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Use of optimization for automatic grouping of beam cross-section dimensions in reinforced concrete building structures

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ABSTRACT

In the development of structural designs, in general, designers avoid varying the size of structural elements, seeking to group them as much as possible. These groupings produce aesthetic effects and facilitate formwork design for reinforced concrete frames, checks, and implementation. Therefore, the elements are pre-grouped into a smaller number of different cross-sections to provide an interesting and practical solution. However, the outcome is highly dependent on this grouping because the dimension of each element and, consequently, the overall cost, will be determined by the element that is the most stressed in each group. This paper minimizes the costs of the beams in reinforced concrete buildings using a grid model. The sizing is performed according to the Brazilian NBR 6118 standard [1], taking into account the flexural, shearing, torsional, and web reinforcements, in addition to checks on the service limit states (deflection and maximum crack opening). In addition to determining the beam height that leads to the lowest global cost, an automatic determination of the optimized group is performed, taking into account the required maximum number of groups. Several numerical analyses were performed using the computational implementation of the developed formulation. This paper presents the results obtained from an analysis of two floors. These results provide evidence that the chosen procedure may provide a significant reduction in the cost of a structure, even for a small number of different cross-sections. Thus, the determination of the optimum dimensions of the elements is less dependent on the designer's experience and sensitivity. The proposed procedure is easy to implement and may generate a significant reduction in the consumption of structural material when incorporated into the daily routine of project offices.

1. Introduction

The sizing of structural elements is an iterative process. The designer, based on his/her experience and intuition, determines the initial size of each element, which must satisfy the resistance and functionality requirements stipulated by current technical standards. Based on that, the design is further improved to reduce costs without compromising safety. Nevertheless, because there is a very large number of acceptable solutions to a given problem, it is unlikely that the best of all possible solutions will be found using this strategy. This is even more difficult with statically indeterminate structures, since a change in one element section will redistribute the efforts in the structure due to the alteration in relative stiffness of the elements. However, the use of a well-defined mathematical model to describe the problem may provide an optimal solution, based on a systematic process, in which the goals, constraints, and design variables are narrowed down. As regards to structural optimization, the smallest weight and lowest cost are the major goals to be achieved, and some constraints exist concerning the fulfillment of current technical standards.

A reduction in the costs for a reinforced concrete structure, if significant, may give construction companies and, especially, structural design offices, an advantage over their competitors. In addition, the rational use of the existing natural resources, provided by optimization, should also be taken into consideration.

Despite their potential application, optimization techniques have only been adopted to a limited degree by design offices. To expedite this process, it is crucial that the mathematical model considers the actual situations faced by designers, and that the result be applied without the need of adjustments that rely on a designer or that have some level of subjectivity.

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Several technical studies have addressed the optimal sizing of reinforced concrete structures, in which calculations are made using various classic optimization techniques. In most studies, the aim is to minimize concrete section costs while simultaneously meeting functional constraints based on design standards and satisfying constraints involving strength criteria. In those studies, the cross-sectional dimensions are often grouped to reduce the number of design variables, thereby lowering manufacturing costs. Nonetheless, the final outcome is highly dependent upon how this grouping is performed.

The aim of this paper is to apply an optimization technique to minimize the costs of beams in reinforced concrete buildings, while grouping structural elements automatically. For this goal to be met, a software program was devised by combining a structural analysis of the floor of a building using a grid model, the sizing of reinforced concrete beams, and a heuristic optimization method known as simulated annealing. The sizing of structural elements in terms of the ultimate and serviceability limit states was based on the Brazilian NBR 6118 technical standard [1], taking into account the flexural stress, shearing, torsional stress, and web reinforcements, in addition to checks on the maximum deflection and crack opening. The optimized grouping of elements was assessed by the so-called cardinality constraints (CC), after determining the maximum number of different cross-sections in the structure.

This paper is an extension of the authors' previous studies on the optimization of reinforced concrete building structures [3] and the use of cardinality constraints, both for steel frames [4] and reinforced concrete frames [16]. In [3], a formulation was developed and implemented to minimize the cost of building floors according to a grid model. This study aimed at identifying pre-sized beam parameters and the importance of the costs related to the forms, concrete, and steel in the optimized cost. The work developed in [4] highlighted the importance of considering the cardinality constraints in the optimization of steel gantries by achieving a significant reduction in the total weight of the structure. In [16], the previously developed formulation for the optimization of reinforced concrete structures was expanded by the incorporation of cardinality constraints. While in [16] the main objective was to introduce the proposed formulation and illustrate the optimization procedure, the present paper focuses on extending the results obtained and investigating how the maximum number of groups of elements influences the optimal costs. The authors are unaware of the existence of a similar study on reinforced concrete structures or the structural model chosen here.

