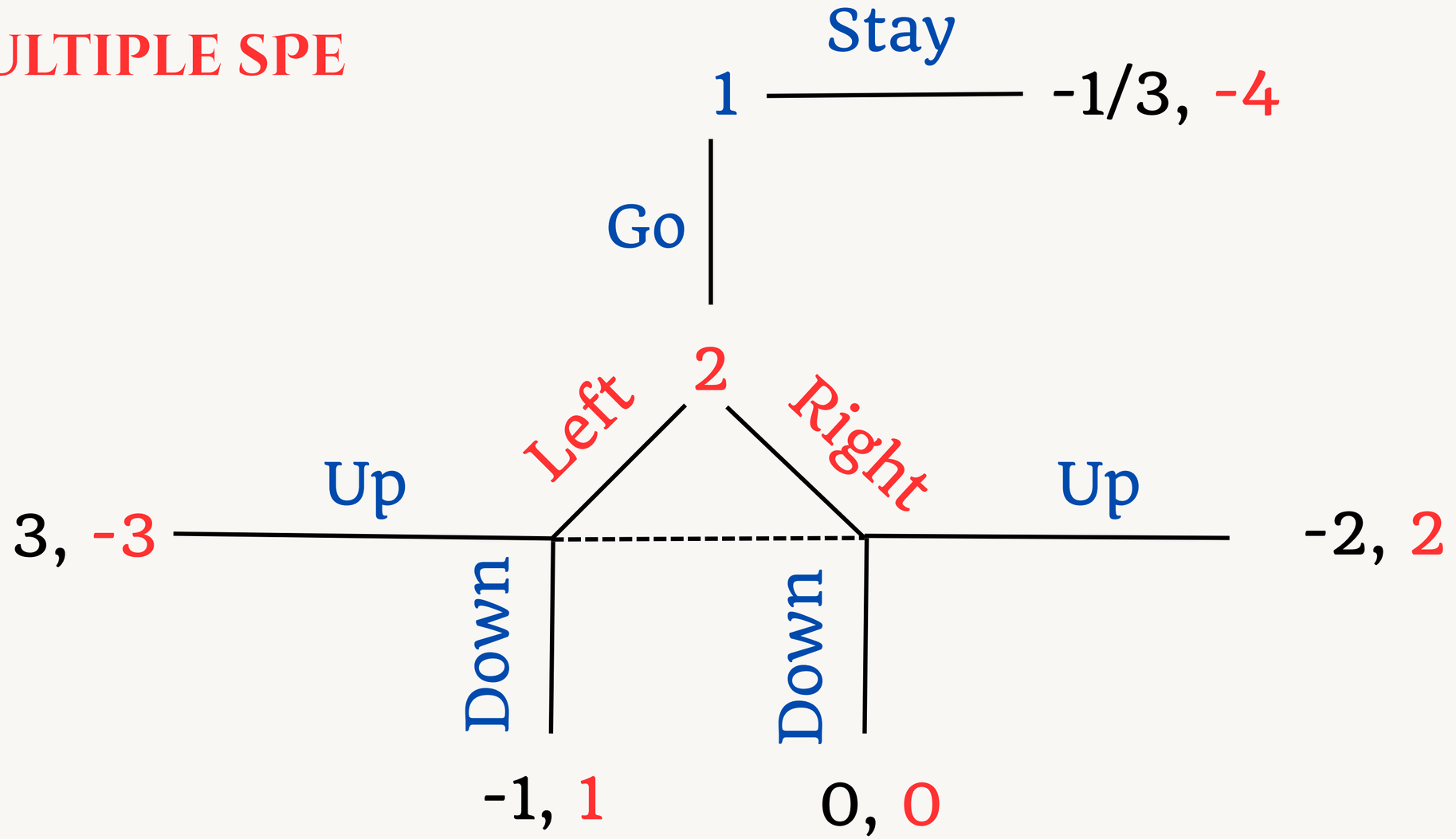
Zero-Sum
Mixed
Strategy game

Player 2

Player 1	Left (1/3)	Right (2/3)
Up (1/6)	3 -3	-2 2
Down (5/6)	-1 1	0 0

Zero-Sum
Mixed
Strategy game

MULTIPLE SPE



Player 2

Player 1	Left (1/3)	Right (2/3)
Up (1/6)	3 -3	-2 2
Down (5/6)	-1 1	0 0

Zero-Sum
Mixed
Strategy game

Player 2

Player 1	Left (1/3)	Right (2/3)
Up (1/6)	3 -3 1/18	-2 2 2/18
Down (5/6)	-1 1 5/18	0 0 10/18

