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# Combined size and shape optimization of structures with a new meta-heuristic algorithm



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#### ABSTRACT

In this study, a new meta-heuristic algorithm called teaching-learning-based optimization (TLBO) is used for the size and shape optimization of structures. The TLBO algorithm is based on the effect of the influence of a teacher on the output of learners in a class. The cross-sectional areas of the bar element and the nodal coordinates of the structural system are the design variables for size and shape optimization, respectively. Displacement, allowable stress and the Euler buckling stress are taken as the constraint for the problem considered. Some truss structures are designed by using this new algorithm to show the efficiency of the TLBO algorithm. The results obtained from this study are compared with those reported in the literature. It is concluded that the TLBO algorithm presented in this study can be effectively used in combined size and shape optimization of the structures.

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# 1. Introduction

Optimization of structure is to obtain a set of design variables that make the weight of structure minimum. In general, design variables are the cross-sectional areas, nodal coordinates and a determined topology. Using design variables, three types of optimization can be performed in the design of structures. These are size optimization, shape optimization and the topology optimization. Among these optimization types, size optimization is preferred to find the minimum weight of the structure under certain constraints. If the other optimization types are used with the size optimization the design variable space will be expanded. The solution of the problem becomes more difficult when the limit of design variables space increase. To overcome this difficulty different optimization algorithms have been presented in the literature. Wang et al. [1] presented a study for truss structure with combined size and shape optimization. A similar study has been made by Gil and Andreu [2] and Kaveh and Kalatjari [3]. Svanberg [4] and Zhou and Xia [5] optimized the truss structures for optimum geometry. Gholizadef et al. [6] made a shape optimization of structures using harmony search. Hasançebi and Erbatur [7], Rahami et al. [8], Tang et al. [9], and Rajan [10] optimized truss structures using genetic algorithm with sizing, geometry and topology design variables. A

master thesis is made by Felix [11] for the shape optimization of trusses. Han [12] presented a shape optimization for general two dimensional structures. Kaveh and Laknejadi [13] made a study for layout optimization of truss structures.

There are different type of optimization problems presented by researches, such as size optimization, size and shape optimization or size, shape and topology optimization. Ahrari and Atai [14] presented a novel truss optimizer based on the principles of the state-of-the-art Evolution Strategies by taking into account the size and shape optimization. Miguel et al. [15] employs the Firefly Algorithm (FA) in the simultaneous optimization of size, shape, and topology in truss structures. They applied the FA to 2D and 3D truss structures. Miguel and Miguel [30] made a study on shape and size optimization of truss structures considering dynamic constraints through modern metaheuristic algorithms (Harmony Search (HS) and Firefly Algorithm (FA)). They used the multiple natural frequency of truss structure as a constraint of an optimization problem. Dede et al. [16] minimized the weight of the truss structures by using adopted Genetic Algorithm (GA). They presented value and binary encodings types in genetic algorithm for discrete and continuous optimization problems and developed a new strategy called as Restricted Range Approach (RRA). Sönmez [17] studied on truss structures taking into account the size optimization with Artificial Bee Colony algorithm (ABC). Sadollah et al. [18] presented a study on size optimization with discrete design variables of truss structures using the Mine Blast Algorithm (MBA). Kaveh and Talatahari [19] made a study on size optimization of space trusses using a Hybrid Big Bang-Big Crunch algorithm

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```
teacher = min ( f ( population_old ) )
difference = rand*( teacher -TF*mean(population_old) )
population_new = population_old + difference

for i = 1:size(population)
    if f(population(i)_new) > f(population(i)_old
        population(i)_new = population(i)_old
    end if
end for
```

Fig. 1. Pseudocode of teacher phase in TLBO.

```
\label{eq:for} \begin{aligned} & \textbf{for} \ i = 1 \colon Pn \\ & \quad randomly \ select \ student_j, \ i \neq j \\ & \quad \textbf{if} \ f(student_i) < f(student_j) \\ & \quad \text{difference} = student_i - student_j \\ & \quad \textbf{else} \\ & \quad \text{difference} = student_j - student_i \\ & \quad \textbf{end} \ \textbf{if} \\ & \quad student_{new-i} = student_i + r.difference \\ & \quad \textbf{end} \ \textbf{for} \end{aligned}
```

Fig. 2. Pseudocode of student phase in TLBO.