The remainder of this paper is organized as follows. Section 2 briefly describes the basis of the structural optimization and the adopted optimization method (simulated annealing), as well as some applications to the optimization of reinforced concrete structures. Section 3 shows the proposition for the optimization problem. Section 4 gives some examples of the application of the proposed technique to a variable number of groups. Section 5 presents the conclusion.

2. Structural optimization

In structural engineering, optimization techniques have been constantly developed and applied to a wide array of problems in an attempt to find the best sets of material, topology, geometry, and/or cross-section dimensions for different structural systems [5].

The algorithms used for the solution of an optimization problem can be either deterministic or probabilistic. Deterministic optimization methods, also called classic methods, in which mathematical programming is included, are usually based on the calculations of first-order derivatives or second-order partial derivatives. Heuristic methods, based on probabilistic algorithms, introduce stochastic data and parameters in the optimization process, solving the problem from a probabilistic perspective.

Mathematical programming methods have some limitations, including difficulty in identifying global optimal solutions because they are dependent on the starting point, difficulty in working with discrete variables, and difficulty in performing non-differentiable functions. An essential characteristic for the application of classic methods is the need for the objective function to be continuous and differentiable in the search space. However, this does not occur in most practical engineering problems, thus hindering their application.

Heuristic methods do not use the calculation of derivatives. Instead, they directly search for solutions in the feasible space. However, these methods require a larger number of assessments of the objective function value, and are computationally more expensive than methods based on mathematical programming. Thus, they should not be used injudiciously, but only for problems for which mathematical programming is a limitation.

Heuristic methods include a large number of algorithms such as simulated annealing, genetic algorithms, ant and bee colony algorithms, harmony search, and particle swarm optimization. Despite this wide variety, genetic algorithms and simulated annealing are still the most popular methods and, therefore, have a larger number of applications [5,6].

Simulated annealing is a heuristic method based on statistical mechanics, which dates back to the annealing process, and was introduced by Kirkpatrick et al. [7]. In the physical process of solid hardening, a material is quickly heated and slowly cooled to eliminate its structural flaws. If the cooling is sufficiently slow, the final configuration of the material will correspond to the minimum energy state. On the other hand, quick cooling will result in a metal with a weak and brittle structure.

In brief, in simulated annealing, a single neighboring state s' of the current solution s is randomly generated in each iteration. The difference (Δ_f) between the quality of the new solution s' and the quality of the current solution s (Eq. (1)) is calculated to assess the acceptance of this new solution s'.

$$\Delta_f = f(s') - f(s) \tag{1}$$

In a minimization problem, if the value of Δ_f is less than zero, the new solution s' is automatically accepted and can be substituted for s. Otherwise, the acceptance of the new solution s' depends on the probability established by the Metropolis criterion, as shown in Eq. (2):

$$p = \exp\left(\frac{-\Delta_f}{T}\right) \tag{2}$$

As the temperature drops throughout the process, there is a higher probability of acceptance of new solutions in the initial stages. This probability decreases throughout the process, reaching a point (when the temperature is close to zero) at which only those movements that improve the cost function are accepted.

Several works published in the past few years successfully used simulated annealing for structural optimization.

Hasançebi and Erbatur [8] used this heuristic and optimized a 942-member truss tower, an 18-member truss, and a 47-member plane truss tower. In the latter two cases, the geometries of the models were optimized, along with their cross-sections. Discrete variables were used. A comparison of the results with those of other studies showed that the proposed simulated annealing algorithm outperformed genetic algorithms. Park and Ryu [9] proposed altering the parameters in order to improve the heuristics. They optimized the weights of two structures usually found in structural optimization problems: 10-member plane trusses and 25-member spatial trusses. Both discrete and continuous variables were used. The authors concluded that the number of necessary iterations in the new simulated annealing algorithm was significantly smaller than that of the conventional algorithm. Kripka [10] optimized plane and spatial trusses, comparing the obtained results with those of different methods. In all cases, the optimal solution provided by simulated annealing was equal to or better than the others. Dagertekin [6] optimized the cross-sections of steel frames using simulated annealing and genetic algorithms. Three simulations were carried out, using frames with 8, 26, and 84 members. Simulated annealing had a slight advantage in all the simulations. Hasançebi et al. [13] proposed an improvement to the simulated annealing algorithm. In order to test the changes, the authors optimized plane and spatial steel frames with 304 and 132 members, respectively. Simulated annealing yielded the smallest weights for both frames compared to the results of other heuristic methods: harmony search and tabu search. Sonmez [14] optimized truss weights with different methods, including simulated annealing, using discrete variables. For the classic example of a 10-member plane truss, the method yielded the best result, as did both the bee colony and ant colony algorithms. It also had a better result than other heuristic methods for 25-member spatial trusses. Particle swarm optimization and genetic algorithms were the other methods used. Quite recently, Kripka and Chamberlain [17] conducted a numerical study to minimize the weights of cold-formed steel frames with and without lips, using the AISI 2007 standard. Flexural, torsional, and torsional–flexural bucklings were regarded as the constraints. The results, experimentally validated in the same study, indicated remarkable reductions in the weights of the analyzed elements.

