

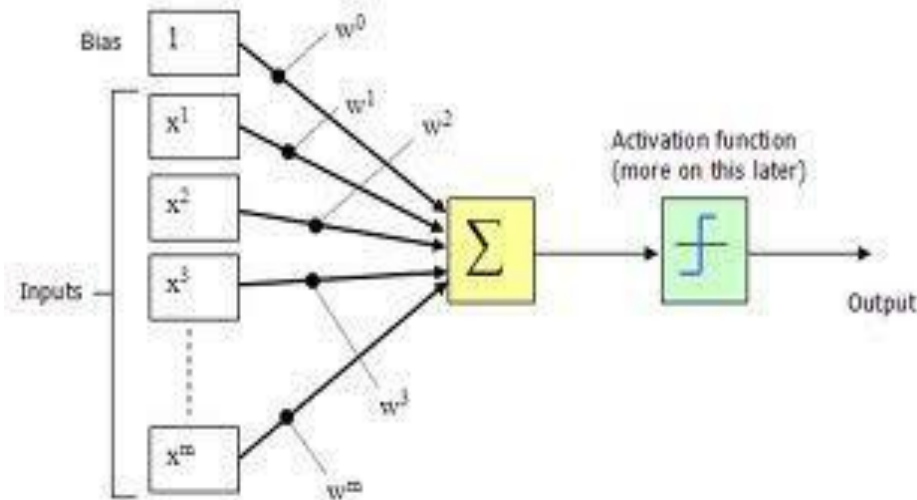
# Generic Features

# Contents

- Neural network
- Covolutional neural network

# Perceptron

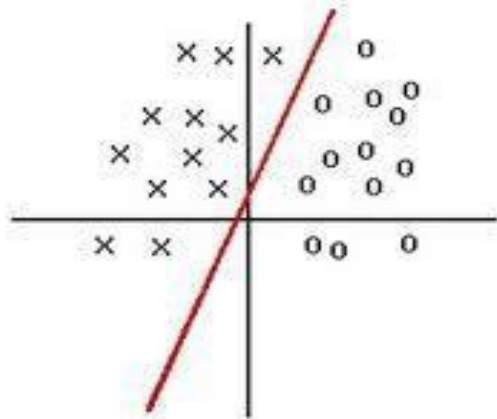
- Perceptron is the basic element of neural network.



- Output 
$$o(\vec{x}) = \text{sng}(\vec{w} \cdot \vec{x}) = \begin{cases} 1 & \text{nếu } \vec{w} \cdot \vec{x} > 0 \\ -1 & \text{nếu ngược lại} \end{cases}$$

# Perceptron

- Perceptron is a hyperplane which separates data into two parts



# Perceptron

- Weight 's updating:

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = \eta(t - o)x_i$$

- If we know or define the loss (error) function  $E$

$$\nabla E(\vec{w}) = \left[ \frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \dots, \frac{\partial E}{\partial w_n} \right]$$

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = \eta \sum_{d \in D} (t_d - o_d)x_{id}$$

# Perceptron

- Error function and weight is updated by gradient of error function

$$E(\vec{w}) = \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2$$

$$\nabla E(\vec{w}) = \left[ \frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \dots, \frac{\partial E}{\partial w_n} \right]$$

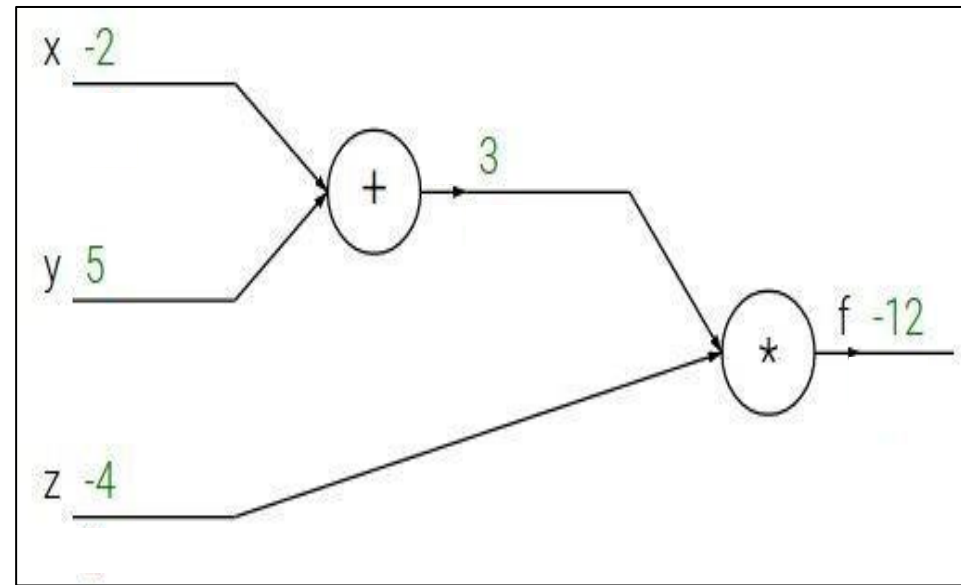
$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = \eta \sum_{d \in D} (t_d - o_d) x_{id}$$

# **Back propagation**

$$f(x, y, z) = (x + y)z$$

e.g.  $x = -2$ ,  $y = 5$ ,  $z = -4$





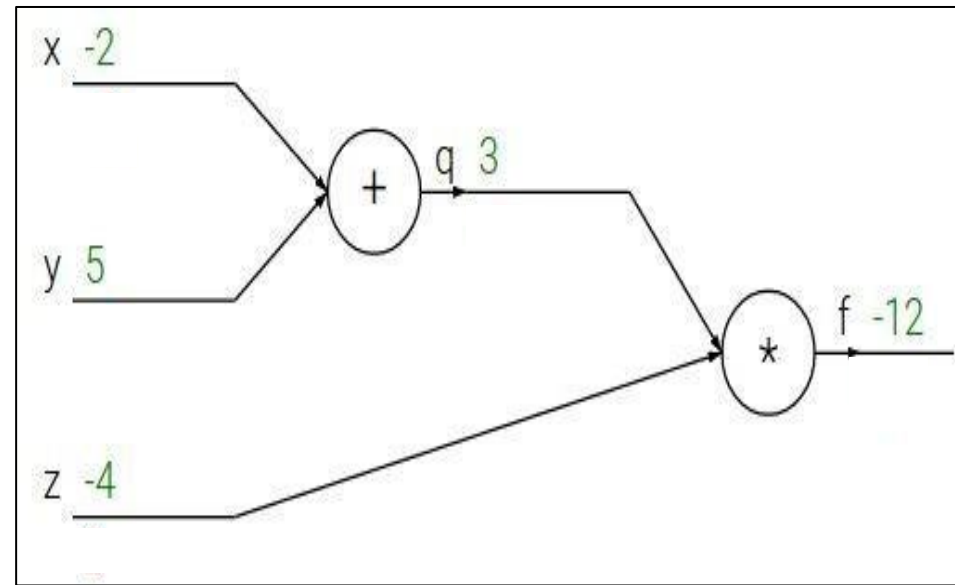
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$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

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Want:  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



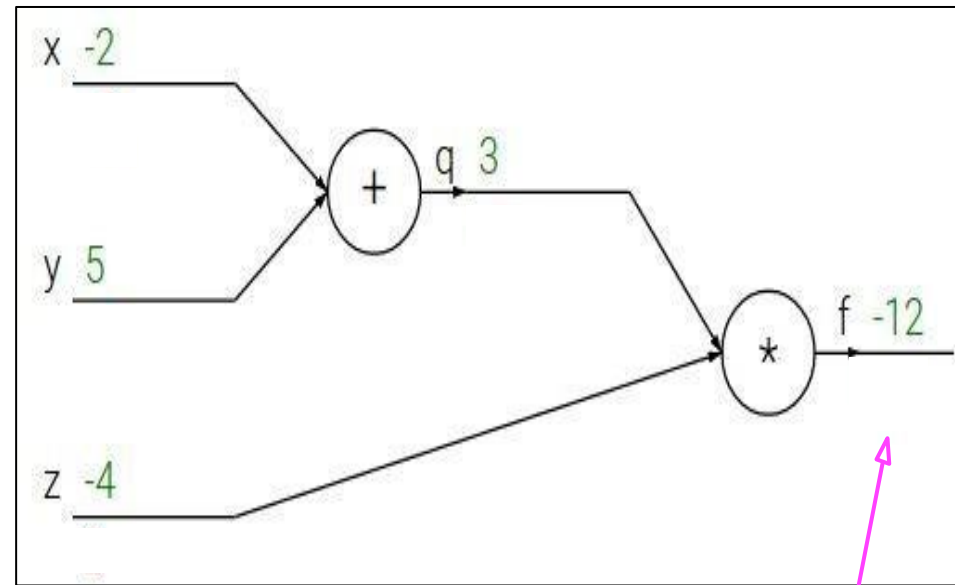
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$$\frac{\partial f}{\partial f}$$

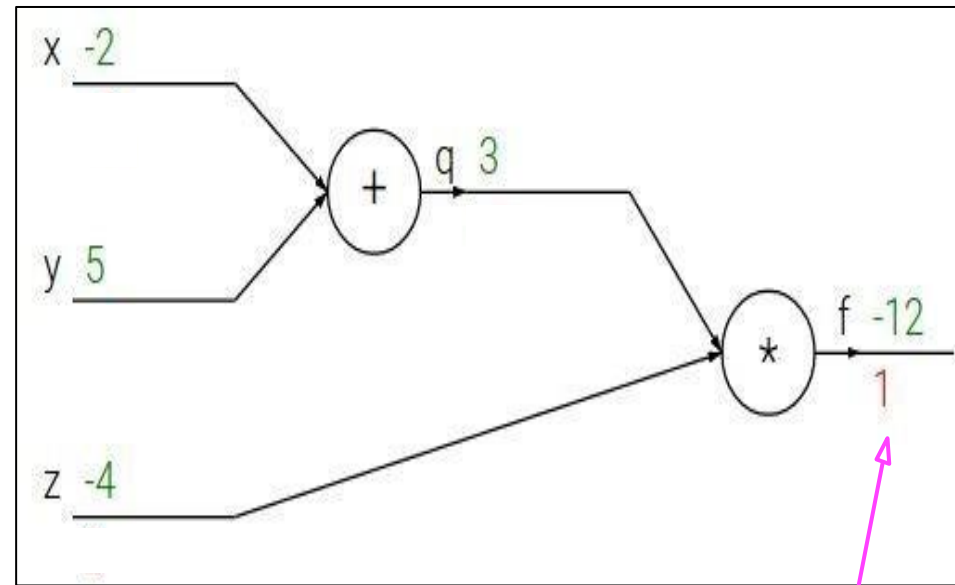
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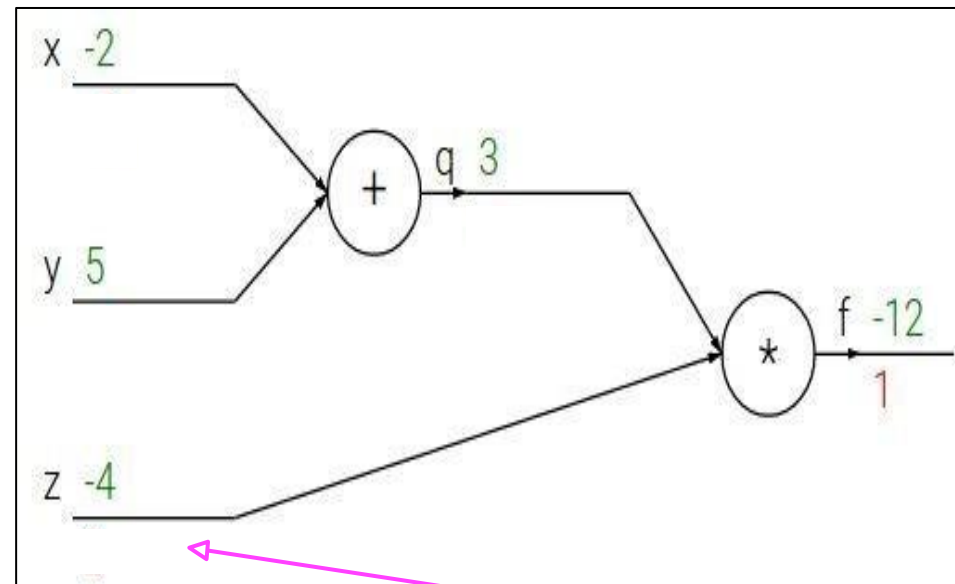
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$$\frac{\partial f}{\partial z}$$

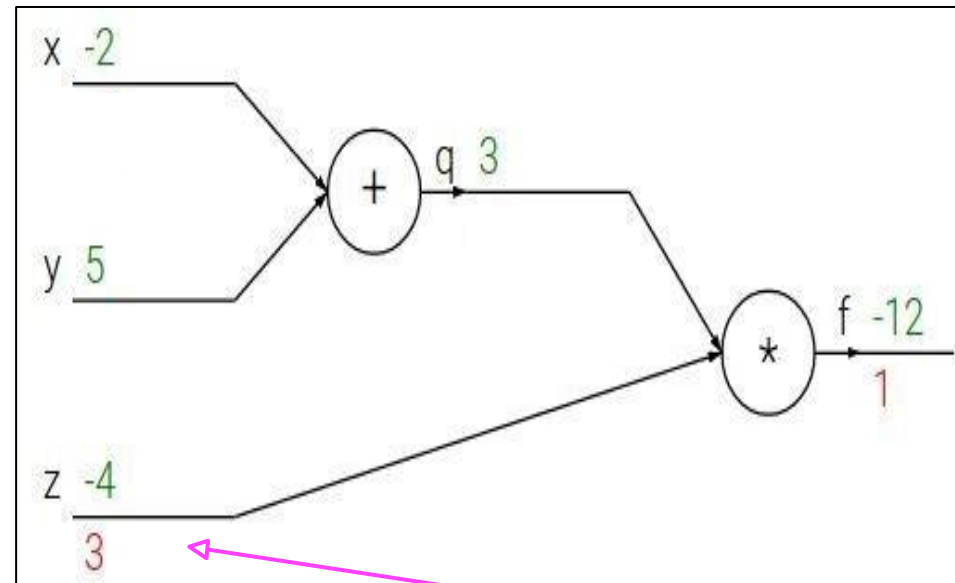
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Want:  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



$$\frac{\partial f}{\partial z}$$

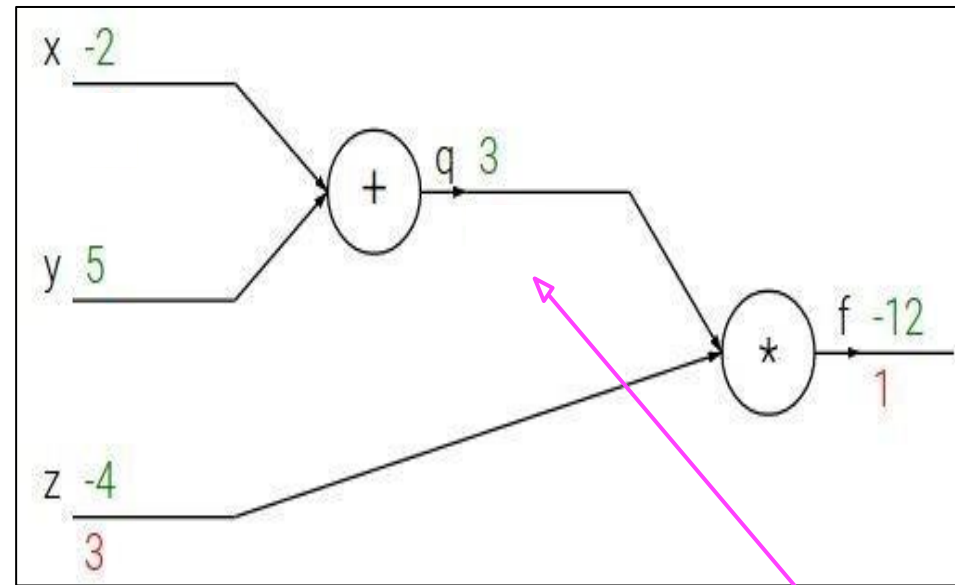
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Want:  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



$$\frac{\partial f}{\partial q}$$

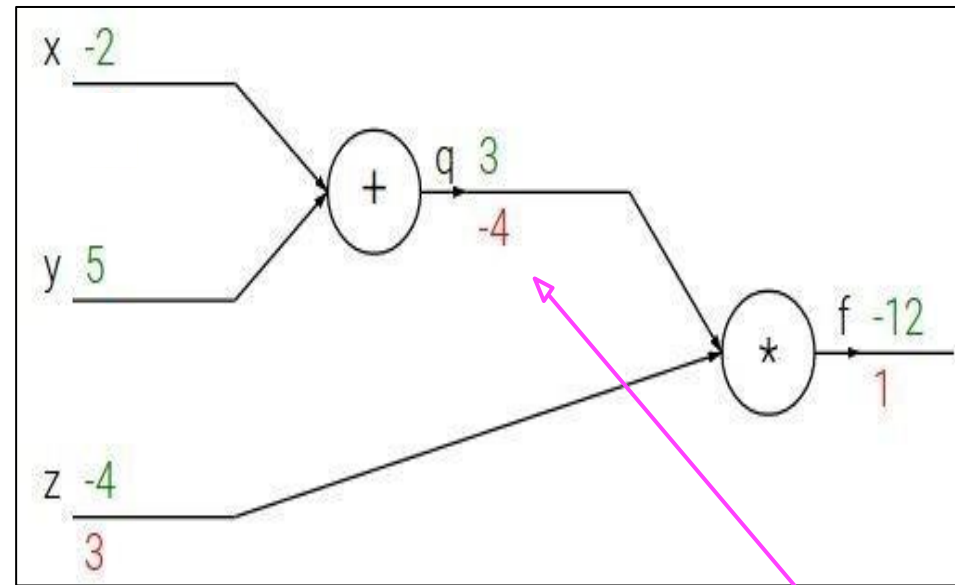
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$$\frac{\partial f}{\partial q}$$

