Team Notebook

SUST_EOF

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Contents

1 DP

1.1 CHT

```
struct CHT {vector<ll> m, b;int ptr = 0;
bool bad(int l1, int l2, int l3) {
return 1.0 * (b[l3] - b[l1]) * (m[l1] - m[l2]) <= 1.0 * (b[
    l2] - b[l1]) * (m[l1] - m[l3]); //(slope dec+query min)
    ,(slope inc+query max)
// return 1.0 * (b[l3] - b[l1]) * (m[l1] - m[l2]) > 1.0 * (b
    [l2] - b[l1]) * (m[l1] - m[l3]); //(slope dec+query max
    ), (slope inc+query min)
}void add(ll _m, ll _b) {m.push_back(_m);b.push_back(_b);int
     s = m.size();while (s >= 3 && bad(s - 3, s - 2, s - 1)
    ) {
s--;m.erase(m.end() - 2);b.erase(b.end() - 2);}}ll f(int i,
    ll x) {return m[i] * x + b[i];
}//(slope dec+query min), (slope inc+query max) -> x
    increasing
//(slope dec+query max), (slope inc+query min) -> x
    decreasing
ll query(ll x) {if (ptr >= m.size()) ptr = m.size() - 1;
while (ptr < m.size() - 1 && f(ptr + 1, x) < f(ptr, x)) ptr
    ++;return f(ptr, x);
}ll bs(int l, int r, ll x) {int mid = (l + r) / 2;
if (mid + 1 < m.size() && f(mid + 1, x) < f(mid, x)) return
    bs(mid + 1, r, x); // > for max
if (mid - 1 >= 0 && f(mid - 1, x) < f(mid, x)) return bs(l,
    mid - 1, x); // > for max
return f(mid, x);}};ll n, c;ll a[N], h[N];ll dp[N];
CHT cht;void Solve() {cin >> n >> c;for (int i = 1; i <= n;
    i++) cin >> h[i];
CHT C;dp[1] = 0;C.add(-2LL * h[1], h[1]*h[1] + dp[1]);for (
    int i = 2; i <= n; i++) {
dp[i] = c + h[i] * h[i] + C.query(h[i]);C.add( -2LL * h[i],
    h[i]*h[i] + dp[i]);}
cout << dp[n] << endl;
}//dp[i] = min(dp[j] + (hj - hi)^2 + c)
```

1.2 Grundy

```
int Grundy(int n){ if (n <= 2) return 0;
if (dp[n] != -1)return dp[n];vector<int>vis(1005, 0);
for (int i = 1 ; i < n ; i++){ if (n - i != i)
{vis[(Grundy(n - i)^Grundy(i))] = 1;}}int p = 0;
while(vis[p]) p++;return dp[n] = p;}//pile divided into two
    unequal pile
```

1.3 Knurth Optimization

```
for (int pos = 1; pos <= N; pos++) Opt[0][pos] = 0;
for (int groupNo = 1; groupNo <= K; groupNo++) Opt[groupNo][
    N + 1] = N;
for (int groupNo = 1; groupNo <= K; groupNo++){ for (int pos
     = N; pos >= 1; pos--){ for (int endOfLast = Opt[
    groupNo - 1][pos];endOfLast <= Opt[groupNo][pos + 1];
    endOfLast++){
          int ret = dp[groupNo - 1][endOfLast] + Cost(
              endOfLast + 1, pos); if (dp[groupNo][pos] <=
              ret) continue; dp[groupNo][pos] = ret;Opt[
              groupNo][pos] = endOfLast;
       }
   }
}
```

1.4 LIS

```
vector<int>dp, lis;int track[n];dp.push_back(a[0]);track[0]
    = 1;
for (int i = 1; i < n; i++){if (a[i] > dp.back()) dp.
    push_back(a[i]), track[i] = dp.size();
else{int ind = lower_bound(dp.begin(), dp.end(), a[i]) - dp.
    begin();
dp[ind] = a[i]; track[i] = ind + 1;}}int len = dp.size();
for (int i = n - 1 ; i >= 0 ; i--){if (track[i] == len) lis.
    push_back(a[i]), len--;
}reverse(lis.begin(), lis.end());
```

1.5 digit dp optimize(1 memset)

```
ll dp[20][1030][2][2];ll casio[20][1030][2][2];int cur;int v
    [20];
string s;int n;ll foo(int pos, int mask, int ok, bool other)
    {
if (pos == -1){int tb = 0;int mxdig = -1;for (int i = 0 ; i
    <= 9 ; i++){
if (mask & (1 << i))tb++, mxdig = max(mxdig, i);}return tb
    == mxdig;}
ll &R = dp[pos][mask][ok][other];if(casio[pos][mask][ok][
    other]==cur)
return R;casio[pos][mask][ok][other] = cur;if(ok && ~R)
    return R;
int dgt = 9;if (!ok){dgt = v[pos];}R = 0;for (int i = 0 ; i
    <= dgt; i++){
int temp = mask;if (other)temp |= (1 << i);else{if (i){other
     = 1;temp |= (1 << i);}
}if (i < dgt || ok)R += foo(pos - 1, temp , true, other);
    else
R += foo(pos - 1, temp , false, other);}return R;}void pro()
    {
memset()ll x;cin >> x;for(int i=0 ; i<20 ; i++){v[i] = x%10;
    x/=10;}
cur++;ll ans = foo(18, 0, 0, 0);cout << ans << endl;}
```

1.6 digit dp

```
const int N=20;int a[N];
ll dp[N][11][2][2]; // digit dp te amar number generate hoy
    emne : 0, 01, 02, 03, 04, 05.....066,0667
ll getsum(int pos, int dig, int n, bool ok, bool other){if(
    pos>n){return 1;}
ll &R = dp[pos][dig][ok][other];if(R!=-1)return R;int
    maxdigit = 9;if(!ok)
maxdigit = a[pos];ll res=0;for(int i=0 ; i<=maxdigit ; i++){
    if(dig == i && other)
continue;if(i>0)other=1;if(i<maxdigit || ok)res+=getsum(pos
    +1, i, n, true, other);else
res+=getsum(pos+1, i, n, false, other);}return R = res;}void
     Solve(){string l, r;
cin>>l>>r;int n = r.size();r='*'+r;for(int i=0 ; i<=n ; i++)
    {a[i]=r[i]-'0';}
memset(dp, -1, sizeof(dp));ll sumr = getsum(1, -1 , n, 0, 0)
    ;}
```

1.7 divide and conquer

```
mt19937 mt_rand(chrono::high_resolution_clock::now().
    time_since_epoch().count());
ll Left = 1, Right = 0;ll cost(ll l, ll r) {while (Right < r
    )Add(++Right);
while (Left > l)Add(--Left);while (Left < l)Remove(Left++);
    while (Right > r)
Remove(Right--);return Totsum;}ll dp[2][N];
void compute(int group, int l, int r, int optl, int optr) {
    if (l > r)return ;
int mid = (l + r) / 2;dp[group & 1][mid] = LLONG_MAX;int
    optnow = optl;
for (int k = optl ; k <= min(mid, optr) ; k++) {ll ret = dp
    [!(group & 1)][k] + cost(k + 1, mid);
if (ret < dp[group & 1][mid]) {dp[group & 1][mid] = ret;
    optnow = k;}}
```

```
compute(group, l, mid - 1, optl, optnow);compute(group, mid
    + 1, r, optnow, optr);}
void Solve() {cin >> n >> k;for (int i = 1 ; i <= n ; i++) {
    cin >> a[i];}
for (int i = 1 ; i <= n ; i++) {dp[1 & 1][i] = cost(1, i);}
    for (int i = 2 ; i <= k ; i++) {
compute(i, 1, n, 1, n);}cout << dp[k & 1][n] << endl;}
```

1.8 sos dp

```
void SOS_DP(){for(int i = 0; i<(1<<N); ++i)F[i] = A[i];
for(int i = 0;i < N; ++i)for(int mask = 0; mask < (1<<N); ++
    mask){
if(mask & (1<<i))F[mask] += F[mask^(1<<i)];}}
//Istiak
const ll MLOG = 20; const ll MAXN = (1 << MLOG); ll dp[sz +
    10], fre[sz + 10], mp[sz + 10];
// forward1: Propagates values from subsets to their
    supersets
void forward1() { for (int bit = 0; bit < MLOG; ++bit) { for
     (int i = 0; i < MAXN; ++i) { if (i & (1 << bit)) { dp[
    i] += dp[i ^ (1 << bit)]; } } } }
// backward1: Reverses the effect of forward1 by removing
    contributions from supersets.This is used when dp[i]
    contains info about all subsets of i, and we want to
    isolate the info for only i.
void backward1() { for (int bit = 0; bit < MLOG; ++bit) {
    for (int i = MAXN - 1; i >= 0; --i) { if (i & (1 << bit
    )) { dp[i] -= dp[i ^ (1 << bit)]; } } } }
// forward2: Propagates values from supersets to their
    subsets
void forward2() { for (int bit = 0; bit < MLOG; ++bit) { for
     (int i = MAXN - 1; i >= 0; --i) { if (i & (1 << bit))
    { dp[i ^ (1 << bit)] += dp[i]; } } } }
// backward2: Reverses the effect of forward2 by removing
    contributions from subsets.This is used when dp[i]
    contains info about all supersets of i, and we want to
    isolate the info for only i.
void backward2() { for (int bit = 0; bit < MLOG; ++bit) {
    for (int i = 0; i < MAXN; ++i) { if (i & (1 << bit)) {
    dp[i ^ (1 << bit)] -= dp[i]; } } } }
memset(dp, 0, sizeof(dp)); memset(fre, 0, sizeof(fre));
    memset(mp, 0, sizeof(mp));
```

2 DataStructure

2.1 BIT range update and query

```
const int N = 3e5 + 9;
struct BIT {
  long long M[N], A[N];
  BIT() {memset(M, 0, sizeof M);memset(A, 0, sizeof A);}
  void update(int i, long long mul, long long add) {
    while (i < N) {M[i] += mul;A[i] += add;i |= (i + 1);}}
  void upd(int l, int r, long long x) {
    update(l, x, -x * (l - 1));update(r, -x, x * r);}
  long long query(int i) {
    long long mul = 0, add = 0;int st = i;
    while (i >= 0) {mul += M[i];add += A[i];i = (i & (i + 1))
        - 1;}
    return (mul * st + add);}
  long long query(int l, int r) {
    return query(r) - query(l - 1);}} t;
```

2.2 BIT2D

```
#include<bits/stdc++.h>using namespace std;const int N =
    1010;
struct BIT2D { long long M[N][N][2], A[N][N][2];BIT2D() {
memset(M, 0, sizeof M); memset(A, 0, sizeof A);}
void upd2(long long t[N][N][2], int x, int y, long long mul,
     long long add) {
for(int i = x; i < N; i += i & -i) { for(int j = y; j < N; j
     += j & -j) {
t[i][j][0] += mul;t[i][j][1] += add;}}}
void upd1(int x, int y1, int y2, long long mul, long long
    add) {
upd2(M, x, y1, mul, -mul * (y1 - 1));upd2(M, x, y2, -mul,
    mul * y2);
upd2(A, x, y1, add, -add * (y1 - 1));upd2(A, x, y2, -add,
    add * y2);}
void upd(int x1, int y1, int x2, int y2, long long val) {
upd1(x1, y1, y2, val, -val * (x1 - 1));upd1(x2, y1, y2, -val
    , val * x2);}
long long query2(long long t[N][N][2], int x, int y) { long
    long mul = 0, add = 0;
for(int i = y; i > 0; i -= i & -i) { mul += t[x][i][0];add
    += t[x][i][1];}
return mul * y + add;}long long query1(int x, int y) {long
    long mul = 0, add = 0;
for(int i = x; i > 0; i -= i & -i) { mul += query2(M, i, y);
     add += query2(A, i, y);}
return mul * x + add; } long long query(int x1, int y1, int
    x2, int y2) {
return query1(x2, y2) - query1(x1 - 1, y2) - query1(x2, y1 -
     1) + query1(x1 - 1, y1 - 1);}
} t; int main() { int n, m; cin >> n >> m; for(int i = 1; i
    <= n; i++) { for(int j = 1; j <= m; j++) {
int k; cin >> k; t.upd(i, j, i, j, k); } } int q; cin >> q;
    while(q--) {
int ty, x1, y1, x2, y2; cin >> ty; if(ty == 1) { long long
    val;
cin >> x1 >> y1 >> x2 >> y2 >> val;
t.upd(x1, y1, x2, y2, val); // add val from top-left(x1, y1)
     to bottom-right (x2, y2);
} else { cin >> x1 >> y1 >> x2 >> y2;
cout << t.query(x1, y1, x2, y2) << '\n'; // output sum from
    top-left(x1, y1) to bottom-right (x2, y2);
}}return 0;}
```

2.3 Centroid decomposition

```
#include<bits/stdc++.h>
using namespace std;
const int N = 1e5 + 9;vector<int> g[N];
int sz[N];int tot, done[N], cenpar[N];
void calc_sz(int u, int p) {tot ++;
sz[u] = 1;for (auto v : g[u]) {if(v == p || done[v])
    continue;
calc_sz(v, u);sz[u] += sz[v];}
}int find_cen(int u, int p) {for (auto v : g[u]) {
if(v == p || done[v]) continue;else if(sz[v] > tot / 2)
    return find_cen(v, u);
}return u;}void decompose(int u, int pre) {
tot = 0;calc_sz(u, pre);int cen = find_cen(u, pre);cenpar[
    cen] = pre;
done[cen] = 1;for(auto v : g[cen]) {if(v == pre || done[v])
    continue;
decompose(v, cen);}}int dep[N];void dfs(int u, int p = 0) {
for(auto v : g[u]) {if(v == p) continue;dep[v] = dep[u] + 1;
dfs(v, u);}}int main() {ios_base::sync_with_stdio(0);
cin.tie(0);int n;cin >> n;for(int i = 1; i < n; i++) {
int u, v;cin >> u >> v;g[u].push_back(v);g[v].push_back(u);}
decompose(1, 0);for(int i = 1; i <= n; i++) g[i].clear();int
     root;
for(int i = 1; i <= n; i++) {g[cenpar[i]].push_back(i);
g[i].push_back(cenpar[i]);if (cenpar[i] == 0) root = i;}
dfs(root);for(int i = 1; i <= n; i++) cout << char(dep[i] +
    'A') << ' ';return 0;}
```

2.4 DSU on tree

```
const int N = 1e5 + 9;
vector<int> g[N];
int ans[N], col[N], sz[N], cnt[N];
bool big[N];
void dfs(int u, int p) {sz[u] = 1;
  for (auto v : g[u]) {if (v == p) continue;dfs(v, u);sz[u]
      += sz[v];}}
void add(int u, int p, int x) {cnt[col[u]] += x;
  for (auto v : g[u]) {if (v == p || big[v] == 1) continue;
      add(v, u, x);}}
void dsu(int u, int p, bool keep) {
int bigchild = -1, mx = -1;
for (auto v : g[u]) {if (v == p) continue;if (sz[v] > mx) mx
     = sz[v], bigchild = v;}
for (auto v : g[u]) {if (v == p || v == bigchild) continue;
    dsu(v, u, 0);}
if (bigchild != -1) dsu(bigchild, u, 1), big[bigchild] = 1;
    add(u, p, 1);ans[u] = cnt[u];
if (bigchild != -1) big[bigchild] = 0;if (keep == 0) add(u,
    p, -1);}
```

2.5 DSU

```
int n,m;
int parent[MAX],Rank[MAX];
void Init(int n){for(int i=0 ;i<=n ;i++)Rank[i]=1,parent[i]=
    i;}
int Find_parent(int v) {if (v == parent[v]) {return v;}
    return parent[v] = Find_parent(parent[v]);}
void Union(int a, int b) {
a = Find_parent(a);b = Find_parent(b);if (a != b) {if (Rank[
    a] > Rank[b]) {swap (a, b);}parent[a] = b;Rank[b] +=
    Rank[a];}}
```

2.6 GP hash table

```
#include <bits/stdc++.h>
using namespace std;
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
struct custom_hash{
static uint64_t splitmix64(uint64_t x){x += 0
    x9e3779b97f4a7c15;x = (x ^ (x >> 30)) * 0
    xbf58476d1ce4e5b9;x = (x ^ (x >> 27)) * 0
    x94d049bb133111eb;return x ^ (x >> 31);}
size_t operator()(uint64_t x) const{static const uint64_t
    FIXED_RANDOM = chrono::steady_clock::now().
    time_since_epoch().count();return splitmix64(x +
    FIXED_RANDOM);}};
gp_hash_table<int, int, custom_hash> mp;
```

