# L1: Controlling How Long A Reasoning Model Thinks With Reinforcement Learning

**Pranjal Aggarwal**Carnegie Mellon University

**Sean Welleck**Carnegie Mellon University

#### **Abstract**

Reasoning language models have shown an uncanny ability to improve performance at test-time by "thinking longer"—that is, by generating longer chain-of-thought sequences and hence using more compute. However, the length of their chain-of-thought reasoning is not controllable, making it impossible to allocate test-time compute to achieve a desired level of performance. We introduce Length Controlled Policy Optimization (LCPO), a simple reinforcement learning method that optimizes for accuracy and adherence to user-specified length constraints. We use LCPO to train L1, a reasoning language model that produces outputs satisfying a length constraint given in its prompt. L1's length control allows for smoothly trading off computational cost and accuracy on a wide range of tasks, and outperforms the state-of-the-art S1 method for length control. Furthermore, we uncover an unexpected short chain-of-thought capability in models trained with LCPO. For instance, our 1.5B L1 model surpasses GPT-40 at equal reasoning lengths. Overall, LCPO enables precise control over reasoning length, allowing for fine-grained allocation of test-time compute and accuracy.

## 1 Introduction

An emerging class of reasoning language models (OpenAI et al., 2024a; DeepSeek-AI et al., 2025) improve performance at test-time by thinking longer when solving complex problems—that is, by generating extended chain-of-thought (Wei et al., 2023) sequences and in turn using more compute. However, current reasoning models have a key limitation: the length of their reasoning is uncontrolled, making it impossible to allocate a test-time compute budget to achieve a target performance level. In some cases, sequences span tens of thousands of tokens, wasting compute, while in others, models stop too early on complex problems.

Recent methods like S1 (Muennighoff et al., 2025) attempt to achieve length control by forcing a model to generate special tokens (e.g., "Wait", "Final Answer") when the gen-

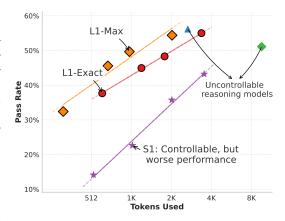


Figure 1: L1 is a length-controllable reasoning model, that achieves higher performance per token than other methods.

eration is too short or too long. However, this rigid, hand-engineered strategy severely degrades performance compared to the underlying model (Figure 1). Other work investigates controlling output lengths in instruction following and general domains (Butcher et al., 2024; Yuan et al., 2024). However, reasoning models face fundamentally new challenges such as much longer output lengths and the need to trade off computational cost for improved performance.

<sup>&</sup>lt;sup>1</sup>Code and models released at https://cmu-l3.github.io/l1

We propose **Length Controlled Policy Optimization (LCP0)**, a simple reinforcement learning (RL)-based method that gives reasoning language models precise and adaptive length control. LCP0 trains models to satisfy two objectives: (1) correctness of the final output, and (2) generating reasoning sequences that meet a length constraint specified in the prompt. In doing so, LCP0-trained models *learn* to satisfy length constraints while optimizing reasoning performance rather than relying on hand-engineered heuristics.

We experiment with two practical constraints: (1) LCP0-Exact, which requires the generated reasoning to be exactly equal to the target length, and (2) LCP0-Max, which requires the output to be no longer than the target length. We use LCP0 to fine-tune a 1.5B-parameter reasoning model based on Qwen-Distilled-R1-1.5B (DeepSeek-AI et al., 2025; Qwen et al., 2025), producing L1-Max and L1-Exact. Our L1 models can precisely trade off token budget and reasoning performance, smoothly interpolating between short, efficient reasoning and longer, more accurate reasoning by simply prompting the model with different length constraints (Figure 1). Crucially, one point on this trade-off curve recovers the original base model's performance, while outperforming S1 (Muennighoff et al., 2025) in performance across the entire range of reasoning lengths (Figure 1). On math reasoning tasks, L1 outperforms S1 by up to 100% relative and 20% absolute, under identical conditions.

Beyond improved length control in the standard math reasoning setting, we find that LCP0-trained models generalize surprisingly well to out-of-distribution tasks, including logical reasoning, and general-knowledge benchmarks like MMLU (Hendrycks et al., 2021a). Furthermore, we show that "long-CoT" models trained with LCP0 become unexpectedly strong "short-CoT" models: when prompted to generate short reasoning traces, LCP0-trained models outperform their original counterparts by significant margins (up to 10% improvement), even at the same generation length. To the best of our knowledge, for the first time we show that a 1.5B model can match the performance of GPT-40 (OpenAI et al., 2024b) despite using the same token budget. In summary, our contributions are:

- We introduce Length Controlled Policy Optimization (LCPO), the first reinforcement learning-based method for training reasoning language models that produce outputs adhering to user-specified length constraints.
- We use LCPO to train L1, which demonstrates high degree of length control and achieves state-of-the-art reasoning accuracy at fixed token budgets on challenging math reasoning benchmarks.
- We show length-control of L1 generalizes beyond math reasoning tasks to diverse out-of-distribution tasks, including logical reasoning, and general-domain benchmarks (MMLU).
- We demonstrate that LCPO-trained models can act as strong short-CoT models, significantly outperforming their non-reasoning counterparts and much larger models such as GPT-4o, despite using the same token budget.

## 2 Related Work

Test-Time Scaling in Large Language Models. Increasing test-time computation has consistently been shown to improve performance in complex reasoning tasks, mathematical problem-solving, and code generation (Wu et al., 2024; Wang et al., 2023; Wei et al., 2023; DeepSeek-AI et al., 2025; Snell et al., 2024). Test-time scaling laws indicate predictable performance gains from increasing inference computation, either by generating more reasoning chains or longer ones (Wu et al., 2024; Snell et al., 2024; OpenAI et al., 2024a). Prominent approaches include parallel sampling of multiple reasoning paths (Wang et al., 2023; Aggarwal et al., 2023), tree-based search (Yao et al., 2023; Wu et al., 2024; Xin et al., 2024), and iterative refinement techniques (Welleck et al., 2023; Madaan et al., 2023; Snell et al., 2024; Welleck et al., 2024). Recent reasoning language models such as "O1" and "R1"-style models (OpenAI et al., 2024a; DeepSeek-AI et al., 2025) simplify test-time scaling by generating extended reasoning traces (longer chains-of-thought). Despite their promising results, these methods lack precise and dynamic control over the length of the generated reasoning chains, resulting in often suboptimal performance or unrealized potential efficiency gains.

Our work complements and extends this line of research by enabling reasoning models to precisely control the length of generated outputs, thereby providing flexibility to calibrate inference compute based on task-specific requirements.

Length Control in Large Language Models. Controlling the length of LLM-generated outputs is an important practical consideration across various generation tasks. Approaches proposed thus far include architectural modifications—such as manipulating positional encodings for exact sequence-length generation (Butcher et al., 2024)—training objective adjustments to explicitly enforce length constraints (Jie et al., 2023; Singhal et al., 2024), or directly training models on instruction-style data explicitly labeled with desired output lengths (Yuan et al., 2024). Previous works on length control largely fall into two use-case categories. The first aims primarily to reduce unnecessary verbosity (as often desired in RLHF-tuned instruction-following models) while the second aims either to impose maximum length budgets or achieve precise token-level length adherence (Jie et al., 2023; Yuan et al., 2024; Singhal et al., 2024). However, existing methods predominantly focus on general-purpose text generation or instruction-following contexts, where cost-quality efficiency trade-offs are less critical or remain unaddressed (Jie et al., 2023; Yuan et al., 2024; Butcher et al., 2024). Our work addresses the new challenges present in reasoning models.

Length control specifically tailored to reasoning tasks remains relatively unexplored. Recent works, such as those by Arora & Zanette (2025) and Kang et al. (2024), emphasize generating shorter reasoning chains for efficiency, but they do not enable explicit length control or precise alignment with user-specified inference budgets. Another work S1 (Muennighoff et al., 2025) introduces "budget-forcing" by imposing a strict token limit: either truncating output at budget exhaustion or inserting a special token ("Wait") to request continued generation until reaching the full length budget. Unfortunately, this strategy presents significant practical drawbacks. Abrupt truncation often interrupts reasoning mid-step, negatively impacting model accuracy and user interpretability. Meanwhile, the repetitive usage of special continuation tokens risks rigid and suboptimal reasoning patterns.