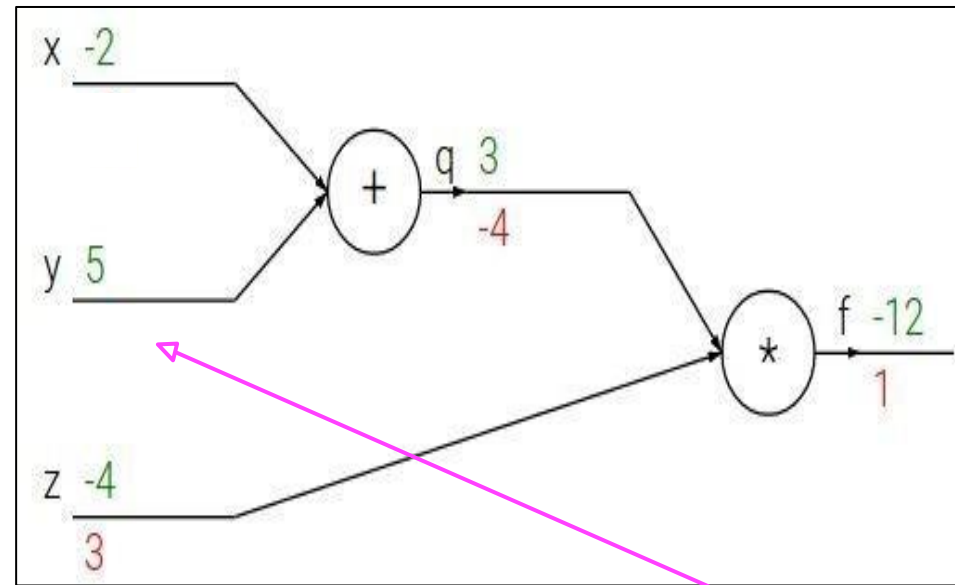
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$$\frac{\partial f}{\partial y}$$



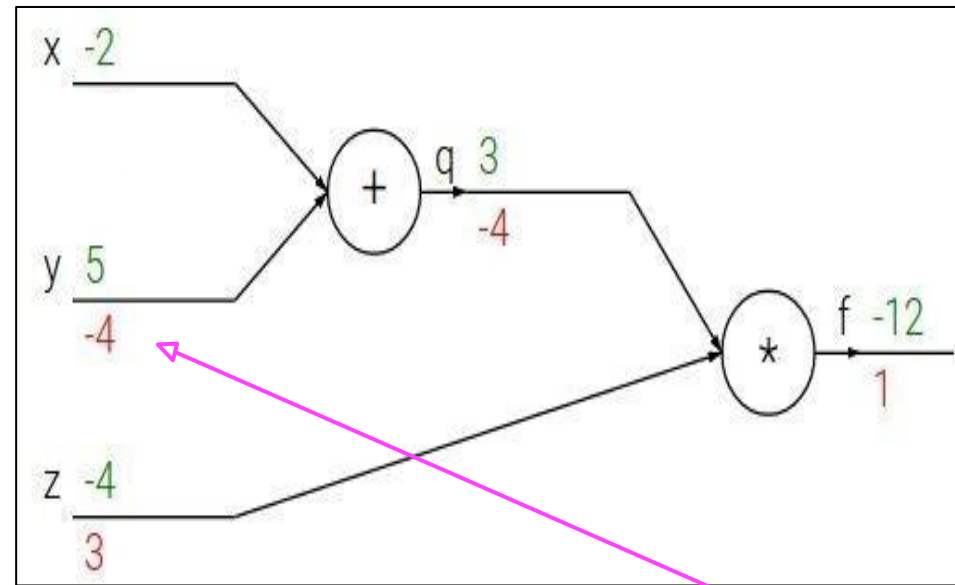
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Want:  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Chain rule:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

$$\frac{\partial f}{\partial y}$$

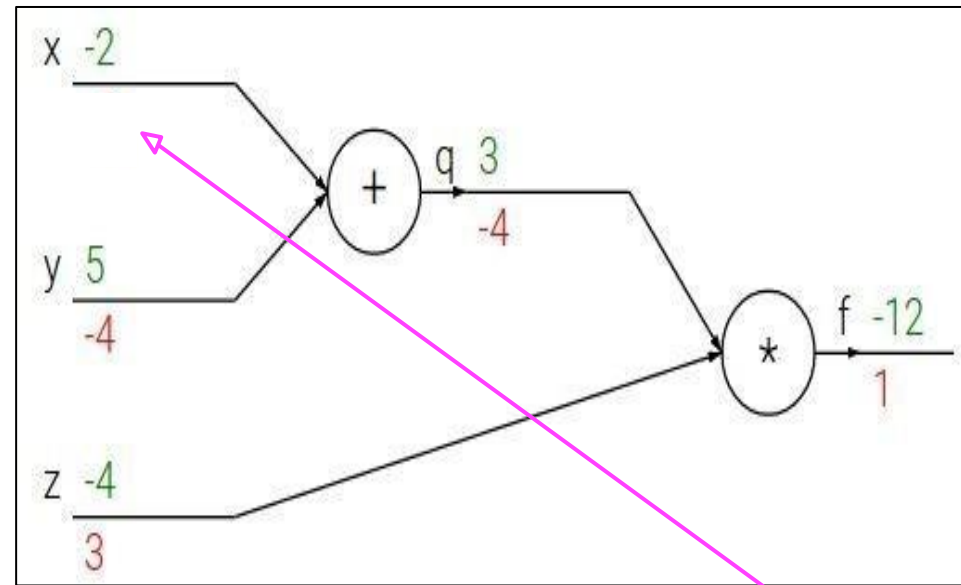
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Want:  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



$$\frac{\partial f}{\partial x}$$

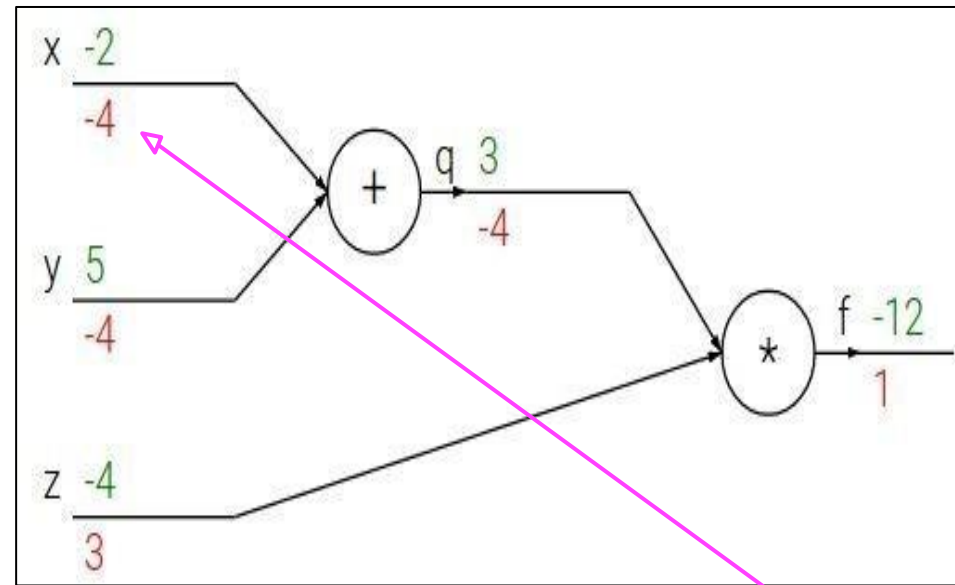
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Want:  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



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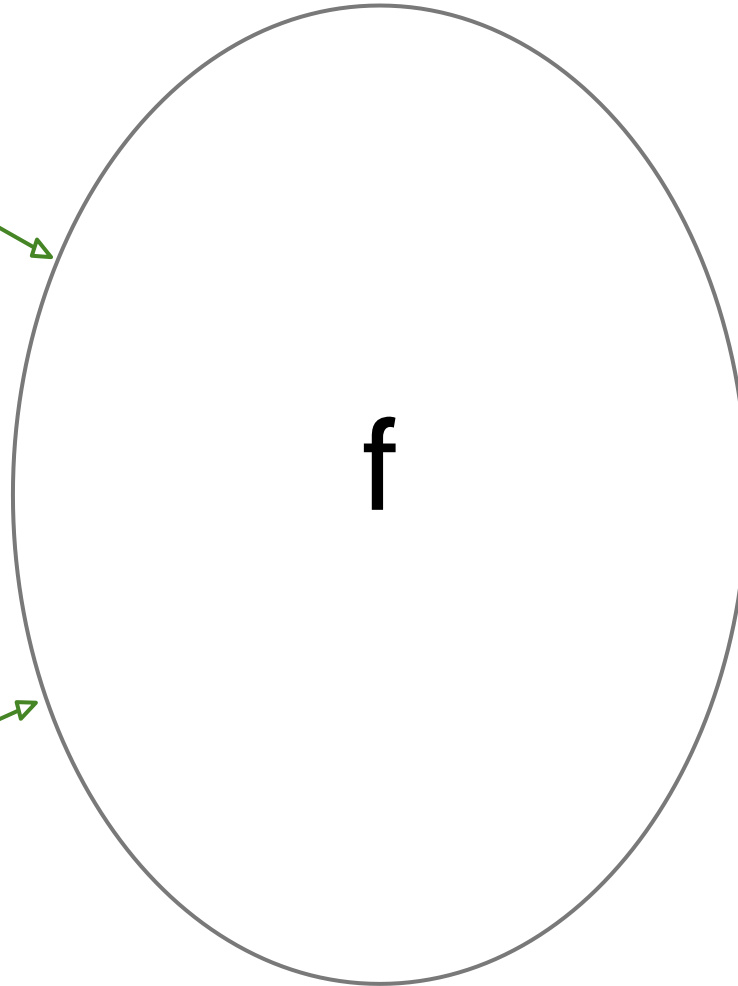
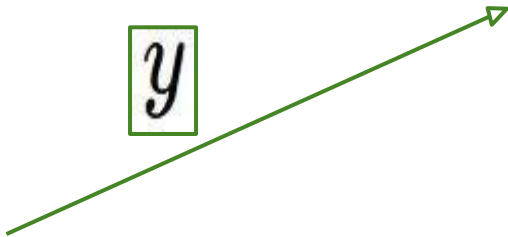
$$\frac{\partial f}{\partial x}$$

activations

$x$



$y$



$z$



activations

$x$

“local gradient”

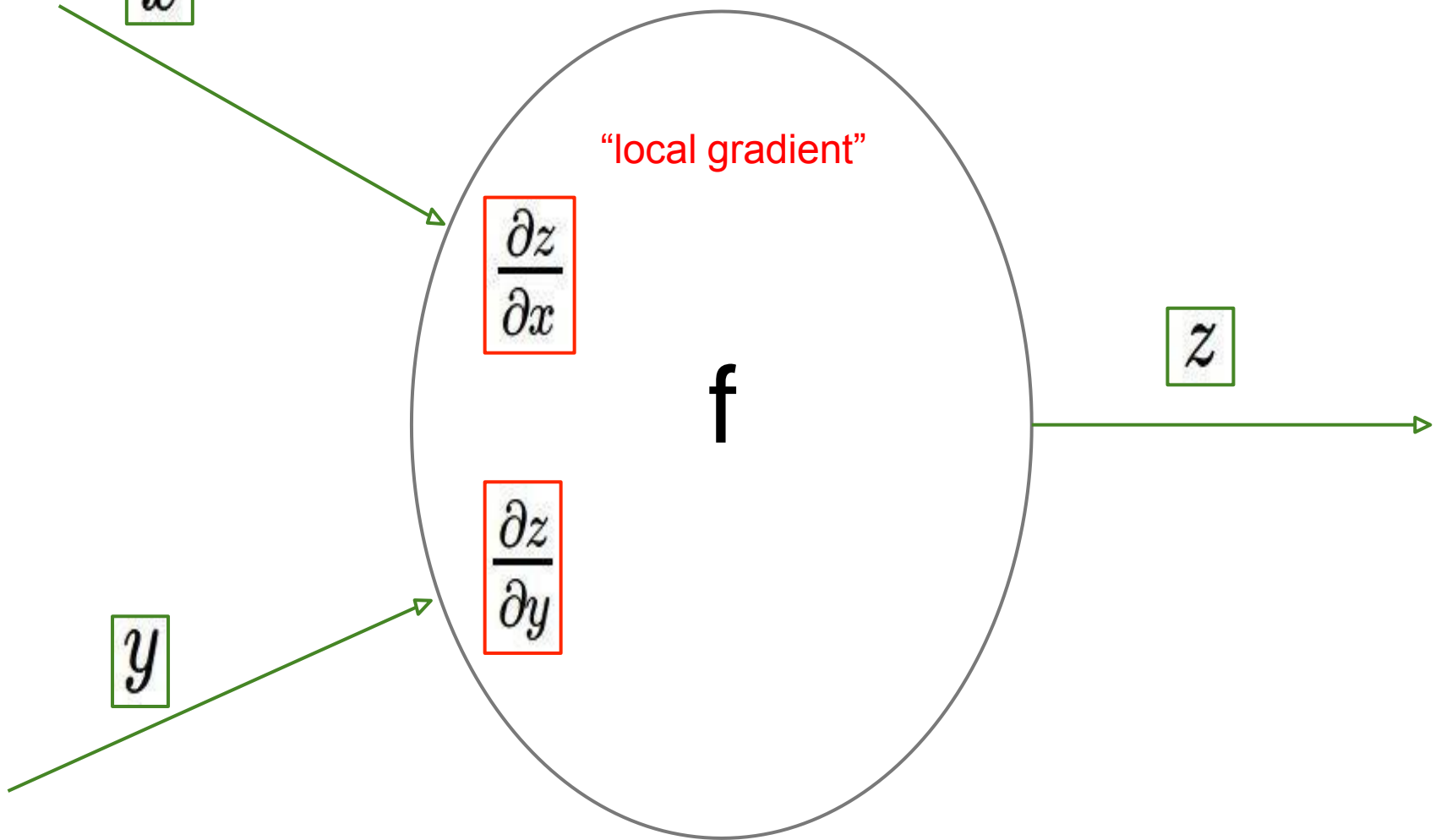
$$\frac{\partial z}{\partial x}$$

**f**

$$\frac{\partial z}{\partial y}$$

$y$

$z$



activations

$$x$$

“local gradient”

