## Team Notebook

## $SUST\_LateForSubmission$

## May 9, 2025

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## 1 DP

#### 1.1 CHT

```
lli inf = LLONG MIN:
//add lines with -m and -b and return -ans to
//make this code work for minimums.(not -x)
struct line {
lli m, b; mutable function<const line*() > succ; bool
     operator < (const line& rhs) const {
 if (rhs.b != inf) return m < rhs.m; const line* s = succ()</pre>
 if (!s) return 0; lli x = rhs.m; return b - s->b < (s->m -
        m) * x;} };
struct CHT : public multiset<line> {
 bool bad(iterator y) { auto z = next(y);
 if (v == begin()) { if (z == end()) return 0: return v ->
       m == z \rightarrow m \&\& v \rightarrow b <= z \rightarrow b;
 auto x = prev(y);
 if (z == end()) return y -> m == x -> m && y -> b <= x ->
 return 1.0 * (x \rightarrow b - y \rightarrow b) * (z \rightarrow m - y \rightarrow m) >= 1.0
       * (y \rightarrow b - z \rightarrow b) * (y \rightarrow m - x \rightarrow m);
 void add(lli m, lli b) {
 auto y = insert({ m, b }); y->succ = [ = ] { return next(y
       ) == end() ? 0 : &*next(v): }:
 if (bad(y)) { erase(y); return; }
 while (next(v) != end() && bad(next(v))) erase(next(v)):
       while (y != begin() && bad(prev(y))) erase(prev(y)); }
 lli query(lli x) { assert(!empty());
    auto 1 = *lower bound((line) { x, inf }):
    return 1.m * x + 1.b;}
};
```

## 1.2 Grundy

```
int Grundy(int n){ if (n <= 2) return 0;
if (dp[n] != -1)return dp[n]; vector<int>vis(1005, 0);
for (int i = 1; i < n; i++){ if (n - i != i)
{vis[(Grundy(n - i)^Grundy(i))] = 1;}}int p = 0;
while(vis[p]) p++; return dp[n] = p;}//pile divided into two
unequal pile</pre>
```

## 1.3 Knurth Optimization

## 1.4 LIS

## 1.5 digit dp optimize(1 memset)

#### 1.6 digit dp

```
const int N=20;int a[N];
11 dp[N][11][2][2]; // digit dp te amar number generate hoy
    emne: 0, 01, 02, 03, 04, 05.....066,0667
11 getsum(int pos, int dig, int n, bool ok, bool other){if(
    pos>n){return 1;}
11 &R = dp[pos][dig][ok][other];if(R!=-1)return R;int
    maxdigit = 9;if(!ok)
maxdigit = a[pos]:ll res=0:for(int i=0 : i<=maxdigit : i++){</pre>
    if(dig == i && other)
continue:if(i>0)other=1:if(i<maxdigit || ok)res+=getsum(pos</pre>
    +1, i, n, true, other);else
res+=getsum(pos+1, i, n, false, other);}return R = res;}void
     Solve(){string 1, r:
cin>>l>>r;int n = r.size();r='*'+r;for(int i=0; i<=n; i++)</pre>
    {a[i]=r[i]-'0':}
memset(dp, -1, sizeof(dp)); ll sumr = getsum(1, -1, n, 0, 0)
```

## 1.7 divide and conquer

```
mt19937 mt_rand(chrono::high_resolution_clock::now().
    time since epoch().count());
11 Left = 1, Right = 0;11 cost(11 1, 11 r) {while (Right < r</pre>
    )Add(++Right);
while (Left > 1)Add(--Left):while (Left < 1)Remove(Left++):
     while (Right > r)
Remove(Right--); return Totsum; } 11 dp[2][N];
void compute(int group, int 1, int r, int optl, int optr) {
     if (1 > r) return:
int mid = (1 + r) / 2;dp[group & 1][mid] = LLONG_MAX;int
     optnow = optl;
for (int k = optl ; k <= min(mid, optr) ; k++) {ll ret = dp</pre>
     \lceil ! (group \& 1) \rceil \lceil k \rceil + cost(k + 1, mid) :
if (ret < dp[group & 1][mid]) {dp[group & 1][mid] = ret;</pre>
     optnow = k;}}
compute(group, 1, mid - 1, optl, optnow);compute(group, mid
     + 1, r, optnow, optr);}
```

```
void Solve() {cin >> n >> k;for (int i = 1 ; i <= n ; i++) {
    cin >> a[i];}
for (int i = 1 ; i <= n ; i++) {dp[1 & 1][i] = cost(1, i);}
    for (int i = 2 ; i <= k ; i++) {
    compute(i, 1, n, 1, n);}cout << dp[k & 1][n] << endl;}</pre>
```

void SOS DP(){for(int i = 0: i < (1 << N): ++i)F[i] = A[i]:

## $1.8 \quad \cos dp$

```
for(int i = 0;i < N; ++i)for(int mask = 0; mask < (1<<N); ++</pre>
    mask){
if(mask & (1<<i))F[mask] += F[mask^(1<<i)];}}</pre>
const 11 MLOG = 20; const 11 MAXN = (1 << MLOG); 11 dp[sz +</pre>
    10], fre[sz + 10], mp[sz + 10];
// forward1: Propagates values from subsets to their
void forward1() { for (int bit = 0; bit < MLOG; ++bit) { for</pre>
      (int i = 0; i < MAXN; ++i) { if (i & (1 << bit)) { dp[
     i] += dp[i ^ (1 << bit)]; } } }
// backward1: Reverses the effect of forward1 by removing
    contributions from supersets. This is used when dp[i]
     contains info about all subsets of i, and we want to
    isolate the info for only i.
void backward1() { for (int bit = 0: bit < MLOG: ++bit) {</pre>
    for (int i = MAXN - 1; i >= 0; --i) { if (i & (1 << bit
    )) { dp[i] -= dp[i ^ (1 << bit)]; } } }
// forward2: Propagates values from supersets to their
     subsets
void forward2() { for (int bit = 0; bit < MLOG; ++bit) { for</pre>
     (int i = MAXN - 1; i >= 0; --i) { if (i & (1 << bit))}
    { dp[i ^ (1 << bit)] += dp[i]; } } }
// backward2: Reverses the effect of forward2 by removing
    contributions from subsets. This is used when dp[i]
     contains info about all supersets of i. and we want to
     isolate the info for only i.
void backward2() { for (int bit = 0; bit < MLOG; ++bit) {</pre>
     for (int i = 0; i < MAXN; ++i) { if (i & (1 << bit)) {</pre>
    dp[i ^ (1 << bit)] -= dp[i]; } } }</pre>
memset(dp, 0, sizeof(dp)); memset(fre, 0, sizeof(fre));
    memset(mp, 0, sizeof(mp));
```

## 2 DataStructure

## 2.1 BIT range update and query

```
const int N = 3e5 + 9;
struct BIT {
  long long M[N], A[N];
BIT() {memset(M, 0, sizeof M);memset(A, 0, sizeof A);}
  void update(int i, long long mul, long long add) {
    while (i < N) {M[i] += mul;A[i] += add;i |= (i + 1);}}
  void upd(int l, int r, long long x) {
    update(1, x, -x * (1 - 1));update(r, -x, x * r);}
  long long query(int i) {
    long long mul = 0, add = 0;int st = i;
    while (i >= 0) {mul += M[i];add += A[i];i = (i & (i + 1)) - 1;}
    return (mul * st + add);}
  long long query(int l, int r) {
    return query(r) - query(1 - 1);} t;
```

## 2.2 BIT2D

```
#include<bits/stdc++.h>using namespace std;const int N =
    1010:
struct BIT2D { long long M[N][N][2], A[N][N][2];BIT2D() {
memset(M, 0, sizeof M); memset(A, 0, sizeof A);}
void upd2(long long t[N][N][2], int x, int y, long long mul,
     long long add) {
for(int i = x; i < N; i += i & -i) { for(int j = y; j < N; j
      += i & -i) {
t[i][j][0] += mul;t[i][j][1] += add;}}}
void upd1(int x, int y1, int y2, long long mul, long long
mul * v2):
upd2(A, x, y1, add, -add * (y1 - 1)); upd2(A, x, y2, -add,
    add * v2):}
void upd(int x1, int y1, int x2, int y2, long long val) {
upd1(x1, y1, y2, val, -val * (x1 - 1)); upd1(x2, y1, y2, -val)
     val * x2):
long long query2(long long t[N][N][2], int x, int y) { long
    long mul = 0, add = 0:
for(int i = y; i > 0; i -= i & -i) { mul += t[x][i][0];add
    += t[x][i][1]:}
return mul * y + add;}long long query1(int x, int y) {long
    long mul = 0, add = 0;
for(int i = x; i > 0; i -= i & -i) { mul += query2(M, i, y);
     add += query2(A, i, y);}
return mul * x + add; } long long query(int x1, int y1, int
    x2, int y2) {
return query1(x2, y2) - query1(x1 - 1, y2) - query1(x2, y1 -
     1) + query1(x1 - 1, y1 - 1);}
```

## 2.3 Centroid decomposition

```
#include<bits/stdc++.h>
using namespace std;
const int N = 1e5 + 9;vector<int> g[N];
int sz[N]:int tot, done[N], cenpar[N]:
void calc sz(int u, int p) {tot ++;
sz[u] = 1; for (auto v : g[u]) {if(v == p || done[v])}
    continue:
calc_sz(v, u); sz[u] += sz[v];
}int find_cen(int u, int p) {for (auto v : g[u]) {
if(v == p || done[v]) continue; else if(sz[v] > tot / 2)
    return find_cen(v, u);
}return u;}void decompose(int u, int pre) {
tot = 0:calc sz(u, pre):int cen = find cen(u, pre):cenpar[
done[cen] = 1;for(auto v : g[cen]) {if(v == pre || done[v])
    continue:
decompose(v, cen);}}int dep[N];void dfs(int u, int p = 0) {
for(auto v : g[u]) {if(v == p) continue:dep[v] = dep[u] + 1:
dfs(v, u);}}int main() {ios_base::sync_with_stdio(0);
cin.tie(0); int n; cin >> n; for(int i = 1; i < n; i++) {
int u, v;cin >> u >> v;g[u].push_back(v);g[v].push_back(u);}
decompose(1, 0); for(int i = 1; i \le n; i++) g[i].clear(); int
for(int i = 1; i <= n; i++) {g[cenpar[i]].push_back(i);</pre>
g[i].push_back(cenpar[i]); if (cenpar[i] == 0) root = i;}
dfs(root):for(int i = 1: i \le n: i++) cout << char(dep[i] +
     'A') << ' ';return 0;}
```

#### 2.4 DSU on tree

```
const int N = 1e5 + 9;
```

```
vector<int> g[N]:
int ans[N], col[N], sz[N], cnt[N];
bool big[N];
void dfs(int u, int p) {sz[u] = 1;
 for (auto v : g[u]) {if (v == p) continue;dfs(v, u);sz[u]
      += sz[v]:}}
void add(int u, int p, int x) {cnt[col[u]] += x;
 for (auto v : g[u]) {if (v == p \mid \mid big[v] == 1) continue;
      add(v. u. x):}}
void dsu(int u, int p, bool keep) {
int bigchild = -1, mx = -1;
for (auto v : g[u]) {if (v == p) continue:if (sz[v] > mx) mx
     = sz[v], bigchild = v;}
for (auto v : g[u]) {if (v == p || v == bigchild) continue;
    dsu(v. u. 0):}
if (bigchild != -1) dsu(bigchild, u, 1), big[bigchild] = 1;
    add(u, p, 1):ans[u] = cnt[u]:
if (bigchild != -1) big[bigchild] = 0; if (keep == 0) add(u,
    p. -1):}
```