(HBB-BC). In their study, HBB-BC is compared to Big Bang-Big Crunch (BB-BC), Genetic Algorithm (GA), Ant Colony Optimization (ABC), Particle Swarm Optimization (PSO), and Harmony Search (HS).

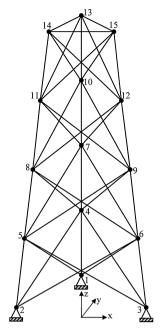


Fig. 4. 39-bar 3D truss structure.

The aim of this study is to find an optimal design for 2D and 3D truss structure with a new meta-heuristic optimization algorithm called Teaching-learning-based optimization (TLBO) under the some constraints. These constraints are the displacements, stresses and Euler buckling stress. In the optimization process, size and shape optimization is taken into account while the topology of the truss structure is fixed. Cross-sectional areas of the bar element and nodal coordinates of structural system are selected as design variables for size and shape optimization, respectively. Five truss structures are designed for numerical example. The results obtained from this study are compared to those of the literature to show the efficiency of the TLBO algorithm.

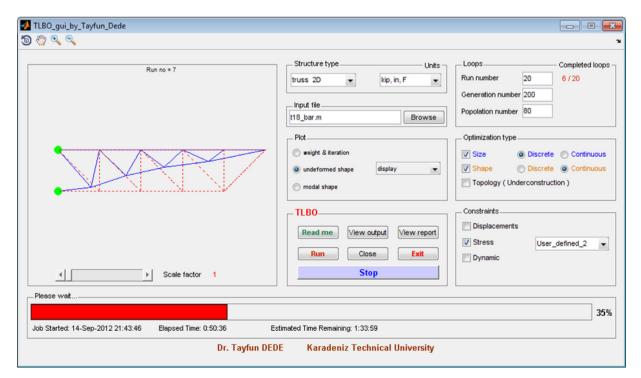


Fig. 3. Data interface of the developed program.

**Table 1**Problem definition for the 39-bar 3D truss structure.

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Material Young's modulus Density Allowable stress	$E$ = 210 GPa $ ho$ = 7800 kg/m $^3$ $\pm$ 240 MPa		
Geometry Coordinate (m) 1 2 3 13 14	$X$ $0$ $-\sqrt{3}/2$ $+\sqrt{3}/2$ $0$ $-0.42/\sqrt{3}$ $+0.42/\sqrt{3}$	Y 1 -0.5 -0.5 0.28 -0.14 -0.14	Z 0 0 0 4 4 4
Design variables Size variables $A_1$ $A_2$ $A_3$ $A_4$ $A_5$ Shape variables $Y_4, Z_4, Y_7, Z_7, Y_{10}, Z_{10}$	Elements defined by nodes (1,4), (2,5), (3,6) (4,7), (5,8), (6,9) (7,10), (8,11), (9,12) (10,13), (11,14), (12,15) Rest of the elements		
Load Node 13 14 15	X 0 0 0	Y 10 kN 10 kN 10 kN	Z 0 0 0
Constraints Displacement Node 13 Euler buckling Stress	$X$ $\sigma_{eb} \leq \frac{-K_e E_e A_e}{L_e^2}$ $\sigma_e \leq \text{allowable stress}$	Y 4 mm	<i>Z</i> -

### 2. The TLBO algorithm

Teaching-learning-based optimization is a new meta-heuristic optimization algorithm based on the natural phenomenon of teaching and learning has been presented by Rao et al. [20,21]. It was firstly used for constrained mechanical design optimization problems. After that, the TLBO used for several purpose of optimization problems, such as minimum weight of truss structure, size optimization, estimation of energy consumption.

TLBO optimization algorithm requires only common controlling parameters like population size and number of generations

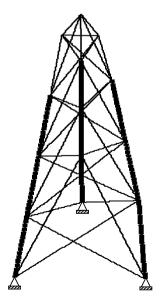


Fig. 5. Optimized geometry of 39-bar 3D truss structure.

**Table 2**Optimal size and shape results for the 39-bar 3D truss structure.

Design variables	Wang et al. [1]	TLBO
Size		
Area (cm²)		
$A_1$	11.01	11.9650
$A_2$	8.63	11.1457
$A_3$	6.69	7.8762
$A_4$	4.11	2.7013
$A_5$	4.37	2.4058
Shape		
Coordinate (in.)		