$$\frac{\partial z}{\partial x}$$

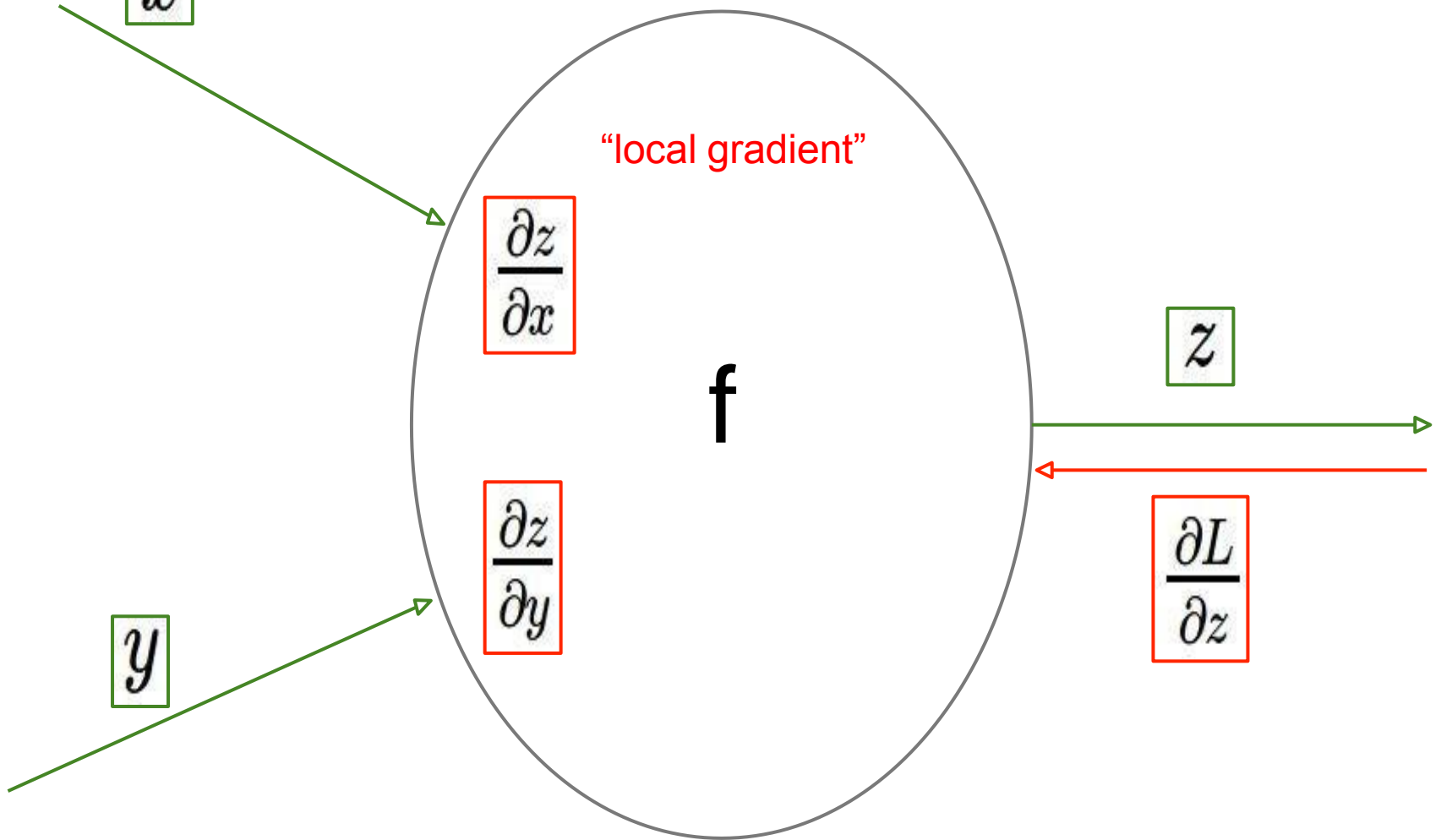
**f**

$$\frac{\partial z}{\partial y}$$

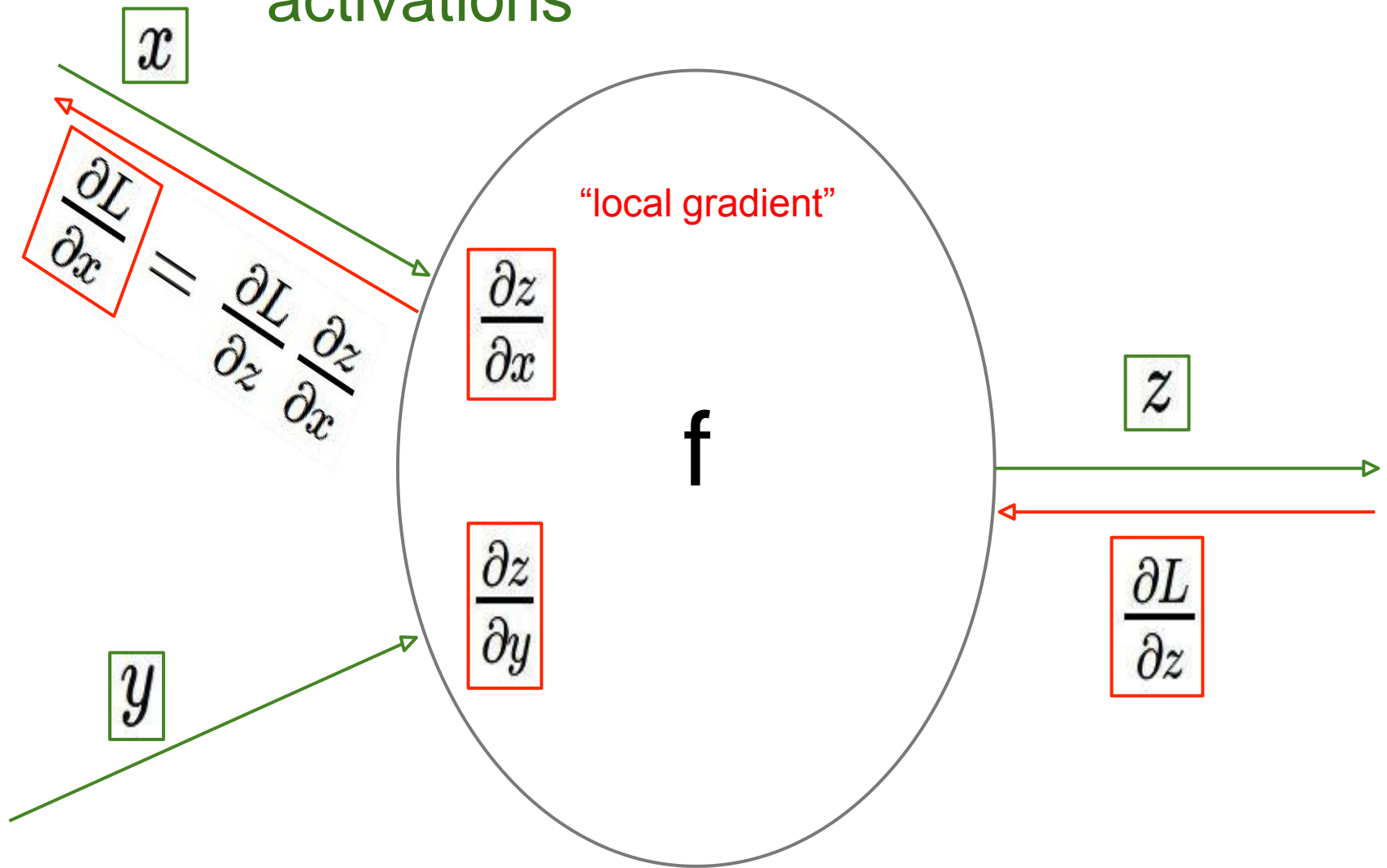
$$y$$

$$z$$

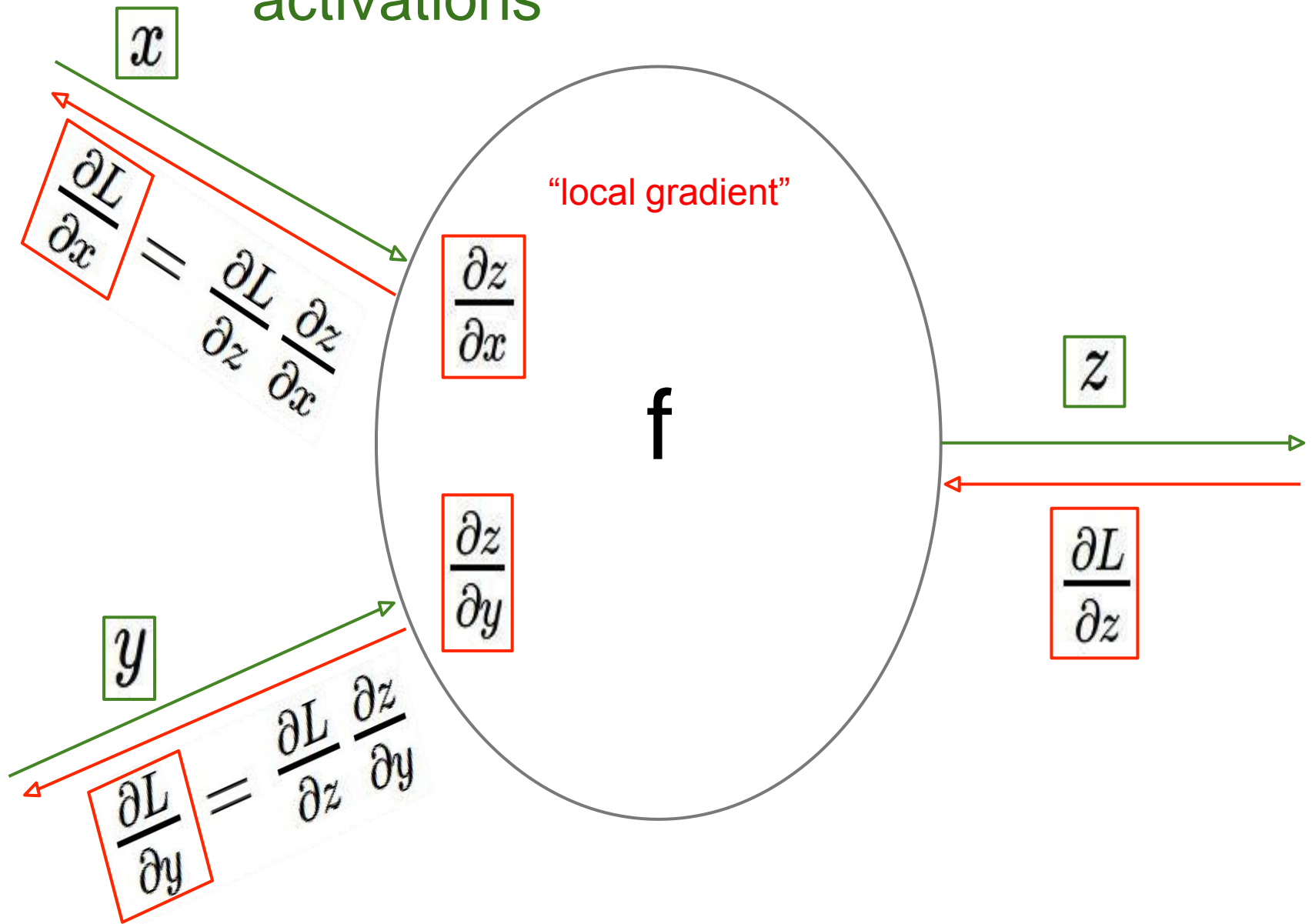
$$\frac{\partial L}{\partial z}$$



activations



activations





activations

$x$

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z} \frac{\partial z}{\partial x}$$

“local gradient”

$$\frac{\partial z}{\partial x}$$

$f$

$$\frac{\partial z}{\partial y}$$

$y$

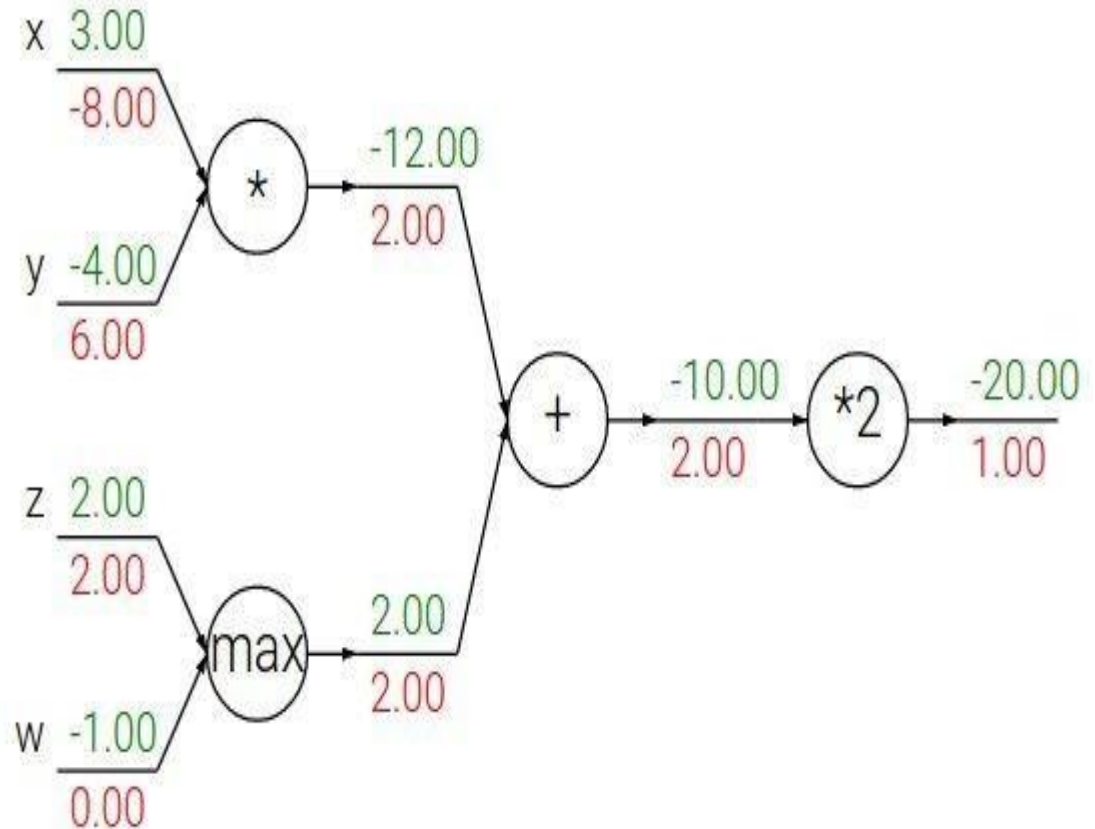
$$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z} \frac{\partial z}{\partial y}$$

$z$

$$\frac{\partial L}{\partial z}$$

# Patterns in backward flow

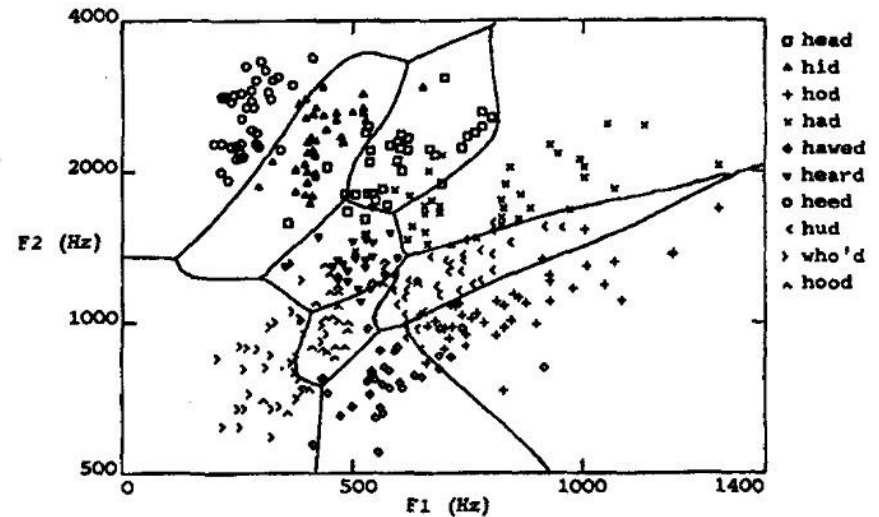
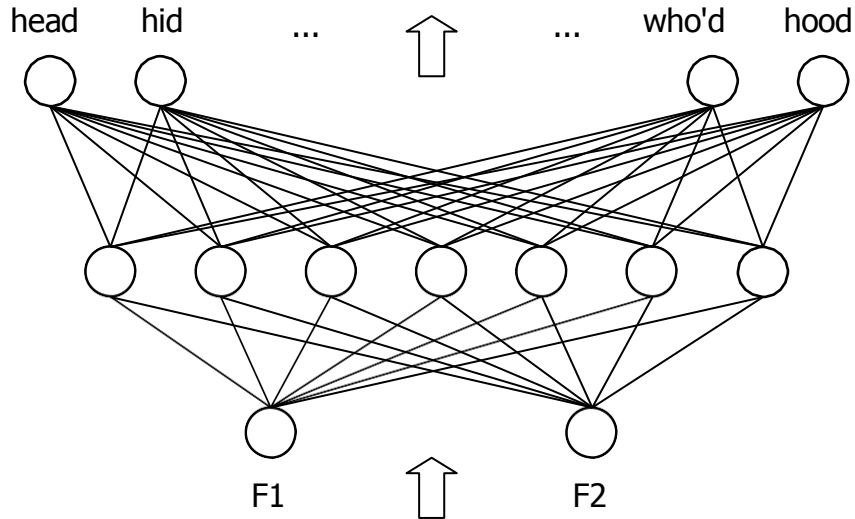
**add** gate: gradient distributor  
**max** gate: gradient router  
**mul** gate: gradient... “switcher”?



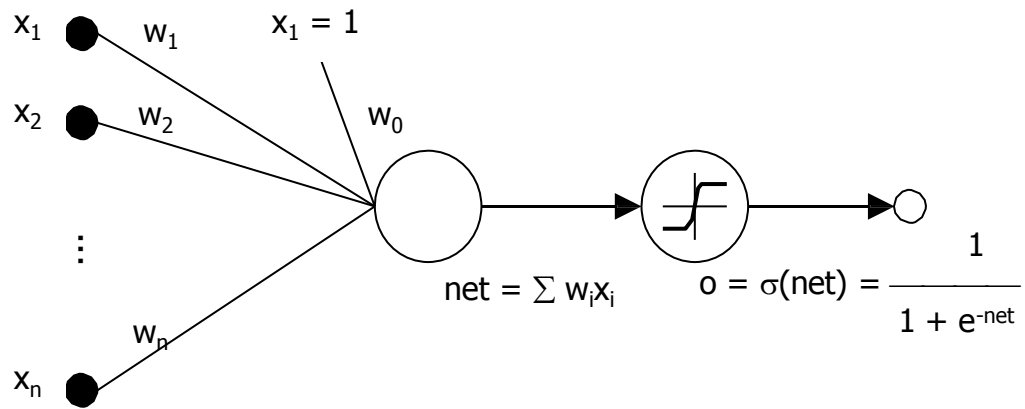
# Multilayer network

Perceptron is a linear classifier.

Multilayer network is a nonlinear classifier.



# Sigmoid function



$$\sigma(y) = \frac{1}{1 + e^{-y}}$$

$$\frac{d\sigma(y)}{dy} = \sigma(y) \cdot (1 - \sigma(y))$$

# Error function

Def :

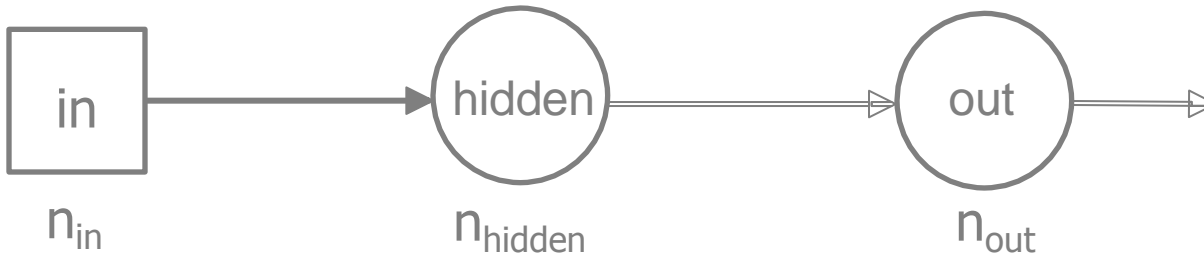
$$E(w) = \frac{1}{2} \sum_{d \in D} \sum_{k \in \text{outputs}} (t_{kd} - o_{kd})^2$$

outputs: set of outputs

$t_{kd}$  ,  $o_{kd}$  : true value and estimated value of outputs

# Forward process

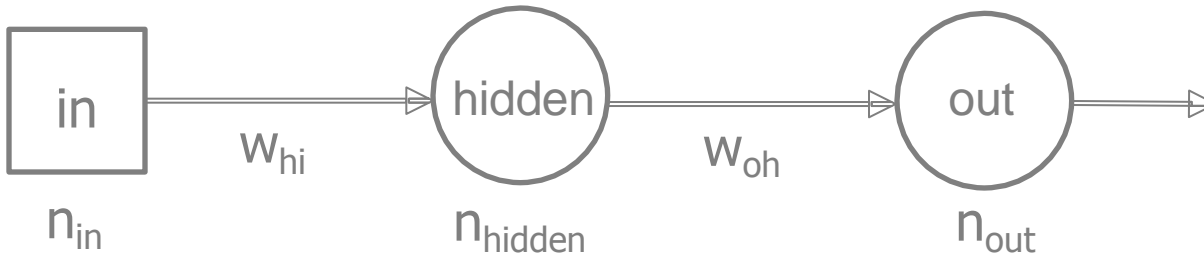
**BackPropagation** (training\_examples,  $\eta$ ,  $n_{in}$ ,  $n_{out}$ ,  $n_{hidden}$ )



$n_{in}$  Inputs,  
 $n_{hidden}$  hidden nodes,  
 $n_{out}$  Outputs.