#### 2.5 DSU

```
int n,m;
int parent[MAX],Rank[MAX];
void Init(int n){for(int i=0 ;i<=n ;i++)Rank[i]=1,parent[i]=
    i;}
int Find_parent(int v) {if (v == parent[v]) {return v;}
    return parent[v] = Find_parent(parent[v]);}
void Union(int a, int b) {
a = Find_parent(a);b = Find_parent(b);if (a != b) {if (Rank[a] > Rank[b]) {swap (a, b);}parent[a] = b;Rank[b] +=
    Rank[a];}}
```

#### 2.6 GP hash table

```
time_since_epoch().count();return splitmix64(x +
   FIXED_RANDOM);}};
gp_hash_table<int, int, custom_hash> mp;
```

# 2.7 Mo on tree (number of distinct in a path) if (type == 1)operation(starttime[lc]);} for (int i = 1; i <= q; i++) {cout << a}

```
const int mod = 1e9 + 7, LG = 18;const int N = 2e5 + 6;const
      int BLOCK SIZE = 450:int a[N]:
vector<int>node[N];int starttime[N], endtime[N];int ft[N];
     int par[N][LG + 1], dep[N], sz[N];int timer = 1;
void dfs(int u, int p = 0){
ft[timer] = u;starttime[u] = timer++;par[u][0] = p;dep[u] =
     dep[p] + 1:sz[u] = 1:
for (int i = 1; i <= LG; i++){par[u][i] = par[par[u][i -</pre>
     1]][i - 1]:}
for (auto v : node[u]){if (v == p) continue;dfs(v, u);sz[u]
     += sz[v];}ft[timer] = u;endtime[u] = timer++;}
int lca(int u, int v){ // ache already}
int freq[N];int colour[N];int res;
void operation(int id){
int curnode = ft[id];int c = a[curnode];
if (freg[curnode] == 0){colour[c]++:
if (colour[c] == 1)res++;}
else{colour[c]--:if (colour[c] == 0)res--:}freq[curnode] ^=
     1:}
void Solve(){
int n, q;
while (cin >> n >> q){
set<11>st;map<11 , 11>m;
for (int i = 1 : i \le n : i++)\{cin >> a[i]:\}int tot = 0:
for (int i = 1; i \le n; i++){if (m[a[i]])a[i] = m[a[i]];
     else{m[a[i]] = ++tot:a[i] = m[a[i]]:}}
for (int i = 1; i < n; i++){int u, v; cin >> u >> v; node[u]
     ].push_back(v);node[v].push_back(u);}
dfs(1);ll ans[q + 1];Query queries[q];
for (int i = 0; i < q; i++){int u, v, c; cin >> u >> v; int
     lc = lca(u, v):
if (dep[u] > dep[v])swap(u, v):
if (lc == u || lc == v)queries[i] = {starttime[u], starttime
     [v], i + 1, 1, lc, -1};
else queries[i] = {endtime[u], starttime[v], i + 1, 1, 1c,
sort(queries, queries + q):
int Left = 1, Right = 0;
for (auto i : queries){
int l = i.l;int r = i.r;int id = i.idx;int c = i.c;int type
     = i.type;int lc = i.lc;
```

```
while (Right < r)operation(++Right);
while (Left > 1)operation(--Left);
while (Left < 1)operation(Left++);
while (Right > r)operation(Right--);
if (type == 1){operation(starttime[lc]);}
ans[id] = res;
if (type == 1)operation(starttime[lc]);}
for (int i = 1; i <= q; i++) {cout << ans[i] << endl;}}}</pre>
```

#### 2.8 Mo's

```
const int mod = 1e9 + 7; const int N = 5e5 + 6; const int
    BLOCK SIZE = 500:
struct Query {
int 1, r, idx, lc, type;
bool operator<(const Query &y) const {</pre>
// Current query x is being compared with other query y
int x_block = 1 / BLOCK_SIZE; int y_block = y.1 / BLOCK_SIZE;
// If x and v both lie in the same block, sort in non
    decreasing order of endpoint
if (x_block == y_block)return r < y.r;</pre>
// x and y lie in different blocks
return x_block < y_block;}};</pre>
11 nc3(11 x){if (x < 3)return 0:return (x * (x - 1) * (x -
    2)) / 6;}
int a[N]; ll last[N]; ll freq[N]; ll res;
void Add(int i){int x = a[i];res -= last[x];freq[x]++;last[x
    ] = nc3(freq[x]);res += last[x];}
void Remove(int i){int x = a[i];res -= last[x];freq[x]--;
    last[x] = nc3(freq[x]);res += last[x];}
void Solve(){
int n, q; cin >> n >> q;
for (int i = 1; i \le n; i ++){cin >> a[i];}
vector<Query>queries;ll ans[q + 1];
for (int i = 1; i \le q; i++){int 1, r;cin >> 1 >> r;
    queries.push_back({1, r, i});}
sort(queries.begin(), queries.end());
int Left = 1, Right = 0;
for (auto i : queries){int l = i.l;int r = i.r;int id = i.
while (Right < r)Add(++Right);</pre>
while (Left > 1)Add(--Left):
while (Left < 1)Remove(Left++);</pre>
while (Right > r)Remove(Right--);
ans[i.idx] = res:}
for (int i = 1 ; i <= q ; i++)cout << ans[i] << endl;}</pre>
//number of triple(l, r) a[i] = a[i] = a[k]
```