$Y_4$	0.805	0.8996
$Z_4$	1.186	1.3507
Y <sub>7</sub>	0.654	0.6917
$Z_7$	2.204	2.3122
Y <sub>10</sub>	0.466	0.4825
Z <sub>10</sub>	3.092	3.3031
Weight (kg)	203.18	154.13

for its working and does not require the determination of any algorithm specific controlling parameters such as mutation ratio and crossover ration as in GA. If the TLBO is used in the optimization process, it requires only population size and number of generation as common controlling parameters.

Like the other optimization algorithm, TLBO also uses a randomly generated initial population. This population consists of an even number of students which are any solutions in TLBO. These students consist of a number of design variables ( $X_i$ ).

A new population is obtained as a result of two phases called teacher phase and student phase in TLBO algorithm. In the teacher phase, the students having minimum objective function (f) value is assigned as a teacher.

The other students in the current population are modified as neighborhood of the teacher. This modification is carried out by using the following equations.

$$student_i = \begin{bmatrix} X_{i,1} & X_{i,2} & \cdots & X_{i,Dn} \end{bmatrix}, \quad i = 1, 2, ..., Pn$$
 (2.1)

$$mean = \lceil mean(X_1) \quad mean(X_2) \quad \cdots \quad (X_{Dn}) \rceil$$
 (2.2)

$$student_{new\ i} = student_i + r.*(teacher - TF*mean)$$
 (2.3)

where, Dn is number of design variables, Pn is size of population, r is a vector created randomly in the range [0,1] and TF is the teaching factor and TF can be either 1 or 2. It should be noted that the size of r must be equal to size of the student for the scalar multiplication given in Eq. (2.3). If the objective function of modified student is greater than the objective function of old student, the new student is not taken into account. The teaching phase is carried out by the hope that the level of students will be updated to the level of teacher. A pseudocode of teacher phase in TLBO is given in Fig. 1.

In student phase, all modified students are compared with each other to increase their knowledge. Implementation of this comparison is given in Fig. 2.

As noted in the teacher phase, the new student obtained from student phase is not taken into account if its objective function is not better. At the end of the last iteration, the student whose

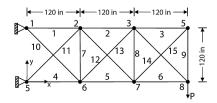


Fig. 6. 15-bar 2D truss structure.

**Table 3**Problem definition for the 15-bar 2D truss structure.

Material	E=10 <sup>4</sup> ksi	
Young's modulus		
Density	$\rho = 0.1  \text{lb/in.}^3$	
Allowable stress	$\pm 25 \mathrm{ksi}$	
Design variables		
Size variables $A_i = 1, 2, \dots, 15$		
	20, 0.270, 0.287, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488,	1.764, 2.142, 2.697, 2.800, 3.131,
	25, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180} in. <sup>2</sup>	
Shape variables		
	$100 \text{ in.} \le X_2 = X_6 \le 140 \text{ in.}$	
	$220 \text{ in.} \le X_3 = X_7 \le 260 \text{ in.}$	
	$100 \text{ in.} \le Y_2 \le 140 \text{ in.}$	
	$100 \text{ in.} \le Y_3 \le 140 \text{ in.}$	
	$50 \text{ in.} \le Y_4 \le 90 \text{ in.}$	
	$-20 \text{ in.} \le Y_6 \le 20 \text{ in.}$	
	$-20 \text{ in.} \le Y_7 \le 20 \text{ in.}$	
	$20 \text{ in.} \le Y_8 \le 60 \text{ in.}$	
Load	V	V
Node	X	Y
8	0	-10 kips
Constraints		
Displacement	<del>-</del>	
Euler buckling	_	
Stress	$\sigma_e \leq$ allowable stress	

objective function is minimum is the best solution of optimization problem. Extensive details about the TLBO algorithm and its implementation had been presented by Rao et al. [20].

# 3. Objective function of problem

The aim of the structural design in this study is to obtain a minimum weight of the truss structure under consideration. For this aim, the objective function for the structure considered is formulated as

$$\min W = \sum_{k=1}^{ng} A_k \sum_{i=1}^{nk} \rho_i L_i$$
 (3.1)

where, W is the objective function which is also the minimum weight of the structure,  $\rho$  is the density of materials, A is the cross-section area of the each member, nk is the number of member belonging to the group k (k=1:nk) in truss structures, and ng is the number of group.