Specifically regarding the optimization of reinforced concrete structures with simulated annealing, Payá-Zaforteza et al. [11] optimized the reinforced concrete frames used in the construction of buildings following five heuristic methods, including simulated annealing and genetic algorithms. Initially, the different methods were tested using a model made up of two bays and four floors. Of these methods, simulated annealing was more efficient in the search for an optimal solution, showing an intermediate processing time. Later, several models were used to fine-tune the method: two-bay frames with two, four, six, and eight heights. Suji et al. [5] optimized fiber-reinforced concrete beams. These beams were reinforced as rationally as possible, given the high price of this material. González-Vidosa et al. [12] optimized four reinforced concrete frames. The first model consisted of a soil barrier system. The second and third models consisted of frames used in road construction, and the fourth optimized model consisted of a 20-member plane frame. Payá-Zaforteza et al. [11] conducted multiobjective optimization using simulated annealing. In addition to assessing cost minimization, a common goal in structural optimization problems, they evaluated three other objectives: maximization of the model constructability, minimization of the environmental impacts, and maximization of the global structural safety. The model used consisted of a 20-member reinforced concrete plane frame for a 4-story building. Their results indicated that, with a small increase in optimal cost, it is possible to produce structures with higher constructability, larger sustainability, and better global structural safety. Later, the same authors used this method once again to optimize a 20-member reinforced concrete plane frame, but they considered only the costs this time. Their aim was to improve the parameters of this method. Bordignon and Kripka [15] used this method for the cost minimization of reinforced concrete column sections, achieving considerable savings compared to the conventional design procedure, genetic algorithms, and mathematical programming. More recently, Garcia-Segura et al. [18] employed a hybrid method, combining simulated annealing and the firefly algorithm for the optimization of I beams. A total of 20 design variables were utilized, including the steel and concrete section dimensions and the characteristic concrete strengths. The results were better than those obtained from the two heuristic methods when applied separately.

3. Problem formulation

In this study, the cost minimization of reinforced concrete beams took into account the influences of the formwork, concrete, and transverse and longitudinal reinforcements. The major optimization problem variable was the cross-sectional beam height. The cross-sectional height of each set of beams was the main design variable, and only discrete values in whole centimeters were used. The beam width was preset. The cost of each material was obtained by multiplying the respective amount by the unit cost of the material. Steel was quantified by mass (kg), concrete as a volume (m^3), and formwork as an area (m^2). Thus, the objective function is given by Eq. (3), where C_t corresponds to the overall cost of the analyzed structure; P_A, P_{Asw}, A_F, and V_C refer to the amounts of CA-50 steel bars, CA-60 steel bars, formworks, and concrete, respectively; and C_A, C_{Asw}, C_F, and C_C represent the unit costs of these materials, respectively.

$$C_t = (P_A \cdot C_A) + (P_{Asw} \cdot C_{Asw}) + (A_F \cdot C_F) + (V_{CC} \cdot C_C) \tag{3}$$

ABNT NBR 6118 [1], the Brazilian standard for the design and installation of reinforced concrete structures, was used for the sizing and detailed description of the structural elements. The corresponding constraints are represented in Eqs. (4)–(15).

The first two constraints (Eqs. (4) and (5)) refer to the serviceability limit states. The maximum deflection of each element, δ, taking into account the long-term effects, should be smaller than the limit deflection δ_{lim}, and the characteristic crack width w_k should be smaller than the stipulated limits $w_{k,lim}$.

$$\delta \leqslant \delta_{lim} \tag{4}$$

$$w_k \leqslant w_{k,lim} \tag{5}$$

The following constraints refer to flexural reinforcements: the ratio between the fractions of the bending moment absorbed by compression and tension in double reinforcements, $M_{As'}$ and M_{As}, respectively, should not exceed 30% (Eq. (6)), in order to prevent a large concentration of reinforcements, hindering concrete placement. The minimum reinforcement ratio, ρ_{min}, should be larger than the ratios defined in Table 17.3 of the NBR 6118 [1] standard (minimum of 0.15%), whereas the maximum ratio should be equivalent to, at most, 4% of the cross-sectional area, A_C (Eq. (7)).