## 2.9 Persistent Segment Tree

```
#include<bits/stdc++.h>using namespace std;
struct nd{long long sum;nd *left;nd *right;nd(long long data
    ){sum=data:}
nd(nd l.nd r){sum=l.sum+r.sum:left=&l:right=&r:}}:
int n;vector<nd>states;
nd build(int start.int end){
if(start==end)return nd(0):int mid=(start+end)/2:return nd(
    build(start,mid),build(mid+1,end));}
nd update(nd root,int start,int end,int pos,int val){if(
    start==end)return nd(val);
int mid=(start+end)/2:return pos<=mid?nd(update(*root.left.</pre>
     start,mid,pos,val),*root.right):nd(*root.left,update(*
    root.right,mid+1,end,pos,val));}
void solve(){cin>>n;states.push_back(build(0,n-1));states.
    push_back(update(states.back(),0,n-1,4,3));return;}
int main(){ios_base::sync_with_stdio(0);cin.tie(0);solve();
    return 0:}
```

## 2.10 Sparse table

```
int Table[N][22], a[N];
void Build(int n){
for (int i = 1 ; i <= n ; i++)Table[i][0] = a[i];
for (int k = 1 ; k < 22 ; k++){
for (int i = 1 ; i + (1 << k) - 1 <= n ; i++)Table[i][k] =
    min(Table[i][k - 1], Table[i + (1 << (k - 1))][k - 1])
    ;}}
int Query(int l, int r){int k = log2(r - l + 1);return min(
    Table[l][k], Table[r - (1 << k) + 1][k]);}</pre>
```

## 2.11 $next_s maller previous somaller$

```
11 Next_smaller[N + 2];1l Prev_smaller[N + 2];
void NEXTSMALLER(){stack<int>st;
for (int i = 1; i <= n; i++){if (st.empty()){st.push(i);}}
else{while (!st.empty() && a[st.top()] > a[i]){Next_smaller[st.top()] = i;st.pop();}st.push(i);}}
while (!st.empty()){Next_smaller[st.top()] = n + 1;st.pop();}}
void PREVSMALLER(){stack<int>st;
for (int i = n; i >= 1; i--){if (st.empty()){st.push(i);}}
else{while (!st.empty() && a[st.top()] > a[i]){Prev_smaller[st.top()] = i;st.pop();}st.push(i);}}
while (!st.empty()){Prev_smaller[st.top()] = 0;st.pop();}}
//priority_queue<int,vector<int>, greater<int> >pq;
```

#### 2.12 ordered set

```
#include<ext/pb_ds/assoc_container.hpp>#include<ext/pb_ds/</pre>
    tree_policy.hpp>using namespace std;using namespace
    __gnu_pbds:
template < class T > using ordered set =
tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>; // find_by_order,
    order of kev
//1 2 2 3 3 7
//greater->>descending order => 7 3 2 1
//less ->>ascending order => 1 2 3 7
//less_equal -> ascending but in duplicate value => 1 2 2 3
    3 7 so this will work as multiset
// finding kth element - 4th query
cout << "Oth element: " << *A.find_by_order(0) << endl;</pre>
// finding number of elements smaller than X - 3rd query
cout << "No. of elems smaller than 6: " << A.order_of_key(6)</pre>
      << endl: //
```

## 2.13 trie xor operation

int Trie[35 \* N][2];int root = 1;int cnt[35 \* N];int cur =

#### 2.14 wavelet tree

```
const int MAXN = (int)3e5 + 9;const int MAXV = (int)1e9 + 9;
    //maximum value of any element in array
//array values can be negative too, use appropriate minimum
    and maximum value
```

```
struct wavelet tree {int lo. hi:wavelet tree *1. *r:int *b.
     *c. bsz. csz: // c holds the prefix sum of elements
wavelet_tree() {lo = 1;hi = 0;bsz = 0;csz = 0, l = NULL;r =
void init(int *from, int *to, int x, int y) {lo = x, hi = y;
     if (from >= to) return:
int mid = (lo + hi) >> 1;auto f = [mid](int x) {return x <=</pre>
b = (int*)malloc((to - from + 2) * sizeof(int));bsz = 0;b[
c = (int*)malloc((to - from + 2) * sizeof(int));csz = 0;c[
     csz++1 = 0:
for (auto it = from; it != to; it++) {b[bsz] = (b[bsz - 1] +
      f(*it));c[csz] = (c[csz - 1] + (*it));bsz++;csz++;}
if (hi == lo) return:
auto pivot = stable_partition(from, to, f);
1 = new wavelet tree():
1->init(from, pivot, lo, mid);
r = new wavelet tree():
r->init(pivot, to, mid + 1, hi):}
//kth smallest element in [1, r]
//for array [1.2.1.3.5] 2nd smallest is 1 and 3rd smallest
int kth(int 1, int r, int k) {if (1 > r) return 0;if (1o ==
    hi) return lo; int inLeft = b[r] - b[1 - 1], lb = b[1 - 1]
     1]. rb = b[r]:
if (k <= inLeft) return this->l->kth(lb + 1, rb, k);return
    this->r->kth(1 - lb, r - rb, k - inLeft):}
//count of numbers in [1, r] Less than or equal to k
int LTE(int 1, int r, int k) {if (1 > r | | k < lo) return 0:
     if (hi \leq k) return r - 1 + 1; int lb = b[1 - 1], rb = b
     [r];return this->l->LTE(lb + 1, rb, k) + this->r->LTE(l
     - lb, r - rb, k):}
//count of numbers in [1, r] equal to k
int count(int 1, int r, int k) {if (1 > r \mid | k < lo \mid | k >
    hi) return 0:if (lo == hi) return r - 1 + 1:int lb = b[
    1 - 1, rb = b[r]:int mid = (lo + hi) >> 1:if (k <= mid
    ) return this->l->count(lb + 1, rb, k); return this->r->
    count(1 - lb, r - rb, k);
//sum of numbers in [l ,r] less than or equal to k
int sum(int 1, int r, int k) {if (1 > r or k < 1o) return 0;
     if (hi \leq k) return c[r] - c[1 - 1]; int 1b = b[1 - 1],
     rb = b[r]:return this->l->sum(lb + 1, rb, k) + this->r
     ->sum(1 - 1b, r - rb, k);
~wavelet_tree() {delete 1;delete r;}};
wavelet tree t:int a[MAXN]:
int main() {
int i, j, k, n, m, q, l, r; cin >> n; for (i = 1; i <= n; i++)</pre>
      cin >> a[i];t.init(a + 1, a + n + 1, -MAXV, MAXV);
```

```
//beware! after the init() operation array a[] will not be
    samecin >> q;
while (q--) {int x;cin >> x;cin >> 1 >> r >> k;
if (x == 0) {//kth smallestcout << t.kth(1, r, k) << endl;}
    else if (x == 1) {//less than or equal to Kcout << t.
    LTE(1, r, k) << endl;} else if (x == 2) {//count
    occurence of K in [1, r]cout << t.count(1, r, k) <<
    endl;}
if (x == 3) {//sum of elements less than or equal to K in [1
    , r]cout << t.sum(1, r, k) << endl;}}return 0;}</pre>
```