In contrast to these prior works, our LCPO is uniquely designed to train reasoning-specialized models for precise and adaptive length control. LCPO uses reinforcement learning so that models *learn* to dynamically allocate inference compute based on constraints provided in a prompt. As our experiments will demonstrate, our method substantially surpasses previous approaches in precision over length control, and performance at varying length budgets.

#### 3 Method

Current reasoning language models lack an explicit mechanism for controlling the length of their generated reasoning traces. This limitation prevents users and downstream applications from explicitly calibrating the inference compute budget (number of generated tokens) according to task-specific requirements or available computational resources.

In this work, we address this limitation by conditioning the model on a target token length provided in the prompt. Formally, given an input prompt x and a target length  $n_{gold}$ , the model is expected to generate a response y whose length  $n_y$  minimizes the absolute difference  $|n_{gold}-n_y|$  while simultaneously producing the correct answer. This formulation directly couples accuracy with output length, ensuring that the generated chain-of-thoughts adhere to user-specified constraints.

**Length Controlled Policy Optimization.** We begin with a pre-trained reasoning language model  $LLM_{\theta}$  and a dataset  $D = \{(x_i, y_{gold,i})\}_{i=1}^N$ , where each instance contains only the input prompt and the final answer (i.e., no intermediate reasoning traces). To enable length control, each prompt  $x_i$  is augmented by appending a target length instruction. In particular, we form

$$x_i^{new} = \text{Concat}(x_i, \text{"Think for } n_{gold,i} \text{ tokens."}),$$

where  $n_{gold,i}$  is sampled uniformly from  $\mathbb{Z}(n_{min},n_{max})$ . This augmentation yields a new dataset  $D^{new} = \{(x_i^{new},y_{gold,i})\}_{i=1}^N$ .

We then update  $LLM_{\theta}$  using a reinforcement learning objective. In our experiments we adopt GRPO (Shao et al., 2024) (though the method is compatible with other RL algorithms). Our reward function combines two terms: a correctness reward  $r_c$  and a length penalty  $r_{length}$ . It is defined as

$$r(y, y_{gold}, n_{gold}) = \mathbb{I}(y = y_{gold}) - \alpha \cdot |n_{gold} - n_y|, \tag{1}$$

where  $\mathbb{I}(\cdot)$  is the indicator function,  $n_y$  is the generated output length, and  $\alpha$  is a scalar that regulates the trade-off between generating the correct answer and meeting the target length. In practice, a lower value of  $\alpha$  prioritizes correctness when it is critical, whereas a higher value enforces stricter adherence to the length constraint. Notably, the reward function serves a dual purpose: (a) it encourages the model to produce correct answers while implicitly favoring concise reasoning traces when shorter outputs are requested, and (b) it consistently motivates the model to match the prescribed target length even when a correct answer could be generated with fewer tokens. We refer to the model trained with this objective as L1-Exact.

At inference, the output length is controlled by selecting a fixed target length  $n_{gold}$  (or a set of lengths) that is appended uniformly to every test prompt.

**Maximum Length Constraint Mode.** We further train a variant of L1 called L1-Max, which flexibly generates outputs of varying lengths while respecting a maximum length constraint. This approach is valuable when users prioritize staying within a computational budget rather than adhering to exact generation lengths. To train L1-Max, we fine-tune the L1-Exact model using the same RL framework but with a modified reward function:

$$r(y, y_{gold}, n_{gold}) = \mathbb{I}(y = y_{gold}) \cdot \text{clip}(\alpha \cdot (n_{gold} - n_y) + \delta, 0, 1), \tag{2}$$

where  $\alpha$  controls the penalty for length violations. This formulation applies a soft constraint that (1) gradually penalizes outputs exceeding the target length rather than imposing a hard cutoff (which is necessary to ensure gradient propagation in GRPO objective), and (2) incentivizes the model to use fewer tokens when possible without sacrificing correctness. The  $\delta=0.5$  term ensures that correct answers with minor budget violations are still preferred over incorrect answers. Further, L1-Max is trained with dual objective: when the prompt requests an exact length, the model uses Equation 1; otherwise, it defaults to the maximum constraint mode using Equation 2.

#### 4 Experimental Setup

Models and Datasets. We conduct training on the DeepScaleR-Preview-Dataset (Luo et al., 2025), a mathematics dataset consisting of 40K question-answer pairs drawn from AIME, AMC, Omni-Math (Gao et al., 2024) and STILL (Min et al., 2024). We evaluate our models on test sets of 4 different reasoning datasets: AIME 2025, MATH (Hendrycks et al., 2021b), AMC, Olympiad-Bench (He et al., 2024), and additionally GPQA (Rein et al., 2023), LSAT (Zhong et al., 2023), and MMLU (Hendrycks et al., 2021a). Our base model is DeepScaleR-1.5B-Preview, a 1.5B-parameter model originally RL fine-tuned (from DeepSeek-R1-Distill-Qwen-1.5B (DeepSeek-AI et al., 2025)) on this dataset with a 24K token context length. Due to compute constraints, we restrict the maximum context length to 4K tokens during training and to 8K tokens during evaluation. The model is further fine-tuned for 700 steps with LCPO-Exact objective (Equation 1), and the resulting model is referred to as L1-Exact. The model is further RL finetuned for 120 steps with the objective mentioned in Equation 2, and the resulting model is referred to as L1-Max.

**Baselines.** We evaluate our proposed method against the following baselines:

- **DeepSeek-R1-Distill-Qwen-1.5B:** is the SFT version of Qwen-2.5-1.5B-Instruct finetuned on reasoning traces of DeepSeek's R1 model. For brevity, we refer to this model as DeepSeek-R1-1.5B.
- **DeepScaleR-1.5B-Preview:** the original model, evaluated without any length control modifications. For brevity, we refer this model as Agentica-24K.

- **DeepScaleR-1.5B-Preview-4K:** a version of Agentic-24K fine-tuned with 4K context length. This is done due to computational constraints of training LCPO with long sequence length (such as 24K used in Agentica-24K). The model therefore serves as a fair comparison to L1. For brevity, we refer to this model as Agentica-4K.
- S1: (Muennighoff et al., 2025) is a budget-forcing method, which controls reasoning length using simple test-time interventions. We implement this method on top of the Agentica-24K model.

**Evaluation Protocol.** We evaluate our approaches along two dimensions. First, we assess the model's ability to adhere to the targeted length by reporting the mean deviation between the generated token length  $n_y$  and the target  $n_{gold}$ . Second, we evaluate the overall performance (i.e., problem-solving accuracy) when generating responses at different target lengths. In our experiments, target lengths are selected from  $\{512, 1024, 2048, 3600\}$  tokens.

**Hyperparameters and Implementation Details.** For GRPO training, we adopt the same hyperparameters as in DeepScaleR-1.5B-Preview. In particular, we use a learning rate of 1e-6 and a batch size of 128. The maximum context length is set to 4K tokens at training time and extended to 8K tokens during evaluation. Training is performed for 700 steps using the VeRL framework (MLSys, 2025). During training, the target length  $n_{gold}$  is sampled uniformly from  $U(n_{min}, n_{max})$ , where we set  $n_{min} = 100$  and  $n_{max} = 4000$ . The balancing parameter α in Equation 1 is fixed at 0.0003. Note that we did not conduct extensive hyperparameter tuning, so one can expect further improvements with additional optimization.

# 5 Results and Analysis

In this section, we report and analyze the effectiveness of the proposed method (LCPO) across various settings and benchmarks. We evaluate our method's relative performance, generalization capability on out-of-domain tasks, controllability of length constraints and competitive performance in short Chain-of-Thought (CoT) setups, and examine learned reasoning behaviors.

**L1** significantly outperforms other length-controlled models while maintaining strong performance. Figure 2 compares performance of L1-Exact and L1-Max with other baselines across varying generation lengths. Both variants of L1 achieve superior performance across all token budgets while maintaining precise length control. Compared to S1, the only other method specifically designed for length control, L1 shows remarkable improvements, over 100-150% relative and 20-25% absolute performance gains at both 512 and 1024 token budgets. This substantial difference can be attributed to two key factors: (1) L1 intelligently adapts its chain-of-thought to fit within specified length constraints without disrupting the reasoning process, while S1 often truncates mid-reasoning; and (2) L1 is explicitly trained to generate high-quality reasoning chains of varying lengths, effectively distilling reasoning patterns from longer chains to shorter ones.