# Forward process

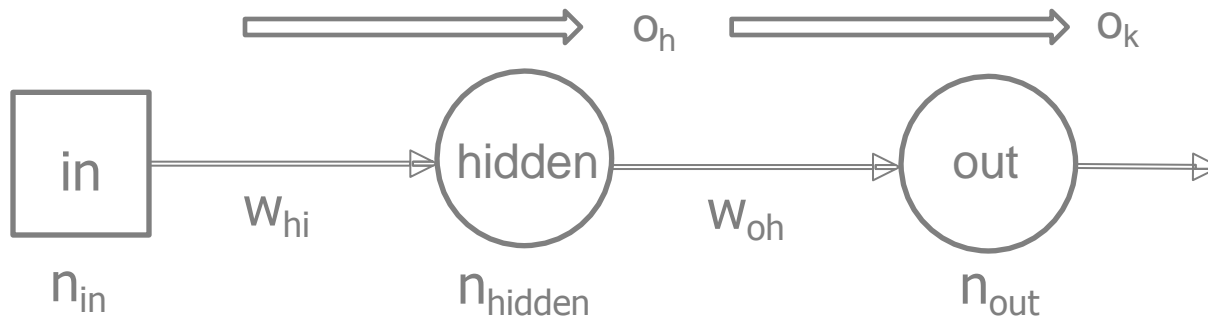
**BackPropagation** (training\_examples,  $\eta$ ,  $n_{in}$ ,  $n_{out}$ ,  $n_{hidden}$ )



Randomize initial weight

# Forward process

**BackPropagation** (training\_examples,  $\eta$ ,  $n_{in}$ ,  $n_{out}$ ,  $n_{hidden}$ )



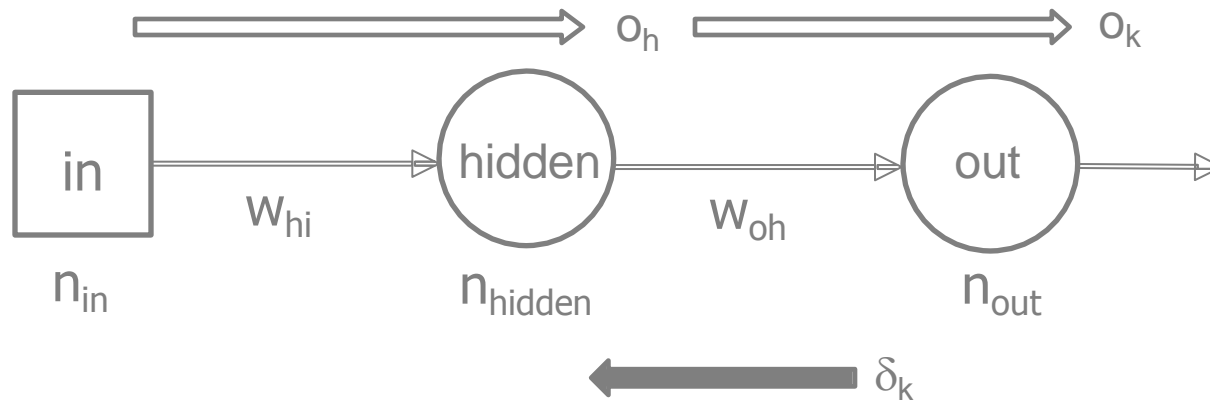
**Push each input ( $x, y$ ) into neuron network :**  
 **$x$  – data;  $y$  - label**

1. For each input  $x$ , calculate output  $o_u$  for each neuron  $u$  of the network.



# Backward process

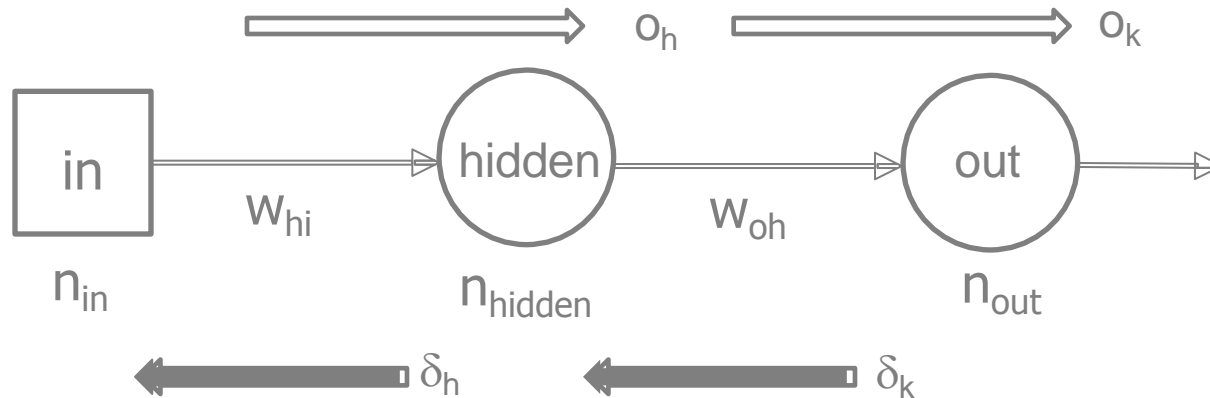
**BackPropagation** (training\_examples,  $\eta$ ,  $n_{in}$ ,  $n_{out}$ ,  $n_{hidden}$ )



2. For each output  $o_k$ , calculate transferred error  $\delta_k$   
$$\delta_k = o_k(1 - o_k)(t_k - o_k)$$

# Backward process

**BackPropagation** (training\_examples,  $\eta$ ,  $n_{in}$ ,  $n_{out}$ ,  $n_{hidden}$ )

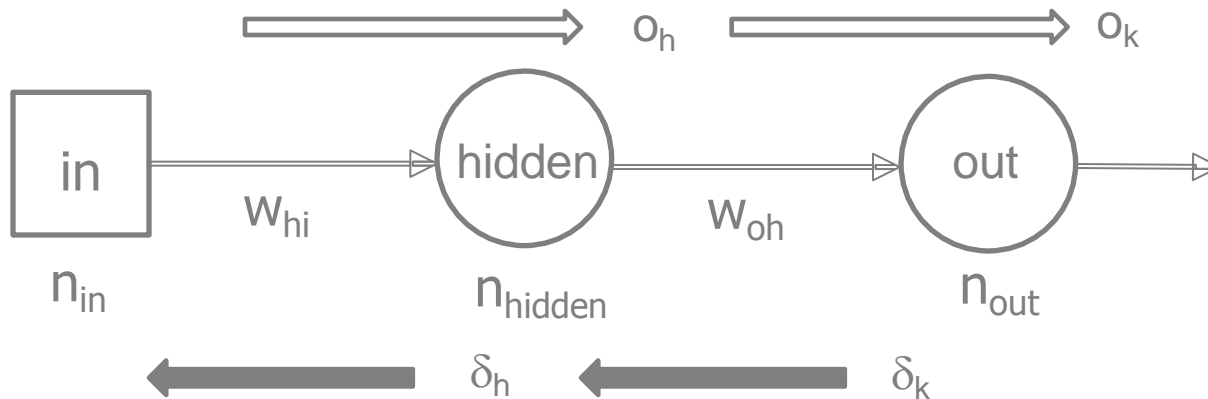


3. For each output of hidden neuron  $o_h$ , calculate transferred error  $\delta_h$

$$\delta_h = o_h(1 - o_h) \sum_{k \in \text{outputs}} w_{kh} \delta_k$$

# Backward process

**BackPropagation** (training\_examples,  $\eta$ ,  $n_{in}$ ,  $n_{out}$ ,  $n_{hidden}$ )

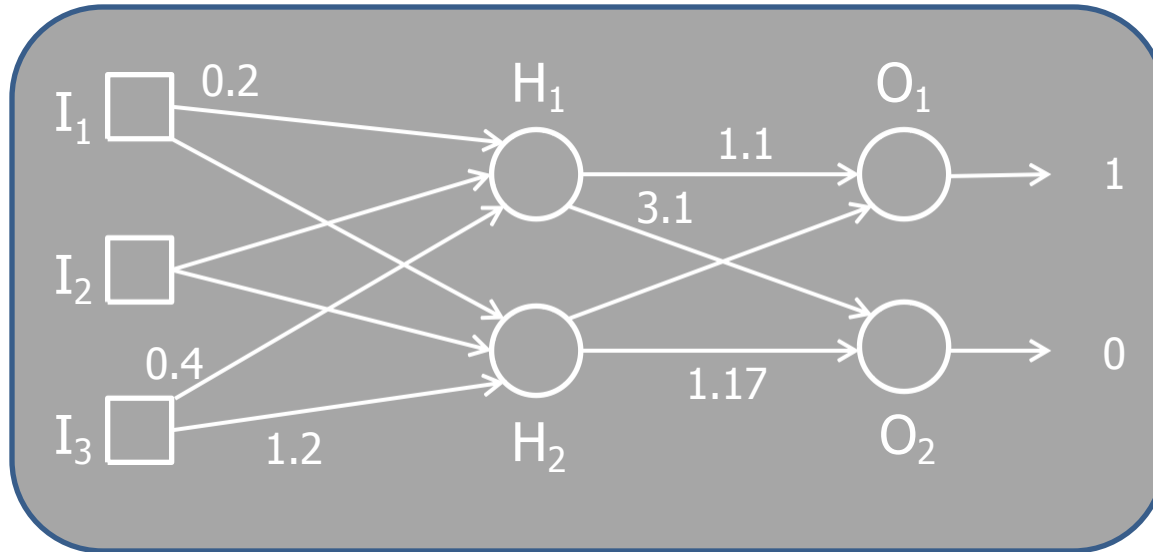


4. Update  $w_{ji}$

$$w_{ji} \leftarrow w_{ji} + \Delta w_{ji}$$

$$\Delta w_{ji} = \eta \delta_j x_{ji}$$

# Examples



Input:  $\vec{x} = (10, 30, 20)$

Target:  $\vec{t} = (1, 0)$

Learning rate:  $\eta = 0.1$

# Examples

1. For each input  $x$ , calculate output  $o_u$  corresponding to neuron  $u$  in network

$$o = \sigma(\text{net}) = \frac{1}{1 + e^{-\text{net}}} \quad \text{with } \text{net} = \sum w_{ji} x_{ji}$$

$$H_1: \text{net}_{H1} = 10 * 0.2 + 30 * (-0.1) + 20 * 0.4 = 7$$

$$o_{H1} = \sigma(\text{net}_{H1}) = 0.9990$$

$$H_2: \text{net}_{H2} = 10 * 0.7 + 30 * (-1.2) + 20 * 1.2 = -5$$

$$o_{H2} = \sigma(\text{net}_{H2}) = 0.0067$$

$$O_1: \text{net}_{O1} = 0.9990 * 1.1 + 0.0067 * 0.1 = 1.0996$$

$$o_{O1} = \sigma(\text{net}_{O1}) = 0.7501$$

$$O_2: \text{net}_{O2} = 0.9990 * 3.1 + 0.0067 * 1.17 = 3.1047$$

$$o_{O2} = \sigma(\text{net}_{O2}) = 0.9571$$

# Examples

2. For each output  $o_k$  of output layer, calculate error  $\delta_k$

$$\delta_k = o_k(1 - o_k)(t_k - o_k)$$

$$\delta_{O1} = o_{O1}(1 - o_{O1})(t_{O1} - o_{O1}) = 0.750 (1 - 0.750)(1 - 0.750) = 0.0469$$

$$\delta_{O2} = o_{O2}(1 - o_{O2})(t_{O2} - o_{O2}) = 0.957 (1 - 0.957)(0 - 0.957) = - 0.0394$$

3. For each neuron of hidden layer, calculate error  $\delta_h$

$$\delta_h = o_h(1 - o_h) \sum_{k \in \text{outputs}} w_{kh} \delta_k$$

$$\begin{aligned} \delta_{H1} &= o_{H1}(1 - o_{H1})[(w_{11} * \delta_{O1}) + (w_{21} * \delta_{O2})] \\ &= 0.999(1 - 0.999)[(1.1 * 0.0469) + (3.1 * (-0.0394))] \\ &= - 0.0000705 \end{aligned}$$

$$\begin{aligned} \delta_{H2} &= o_{H2}(1 - o_{H2})[(w_{12} * \delta_{O1}) + (w_{22} * \delta_{O2})] \\ &= 0.0067(1 - 0.0067)[(0.1 * 0.0469) + (1.17 * (-0.0394))] \\ &= - 0.000275 \end{aligned}$$

# Examples

4. Update weight Output-Hidden  $w_{ji}$

$$w_{ji} \leftarrow w_{ji} + \Delta w_{ji}$$

$$\Delta w_{ji} = \eta \delta_j x_{ji}$$

Hidden	Output	$\eta$	$\delta_o$	$o_H = x_{ji}$	$\Delta = \eta \delta_o x_{ji}$	Old W	New W
$H_1$	$O_1$	0.1	0.0469	0.999	0.000469	1.1	1.100469
$H_1$	$O_2$	0.1	- 0.0394	0.999	-0.00394	3.1	3.09606
$H_2$	$O_1$	0.1	0.0469	0.0067	0.0000314	0.1	0.1000314
$H_2$	$O_2$	0.1	- 0.0394	0.0067	-0.0000264	1.17	1.1699736

# Examples

## 4. Update weight $w_{ji}$ in Hidden - Input

Input	Hidden	$\eta$	$\delta_H$	$x_I$	$\Delta = \eta \delta_O x_{ji}$	Old W	New W
$I_1$	$H_1$	0.1	-0.0000705	10	-0.0000705	0.2	0.1999295
$I_1$	$H_2$	0.1	-0.000275	10	-0.000275	0.7	0.699725
$I_2$	$H_1$	0.1	-0.0000705	30	-0.0002115	-0.1	-0.1000705
$I_2$	$H_2$	0.1	-0.000275	30	-0.000825	-1.2	-1.200825
$I_3$	$H_1$	0.1	-0.0000705	20	-0.000141	0.4	0.399859
$I_3$	$H_2$	0.1	-0.000275	20	-0.00055	1.2	1.19945



# Examples

Updated value

$$\Delta \mathbf{w}_{ji}(\mathbf{n}) = \eta \delta_i \mathbf{x}_{ji} + \alpha \Delta \mathbf{w}_{ji}(\mathbf{n}-1)$$

$n$ : iteration number

$0 \leq \alpha < 1$ : momentum value

# Equations

- $E_d = \frac{1}{2} \sum_{k \in \text{outputs}} (t_k - o_k)^2$

$$\Delta w_{ji} = -\eta \frac{\partial E_d}{\partial w_{ji}}$$

$$\frac{\partial E_d}{\partial w_{ji}} = \frac{\partial E_d}{\partial \text{net}_j} \frac{\partial \text{net}_j}{\partial w_{ji}} = \frac{\partial E_d}{\partial \text{net}_j} x_{ji}$$

# Equations

Case 1: Updated weights of Output layer

$$\frac{\partial E_d}{\partial \text{net}_j} = \frac{\partial E_d}{\partial o_j} \frac{\partial o_j}{\partial \text{net}_j}$$

$$\begin{aligned} \frac{\partial E_d}{\partial o_j} &= \frac{\partial}{\partial o_j} \frac{1}{2} \sum_{k \in \text{outputs}} (t_k - o_k)^2 = \frac{\partial}{\partial o_j} \frac{1}{2} (t_j - o_j)^2 \quad \left( \frac{\partial}{\partial o_j} (t_k - o_k)^2 = 0 \text{ với } k \neq j \right) \\ &= 2 \cdot \frac{1}{2} (t_j - o_j) \frac{\partial (t_j - o_j)}{\partial o_j} = -(t_j - o_j) \end{aligned}$$

$$\frac{\partial o_j}{\partial \text{net}_j} = \frac{\partial \sigma(\text{net}_j)}{\partial \text{net}_j} = o_j(1 - o_j)$$

$$\frac{\partial E_d}{\partial \text{net}_j} = -(t_j - o_j) o_j(1 - o_j) \Rightarrow \boxed{\Delta w_{ji} = \eta (t_j - o_j) o_j(1 - o_j) x_{ji}}$$

# Equations

Case 2: Updated weights of Hidden – Input

$$\begin{aligned}\frac{\partial E_d}{\partial \text{net}_j} &= \sum_{k \in \text{Downstream}(j)} \frac{\partial E_d}{\partial \text{net}_k} \frac{\partial \text{net}_k}{\partial \text{net}_j} \\&= \sum_{k \in \text{Downstream}(j)} -\delta_k \frac{\partial \text{net}_k}{\partial \text{net}_j} = \sum_{k \in \text{Downstream}(j)} -\delta_k \frac{\partial \text{net}_k}{\partial o_j} \frac{\partial o_j}{\partial \text{net}_j} \\&= \sum_{k \in \text{Downstream}(j)} -\delta_k w_{kj} \frac{\partial o_j}{\partial \text{net}_j} = \sum_{k \in \text{Downstream}(j)} -\delta_k w_{kj} o_j(1 - o_j)\end{aligned}$$

$$\text{Let } \delta_j = - \frac{\partial E_d}{\partial \text{net}_j} \Rightarrow \delta_j = o_j(1 - o_j) \sum_{k \in \text{Downstream}(j)} -\delta_k w_{kj}$$

$$\Delta w_{ji} = \eta \delta_j x_{ji}$$

# Convolutional Networks

Neural Networks that use convolution in place of general matrix multiplication in atleast one layer

Next:

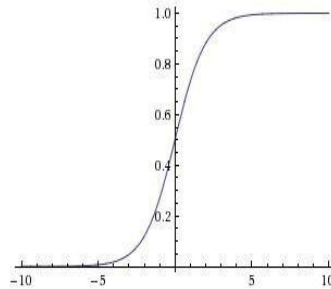
- What is convolution?
- What is pooling?
- What is the motivation for such architectures (remember LeNet?)