#### 2.15 xor basis

```
struct XorBasis { vector<11> basis;
11 N = 0, tmp = 0; void add(11 x) {
N++; tmp \mid= x;for (auto &i : basis) x = min(x, x ^ i);if (!x
     ) return:
for (auto &i : basis) if ((i ^ x) < i) i ^= x;basis.</pre>
     push back(x):
sort(basis.begin(), basis.end());}
11 size() {return (11)basis.size():
}void clear() {N = 0; tmp = 0;basis.clear();
}bool possible(ll x) {for (auto &i : basis) x = min(x, x ^ i
return !x;}ll maxxor(ll x = 0) {for (auto &i : basis) x =
     max(x, x^i);
return x:}ll minxor(ll x = 0) {
for (auto &i : basis) x = min(x, x^i);
return x;}ll cntxor(ll x) {if (!possible(x)) return OLL;//
     return (1LL<<(N-size()));</pre>
11 ans = 1LL;for (int i = 0; i < N - size(); i++)ans = (ans</pre>
return ans;}ll sumOfAll() {ll ans = tmp * (1LL << (N - 1));</pre>
return ans:}ll kth(ll k) {ll sz = size():if (k > (1LL << sz)</pre>
    ) return -1:
k--; ll ans = 0; for (ll i = 0; i < sz; i++) if (k >> i & 1)
     ans ^= basis[i]:
return ans;}} xb;
```

## 3 Extra

## 3.1 Ordered Set Template

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
```

```
#define ordered_set tree<lli, null_type,less<lli>,
    rb_tree_tag,tree_order_statistics_node_update>
```

## 3.2 Random Number Template

#### 3.3 build

## 3.4 equation

```
Some properties of bitwise operations:

a|b = ab + a&b , a (a&b) = (a|b) b ,

b (a&b) = (a|b) a ,(a&b) (a|b) = ab

Addition: a+b = a|b + a&b, a+b = ab + 2(a&b)

Subtraction:

a-b = (a (a&b))-((a|b) a),a-b = ((a|b) b)-((a|b) a)

,a-b = (a (a&b))-(b (a&b)),a-b = ((a|b)b)-(b (a&b))
```

#### 3.5 pragma

```
// #pragma GCC optimize("03,unroll-loops,0fast")
// #pragma GCC target("avx2")
```

## 4 Geo

## 4.1 $geo_template_2$

```
#include <bits/stdc++.h>using namespace std;
// https://victorlecomte.com/cp-geo.pdf
const int N = 3e5 + 9;const double inf = 1e100;const double
    eps = 1e-9:const double PI = acos((double)-1.0):
int sign(double x) { return (x > eps) - (x < -eps); }</pre>
struct PT{double x, y;PT() { x = 0, y = 0; }PT(double x,
    double y) : x(x), y(y) {}PT(const PT \& p) : x(p.x), y(p.x)
    y) {}PT operator+(const PT &a) const { return PT(x + a.
    x, y + a.y; }
PT operator-(const PT &a) const { return PT(x - a.x, y - a.y
    ); }PT operator*(const double a) const { return PT(x *
    a. v * a): }
friend PT operator*(const double &a. const PT &b) { return
    PT(a * b.x, a * b.y); }PT operator/(const double a)
    const { return PT(x / a, y / a); }bool operator==(PT a)
     const { return sign(a.x - x) == 0 && sign(a.y - y) ==
    0: }
bool operator!=(PT a) const { return !(*this == a); }bool
    operator<(PT a) const { return sign(a.x - x) == 0 ? y <
     a.y : x < a.x; }
bool operator>(PT a) const { return sign(a.x - x) == 0 ? y >
     a.v : x > a.x; }double norm() { return sqrt(x * x + y
double norm2() { return x * x + y * y; }PT perp() { return
    PT(-v, x); }double arg() { return atan2(v, x); }
PT truncate(double r) { // returns a vector with norm r and
    having same directiondouble k = norm(); if (!sign(k))
    return *this;r /= k;return PT(x * r, y * r);}};
istream &operator>>(istream &in, PT &p) { return in >> p.x
    >> p.v: }
ostream &operator<<(ostream &out, PT &p) { return out << "("
     << p.x << "," << p.y << ")"; }
inline double dot(PT a, PT b) { return a.x * b.x + a.y * b.y
inline double dist2(PT a, PT b) { return dot(a - b, a - b);
inline double dist(PT a, PT b) { return sqrt(dot(a - b, a -
    b)); }
```