Like the other optimization algorithm the initial population in TLBO is constituted by randomly using the design variables determined for the problem under consideration. Length of the individual in a population is equal to the summation of the length of the size variables and the length of shape variables.

Student\_size(
$$i$$
) = { $A_1, A_2, ..., A_n$ }  $i = 1 : Pn, n = 1 : ng$  (3.2)

Student\_shape(i) = 
$$\{N_1, N_2, ..., N_m\}$$
  $i = 1 : Pn, m = 1 : ng_N$  (3.3)

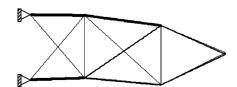


Fig. 7. Optimized geometry of 15-bar 2D truss structure.

Student(i) = {Student\_size, Student\_shape}  
= {
$$A_1, A_2, ..., A_n, N_1, N_2, ..., N_m$$
} (3.4)

where  $ng_N$  is the number of groups for the selected shifted nodal coordinates of structure.

In shape optimization, each design variables must be taken from the upper and lower boundaries of a reasonable nodal coordinate value to avoid the overlapping the nodes.

$$N_{1-m} \le N_m \le N_{u-m} \quad m = 1 : ng_{xy}$$
 (3.5)

where  $N_{1-m}$  and  $N_{u-m}$  are the lower and the upper boundaries for the shifted node  $N_m$ , respectively.

When the truss structure is optimized, the displacement and stress constraints are taken into account.

$$\delta_i \leq \delta_u, \quad c_{\delta,i} = \frac{\delta_i}{\delta_u} \rightarrow i = 1, 2, ..., p$$
 (3.6)

$$\sigma_j \le \sigma_u, \quad c_{\sigma,j} = \frac{\sigma_j}{\sigma_u} \to j = 1, 2, ..., ne$$
 (3.7)

where  $\delta_i$  and  $\delta_u$  are the calculated and allowable displacement for point i, respectively. p is the number of points with restricted displacements and ne is the number of elements.  $\sigma_j$  and  $\sigma_u$  are the calculated and allowable stress or Euler buckling stress for member j, respectively.

The objective function must be changed as independent of constraints. For this aim, a penalty function calculating value of violation of constraints is determined. By means of this function, the objective function is changed to a function including constraints.

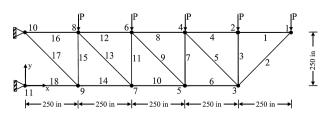


Fig. 8. 18-bar 2D truss structure.

**Table 4**Optimal size and shape results for the 15-bar 2D truss structure.

Design variables	Rahami et al. [8]	Hwang and He [22]	Wu and Chow [23,24]	Tang et al. [9]	Miguel et al. [15] Mode I	TLBO
Size						
Area (in.²)						
$A_1$	1.081	0.954	1.174	1.081	0.954	1.081
$A_2$	0.539	1.081	0.954	0.539	0.539	0.954
$A_3$	0.287	0.440	0.440	0.287	0.220	0.141
$A_4$	0.954	1.174	1.333	0.954	0.954	1.081
$A_5$	0.539	1.488	0.954	0.954	0.539	0.539
$A_6$	0.141	0.270	0.174	0.220	0.220	0.347
$A_7$	0.111	0.270	0.440	0.111	0.111	0.111
$A_8$	0.111	0.347	0.440	0.111	0.111	0.174
$A_9$	0.539	0.220	1.081	0.287	0.287	0.141
$A_{10}$	0.440	0.440	1.333	0.220	0.440	0.270
$A_{11}$	0.539	0.220	0.174	0.440	0.440	0.220
$A_{12}$	0.270	0.440	0.174	0.440	0.220	0.141
$A_{13}$	0.220	0.347	0.347	0.111	0.220	0.440
$A_{14}$	0.141	0.270	0.347	0.220	0.220	0.347
A <sub>15</sub>	0.287	0.220	0.440	0.347	0.220	0.141
Shape						
Coordinate (in.)						