# **Activation function**

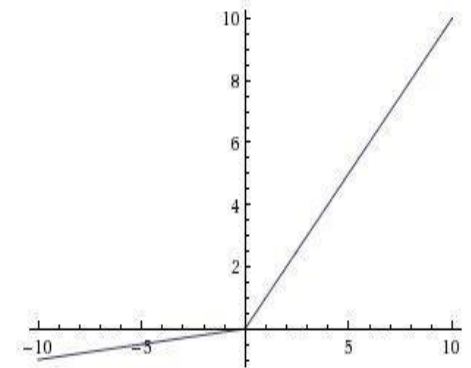
# Activation Functions

**Sigmoid**

$$\sigma(x) = 1/(1 + e^{-x})$$



**Leaky  
ReLU**

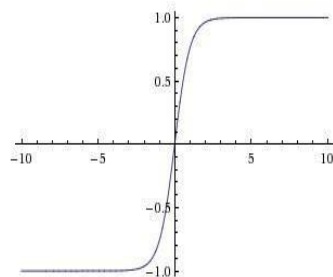


**Maxou**

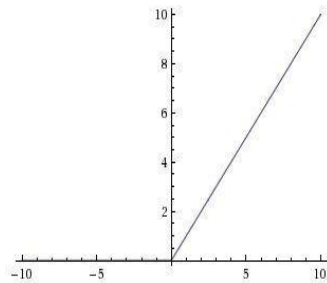
**t ELU**

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

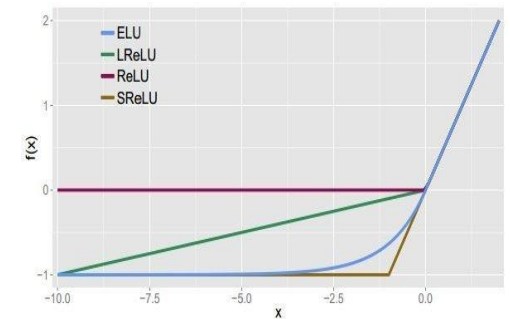
**tanh**     $\tanh(x)$



**ReLU**     $\max(0, x)$

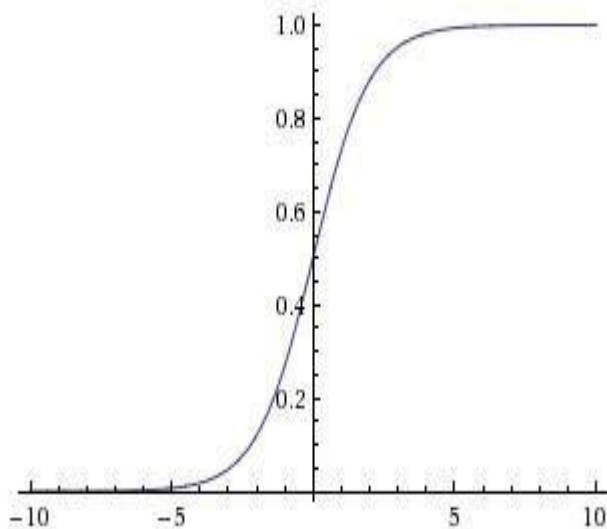


$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha (\exp(x) - 1) & \text{if } x \leq 0 \end{cases}$$



# Activation Functions

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

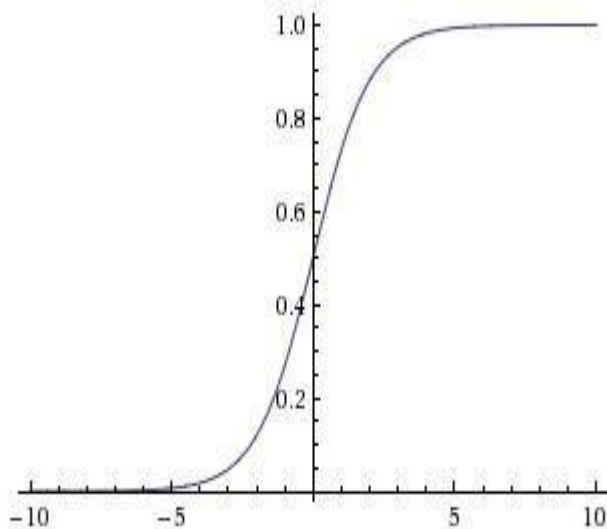


## Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$



# Activation Functions

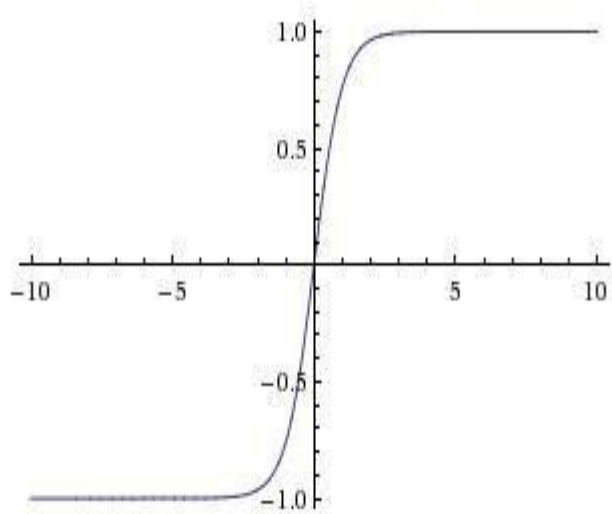


**Sigmoid**

$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
  - Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron
1. Saturated neurons “kill” the gradients
  2. Sigmoid outputs are not zero-centered
  3.  $\exp()$  is a bit compute expensive

# Activation Functions

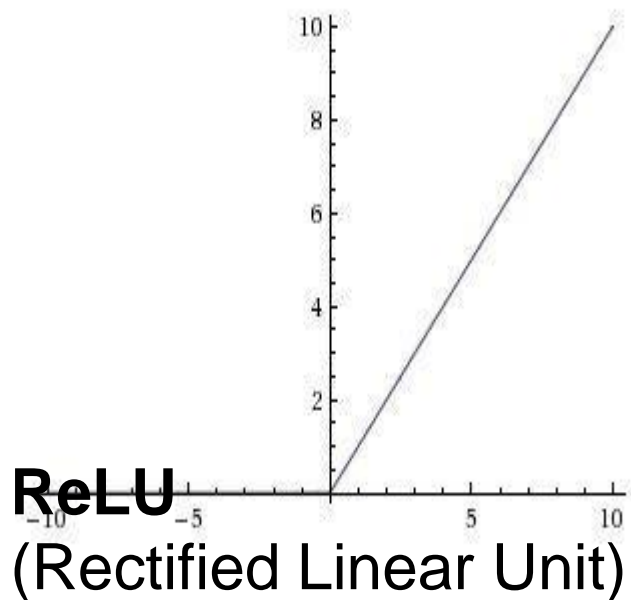


**$\tanh(x)$**

- Squashes numbers to range  $[-1,1]$
- zero centered (nice)
- still kills gradients when saturated :(

[LeCun et al., 1991]

# Activation Functions



Computes  $f(x) = \max(0, x)$

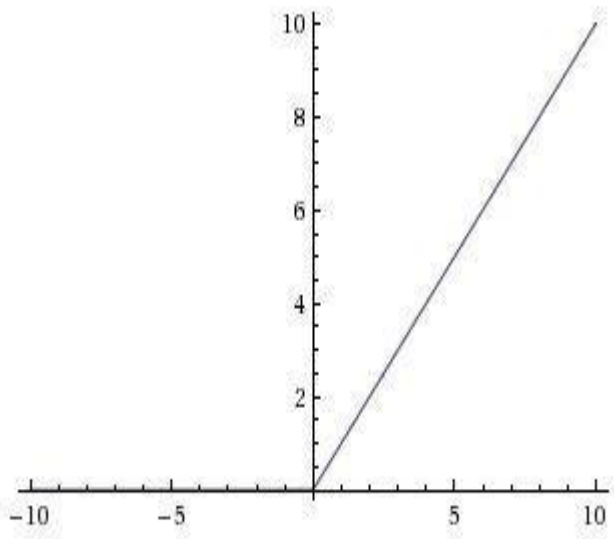
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

[Krizhevsky et al., 2012]

# Activation Functions

Computes  $f(x) = \max(0, x)$

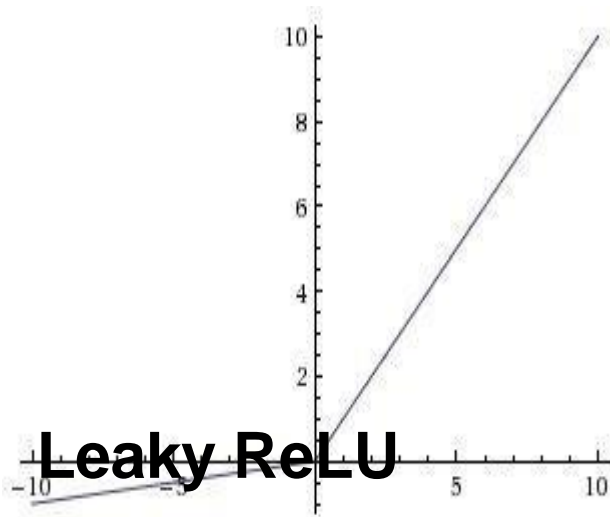
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)



**ReLU**  
(Rectified Linear Unit)

- Not zero-centered output
- ReLU units can “die”

# Activation Functions

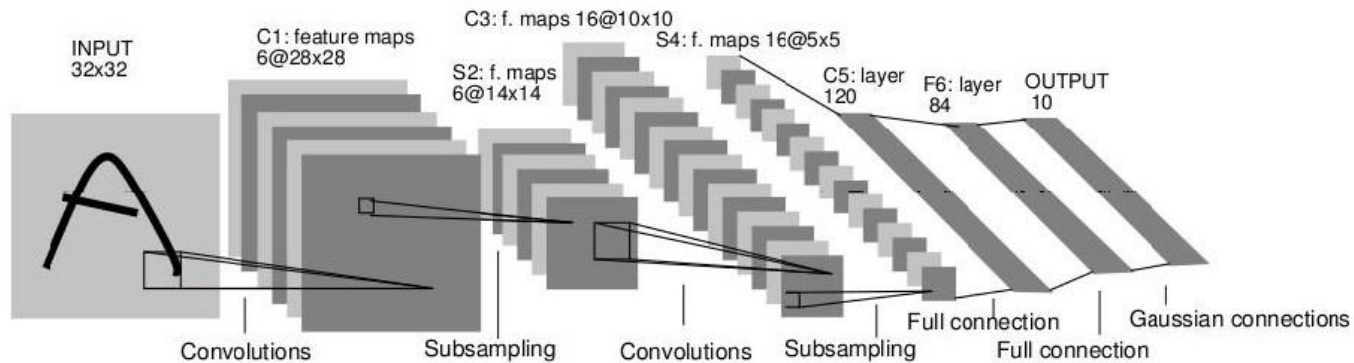


- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

[Mass et al., 2013] [He et al., 2015]

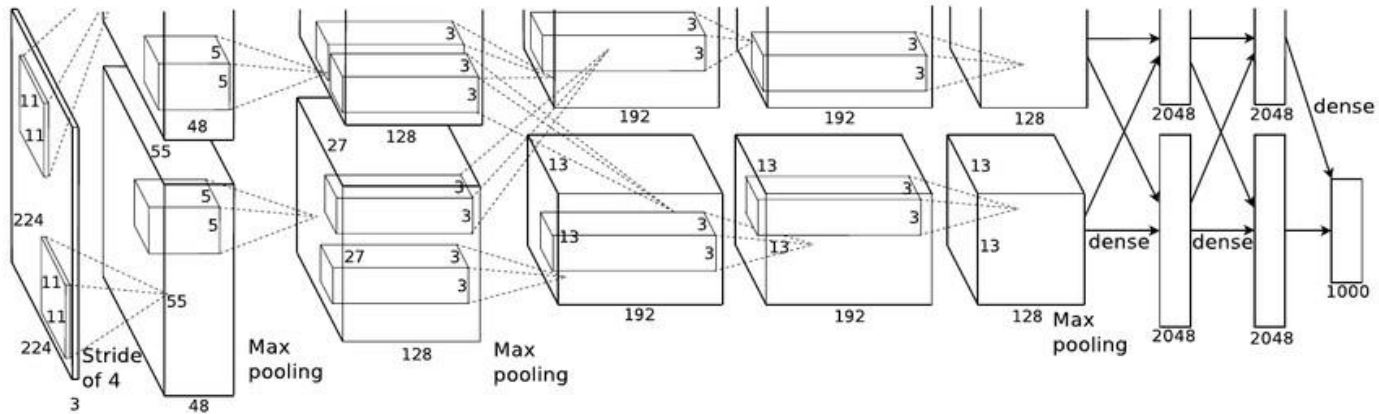
$$f(x) = \max(0.01x, x)$$

# LeNet-5 (LeCun, 1998)



The original Convolutional Neural Network model goes back to 1989 (LeCun)

# AlexNet (Krizhevsky, Sutskever, Hinton 2012)



ImageNet 2012 15.4% error rate

# Convolutional Neural Network

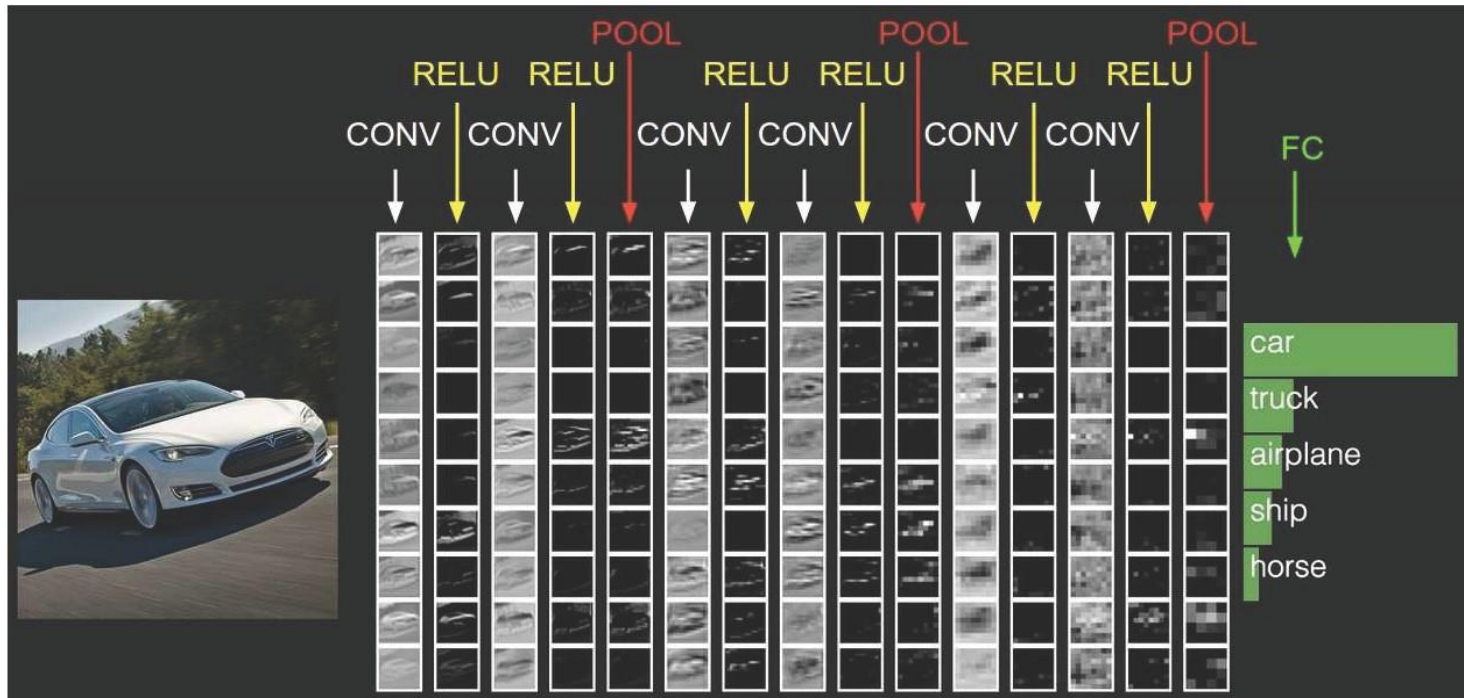


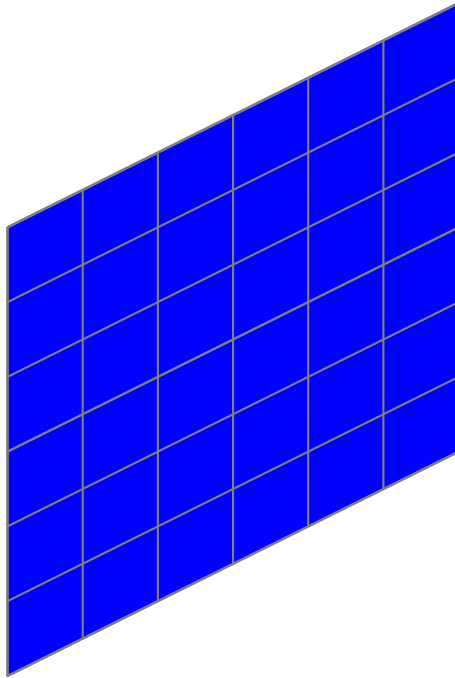
Figure: Andrej Karpathy



# Convolution

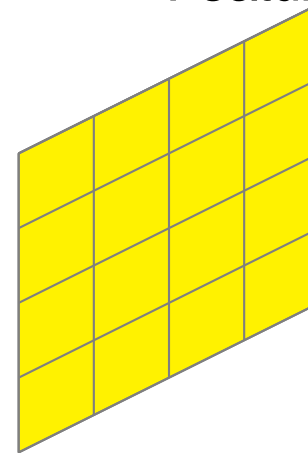
Kernel

$w_7$	$w_8$	$w_9$
$w_4$	$w_5$	$w_6$
$w_1$	$w_2$	$w_3$



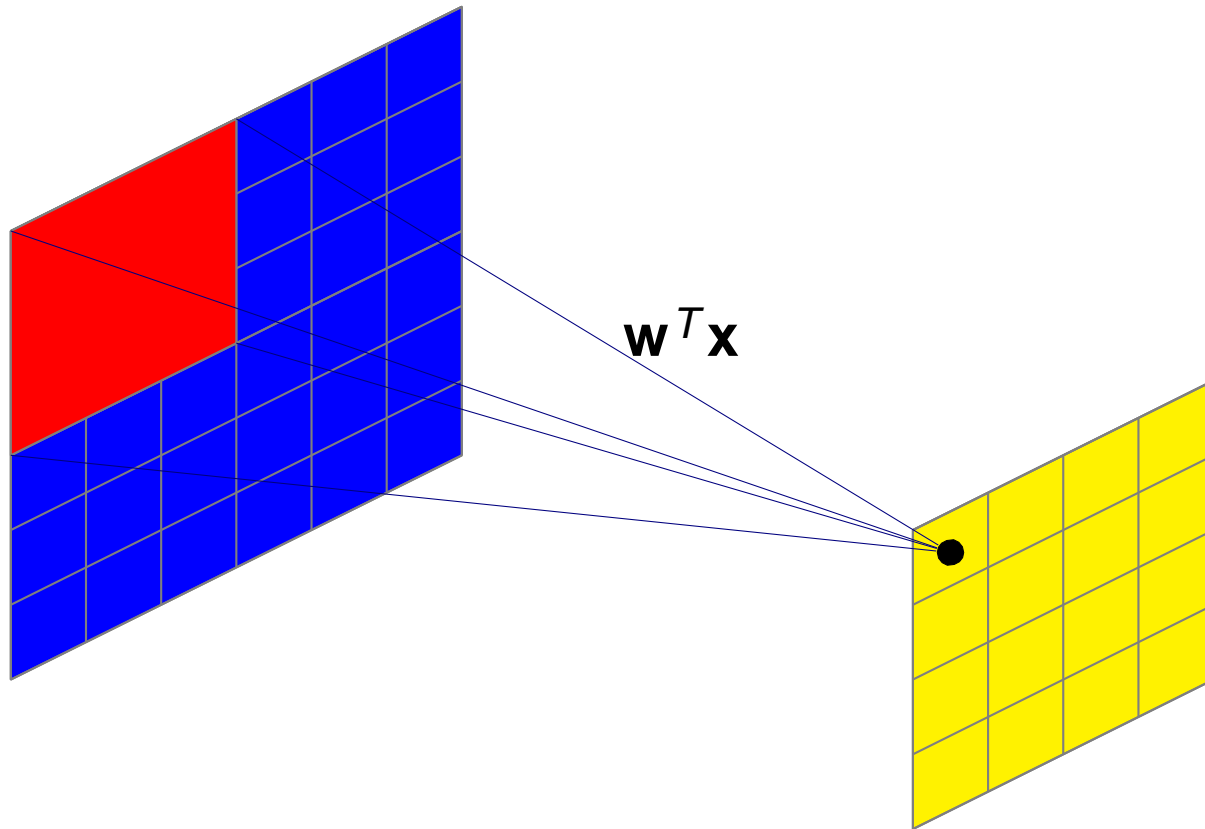
Grayscale Image

Feature Map

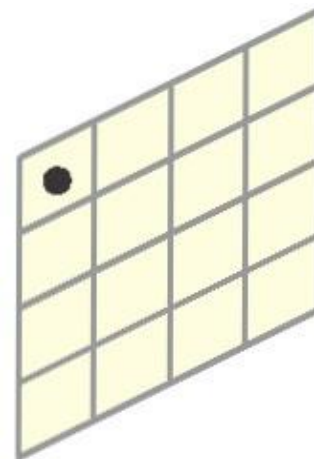
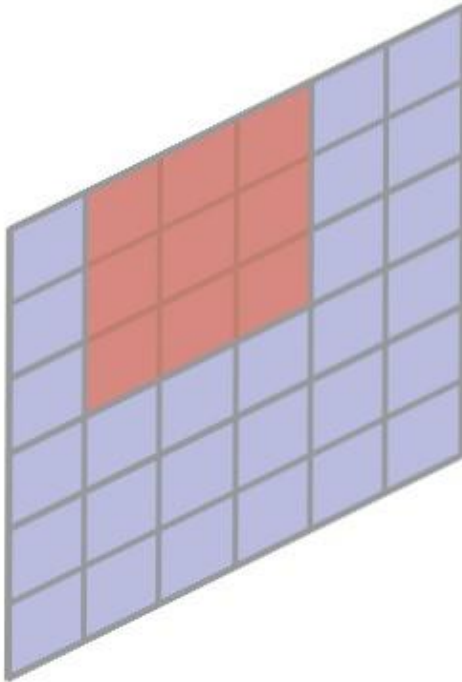