$$\frac{M_{As'}}{M_{As}} \leqslant 0.30 \tag{6}$$

$$\rho_{min} \cdot A_C \leqslant A_s + A'_s \leqslant 4\% \cdot A_C \tag{7}$$

To check for shearing, the NBR 6118 [1] standard determines that the strain concrete should withstand in compressed struts, V_{Rd2}, should be greater than the respective stress V_{Sd}, as outlined in Eq. (8), and the strength of the concrete and reinforcements in the tensioned struts, V_{Rd3}, should be larger than the working stress V_{Sd}, as shown in Eq. (9).

$$V_{Sd} \leqslant V_{Rd2} \tag{8}$$

$$V_{Sd} \leqslant V_{Rd3} \tag{9}$$

In regard to torsion, the first constraint (Eq. (10)) defines that the working shearing stress V_{Sd} should not exceed 70% of the structural resistance V_{Rd2}, so that compatibility torsion can be excluded from the analysis, which includes only the equilibrium torsion. Moreover, the working torsional moment T_{Sd} must be smaller than

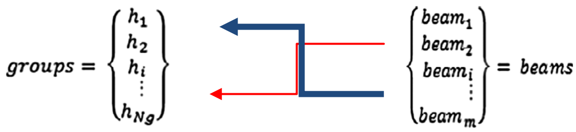

Fig. 1. Height assignment to m beams according to Ng groups.

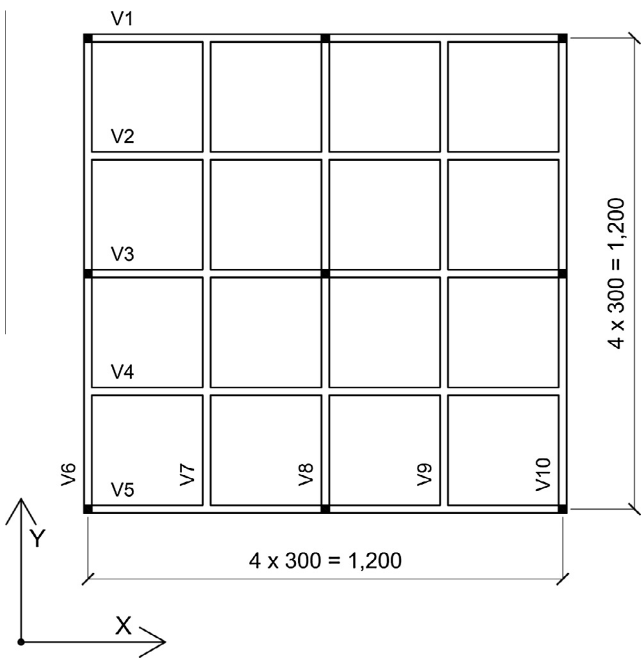

Fig. 2. Example 1: Sketch (dimensions in centimeters).

the moment of resistance of the compressed concrete diagonals T_{Rd2} (Eq. (11)), as well as the moments supported by the stirrups T_{Rd3} (Eq. (12)) and longitudinal bars (Eq. (13)).

$$V_{Sd} \leqslant 0.7 \cdot V_{Rd2} \tag{10}$$

$$T_{Sd} \leqslant T_{Rd2} \tag{11}$$

$$T_{Sd} \leqslant T_{Rd3} \tag{12}$$

$$T_{Sd} \leqslant T_{Rd4} \tag{13}$$

For the combined shear and torsion, the NBR 6118 [1] standard establishes that the requirement outlined in Eq. (14) be met, representing one more problem constraint. The constraint in Eq. (15) refers to the minimum reinforcement ratio, taking into account the shear and torsion.