$X_2$	101.5775	118.346	123.189	133.612	114.967	100.0042
$X_3$	227.9112	225.209	231.595	234.752	247.040	241.0473
$Y_2$	134.7986	119.046	107.189	100.449	125.919	118.8228
Y <sub>3</sub>	128.2206	105.086	119.175	104.738	111.067	100.0829
$Y_4$	54.8630	63.375	60.462	73.762	58.298	50.0000
$Y_6$	-16.4484	-20.0	16.728	-10.067	-17.564	3.1411
Y <sub>7</sub>	-16.4484	-20.0	15.565	-1.339	-5.821	-9.6997
Y <sub>8</sub>	54.8572	57.722	36.645	50.402	31.465	46.8963
Weight (lb)	76.6854	104.573	120.52	79.820	75.55	76.6519

Penalty function is calculated as the summation of the violation of displacement, and stresses.

$$C = \sum_{i=1}^{p} c_{\delta,i} + \sum_{j=1}^{ne} c_{\sigma,j}$$
 (3.8)

Objective function is changed to penalized objective function by adding penalty function to it. The penalized objective function,  $\Phi(x)$ , can be formulated as

$$\Phi(x) = W(x)[1+C]$$
 (3.9)

**Table 5**Problem definition for the 18-bar 2D truss structure.

Material		
Young's modulus	$E = 10^4 \text{ ksi}$	
Density	$\rho = 0.1 \text{ lb/in.}^3$	
Allowable stress	±20 ksi	
Design variables		
Size variables	Elements	
$A_1$	1, 4, 8, 12, 16	
$A_2$	2, 6, 10, 14, 18	
$A_3$	3, 7, 11, 15	
$A_4$	5, 9, 13, 17	
Discrete area set = {2.0, 2.25,	.,21.5, 21.75} in. <sup>2</sup>	
Shape variables		
-225	5 in. $\leq Y_3$ , $Y_5$ , $Y_7$ , $Y_9 \leq 245$ in.	
	775 in. $\leq X_3 \leq 1225$ in	
	$525 \text{ in.} \le X_5 \le 925 \text{ in}$	
	$275 \text{ in.} \le X_7 \le 725 \text{ in.}$	
	$25 \text{ in.} \le X_9 \le 475 \text{ in.}$	
Load		
Node	X	Y
1, 2, 4, 6, 8	0	$-10 \mathrm{kips}$
Construction		•
Constraints		
Displacement	_ αE <sub>e</sub> A <sub>e</sub>	
Euler buckling	$\sigma_{eb} \leq rac{lpha E_e A_e}{L_e^2}$	
Stress	$\sigma_e \leq$ allowable stress	

At the end of the optimization process, penalized objective function,  $\Phi(x)$  must be equal to the objective function W(x).

The developed program can take into account the size optimization, shape optimization or both size and shape optimization with different constraints by using TLBO. Fig. 3 shows the general data interface of the developed program.

As seen from this figure, the developed program has different properties such as optimization types and constraints types. This program also can solve structure types like truss with the finite element analysis. As mentioned before, the TLBO was firstly used for constrained mechanical design optimization problems. But, in this study, it is used for the optimization of structural design problem.

# 4. Numerical examples

To demonstrate the efficiency of this new meta-heuristic algorithm 2D and 3D truss structures taken from literature are considered. These are 39-bar 3D truss, 72-bar 3D truss, 15-bar 2D truss, 18-bar 2D truss and 200-bar 2D truss structures. Among these truss structures, 72-bar and 200-bar truss structures are solved for size optimization and the others are solved both size and shape optimization.

# 4.1. Example 1

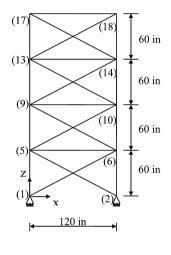
A 39-bar 3D truss structure is considered as given in Fig. 4. This truss structure is previously designed by Wang et al. [1]. The

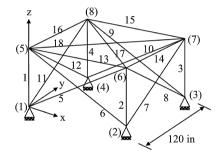


Fig. 9. Optimized geometry of 18-bar 2D truss structure.

**Table 6**Optimal size and shape results for the 18-bar 2D truss structure.

Design variables	Hasançebi and Erbatur [7]	Kaveh and Kalatjari [3]	Rahami et al. [8]	Ahrari and Atai [14]	TLBO
Size					
Area (in.²)					
$A_1$	12.25	12.25	12.75	12.4349	12.25
$A_2$	17.50	18.0	18.50	17.8010	17.50
$A_3$	5.75	5.25	4.75	5.28260	5.75
$A_4$	4.25	4.25	3.25	3.79195	4.25
Shape					
Coordinate (in.)					