- Convolve image with kernel having weights  $\mathbf{w}$  (learned by backpropagation)

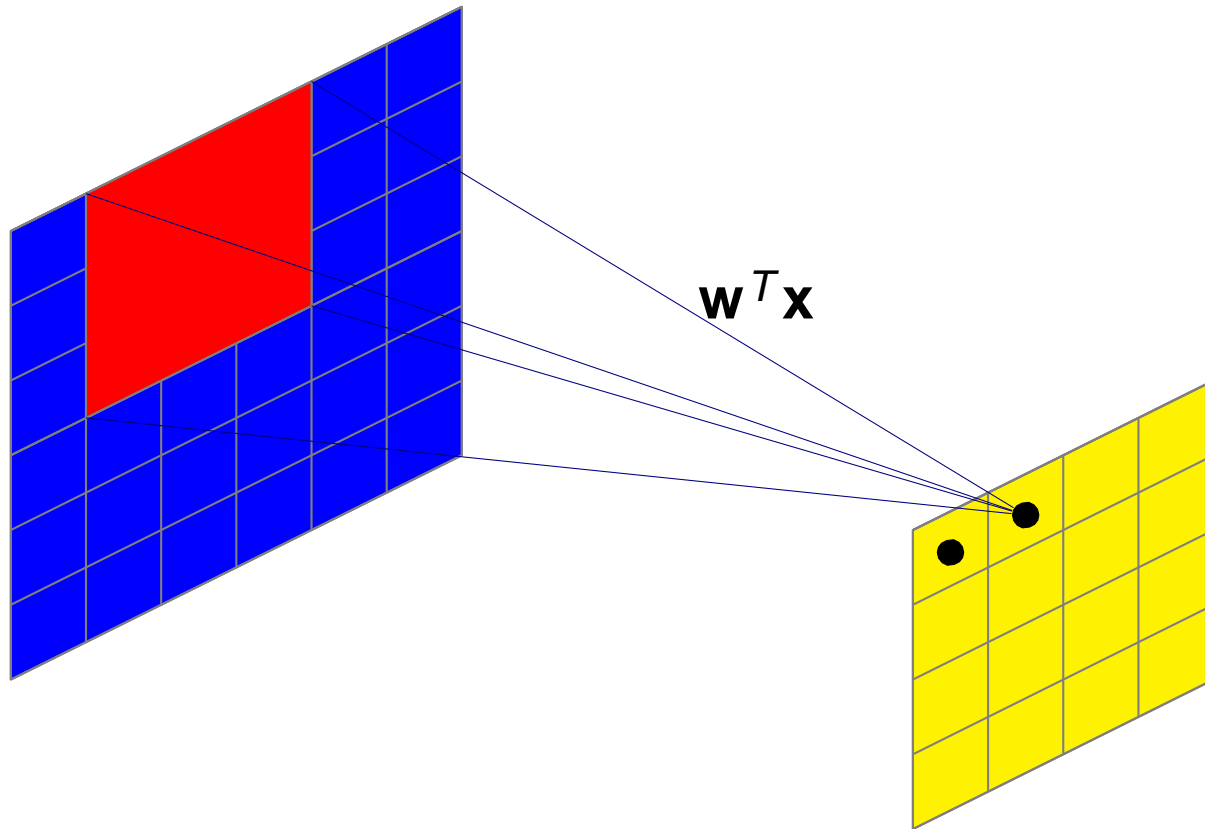
# Convolution



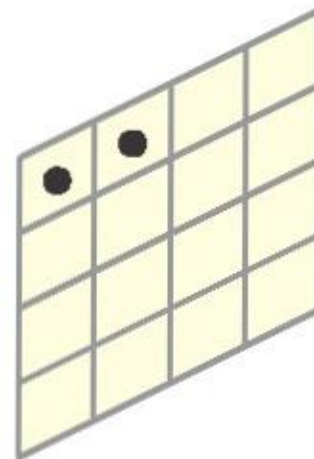
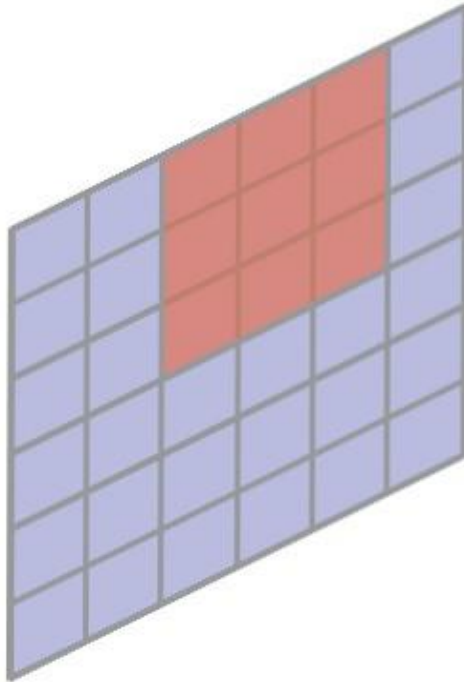
# Convolution



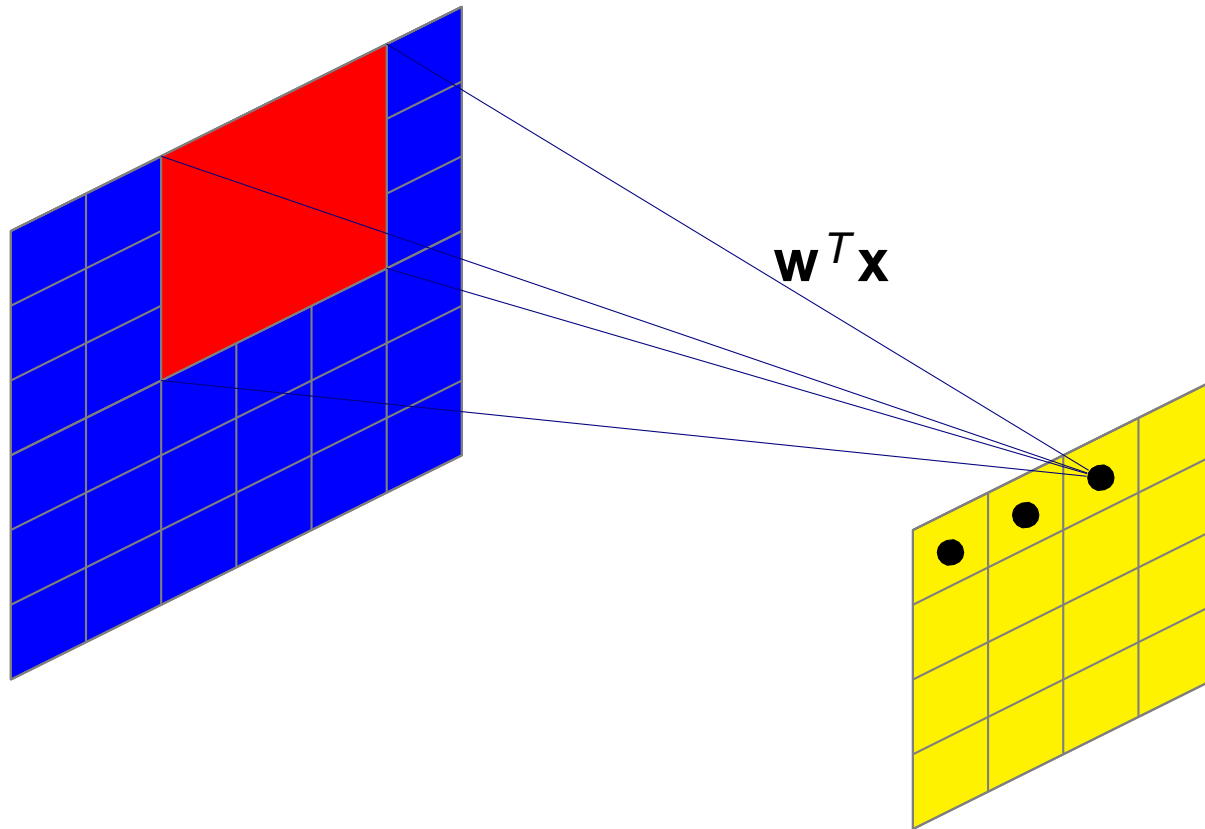
# Convolution



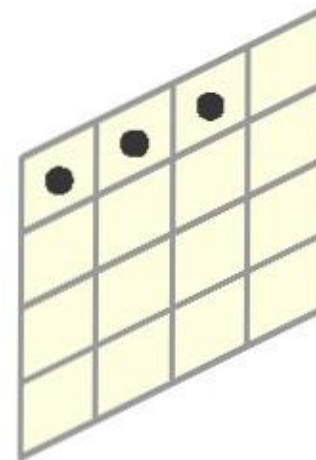
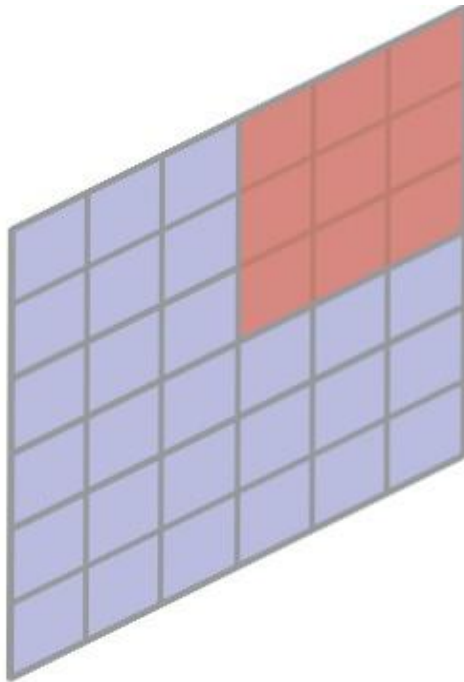
# Convolution



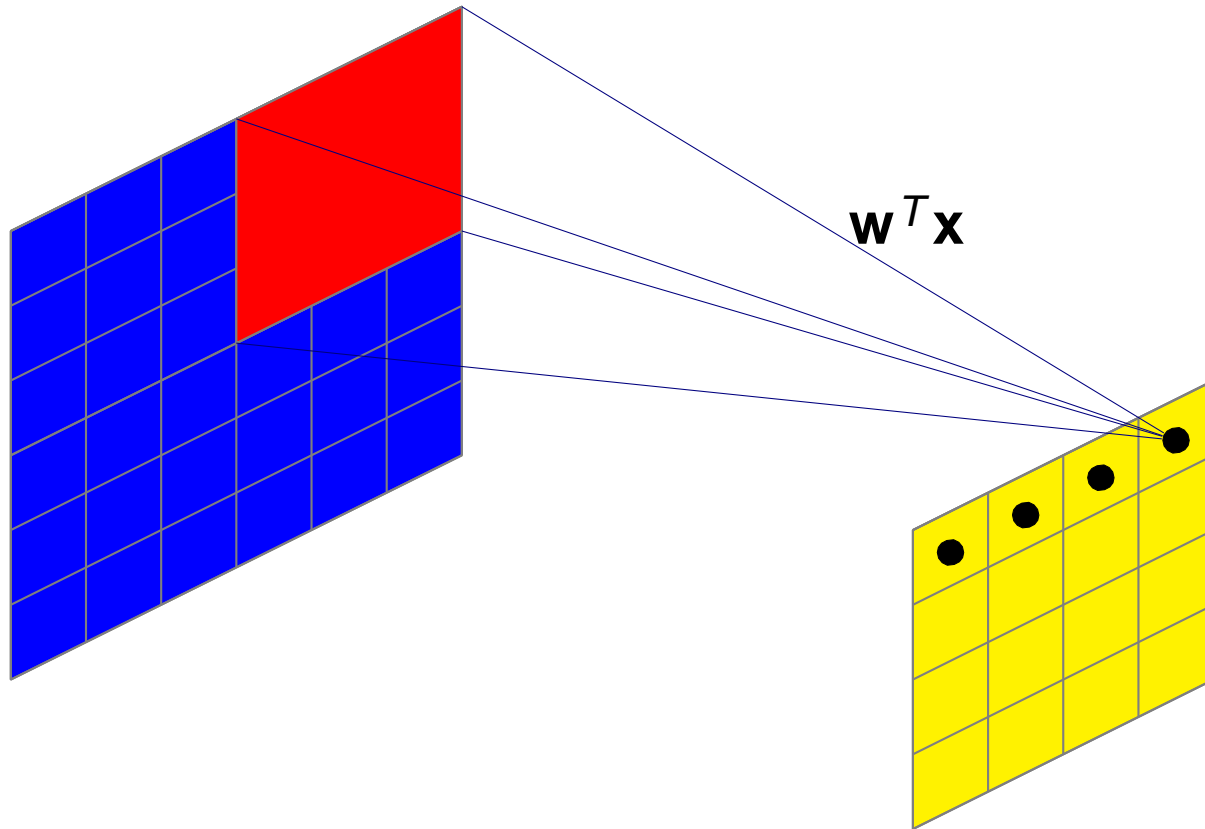
# Convolution



# Convolution

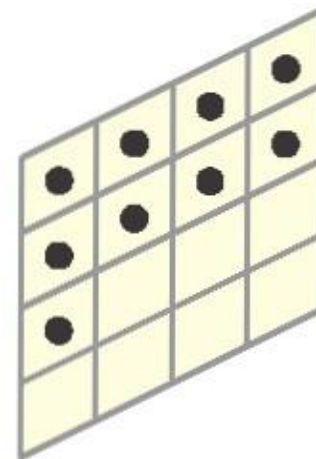
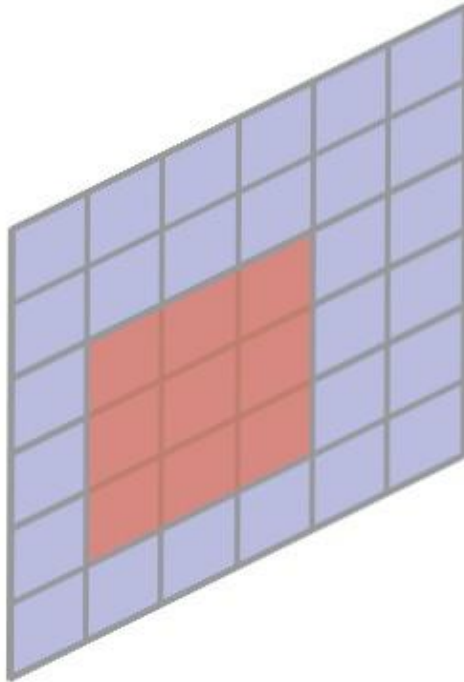


# Convolution

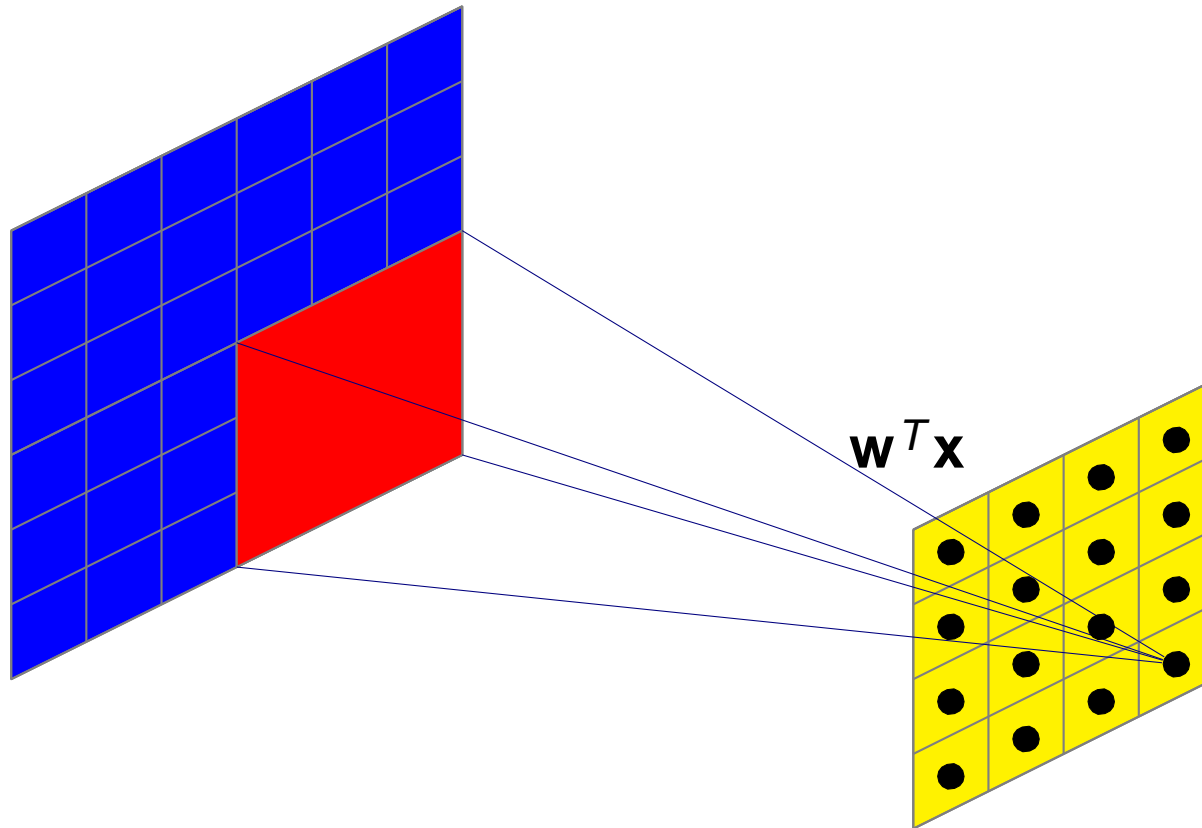




# Convolution



# Convolution



- What is the number of parameters?