2.7 Mo on tree (number of distinct in a path)

```
const int mod = 1e9 + 7, LG = 18;const int N = 2e5 + 6;const
     int BLOCK_SIZE = 450;int a[N];
vector<int>node[N];int starttime[N], endtime[N];int ft[N];
    int par[N][LG + 1], dep[N], sz[N];int timer = 1;
void dfs(int u, int p = 0){
ft[timer] = u;starttime[u] = timer++;par[u][0] = p;dep[u] =
    dep[p] + 1;sz[u] = 1;
for (int i = 1; i <= LG; i++){par[u][i] = par[par[u][i -
    1]][i - 1];}
for (auto v : node[u]){if (v == p) continue;dfs(v, u);sz[u]
    += sz[v];}ft[timer] = u;endtime[u] = timer++;}
int lca(int u, int v){ // ache already}
int freq[N];int colour[N];int res;
void operation(int id){
int curnode = ft[id];int c = a[curnode];
if (freq[curnode] == 0){colour[c]++;
if (colour[c] == 1)res++;}
else{colour[c]--;if (colour[c] == 0)res--;}freq[curnode] ^=
    1;}
void Solve(){
int n, q;
while (cin >> n >> q){
set<ll>st;map<ll , ll>m;
for (int i = 1 ; i <= n ; i++){cin >> a[i];}int tot = 0;
for (int i = 1 ; i <= n ; i++){if (m[a[i]])a[i] = m[a[i]];
    else{m[a[i]] = ++tot;a[i] = m[a[i]];}}
for (int i = 1 ; i < n ; i++){int u, v;cin >> u >> v;node[u
    ].push_back(v);node[v].push_back(u);}
dfs(1);ll ans[q + 1];Query queries[q];
for (int i = 0 ; i < q ; i++){int u, v, c;cin >> u >> v;int
    lc = lca(u, v);
if (dep[u] > dep[v])swap(u, v);
if (lc == u || lc == v)queries[i] = {starttime[u], starttime
    [v], i + 1, 1, lc, -1};
else queries[i] = {endtime[u], starttime[v], i + 1, 1, lc,
    1};}
sort(queries, queries + q);
int Left = 1, Right = 0;
for (auto i : queries){
int l = i.l;int r = i.r;int id = i.idx;int c = i.c;int type
    = i.type;int lc = i.lc;
while (Right < r)operation(++Right);
while (Left > l)operation(--Left);
while (Left < l)operation(Left++);
while (Right > r)operation(Right--);
if (type == 1){operation(starttime[lc]);}
ans[id] = res;
if (type == 1)operation(starttime[lc]);}
for (int i = 1 ; i <= q ; i++) {cout << ans[i] << endl;}}}
```

2.8 Mo's

```
const int mod = 1e9 + 7;const int N = 5e5 + 6;const int
    BLOCK_SIZE = 500;
struct Query {
int l, r, idx, lc, type;
bool operator<(const Query &y) const {
// Current query x is being compared with other query y
int x_block = l / BLOCK_SIZE;int y_block = y.l / BLOCK_SIZE;
// If x and y both lie in the same block, sort in non
    decreasing order of endpoint
if (x_block == y_block)return r < y.r;
// x and y lie in different blocks
return x_block < y_block;}};
ll nc3(ll x){if (x < 3)return 0;return (x * (x - 1) * (x -
    2)) / 6;}
int a[N];ll last[N];ll freq[N];ll res;
void Add(int i){int x = a[i];res -= last[x];freq[x]++;last[x
    ] = nc3(freq[x]);res += last[x];}
void Remove(int i){int x = a[i];res -= last[x];freq[x]--;
    last[x] = nc3(freq[x]);res += last[x];}
void Solve(){
int n, q;cin >> n >> q;
for (int i = 1 ; i <= n ; i++){cin >> a[i];}
vector<Query>queries;ll ans[q + 1];
for (int i = 1 ; i <= q ; i++){int l, r;cin >> l >> r;
    queries.push_back({l, r, i});}
sort(queries.begin(), queries.end());
int Left = 1, Right = 0;
for (auto i : queries){int l = i.l;int r = i.r;int id = i.
    idx;
while (Right < r)Add(++Right);
while (Left > l)Add(--Left);
while (Left < l)Remove(Left++);
while (Right > r)Remove(Right--);
ans[i.idx] = res;}
```

```
for (int i = 1 ; i <= q ; i++)cout << ans[i] << endl;}
//number of triple(1, r) a[i] = a[j] = a[k]
```

2.9 Persistent Segment Tree

```
#include<bits/stdc++.h>using namespace std;
struct nd{long long sum;nd *left;nd *right;nd(long long data
    ){sum=data;}
nd(nd l,nd r){sum=l.sum+r.sum;left=&l;right=&r;}};
int n;vector<nd>states;
nd build(int start,int end){
if(start==end)return nd(0);int mid=(start+end)/2;return nd(
    build(start,mid),build(mid+1,end));}
nd update(nd root,int start,int end,int pos,int val){if(
    start==end)return nd(val);
int mid=(start+end)/2;return pos<=mid?nd(update(*root.left,
    start,mid,pos,val),*root.right):nd(*root.left,update(*
    root.right,mid+1,end,pos,val));}
void solve(){cin>>n;states.push_back(build(0,n-1));states.
    push_back(update(states.back(),0,n-1,4,3));return;}
int main(){ios_base::sync_with_stdio(0);cin.tie(0);solve();
    return 0;}
```

2.10 Sparse table

```
int Table[N][22], a[N];
void Build(int n){
for (int i = 1 ; i <= n ; i++)Table[i][0] = a[i];
for (int k = 1 ; k < 22 ; k++){
for (int i = 1 ; i + (1 << k) - 1 <= n ; i++)Table[i][k] =
    min(Table[i][k - 1], Table[i + (1 << (k - 1))][k - 1])
    ;}}
int Query(int l, int r){int k = log2(r - l + 1);return min(
    Table[l][k], Table[r - (1 << k) + 1][k]);}
```

2.11 next$_s$*mallerpreviousso*maller

```
ll Next_smaller[N + 2];ll Prev_smaller[N + 2];
void NEXTSMALLER(){stack<int>st;
for (int i = 1; i <= n; i++){if (st.empty()){st.push(i);}
else{while (!st.empty() && a[st.top()] > a[i]){Next_smaller[
    st.top()] = i;st.pop();}st.push(i);}}
while (!st.empty()){Next_smaller[st.top()] = n + 1;st.pop()
    ;}}
void PREVSMALLER(){stack<int>st;
for (int i = n; i >= 1; i--){if (st.empty()){st.push(i);}
else{while (!st.empty() && a[st.top()] > a[i]){Prev_smaller[
    st.top()] = i;st.pop();}st.push(i);}}
while (!st.empty()){Prev_smaller[st.top()] = 0;st.pop();}}
//priority_queue<int,vector<int>, greater<int> >pq;
```

2.12 ordered set

```
#include<ext/pb_ds/assoc_container.hpp>#include<ext/pb_ds/
    tree_policy.hpp>using namespace std;using namespace
    __gnu_pbds;
template<class T> using ordered_set =
tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>; // find_by_order,
    order_of_key
//1 2 2 3 3 7
//greater->>descending order => 7 3 2 1
//less ->>ascending order => 1 2 3 7
//less_equal -> ascending but in duplicate value => 1 2 2 3
    3 7 so this will work as multiset
// finding kth element - 4th query
cout << "0th element: " << *A.find_by_order(0) << endl;
// finding number of elements smaller than X - 3rd query
cout << "No. of elems smaller than 6: " << A.order_of_key(6)
     << endl; //
```

2.13 trie xor operation

```
int Trie[35 * N][2];int root = 1;int cnt[35 * N];int cur =
    1;
void Update(ll x, ll value){int start = root;cnt[start] +=
    value;
for (int i = 32 ; i >= 0 ; i--){bool bit = x & (1LL << i);
if (Trie[start][bit] == 0){Trie[start][bit] = ++cur;}
start = Trie[start][bit];cnt[start] += value;}}
ll MaxQuery(ll x){int start = root;ll ans = 0;
for (int i = 32 ; i >= 0 ; i--){bool bit = x & (1LL << i);
if (Trie[start][1 ^ bit] == 0 || cnt[Trie[start][1 ^ bit]]
    == 0){ans = ans;}
else {ans += (1LL << i);bit ^= 1;}start = Trie[start][bit];}
    return ans;}
ll MinQuery(ll x) {int start = root;ll ans = 0;
for (int i = 32 ; i >= 0 ; i--) {bool bit = x & (1LL << i);
if (Trie[start][bit]) {ans = ans;}
else {ans += (1LL << i);bit ^= 1;}start = Trie[start][bit];}
    return ans;}
```

2.14 wavelet tree

```
const int MAXN = (int)3e5 + 9;const int MAXV = (int)1e9 + 9;
     //maximum value of any element in array
//array values can be negative too, use appropriate minimum
    and maximum value
struct wavelet_tree {int lo, hi;wavelet_tree *l, *r;int *b,
    *c, bsz, csz; // c holds the prefix sum of elements
wavelet_tree() {lo = 1;hi = 0;bsz = 0;csz = 0, l = NULL;r =
    NULL;}
void init(int *from, int *to, int x, int y) {lo = x, hi = y;
    if (from >= to) return;
int mid = (lo + hi) >> 1;auto f = [mid](int x) {return x <=
    mid;};
b = (int*)malloc((to - from + 2) * sizeof(int));bsz = 0;b[
    bsz++] = 0;
c = (int*)malloc((to - from + 2) * sizeof(int));csz = 0;c[
    csz++] = 0;
for (auto it = from; it != to; it++) {b[bsz] = (b[bsz - 1] +
     f(*it));c[csz] = (c[csz - 1] + (*it));bsz++;csz++;}
if (hi == lo) return;
auto pivot = stable_partition(from, to, f);
l = new wavelet_tree();
l->init(from, pivot, lo, mid);
r = new wavelet_tree();
r->init(pivot, to, mid + 1, hi);}
//kth smallest element in [l, r]
//for array [1,2,1,3,5] 2nd smallest is 1 and 3rd smallest
    is 2
int kth(int l, int r, int k) {if (l > r) return 0;if (lo ==
    hi) return lo;int inLeft = b[r] - b[l - 1], lb = b[l -
    1], rb = b[r];
if (k <= inLeft) return this->l->kth(lb + 1, rb, k);return
    this->r->kth(l - lb, r - rb, k - inLeft);}
//count of numbers in [l, r] Less than or equal to k
int LTE(int l, int r, int k) {if (l > r || k < lo) return 0;
    if (hi <= k) return r - l + 1;int lb = b[l - 1], rb = b
    [r];return this->l->LTE(lb + 1, rb, k) + this->r->LTE(l
     - lb, r - rb, k);}
//count of numbers in [l, r] equal to k
int count(int l, int r, int k) {if (l > r || k < lo || k >
    hi) return 0;if (lo == hi) return r - l + 1;int lb = b[
    l - 1], rb = b[r];int mid = (lo + hi) >> 1;if (k <= mid
    ) return this->l->count(lb + 1, rb, k);return this->r->
    count(l - lb, r - rb, k);}
//sum of numbers in [l ,r] less than or equal to k
int sum(int l, int r, int k) {if (l > r or k < lo) return 0;
    if (hi <= k) return c[r] - c[l - 1];int lb = b[l - 1],
    rb = b[r];return this->l->sum(lb + 1, rb, k) + this->r
    ->sum(l - lb, r - rb, k);}
```

```
~wavelet_tree() {delete l;delete r;}};
wavelet_tree t;int a[MAXN];
int main() {
int i, j, k, n, m, q, l, r;cin >> n;for (i = 1; i <= n; i++)
     cin >> a[i];t.init(a + 1, a + n + 1, -MAXV, MAXV);
//beware! after the init() operation array a[] will not be
    samecin >> q;
while (q--) {int x;cin >> x;cin >> l >> r >> k;
if (x == 0) {//kth smallestcout << t.kth(l, r, k) << endl;}
    else if (x == 1) {//less than or equal to Kcout << t.
    LTE(l, r, k) << endl;} else if (x == 2) {//count
    occurence of K in [l, r]cout << t.count(l, r, k) <<
    endl;}
if (x == 3) {//sum of elements less than or equal to K in [l
    , r]cout << t.sum(l, r, k) << endl;}}return 0;}
```

2.15 xor basis

```
struct XorBasis { vector<ll> basis;
ll N = 0, tmp = 0; void add(ll x) {
N++; tmp |= x;for (auto &i : basis) x = min(x, x ^ i);if (!x
    ) return;
for (auto &i : basis) if ((i ^ x) < i) i ^= x;basis.
    push_back(x);
sort(basis.begin(), basis.end());}
ll size() {return (ll)basis.size();
}void clear() {N = 0; tmp = 0;basis.clear();
}bool possible(ll x) {for (auto &i : basis) x = min(x, x ^ i
    );
return !x;}ll maxxor(ll x = 0) {for (auto &i : basis) x =
    max(x, x ^ i);
return x;}ll minxor(ll x = 0) {
for (auto &i : basis) x = min(x, x ^ i);
return x;}ll cntxor(ll x) {if (!possible(x)) return 0LL;//
    return (1LL<<(N-size()));
ll ans = 1LL;for (int i = 0; i < N - size(); i++)ans = (ans
    * 2) % MOD;
return ans;}ll sumOfAll() {ll ans = tmp * (1LL << (N - 1));
return ans;}ll kth(ll k) {ll sz = size();if (k > (1LL << sz)
    ) return -1;
k--; ll ans = 0;for (ll i = 0; i < sz; i++) if (k >> i & 1)
    ans ^= basis[i];
return ans;}} xb;
```

3 Extra

3.1 build

```
//windows
{
"shell_cmd": "g++.exe -std=c++17 \"$file\" -o \"
    $file_base_name\".exe && \"$file_base_name.exe\"<\"D://
    c programes/in.txt\">\"D://c programes/out.txt\"",
"shell":true,
"working_dir":"$file_path",
"selector":"source.c, source.cpp, source.c++"
}


//linux
{
"cmd" : ["g++ -std=c++17 \"$file_name\" && timeout 10s ./a.
    out <~/Code/input.txt | head -n 2000000 | head -c
    50000000 >~/Code/output.txt"],
"selector" : "source.c,source.cpp,source.c++",
"shell": true,
"working_dir" : "$file_path"
}
```

3.2 equation

```
Some properties of bitwise operations:
a|b =  ab  + a&b ,  a (a&b) = (a|b) b ,
 b (a&b) = (a|b) a  ,(a&b)  (a|b) =  ab
Addition: a+b = a|b + a&b, a+b = ab + 2(a&b)
Subtraction:
a-b = ( a (a&b))-((a|b) a ),a-b = ((a|b) b )-((a|b) a )
,a-b = ( a (a&b))-( b (a&b)),a-b = ((a|b) b )-( b (a&b))
```

3.3 pragma

```
// #pragma GCC optimize("O3,unroll-loops,Ofast")
// #pragma GCC target("avx2")
```