Moreover, with L1, we observe a log-linear scaling pattern, similar to the prior works O1 and S1 by OpenAI—performance improves linearly with respect to the log-length of generated reasoning chains. However, this scaling curve for L1 exhibits a notably smaller slope (0.24 vs. 0.37 slope of S1), indicating substantially improved effectiveness at lower token ranges.

L1-Exact performs approximately 1% below Agentica-4K, which is the same underlying model as L1, but trained without length constraints. However, this difference is primarily observed in the AIME dataset, where unconstrained models can generate very long chains for complex problems. Additionally, L1-Exact allocates the same token budget to all problems regardless of difficulty, potentially using extra tokens on simpler problems. L1-Max effectively alleviates this challenge, matching the performance of Agentica-4K by optimizing token usage based on problem difficulty while respecting the upper ceiling. In doing so, it outperforms even L1-Exact often by up to 2x fewer tokens. L1-Max is particularly valuable when exact token counts are less desirable than a worst-case compute budget. Finally, the

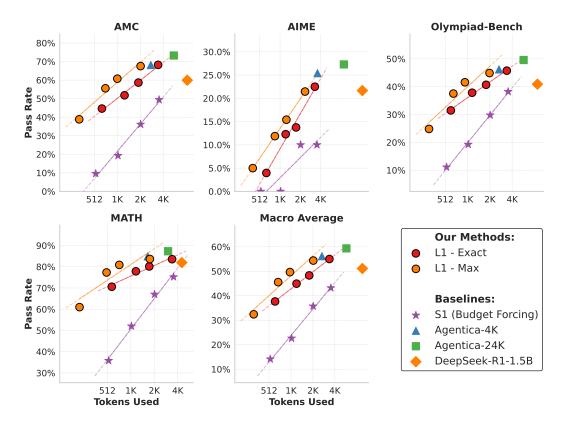


Figure 2: **Performance of L1 under varying token budgets.** Pass rate (y-axis) across multiple benchmarks as a function of tokens used (x-axis). L1-Exact and L1-Exact significantly outperform length-controlled baselines (S1) while approaching the performance of unconstrained models.

scaling trends suggest that with longer context training, L1 would match or even surpass Agentica-24K's performance while maintaining a high degree of length control.

L1 generalizes effectively to out-of-domain (OOD) tasks. We evaluate L1's ability to generalize length control capabilities to domains outside its RL training distribution. We categorize out-of-domain (OOD) datasets: general reasoning datasets GPQA and LSAT that were not explicitly used in L1's training but is likely within DeepSeek-R1-1.5B's training domain and MMLU, which likely falls even outside DeepSeek-R1-1.5B's training distribution.

Figure 3 confirms that L1 generalizes robustly to new domains: performance consistently scales positively with token budget for OOD general reasoning datasets, approaching or matching Agentica-4K benchmarks despite explicit length control constraints. For GPQA and LSAT, we observe the same linear performance scaling trend as in our primary datasets, with L1 matching Agentica-4K's performance at comparable token budgets. This generalization is particularly impressive given that L1 was not explicitly trained on these tasks. For MMLU, we see a less pronounced linear scaling relationship ( $R^2 = 0.66$ ), likely because these knowledge-focused questions benefit less from extended reasoning.

**L1 follows length constraints with high precision.** We quantitatively assess L1's ability to follow length constraints across various mathematical reasoning datasets. As shown in Figure 4, our model maintains consistent control across all token budgets (512, 1024, 2048, and 3600 tokens), with observed output lengths usually closely matching the requested lengths. Further, in Figure 5, we show the mean error:  $(\frac{E_{x\sim D}[n_{generated}]-n_{gold}}{n_{gold}})$ , which captures the average deviation from target lengths across the dataset. The figure demonstrates that mean

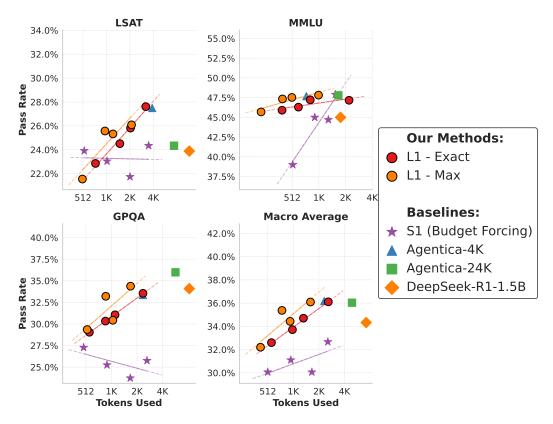


Figure 3: **Out-of-domain (OOD) generalization of L1.** L1 generalizes its length control to out-of-domain tasks, showing linear scaling of pass rate (y-axis) over generation length (x-axis) matching or exceeding performance of various baselines.

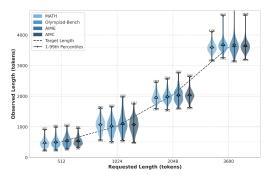


Figure 4: **Length control precision** across different datasets. The plot show distribution of response lengths at different requested lengths. Our model maintains consistent length control, with output lengths closely matching the requested lengths.

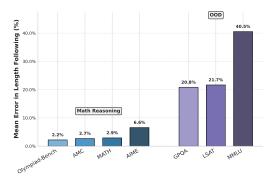


Figure 5: **Mean Error in Length of generated chains** across different datasets. L1 maintains around 3% average deviation on math reasoning datasets and higher but acceptable deviation on OOD tasks.

error is low: close to 3% for all math reasoning datasets. Although OOD datasets exhibit predictably higher errors (20-40%), these remain preferable over uncontrolled prompting. Further Analysis in Appendix A.2 demonstrates that larger errors primarily appear at higher token budgets on tasks like MMLU, where the longer chain of thoughts is mostly unnecessary. Additionally, in Appendix A.1, we show ethat rror can be further reduced significantly with extended RL training.

Model	AIME		MATH		AMC		Olympid-Bench		Average	
	# Tokens	Accuracy	# Tokens	Accuracy	# Tokens	Accuracy	# Tokens	Accuracy	# Tokens	Accuracy
Qwen-1.5	877	7.7	554	73.4	791	47.5	784	35.4	752	41.0
L1-Exact	880	9.2	583	70.6	801	49.7	770	35.5	758	41.2
L1-Max	858	11.9	541	78.3	757	55.9	722	38.5	720	46.2
Llama-3.3-70B	1024	3.8	573	74.9	838	48.6	859	37.3	824	41.2
L1-Exact	983	10.6	583	70.6	801	49.7	913	36.9	820	41.9
L1-Max	1059	15.0	631	79.4	921	59.2	853	40.5	866	48.5
GPT-4o	867	10.0	591	78.4	990	51.8	911	42.4	840	45.6
L1-Exact	880	10.0	583	70.6	1060	51.3	913	36.9	859	42.2
L1-Max	858	11.9	631	79.4	921	59.2	853	40.5	816	47.8

Table 1: **Short CoT performance across math benchmarks.** L1-Max surpasses its non-reasoning base model and competes with frontier models under identical short CoT lengths. \* Our analysis reveals model is significantly contaminated, See results on AIME2025 for details.

**Long CoT Models are secretly Strong Short CoT Models.** Given L1's strong performance at lower token budgets, we conducted a focused evaluation comparing it to both its base non-reasoning model (Qwen-2.5-1.5B-Instruct) and significantly larger non-reasoning models (GPT-4o and Llama-3.3-70B) at comparable *generation lengths*. Table 1 presents these results, showing that L1 consistently outperforms or matches all models across all datasets despite using equivalent token budgets. Further, on average, L1 is 5% better than its non-reasoning counterpart, and even outperforms GPT-4o by 2% on average.

This finding is remarkable, as to the best of our knowledge, this is the first demonstration that a 1.5B model can outperform frontier models such as GPT-4o, despite using the *same generation length*. Overall, the results signify that with suitable RL training, long CoT models can be adaptively used as short CoT models, while significantly outperforming their base counterparts at the same generation length.