$X_3$	910.0	913.0	917.4475	911.679	906.9373
$Y_3$	179.0	186.8	193.7899	185.547	179.8866
$X_5$	638.0	650.0	654.3243	643.714	637.0087
Y <sub>5</sub>	141.0	150.5	159.9436	147.199	142.6170
$X_7$	408.0	418.8	424.4821	413.964	408.6414
Y <sub>7</sub>	91.0	97.4	108.5779	97.871	94.1563
$X_9$	198.0	204.8	208.4691	202.283	199.6503
Y <sub>9</sub>	24.0	26.7	37.6349	29.767	25.3657
Weight (lb)	4533.24	4547.9	4530.7	4505.97	4528.7969





(a) profile view

(b) element and node numbering system

Fig. 10. 72-bar space truss structure.

material properties, geometry properties, load and the constraints are given in Table 1. In this table, K is taken as  $\pi/4$  for circular section. The numbers of groups are 5 and 6 for cross-sectional areas and shifted nodal coordinates, respectively.

The design variables are continuous for the cross-sectional areas and shifted nodal coordinates. Optimal shape after the optimization with TLBO is given in Fig. 5. The comparison of results with those of the other reference is given in Table 2. For the TLBO algorithm the number of function evaluation (nfe) is calculated as by Eq. (4.1). In this equation, Gn is number of generation where the best solution obtained and Pn is the number of population. For this example, population size is 40, maximum generation number is 100 and the best solution is obtained after the generation 94. So, the number of function evaluation is 7560.

$$nfe = 2 * Gn * Pn + Pn \tag{4.1}$$

**Table 7**Multiple loading (kip) conditions for the 72-bar space truss structure.

Case	Node	Fx	Fy	Fz
1	17	0.0	0.0	-5.0
	18	0.0	0.0	-5.0
	19	0.0	0.0	-5.0
	20	0.0	0.0	-5.0
2	17	5.0	5.0	-5.0

### 4.2. Example 2

In this example, the 15-bar 2D truss structure is considered as given in Fig. 6. This truss structure is previously designed by Rahami et al. [8], Tang et al. [9], Hwang and He [22], Wu and Chow [23,24], and Miguel et al. [15]. The material properties, geometry properties, load and the constraints are given in the Table 3.

The number of groups are 15 and 8 for cross-sectional areas and shifted nodal coordinates, respectively. The design variables are discrete for the cross-sectional areas while it is continuous for the shifted nodal coordinates. Optimal shape after the optimization with TLBO is given in Fig. 7. The comparison of results with those of the other references is given in Table 4. The number of populations are 80, 80, 30, 40, 10, and 80 for the Rahami et al. [8], Hwang and He [22], Wu and Chow [23]; Wu and Chow [24], Tang et al. [9], Miguel et al. [15], and this study, respectively. The maximum generation numbers are 100, 200, 200, 800, and 200 for the Rahami et al. [8], Hwang and He [22], Tang et al. [9], Miguel et al. [15] and this study, respectively. The best solution is obtained in the generation 191 with TLBO. So, the number of function evaluation is 30,640.

# 4.3. Example 3

In this example, the 18-bar 2D truss structure is considered as given in Fig. 8. This truss structure is previously designed by