```
inline double cross2(PT a, PT b, PT c) { return cross(b - a,
inline int orientation(PT a, PT b, PT c) { return sign(cross
     (b - a, c - a): }
PT perp(PT a) { return PT(-a.v, a.x); }
PT rotateccw90(PT a) { return PT(-a.v, a.x); }
PT rotatecw90(PT a) { return PT(a.y, -a.x); }
PT rotateccw(PT a, double t) { return PT(a.x * cos(t) - a.y
     * \sin(t). a.x * \sin(t) + a.v * \cos(t): }
PT rotatecw(PT a. double t) { return PT(a.x * cos(t) + a.v *
      sin(t), -a.x * sin(t) + a.y * cos(t)); }
double SQ(double x) { return x * x: }
double rad_to_deg(double r) { return (r * 180.0 / PI); }
double deg_to_rad(double d) { return (d * PI / 180.0); }
double get angle(PT a, PT b){double costheta = dot(a, b) / a
     .norm() / b.norm();return acos(max((double)-1.0, min())
    double)1.0. costheta)));}
bool is_point_in_angle(PT b, PT a, PT c, PT p){ // does
    point p lie in angle <bac
assert(orientation(a, b, c) != 0):if (orientation(a, c, b) <
     0)swap(b, c);
return orientation(a, c, p) >= 0 && orientation(a, b, p) <=
bool half(PT p){return p.y > 0.0 || (p.y == 0.0 && p.x <
void polar_sort(vector<PT> &v){ // sort points in
     counterclockwise
sort(v.begin(), v.end(), [](PT a, PT b){ return make tuple(
    half(a), 0.0, a.norm2()) < make_tuple(half(b), cross(a,
     b), b.norm2()); });}
void polar sort(vector<PT> &v. PT o){ // sort points in
    counterclockwise with respect to point o
sort(v.begin(), v.end(), [&](PT a, PT b){ return make_tuple(
    half(a - o), 0.0, (a - o).norm2()) < make tuple(half(b
     - o), cross(a - o, b - o), (b - o).norm2()); });}
struct line{PT a, b; // goes through points a and bPT v;
    double c; // line form: direction vec [cross] (x, y) =
line() {}// direction vector v and offset cline(PT v, double
     c) : v(v), c(c){auto p = get_points();a = p.first;b =
    p.second:}
// equation ax + by + c = 0
line(double _a, double _b, double _c) : v(\{b, -a\}), c(-c)
    {auto p = get points():a = p.first:b = p.second:}
// goes through points p and q
line(PT p, PT q) : v(q - p), c(cross(v, p)), a(p), b(q) {}
pair<PT, PT> get_points()
```

```
inline double cross(PT a, PT b) { return a.x * b.y - a.y * b | { // extract any two points from this linePT p, q;double a = | r = dot(c - a, b - a) / r;if (r < 0) return a;if (r > 1)
                                                                    -v.v. b = v.x: // ax + bv = cif (sign(a) == 0){p = PT
                                                                    (0, c / b);q = PT(1, c / b);
                                                               else if (sign(b) == 0){p = PT(c / a, 0);q = PT(c / a, 1);}
                                                               else{p = PT(0, c / b); q = PT(1, (c - a) / b);}return {p, q}
                                                               // ax + by + c = Oarray<double, 3> get_abc(){double a = -v.y
                                                                    b = v.x:return \{a, b, -c\}:
                                                               // 1 if on the left, -1 if on the right, 0 if on the line
                                                              int side(PT p) { return sign(cross(v, p) - c); }
                                                               // line that is perpendicular to this and goes through point
                                                               line perpendicular_through(PT p) { return {p, p + perp(v)};
                                                               // translate the line by vector t i.e. shifting it by vector
                                                              line translate(PT t) { return {v, c + cross(v, t)}; }
                                                               // compare two points by their orthogonal projection on this
                                                               // a projection point comes before another if it comes first
                                                                    according to vector v
                                                              bool cmp_by_projection(PT p, PT q) { return dot(v, p) < dot(</pre>
                                                                   v, q); }
                                                               line shift_left(double d){PT z = v.perp().truncate(d);return
                                                                    line(a + z, b + z):}}:
                                                               // find a point from a through b with distance d
                                                               PT point_along_line(PT a, PT b, double d){assert(a != b);
                                                                   return a + (((b - a) / (b - a).norm()) * d);}
                                                               // projection point c onto line through a and b assuming a
                                                                    l = h
                                                              PT project_from_point_to_line(PT a, PT b, PT c){return a + (
                                                                   b - a) * dot(c - a, b - a) / (b - a).norm2();}
                                                               // reflection point c onto line through a and b assuming a
                                                               PT reflection_from_point_to_line(PT a, PT b, PT c){PT p =
                                                                   project from point to line(a, b, c):return p + p - c:}
                                                               // minimum distance from point c to line through a and b
                                                               double dist_from_point_to_line(PT a, PT b, PT c){return fabs
                                                                    (cross(b - a, c - a) / (b - a).norm());
                                                               // returns true if point p is on line segment ab
                                                              bool is point on seg(PT a, PT b, PT p){if (fabs(cross(p - b,
                                                                    a - b)) < eps){
                                                               if (p.x < min(a.x, b.x) - eps \mid\mid p.x > max(a.x, b.x) + eps)
                                                                   return false:
                                                               if (p.y < min(a.y, b.y) - eps \mid\mid p.y > max(a.y, b.y) + eps)
                                                                    return false:return true:}return false:}
                                                               // minimum distance point from point c to segment ab that
                                                                    lies on segment ab
                                                               PT project_from_point_to_seg(PT a, PT b, PT c){double r =
                                                                    dist2(a, b):if (sign(r) == 0)return a:
```