4 Geo

4.1 geo$_template_2$

```
#include <bits/stdc++.h>using namespace std;
// https://victorlecomte.com/cp-geo.pdf
const int N = 3e5 + 9;const double inf = 1e100;const double
    eps = 1e-9;const double PI = acos((double)-1.0);
int sign(double x) { return (x > eps) - (x < -eps); }
struct PT{double x, y;PT() { x = 0, y = 0; }PT(double x,
    double y) : x(x), y(y) {}PT(const PT &p) : x(p.x), y(p.
    y) {}PT operator+(const PT &a) const { return PT(x + a.
    x, y + a.y); }
PT operator-(const PT &a) const { return PT(x - a.x, y - a.y
    ); }PT operator*(const double a) const { return PT(x *
    a, y * a); }
friend PT operator*(const double &a, const PT &b) { return
    PT(a * b.x, a * b.y); }PT operator/(const double a)
    const { return PT(x / a, y / a); }bool operator==(PT a)
     const { return sign(a.x - x) == 0 && sign(a.y - y) ==
    0; }
bool operator!=(PT a) const { return !(*this == a); }bool
    operator<(PT a) const { return sign(a.x - x) == 0 ? y <
     a.y : x < a.x; }
bool operator>(PT a) const { return sign(a.x - x) == 0 ? y >
     a.y : x > a.x; }double norm() { return sqrt(x * x + y
    * y); }
double norm2() { return x * x + y * y; }PT perp() { return
    PT(-y, x); }double arg() { return atan2(y, x); }
PT truncate(double r){ // returns a vector with norm r and
    having same directiondouble k = norm();if (!sign(k))
    return *this;r /= k;return PT(x * r, y * r);}};
istream &operator>>(istream &in, PT &p) { return in >> p.x
    >> p.y; }
ostream &operator<<(ostream &out, PT &p) { return out << "("
     << p.x << "," << p.y << ")"; }
inline double dot(PT a, PT b) { return a.x * b.x + a.y * b.y
    ; }
inline double dist2(PT a, PT b) { return dot(a - b, a - b);
    }
inline double dist(PT a, PT b) { return sqrt(dot(a - b, a -
    b)); }
inline double cross(PT a, PT b) { return a.x * b.y - a.y * b
    .x; }
inline double cross2(PT a, PT b, PT c) { return cross(b - a,
     c - a); }
inline int orientation(PT a, PT b, PT c) { return sign(cross
    (b - a, c - a)); }
PT perp(PT a) { return PT(-a.y, a.x); }
PT rotateccw90(PT a) { return PT(-a.y, a.x); }
```

```
PT rotatecw90(PT a) { return PT(a.y, -a.x); }
PT rotateccw(PT a, double t) { return PT(a.x * cos(t) - a.y
    * sin(t), a.x * sin(t) + a.y * cos(t)); }
PT rotatecw(PT a, double t) { return PT(a.x * cos(t) + a.y *
     sin(t), -a.x * sin(t) + a.y * cos(t)); }
double SQ(double x) { return x * x; }
double rad_to_deg(double r) { return (r * 180.0 / PI); }
double deg_to_rad(double d) { return (d * PI / 180.0); }
double get_angle(PT a, PT b){double costheta = dot(a, b) / a
    .norm() / b.norm();return acos(max((double)-1.0, min((
    double)1.0, costheta)));}
bool is_point_in_angle(PT b, PT a, PT c, PT p){ // does
    point p lie in angle <bac
assert(orientation(a, b, c) != 0);if (orientation(a, c, b) <
     0)swap(b, c);
return orientation(a, c, p) >= 0 && orientation(a, b, p) <=
    0;}
bool half(PT p){return p.y > 0.0 || (p.y == 0.0 && p.x <
    0.0);}
void polar_sort(vector<PT> &v){ // sort points in
    counterclockwise
sort(v.begin(), v.end(), [](PT a, PT b){ return make_tuple(
    half(a), 0.0, a.norm2()) < make_tuple(half(b), cross(a,
     b), b.norm2()); });}
void polar_sort(vector<PT> &v, PT o){ // sort points in
    counterclockwise with respect to point o
sort(v.begin(), v.end(), [&](PT a, PT b){ return make_tuple(
    half(a - o), 0.0, (a - o).norm2()) < make_tuple(half(b
    - o), cross(a - o, b - o), (b - o).norm2()); });}
struct line{PT a, b; // goes through points a and bPT v;
    double c; // line form: direction vec [cross] (x, y) =
    c
line() {}// direction vector v and offset cline(PT v, double
     c) : v(v), c(c){auto p = get_points();a = p.first;b =
    p.second;}
// equation ax + by + c = 0
line(double _a, double _b, double _c) : v({_b, -_a}), c(-_c)
    {auto p = get_points();a = p.first;b = p.second;}
// goes through points p and q
line(PT p, PT q) : v(q - p), c(cross(v, p)), a(p), b(q) {}
pair<PT, PT> get_points()
{ // extract any two points from this linePT p, q;double a =
     -v.y, b = v.x; // ax + by = cif (sign(a) == 0){p = PT
    (0, c / b);q = PT(1, c / b);}
else if (sign(b) == 0){p = PT(c / a, 0);q = PT(c / a, 1);}
else{p = PT(0, c / b);q = PT(1, (c - a) / b);}return {p, q
    };}
// ax + by + c = 0array<double, 3> get_abc(){double a = -v.y
    , b = v.x;return {a, b, -c};}
// 1 if on the left, -1 if on the right, 0 if on the line
int side(PT p) { return sign(cross(v, p) - c); }
// line that is perpendicular to this and goes through point
     p
line perpendicular_through(PT p) { return {p, p + perp(v)};
    }
// translate the line by vector t i.e. shifting it by vector
     t
line translate(PT t) { return {v, c + cross(v, t)}; }
// compare two points by their orthogonal projection on this
     line
// a projection point comes before another if it comes first
     according to vector v
bool cmp_by_projection(PT p, PT q) { return dot(v, p) < dot(
    v, q); }
line shift_left(double d){PT z = v.perp().truncate(d);return
     line(a + z, b + z);}};
// find a point from a through b with distance d
PT point_along_line(PT a, PT b, double d){assert(a != b);
    return a + (((b - a) / (b - a).norm()) * d);}
// projection point c onto line through a and b assuming a
    != b
PT project_from_point_to_line(PT a, PT b, PT c){return a + (
    b - a) * dot(c - a, b - a) / (b - a).norm2();}
// reflection point c onto line through a and b assuming a
    != b
PT reflection_from_point_to_line(PT a, PT b, PT c){PT p =
    project_from_point_to_line(a, b, c);return p + p - c;}
// minimum distance from point c to line through a and b
double dist_from_point_to_line(PT a, PT b, PT c){return fabs
    (cross(b - a, c - a) / (b - a).norm());}
// returns true if point p is on line segment ab
bool is_point_on_seg(PT a, PT b, PT p){if (fabs(cross(p - b,
     a - b)) < eps){
if (p.x < min(a.x, b.x) - eps || p.x > max(a.x, b.x) + eps)
    return false;
if (p.y < min(a.y, b.y) - eps || p.y > max(a.y, b.y) + eps)
    return false;return true;}return false;}
// minimum distance point from point c to segment ab that
    lies on segment ab
PT project_from_point_to_seg(PT a, PT b, PT c){double r =
    dist2(a, b);if (sign(r) == 0)return a;
r = dot(c - a, b - a) / r;if (r < 0)return a;if (r > 1)
    return b;return a + (b - a) * r;}
// minimum distance from point c to segment ab
double dist_from_point_to_seg(PT a, PT b, PT c){return dist(
    c, project_from_point_to_seg(a, b, c));}
// 0 if not parallel, 1 if parallel, 2 if collinear
int is_parallel(PT a, PT b, PT c, PT d){double k = fabs(
    cross(b - a, d - c));
if (k < eps){if (fabs(cross(a - b, a - c)) < eps && fabs(
    cross(c - d, c - a)) < eps)return 2;
else return 1;}else return 0;}
// check if two lines are same
bool are_lines_same(PT a, PT b, PT c, PT d){
if (fabs(cross(a - c, c - d)) < eps && fabs(cross(b - c, c -
     d)) < eps)return true;return false;}
// bisector vector of <abc
PT angle_bisector(PT &a, PT &b, PT &c){PT p = a - b, q = c -
     b;return p + q * sqrt(dot(p, p) / dot(q, q));}
// 1 if point is ccw to the line, 2 if point is cw to the
    line, 3 if point is on the line
int point_line_relation(PT a, PT b, PT p){int c = sign(cross
    (p - a, b - a));
if (c < 0)return 1;if (c > 0)return 2;return 3;}
// intersection point between ab and cd assuming unique
    intersection exists
bool line_line_intersection(PT a, PT b, PT c, PT d, PT &ans)
    {
double a1 = a.y - b.y, b1 = b.x - a.x, c1 = cross(a, b);
    double a2 = c.y - d.y, b2 = d.x - c.x, c2 = cross(c, d)
    ;
double det = a1 * b2 - a2 * b1;if (det == 0)return 0;
ans = PT((b1 * c2 - b2 * c1) / det, (c1 * a2 - a1 * c2) /
    det);return 1;}
// intersection point between segment ab and segment cd
    assuming unique intersection exists
bool seg_seg_intersection(PT a, PT b, PT c, PT d, PT &ans){
double oa = cross2(c, d, a), ob = cross2(c, d, b);double oc
    = cross2(a, b, c), od = cross2(a, b, d);if (oa * ob < 0
     && oc * od < 0)
{ans = (a * ob - b * oa) / (ob - oa);return 1;}elsereturn
    0;}
// intersection point between segment ab and segment cd
    assuming unique intersection may not exists
// se.size()==0 means no intersection
// se.size()==1 means one intersection
// se.size()==2 means range intersection
set<PT> seg_seg_intersection_inside(PT a, PT b, PT c, PT d){
PT ans;if (seg_seg_intersection(a, b, c, d, ans))return {ans
    };set<PT> se;
if (is_point_on_seg(c, d, a))se.insert(a);if (
    is_point_on_seg(c, d, b))se.insert(b);
if (is_point_on_seg(a, b, c))se.insert(c);
if (is_point_on_seg(a, b, d))se.insert(d);return se;}
// intersection between segment ab and line cd
// 0 if do not intersect, 1 if proper intersect, 2 if
    segment intersect
int seg_line_relation(PT a, PT b, PT c, PT d){
double p = cross2(c, d, a);double q = cross2(c, d, b);
```

```
if (sign(p) == 0 && sign(q) == 0)return 2;
else if (p * q < 0)return 1;else return 0;}
// intersection between segament ab and line cd assuming
    unique intersection exists
bool seg_line_intersection(PT a, PT b, PT c, PT d, PT &ans){
bool k = seg_line_relation(a, b, c, d);assert(k != 2);if (k)
    line_line_intersection(a, b, c, d, ans);return k;}
// minimum distance from segment ab to segment cd
double dist_from_seg_to_seg(PT a, PT b, PT c, PT d){PT dummy
    ;if (seg_seg_intersection(a, b, c, d, dummy))return
    0.0;
else return min({dist_from_point_to_seg(a, b, c),
    dist_from_point_to_seg(a, b, d),dist_from_point_to_seg(
    c, d, a), dist_from_point_to_seg(c, d, b)});}
// minimum distance from point c to ray (starting point a
    and direction vector b)
double dist_from_point_to_ray(PT a, PT b, PT c){b = a + b;
    double r = dot(c - a, b - a);if (r < 0.0)return dist(c,
     a);return dist_from_point_to_line(a, b, c);}
// starting point as and direction vector ad
bool ray_ray_intersection(PT as, PT ad, PT bs, PT bd){
double dx = bs.x - as.x, dy = bs.y - as.y;double det = bd.x
    * ad.y - bd.y * ad.x;if (fabs(det) < eps)return 0;
    double u = (dy * bd.x - dx * bd.y) / det;
double v = (dy * ad.x - dx * ad.y) / det;
if (sign(u) >= 0 && sign(v) >= 0)return 1;elsereturn 0;}
double ray_ray_distance(PT as, PT ad, PT bs, PT bd){if (
    ray_ray_intersection(as, ad, bs, bd))return 0.0;
double ans = dist_from_point_to_ray(as, ad, bs);ans = min(
    ans, dist_from_point_to_ray(bs, bd, as));return ans;}
struct circle{PT p;double r;
circle() {}circle(PT _p, double _r) : p(_p), r(_r) {};
// center (x, y) and radius r
circle(double x, double y, double _r) : p(PT(x, y)), r(_r)
    {};
// circumcircle of a triangle
// the three points must be unique
circle(PT a, PT b, PT c){b = (a + b) * 0.5;c = (a + c) *
    0.5;line_line_intersection(b, b + rotatecw90(a - b), c,
     c + rotatecw90(a - c), p);r = dist(a, p);}
// inscribed circle of a triangle
// pass a bool just to differentiate from circumcircle
circle(PT a, PT b, PT c, bool t){line u, v;double m = atan2(
    b.y - a.y, b.x - a.x), n = atan2(c.y - a.y, c.x - a.x);
u.a = a;u.b = u.a + (PT(cos((n + m) / 2.0), sin((n + m) /
    2.0)));v.a = b;
m = atan2(a.y - b.y, a.x - b.x), n = atan2(c.y - b.y, c.x -
    b.x);v.b = v.a + (PT(cos((n + m) / 2.0), sin((n + m) /
    2.0)));
line_line_intersection(u.a, u.b, v.a, v.b, p);r =
    dist_from_point_to_seg(a, b, p);}
bool operator==(circle v) { return p == v.p && sign(r - v.r)
     == 0; }double area() { return PI * r * r; }double
    circumference() { return 2.0 * PI * r; }};
// 0 if outside, 1 if on circumference, 2 if inside circle
int circle_point_relation(PT p, double r, PT b){double d =
    dist(p, b);if (sign(d - r) < 0)return 2;if (sign(d - r)
     == 0)return 1;return 0;}
// 0 if outside, 1 if on circumference, 2 if inside circle
int circle_line_relation(PT p, double r, PT a, PT b){double
    d = dist_from_point_to_line(a, b, p);if (sign(d - r) <
    0)return 2;
if (sign(d - r) == 0)return 1;return 0;}
// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> circle_line_intersection(PT c, double r, PT a, PT
     b){vector<PT> ret;
b = b - a;a = a - c;double A = dot(b, b), B = dot(a, b);
    double C = dot(a, a) - r * r, D = B * B - A * C;
if (D < -eps)return ret;
ret.push_back(c + a + b * (-B + sqrt(D + eps)) / A);
if (D > eps)ret.push_back(c + a + b * (-B - sqrt(D)) / A);
    return ret;}
// 5 - outside and do not intersect
// 4 - intersect outside in one point
// 3 - intersect in 2 points
// 2 - intersect inside in one point
// 1 - inside and do not intersect
int circle_circle_relation(PT a, double r, PT b, double R){
    double d = dist(a, b);
if (sign(d - r - R) > 0)return 5;
if (sign(d - r - R) == 0)return 4;
double l = fabs(r - R);
if (sign(d - r - R) < 0 && sign(d - l) > 0)return 3;
if (sign(d - l) == 0)return 2;
if (sign(d - l) < 0)return 1;assert(0);return -1;}
vector<PT> circle_circle_intersection(PT a, double r, PT b,
    double R){
if (a == b && sign(r - R) == 0)return {PT(1e18, 1e18)};
    vector<PT> ret;double d = sqrt(dist2(a, b));
if (d > r + R || d + min(r, R) < max(r, R))return ret;double
     x = (d * d - R * R + r * r) / (2 * d);
double y = sqrt(r * r - x * x);PT v = (b - a) / d;
ret.push_back(a + v * x + rotateccw90(v) * y);if (y > 0)ret.
    push_back(a + v * x - rotateccw90(v) * y);return ret;}
// returns two circle c1, c2 through points a, b and of
    radius r
// 0 if there is no such circle, 1 if one circle, 2 if two
    circle
int get_circle(PT a, PT b, double r, circle &c1, circle &c2)
    {vector<PT> v = circle_circle_intersection(a, r, b, r);
int t = v.size();if (!t)return 0;
c1.p = v[0], c1.r = r;if (t == 2)c2.p = v[1], c2.r = r;
    return t;}
// returns two circle c1, c2 which is tangent to line u,
    goes through
// point q and has radius r1; 0 for no circle, 1 if c1 = c2
    , 2 if c1 != c2
int get_circle(line u, PT q, double r1, circle &c1, circle &
    c2){
double d = dist_from_point_to_line(u.a, u.b, q);if (sign(d -
     r1 * 2.0) > 0)return 0;
if (sign(d) == 0){cout << u.v.x << ' ' << u.v.y << '\n';
c1.p = q + rotateccw90(u.v).truncate(r1);c2.p = q +
    rotatecw90(u.v).truncate(r1);c1.r = c2.r = r1;return
    2;}
line u1 = line(u.a + rotateccw90(u.v).truncate(r1), u.b +
    rotateccw90(u.v).truncate(r1));line u2 = line(u.a +
    rotatecw90(u.v).truncate(r1), u.b + rotatecw90(u.v).
    truncate(r1));
circle cc = circle(q, r1);PT p1, p2;vector<PT> v;v =
    circle_line_intersection(q, r1, u1.a, u1.b);
if (!v.size())v = circle_line_intersection(q, r1, u2.a, u2.b
    );v.push_back(v[0]);p1 = v[0], p2 = v[1];
c1 = circle(p1, r1);if (p1 == p2){c2 = c1;return 1;}c2 =
    circle(p2, r1);return 2;}
// returns the circle such that for all points w on the
    circumference of the circle
// dist(w, a) : dist(w, b) = rp : rq
// rp != rq
// https://en.wikipedia.org/wiki/Circles_of_Apollonius
