... University Chittagong
Department of Computer Science and Engineering
B. Sc. in CSE Final Examination, Spring 2023

CSE-4747 **Mathematical Analysis for Computer Science**

Time: 2 Hours 30 Minutes Total Marks: 50

[Answer *all* the following questions. Figures in the right-hand margin indicate full marks. Use separate answer scripts for Group-A and Group-B.]

Group A

CO DL

1.a. Farisha just graduated from high school. She was accepted to three reputable colleges. CO3 N 5

- With probability 4/12, she attends Yale.
- With probability 5/12, she attends MIT.
- With probability 3/12, she attends Little Hoop Community College.

Farisha is either happy or unhappy in college.

- If she attends Yale, she is happy with probability 4/12.
- If she attends MIT, she is happy with probability 7/12.
- If she attends Little Hoop, she is happy with probability 11/12.

I. Draw the tree diagram. On the diagram, fill in the edge probabilities, and at each leaf write the probability of the corresponding outcome.

II. What is the probability that Farisha is happy in college?

III. What is the probability that Farisha attends Yale, given that she is happy in college

1.b. There is an unpleasant, degenerative disease called Beaver Fever which causes people to tell math jokes unrelentingly in social settings, believing other people will think they're funny. Fortunately, Beaver Fever is rare, afflicting only about 1 in 1000 people. Doctor Islam has a fairly reliable diagnostic test to determine who is going to suffer from this disease: CO3 N 5

- If a person will suffer from Beaver Fever, the probability that Dr. Islam diagnoses this is 0.99.
- If a person will not suffer from Beaver Fever, the probability that Dr. Islam diagnoses this is 0.97.

Let B be the event that a randomly chosen person will suffer Beaver Fever, and Y be the event that Dr. Islam's diagnosis is "Yes, this person will suffer from Beaver Fever," with B^c and Y^c being the complements of these events.

Now find Pr[B], Pr[Y|B], and $Pr[Y^c|B^c]$

2.a. Every minute a data center sees a single disk failure with probability p. Assume that more than a single failure does not occur in a minute, and occurrences of failure are independent from minute to minute. What is the probability that there will be no disk failures in a given day? What is the probability that there will be no more than two disk failures in a given day? In addition, starting at an arbitrary point in time, what is the expected duration an observer has to wait before encountering the first disk failure? CO2 N 5

OR

The probability that you manage to get ready on time at the start of a given day is 0.7 and when you are ready on time you can then take the university bus. Otherwise, you have to take a public/local bus to get to the campus. The university bus is equally likely to take 41, 42, ··, or 50 minutes to reach the campus, whereas the local/public bus is equally likely to take 46, 47, or 55 minutes (integral measures in both cases).

What is the expected time for you to get to the campus on any given day? What is the probability
that you'd manage to get to the campus within 45 minutes on any given day?

2.b. Let we flip a fair coin 5 times. X is a random number that represents the number of heads that come up independently from the experiment. Now find out the probability distribution of the random variables using binomial distribution. Also, plot the probability distribution function. CO3 N 5

OR

A fair 6-sided die is rolled twice. Consider the following three events A, B and C:

A ≡ 1st roll is 1, 2 or 3.

B ≡ 1st roll is 3, 4 or 5.

C ≡ the sum of the two rolls is 9.

Determine the sample space and the outcome probabilities.

Then express the three events on that basis before showing whether they are pairwise independent and/or mutually independent.

Group B

3.a. A local area network has its n nodes share a single wireless channel. Each of these nodes transmits its packet to the access point (AP) at any given time-slot with probability p (independent of the actions of the other nodes and occurrences in the prior slots). Whenever more than one node transmits in a particular slot, collision occurs and the AP cannot correctly receive any of the transmitted packets. When no nodes transmit during a particular slot, it goes idle. CO3 N 5

Now answer the following.

i. Find the probability that a particular node's transmission in a given slot will be successful.

ii. Find the expected number of nodes that refrain from transmitting in a given slot.

iii. Find the probability that two successive slots on the channel go idle.