**Table 8**Optimal design comparison for the 72-bar space truss for size optimization.

Design variables (in. <sup>2</sup> )		Lee et al. [28]	Li et al. [31]	Wu and Chow [24]	Sadollah et al. [18]	This study
No	Members	HS	HPSO	SSGA-4P	MBA	TLBO
1	A1-A4	1.9	2.1	1.5	2.0	1.9
2	A5-A12	0.5	0.6	0.7	0.6	0.5
3	A13-A16	0.1	0.1	0.1	0.4	0.1
4	A17-A18	0.1	0.1	0.1	0.6	0.1
5	A19-A22	1.4	1.4	1.3	0.5	1.4
6	A23-A30	0.6	0.5	0.5	0.5	0.5
7	A31-A34	0.1	0.1	0.2	0.1	0.1
8	A35-A36	0.1	0.1	0.1	0.1	0.1
9	A37-A40	0.6	0.5	0.5	1.4	0.5
10	A41-A48	0.5	0.5	0.5	0.5	0.5
11	A49-A52	0.1	0.1	0.1	0.1	0.1
12	A53-A54	0.1	0.1	0.2	0.1	0.1
13	A55-A58	0.2	0.2	0.2	1.9	0.2
14	A59-A66	0.5	0.5	0.5	0.5	0.6
15	A67-A70	0.4	0.3	0.5	0.1	0.4
16	A71-A72	0.6	0.7	0.7	0.1	0.6
Weight (lb)	387.94	388.94	400.66	385.54	385.54	
Maximum iteration	-	1000	-	1000	80	
Population size	-	50	60	50	50	
Independent run	-	100	-	15	20	

Hasançebi and Erbatur [25], Kaveh and Kalatjari [3], Ahrari and Atai [14], and Rahami et al. [8]. The material properties, geometry properties, load and the constraints are given in the Table 5. In this table, the buckling coefficient  $\alpha$  is taken as 4.

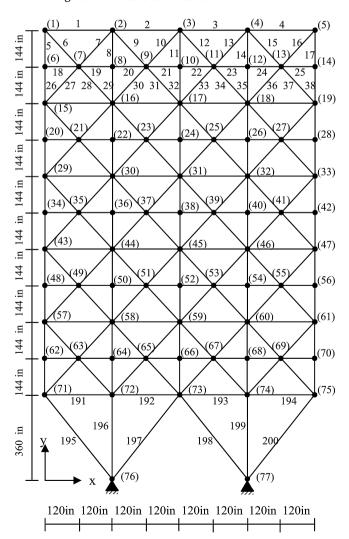


Fig. 11. Configuration of 200-bar plane truss structure.

The numbers of groups are 4 and 8 for cross-sectional areas and shifted nodal coordinates, respectively. The design variables are discrete for the cross-sectional areas while it is continuous for the shifted nodal coordinates. Optimal shape after the optimization with TLBO is given in Fig. 9. The comparison of results with those of the other references is given in Table 6. The number of populations are 50, 80, and 80 for Kaveh and Kalatjari [3], Rahami et al. [8], and this study, respectively. The maximum generation numbers are 100, 100, and 200 for Kaveh and Kalatjari [3], Rahami et al. [8], and this study, respectively.

# 4.4. Example 4

72-bar space truss structure is given in Fig. 10. Loading conditions for this structure are given in Table 7. Modulus of elasticity is 10,000 ksi and density of material 0.1 lb/in.<sup>3</sup>. The allowable stress for all members is  $\pm 25$  ksi and allowable displacement is  $\pm 0.25$  in at nodes 17, 18, 19, and 20. Members of this structure are categorized into 16 groups. This grouping can be seen from the first column of Table 8. The discrete variables are selected from the set  $D = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0, 3.1, 3.2\} (in.<sup>2</sup>).$ 

Under the multiple loading conditions, the best solution vector obtained in this study is [1.9, 0.5, 0.1, 0.1, 1.4, 0.5, 0.1, 0.1, 0.5, 0.5, 0.1, 0.1, 0.2, 0.6, 0.4, 0.6]. These results are compared with the results given in the literature in Table 8. The maximum generation

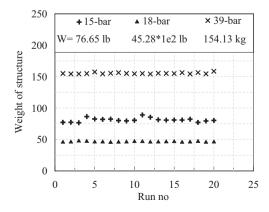


Fig. 12. Weight of the structure with run no.