```
return b:return a + (b - a) * r:}
// minimum distance from point c to segment ab
double dist from point to seg(PT a, PT b, PT c){return dist(
    c, project_from_point_to_seg(a, b, c));}
// 0 if not parallel, 1 if parallel, 2 if collinear
int is_parallel(PT a, PT b, PT c, PT d){double k = fabs(
    cross(b - a, d - c)):
if (k < eps){if (fabs(cross(a - b, a - c)) < eps && fabs(</pre>
    cross(c - d, c - a)) < eps)return 2;
else return 1:}else return 0:}
// check if two lines are same
bool are_lines_same(PT a, PT b, PT c, PT d){
if (fabs(cross(a - c, c - d)) < eps && fabs(cross(b - c, c -
     d)) < eps)return true:return false:}
// bisector vector of <abc
PT angle bisector(PT &a. PT &b. PT &c){PT p = a - b. q = c -
     b; return p + q * sqrt(dot(p, p) / dot(q, q));}
// 1 if point is ccw to the line, 2 if point is cw to the
    line, 3 if point is on the line
int point_line_relation(PT a, PT b, PT p){int c = sign(cross
     (p - a, b - a)):
if (c < 0)return 1;if (c > 0)return 2;return 3;}
// intersection point between ab and cd assuming unique
    intersection exists
bool line_line_intersection(PT a, PT b, PT c, PT d, PT &ans)
double a1 = a.v - b.v. b1 = b.x - a.x. c1 = cross(a, b):
    double a2 = c.v - d.v, b2 = d.x - c.x, c2 = cross(c, d)
double det = a1 * b2 - a2 * b1:if (det == 0)return 0:
ans = PT((b1 * c2 - b2 * c1) / det, (c1 * a2 - a1 * c2) /
    det):return 1:}
// intersection point between segment ab and segment cd
    assuming unique intersection exists
bool seg seg intersection(PT a, PT b, PT c, PT d, PT &ans){
double oa = cross2(c, d, a), ob = cross2(c, d, b);double oc
    = cross2(a, b, c), od = cross2(a, b, d); if (oa * ob < 0
     \&\& oc * od < 0)
\{ans = (a * ob - b * oa) / (ob - oa); return 1;\} elsereturn
// intersection point between segment ab and segment cd
    assuming unique intersection may not exists
// se.size()==0 means no intersection
// se.size()==1 means one intersection
// se.size()==2 means range intersection
set<PT> seg_seg_intersection_inside(PT a, PT b, PT c, PT d){
PT ans:if (seg seg intersection(a, b, c, d, ans))return {ans
    }:set<PT> se:
```

```
if (is_point_on_seg(c, d, a))se.insert(a);if (
    is_point_on_seg(c, d, b))se.insert(b);
if (is_point_on_seg(a, b, c))se.insert(c);
if (is point on seg(a, b, d))se.insert(d):return se:}
// intersection between segment ab and line cd
// 0 if do not intersect, 1 if proper intersect, 2 if
     segment intersect
int seg_line_relation(PT a, PT b, PT c, PT d){
double p = cross2(c, d, a);double q = cross2(c, d, b);
if (sign(p) == 0 && sign(q) == 0)return 2;
else if (p * q < 0)return 1;else return 0;}</pre>
// intersection between segament ab and line cd assuming
    unique intersection exists
bool seg_line_intersection(PT a, PT b, PT c, PT d, PT &ans){
bool k = seg_line_relation(a, b, c, d);assert(k != 2);if (k)
    line_line_intersection(a, b, c, d, ans); return k;}
// minimum distance from segment ab to segment cd
double dist_from_seg_to_seg(PT a, PT b, PT c, PT d){PT dummy
     :if (seg seg intersection(a, b, c, d, dummy))return
    0.0:
else return min({dist_from_point_to_seg(a, b, c),
    dist_from_point_to_seg(a, b, d),dist_from_point_to_seg(
    c, d, a), dist_from_point_to_seg(c, d, b)});}
// minimum distance from point c to ray (starting point a
     and direction vector b)
double dist_from_point_to_ray(PT a, PT b, PT c){b = a + b;
    double r = dot(c - a, b - a); if (r < 0.0) return dist(c, a)
      a):return dist from point to line(a, b, c):}
// starting point as and direction vector ad
bool ray ray intersection(PT as, PT ad, PT bs, PT bd){
double dx = bs.x - as.x, dy = bs.y - as.y;double det = bd.x
    * ad.y - bd.y * ad.x; if (fabs(det) < eps)return 0;
    double u = (dv * bd.x - dx * bd.v) / det:
double v = (dv * ad.x - dx * ad.v) / det;
if (sign(u) >= 0 && sign(v) >= 0)return 1;elsereturn 0;}
double ray ray distance(PT as. PT ad. PT bs. PT bd){if (
    ray_ray_intersection(as, ad, bs, bd))return 0.0;
double ans = dist_from_point_to_ray(as, ad, bs);ans = min(
    ans, dist_from_point_to_ray(bs, bd, as));return ans;}
struct circle{PT p:double r:
circle() {}circle(PT _p, double _r) : p(_p), r(_r) {};
// center (x, y) and radius r
circle(double x, double y, double _r) : p(PT(x, y)), r(_r)
// circumcircle of a triangle
// the three points must be unique
circle(PT a, PT b, PT c)\{b = (a + b) * 0.5; c = (a + c) *
    0.5; line_line_intersection(b, b + rotatecw90(a - b), c,
     c + rotatecw90(a - c), p); r = dist(a, p);}
// inscribed circle of a triangle
```