3.b. If an aircraft is present in a certain area, a radar detects it and generates an alarm signal with probability 0.99. If an aircraft is not present, the radar generates (false) alarm, with probability 0.10. We assume that an aircraft is present with probability 0.04. What is the probability that an aircraft is indeed present, given that an alarm is generated? CO3 N 5

OR

A lie detector is 90 percent effective in detecting a lie when the person is, in fact, lying. However, the detector also yields a "false positive" result for 1 percent of the truthful persons tested. (That is, if a person being tested is truthful, then, with probability 0.01, the test result will indicate that he is lying.) If 15 percent of the population actually lie, what is the probability that a person is indeed lying given that his test result indicates him to be lying?

4.a. The transition matrix for people voting for candidates from various political parties in an election year is given below. If a person votes for the candidate from one party in an election, that person may vote for the same party in the next election or may switch to vote for a candidate from another party in the next election. Democrats, Republicans, and Independents are denoted by the letters D, R, and I. CO3 N 5

$$\text{This Election}\quad \begin{array}{c} \\ D \\ R \\ I \end{array}\overset{\text{Next Election}}{\begin{array}{c} \begin{array}{ccc} D & R & I \end{array} \\ \begin{bmatrix} .6 & .3 & .1 \\ .3 & .6 & .1 \\ .2 & .2 & .6 \end{bmatrix} \end{array}}$$

Assume there is an election every year so that the transition period is 1 year.

I. Find the probability that a person who votes Democratic in the current election will vote Republican in the next election.

II. Find the probability that a person who votes Democratic in the current election will vote Republican in the election two years from now.

4.b. How can you measure the importance of pages (Page Rank) using Markov chain? When a Markob chain is ergodic? CO2 C 5

OR

A Web graph is made of Webpages linking to each other via hyperlinks. Find a Markov chain based model that produces relative importance or ranks of the pages in a Web graph. Remark on the ways to handle edge cases such as isolated pages.

5.a. Derive the steady state probability of the M/M/1 queuing system. CO3 N 3

5.b. On a network gateway, measurements show that the packets arrive at a mean rate of 125 packets per second (pps) and the gateway takes about two milliseconds to forward them. Using an M/M/1 model, CO3 N 6

I. Analyze the gateway (Gateway Utilization, Steady state probability Mean Number of packets in the gateway, Mean time spent in the gateway.

II. What is the probability of buffer overflow if the gateway had only 12 buffers?

III. How many buffers do we need to keep packet loss below one packet per million?

5.c. A monitor on a disk server showed that the average time to satisfy an I/O request was 100 milliseconds. The I/O rate was about 100 requests per second. What was the mean number of requests at the disk server? CO3 N 1

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[Answer all the questions. Write concisely, keeping your handwriting legible. Figures in the right hand margin indicate full marks.]

Group A

1.

a) A fair coin is flipped three times. Consider the following three events A, B and C: **5**

$A \equiv$ more heads than tails showed up.

$B \equiv$ more tails than heads showed up.

$C \equiv$ same side showed up in all three flips.

With this backdrop, show that *pairwise independence does not necessarily imply mutual independence.*

b) A prize is hidden behind, uniformly at random, one of the four closed, identical doors in a game show. A contestant initially picks a door uniformly at random. Then the host of the show reveals one of the other three doors that do not hide the prize. At this stage the contestant is given a choice of finally picking a door out of the three unopened doors (including the one she already has picked). She is hesitant whether sticking to her original pick is any worse than switching to one of the two other (unopened) doors. Analyze her options and conclude which of them, if any, yields higher likelihood of her winning the prize. **5**

2.

a) A computer program crashes at the end of each hour of use with probability p, if it has not crashed already. What is the probability that there will be no crash in a given day (assuming, the day starts with the program remaining operational)? In addition, what is the expected time until the program crashes? **4**

OR

Your uncle gives you the following present for your doing well in the studies. First he has you flip a coin that lands heads with probability 1/3. If it lands heads, he gives you \$10. If it's tails he has you roll a (fair, regular, six-sided) die, and he gives you a number of dollars equal to whatever the die shows. What is the probability that you would get more than four dollars? What is the expected reward?

b) A rat is trapped in a maze. Initially it has to choose one of two directions. If it goes to the right, then it will wander around in the maze for three minutes and will then return to its initial position. If it goes to the left, then with probability 1/3 it will depart the maze after two minutes of traveling, and with probability 2/3 it will return to its initial position after five minutes of traveling. Assuming that the rat is at all times equally likely to go to the left or the right, what is the expected number of minutes that it will be trapped in the maze? **4**

OR

If an aircraft is present in a certain area, a radar detects it and generates an alarm signal with probability 0.99. If an aircraft is not present, the radar generates (false) alarm, with probability 0.10. We assume that an aircraft is present with probability 0.04. What is the probability that an aircraft is indeed present, given that an alarm is generated?

c) Briefly explain the Simpson's paradox. **2**

OR

What is linearity of expectation?