circle get_apollonius_circle(PT p, PT q, double rp, double
    rq){rq *= rq;rp *= rp;double a = rq - rp;
assert(sign(a));double g = rq * p.x - rp * q.x;g /= a;double
     h = rq * p.y - rp * q.y;h /= a;double c = rq * p.x * p
    .x - rp * q.x * q.x + rq * p.y * p.y - rp * q.y * q.y;
c /= a;PT o(g, h);double r = g * g + h * h - c;r = sqrt(r);
    return circle(o, r);}
// returns area of intersection between two circles
double circle_circle_area(PT a, double r1, PT b, double r2){
    double d = (a - b).norm();if (r1 + r2 < d + eps)return
    0;
if (r1 + d < r2 + eps)return PI * r1 * r1;
if (r2 + d < r1 + eps)return PI * r2 * r2;
double theta_1 = acos((r1 * r1 + d * d - r2 * r2) / (2 * r1
    * d)),theta_2 = acos((r2 * r2 + d * d - r1 * r1) / (2 *
     r2 * d));
return r1 * r1 * (theta_1 - sin(2 * theta_1) / 2.) + r2 * r2
     * (theta_2 - sin(2 * theta_2) / 2.);}
```

```
// tangent lines from point q to the circle
int tangent_lines_from_point(PT p, double r, PT q, line &u,
    line &v){
int x = sign(dist2(p, q) - r * r);if (x < 0)return 0; //
    point in cricleif (x == 0)
{ // point on circle
u = line(q, q + rotateccw90(q - p));v = u;return 1;}
double d = dist(p, q);double l = r * r / d;double h = sqrt(r
     * r - l * l);u = line(q, p + ((q - p).truncate(l) + (
    rotateccw90(q - p).truncate(h))));
v = line(q, p + ((q - p).truncate(l) + (rotatecw90(q - p).
    truncate(h))));return 2;}
// returns outer tangents line of two circles
// if inner == 1 it returns inner tangent lines
int tangents_lines_from_circle(PT c1, double r1, PT c2,
    double r2, bool inner, line &u, line &v){
if (inner)r2 = -r2;PT d = c2 - c1;double dr = r1 - r2, d2 =
    d.norm2(), h2 = d2 - dr * dr;
if (d2 == 0 || h2 < 0){assert(h2 != 0);return 0;}vector<pair
    <PT, PT>> out;for (int tmp : {-1, 1}){PT v = (d * dr +
    rotateccw90(d) * sqrt(h2) * tmp) / d2;out.push_back({c1
     + v * r1, c2 + v * r2});}
u = line(out[0].first, out[0].second);
if (out.size() == 2)v = line(out[1].first, out[1].second);
    return 1 + (h2 > 0);}
// O(n^2 log n)
// https://vjudge.net/problem/UVA-12056
struct CircleUnion
{int n;double x[2020], y[2020], r[2020];int covered[2020];
    vector<pair<double, double>> seg, cover;double arc, pol
    ;inline int sign(double x) { return x < -eps ? -1 : x >
     eps; }inline int sign(double x, double y) { return
    sign(x - y); }
inline double SQ(const double x) { return x * x; }inline
    double dist(double x1, double y1, double x2, double y2)
     { return sqrt(SQ(x1 - x2) + SQ(y1 - y2)); }
inline double angle(double A, double B, double C)
{double val = (SQ(A) + SQ(B) - SQ(C)) / (2 * A * B);if (val
    < -1)val = -1;
if (val > +1)val = +1;return acos(val);}
CircleUnion(){n = 0;seg.clear(), cover.clear();arc = pol =
    0;}
void init(){n = 0;seg.clear(), cover.clear();arc = pol = 0;}
void add(double xx, double yy, double rr)
{x[n] = xx, y[n] = yy, r[n] = rr, covered[n] = 0, n++;}
void getarea(int i, double lef, double rig){
arc += 0.5 * r[i] * r[i] * (rig - lef - sin(rig - lef));
double x1 = x[i] + r[i] * cos(lef), y1 = y[i] + r[i] * sin(
    lef);
double x2 = x[i] + r[i] * cos(rig), y2 = y[i] + r[i] * sin(
    rig);pol += x1 * y2 - x2 * y1;}
double solve(){for (int i = 0; i < n; i++)
{for (int j = 0; j < i; j++){if (!sign(x[i] - x[j]) && !sign
    (y[i] - y[j]) && !sign(r[i] - r[j]))
{r[i] = 0.0;break;}}}
for (int i = 0; i < n; i++){for (int j = 0; j < n; j++){if (
    i != j && sign(r[j] - r[i]) >= 0 && sign(dist(x[i], y[i
    ], x[j], y[j]) - (r[j] - r[i])) <= 0){
covered[i] = 1;break;}}}for (int i = 0; i < n; i++){
if (sign(r[i]) && !covered[i]){seg.clear();
for (int j = 0; j < n; j++){if (i != j){
double d = dist(x[i], y[i], x[j], y[j]);
if (sign(d - (r[j] + r[i])) >= 0 || sign(d - abs(r[j] - r[i
    ])) <= 0)
{continue;}double alpha = atan2(y[j] - y[i], x[j] - x[i]);
double beta = angle(r[i], d, r[j]);pair<double, double> tmp(
    alpha - beta, alpha + beta);
if (sign(tmp.first) <= 0 && sign(tmp.second) <= 0){
seg.push_back(pair<double, double>(2 * PI + tmp.first, 2 *
    PI + tmp.second));}
else if (sign(tmp.first) < 0){seg.push_back(pair<double,
    double>(2 * PI + tmp.first, 2 * PI));seg.push_back(pair
    <double, double>(0, tmp.second));}
else{seg.push_back(tmp);}}}
sort(seg.begin(), seg.end());double rig = 0;
for (vector<pair<double, double>>::iterator iter = seg.begin
    (); iter != seg.end(); iter++){
if (sign(rig - iter->first) >= 0){rig = max(rig, iter->
    second);}
else{getarea(i, rig, iter->first);rig = iter->second;}}
if (!sign(rig)){arc += r[i] * r[i] * PI;}
else{getarea(i, rig, 2 * PI);}}}return pol / 2.0 + arc;}} CU
    ;
double area_of_triangle(PT a, PT b, PT c){return fabs(cross(
    b - a, c - a) * 0.5);}
// -1 if strictly inside, 0 if on the polygon, 1 if strictly
     outside
int is_point_in_triangle(PT a, PT b, PT c, PT p){
if (sign(cross(b - a, c - a)) < 0)swap(b, c);
int c1 = sign(cross(b - a, p - a));int c2 = sign(cross(c - b
    , p - b));int c3 = sign(cross(a - c, p - c));
if (c1 < 0 || c2 < 0 || c3 < 0)return 1;if (c1 + c2 + c3 !=
    3)return 0;return -1;}
double perimeter(vector<PT> &p){
double ans = 0;int n = p.size();
for (int i = 0; i < n; i++)ans += dist(p[i], p[(i + 1) % n])
    ;return ans;}
double area(vector<PT> &p){double ans = 0;int n = p.size();
    or (int i = 0; i < n; i++)ans += cross(p[i], p[(i + 1)
    % n]);return fabs(ans) * 0.5;}
// centroid of a (possibly non-convex) polygon,
// assuming that the coordinates are listed in a clockwise
    or
// counterclockwise fashion. Note that the centroid is often
     known as
// the "center of gravity" or "center of mass".
PT centroid(vector<PT> &p){int n = p.size();PT c(0, 0);
    double sum = 0;
for (int i = 0; i < n; i++)sum += cross(p[i], p[(i + 1) % n
    ]);double scale = 3.0 * sum;
for (int i = 0; i < n; i++){int j = (i + 1) % n;c = c + (p[i
    ] + p[j]) * cross(p[i], p[j]);}return c / scale;}
// 0 if cw, 1 if ccw
bool get_direction(vector<PT> &p){double ans = 0;int n = p.
    size();
for (int i = 0; i < n; i++)ans += cross(p[i], p[(i + 1) % n
    ]);if (sign(ans) > 0)return 1;return 0;}
// it returns a point such that the sum of distances
// from that point to all points in p is minimum
// O(n log^2 MX)
PT geometric_median(vector<PT> p){auto tot_dist = [&](PT z){
    double res = 0;
for (int i = 0; i < p.size(); i++)res += dist(p[i], z);
    return res;};
auto findY = [&](double x){double yl = -1e5, yr = 1e5;for (
    int i = 0; i < 60; i++){
double ym1 = yl + (yr - yl) / 3;double ym2 = yr - (yr - yl)
    / 3;double d1 = tot_dist(PT(x, ym1));
double d2 = tot_dist(PT(x, ym2));if (d1 < d2)yr = ym2;
else yl = ym1;}return pair<double, double>(yl, tot_dist(PT(x
    , yl)));};double xl = -1e5, xr = 1e5;
for (int i = 0; i < 60; i++){double xm1 = xl + (xr - xl) /
    3;
double xm2 = xr - (xr - xl) / 3;double y1, d1, y2, d2;auto z
     = findY(xm1);y1 = z.first;d1 = z.second;z = findY(xm2)
    ;
y2 = z.first;d2 = z.second;
if (d1 < d2)xr = xm2;elsexl = xm1;}return {xl, findY(xl).
    first};}
vector<PT> convex_hull(vector<PT> &p){
if (p.size() <= 1)return p;
vector<PT> v = p;sort(v.begin(), v.end());vector<PT> up, dn;
for (auto &p : v){
while (up.size() > 1 && orientation(up[up.size() - 2], up.
    back(), p) >= 0){up.pop_back();}
while (dn.size() > 1 && orientation(dn[dn.size() - 2], dn.
    back(), p) <= 0){dn.pop_back();}
up.push_back(p);dn.push_back(p);}
```

```
v = dn;if (v.size() > 1)v.pop_back();reverse(up.begin(), up.
    end());up.pop_back();
for (auto &p : up){v.push_back(p);}
if (v.size() == 2 && v[0] == v[1])v.pop_back();return v;}
// checks if convex or not
bool is_convex(vector<PT> &p){bool s[3];s[0] = s[1] = s[2] =
    0;int n = p.size();for (int i = 0; i < n; i++){int j =
    (i + 1) % n;int k = (j + 1) % n;
s[sign(cross(p[j] - p[i], p[k] - p[i])) + 1] = 1;if (s[0] &&
    s[2])return 0;}return 1;}
// -1 if strictly inside, 0 if on the polygon, 1 if strictly
    outside
// it must be strictly convex, otherwise make it strictly
    convex first
int is_point_in_convex(vector<PT> &p, const PT &x){ // O(log
    n)int n = p.size();assert(n >= 3);
int a = orientation(p[0], p[1], x), b = orientation(p[0], p[
    n - 1], x);if (a < 0 || b > 0)return 1;int l = 1, r = n
    - 1;while (l + 1 < r){
int mid = l + r >> 1;if (orientation(p[0], p[mid], x) >= 0)l
    = mid;elser = mid;}int k = orientation(p[l], p[r], x);
    if (k <= 0)return -k;
if (l == 1 && a == 0)return 0;if (r == n - 1 && b == 0)
    return 0;return -1;}
bool is_point_on_polygon(vector<PT> &p, const PT &z){int n =
    p.size();
for (int i = 0; i < n; i++){if (is_point_on_seg(p[i], p[(i +
    1) % n], z))return 1;}return 0;}
// returns 1e9 if the point is on the polygon
int winding_number(vector<PT> &p, const PT &z){ // O(n)
if (is_point_on_polygon(p, z))return 1e9;
int n = p.size(), ans = 0;
for (int i = 0; i < n; ++i){int j = (i + 1) % n;bool below =
    p[i].y < z.y;
if (below != (p[j].y < z.y)){
auto orient = orientation(z, p[j], p[i]);
if (orient == 0)return 0;
if (below == (orient > 0))ans += below ? 1 : -1;}}return ans
    ;}
// -1 if strictly inside, 0 if on the polygon, 1 if strictly
    outside
int is_point_in_polygon(vector<PT> &p, const PT &z){ // O(n)
int k = winding_number(p, z);return k == 1e9 ? 0 : k == 0 ?
    1: -1;}
// id of the vertex having maximum dot product with z
// polygon must need to be convex
// top - upper right vertex
// for minimum dot product negate z and return -dot(z, p[id
    ])
int extreme_vertex(vector<PT> &p, const PT &z, const int top
    ){ // O(log n)
int n = p.size();if (n == 1)return 0;
double ans = dot(p[0], z);int id = 0;
if (dot(p[top], z) > ans)ans = dot(p[top], z), id = top;
int l = 1, r = top - 1;
while (l < r){int mid = l + r >> 1;
if (dot(p[mid + 1], z) >= dot(p[mid], z))l = mid + 1;
else r = mid;}
if (dot(p[l], z) > ans)ans = dot(p[l], z), id = l;l = top +
    1, r = n - 1;while (l < r){
int mid = l + r >> 1;if (dot(p[(mid + 1) % n], z) >= dot(p[
    mid], z))l = mid + 1;elser = mid;}l %= n;
if (dot(p[l], z) > ans)ans = dot(p[l], z), id = l;return id
    ;}
// maximum distance from any point on the perimeter to
    another point on the perimeter
double diameter(vector<PT> &p){int n = (int)p.size();if (n
    == 1)return 0;if (n == 2)return dist(p[0], p[1]);double
     ans = 0;int i = 0, j = 1;
while (i < n){
while (cross(p[(i + 1) % n] - p[i], p[(j + 1) % n] - p[j])
    >= 0){ans = max(ans, dist2(p[i], p[j]));j = (j + 1) % n
    ;}
ans = max(ans, dist2(p[i], p[j]));i++;}return sqrt(ans);}
// minimum distance between two parallel lines (non
    necessarily axis parallel)
// such that the polygon can be put between the lines
double width(vector<PT> &p){int n = (int)p.size();if (n <=
    2)return 0;double ans = inf;int i = 0, j = 1;while (i <
     n){
while (cross(p[(i + 1) % n] - p[i], p[(j + 1) % n] - p[j])
    >= 0)j = (j + 1) % n;ans = min(ans,
    dist_from_point_to_line(p[i], p[(i + 1) % n], p[j]));i
    ++;}return ans;}
// minimum perimeter
double minimum_enclosing_rectangle(vector<PT> &p){int n = p.
    size();
if (n <= 2)return perimeter(p);int mndot = 0;double tmp =
    dot(p[1] - p[0], p[0]);
for (int i = 1; i < n; i++){if (dot(p[1] - p[0], p[i]) <=
    tmp){tmp = dot(p[1] - p[0], p[i]);mndot = i;}}
double ans = inf;int i = 0, j = 1, mxdot = 1;
while (i < n){PT cur = p[(i + 1) % n] - p[i];while (cross(
    cur, p[(j + 1) % n] - p[j]) >= 0)j = (j + 1) % n;while
    (dot(p[(mxdot + 1) % n], cur) >= dot(p[mxdot], cur))
    mxdot = (mxdot + 1) % n;
while (dot(p[(mndot + 1) % n], cur) <= dot(p[mndot], cur))
    mndot = (mndot + 1) % n;ans = min(ans, 2.0 * ((dot(p[
    mxdot], cur) / cur.norm() - dot(p[mndot], cur) / cur.
    norm()) + dist_from_point_to_line(p[i], p[(i + 1) % n],
     p[j])));i++;}return ans;}
// given n points, find the minimum enclosing circle of the
    points
// call convex_hull() before this for faster solution
// expected O(n)
circle minimum_enclosing_circle(vector<PT> &p){
    random_shuffle(p.begin(), p.end());int n = p.size();
    circle c(p[0], 0);
for (int i = 1; i < n; i++){if (sign(dist(c.p, p[i]) - c.r)
    > 0){c = circle(p[i], 0);
for (int j = 0; j < i; j++){
if (sign(dist(c.p, p[j]) - c.r) > 0){
c = circle((p[i] + p[j]) / 2, dist(p[i], p[j]) / 2);
for (int k = 0; k < j; k++){if (sign(dist(c.p, p[k]) - c.r)
    > 0){c = circle(p[i], p[j], p[k]);}}}}}return c;}
// returns a vector with the vertices of a polygon with
    everything
// to the left of the line going from a to b cut away.
vector<PT> cut(vector<PT> &p, PT a, PT b){vector<PT> ans;int
     n = (int)p.size();
for (int i = 0; i < n; i++){double c1 = cross(b - a, p[i] -
    a);double c2 = cross(b - a, p[(i + 1) % n] - a);
if (sign(c1) >= 0)ans.push_back(p[i]);if (sign(c1 * c2) < 0)
    {if (!is_parallel(p[i], p[(i + 1) % n], a, b)){PT tmp;
    line_line_intersection(p[i], p[(i + 1) % n], a, b, tmp)
    ;ans.push_back(tmp);}}}return ans;}
// not necessarily convex, boundary is included in the
    intersection
// returns total intersected length
// it returns the sum of the lengths of the portions of the
    line that are inside the polygon
double polygon_line_intersection(vector<PT> p, PT a, PT b){
int n = p.size();p.push_back(p[0]);line l = line(a, b);
    double ans = 0.0;vector<pair<double, int>> vec;for (int
     i = 0; i < n; i++){int s1 = orientation(a, b, p[i]);
    int s2 = orientation(a, b, p[i + 1]);
if (s1 == s2)continue;line t = line(p[i], p[i + 1]);PT inter
     = (t.v * l.c - l.v * t.c) / cross(l.v, t.v);double tmp
     = dot(inter, l.v);int f;
if (s1 > s2)f = s1 && s2 ? 2 : 1;
else f = s1 && s2 ? -2 : -1;vec.push_back(make_pair((f > 0 ?
     tmp - eps : tmp + eps), f)); // keep eps very small
    like 1e-12}
sort(vec.begin(), vec.end());for (int i = 0, j = 0; i + 1 <
    (int)vec.size(); i++){j += vec[i].second;
if (j)ans += vec[i + 1].first - vec[i].first; // if this
    portion is inside the polygon// else ans = 0; // if we
    want the maximum intersected length which is totally
    inside the polygon, uncomment this and take the maximum
```

```
     of ans}
ans = ans / sqrt(dot(l.v, l.v));p.pop_back();return ans;}
// given a convex polygon p, and a line ab and the top
     vertex of the polygon
// returns the intersection of the line with the polygon
// it returns the indices of the edges of the polygon that
     are intersected by the line
// so if it returns i, then the line intersects the edge (p[
     i], p[(i + 1) % n])
array<int, 2> convex_line_intersection(vector<PT> &p, PT a,
     PT b, int top){int end_a = extreme_vertex(p, (a - b).
     perp(), top);int end_b = extreme_vertex(p, (b - a).perp
     (), top);
auto cmp_l = [&](int i){ return orientation(a, p[i], b); };
     if (cmp_l(end_a) < 0 || cmp_l(end_b) > 0)return {-1,
     -1}; // no intersectionarray<int, 2> res;for (int i =