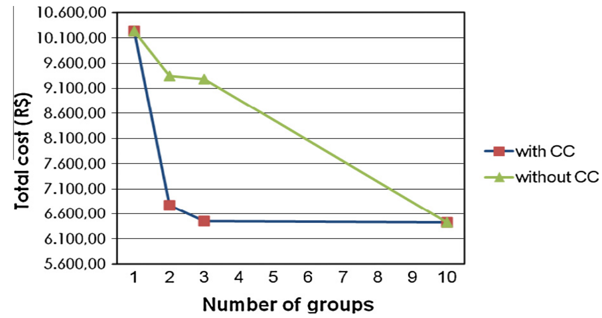

Fig. 3. Example 1: Optimal cost for different numbers of groups, with and without cardinality constraints.

$$\frac{V_{Sd}}{V_{Rd2}} + \frac{T_{Sd}}{T_{Rd2}} \leqslant 1 \tag{14}$$

$$\rho_{SW_{min}} = \frac{A_{SW}}{b_w \cdot s} \geqslant 0.2 \cdot \frac{f_{ct,m}}{f_{ywk}} \tag{15}$$

In addition to the constraints mentioned earlier, the present paper also includes cardinality constraints, aimed at the optimal grouping of elements. The maximum number of different cross-sections is limited by Ng, or the number of groups, and each beam is assigned to one of these groups for the determination of the section height, according to Fig. 1. Note that, if Ng is equivalent to the total number of beams, the usual problem reappears. It is observed that the same height may be assigned to more than one element group. Barbosa et al. [2] presented a special encoding for a genetic algorithm where cardinality constraints are considered in order to find the best member grouping for truss structures. This reference provides a more detailed explanation about the use of cardinality constraints for this purpose.

4. Results

The developed formulation was implemented by associating the simulated annealing optimization method with beam analysis according to the grid model. In general, beam optimization starts with the cross-sections defined in the original project, or based on practical pre-sizing rules (e.g., defining the height as a span ratio). The number of different cross-sections and the way they are grouped are also the initial data for the problem. The structural cost is computed for these dimensions, as well as the efforts and displacements. If any of these constraints is violated, the structural cost is artificially increased in a procedure known as penalty. Consequently, new solutions are generated by changing the element section value, and the process is repeated until any

Table 1
Example 1: One to ten variables.

Ng	h_1 (m)	h_2 (m)	h_3 (m)	h_4 (m)	h_5 (m)	h_6 (m)	h_7 (m)	h_8 (m)	h_9 (m)	h_{10} (m)	Cost (R$)	Cost/cost 1 (%)
1	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	10240.84	1.000
2	0.49	0.80	0.80	0.80	0.49	0.49	0.80	0.80	0.80	0.49	9354.28	0.913
3	0.49	0.69	0.69	0.69	0.49	0.49	0.89	0.89	0.89	0.49	9280.83	0.906
10	0.40	0.15	1.22	0.15	0.40	0.32	0.37	0.36	0.37	0.32	6420.95	0.627

Table 2
Example 1: Analysis with cardinality constraints.

Ng	h_1 (m)	h_2 (m)	h_3 (m)	h_4 (m)	h_5 (m)	h_6 (m)	h_7 (m)	h_8 (m)	h_9 (m)	h_{10} (m)	Cost (R$)
2	0.34	0.34	1.29	0.34	0.34	0.34	0.34	0.34	0.34	0.34	6776.27
3	0.37	0.14	1.23	0.14	0.37	0.37	0.37	0.37	0.37	0.37	6450.35