3.

a) Each of the n nodes in a local area network transmits its packet in the shared channel at any given slot with probability p. Whenever more than one node transmits in a particular slot, collision occurs and no transmitted packet can be successfully received.

   i. Find the probability that a particular node's transmission in a given slot will encounter collision. **2**
   ii. Find the expected number of nodes that transmit in a given slot. **2**
   iii. Find the probability that two successive slots on the channel see successful transmissions. **2**
   iv.

b) Zayed, Bakr and Hasan decide to play a game. Each player puts \$2 on the table and secretly writes down either "heads" or "tails". Then one of them tosses a fair coin. The \$6 on the table is divided evenly among the players who correctly predicted the outcome of the coin toss. If everyone guessed incorrectly, then everyone takes their money back. Now, explain the outcome when Zayed and Hasan are colluding and always make opposite guesses. **4**

4.

a) What is Markov property? **2**

b) Formulate a mathematical model to deduce the probability that a gambler reaches his target of \$T starting with \$n ($T > n$), before ever going broke. In each round, he wins \$1 with probability p or loses the same amount with probability $1 - p$. His play stops if he ever goes broke, i.e., reaches \$0. **4**

c) How can random walk be employed to calculate the ranks of the pages in a Web-graph? Illustrate how the scheme works with a Web-graph of 4 webpages and at least 7 hyperlinks (directed edges). **4**

5.

a) In this corona situation IIUC giving loan to five talented and needy students. According to the recent situation. the probability of a students living in these conditions for CGPA 3.7 or more is 2/3 Calculate the probability that after 3.7 CGPA. *[Hint. you can use Binomial distribution.]* **5**

(a) All five-students needy.
(b) Exactly two students are needy
(c) at least three students are needy.

OR

A prisoner in a dark dungeon discovers three tunnels leading from his cell. The first tunnel reaches a dead-end after 50 feet and the second tunnel reaches a dead-end after 20 feet, but the third tunnel leads to freedom after 100 feet. Each day, the prisoner picks a tunnel uniformly at random and crawls along it. If he reaches a dead-end, he has to crawl back to his cell. Find the expected distance that he crawls before he reaches freedom

b) Packets are transmitted in slots over a communication channel that is either in good or bad/noisy condition. Packets transmitted during any bad/noisy slots get lost. The bad/noisy channel remains so in the next slot with probability 0.4 and turns good with probability 0.6 and. On the other hand, the channel retains the good condition in the next slot with probability 0.8 and becomes bad/noisy with probability 0.2. **5**

Find the steady-state probability that the channel will be found in a good or bad/noisy condition.
If the channel is seen to be good in the k-th slot, what is the probability that it will remain good during the $(k+4)$th slot?

OR

Auto vehicles arrive at a petrol pump, having one petrol unit, in Poisson fashion with an average of 10 units per hour. The service is distributed exponentially with a mean of 3 minutes. Find the following:

   i. Average waiting time for customer
   ii. Average length of queue
   iii. Probability that a customer arriving at the pump will have to wait.
   iv. The utilization factor for the pump unit.
   v. Probability that the number of customers in the system is 2

Sub: Macs. (Sug)

Day

Time: Date: / /

MIT Book

* four step method [এটা step করে দেখাতে হবে]
  (monty hall)
* Strange Dice.
* Set theory law

(P. 16.5), (P. 16.9)

* conditional Probability.
  → medical-testing.
* Theorem 17.4.1 (Bayes' Rule).
* The law of total probability ***.
* Independence * Mutual Independence
* Pairwise independence.

(P 17.7), (P 17.12), (P 17.18) ****, (P 17.23), (P 17.26) ****

* Random variable
  PDF, CDF,
  → Bernoulli Distribution *****.
  → Binomial Distribution *****
  → Great expectation detail. **.

Sub:

Time: Date: /

→ Expectation of a Binomial Distribution.
(P(18.11), (P 18.19) **, (P 18.20)
(Head check problem) probability to আছে,

* Random Walk.
* Markov Chain **** আসবেই
* Queuing theory **** n