     0; i < 2; i++){int lo = end_b, hi = end_a, n = p.size()
     ;
while ((lo + 1) % n != hi){int m = ((lo + hi + (lo < hi ? 0
     : n)) / 2) % n;(cmp_l(m) == cmp_l(end_b) ? lo : hi) = m
     ;}res[i] = (lo + !cmp_l(hi)) % n;
swap(end_a, end_b);}
if (res[0] == res[1])return {res[0], -1}; // touches the
     vertex res[0]if (!cmp_l(res[0]) && !cmp_l(res[1]))
switch ((res[0] - res[1] + (int)p.size() + 1) % p.size()){
     case 0:return {res[0], res[0]}; // touches the edge (
     res[0], res[0] + 1)case 2:return {res[1], res[1]}; //
     touches the edge (res[1], res[1] + 1)}
return res; // intersects the edges (res[0], res[0] + 1) and
      (res[1], res[1] + 1)}

pair<PT, int> point_poly_tangent(vector<PT> &p, PT Q, int
     dir, int l, int r){
while (r - l > 1){int mid = (l + r) >> 1;bool pvs =
     orientation(Q, p[mid], p[mid - 1]) != -dir;
bool nxt = orientation(Q, p[mid], p[mid + 1]) != -dir;
if (pvs && nxt)return {p[mid], mid};
if (!(pvs || nxt)){auto p1 = point_poly_tangent(p, Q, dir,
     mid + 1, r);
auto p2 = point_poly_tangent(p, Q, dir, l, mid - 1);
return orientation(Q, p1.first, p2.first) == dir ? p1 : p2;}
if (!pvs){if (orientation(Q, p[mid], p[l]) == dir)r = mid -
     1;
else if (orientation(Q, p[l], p[r]) == dir)r = mid - 1;elsel
      = mid + 1;}
if (!nxt){if (orientation(Q, p[mid], p[l]) == dir)l = mid +
     1;
else if (orientation(Q, p[l], p[r]) == dir)r = mid - 1;
else l = mid + 1;}}
pair<PT, int> ret = {p[l], l};
for (int i = l + 1; i <= r; i++)ret = orientation(Q, ret.
     first, p[i]) != dir ? make_pair(p[i], i) : ret;
return ret;}
// (ccw, cw) tangents from a point that is outside this
     convex polygon
// returns indexes of the points
// ccw means the tangent from Q to that point is in the same
      direction as the polygon ccw direction
pair<int, int> tangents_from_point_to_polygon(vector<PT> &p,
      PT Q){int ccw = point_poly_tangent(p, Q, 1, 0, (int)p.
     size() - 1).second;int cw = point_poly_tangent(p, Q,
     -1, 0, (int)p.size() - 1).second;return make_pair(ccw,
     cw);}
// minimum distance from a point to a convex polygon
// it assumes point lie strictly outside the polygon
double dist_from_point_to_polygon(vector<PT> &p, PT z){
     double ans = inf;int n = p.size();if (n <= 3){for (int
     i = 0; i < n; i++)ans = min(ans, dist_from_point_to_seg
     (p[i], p[(i + 1) % n], z));return ans;}
auto [r, l] = tangents_from_point_to_polygon(p, z);if (l > r
     )r += n;
while (l < r){int mid = (l + r) >> 1;double left = dist2(p[
     mid % n], z), right = dist2(p[(mid + 1) % n], z);ans =
     min({ans, left, right});if (left < right)r = mid;elsel
     = mid + 1;}
ans = sqrt(ans);ans = min(ans, dist_from_point_to_seg(p[l %
     n], p[(l + 1) % n], z));ans = min(ans,
     dist_from_point_to_seg(p[l % n], p[(l - 1 + n) % n], z)
     );return ans;}
// minimum distance from convex polygon p to line ab
// returns 0 is it intersects with the polygon
// top - upper right vertex
double dist_from_polygon_to_line(vector<PT> &p, PT a, PT b,
     int top){ // O(log n)
PT orth = (b - a).perp();
if (orientation(a, b, p[0]) > 0)orth = (a - b).perp();
int id = extreme_vertex(p, orth, top);if (dot(p[id] - a,
     orth) > 0)return 0.0;// if orth and a are in the same
     half of the line, then poly and line intersects
return dist_from_point_to_line(a, b, p[id]); // does not
     intersect}
// minimum distance from a convex polygon to another convex
     polygon
// the polygon doesnot overlap or touch
// tested in https://toph.co/p/the-wall
double dist_from_polygon_to_polygon(vector<PT> &p1, vector<
     PT> &p2){ // O(n log n)
double ans = inf;
for (int i = 0; i < p1.size(); i++){ans = min(ans,
     dist_from_point_to_polygon(p2, p1[i]));}
for (int i = 0; i < p2.size(); i++){ans = min(ans,
     dist_from_point_to_polygon(p1, p2[i]));}
return ans;}
// maximum distance from a convex polygon to another convex
     polygon
double maximum_dist_from_polygon_to_polygon(vector<PT> &u,
     vector<PT> &v){ // O(n)
int n = (int)u.size(), m = (int)v.size();double ans = 0;
if (n < 3 || m < 3){
for (int i = 0; i < n; i++){
for (int j = 0; j < m; j++)ans = max(ans, dist2(u[i], v[j]))
     ;}return sqrt(ans);}
if (u[0].x > v[0].x)swap(n, m), swap(u, v);int i = 0, j = 0,
      step = n + m + 10;
while (j + 1 < m && v[j].x < v[j + 1].x)j++;
while (step--){if (cross(u[(i + 1) % n] - u[i], v[(j + 1) %
     m] - v[j]) >= 0)j = (j + 1) % m;elsei = (i + 1) % n;ans
      = max(ans, dist2(u[i], v[j]));}
return sqrt(ans);}

// calculates the area of the union of n polygons (not
     necessarily convex).
// the points within each polygon must be given in CCW order
     .
// complexity: O(N^2), where N is the total number of points
double rat(PT a, PT b, PT p){return !sign(a.x - b.x) ? (p.y
     - a.y) / (b.y - a.y) : (p.x - a.x) / (b.x - a.x);};
double polygon_union(vector<vector<PT>> &p){
int n = p.size();double ans = 0;
for (int i = 0; i < n; ++i){
for (int v = 0; v < (int)p[i].size(); ++v){
PT a = p[i][v], b = p[i][(v + 1) % p[i].size()];vector<pair<
     double, int>> segs;segs.emplace_back(0, 0), segs.
     emplace_back(1, 0);
for (int j = 0; j < n; ++j){
if (i != j){for (size_t u = 0; u < p[j].size(); ++u){
PT c = p[j][u], d = p[j][(u + 1) % p[j].size()];int sc =
     sign(cross(b - a, c - a)), sd = sign(cross(b - a, d - a
     ));if (!sc && !sd){
if (sign(dot(b - a, d - c)) > 0 && i > j){segs.emplace_back(
     rat(a, b, c), 1), segs.emplace_back(rat(a, b, d), -1)
     ;}}else{
double sa = cross(d - c, a - c), sb = cross(d - c, b - c);
if (sc >= 0 && sd < 0)segs.emplace_back(sa / (sa - sb), 1);
     else if (sc < 0 && sd >= 0)segs.emplace_back(sa / (sa -
      sb), -1);}}}
sort(segs.begin(), segs.end());double pre = min(max(segs[0].
     first, 0.0), 1.0), now, sum = 0;int cnt = segs[0].
     second;
```

```
for (int j = 1; j < segs.size(); ++j){now = min(max(segs[j].
    first, 0.0), 1.0);if (!cnt)sum += now - pre;cnt += segs
    [j].second;pre = now;}ans += cross(a, b) * sum;}}return
     ans * 0.5;}
// contains all points p such that: cross(b - a, p - a) >= 0
struct HP{PT a, b;HP() {}HP(PT a, PT b) : a(a), b(b) {}HP(
    const HP &rhs) : a(rhs.a), b(rhs.b) {}
int operator<(const HP &rhs) const{PT p = b - a;PT q = rhs.b
     - rhs.a;
int fp = (p.y < 0 || (p.y == 0 && p.x < 0));int fq = (q.y <
    0 || (q.y == 0 && q.x < 0));
if (fp != fq)return fp == 0;if (cross(p, q))return cross(p,
    q) > 0;return cross(p, rhs.b - a) < 0;}PT
    line_line_intersection(PT a, PT b, PT c, PT d)
{b = b - a;d = c - d;c = c - a;return a + b * cross(c, d) /
    cross(b, d);}
PT intersection(const HP &v){return line_line_intersection(a
    , b, v.a, v.b);}};
int check(HP a, HP b, HP c){return cross(a.b - a.a, b.
    intersection(c) - a.a) > -eps; //-eps to include
    polygons of zero area (straight lines, points)}
// consider half-plane of counter-clockwise side of each
    line
// if lines are not bounded add infinity rectangle
// returns a convex polygon, a point can occur multiple
    times though
// complexity: O(n log(n))
vector<PT> half_plane_intersection(vector<HP> h){sort(h.
    begin(), h.end());vector<HP> tmp;
for (int i = 0; i < h.size(); i++){if (!i || cross(h[i].b -
    h[i].a, h[i - 1].b - h[i - 1].a)){tmp.push_back(h[i])
    ;}}
h = tmp;vector<HP> q(h.size() + 10);int qh = 0, qe = 0;
for (int i = 0; i < h.size(); i++){
while (qe - qh > 1 && !check(h[i], q[qe - 2], q[qe - 1]))qe
    --;
while (qe - qh > 1 && !check(h[i], q[qh], q[qh + 1]))qh++;q[
    qe++] = h[i];}
while (qe - qh > 2 && !check(q[qh], q[qe - 2], q[qe - 1]))qe
    --;
while (qe - qh > 2 && !check(q[qe - 1], q[qh], q[qh + 1]))qh
    ++;
vector<HP> res;
for (int i = qh; i < qe; i++)res.push_back(q[i]);vector<PT>
    hull;
if (res.size() > 2){for (int i = 0; i < res.size(); i++){
    hull.push_back(res[i].intersection(res[(i + 1) % ((int)
    res.size())]));}}
return hull;}

// rotate the polygon such that the (bottom, left)-most
    point is at the first position
void reorder_polygon(vector<PT> &p){int pos = 0;for (int i =
     1; i < p.size(); i++){if (p[i].y < p[pos].y || (sign(p
    [i].y - p[pos].y) == 0 && p[i].x < p[pos].x))pos = i;}
    rotate(p.begin(), p.begin() + pos, p.end());}
// a and b are convex polygons
// returns a convex hull of their minkowski sum
// min(a.size(), b.size()) >= 2
// https://cp-algorithms.com/geometry/minkowski.html
vector<PT> minkowski_sum(vector<PT> a, vector<PT> b){
reorder_polygon(a);reorder_polygon(b);
int n = a.size(), m = b.size();int i = 0, j = 0;a.push_back(
    a[0]);a.push_back(a[1]);b.push_back(b[0]);b.push_back(b
    [1]);vector<PT> c;
while (i < n || j < m){c.push_back(a[i] + b[j]);double p =
    cross(a[i + 1] - a[i], b[j + 1] - b[j]);if (sign(p) >=
    0)++i;if (sign(p) <= 0)++j;}
return c;}
// returns the area of the intersection of the circle with
    center c and radius r
// and the triangle formed by the points c, a, b
double _triangle_circle_intersection(PT c, double r, PT a,
    PT b){
double sd1 = dist2(c, a), sd2 = dist2(c, b);
if (sd1 > sd2)swap(a, b), swap(sd1, sd2);double sd = dist2(a
    , b);double d1 = sqrtl(sd1), d2 = sqrtl(sd2), d = sqrt(
    sd);double x = abs(sd2 - sd - sd1) / (2 * d);double h =
     sqrtl(sd1 - x * x);
if (r >= d2)return h * d / 2;double area = 0;
if (sd + sd1 < sd2){if (r < d1)area = r * r * (acos(h / d2)
    - acos(h / d1)) / 2;else{area = r * r * (acos(h / d2) -
     acos(h / r)) / 2;double y = sqrtl(r * r - h * h);area
    += h * (y - x) / 2;}}
else{if (r < h)area = r * r * (acos(h / d2) + acos(h / d1))
    / 2;else{area += r * r * (acos(h / d2) - acos(h / r)) /
     2;double y = sqrtl(r * r - h * h);area += h * y / 2;if
     (r < d1){area += r * r * (acos(h / d1) - acos(h / r))
    / 2;area += h * y / 2;}elsearea += h * x / 2;}}
return area;}
// intersection between a simple polygon and a circle
double polygon_circle_intersection(vector<PT> &v, PT p,
    double r){int n = v.size();double ans = 0.00;PT org =
    {0, 0};
for (int i = 0; i < n; i++){int x = orientation(p, v[i], v[(
    i + 1) % n]);if (x == 0)continue;double area =
    _triangle_circle_intersection(org, r, v[i] - p, v[(i +
    1) % n] - p);if (x < 0)ans -= area;elseans += area;}
return abs(ans);}

// find a circle of radius r that contains as many points as
     possible
// O(n^2 log n);
double maximum_circle_cover(vector<PT> p, double r, circle &
    c){int n = p.size();int ans = 0;int id = 0;double th =
    0;
for (int i = 0; i < n; ++i){
// maximum circle cover when the circle goes through this
    point
vector<pair<double, int>> events = {{-PI, +1}, {PI, -1}};
for (int j = 0; j < n; ++j){if (j == i)continue;double d =
    dist(p[i], p[j]);if (d > r * 2)continue;
double dir = (p[j] - p[i]).arg();double ang = acos(d / 2 / r
    );double st = dir - ang, ed = dir + ang;
if (st > PI)st -= PI * 2;if (st <= -PI)st += PI * 2;if (ed >
     PI)ed -= PI * 2;if (ed <= -PI)ed += PI * 2;
events.push_back({st - eps, +1}); // take care of precisions
    !
events.push_back({ed, -1});if (st > ed){events.push_back({-
    PI, +1});events.push_back({+PI, -1});}}sort(events.
    begin(), events.end());int cnt = 0;for (auto &&e :
    events){cnt += e.second;if (cnt > ans){ans = cnt;id = i
    ;th = e.first;}}}PT w = PT(p[id].x + r * cos(th), p[id
    ].y + r * sin(th));c = circle(w, r); // best_circle
return ans;}
// radius of the maximum inscribed circle in a convex
    polygon
double maximum_inscribed_circle(vector<PT> p){int n = p.size
    ();if (n <= 2)return 0;double l = 0, r = 20000;
while (r - l > eps){double mid = (l + r) * 0.5;vector<HP> h;
    const int L = 1e9;h.push_back(HP(PT(-L, -L), PT(L, -L))
    );h.push_back(HP(PT(L, -L), PT(L, L)));h.push_back(HP(
    PT(L, L), PT(-L, L)));h.push_back(HP(PT(-L, L), PT(-L,
    -L)));for (int i = 0; i < n; i++){PT z = (p[(i + 1) % n
    ] - p[i]).perp();z = z.truncate(mid);PT y = p[i] + z, q
     = p[(i + 1) % n] + z;h.push_back(HP(p[i] + z, p[(i +
    1) % n] + z));}vector<PT> nw = half_plane_intersection(
    h);if (!nw.empty())l = mid;elser = mid;}
return l;}
// ear decomposition, O(n^3) but faster
vector<vector<PT>> triangulate(vector<PT> p){
vector<vector<PT>> v;
while (p.size() >= 3){
for (int i = 0, n = p.size(); i < n; i++){int pre = i == 0 ?
     n - 1 : i - 1;;int nxt = i == n - 1 ? 0 : i + 1;int
    ori = orientation(p[i], p[pre], p[nxt]);if (ori < 0){
    int ok = 1;for (int j = 0; j < n; j++){if (j == i || j
    == pre || j == nxt)continue;if (is_point_in_triangle(p[
    i], p[pre], p[nxt], p[j]) < 1){ok = 0;break;}}if (ok){v
    .push_back({p[pre], p[i], p[nxt]});p.erase(p.begin() +
```

```
    i);break;}}}}
return v;}

struct star{
int n;   // number of sides of the star
double r; // radius of the circumcircle
star(int _n, double _r){n = _n;r = _r;}
double area(){double theta = PI / n;double s = 2 * r * sin(
    theta);double R = 0.5 * s / tan(theta);double a = 0.5 *
     n * s * R;double a2 = 0.25 * s * s / tan(1.5 * theta);
    return a - n * a2;}};

// given a list of lengths of the sides of a polygon in
    counterclockwise order
// returns the maximum area of a non-degenerate polygon that
     can be formed using those lengths
double get_maximum_polygon_area_for_given_lengths(vector<
    double> v){
if (v.size() < 3){return 0;}
int m = 0;double sum = 0;
for (int i = 0; i < v.size(); i++){if (v[i] > v[m]){m = i;}
    sum += v[i];}
if (sign(v[m] - (sum - v[m])) >= 0){return 0; // no non-
    degenerate polygon is possible}
// the polygon should be a circular polygon
// that is all points are on the circumference of a circle
double l = v[m] / 2, r = 1e6; // fix it correctlyint it =
    60;
auto ang = [](double x, double r) { // x = length of the
    chord, r = radius of the circle
return 2 * asin((x / 2) / r);};
auto calc = [=](double r){double sum = 0;for (auto x : v){
    sum += ang(x, r);}return sum; };
// compute the radius of the circle
while (it--){double mid = (l + r) / 2;if (calc(mid) <= 2 *
    PI){r = mid;}else{l = mid;}}
if (calc(r) <= 2 * PI - eps){ // the center of the circle is
     outside the polygon
auto calc2 = [&](double r){double sum = 0;for (int i = 0; i
    < v.size(); i++){double x = v[i];double th = ang(x, r);
    if (i != m){sum += th;}else{sum += 2 * PI - th;}}return
     sum;};l = v[m] / 2;r = 1e6;it = 60;while (it--){
double mid = (l + r) / 2;if (calc2(mid) > 2 * PI){r = mid;}
    else{l = mid;}}
auto get_area = [=](double r){double ans = 0;
for (int i = 0; i < v.size(); i++){double x = v[i];double
    area = r * r * sin(ang(x, r)) / 2;if (i != m){ans +=
    area;}else{ans -= area;}}return ans;};return get_area(r
    );}else{ // the center of the circle is inside the
    polygon
auto get_area = [=](double r){double ans = 0;for (auto x : v
    ){ans += r * r * sin(ang(x, r)) / 2;}return ans;};
    return get_area(r);}}
```