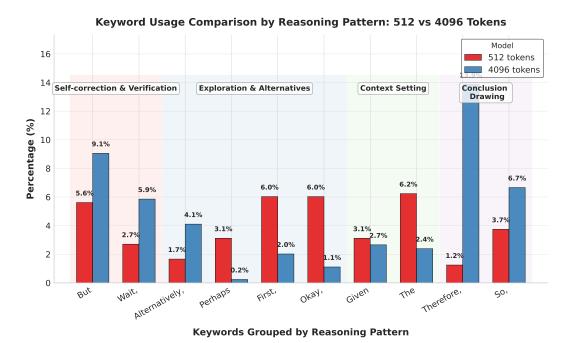


Figure 6: **Keyword usage by reasoning pattern at 512 vs. 4096 tokens.** L1 allocates more tokens to self-correction and conclusion drawing at higher budgets, reflecting adaptive reasoning strategies.

L1 employs distinct reasoning strategies at different token budgets. To understand how L1 changes its reasoning approach across different length constraints, we analyzed

how frequently certain reasoning-related terms appear in outputs of different lengths. Specifically, we calculated the normalized occurrence rate of most common reasoning terms in 512-token outputs compared to 4096-token outputs, showing how the model's reasoning strategies shift when given different length constraints. Figure 6 organizes these keywords into four distinct reasoning patterns: "Self-Correction and Verification," "Exploration and Alternatives," "Context Setting," and "Conclusion Drawing."

Figure 6 shows that self-correction and verification keywords appear approximately twice as frequently in 4096token outputs compared to 512-token outputs. Similarly, conclusion-drawing terms increase 2-10x with increased token budget, indicating more thorough solution validation and completion. Interestingly, most explorationrelated keywords decrease in relative frequency at higher token counts, with "Alternatively" being a notable exception. Overall, we observe that smaller CoTs have reasoning patterns similar to their longer counterparts, but with changed relative frequencies that favor more self-verification and conclusion drawing in longer chains-of-thought.

Further, Figure 7 shows the ratio of thinking tokens (those within <think> tags) to solution tokens for different generation lengths. We observe that the ratio is relatively stable across different generation lengths. This implies for

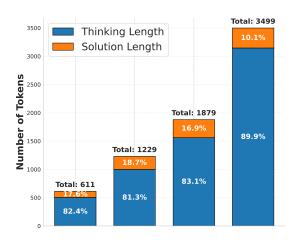


Figure 7: **Distribution of tokens for thinking vs. solution across chain-of-thought lengths.** L1 maintains a mostly consistent ratio of thinking tokens to solution tokens, indicating efficient scaling of intermediate steps without making final answers overly verbose.

shorter CoTs, the model generally provides short solutions (often just outputting the final answer), which helps save tokens. As generation length increases, we notice a stabilized response length in the last two bars, implying the model scales its thinking tokens without making the final solution overly verbose.

#### 6 Conclusion

In this work, we introduced Length Controlled Policy Optimization (LCPO), a simple yet powerful reinforcement learning-based method enabling adaptive control over the length of reasoning chains in language models. We use LCPO to train L1, a reasoning language model, optimized to produce outputs that adhere to length constraints given in its prompt. LCPO significantly surpasses previous test-time scaling methods, achieving over 100% relative and 20% absolute improvements in mathematical reasoning tasks compared to prior length-control approaches.

Further, we demonstrated that L1 generalizes robustly beyond its training distribution, extending its length-controlled capabilities to out-of-domain tasks. Furthermore, our analysis revealed an intriguing phenomenon: models trained to generate longer reasoning chains become unexpectedly strong short-CoT reasoners, outperforming significantly larger frontier models such as GPT-40 at identical generation lengths. By providing length control using simple prompt, LCPO opens promising avenues toward more efficient, flexible, and scalable reasoning models.

#### References

Pranjal Aggarwal, Aman Madaan, Yiming Yang, and Mausam. Let's sample step by step: Adaptive-consistency for efficient reasoning and coding with LLMs. In Houda Bouamor, Juan Pino, and Kalika Bali (eds.), Proceedings of the 2023 Conference on Empirical Methods in Natural Language Processing, pp. 12375–12396, Singapore, December 2023. Association for Computational Linguistics. doi: 10.18653/v1/2023.emnlp-main.761. URL https://aclanthology.org/2023.emnlp-main.761/.

Daman Arora and Andrea Zanette. Training language models to reason efficiently, 2025. URL https://arxiv.org/abs/2502.04463.

Bradley Butcher, Michael O'Keefe, and James Titchener. Precise length control in large language models, 2024. URL https://arxiv.org/abs/2412.11937.

DeepSeek-AI, Daya Guo, Dejian Yang, Haowei Zhang, Junxiao Song, Ruoyu Zhang, Runxin Xu, Qihao Zhu, Shirong Ma, Peiyi Wang, Xiao Bi, Xiaokang Zhang, Xingkai Yu, Yu Wu, Z. F. Wu, Zhibin Gou, Zhihong Shao, Zhuoshu Li, Ziyi Gao, Aixin Liu, Bing Xue, Bingxuan Wang, Bochao Wu, Bei Feng, Chengda Lu, Chenggang Zhao, Chengqi Deng, Chenyu Zhang, Chong Ruan, Damai Dai, Deli Chen, Dongjie Ji, Erhang Li, Fangyun Lin, Fucong Dai, Fuli Luo, Guangbo Hao, Guanting Chen, Guowei Li, H. Zhang, Han Bao, Hanwei Xu, Haocheng Wang, Honghui Ding, Huajian Xin, Huazuo Gao, Hui Qu, Hui Li, Jianzhong Guo, Jiashi Li, Jiawei Wang, Jingchang Chen, Jingyang Yuan, Junjie Qiu, Junlong Li, J. L. Cai, Jiaqi Ni, Jian Liang, Jin Chen, Kai Dong, Kai Hu, Kaige Gao, Kang Guan, Kexin Huang, Kuai Yu, Lean Wang, Lecong Zhang, Liang Zhao, Litong Wang, Liyue Zhang, Lei Xu, Leyi Xia, Mingchuan Zhang, Minghua Zhang, Minghui Tang, Meng Li, Miaojun Wang, Mingming Li, Ning Tian, Panpan Huang, Peng Zhang, Qiancheng Wang, Qinyu Chen, Qiushi Du, Ruiqi Ge, Ruisong Zhang, Ruizhe Pan, Runji Wang, R. J. Chen, R. L. Jin, Ruyi Chen, Shanghao Lu, Shangyan Zhou, Shanhuang Chen, Shengfeng Ye, Shiyu Wang, Shuiping Yu, Shunfeng Zhou, Shuting Pan, S. S. Li, Shuang Zhou, Shaoqing Wu, Shengfeng Ye, Tao Yun, Tian Pei, Tianyu Sun, T. Wang, Wangding Zeng, Wanjia Zhao, Wen Liu, Wenfeng Liang, Wenjun Gao, Wenqin Yu, Wentao Zhang, W. L. Xiao, Wei An, Xiaodong Liu, Xiaohan Wang, Xiaokang Chen, Xiaotao Nie, Xin Cheng, Xin Liu, Xin Xie, Xingchao Liu, Xinyu Yang, Xinyuan Li, Xuecheng Su, Xuheng Lin, X. Q. Li, Xiangyue Jin, Xiaojin Shen, Xiaosha Chen, Xiaowen Sun, Xiaoxiang Wang, Xinnan Song, Xinyi Zhou, Xianzu Wang, Xinxia Shan, Y. K. Li, Y. Q. Wang, Y. X. Wei, Yang Zhang, Yanhong Xu, Yao Li, Yao Zhao, Yaofeng Sun, Yaohui Wang, Yi Yu, Yichao Zhang, Yifan Shi, Yiliang Xiong, Ying He, Yishi Piao, Yisong Wang, Yixuan Tan, Yiyang Ma, Yiyuan Liu, Yongqiang Guo, Yuan Ou, Yuduan Wang, Yue Gong, Yuheng Zou, Yujia He, Yunfan Xiong, Yuxiang Luo, Yuxiang You, Yuxuan Liu, Yuyang Zhou, Y. X. Zhu, Yanhong Xu, Yanping Huang, Yaohui Li, Yi Zheng, Yuchen Zhu, Yunxian Ma, Ying Tang, Yukun Zha, Yuting Yan, Z. Z. Ren, Zehui Ren, Zhangli Sha, Zhe Fu, Zhean Xu, Zhenda Xie, Zhengyan Zhang, Zhewen Hao, Zhicheng Ma, Zhigang Yan, Zhiyu Wu, Zihui Gu, Zijia Zhu, Zijun Liu, Zilin Li, Ziwei Xie, Ziyang Song, Zizheng Pan, Zhen Huang, Zhipeng Xu, Zhongyu Zhang, and Zhen Zhang. Deepseek-r1: Incentivizing reasoning capability in llms via reinforcement learning, 2025. URL https://arxiv.org/abs/2501.12948.