Zero-Sum
Mixed
Strategy game

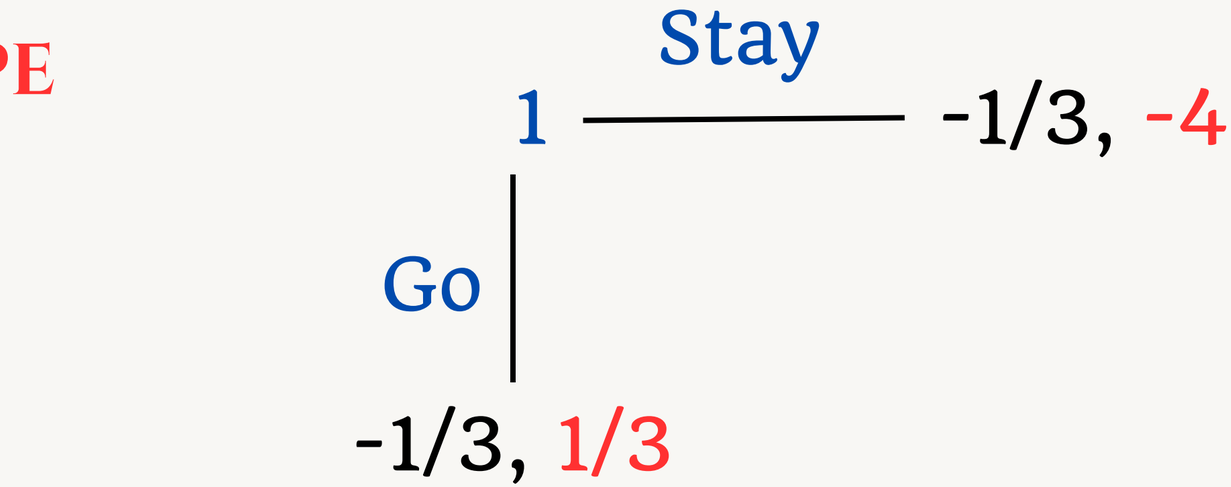
Zero-Sum
Mixed
Strategy game

Player 2

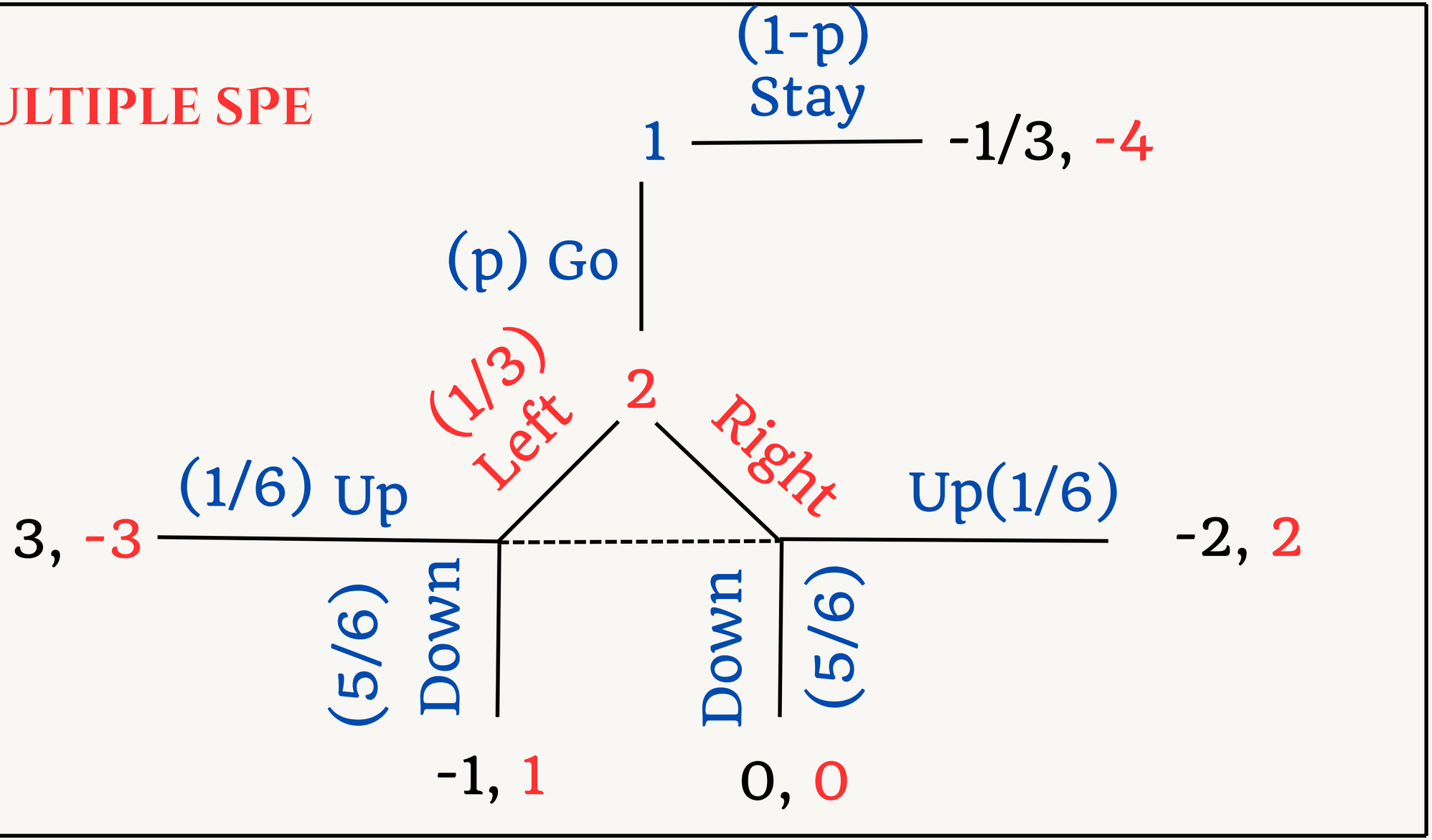
Player 1	Left (1/3)	Right (2/3)
Up (1/6)	3 * 1/18	-2 * 2/18
Down (5/6)	-1 * 5/18	0 * 10/18