5 NumberTheory

5.1 CRT

```
using T = __int128;
// ax + by = __gcd(a, b)
// returns __gcd(a, b)
T extended_euclid(T a, T b, T &x, T &y) {
T xx = y = 0;T yy = x = 1;
while (b) {T q = a / b;T t = b; b = a % b; a = t;
t = xx; xx = x - q * xx; x = t;t = yy; yy = y - q * yy; y =
    t;}
return a;}// finds x such that x % m1 = a1, x % m2 = a2. m1
    and m2 may not be coprime
// here, x is unique modulo m = lcm(m1, m2). returns (x, m).
     on failure, m = -1.
pair<T, T> CRT(T a1, T m1, T a2, T m2) {
T p, q; T g = extended_euclid(m1, m2, p, q);
if (a1 % g != a2 % g) return make_pair(0, -1); T m = m1 / g
    * m2;
p = (p % m + m) % m;q = (q % m + m) % m;
return make_pair((p * a2 % m * (m1 / g) % m + q * a1 % m * (
    m2 / g) % m) % m, m);}
```

5.2 bigmod

```
ll bigmod(ll a, ll b, ll n) {ll res = 1;
if (b == 0) return 1; a = a % n; if (a == 0)
return 0;while (b > 0) {if (b % 2)res = (res * a) % n;
b = b / 2; a = (a * a) % n;}return res;}
```

5.3 extended euclid

```
#include <bits/stdc++.h>
using namespace std;int x, y;
int gcdExtended(int a, int b, int *x, int *y) {
if (b==0) {*x = 1;*y = 0;return a;}
int x1, y1;int gcd = gcdExtended(b,a%b,&x1,&y1);
*x =y1;*y =x1-y1*(a/b);return gcd;}
int main() { int a = 50, b = 10;
cout<<"gcd "<<gcdExtended(a, b, &x, &y)<<endl;;
cout<<x<<" " <<y<<endl;return 0;}
```

5.4 fft

```
const double PI = acos(-1);struct base { double a, b;base(
    double a = 0, double b = 0) : a(a), b(b) {}
const base operator + (const base &c) const{ return base(a +
     c.a, b + c.b); }
const base operator - (const base &c) const{ return base(a -
     c.a, b - c.b); }
const base operator * (const base &c) const{ return base(a *
     c.a - b * c.b, a * c.b + b * c.a); }
};void fft(vector<base> &p, bool inv = 0) {int n = p.size(),
     i = 0;for(int j = 1; j < n - 1; ++j) {
for(int k = n >> 1; k > (i ^= k); k >>= 1);if(j < i) swap(p[
    i], p[j]);}
for(int l = 1, m; (m = l << 1) <= n; l <<= 1) {double ang =
    2 * PI / m;
base wn = base(cos(ang), (inv ? 1. : -1.) * sin(ang)), w;for
    (int i = 0, j, k; i < n; i += m) {
for(w = base(1, 0), j = i, k = i + l; j < k; ++j, w = w * wn
    ) {
base t = w * p[j + l];p[j + l] = p[j] - t;p[j] = p[j] + t
    ;}}}
if(inv) for(int i = 0; i < n; ++i) p[i].a /= n, p[i].b /= n
    ;}
vector<long long> multiply(vector<int> &a, vector<int> &b) {
int n = a.size(), m = b.size(), t = n + m - 1, sz = 1;while(
    sz < t) sz <<= 1;
vector<base> x(sz), y(sz), z(sz);for(int i = 0 ; i < sz; ++i
    ) {
x[i] = i < (int)a.size() ? base(a[i], 0) : base(0, 0);
y[i] = i < (int)b.size() ? base(b[i], 0) : base(0, 0);}
fft(x), fft(y);for(int i = 0; i < sz; ++i) z[i] = x[i] * y[i
    ];fft(z, 1);
vector<long long> ret(sz);for(int i = 0; i < sz; ++i) ret[i]
     = (long long) round(z[i].a);
// while((int)ret.size() > 1 && ret.back() == 0) ret.
    pop_back();
return ret;}
```

5.5 linear diphantine equation

```
#include<bits/stdc++.h> using namespace std;using ll = long
    long;
ll extended_euclid(ll a, ll b, ll &x, ll &y) {
```

```
ll xx = y = 0;ll yy = x = 1;while (b) {
ll q = a / b;ll t = b; b = a % b; a = t;
t = xx; xx = x - q * xx; x = t;
t = yy; yy = y - q * yy; y = t;}return a;}
// a*x+b*y=c. returns valid x and y if possible.
// all solutions are of the form (x0 + k * b / g, y0 - k * b
    / g)
bool find_any_solution (ll a, ll b, ll c, ll &x0, ll &y0, ll
    &g) {
if (a == 0 and b == 0) {if (c) return false;
x0 = y0 = g = 0; return true;}
g = extended_euclid (abs(a), abs(b), x0, y0);if (c % g != 0)
    return false;
x0 *= c / g;y0 *= c / g;if (a < 0) x0 *= -1;if (b < 0) y0 *=
    -1;
return true;}void shift_solution(ll &x, ll &y, ll a, ll b,
    ll cnt) {
x += cnt * b;y -= cnt * a;}
// returns the number of solutions where x is in the range[
    minx, maxx] and y is in the range[miny, maxy]
ll find_all_solutions(ll a, ll b, ll c, ll minx, ll maxx, ll
    miny,ll maxy) {ll x, y, g;
if (find_any_solution(a, b, c, x, y, g) == 0) return 0;if (a
    == 0 and b == 0) {
assert(c == 0);return 1LL * (maxx - minx + 1) * (maxy - miny
    + 1);}
if (a == 0) {return (maxx - minx + 1) * (miny <= c / b and c
    / b <= maxy);}
if (b == 0) {return (maxy - miny + 1) * (minx <= c / a and c
    / a <= maxx);}
a /= g, b /= g;ll sign_a = a > 0 ? +1 : -1;ll sign_b = b > 0
    ? +1 : -1;
shift_solution(x, y, a, b, (minx - x) / b);if (x < minx)
    shift_solution(x, y, a, b, sign_b);
if (x > maxx) return 0;ll lx1 = x;shift_solution(x, y, a, b,
    (maxx - x) / b);
if (x > maxx) shift_solution (x, y, a, b, -sign_b);ll rx1 =
    x;shift_solution(x, y, a, b, -(miny - y) / a);
if (y < miny) shift_solution (x, y, a, b, -sign_a);if (y >
    maxy) return 0;
ll lx2 = x;shift_solution(x, y, a, b, -(maxy - y) / a);if (y
    > maxy) shift_solution(x, y, a, b, sign_a);
ll rx2 = x;if (lx2 > rx2) swap (lx2, rx2);ll lx = max(lx1,
    lx2);ll rx = min(rx1, rx2);
if (lx > rx) return 0;return (rx - lx) / abs(b) + 1;}
int32_t main() { ios_base::sync_with_stdio(0);cin.tie(0);int
    t, cs = 0; cin >> t;
while (t--) {ll a, b, c, x1, x2, y1, y2; cin >> a >> b >> c
    >> x1 >> x2 >> y1 >> y2;
cout << "Case " << ++cs << ": " << find_all_solutions(a, b,
    -c, x1, x2, y1, y2) << '\n';}
return 0;}// https://lightoj.com/problem/solutions-to-an-
    equation
```

5.6 linear sieve

```
const ll N = 1e7 + 7;bool isPrime[N];vector < ll > p;
void lin_sieve () {ll i;for (i = 2; i < N; i++) {
if (!isPrime[i]) p.push_back (i);for (ll j : p) {
if (i * j >= N) break;isPrime[i * j] = 1;if (i % j == 0)
    break;}}}
```

5.7 mat expo

```
mt19937 mt_rand(chrono::high_resolution_clock::now().
    time_since_epoch().count());
const int mod = 1e9 + 7;// const int N = 5e5 + 6;
// No of terms in the Recurrence Relation.
const int N = 4;const long long M = 1000000007;
// Multiplies two matrices A and B and stores the result in
    A.
void multiply (long long A[N][N], long long B[N][N]){long
    long R[N][N];
// Multiply A and B and store result in R.
for (int i = 0; i < N; i++){for (int j = 0; j < N; j++){
R[i][j] = 0;for (int k = 0; k < N; k++){R[i][j] = (R[i][j] +
     A[i][k] * B[k][j]) % M;}}}
// Copy contents of R in A.
for (int i = 0; i < N; i++){for (int j = 0; j < N; j++){A[i
    ][j] = R[i][j];}}}
// Raise matrix A to the power of n in O(log n).
void power_matrix (long long A[N][N], ll n){long long B[N][N
    ];// B = Identity Matrix.
for (int i = 0; i < N; i++){for (int j = 0; j < N; j++){B[i
    ][j] = A[i][j];}}
// A = A * A ^ (n - 1).
n = n - 1;while (n > 0){// If n is odd, A = A * B.
if (n & 1)multiply (A, B);// B = B * B.
multiply (B, B);// n = n / 2.
n = n >> 1;}}// A = Coefficient Matrix, B = Base Matrix.
// It returns the nth term of the recurrence relation formed
     from A and B in O(log n).
long long solve_recurrence (long long A[N][N], long long B[N
    ][1], ll n){//Base Cases.
if (n < N)return B[N - 1 - n][0];// A = A ^ (n - N + 1).
power_matrix (A, n - N + 1);long long result = 0;for (int i
    = 0; i < N; i++)
result = (result + A[0][i] * B[i][0]) % M;return result;}
void Solve(){
/*
The recurrence relation used here is: -
R(n) = 2 * R(n-1) + R(n-2) + 3 * R(n-3) + 3.
Base Cases: R(0) = 1, R(1) = 2, R(2) = 3.
*/
// Forming the Coefficient Matrix
long long A[N][N] = {{3, 2, 1, 1}, {1, 0, 0, 0}, {0, 1, 0,
    0}, {0, 0, 0, 1}};
//Forming the Base Matrix
long long B[N][1] = {{3}, {2}, {1}, {3}};ll n;cin >> n;if (n
     <= 2){
cout << n + 1 << endl;return ;}long long R_n =
    solve_recurrence (A, B, n + 1);
cout << R_n << endl;}
```

5.8 mobius

```
void mobius() {mob[1] = 1;for (int i = 2; i < N; i++){
mob[i]--;for (int j = i + i; j < N; j += i) {mob[j] -= mob[i
    ];}}}
```

5.9 ncr for mod

```
ll fact[200008];ll bigmod(ll b, ll p) {if(p == 0) return 1;
ll h = bigmod(b, p/2);h = h * h % mod;if(p&1) h = h * b %
    mod;
return h;}ll ncr(ll n, ll r) {if(n<r)return 0;
return fact[n] * bigmod(fact[r] * fact[n-r] % mod, mod - 2)
    % mod;}
void Fact(){fact[0] = 1;for(int i=1; i<=200002; i++)
fact[i] = fact[i-1] * i % mod;}
```

5.10 ntt

```
const int N = 1 << 20;const int mod = 998244353;const int
    root = 3;
int lim, rev[N], w[N], wn[N], inv_lim;void reduce(int &x) {
    x = (x + mod) % mod; }
int POW(int x, int y, int ans = 1) {for (; y; y >>= 1, x = (
    long long) x * x % mod)
if (y & 1) ans = (long long) ans * x % mod;return ans;}void
    precompute(int len) {
```

```
lim = wn[0] = 1; int s = -1;while (lim < len) lim <<= 1, ++s
    ;
for (int i = 0; i < lim; ++i) rev[i] = rev[i >> 1] >> 1 | (i
     & 1) << s;
const int g = POW(root, (mod - 1) / lim);inv_lim = POW(lim,
    mod - 2);
for (int i = 1; i < lim; ++i) wn[i] = (long long) wn[i - 1]
    * g % mod;}
void ntt(vector<int> &a, int typ) {
for (int i = 0; i < lim; ++i) if (i < rev[i]) swap(a[i], a[
    rev[i]]);
for (int i = 1; i < lim; i <<= 1) {for (int j = 0, t = lim /
     i / 2; j < i; ++j)
w[j] = wn[j * t];for (int j = 0; j < lim; j += i << 1) {for
    (int k = 0; k < i; ++k) {
const int x = a[k + j], y = (long long) a[k + j + i] * w[k]
    % mod;
reduce(a[k + j] += y - mod), reduce(a[k + j + i] = x - y)
    ;}}}if (!typ) {
reverse(a.begin() + 1, a.begin() + lim);for (int i = 0; i <
    lim; ++i)
a[i] = (long long) a[i] * inv_lim % mod;}}
vector<int> multiply(vector<int> &f, vector<int> &g) {
if (f.empty() or g.empty()) return {};int n = (int)f.size()
    + (int)g.size() - 1;
if (n == 1) return {(int)((long long) f[0] * g[0] % mod)};
    precompute(n);
vector<int> a = f, b = g;a.resize(lim); b.resize(lim);
ntt(a, 1), ntt(b, 1);for (int i = 0; i < lim; ++i)
a[i] = (long long) a[i] * b[i] % mod;ntt(a, 0);a.resize(n +
    1);return a;}
```

5.11 phi

```
// for n<=1e6 by nloglogn
void phi(int n){vector<int>phi(n+1) ;
for(int i = 0; i <= n; i++)phi[i] = i;for(int i = 2; i <=n;
    i++){
if(phi[i] == i){for(int j = i; j <= n; j += i){
phi[j] -= phi[j]/i ;}}}}
// for single number by sqrt(n)
int phi(int n){int res = n ;for(int i = 2; i * i <= n ; i++)
    {
if(n % i == 0){while(n % i == 0) n /= i ;res -= res / i ;}}
if(n > 1) res -= res / n ;return res ;}
```

5.12 pollard rho

```
using ll = long long;namespace PollardRho {
mt19937 rnd(chrono::steady_clock::now().time_since_epoch().
    count());
const int P = 1e6 + 9;ll seq[P];int primes[P], spf[P];
inline ll add_mod(ll x, ll y, ll m) {return (x += y) < m ? x
     : x - m;}
inline ll mul_mod(ll x, ll y, ll m) {ll res = __int128(x) *
    y % m;
return res;// ll res = x * y - (ll)((long double)x * y / m +
     0.5) * m;
// return res < 0 ? res + m : res;
}inline ll pow_mod(ll x, ll n, ll m) {ll res = 1 % m;
for (; n; n >>= 1) {if (n & 1) res = mul_mod(res, x, m);
x = mul_mod(x, x, m);}return res;}
// O(it * (logn)^3), it = number of rounds performed
inline bool miller_rabin(ll n) {if (n <= 2 || (n & 1 ^ 1))
    return (n == 2);
if (n < P) return spf[n] == n;ll c, d, s = 0, r = n - 1;for
    (; !(r & 1); r >>= 1, s++) {}
// each iteration is a round
for (int i = 0; primes[i] < n && primes[i] < 32; i++) {c =
    pow_mod(primes[i], r, n);
for (int j = 0; j < s; j++) {d = mul_mod(c, c, n);if (d == 1
     && c != 1 && c != (n - 1)) return false;
c = d;}if (c != 1) return false;}return true;} void init() {
    int cnt = 0;
for (int i = 2; i < P; i++) { if (!spf[i]) primes[cnt++] =
    spf[i] = i;
for (int j = 0, k; (k = i * primes[j]) < P; j++) { spf[k] =
    primes[j];
if (spf[i] == spf[k]) break;}}}// returns O(n^(1/4))
ll pollard_rho(ll n) {while (1) {ll x = rnd() % n, y = x, c
    = rnd() % n, u = 1, v, t = 0;
ll *px = seq, *py = seq;while (1) {*py++ = y = add_mod(
    mul_mod(y, y, n), c, n);
*py++ = y = add_mod(mul_mod(y, y, n), c, n);if ((x = *px++)
    == y) break;v = u;
u = mul_mod(u, abs(y - x), n);if (!u) return __gcd(v, n);
if (++t == 32) {t = 0;if ((u = __gcd(u, n)) > 1 && u < n)
    return u;}}
if (t && (u = __gcd(u, n)) > 1 && u < n) return u;}}
vector<ll> factorize(ll n) {if (n == 1) return vector <ll>()
    ;if (miller_rabin(n)) return vector<ll> {n};
vector <ll> v, w; while (n > 1 && n < P) { v.push_back(spf[n
    ]);n /= spf[n];}
if (n >= P) { ll x = pollard_rho(n); v = factorize(x); w =
    factorize(n / x);
v.insert(v.end(), w.begin(), w.end());}return v;}}
int32_t main() { ios_base::sync_with_stdio(0); cin.tie(0);
    PollardRho::init();
int t; cin >> t; while (t--) { ll n; cin >> n; auto f =
    PollardRho::factorize(n);
sort(f.begin(), f.end()); cout << f.size() << ' '; for (auto
     x: f) cout << x << ' '; cout << '\n';}return 0;}
// https://judge.yosupo.jp/problem/factorize
```

5.13 power tower

```
const int N = 1e5 + 9;using ll = long long;map<ll, ll> mp;
ll phi(ll n) { if (mp.count(n)) return mp[n];ll ans = n, m =
     n;
for (ll i = 2; i * i <= m; i++) { if (m % i == 0) {while (m
    % i == 0) m /= i;
ans = ans / i * (i - 1);}}if (m > 1) ans = ans / m * (m - 1)
    ;return mp[n] = ans;}
inline ll MOD(ll x, ll m) { if (x < m) return x;return x % m
     + m;}
ll power(ll n, ll k, ll mod) { ll ans = MOD(1, mod); while (
    k) {
if (k & 1) ans = MOD(ans * n, mod); n = MOD(n * n, mod);k
    >>= 1;}return ans;}
int a[N]; // if x >= log2(m), then a^x = a^(MOD(x, phi(m)))
    % m
ll yo(ll l, ll r, ll m) { if (l == r) return MOD(a[l], m);if
     (m == 1) return 1;
return power(a[l], yo(l + 1, r, phi(m)), m);}
int32_t main() { ios_base::sync_with_stdio(0); cin.tie(0);
int n, m; cin >> n >> m; for (int i = 1; i <= n; i++) { cin
    >> a[i];}
int q; cin >> q; while (q--) {int l, r; cin >> l >> r;cout
    << yo(l, r, m) % m << '\n';}
return 0;} // https://codeforces.com/contest/906/problem/D
```