Bofei Gao, Feifan Song, Zhe Yang, Zefan Cai, Yibo Miao, Qingxiu Dong, Lei Li, Chenghao Ma, Liang Chen, Runxin Xu, Zhengyang Tang, Benyou Wang, Daoguang Zan, Shanghaoran Quan, Ge Zhang, Lei Sha, Yichang Zhang, Xuancheng Ren, Tianyu Liu, and Baobao Chang. Omni-math: A universal olympiad level mathematic benchmark for large language models, 2024. URL https://arxiv.org/abs/2410.07985.

Chaoqun He, Renjie Luo, Yuzhuo Bai, Shengding Hu, Zhen Leng Thai, Junhao Shen, Jinyi Hu, Xu Han, Yujie Huang, Yuxiang Zhang, Jie Liu, Lei Qi, Zhiyuan Liu, and Maosong Sun. Olympiadbench: A challenging benchmark for promoting agi with olympiadlevel bilingual multimodal scientific problems, 2024. URL https://arxiv.org/abs/2402.14008.

Dan Hendrycks, Collin Burns, Steven Basart, Andy Zou, Mantas Mazeika, Dawn Song, and Jacob Steinhardt. Measuring massive multitask language understanding, 2021a. URL https://arxiv.org/abs/2009.03300.

- Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul Arora, Steven Basart, Eric Tang, Dawn Song, and Jacob Steinhardt. Measuring mathematical problem solving with the math dataset, 2021b. URL https://arxiv.org/abs/2103.03874.
- Renlong Jie, Xiaojun Meng, Lifeng Shang, Xin Jiang, and Qun Liu. Prompt-based length controlled generation with reinforcement learning. arXiv preprint arXiv:2308.12030, 2023.
- Yu Kang, Xianghui Sun, Liangyu Chen, and Wei Zou. C3ot: Generating shorter chain-of-thought without compromising effectiveness, 2024. URL https://arxiv.org/abs/2412.11664.
- Michael Luo, Sijun Tan, Justin Wong, Xiaoxiang Shi, William Y. Tang,
  Manan Roongta, Colin Cai, Jeffrey Luo, Tianjun Zhang, Li Erran Li,
  Raluca Ada Popa, and Ion Stoica. Deepscaler: Surpassing o1-preview
  with a 1.5b model by scaling rl. https://pretty-radio-b75.notion.site/
  DeepScaleR-Surpassing-01-Preview-with-a-1-5B-Model-by-Scaling-RL-19681902c1468005bed8ca303013a4e2,
  2025. Notion Blog.
- Aman Madaan, Niket Tandon, Prakhar Gupta, Skyler Hallinan, Luyu Gao, Sarah Wiegreffe, Uri Alon, Nouha Dziri, Shrimai Prabhumoye, Yiming Yang, Shashank Gupta, Bodhisattwa Prasad Majumder, Katherine Hermann, Sean Welleck, Amir Yazdanbakhsh, and Peter Clark. Self-refine: Iterative refinement with self-feedback, 2023. URL https://arxiv.org/abs/2303.17651.
- Yingqian Min, Zhipeng Chen, Jinhao Jiang, Jie Chen, Jia Deng, Yiwen Hu, Yiru Tang, Jiapeng Wang, Xiaoxue Cheng, Huatong Song, Wayne Xin Zhao, Zheng Liu, Zhongyuan Wang, and Ji-Rong Wen. Imitate, explore, and self-improve: A reproduction report on slow-thinking reasoning systems, 2024. URL https://arxiv.org/abs/2412.09413.
- Bytedance Seed MLSys. verl: Volcano engine reinforcement learning for llms. https://github.com/volcengine/verl, 2025. Accessed: 2025-02-28.
- Niklas Muennighoff, Zitong Yang, Weijia Shi, Xiang Lisa Li, Li Fei-Fei, Hannaneh Hajishirzi, Luke Zettlemoyer, Percy Liang, Emmanuel Candès, and Tatsunori Hashimoto. s1: Simple test-time scaling, 2025. URL https://arxiv.org/abs/2501.19393.
- OpenAI,:, Aaron Jaech, Adam Kalai, Adam Lerer, Adam Richardson, Ahmed El-Kishky, Aiden Low, Alec Helyar, Aleksander Madry, Alex Beutel, Alex Carney, Alex Iftimie, Alex Karpenko, Alex Tachard Passos, Alexander Neitz, Alexander Prokofiev, Alexander Wei, Allison Tam, Ally Bennett, Ananya Kumar, Andre Saraiva, Andrea Vallone, Andrew Duberstein, Andrew Kondrich, Andrey Mishchenko, Andy Applebaum, Angela Jiang, Ashvin Nair, Barret Zoph, Behrooz Ghorbani, Ben Rossen, Benjamin Sokolowsky, Boaz Barak, Bob McGrew, Borys Minaiev, Botao Hao, Bowen Baker, Brandon Houghton, Brandon McKinzie, Brydon Eastman, Camillo Lugaresi, Cary Bassin, Cary Hudson, Chak Ming Li, Charles de Bourcy, Chelsea Voss, Chen Shen, Chong Zhang, Chris Koch, Chris Orsinger, Christopher Hesse, Claudia Fischer, Clive Chan, Dan Roberts, Daniel Kappler, Daniel Levy, Daniel Selsam, David Dohan, David Farhi, David Mely, David Robinson, Dimitris Tsipras, Doug Li, Dragos Oprica, Eben Freeman, Eddie Zhang, Edmund Wong, Elizabeth Proehl, Enoch Cheung, Eric Mitchell, Eric Wallace, Erik Ritter, Evan Mays, Fan Wang, Felipe Petroski Such, Filippo Raso, Florencia Leoni, Foivos Tsimpourlas, Francis Song, Fred von Lohmann, Freddie Sulit, Geoff Salmon, Giambattista Parascandolo, Gildas Chabot, Grace Zhao, Greg Brockman, Guillaume Leclerc, Hadi Salman, Haiming Bao, Hao Sheng, Hart Andrin, Hessam Bagherinezhad, Hongyu Ren, Hunter Lightman, Hyung Won Chung, Ian Kivlichan, Ian O'Connell, Ian Osband, Ignasi Clavera Gilaberte, Ilge Akkaya, Ilya Kostrikov, Ilya Sutskever, Irina Kofman, Jakub Pachocki, James Lennon, Jason Wei, Jean Harb, Jerry Twore, Jiacheng Feng, Jiahui Yu, Jiayi Weng, Jie Tang, Jieqi Yu, Joaquin Quiñonero Candela, Joe Palermo, Joel Parish, Johannes Heidecke, John Hallman, John Rizzo, Jonathan Gordon, Jonathan Uesato, Jonathan Ward, Joost Huizinga, Julie Wang, Kai Chen, Kai Xiao, Karan Singhal, Karina Nguyen, Karl Cobbe, Katy Shi, Kayla Wood, Kendra Rimbach, Keren Gu-Lemberg, Kevin Liu, Kevin Lu, Kevin Stone, Kevin Yu, Lama Ahmad, Lauren Yang, Leo Liu, Leon Maksin, Leyton Ho, Liam Fedus, Lilian Weng,