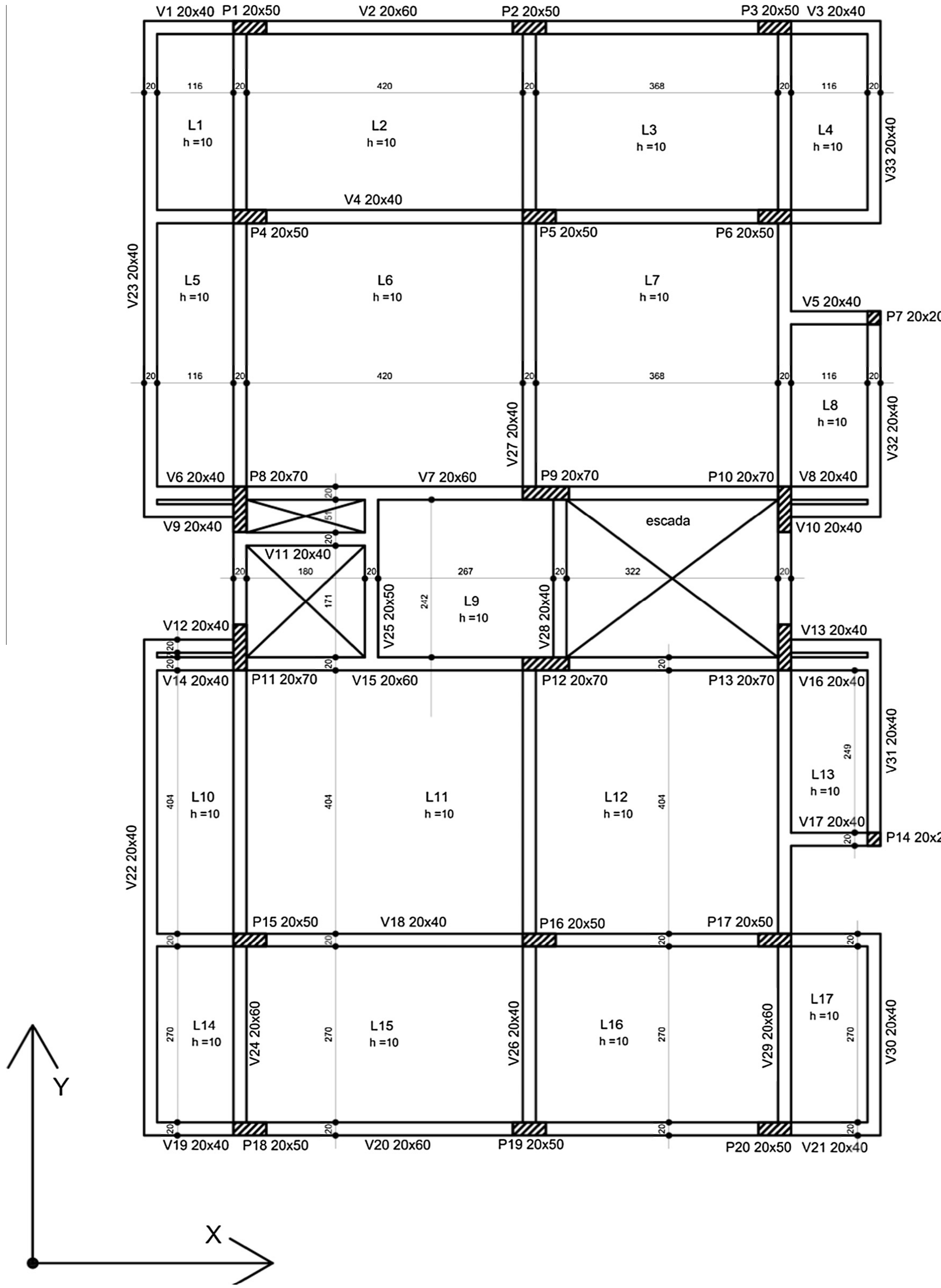

Fig. 4. Floor of example 2 (dimensions in centimeters).

interruption criteria are met. In relation to the procedure normally adopted for the optimization routines, the difference highlighted in this paper is the fact that the establishment of the grouping–and not just the beam height–varies during the process. In the end, a maximum number of different *Ng* heights is obtained.

Some simulations that were conducted based on computational programming for beam cost minimization are discussed next, placing special emphasis on the effect of the optimized grouping on the outcomes. In the grid model, loads are applied perpendicularly to the plane formed by the floor beams, and the self weight is computed automatically. In all the analyses, the beams were considered to be simply supported by the columns. The effects of actions such as wind and earthquakes were not computed using this structural model.

In relation to the applied material, the characteristic concrete strength (*fck*) was equivalent to 25 MPa, with the following unit costs, denominated in Brazilian currency (R$): CA-50 steel bars = R$3.97/kg, CA-60 steel bars = R$3.89/kg, formworks = R$8.68/m^2, and concrete = R$233.55/m^3.

4.1. Example 1

The first structure was adapted from Kripka [19], and consisted of 40 elements, with 10 beams supported by eight columns and a floor with double symmetry (in relation to the dimension of the element and applied loads), as shown in Fig. 2. In this structure, loads were transferred from the slabs, which were analyzed separately, to form an evenly distributed total load of 4.5 kN/m^2 (of which, 1.5 kN/m^2 was due to variable vertical loading), using an evenly distributed load of 3.6 kN/m throughout the perimeter beams (beams V1, V5, V6, and V10), in addition to the weight of the beams themselves (computed automatically). In all the beams, the cross-sectional base width was set at 0.20 m.

Initially, all the beams were limited to the same cross-sectional height, yielding an optimal cost equivalent to the height of 0.77 m. Taking this height as a benchmark, several analyses were conducted for a variable number of groups with different cross-sections (*Ng*) in order to verify the effective reduction in the optimized cost, namely,

- external beams with height h_1 and internal beams with height h_2, thus consisting of two design variables;
- external beams with height h_1, horizontal internal beams (V2 through V4) with height h_2, and vertical internal beams (V7 through V9) with height h_3 (three variables);
- beams with different heights (10 design variables).