5.14 sieve all

```
vector<bool>prime(N, true);vector<int>vec ;
void seive() { prime[0] = false;prime[1] = false ;
for (int i = 2; i * i < N; i++) {if (prime[i]) {
for (int j = i * i; j < N; j += i) {prime[j] = false ;}}}
for (int i = 2; i < N; i++) if (prime[i])vec.push_back(i);}
void pro() { int n; cin >> n ; int ans = 1 ;
for (auto it : vec) { if (it * it > n)break ;
if (n % it == 0) { int cnt = 1 ; while (n % it == 0) {
n /= it ; cnt++ ;}ans *= cnt ;}}if (n > 1) ans *= 2 ;
cout << ans - 1 << endl;}//// Segmented Sieve
```

```
void pro() { int n, m;cin >> n >> m; bool ara[m - n + 1] ;
memset(ara, true, sizeof(ara)); for (auto it : sve){ if (it
    * it > m)break;
int fmpl = (n + it - 1) / it; fmpl *= it ;int strt = max(
    fmpl, it * it);
for (int j = strt; j <= m; j += it) { ara[j - n] = false ;}}
if (n == 1)ara[0] = false ;for (int i = n; i <= m; i++)if (
    ara[i - n]) cout << i << endl;}
```

5.15 totient

```
#include<bits/stdc++.h>using namespace std ;
const double pi = 2 * acos (0.0) ;const int N=5000006 ;
const int INF=INT_MAX;const int mod=1000000007 ;
vector<int>phi(N,0) ;void totient_seive(){
for(int i=1; i<N; i++)phi[i]=i;for(int i=2; i<N; i++){
if(phi[i]==i) { for(int j=i; j<N; j+=i) {
phi[j]= ( phi[j] - (phi[j]/i) ) ;}}}}
//FOR ANY SINGLE NUMBER ___ CALCULTAING THE VALUE OF PHI
    USING SQRT COMPLEXITY
const int N=1000009 ;const int INF=INT_MAX ;const int mod
    =1000000007 ;
int sqrt_phi(int n){ int ans=n ; for(int i=2; i*i<=n; i++){
if(n%i==0){while(n%i==0)n/=i ;ans-=(ans/i) ;}}
if(n>1) ans-=(ans/n) ;return ans ;}
//CALCULATING PHI VALUE USING SUM OF PHI....
//SUM OF TOTAIENT VALUE FOR ALL DIVISOR OF N IS EQUAL TO N
// PHI(10)+PHI(5)+PHI(2)+PHI(1)
//= 4 + 4 + 1 + 1
// 10
const int N=10000007 ;const int INF=INT_MAX;const int mod
    =1000000007 ;
vector<int>phi(N,0) ;void calcphi(){phi[0]=0 ;phi[1]=1 ;
for(int i=2; i<N; i++) phi[i]=i-1 ;
//(loop er modde 1 divisor hisabe ani nai.....tai 1 er
    contribution bad)
for(int i=2; i<N; i++){ for(int j=2*i; j<N; j+=i) {phi[j]-=
    phi[i] ;}}}
```

6 String

6.1 Aho

```
#include<bits/stdc++.h>
using namespace std;const int N = 1e5 + 9;//credit: Alpha_Q
struct AC { int N, P; const int A = 26; vector <vector <int
    >> next;
vector <int> link, out_link; vector <vector <int>> out;
AC(): N(0), P(0) {node();} int node() { next.emplace_back(A,
     0);
link.emplace_back(0); out_link.emplace_back(0);
out.emplace_back(0); return N++;} inline int get (char c) {
    return c - 'a';
} int add_pattern (const string T) {int u = 0; for (auto c :
     T) {
if (!next[u][get(c)]) next[u][get(c)] = node(); u = next[u][
    get(c)];}
out[u].push_back(P);return P++;}void compute() {queue <int>
    q;
for (q.push(0); !q.empty();) {int u = q.front(); q.pop();
for (int c = 0; c < A; ++c) { int v = next[u][c];
if (!v) next[u][c] = next[link[u]][c];else {link[v] = u ?
    next[link[u]][c] : 0;
out_link[v] = out[link[v]].empty() ? out_link[link[v]] :
    link[v];q.push(v);}}}}
int advance (int u, char c) { while (u && !next[u][get(c)])
    u = link[u];
u = next[u][get(c)]; return u;}};
int32_t main() { ios_base::sync_with_stdio(0);
cin.tie(0); auto st = clock(); int t, cs = 0; cin >> t;
while (t--) { int n; cin >> n; vector<string> v;
for (int i = 0; i < n; i++) { string s; cin >> s;
v.push_back(s); } sort(v.begin(), v.end());
v.erase(unique(v.begin(), v.end()), v.end());AC aho;
vector<int> len(n + 3, 0);for (auto s: v) {len[aho.
    add_pattern(s)] = s.size();}
aho.compute();string s; cin >> s;n = s.size();
vector<int> dp(n, n + 10);int u = 0;for (int i = 0; i < n; i
    ++) {
char c = s[i];u = aho.advance(u, c);for (int v = u; v; v =
    aho.out_link[v]) {
for (auto p : aho.out[v]) { dp[i] = min(dp[i], (i - len[p]
    >= 0 ? dp[i - len[p]] : 0) + 1);}}}
cout << "Case " << ++cs << ": ";if (dp[n - 1] == n + 10) {
    cout << "impossible\n";}
else { cout << dp[n - 1] << '\n';}}cout << 1.0 * (clock() -
    st) / 1000 << '\n';return 0;}
```

6.2 String matching using bitset

```
#include<bits/stdc++.h>
using namespace std;const int N = 1e5 + 9;
vector<int> v;bitset<N>bs[26], oc;
int main() {int i, j, k, n, q, l, r;string s, p;
cin >> s;for(i = 0; s[i]; i++) bs[s[i] - 'a'][i] = 1;
cin >> q;while(q--) {cin >> p;oc.set();
for(i = 0; p[i]; i++) oc &= (bs[p[i] - 'a'] >> i);
cout << oc.count() << endl; // number of occurences
int ans = N, sz = p.size();int pos = oc._Find_first();
v.push_back(pos);pos = oc._Find_next(pos);while(pos < N) {
v.push_back(pos);pos = oc._Find_next(pos);}
for(auto x : v) cout << x << ' '; // position of occurences
cout << endl;v.clear();cin >> l >> r; // number of
    occurences from l to r,where l and r is 1-indexed
if(sz > r - l + 1) cout << 0 << endl;else cout << (oc >> (l
    - 1)).count() - (oc >> (r - sz + 1)).count() << endl;
}return 0;}
```

6.3 Trie

```
struct node {bool endmark;node* next[26 + 1];node(){
endmark = false;for (int i = 0; i < 26; i++)next[i] = NULL;}
} * root;void insert(char* str, int len){node* curr = root;
for (int i = 0; i < len; i++) {int id = str[i] - 'a';
if (curr->next[id] == NULL)curr->next[id] = new node();
curr = curr->next[id];}curr->endmark = true;}
bool search(char* str, int len){node* curr = root;for (int i
     = 0; i < len; i++) {
int id = str[i] - 'a';if (curr->next[id] == NULL)return
    false;curr = curr->next[id];
}return curr->endmark;}void del(node* cur){for (int i = 0; i
     < 26; i++)if (cur->next[i])
del(cur->next[i]);delete (cur);}puts("ENTER NUMBER OF WORDS"
    );
root = new node();int num_word;cin >> num_word;for (int i =
    1; i <= num_word; i++) {
char str[50];scanf("%s", str);insert(str, strlen(str));}
puts("ENTER NUMBER OF QUERY");int query;cin >> query;
for (int i = 1; i <= query; i++) {char str[50];scanf("%s",
    str);if (search(str, strlen(str)))
puts("FOUND");elseputs("NOT FOUND");}del(root);return 0;
```

6.4 kmp

```
// returns the longest proper prefix array of pattern p
// where lps[i]=longest proper prefix which is also suffix
    of p[0...i]
vector<int> build_lps(string p) {int sz = p.size();vector<
    int> lps;
lps.assign(sz + 1, 0);int j = 0;lps[0] = 0;for (int i = 1; i
     < sz; i++) {
```

```
while (j >= 0 && p[i] != p[j]) {if (j >= 1) j = lps[j - 1];
    else j = -1;}
j++;lps[i] = j;}return lps;}vector<int>ans;
// returns matches in vector ans in 0-indexed
void kmp(vector<int> lps, string s, string p) {int psz = p.
    size(), sz = s.size();
int j = 0;for (int i = 0; i < sz; i++) {while (j >= 0 && p[j
    ] != s[i])
if (j >= 1) j = lps[j - 1];else j = -1;j++;if (j == psz) {
j = lps[j - 1];// pattern found in string s at position i-
    psz+1
ans.push_back(i - psz + 1);}
// after each loop we have j=longest common suffix of s[0..i
    ] which is also prefix of p}}
```

6.5 manacher

```
struct Manacher {vector<int> p[2];
// p[1][i] = (max odd length palindrome centered at i) / 2 [
    floor division]
// p[0][i] = same for even, it considers the right center
// e.g. for s = "abbabba", p[1][3] = 3, p[0][2] = 2
Manacher(string s) {int n = s.size();p[0].resize(n + 1);
p[1].resize(n);for (int z = 0; z < 2; z++) {
for (int i = 0, l = 0, r = 0; i < n; i++) {int t = r - i + !
    z;
if (i < r) p[z][i] = min(t, p[z][l + t]);int L = i - p[z][i
    ], R = i + p[z][i] - !z;
while (L >= 1 && R + 1 < n && s[L - 1] == s[R + 1])p[z][i
    ]++, L--, R++;
if (R > r) l = L, r = R;}}}bool is_palindrome(int l, int r)
    {
int mid = (l + r + 1) / 2, len = r - l + 1;return 2 * p[len
    % 2][mid] + len % 2 >= len;}};
```

6.6 palindrome$_{h}ashing$

```
#include <bits/stdc++.h>
using namespace std;
vector<vector<long long>> HASH, REV_HASH, POW;
vector<int> BASE = {1231, 1567}, MOD = {1000000000 + 7,
    1000000000 + 9};
#define lim 1000006 string text, pattern;void init(){
POW = vector<vector<long long>>(2, vector<long long>(lim));
POW[0][0] = POW[1][0] = 1;for (int b = 0; b < 2; b++)
for (int j = 1; j < lim; j++)POW[b][j] = (POW[b][j - 1] *
    BASE[b]) % MOD[b];return;
}void initHash(string str){int len = str.size();HASH[0][0] =
     HASH[1][0] = 0;
for (int b = 0; b < 2; b++)for (int i = 1; i <= len; i++)
HASH[b][i] = (HASH[b][i - 1] * BASE[b] + (str[i - 1] - 'a' +
     1)) % MOD[b];
REV_HASH[0][len + 1] = REV_HASH[1][len + 1] = 0;for (int b =
     0; b < 2; b++)
for (int i = len; i; i--)REV_HASH[b][i] = (REV_HASH[b][i +
    1] * BASE[b] + (str[i - 1] - 'a' + 1)) % MOD[b];
return;}long long getHash(int left, int right, int hsh){int
    len = (right - left + 1);
long long ret = (HASH[hsh][right] - HASH[hsh][left - 1] *
    POW[hsh][len]) % MOD[hsh];
if (ret < 0)ret += MOD[hsh];return ret;}
pair<long long, long long> getHash(int left, int right){long
     long hsh0 = getHash(left, right, 0);
long long hsh1 = getHash(left, right, 1);return {hsh0, hsh1
    };}
long long getRevHash(int left, int right, int hsh){int len =
     (right - left + 1);
long long ret = (REV_HASH[hsh][left] - REV_HASH[hsh][right +
     1] * POW[hsh][len]) % MOD[hsh];
if (ret < 0)ret += MOD[hsh];return ret;}pair<long long, long
     long> getRevHash(int left, int right){
long long hsh0 = getRevHash(left, right, 0);long long hsh1 =
     getRevHash(left, right, 1);
return {hsh0, hsh1};}bool palindrome(int l, int r){return
    getHash(l, r) == getRevHash(l, r);}
void solve(){string s = "aaabbabbaaac";HASH = vector<vector<
    long long>>(2, vector<long long>(s.size() + 5));
REV_HASH = vector<vector<long long>>(2, vector<long long>(s.
    size() + 5));initHash(s);
cout << (palindrome(1, s.size()) ? "YES\n" : "NO\n");cout <<
     (palindrome(1, s.size() - 1) ? "YES\n" : "NO\n");
return;}int32_t main(){ios_base::sync_with_stdio(0);cin.tie
    (0);init();solve();return 0;}
```

6.7 pallindromic tree

```
struct PalindromicTree {struct node {
int nxt[26], len, st, en, link, diff, slink, cnt, oc;};
string s;vector<node> t;int sz, last;PalindromicTree() {}
PalindromicTree(string _s) {s = _s;int n = s.size();t.clear
    ();
t.resize(n + 9);sz = 2, last = 2;t[1].len = -1, t[1].link =
    1;
t[2].len = 0, t[2].link = 1;t[1].diff = t[2].diff = 0;
t[1].slink = 1;t[2].slink = 2;}int extend(int pos) {
// returns 1 if it creates a new palindrome
int cur = last, curlen = 0;int ch = s[pos] - 'a';
while (1) {curlen = t[cur].len;
if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos])
    break;cur = t[cur].link;
}if (t[cur].nxt[ch]) {last = t[cur].nxt[ch];t[last].oc++;
    return 0;}
sz++;last = sz;t[sz].oc = 1;t[sz].len = t[cur].len + 2;t[cur
    ].nxt[ch] = sz;
t[sz].en = pos;t[sz].st = pos - t[sz].len + 1;
if (t[sz].len == 1) {t[sz].link = 2;t[sz].cnt = 1;
t[sz].diff = 1;t[sz].slink = 2;return 1;}
while (1) {cur = t[cur].link;curlen = t[cur].len;
if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos])
    {
t[sz].link = t[cur].nxt[ch];break;}}t[sz].cnt = 1 + t[t[sz].
    link].cnt;
t[sz].diff = t[sz].len - t[t[sz].link].len;if (t[sz].diff ==
     t[t[sz].link].diff) t[sz].slink = t[t[sz].link].slink;
else t[sz].slink = t[sz].link;return 1;}void
    calc_occurrences() {
for (int i = sz; i >= 3; i--) t[t[i].link].oc += t[i].oc;
}vector<array<int, 2>> minimum_partition() { //(even, odd),
    1 indexed
int n = s.size();vector<array<int, 2>> ans(n + 1, {0, 0}),
    series_ans(n + 5, {0, 0});
ans[0][1] = series_ans[2][1] = 1e9;for (int i = 1; i <= n; i
    ++) {
extend(i - 1);for (int k = 0; k < 2; k++) {ans[i][k] = 1e9;
for (int v = last; t[v].len > 0; v = t[v].slink) {series_ans
    [v][!k] = ans[i - (t[t[v].slink].len + t[v].diff)][!k];
if (t[v].diff == t[t[v].link].diff) series_ans[v][!k] = min(
    series_ans[v][!k], series_ans[t[v].link][!k]);
ans[i][k] = min(ans[i][k], series_ans[v][!k] + 1);}}}return
    ans;}} t;
int32_t main() {ios_base::sync_with_stdio(0);cin.tie(0);
    string s;cin >> s;PalindromicTree t(s);
for (int i = 0; i < s.size(); i++) t.extend(i);t.
    calc_occurrences();long long ans = 0;
for (int i = 3; i <= t.sz; i++) ans += t.t[i].oc;cout << ans
    << '\n';return 0;}
```

6.8 suffix array occurence of substr in own string

```
int Table[N][20], a[N];void Build(vector<int>lcp){
int n = lcp.size();for (int i = 1 ; i <= n ; i++)
Table[i][0] = lcp[i - 1];for (int k = 1 ; k < 20 ; k++){
for (int i = 1 ; i + (1 << k) - 1 <= n ; i++)
```

```
Table[i][k] = min(Table[i][k - 1], Table[i + (1 << (k - 1))
    ][k - 1]);}}
int Query(int l, int r){l++, r++;int k = log2(r - l + 1);
return min(Table[l][k], Table[r - (1 << k) + 1][k]);}
pair<int, int>FindRight(int low, int high, int val) // Find
    maximum R such that lcp(low, low+1...)>Val and return
    lcp(low, R)
{int l = low, r = high, mid;int ans = low - 1 ;while (l <= r
    ){
mid = (l + r) / 2;if (Query(low , mid) > val){ans = mid, l =
     mid + 1;}
else r = mid - 1;}if (ans == low - 1)return {low, -1};
else return {ans + 1, Query(low, ans)};}
void Solve(){string s;cin >> s;SuffixArray ehhe(s);
ll n = s.size();vector<int>p = ehhe.sa;vector<int>lcp;
lcp = ehhe.lcp;Build(lcp);ll ans = 0;
for (int i = 0 ; i < n ; i++){int high = n - 1;int pans = i
    ? lcp[i - 1] : 0;
int len = n - p[i];while (pans < len){
pair<int, int> pt = FindRight(i, high, pans); // pt = {
    maximum r such that lcp(i, r)>val, lcp(i, r)}
int right = pt.f;ll templ = right - i + 1;
if (pt.f == i) pt.s = len;ll contr = (pt.s - pans);
ans += (contr * (templ * (templ))); // len of contr occurs
    templ times
high = pt.f;pans = pt.s;}}cout << ans << endl;}
// Problem link : https://codeforces.com/contest/802/problem
    /I
```