Linden Li, Lindsay McCallum, Lindsey Held, Lorenz Kuhn, Lukas Kondraciuk, Lukasz Kaiser, Luke Metz, Madelaine Boyd, Maja Trebacz, Manas Joglekar, Mark Chen, Marko Tintor, Mason Meyer, Matt Jones, Matt Kaufer, Max Schwarzer, Meghan Shah, Mehmet Yatbaz, Melody Y. Guan, Mengyuan Xu, Mengyuan Yan, Mia Glaese, Mianna Chen, Michael Lampe, Michael Malek, Michele Wang, Michelle Fradin, Mike McClay, Mikhail Pavlov, Miles Wang, Mingxuan Wang, Mira Murati, Mo Bavarian, Mostafa Rohaninejad, Nat McAleese, Neil Chowdhury, Neil Chowdhury, Nick Ryder, Nikolas Tezak, Noam Brown, Ofir Nachum, Oleg Boiko, Oleg Murk, Olivia Watkins, Patrick Chao, Paul Ashbourne, Pavel Izmailov, Peter Zhokhov, Rachel Dias, Rahul Arora, Randall Lin, Rapha Gontijo Lopes, Raz Gaon, Reah Miyara, Reimar Leike, Renny Hwang, Rhythm Garg, Robin Brown, Roshan James, Rui Shu, Ryan Cheu, Ryan Greene, Saachi Jain, Sam Altman, Sam Toizer, Sam Toyer, Samuel Miserendino, Sandhini Agarwal, Santiago Hernandez, Sasha Baker, Scott McKinney, Scottie Yan, Shengjia Zhao, Shengli Hu, Shibani Santurkar, Shraman Ray Chaudhuri, Shuyuan Zhang, Siyuan Fu, Spencer Papay, Steph Lin, Suchir Balaji, Suvansh Sanjeev, Szymon Sidor, Tal Broda, Aidan Clark, Tao Wang, Taylor Gordon, Ted Sanders, Tejal Patwardhan, Thibault Sottiaux, Thomas Degry, Thomas Dimson, Tianhao Zheng, Timur Garipov, Tom Stasi, Trapit Bansal, Trevor Creech, Troy Peterson, Tyna Eloundou, Valerie Qi, Vineet Kosaraju, Vinnie Monaco, Vitchyr Pong, Vlad Fomenko, Weiyi Zheng, Wenda Zhou, Wes McCabe, Wojciech Zaremba, Yann Dubois, Yinghai Lu, Yining Chen, Young Cha, Yu Bai, Yuchen He, Yuchen Zhang, Yunyun Wang, Zheng Shao, and Zhuohan Li. Openai o1 system card, 2024a. URL https://arxiv.org/abs/2412.16720.

OpenAI, Josh Achiam, Steven Adler, Sandhini Agarwal, Lama Ahmad, Ilge Akkaya, Florencia Leoni Aleman, Diogo Almeida, Janko Altenschmidt, Sam Altman, Shyamal Anadkat, Red Avila, Igor Babuschkin, Suchir Balaji, Valerie Balcom, Paul Baltescu, Haiming Bao, Mohammad Bavarian, Jeff Belgum, Irwan Bello, Jake Berdine, Gabriel Bernadett-Shapiro, Christopher Berner, Lenny Bogdonoff, Oleg Boiko, Madelaine Boyd, Anna-Luisa Brakman, Greg Brockman, Tim Brooks, Miles Brundage, Kevin Button, Trevor Cai, Rosie Campbell, Andrew Cann, Brittany Carey, Chelsea Carlson, Rory Carmichael, Brooke Chan, Che Chang, Fotis Chantzis, Derek Chen, Sully Chen, Ruby Chen, Jason Chen, Mark Chen, Ben Chess, Chester Cho, Casey Chu, Hyung Won Chung, Dave Cummings, Jeremiah Currier, Yunxing Dai, Cory Decareaux, Thomas Degry, Noah Deutsch, Damien Deville, Arka Dhar, David Dohan, Steve Dowling, Sheila Dunning, Adrien Ecoffet, Atty Eleti, Tyna Eloundou, David Farhi, Liam Fedus, Niko Felix, Simón Posada Fishman, Juston Forte, Isabella Fulford, Leo Gao, Elie Georges, Christian Gibson, Vik Goel, Tarun Gogineni, Gabriel Goh, Rapha Gontijo-Lopes, Jonathan Gordon, Morgan Grafstein, Scott Gray, Ryan Greene, Joshua Gross, Shixiang Shane Gu, Yufei Guo, Chris Hallacy, Jesse Han, Jeff Harris, Yuchen He, Mike Heaton, Johannes Heidecke, Chris Hesse, Alan Hickey, Wade Hickey, Peter Hoeschele, Brandon Houghton, Kenny Hsu, Shengli Hu, Xin Hu, Joost Huizinga, Shantanu Jain, Shawn Jain, Joanne Jang, Angela Jiang, Roger Jiang, Haozhun Jin, Denny Jin, Shino Jomoto, Billie Jonn, Heewoo Jun, Tomer Kaftan, Łukasz Kaiser, Ali Kamali, Ingmar Kanitscheider, Nitish Shirish Keskar, Tabarak Khan, Logan Kilpatrick, Jong Wook Kim, Christina Kim, Yongjik Kim, Jan Hendrik Kirchner, Jamie Kiros, Matt Knight, Daniel Kokotajlo, Łukasz Kondraciuk, Andrew Kondrich, Aris Konstantinidis, Kyle Kosic, Gretchen Krueger, Vishal Kuo, Michael Lampe, Ikai Lan, Teddy Lee, Jan Leike, Jade Leung, Daniel Levy, Chak Ming Li, Rachel Lim, Molly Lin, Stephanie Lin, Mateusz Litwin, Theresa Lopez, Ryan Lowe, Patricia Lue, Anna Makanju, Kim Malfacini, Sam Manning, Todor Markov, Yaniv Markovski, Bianca Martin, Katie Mayer, Andrew Mayne, Bob McGrew, Scott Mayer McKinney, Christine McLeavey, Paul McMillan, Jake McNeil, David Medina, Aalok Mehta, Jacob Menick, Luke Metz, Andrey Mishchenko, Pamela Mishkin, Vinnie Monaco, Evan Morikawa, Daniel Mossing, Tong Mu, Mira Murati, Oleg Murk, David Mély, Ashvin Nair, Reiichiro Nakano, Rajeev Nayak, Arvind Neelakantan, Richard Ngo, Hyeonwoo Noh, Long Ouyang, Cullen O'Keefe, Jakub Pachocki, Alex Paino, Joe Palermo, Ashley Pantuliano, Giambattista Parascandolo, Joel Parish, Emy Parparita, Alex Passos, Mikhail Pavlov, Andrew Peng, Adam Perelman, Filipe de Avila Belbute Peres, Michael Petrov, Henrique Ponde de Oliveira Pinto, Michael, Pokorny, Michelle Pokrass, Vitchyr H. Pong, Tolly Powell, Alethea Power, Boris Power, Elizabeth Proehl, Raul Puri, Alec Radford, Jack Rae, Aditya Ramesh, Cameron Raymond, Francis Real, Kendra Rimbach, Carl Ross, Bob Rotsted, Henri Roussez, Nick Ryder, Mario

Saltarelli, Ted Sanders, Shibani Santurkar, Girish Sastry, Heather Schmidt, David Schnurr, John Schulman, Daniel Selsam, Kyla Sheppard, Toki Sherbakov, Jessica Shieh, Sarah Shoker, Pranav Shyam, Szymon Sidor, Eric Sigler, Maddie Simens, Jordan Sitkin, Katarina Slama, Ian Sohl, Benjamin Sokolowsky, Yang Song, Natalie Staudacher, Felipe Petroski Such, Natalie Summers, Ilya Sutskever, Jie Tang, Nikolas Tezak, Madeleine B. Thompson, Phil Tillet, Amin Tootoonchian, Elizabeth Tseng, Preston Tuggle, Nick Turley, Jerry Tworek, Juan Felipe Cerón Uribe, Andrea Vallone, Arun Vijayvergiya, Chelsea Voss, Carroll Wainwright, Justin Jay Wang, Alvin Wang, Ben Wang, Jonathan Ward, Jason Wei, CJ Weinmann, Akila Welihinda, Peter Welinder, Jiayi Weng, Lilian Weng, Matt Wiethoff, Dave Willner, Clemens Winter, Samuel Wolrich, Hannah Wong, Lauren Workman, Sherwin Wu, Jeff Wu, Michael Wu, Kai Xiao, Tao Xu, Sarah Yoo, Kevin Yu, Qiming Yuan, Wojciech Zaremba, Rowan Zellers, Chong Zhang, Marvin Zhang, Shengjia Zhao, Tianhao Zheng, Juntang Zhuang, William Zhuk, and Barret Zoph. Gpt-4 technical report, 2024b. URL https://arxiv.org/abs/2303.08774.