Table 1 briefly lists the results obtained, indicating the overall floor cost (in Brazilian currency) in the penultimate column and the relative reduction to the optimal height of 0.77 m in the last column. Note that, for two and three types of sections, the cost reduction is slightly lower than 10%. For 10 variables, however, this reduction amounts to nearly 38%. Though it constitutes a highly significant reduction in cost, this last solution is difficult to implement in practice.

It is interesting to note that, when the structure has load symmetry in relation to the span and beam widths in both perpendicular directions, such as in example 1, the designer usually keeps the symmetry in relation to the beam height. However, the optimal heights for 3 and 10 variables are not identical in both directions. For instance, when different heights are allowed for internal beams in both perpendicular directions, the 9-cm increase in beam height in one direction allows a decrease of 11 cm in the cross-sectional direction, without decreasing stiffness. This behavior is in line with that obtained in [19], when it was concluded that symmetrical heights in both directions do not correspond to the global optimum (i.e., to the lowest possible cost).

The floor in example 1 was reoptimized by including cardinality constraints for the same maximum numbers of groups. As noted above, in the optimization procedure, each element is assigned to a group, instead of generating new solutions by merely assigning a new height to the elements. Table 2 presents the new heights and optimized groups. It is observed that, for example, for *Ng* = 2, the best solution corresponds to a group composed only of element 3, while the other one is composed of the remaining elements.

Note that, when two groups are chosen, there is a substantial cost reduction, in relation to the case when a single height is used for all the elements (R$10240.84). This reduction amounts to approximately 33.8%, which is slightly higher than that in the case of *Ng* = 10. The variation in the optimal cost, with and without cardinality constraints, is shown in the graph of Fig. 3. It is understood that the adequate grouping of the elements provides a substantial reduction in cost for a small number of different cross-sections.

Table 4
Results for example 2 (with and without cardinality constraints (CC)).

	Ng = 3	*Ng* = 5	*Ng* = 10
Optimized cost (R$) without CC	5441.68	5244.79	5089.09
Optimized cost (R$) with CC	4770.39	4700.60	4686.04
Additional reduction in overall cost (%)	12.3	10.4	7.9

4.2. Example 2

The second structure consists of a typical floor of a building, adapted from Medeiros and Kripka [3] and shown in Fig. 4. The floor has 71 elements for 33 beams. The floor beams were sized according to three groups proposed by the engineer who performed the calculations, resulting in heights of 40, 60, and 70 cm, with all of them having a fixed width of 20 cm.

Initially, the same three groups of beams applied in the original project were used (only three cross-sectional heights). After that, the beams were classified into 5, 10, and 33 groups. Table 3 lists the overall costs of the floor beams, as well as the cost reduction compared to the originally proposed arrangement.

The results obtained for the same three groups of beams from the original structure indicate a remarkable reduction (23.9%) in the overall cost of floor beams. Note, once again, that even though the analyses of an increasing number of different sections have allowed for a larger cost reduction (up to 34.1%), their practical application is unlikely.

Similar analyses were performed to assess the automatic grouping of elements using cardinality constraints. Table 4 compares the results with and without cardinality constraints. The results displayed in the last line illustrate the additional reduction in the case in which groups are not determined automatically. It is interesting to note that the less these beam sections vary independently, the more significant the additional cost reduction provided by the cardinality constraints. Obviously, when *Ng* = 33, the results with and without constraints are coincident (i.e., there is no additional reduction).

The results for *Ng* = 3 directly compare the grouping proposed in the original design with that obtained using cardinality constraints. It can be observed that, even for a smaller number of groups than that of the original design, it is possible to reduce the costs remarkably. In order to highlight this, an analysis with *Ng* = 2 was also performed. Fig. 5 illustrates the optimized groupings for two and three types of sections. The difference in cost between these groups is negligible (around 3%), with the possibility that some less required elements have a lower height.

Table 3
Results obtained for example 2.

	Initial (*Ng* = 3)	Optimized (*Ng* = 3)	Optimized (*Ng* = 5)	Optimized (*Ng* = 10)	Optimized (*Ng* = 33)
Overall cost (R$)	7111.93	5441.68	5244.79	5089.09	4682.88
Reduction compared to the original structure (%)	–	23.9	26.3	28.4	34.1