**Table 9**Optimal design comparison for the 200-bar planar truss under multiple load case.

Design variables	Group members	Optimal cross-sectional areas (in. <sup>2</sup> )				
		Coello and Christiansen [29]	Lee and Geem [27]	Toğan and Daloğlu [26]	Sönmez [17]	This study TLBO
1	1,2,3,4	=	0.1253	0.347	0.1039	0.113546
2	5,8,11,14,17	_	1.0157	1.081	0.9463	0.948427
3	19,20,21,22,23,24	_	0.1069	0.100	0.1037	0.107798
4	18,25,56,63,94,101,132,139,170,177	_	0.1096	0.100	0.1126	0.100009
5	26,29,32,35,38	_	1.9369	2.142	1.9520	1.934462
6	6,7,9,10,12,13,15,16,27,28,30,31,33	_	0.2686	0.347	0.2930	0.288872
7	34,36,37	_	0.1042	0.100	0.1064	0.211586
8	39,40,41,42	_	2.9731	3.565	3.1249	3.090253
9	43,46,49,52,55	_	0.1309	0.347	0.1077	0.103114
10	57,58,59,60,61,62	_	4.1831	4.805	4.1286	4.090254
11	64,67,70,73,76	_	0.3967	0.440	0.4250	0.450150
12	44,45,47,48,50,51,53,54,65,66,68,69	_	0.4416	0.440	0.1046	0.100707
13	71,72,74,75	_	5.1873	5.952	5.4803	5.479848
14	77,78,79,80	_	0.1912	0.347	0.1060	0.101144
15	81,84,87,90,93	_	6.2410	6.572	6.4853	6.479849
16	95,96,97,98,99,100	_	0.6994	0.954	0.5600	0.532949
17	102,105,108,111,114	-	0.1158	0.347	0.1825	0.132492
18	82,83,85,86,88,89,91,92,103,104,106	-	7.7643	8.525	8.0445	7.944450
19	107,109,110,112,113	-	0.1000	0.100	0.1026	0.100486
20	115,116,117,118	-	8.8279	9.300	9.0334	8.944437
21	119,122,125,128,131	-	0.6986	0.954	0.7844	0.701077
22	133,134,135,136,137,138	-	1.5563	1.764	0.7506	1.377693
23	140,143,146,149,152	-	10.9806	13.300	11.3057	11.239401
24	120,121,123,124,126,127,129,130,141	-	0.1317	0.347	0.2208	0.228718
25	142,144,145,147,148,150,151	-	12.1429	13.300	12.2730	12.239392
26	153,154,155,156	-	1.6373	2.142	1.4055	1.684935
27	157,160,163,166,169	-	5.0023	4.805	5.1600	4.913586
28	171,172,173,174,175,176	-	9.3545	9.300	9.9930	9.718956
29	178,181,184,187,190	-	15.0919	17.170	14.70144	15.021916
Weight (lb) Constraint violation		36,167.7300 -	25,447.10000 0.40023	28,554.1400 None	25,533.79 None	25,664.002 None

number, number of population and the number of independent run values are given in this table for related studies. Sadollah et al. [18] optimized these examples with the number of function evaluation as 9450, while it is 7250 in the TLBO.

# 4.5. Example 5

200-bar space truss structure is given in Fig. 11. The members of this structure are categorized into 29 groups as in Toğan and Daloğlu [26], Lee and Geem [27], Coello and Christiansen [29], and Sönmez [17]. The details of grouping is given in Table 8. Material properties and constraints used in this study are as follows: Modulus of elasticity is 30,000 ksi and density of material is 0.283 lb/in³. The allowable stress for all members is  $\pm 10$  ksi and there is no limitation for displacement of free nodes. This structure is subjected to three different load conditions and they are as follows:

*Load case 1*: 1 kip acting in the positive *x* direction at nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62, and 71.

Load case 2: 10 kips acting in the negative y direction at nodes 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 34, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 50, 52, 54, 56, 57, 58, 59, 60, 61, 62, 64, 66, 68, 70, 71, 72, 73, 74, and 75. Load case 3: Cases 1 and 2 are combined.

Optimum results are compared with the results given in literature are in Table 9. As seen from this table, the solution with no violations obtained from this study is better than the other results given in literature. The number of function evaluations are 1,450,000, 51,360, and 48,000 for the Sönmez [17], Toğan and Daloğlu [26], and Lee and Geem [27], respectively.

Also, 20 run are made for each design of some truss structures. The weight of each structure according to the run number is shown in Fig. 12.

# 5. Conclusion

The TLBO algorithm is implemented in this paper for the size and shape optimization of 2D and 3D truss structures. This new optimization algorithm consists of two main phases, i.e. teacher phase and student phase. Like other nature-inspired algorithms, TLBO is also a population-based method using a population of solutions to proceed to the global solution. For TLBO, the population is considered as a group of learners or a class of learners.

The design results are compared with the previous studies to demonstrate the efficiency of the TLBO algorithm. It is concluded that the TLBO algorithm presented in this study can be effectively used in the design of truss structures.

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