Qwen, :, An Yang, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chengyuan Li, Dayiheng Liu, Fei Huang, Haoran Wei, Huan Lin, Jian Yang, Jianhong Tu, Jianwei Zhang, Jianxin Yang, Jiaxi Yang, Jingren Zhou, Junyang Lin, Kai Dang, Keming Lu, Keqin Bao, Kexin Yang, Le Yu, Mei Li, Mingfeng Xue, Pei Zhang, Qin Zhu, Rui Men, Runji Lin, Tianhao Li, Tianyi Tang, Tingyu Xia, Xingzhang Ren, Xuancheng Ren, Yang Fan, Yang Su, Yichang Zhang, Yu Wan, Yuqiong Liu, Zeyu Cui, Zhenru Zhang, and Zihan Qiu. Qwen2.5 technical report, 2025. URL https://arxiv.org/abs/2412.15115.

David Rein, Betty Li Hou, Asa Cooper Stickland, Jackson Petty, Richard Yuanzhe Pang, Julien Dirani, Julian Michael, and Samuel R. Bowman. Gpqa: A graduate-level google-proof q&a benchmark, 2023. URL https://arxiv.org/abs/2311.12022.

Zhihong Shao, Peiyi Wang, Qihao Zhu, Runxin Xu, Junxiao Song, Xiao Bi, Haowei Zhang, Mingchuan Zhang, Y. K. Li, Y. Wu, and Daya Guo. Deepseekmath: Pushing the limits of mathematical reasoning in open language models, 2024. URL https://arxiv.org/abs/2402.03300.

Prasann Singhal, Tanya Goyal, Jiacheng Xu, and Greg Durrett. A long way to go: Investigating length correlations in rlhf, 2024. URL https://arxiv.org/abs/2310.03716.

Charlie Snell, Jaehoon Lee, Kelvin Xu, and Aviral Kumar. Scaling Ilm test-time compute optimally can be more effective than scaling model parameters, 2024. URL https://arxiv.org/abs/2408.03314.

Xuezhi Wang, Jason Wei, Dale Schuurmans, Quoc Le, Ed Chi, Sharan Narang, Aakanksha Chowdhery, and Denny Zhou. Self-consistency improves chain of thought reasoning in language models, 2023. URL https://arxiv.org/abs/2203.11171.

Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Brian Ichter, Fei Xia, Ed Chi, Quoc Le, and Denny Zhou. Chain-of-thought prompting elicits reasoning in large language models, 2023. URL https://arxiv.org/abs/2201.11903.

Sean Welleck, Ximing Lu, Peter West, Faeze Brahman, Tianxiao Shen, Daniel Khashabi, and Yejin Choi. Generating sequences by learning to self-correct. In <a href="mailto:The Eleventh International Conference">The Eleventh International Conference on Learning Representations</a>, 2023. URL <a href="https://openreview.net/forum?id=hH36JeQZDa0">https://openreview.net/forum?id=hH36JeQZDa0</a>.

Sean Welleck, Amanda Bertsch, Matthew Finlayson, Hailey Schoelkopf, Alex Xie, Graham Neubig, Ilia Kulikov, and Zaid Harchaoui. From decoding to meta-generation: Inference-time algorithms for large language models, 2024. URL https://arxiv.org/abs/2406.16838

Yangzhen Wu, Zhiqing Sun, Shanda Li, Sean Welleck, and Yiming Yang. Inference scaling laws: An empirical analysis of compute-optimal inference for problem-solving with language models, 2024. URL https://arxiv.org/abs/2408.00724.

- Huajian Xin, Z. Z. Ren, Junxiao Song, Zhihong Shao, Wanjia Zhao, Haocheng Wang, Bo Liu, Liyue Zhang, Xuan Lu, Qiushi Du, Wenjun Gao, Qihao Zhu, Dejian Yang, Zhibin Gou, Z. F. Wu, Fuli Luo, and Chong Ruan. Deepseek-prover-v1.5: Harnessing proof assistant feedback for reinforcement learning and monte-carlo tree search, 2024. URL https://arxiv.org/abs/2408.08152.
- Shunyu Yao, Dian Yu, Jeffrey Zhao, Izhak Shafran, Thomas L. Griffiths, Yuan Cao, and Karthik Narasimhan. Tree of thoughts: Deliberate problem solving with large language models, 2023. URL https://arxiv.org/abs/2305.10601.
- Weizhe Yuan, Ilia Kulikov, Ping Yu, Kyunghyun Cho, Sainbayar Sukhbaatar, Jason Weston, and Jing Xu. Following length constraints in instructions, 2024. URL https://arxiv.org/abs/2406.17744.
- Wanjun Zhong, Ruixiang Cui, Yiduo Guo, Yaobo Liang, Shuai Lu, Yanlin Wang, Amin Saied, Weizhu Chen, and Nan Duan. Agieval: A human-centric benchmark for evaluating foundation models, 2023. URL https://arxiv.org/abs/2304.06364.

### A Results

#### A.1 Extended training further improves length constraint precision.

Table 2: Length Error Comparison Between Methods

Method	Mean Error (%)↓	RMSE Error (%)↓						
Math Reasoning								
L1-Exact	3.01	18.44						
L1-Exact +	3.15	10.04						
OOD-1 (General Reasoning)								
L1-Exact	21.22	31.37						
L1-Exact +	10.89	16.77						
OOD-2 (General knowledge)								
L1-Exact	40.54	42.61						
L1-Exact +	23.57	33.48						

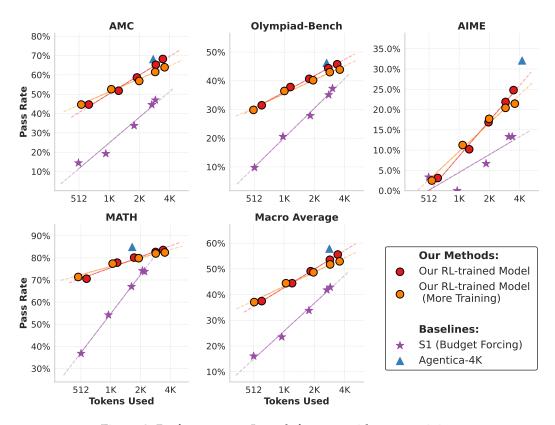


Figure 8: Performance vs Length for LCPO with more training

Motivated by the remaining gap in length control precision, we investigate if further RL training would improve length adherence. In particular, we further fine-tune L1-Exact for 500 RL steps. We refer to the derived model as L1-Exact+ Interestingly, Table 2 reveals substantial precision improvements—particularly steep RMSE reductions—from additional training across both math and OOD datasets. Yet, greater length precision appears to slightly lower performance at high token ranges (Figure 8). This effect likely stems from the model's stricter adherence to length constraints, which reduces its flexibility to generate longer CoTs to more challenging problems. Nevertheless, the model maintains its overall performance trends, suggesting an interesting direction for future work: enabling users to dynamically

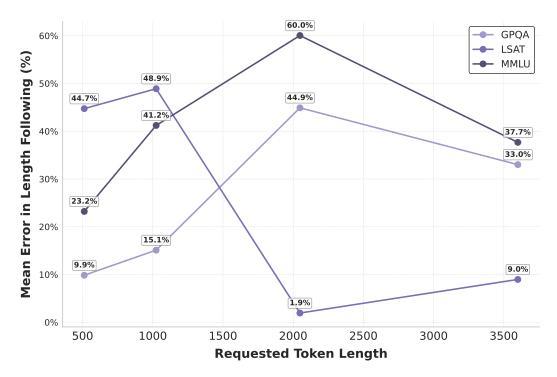


Figure 9: Length error vs token length for OOD datasets. Datasets like MMLU show higher error for longer chains, likely because longer chains are not particularly useful for these datasets.

control the tradeoff between strict length adherence and optimal performance based on their specific requirements.

#### A.2 LCPO follows length constraints with high precision.

In Section 5, we show that LCPO follows length constraints with high precision, although with larger errors in OOD datasets. In this section, we particularly analyze the generation length at which error occurs. Figure 9 shows error as a function of generated token length for 3 different OOD datasets.