# Output Size

- We used stride of 1, kernel with receptive field of size 3 by 3
- Output size:

$$\frac{N-K}{S} + 1$$

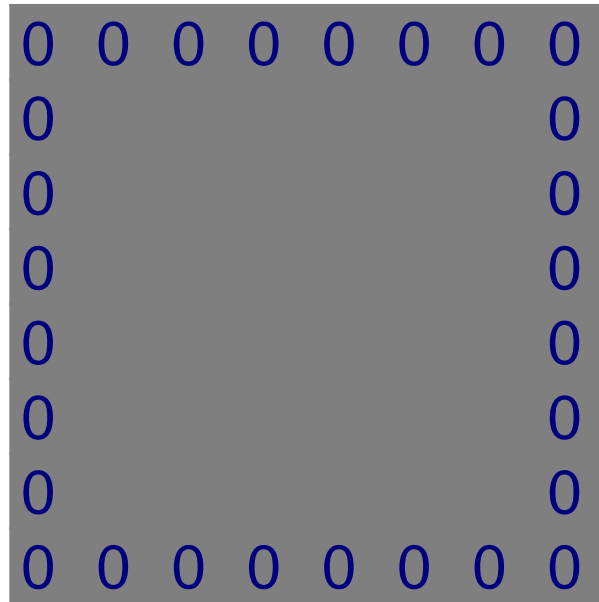
In previous example:  $N = 6, K = 3, S = 1$ , Output size = 4

For  $N = 8, K = 3, S = 1$ , output size is 6

# Zero Padding

Often, we want the output of a convolution to have the same size as the input. Solution: Zero padding.

In our previous example:



Common to see convolution layers with stride of 1, filters of size  $K$ , and zero padding with  $\frac{K-1}{2}$  to preserve size

# In practice

We have only considered a 2-D image as a running example  
But we could operate on volumes (e.g. RGB Images would be depth 3 input, filter would have same depth)

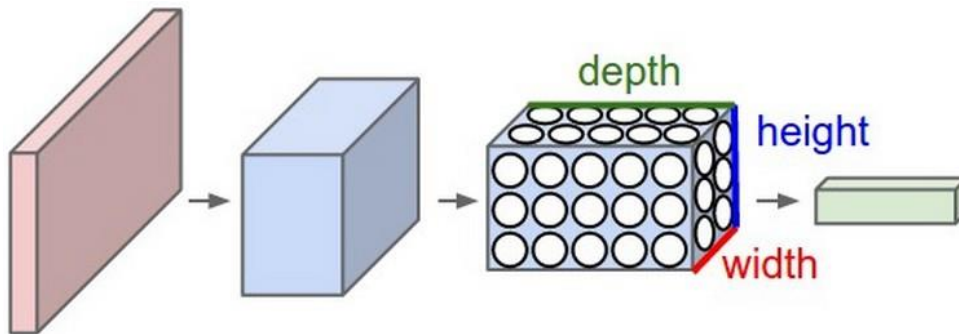


Image from Wikipedia

# Output Size

For convolutional layer:

- Suppose input is of size  $W_1 \times H_1 \times D_1$
- Filter size is  $K$  and stride  $S$
- We obtain another volume of dimensions  $W_2 \times H_2 \times D_2$
- As before:

$$W_2 = \frac{W_1 - K}{S} + 1 \text{ and } H_2 = \frac{H_1 - K}{S} + 1$$

- Depths will be equal

# Convolutional Layer Parameters

Example volume:  $28 \times 28 \times 3$  (RGB Image)

100  $3 \times 3$  filters, stride 1

What is the zero padding needed to preserve size?

Number of parameters in this layer?

For every filter:  $3 \times 3 \times 3 + 1 = 28$  parameters

Total parameters:  $100 \times 28 = 2800$

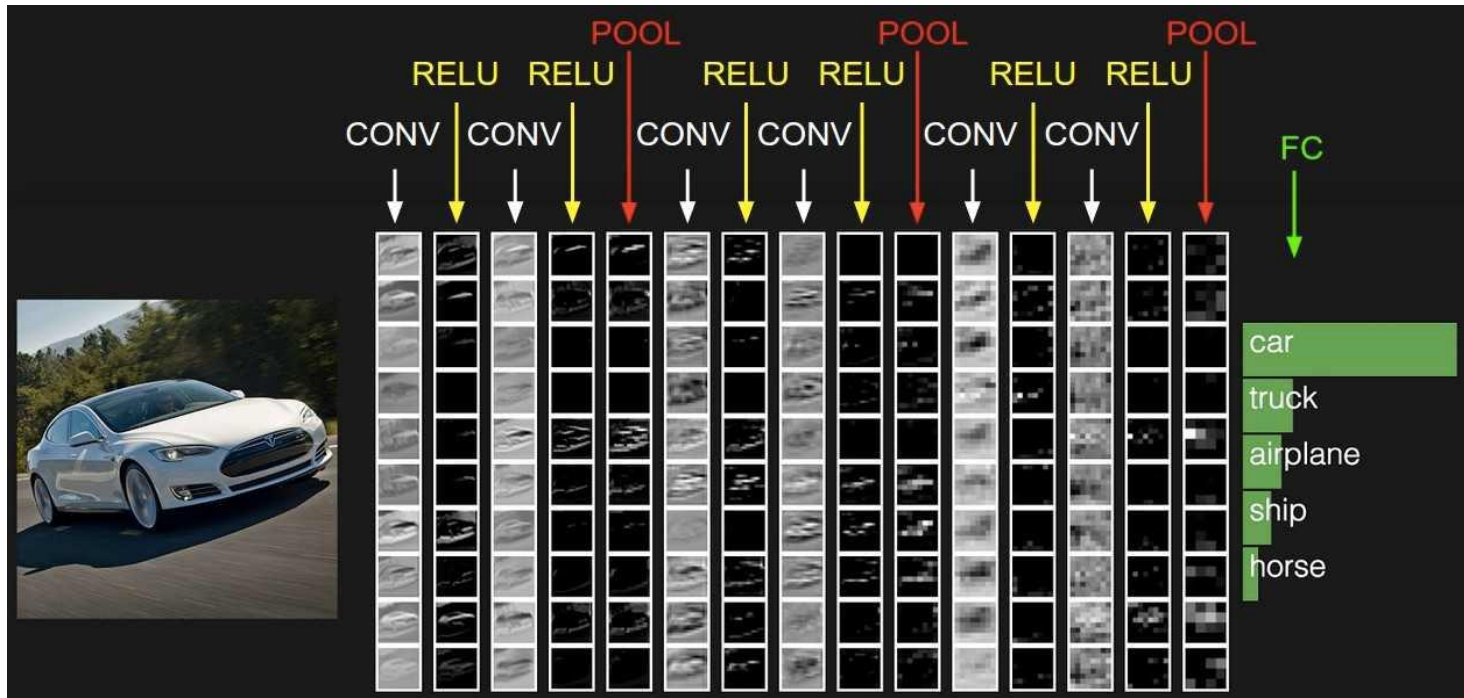
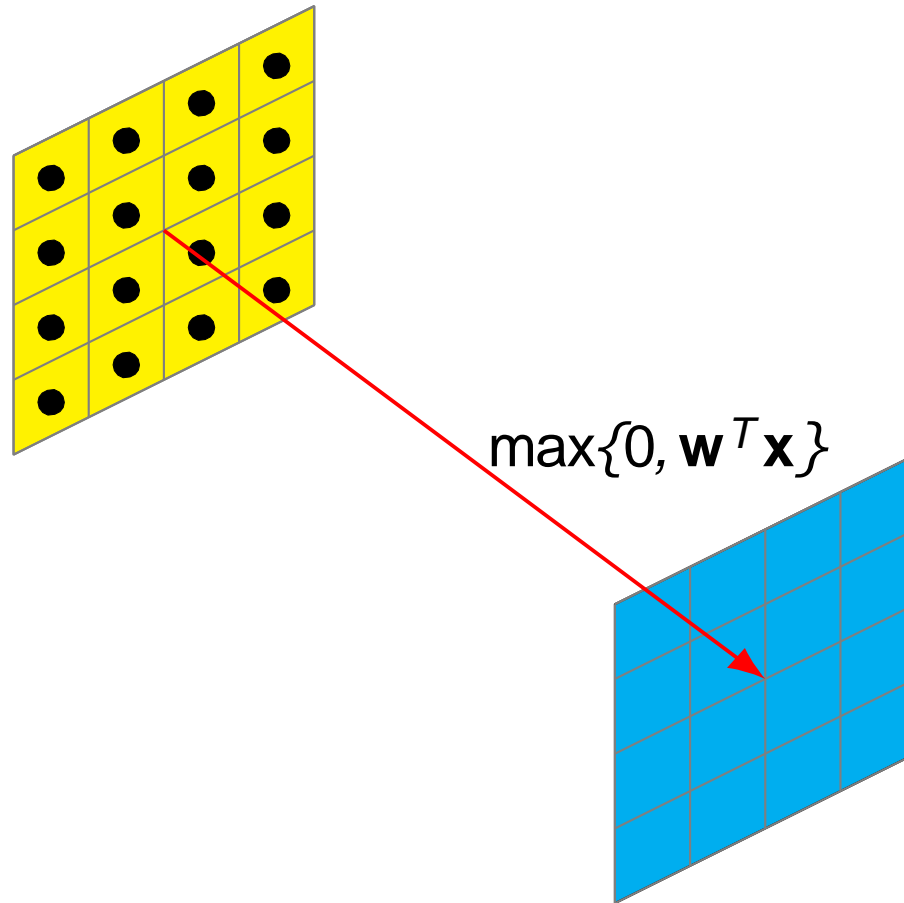


Figure: Andrej Karpathy



# Non-Linearity



- After obtaining feature map, apply an elementwise non-linearity to obtain a transformed feature map (same size)

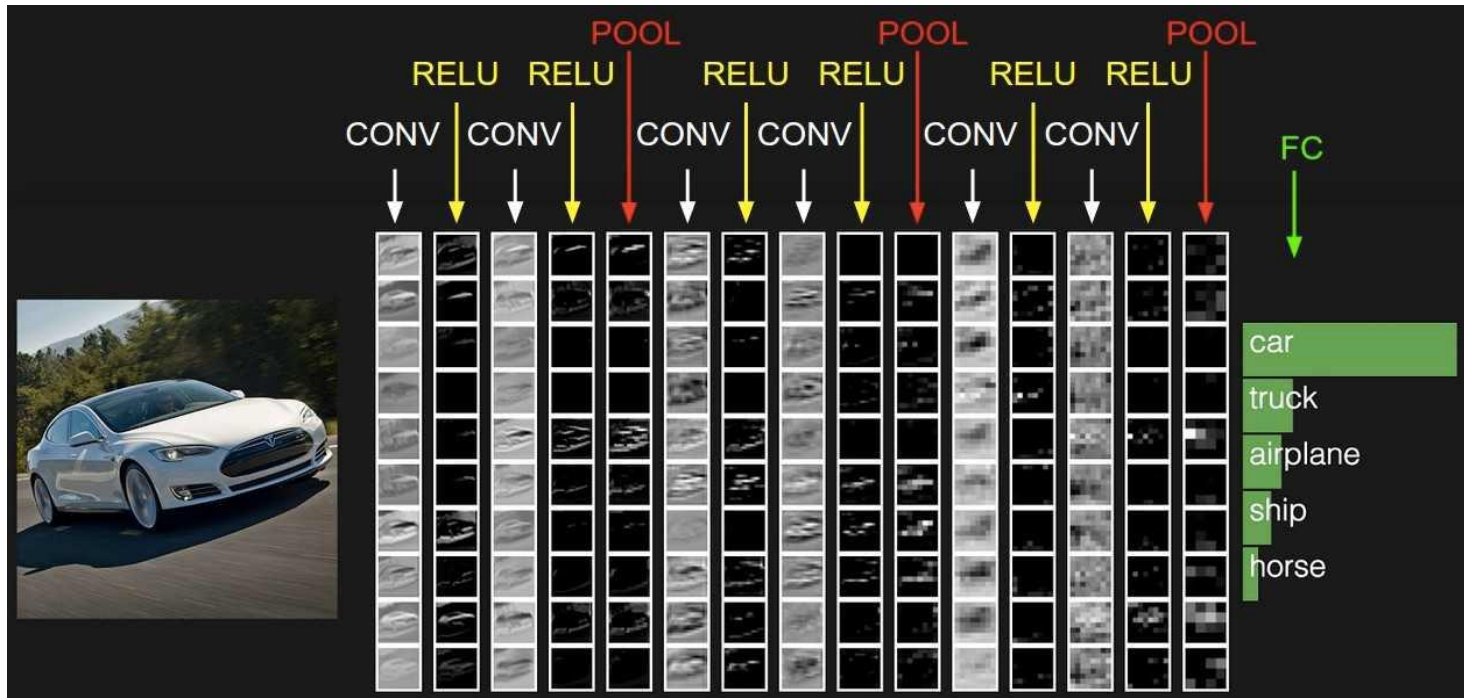
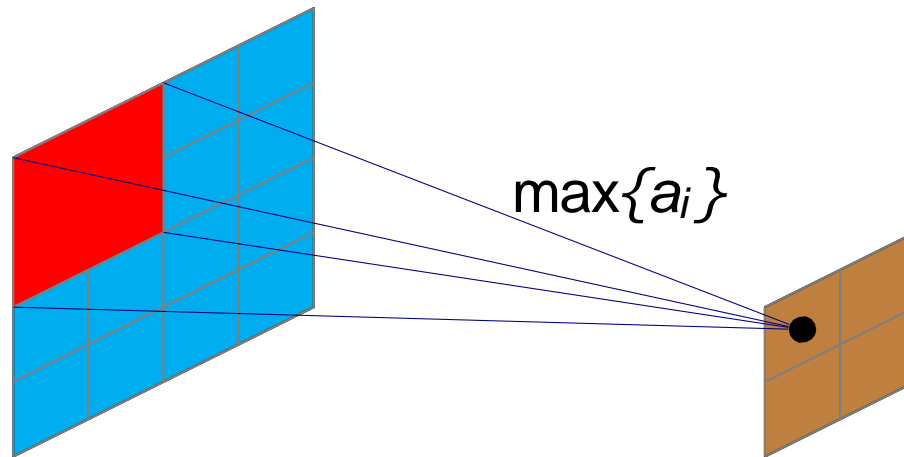
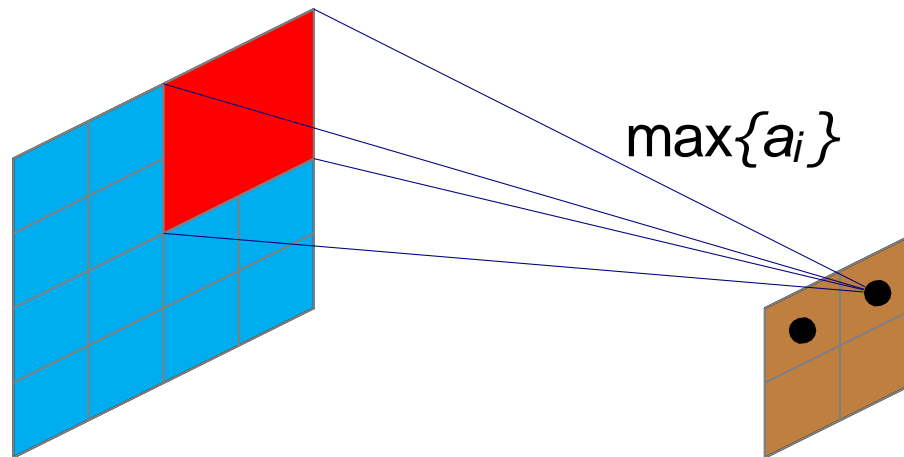