Sum = -1/3

MULTIPLE SPE



MULTIPLE SPE



GAMES WITH STAGES

SIMPLE GAME

1. Play prisoner’s dilemma. Earn payoffs.
2. Then play stag hunt. Earn payoffs.
3. Finish with battle of the sexes. Earn payoffs.
4. Total payoffs are the sum of each game.

LOTS OF POSSIBILITIES

1. Prisoner's dilemma
2. battle of sexes

1. Stag hunt
2. Stag hunt
3. Stag hunt
4. Stag hunt
5. Stag hunt

1. Stag hunt
2. Stag hunt
3. Battle of sexes
4. Matching Pennies

COMMON CONNECTIONS

1. Simultaneous move games in every stage
2. Payoffs from one stage do not directly affect payoffs from another stage
3. Players know each other's previous stage
4. Incredibly hard to draw out.

THEOREM 1

- In the final stage (subgame), players must play a Nash equilibrium in all subgame perfect equilibria.

LOTS OF POSSIBILITIES

1. Prisoner's dilemma
2. battle of sexes

1. Stag hunt
2. Stag hunt
3. Stag hunt
4. Stag hunt
5. Stag hunt

1. Stag hunt
2. Stag hunt
3. Battle of sexes
4. Matching Pennies

THEOREM 1

- In the final stage (subgame), players must play a Nash equilibrium in all subgame perfect equilibria.

Why?

- All payoffs from before are locked in.
- Must maximize in last period.
- Nash equilibria are the only strategy sets that have both players maximize

THEOREM 2

- Playing Nash equilibria in every period is a subgame perfect equilibrium.

LOTS OF POSSIBILITIES

1. Prisoner's dilemma
2. battle of sexes

1. Stag hunt
2. Stag hunt
3. Stag hunt
4. Stag hunt
5. Stag hunt

1. Stag hunt
2. Stag hunt
3. Battle of sexes
4. Matching Pennies

THEOREM 2

- Playing Nash equilibria in every period is a subgame perfect equilibrium.

Why?

- If we are only playing Nash equilibria, yesterday's strategies don't affect today's strategies
- So we must always be maximizing for today
- Must be Nash equilibria

THEOREM 2

- Playing Nash equilibria in every period is a subgame perfect equilibrium.
- Not the only subgame perfect equilibria.
  - More equilibria possible when players use strategies that respond to previous play
  - Makes cooperation possible where it wouldn't be otherwise.

Free money!

You \ Your friend	Yes	No
Yes	1 1	0 0
no	0 0	0 0

??

Free money!

You \ Your friend	Yes	No
Yes	1 1	0 0
no	0 0	0 0

Ans - Punishment strategies

MID - EVALUATION