6.9 z algo

```
// An element Z[i] of Z array stores length of the longest
    substring
// starting from str[i] which is also a prefix of str[0..n
    -1].
// The first entry of Z array is meaning less as complete
    string is always prefix of itself.
// Here Z[0]=0.
vector<int> z_function(string s) {int n = (int) s.length();
    vector<int> z(n);
for (int i = 1, l = 0, r = 0; i < n; ++i) {if (i <= r)z[i] =
     min (r - i + 1, z[i - l]);
while (i + z[i] < n && s[z[i]] == s[i + z[i]])++z[i];
if (i + z[i] - 1 > r)l = i, r = i + z[i] - 1;}return z;}
```

7 Tree

7.1 Articulation bridge

```
vector<int>node[10003];int lowtime[10003],intime[10003],vis
    [10003];
vector<pair<int,int>>edge;int timer;
void dfs(int p, int parent){intime[p]=lowtime[p]=timer;timer
    ++;
vis[p]=1;for(int child:node[p]){if(child==parent)continue;
if(vis[child]){lowtime[p]=min(lowtime[p], intime[child]); //
     node - child is a back edge
}else{// node - child is a forward edge
dfs(child,p);if(lowtime[child] > intime[p])edge.push_back({p
    , child});
lowtime[p] = min(lowtime[p] , lowtime[child]);}}}
```

7.2 Articulation point

```
vector<int>node[10003];int lowtime[10003],intime[10003],vis
    [10003];
set<int>cut_vertex; int timer;
void IS_CUTPOINT(int x){cut_vertex.insert(x);}
void dfs(int p, int parent){intime[p]=lowtime[p]=timer;
timer++;int children=0;vis[p]=1;for(int child:node[p]){
if(child==parent) continue;if(vis[child]){
lowtime[p]=min(lowtime[p], intime[child]); // node - child
    is a back edge
}else{// node - child is a forward edge
dfs(child,p);if(lowtime[child] >= intime[p] && parent!=-1)
IS_CUTPOINT(p);lowtime[p] = min(lowtime[p] , lowtime[child])
    ;
children++;}}if(parent == -1 && children>1)//for root
IS_CUTPOINT(p);}
```

7.3 Dijkstra

```
priority_queue< pair<ll, ll>, vector<pair<ll , ll> >,
    greater<pair<ll , ll > > > pq;
int parrent[100003];void dijsktra(ll p){parrent[1] = 1;pq.
    push({0, p});
dis[p] = 0;ans.push_back(p);while (!pq.empty()){
ll curr_node = pq.top().second;ll curr_dis = pq.top().first;
pq.pop();for ( pair<ll , ll> child : node[curr_node]){
if (child.second + curr_dis < dis[child.first]){
parrent[child.first] = curr_node;dis[child.first] = child.
    second + curr_dis;
pq.push({dis[child.first], child.first});}}}}
```

7.4 Eular tour

```
vector<int>node[N];int Intime[N], Outtime[N], Level[N] , a[N
    ];
int timer = 1;int n, q;void EulerTour(int p, int par, int d)
    {
Intime[p] = timer++; Level[p] = d; for (auto i : node[p]){if
     (i == par)
continue;EulerTour(i, p, Level[p] + 1);} Outtime[p] = timer;
//if timer++ then intime[u] to intime[v] can be find path
    query
}struct BIT {} T[2];void Solve(){ cin >> n >> q;
for (int i = 1 ; i <= n ; i++) cin >> a[i];
for (int i = 1 ; i < n ; i++){ int u, v; cin >> u >> v;
node[u].push_back(v); node[v].push_back(u);}EulerTour(1, 0,
    0);
for (int i = 1 ; i <= n ; i++){T[Level[i] % 2].upd(Intime[i
    ], Intime[i], a[i]);
T[!(Level[i] % 2)].upd(Intime[i], Intime[i], 0);}while (q--)
    { int type;
cin >> type; if (type == 1){ ll x, val; cin >> x >> val;
T[Level[x] % 2].upd(Intime[x], Outtime[x]-1, val);
T[!(Level[x] % 2)].upd(Intime[x], Outtime[x]-1, -val);} else
     {ll x;
cin >> x; cout << T[Level[x] % 2].query(Intime[x], Intime[x
    ]) << endl;}}}
```

7.5 Floyd Warshall

```
ll vis[504][504];void warshall(ll n){
for (int k = 1; k <= n; k++){for (int i = 1; i <= n; i++)
{for (int j = 1; j <= n; j++){
dis[i][j] = min(dis[i][j], dis[i][k] + dis[k][j]);}}}}
```

7.6 HLD(update on edge)

```
ll n, q;vector<ll>node[N];ll a[N];
struct SegmentTree { vector<ll> tree;
vector<ll> lazy; vector<ll> aa; SegmentTree() {
tree.resize(4 * N); lazy.resize(4 * N); aa.resize(4 * N);}
void build(ll node, ll b, ll e) { if (b == e) {
tree[node] = 0; lazy[node] = -1;return;}
ll mid = (b + e) >> 1; build(2 * node, b, mid);
```

```
build(2 * node + 1, mid + 1, e); lazy[node] = -1;
tree[node] = tree[2 * node] + tree[2 * node + 1];}
void push(ll node, ll b, ll e) { tree[node] = (e - b + 1) *
    lazy[node];
if (b != e) { lazy[2 * node] = lazy[2 * node + 1] = lazy[
    node];}
lazy[node] = -1;} void update(ll node, ll b, ll e, ll l, ll
    r, ll x) {
if (lazy[node] != -1) push(node, b, e); if (l > e || r < b)
    return;
if (l <= b && r >= e) { tree[node] = (e - b + 1) * x;
if (b != e) { lazy[2 * node] = lazy[2 * node + 1] = x; }
lazy[node] = -1;return;}ll mid = (b + e) >> 1;
update(2 * node, b, mid, l, r, x); update(2 * node + 1, mid
    + 1, e, l, r, x);
tree[node] = tree[2 * node] + tree[2 * node + 1];}
ll query(ll node, ll b, ll e, ll l, ll r) { if (lazy[node]
    != -1)
push(node, b, e); if (l > e || r < b) return 0; if (l <= b
    && r >= e) {return tree[node];}ll mid = (b + e) >> 1;
   return query(2 * node, b, mid, l, r) + query(2 * node +
       1, mid + 1, e, l, r);
  }
};SegmentTree st;ll par[N][LG + 1], dep[N], sz[N];void dfs(
    int u, int p = 0){
par[u][0] = p;dep[u] = dep[p] + 1;sz[u] = 1;for (int i = 1;
    i <= LG; i++){
par[u][i] = par[par[u][i - 1]][i - 1];}for (auto v : node[u
    ]){
if (v == p) continue;dfs(v, u);sz[u] += sz[v];}}
int lca(int u, int v){
  // ache already}
int intime[N], head[N];int timer = 1;map<ll, ll>alledge[N];
void decompose(int p, int parent, int Head_node){intime[p] =
     timer++;
head[p] = Head_node;st.update(1, 1, n, intime[p], intime[p],
     alledge[parent][p]);
int heavysize = -1, heavychild = -1;for (auto i : node[p]){
    if (i != parent){
if (sz[i] > heavysize)heavysize = sz[i], heavychild = i;}}if
    (heavychild == -1)
return ; decompose(heavychild, p, Head_node);for (auto i :
    node[p]){
if (i == heavychild || i == parent)continue;decompose(i, p,
    i);}}
ll sumpath(int u, int v){ll ans = 0;
//cout << "here " << u << " " << v << endl;
if (u == v) return 0; while (head[u] != head[v]){ if (dep[
    head[u]] > dep[head[v]])
swap(u, v); ans += st.query(1, 1, n, intime[head[v]], intime
    [v]);v = par[head[v]][0];}
if (dep[u] > dep[v])swap(u, v);if (u != v)ans += st.query(1,
     1, n, intime[u] + 1, intime[v]);
return ans;}void reset(int n){for (int i = 0 ; i <= n ; i++)
    {
intime[i] = head[i] = dep[i] = sz[i] = 0;a[i] = 0;node[i].
    clear();
timer = 1;alledge[i].clear();}}void Solve(){cin >> n;//reset
    (n);
st.build(1, 1, n);vector<pair<int, int> > edge;for (int i =
    1 ; i < n ; i++){
ll u, v, w;cin >> u >> v >> w;alledge[u][v] = w;alledge[v][u
    ] = w;node[u].push_back(v);
node[v].push_back(u);edge.push_back({u, v});}dfs(1);
    decompose(1, 0, 1);int q;
cin >> q;while (q--){int type;cin >> type;if (type == 1){
ll id, x;cin >> id >> x;id--;int p = edge[id].f;int q = edge
    [id].s;
if (dep[p] > dep[q]) swap(p, q);st.update(1, 1, n, intime[q
    ], intime[q], x);
alledge[p][q] = x;alledge[q][p] = x}else{int u, v;cin >> u
    >> v;
int l = lca(u, v);if (u == v){cout << 0 << endl;continue;}
if (l == u || l == v){if (dep[u] > dep[v]) swap(u, v); cout
    << sumpath(u, v) << endl;}
else{cout << sumpath(1, u) + sumpath(1, v) << endl;}}}}
```

7.7 HLD(update on node)

```
int n,q;vector<int>node[N];int a[N];
struct SegmentTree {
   vector<int>tree;vector<int>lazy;vector<int>aa;
   SegmentTree() {tree.resize(4*N);lazy.resize(4*N);aa.
       resize(4*N);}
   void build(int node,int b,int e) {if(b==e){tree[node]=0;
       lazy[node]=-1;return;}int mid=(b+e)>>1;build(2*node,
       b,mid);build(2*node+1,mid+1,e);lazy[node]=-1;tree[
       node]=max(tree[2*node],tree[2*node+1]);}
   void push(int node,int b,int e) {tree[node]=lazy[node];if
       (b!=e){lazy[2*node]=lazy[2*node+1]=lazy[node];}lazy[
       node]=-1;}
   void update(int node,int b,int e,int l,int r,int x) {if(
       lazy[node]!=-1)push(node,b,e);if(l>e||r<b)return;if(
       l<=b&&r>=e){tree[node]=x;if(b!=e){lazy[2*node]=lazy
       [2*node+1]=x;}lazy[node]=-1;return;}int mid=(b+e)
       >>1;update(2*node,b,mid,l,r,x);update(2*node+1,mid
       +1,e,l,r,x);tree[node]=max(tree[2*node],tree[2*node
       +1]);}
   int query(int node,int b,int e,int l,int r) {if(lazy[node
       ]!=-1)push(node,b,e);if(l>e||r<b)return 0;if(l<=b&&r
       >=e){return tree[node];}int mid=(b+e)>>1;return max(
       query(2*node,b,mid,l,r),query(2*node+1,mid+1,e,l,r))
       ;}
};
SegmentTree st;int par[N][LG+1],dep[N],sz[N];
void dfs(int u,int p=0){par[u][0]=p;dep[u]=dep[p]+1;sz[u]=1;
    for(int i=1;i<=LG;i++){par[u][i]=par[par[u][i-1]][i
    -1];}for(auto v:node[u]){if(v==p)continue;dfs(v,u);sz[u
    ]+=sz[v];}}
int lca(int u,int v){if(dep[u]<dep[v])swap(u,v);for(int k=LG
    ;k>=0;k--)if(dep[par[u][k]]>=dep[v])u=par[u][k];if(u==v
    )return u;for(int k=LG;k>=0;k--)if(par[u][k]!=par[v][k
    ])u=par[u][k],v=par[v][k];return par[u][0];}
int intime[N],head[N];int timer=1;
void decompose(int p,int parent,int Head_node){intime[p]=
    timer++;head[p]=Head_node;st.update(1,1,n,intime[p],
    intime[p],a[p]);int heavysize=-1,heavychild=-1;for(auto
     i:node[p]){if(i!=parent){if(sz[i]>heavysize)heavysize=
    sz[i],heavychild=i;}}if(heavychild==-1)return;decompose
    (heavychild,p,Head_node);for(auto i:node[p]){if(i==
    heavychild||i==parent)continue;decompose(i,p,i);}}
int maxnode(int u,int v){int ans=0;while(head[u]!=head[v]){
    if(dep[head[u]]>dep[head[v]])swap(u,v);ans=max(ans,st.
    query(1,1,n,intime[head[v]],intime[v]));v=par[head[v
    ]][0];}if(dep[u]>dep[v])swap(u,v);ans=max(ans,st.query
    (1,1,n,intime[u],intime[v]));return ans;}
void Solve(){cin>>n>>q;for(int i=1;i<=n;i++){cin>>a[i];}st.
    build(1,1,n);for(int i=1;i<n;i++){int u,v;cin>>u>>v;
    node[u].push_back(v);node[v].push_back(u);}dfs(1);
    decompose(1,0,1);while(q--){int type;cin>>type;if(type
    ==1){int u,x;cin>>u>>x;st.update(1,1,n,intime[u],intime
    [u],x);}else{int u,v;cin>>u>>v;int l=lca(u,v);cout<<max
    (maxnode(l,u),maxnode(l,v))<<" ";}}}
```

7.8 Inverse Graph

```
void bfs(int u){queue<int>que ;que.push(u) ;wh_cmpnnt[u] =
    cmpnnt ;while(!que.empty()){auto u = que.front() ;que.
    pop();vector<int>restricted;for(auto v: graph[u]){if(!
    wh_cmpnnt[v]){restricted.push_back(v);adj[v] = 1 ;}}for
    (auto v: not_visited){if(v == u || adj[v]) continue ;
    que.push(v) ;wh_cmpnnt[v] = cmpnnt ;}for(auto v:
    restricted)adj[v] = 0 ;not_visited = restricted ;}}
```

7.9 LCA

```
const int N = 3e5 + 9, LG = 18;vector<int> g[N];int par[N][
    LG + 1], dep[N], sz[N];
void dfs(int u, int p = 0) {par[u][0] = p;dep[u] = dep[p] +
    1;sz[u] = 1;
for (int i = 1; i <= LG; i++) par[u][i] = par[par[u][i -
    1]][i - 1];
for (auto v: g[u]) if (v != p) {dfs(v, u);sz[u] += sz[v];}}
int lca(int u, int v) {
if (dep[u] < dep[v]) swap(u, v);
for (int k = LG; k >= 0; k--) if (dep[par[u][k]] >= dep[v])
    u = par[u][k];
if (u == v) return u;
for (int k = LG; k >= 0; k--) if (par[u][k] != par[v][k]) u
    = par[u][k], v = par[v][k];return par[u][0];}
int kth(int u, int k) {assert(k >= 0);
for (int i = 0; i <= LG; i++) if (k & (1 << i)) u = par[u][i
    ];return u;}
int dist(int u, int v) {
int l = lca(u, v);return dep[u] + dep[v] - (dep[l] << 1);}
//kth node from u to v, 0th node is u
int go(int u, int v, int k) {
int l = lca(u, v);int d = dep[u] + dep[v] - (dep[l] << 1);
    assert(k <= d);if (dep[l] + k <= dep[u]) return kth(u,
    k);k -= dep[u] - dep[l];return kth(v, dep[v] - dep[l] -
    k);}
```

```
int32_t main() {int n; cin >> n;for (int i = 1; i < n; i++)
    {int u, v; cin >> u >> v;g[u].push_back(v);g[v].
    push_back(u);}dfs(1);int q; cin >> q;while (q--) {int u
    , v; cin >> u >> v;cout << dist(u, v) << '\n';}return
    0;}
```

7.10 dfs$_tree$

```
void dfs(int u, int p){dp[u] = 0 ;for(auto v: graph[u]){if(v
    == p)continue ;if(level[v] == 0){level[v] = level[u] +
    1 ;vp.push_back({u, v}) ;dfs(v, u) ;dp[u] += dp[v] ;}
    else if(level[v] < level[u]){vp.push_back({u, v}) ;dp[u
    ]++ ;}else{dp[u]-- ;}}if(level[u] > 1 && dp[u] == 0)
    bridges++ ;}
```

7.11 strongly connected components

```
vector<int>node[10000], transpose_node[10000];
int visit[10000], out_time[10000], in_time[10000];
vector<int>order;//by out_time we can sort by out_time but
    stack/vector reduce complexity
vector<int>SCC;int timer;void dfs(int x) {
visit[x] = 1;in_time[x] = ++timer;for (auto i : node[x])
if (!visit[i]) dfs(i); out_time[x] = ++timer;
order.push_back(x); // all the child of this node (X)
    already visited
}void dfs_for_scc(int x) //This dfs for find scc
{visit[x] = 1;SCC.push_back(x);for (auto child :
    transpose_node[x])
if (!visit[child]) dfs_for_scc(child);}while (m--){
cin >> a >> b;node[a].push_back(b);transpose_node[b].
    push_back(a);
//to find scc we need to run dfs is transpose graph of main
    graph
}for (int i = 0 ; i <= n ; i++)if (visit[i] == 0) dfs(i);
for (int i = 0 ; i <= n ; i++) visit[i] = 0;
cout << "here is node list by order of out time\n";
for (int i = n - 1 ; i >= 0 ; i--)
cout << order[i] << " out time is -> " << out_time[order[i]]
     << endl;
for (int i = n - 1 ; i >= 0 ; i--) {if (visit[order[i]] ==
    0) //order[i] is by largest outime
{SCC.clear(); //previous SCC cleard
dfs_for_scc(order[i]); // for finding scc we run dfs in
    transpose graph
cout << "Strongly Connected Components are \n";for (auto
    child : SCC)
cout << child << " ";cout << endl;}}
```