Figure: Andrej Karpathy

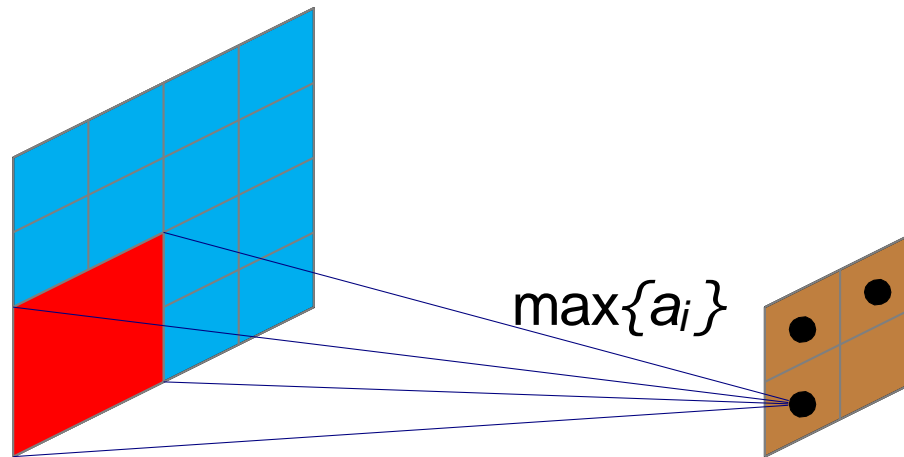
# Pooling



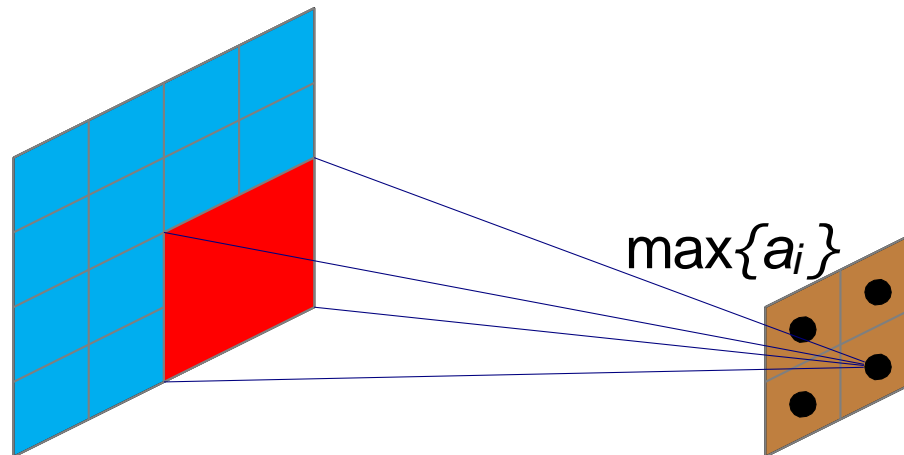
# Pooling



# Pooling

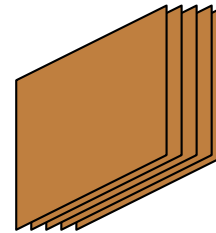
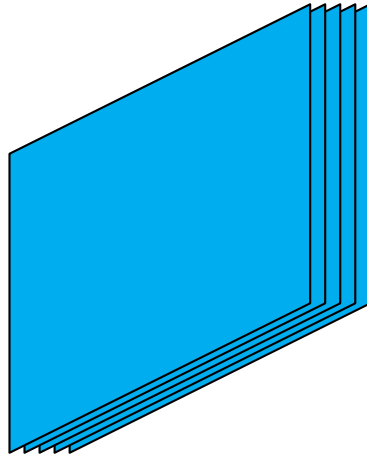


# Pooling



- Other options: Average pooling, L2-norm pooling, random pooling

# Pooling



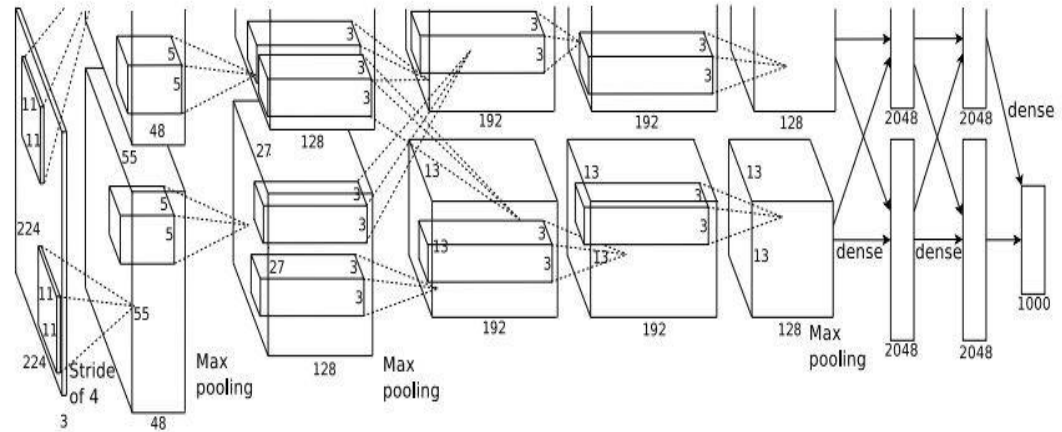
- We have multiple feature maps, and get an equal number of subsampled maps
- This changes if cross channel pooling is done

# **AlexNet example**



# Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

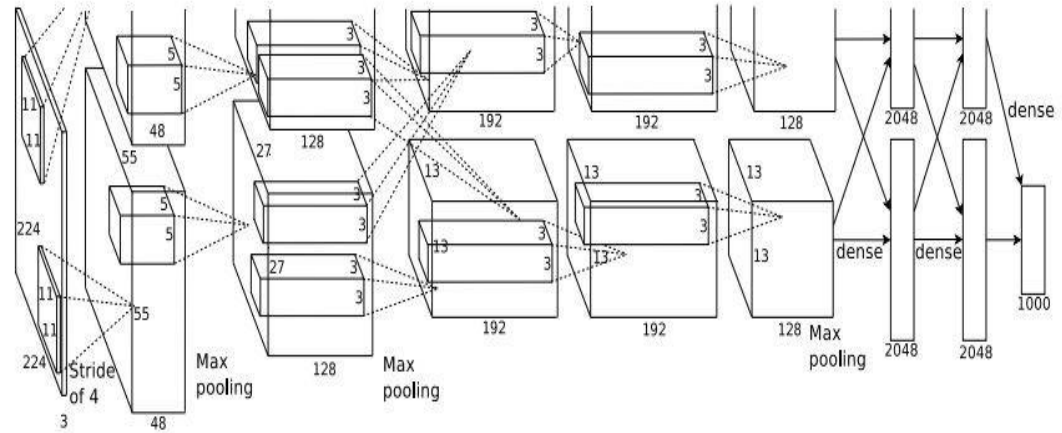
**First layer (CONV1):** 96 11x11 filters applied at stride 4

=>

Q: what is the output volume size? Hint:  $(227-11)/4+1 = 55$

# Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

**First layer (CONV1):** 96 11x11 filters applied at stride 4

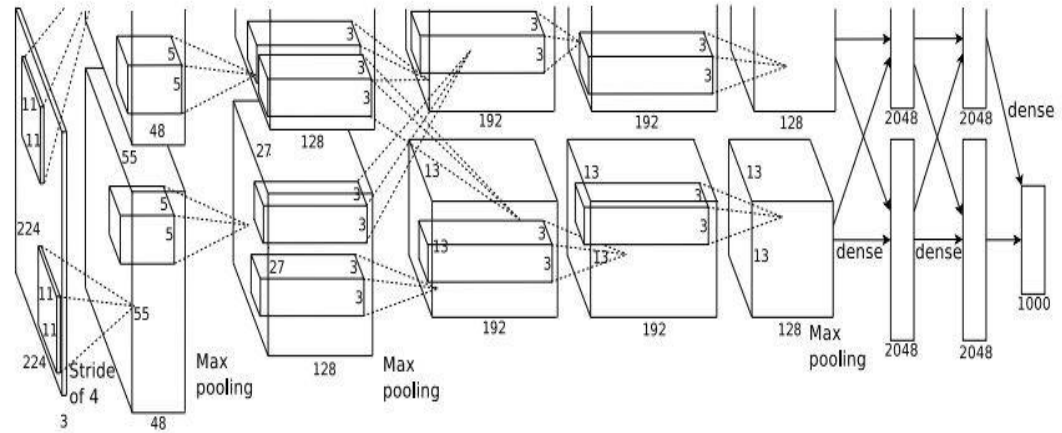
=>

Output volume **[55x55x96]**

Q: What is the total number of parameters in this layer?

# Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

**First layer (CONV1):** 96 11x11 filters applied at stride 4

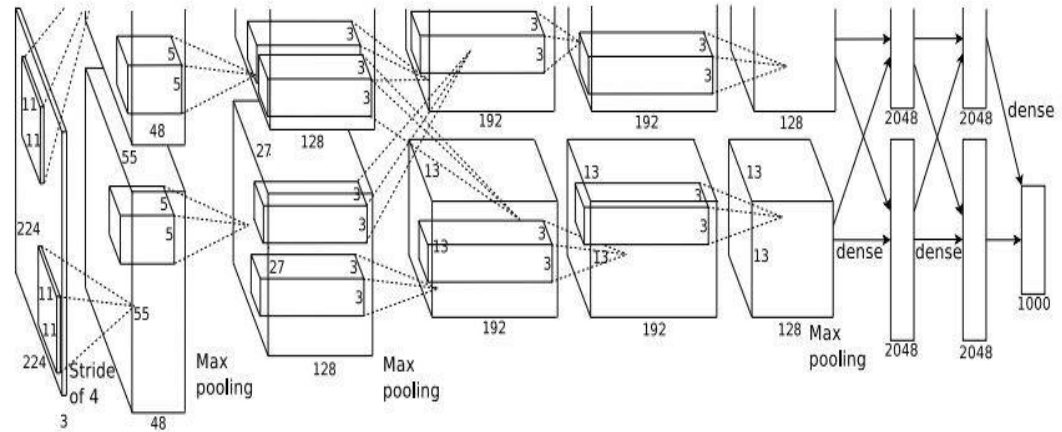
=>

Output volume **[55x55x96]**

Parameters:  $(11*11*3)*96 = \mathbf{35K}$

# Case Study: AlexNet

[Krizhevsky et al. 2012]



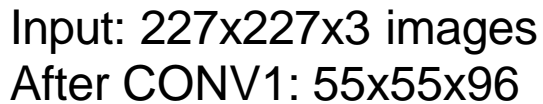
Input: 227x227x3 images

After CONV1: 55x55x96

**Second layer** (POOL1): 3x3 filters applied at stride 2

Q: what is the output volume size? Hint:  $(55-3)/2+1 = 27$

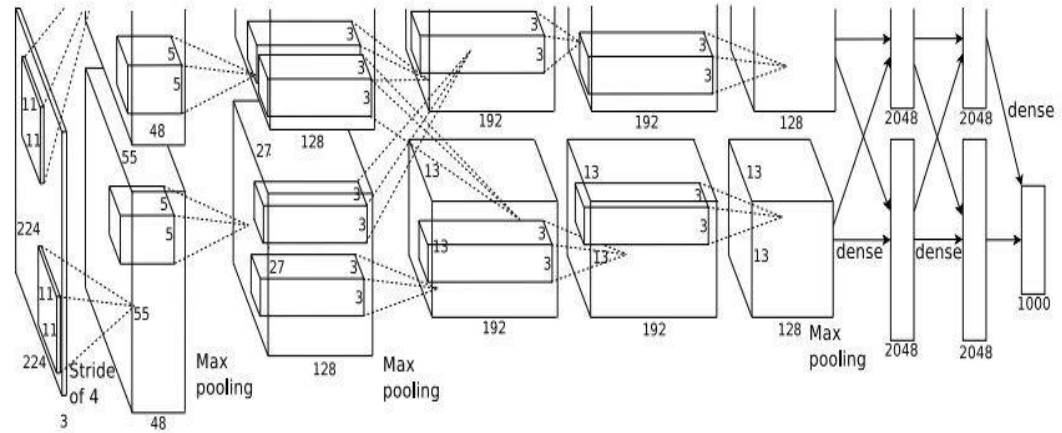
[Krizhevsky et al. 2012]



Q: what is the number of parameters in this layer?

# Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

After CONV1: 55x55x96

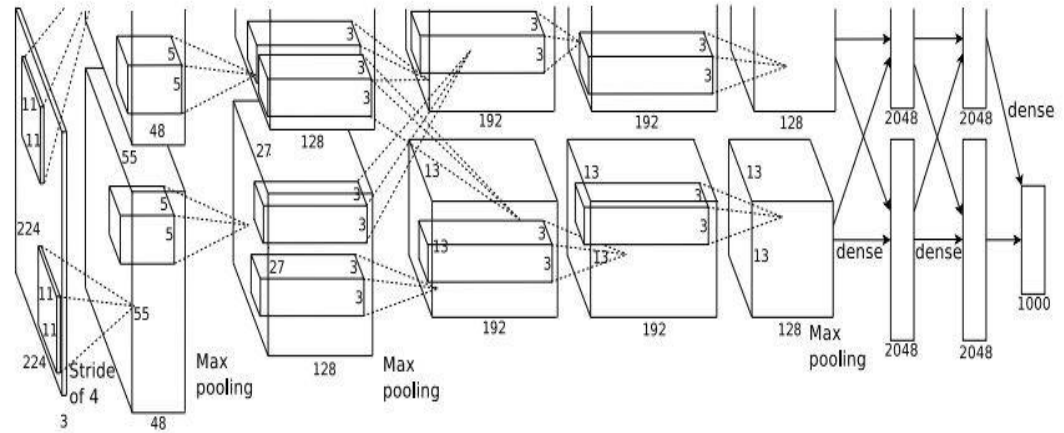
**Second layer (POOL1):** 3x3 filters applied at stride 2

Output volume: 27x27x96

Parameters: 0!

# Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

After CONV1: 55x55x96

After POOL1: 27x27x96

...

# Case Study: AlexNet

[Krizhevsky et al. 2012]

Full (simplified) AlexNet architecture:

[227x227x3] INPUT

[55x55x96] **CONV1**: 96 11x11 filters at stride 4, pad 0

[27x27x96] **MAX POOL1**: 3x3 filters at stride 2

[27x27x96] **NORM1**: Normalization layer

[27x27x256] **CONV2**: 256 5x5 filters at stride 1, pad 2

[13x13x256] **MAX POOL2**: 3x3 filters at stride 2

[13x13x256] **NORM2**: Normalization layer

[13x13x384] **CONV3**: 384 3x3 filters at stride 1, pad 1

[13x13x384] **CONV4**: 384 3x3 filters at stride 1, pad 1

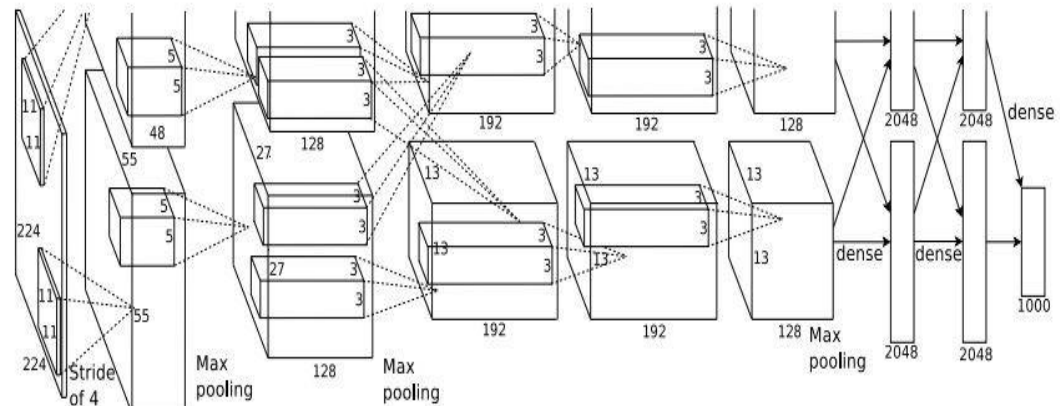
[13x13x256] **CONV5**: 256 3x3 filters at stride 1, pad 1

[6x6x256] **MAX POOL3**: 3x3 filters at stride 2

[4096] **FC6**: 4096 neurons

[4096] **FC7**: 4096 neurons

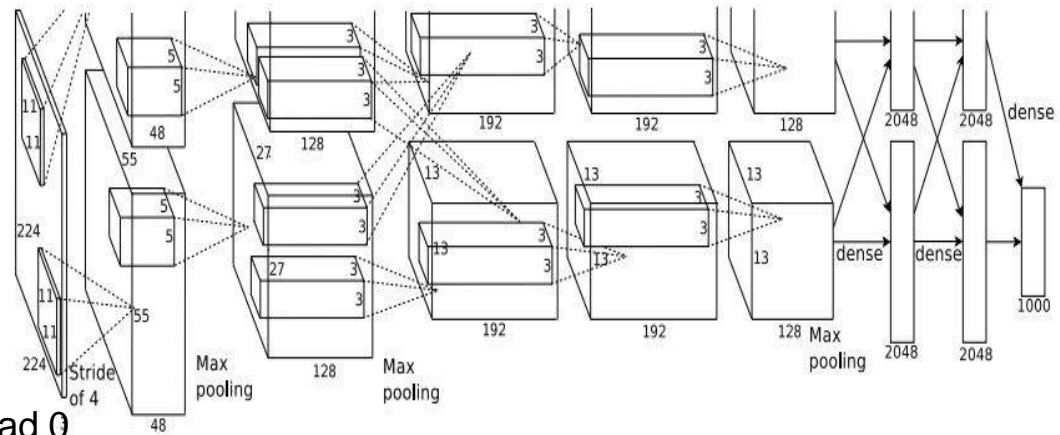
[1000] **FC8**: 1000 neurons (class scores)





# Case Study: AlexNet

[Krizhevsky et al. 2012]



Full (simplified) AlexNet architecture:

[227x227x3] INPUT

[55x55x96] **CONV1**: 96 11x11 filters at stride 4, pad 0

[27x27x96] **MAX POOL1**: 3x3 filters at stride 2

[27x27x96] **NORM1**: Normalization layer

[27x27x256] **CONV2**: 256 5x5 filters at stride 1, pad 2

[13x13x256] **MAX POOL2**: 3x3 filters at stride 2

[13x13x256] **NORM2**: Normalization layer

[13x13x384] **CONV3**: 384 3x3 filters at stride 1, pad 1

[13x13x384] **CONV4**: 384 3x3 filters at stride 1, pad 1

[13x13x256] **CONV5**: 256 3x3 filters at stride 1, pad 1

[6x6x256] **MAX POOL3**: 3x3 filters at stride 2

[4096] **FC6**: 4096 neurons

[4096] **FC7**: 4096 neurons

[1000] **FC8**: 1000 neurons (class scores)

## Details/Retrospectives:

- first use of ReLU
- used Norm layers (not common anymore)
- heavy data augmentation
- dropout 0.5
- batch size 128
- SGD Momentum 0.9
- Learning rate 1e-2, reduced by 10 manually when val accuracy plateaus
- L2 weight decay 5e-4
- 7 CNN ensemble: 18.2% -> 15.4%