Model	AIME2024	AIME2025	Delta
Qwen-Math-1.5	12.9	7.7	-5.2
GPT-40	6.7	10.0	+3.3
LLama-3.3-70B	27.3	3.8	-23.5
DeepSeek-R1-1.5B	29.2	21.7	-7.5
Agentica-24K	40.2	27.3	-12.9
Agentica-4K	32.1	25.4	-6.7
L1-Exact (ours)	24.8	22.5	-2.3
L1-Max (ours)	27.3	21.5	-5.8
L1-Exact (1024 tokens)	10.2	12.3	+2.1
L1-Max (1024 tokens)	16.3	11.9	-4.4

Table 3: Comparison of model performance on AIME2024 and AIME2025 benchmarks.

**Memorization or Generalization?** Given the surprisingly strong performance of LCPO, particularly at lower token budgets, a natural question arises regarding whether the model has memorized solutions, suggesting potential train-test leakage. To address this concern,

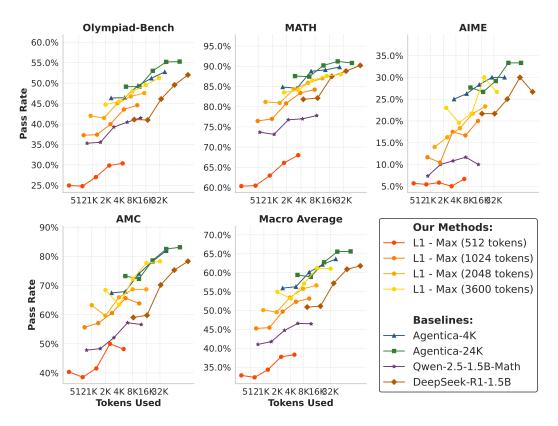


Figure 10: Parallel vs Sequential Scaling

we specifically evaluate on the AIME2025 dataset, an exam released after the training of all evaluated models. Results in Table 3 demonstrate that the performance drop for LCPO is substantially smaller ( $\approx$  2%) compared to other models such as Agentica-24K (-12.9%), DeepSeek-R1-1.5B (-7.5%), and even Llama-3.3-70B (-23.5%). These results provide strong evidence that despite using shorter CoT lengths, our model demonstrates generalization rather than relying solely on memorization.

#### A.3 Parallel vs Sequential Scaling

Figure 10 compares how sequential scaling (increasing CoT length) and parallel scaling (Majority Voting) vary in performance for the same incurred cost (total tokens generated). There are a few interesting observations from the graph: First, parallel scaling is almost always worse than sequential scaling, when compared at the same token budget. Second, parallel scaling can provide significant improvements, to all reasoning models (eg, more than 10% in some cases). Third, while Qwen-2.5-1.5B-Math, barely benefits from majority voting, L1-Max (1024 tokens), despite using similar number of generation tokens in each task, benefits significantly. This is noteworthy, since for many cases, it is desirable to have low token lengths, because of both reduced latency, and avoiding the quadratic cost of generation. In such cases, LCP0-Max clearly severs as a better alternative to parallel scaling of non-reasoning models. Finally, LCP0-Max (3600 tokens) matches the performance of Agentica-4K baseline across parallel scaling. We do note a small dip in the last point, but this is because only 1 seed was used for parallel scaling, and as such the difference is not statistically significant.

#### A.4 Qualitative Examples

In Figures 11 and 12, we show two examples of L1-Exact solving the same problem but with different requested generation lengths: 512 and 3600 tokens. While the model with 512

tokens is unable to solve the problem correctly, the model with longer generation arrives at the correct answer. We refer interested readers to our website, which has more such interesting examples.

# Example Question Find the largest possible real part of $[(75 + 117i)z + \frac{96 + 144i}{z}]$ where z is a complex number with |z| = 4. Let's think step by step and output the final answer within boxed{}. Think for 512 tokens. Correct Answer 540 Correctness × Incorrect **Tokens Requested** 512 Model Response Okay, so I have this problem: I need to find the largest possible real part of the expression (75 + 117i)z + (96 + 144i)/z, where z is a complex number with |z| = 4. Hmm, let's break this down. First, since |z| = 4, z can be written as $4e^{i\theta}$ for some angle $\theta$ . That might help simplify things. Let me denote $z = 4e^{i\theta}$ . Then, $1/z = (1/4)e^{-i\theta}$ because |z| = 4. So, the expression becomes $(75 + 117i) \cdot 4e^{i\theta} + (96 + 144i) \cdot (1/4)e^{-i\theta}$ . Simplifying, that's $4(75 + 117i)e^{i\theta} + (24 + 36i)e^{-i\theta}$ . Now, I need to find the real part of this expression. Let's denote Re[(75 + 117i)z + (96 + 144i)/z]. Breaking it down, Re[(75 + 117i)z] + Re[(96 + 144i)/z]. Since $z = 4e^{i\theta}$ , Re $[(75 + 117i) \cdot 4e^{i\theta}] + \text{Re}[(96 + 144i) \cdot (1/4)e^{-i\theta}]$ . This becomes $4\text{Re}[(75 + 117i)e^{i\theta}] + (1/4)\text{Re}[(96 + 144i)e^{-i\theta}].$ Now, let's compute $Re[(75 + 117i)e^{i\theta}]$ . This is $75 \cos \theta + 117 \sin \theta$ . Similarly, Re[ $(96 + 144i)e^{-i\theta}$ ] is $96\cos\theta - 144\sin\theta$ . So, putting it all together, the real part is $4(75\cos\theta + 117\sin\theta) +$ $(1/4)(96\cos\theta - 144\sin\theta).$ Simplifying, that's $300 \cos \theta + 468 \sin \theta + 24 \cos \theta - 36 \sin \theta$ . Combining like terms, we get $(300 + 24)\cos\theta + (468 - 36)\sin\theta = 324\cos\theta +$ Now, we need to find the maximum of $324 \cos \theta + 432 \sin \theta$ . This can be written as $R\cos(\theta - \phi)$ , where $R = \sqrt{324^2 + 432^2}$ . Calculating R: $324^2 = 104,976, 432^2 = 186,624$ . Sum is 391,600. R = $\sqrt{391,600} \approx 625.6$ . Thus, the maximum real part is approximately 625.6. The largest possible real part is 625.6.

Figure 11: Example of model response with 512 tokens. Note that the model's answer is incorrect.

#### Example

#### Question

Find the largest possible real part of  $[(75 + 117i)z + \frac{96 + 144i}{z}]$  where z is a complex number with |z| = 4. Let's think step by step and output the final answer within  $boxed\{\}$ . Think for 3600 tokens.

#### Correct Answer

540

#### Correctness

**✓** Correct

#### **Tokens Requested**

3600

#### **Model Response**

#### Thinking Process (not shown to user)

Okay, so I have this problem: I need to find the largest possible real part of the expression (75 + 117i)z + (96 + 144i)/z, where z is a complex number with |z|=4. Hmm, that seems a bit involved, but let's break it down step by step. First, let me understand what is given. We have a complex number z with magnitude 4, so z can be written in the form  $z=4e^{i\theta}$ , where  $\theta$  is the argument of z. That might be useful because it allows us to express z in terms of its angle, which could simplify the expression.

The expression we need to consider is  $(75 + 117i)z + \frac{96 + 144i}{z}$ . We are asked to find the largest possible real part of this expression. So, let's denote the expression as  $E = (75 + 117i)z + \frac{96 + 144i}{z}$ .

[Reasoning process truncated for brevity ...]

Then, adding 104,976 + 186,624 = 291,600.  $\sqrt{291,600} = \sqrt{2916}*100 = \sqrt{2916}*10$ . Now,  $\sqrt{2916}$ : let's see,  $54^2 = 2916$ , because  $50^2 = 2500, 54^2 = 2916$ . So,  $\sqrt{2916} = 54$ , thus  $\sqrt{2916}*100 = 54*10 = 540$ . So, that seems correct. Therefore, the maximum value of Re(E) is 540.

Therefore, the largest possible real part is 540.

Given the expression  $(75+117i)z+\frac{96+144i}{z}$  where z is a complex number with |z|=4, we need to find the largest possible real part of this expression. First, we express z as  $4e^{i\theta}$  since |z|=4. Then, we substitute z and  $\frac{1}{z}$  into the expression:

$$(75+117i)z + \frac{96+144i}{z}$$

[Output truncated for brevity ...] Thus, the largest possible real part is:

540

Figure 12: Example of model response with 3600 tokens. With more tokens, the model correctly calculates the answer.