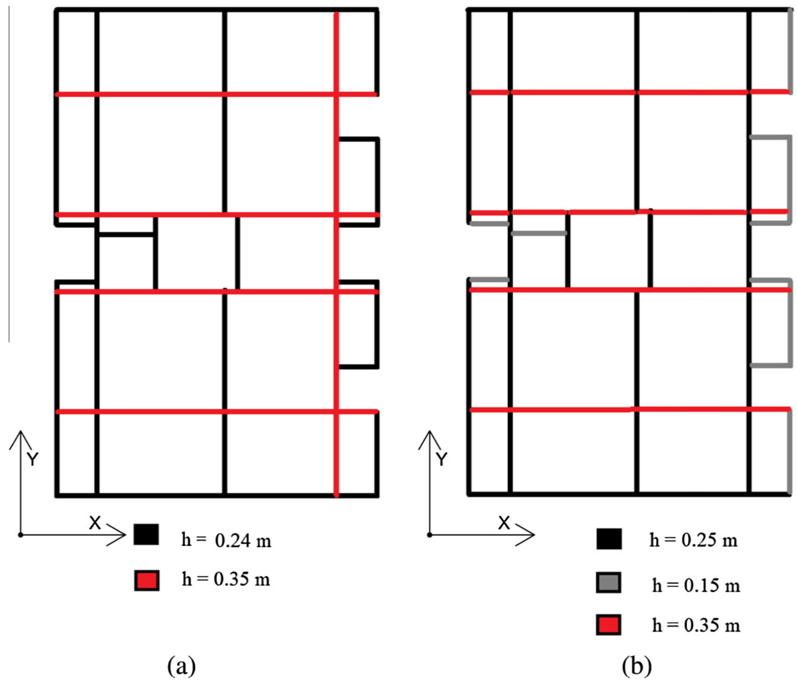

Fig. 5. Optimal heights for: (a) $Ng = 2$ (R$4936) and (b) $Ng = 3$ (R$4770).

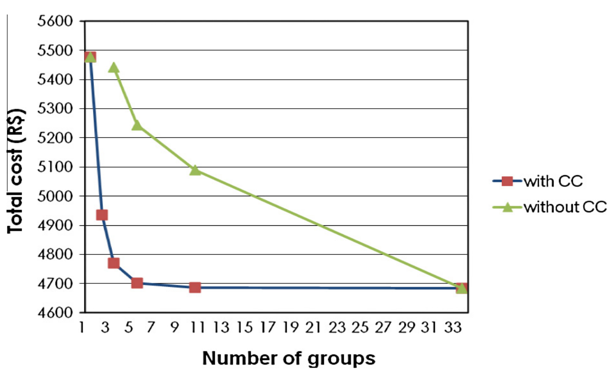

Fig. 6. Optimized cost variation (with and without cardinality constraints).

The results for example 2, including the analysis of the cardinality constraints with $Ng = 2$, are summarized in Fig. 6. This clearly shows how the grouping of elements significantly affects the final outcomes, and that the efficiency of such a grouping basically depends on the designer's experience in assigning the beams to a given group. Cardinality constraints, by allowing the automatic grouping of elements, may drastically reduce this dependence.

5. Conclusions

The aim of the present paper was to apply the automatic grouping of elements, using cardinality constraints, to optimize the cost of reinforced concrete structures assessed by a grid model. Bearing that in mind, an optimization technique was implemented computationally by combining simulated annealing optimization method with a software program for the analysis of structures using the grid model. The sizing of structural elements in terms of the ultimate and serviceability limit states was based on the Brazilian NBR 6118 technical standard [1]. In contrast to the generally adopted formulations in the optimization routines, where only the optimized sections are obtained, this study allowed element groups to be defined in a way that provided the lowest total cost for the structure by including cardinality restrictions. The authors are unaware of the existence of a similar study on reinforced concrete structures or the structural model chosen here.

Based on the formulation implementation and the results obtained from the analyses of the two structures presented, it was possible to conclude the following:

- The simulated annealing method was efficient at obtaining optimized structures, making it possible to work with problems involving discrete variables.
- The developed formulation provided a significant reduction in the cost of a structure compared to a procedure that does not determine the optimized grouping.
- When the cardinality constraints were not considered, the results were highly dependent on the grouping performed by the designer, and, therefore, on their expertise and intuition.
- The economic benefits provided were obtained while observing all project constraints, i.e., without undermining safety.
- Even for a small number of groups, the economic benefit provided by the use of the cardinality restrictions was significant. In the examples analyzed, the results obtained for small groups ($Ng = 2$ and $Ng = 3$) were similar to those obtained when each element took a different section (Ng equal to the total number of elements). This last situation does not correspond to a practically applicable solution because of both construction difficulties and esthetic issues.

The developed formulation was implemented according to the Brazilian standard. For the region where the study was performed, exceptional actions such as earthquakes are not considered. Thus, regarding the examples presented, different dimensions for the elements may be obtained if other technical standards, loads, and structural models are used. However, it is understood that the eventual percentage differences in the results do not invalidate the proposed procedure nor the conclusions presented here.

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