```
// pass a bool just to differentiate from circumcircle
 circle(PT a, PT b, PT c, bool t){line u, v;double m = atan2(
     b.y - a.y, b.x - a.x), n = atan2(c.y - a.y, c.x - a.x);
 u.a = a:u.b = u.a + (PT(cos((n + m) / 2.0), sin((n + m) / 2.0)))
     (2.0)):v.a = b:
m = atan2(a.v - b.v, a.x - b.x), n = atan2(c.v - b.v, c.x - b.x)
     b.x; v.b = v.a + (PT(cos((n + m) / 2.0), sin((n + m) / 2.0))
     2.0))):
line line intersection(u.a, u.b, v.a, v.b, p):r =
     dist_from_point_to_seg(a, b, p);}
 bool operator==(circle v) { return p == v.p && sign(r - v.r)
       == 0: }double area() { return PI * r * r: }double
     circumference() { return 2.0 * PI * r; }};
 // 0 if outside, 1 if on circumference, 2 if inside circle
 int circle_point_relation(PT p, double r, PT b){double d =
     dist(p, b); if (sign(d - r) < 0) return 2; if (sign(d - r))
      == 0)return 1:return 0:}
// 0 if outside, 1 if on circumference, 2 if inside circle
 int circle line relation(PT p. double r. PT a. PT b){double
     d = dist_from_point_to_line(a, b, p);if (sign(d - r) <</pre>
     0)return 2;
if (sign(d - r) == 0)return 1;return 0;}
 // compute intersection of line through points a and b with
 // circle centered at c with radius r > 0
 vector<PT> circle_line_intersection(PT c, double r, PT a, PT
       b){vector<PT> ret:
b = b - a; a = a - c; double A = dot(b, b), B = dot(a, b);
     double C = dot(a, a) - r * r, D = B * B - A * C:
 if (D < -eps)return ret;</pre>
 ret.push back(c + a + b * (-B + sart(D + eps)) / A):
if (D > eps)ret.push_back(c + a + b * (-B - sqrt(D)) / A);
     return ret:}
 // 5 - outside and do not intersect
// 4 - intersect outside in one point
// 3 - intersect in 2 points
// 2 - intersect inside in one point
 // 1 - inside and do not intersect
 int circle_circle_relation(PT a, double r, PT b, double R){
     double d = dist(a, b);
 if (sign(d - r - R) > 0) return 5:
 if (sign(d - r - R) == 0)return 4:
 double 1 = fabs(r - R);
 if (sign(d - r - R) < 0 \&\& sign(d - 1) > 0) return 3:
 if (sign(d - 1) == 0)return 2;
 if (sign(d - 1) < 0)return 1;assert(0);return -1;}</pre>
 vector<PT> circle circle intersection(PT a, double r, PT b.
if (a == b && sign(r - R) == 0)return {PT(1e18, 1e18)};
     vector<PT> ret;double d = sqrt(dist2(a, b));
```

```
if (d > r + R \mid | d + min(r, R) < max(r, R)) return ret; double
     x = (d * d - R * R + r * r) / (2 * d):
double y = sqrt(r * r - x * x); PT v = (b - a) / d;
ret.push back(a + v * x + rotateccw90(v) * v):if (v > 0)ret.
    push_back(a + v * x - rotateccw90(v) * y);return ret;}
// returns two circle c1, c2 through points a, b and of
    radius r
// 0 if there is no such circle, 1 if one circle, 2 if two
int get_circle(PT a, PT b, double r, circle &c1, circle &c2)
    {vector<PT> v = circle circle intersection(a, r, b, r);
int t = v.size():if (!t)return 0:
c1.p = v[0], c1.r = r; if (t == 2)c2.p = v[1], c2.r = r;
    return t:}
// returns two circle c1, c2 which is tangent to line u,
    goes through
// point q and has radius r1: 0 for no circle, 1 if c1 = c2
     , 2 if c1 != c2
int get_circle(line u, PT q, double r1, circle &c1, circle &
double d = dist_from_point_to_line(u.a, u.b, q);if (sign(d -
     r1 * 2.0) > 0)return 0:
if (sign(d) == 0){cout << u.v.x << ' ' ' << u.v.v << '\n';</pre>
c1.p = q + rotateccw90(u.v).truncate(r1);c2.p = q +
    rotatecw90(u.v).truncate(r1);c1.r = c2.r = r1;return
line u1 = line(u.a + rotateccw90(u.v).truncate(r1), u.b +
    rotateccw90(u.v).truncate(r1)):line u2 = line(u.a +
    rotatecw90(u.v).truncate(r1), u.b + rotatecw90(u.v).
    truncate(r1)):
circle cc = circle(q, r1);PT p1, p2;vector<PT> v;v =
     circle_line_intersection(q, r1, u1.a, u1.b);
if (!v.size())v = circle line intersection(g, r1, u2.a, u2.b
    ); v.push_back(v[0]); p1 = v[0], p2 = v[1];
c1 = circle(p1, r1); if (p1 == p2){c2 = c1; return 1;}c2 =
    circle(p2, r1):return 2:}
// returns the circle such that for all points w on the
    circumference of the circle
// dist(w, a) : dist(w, b) = rp : rq
// rp != rq
// https://en.wikipedia.org/wiki/Circles of Apollonius
circle get_apollonius_circle(PT p, PT q, double rp, double
    rq){rq *= rq;rp *= rp;double a = rq - rp;
assert(sign(a));double g = rq * p.x - rp * q.x;g /= a;double
     h = rq * p.v - rp * q.v;h /= a;double c = rq * p.x * p
     .x - rp * q.x * q.x + rq * p.y * p.y - rp * q.y * q.y;
c \neq a; PT \circ (g, h); double r = g * g + h * h - c; r = sqrt(r);
    return circle(o, r):}
// returns area of intersection between two circles
```

```
double d = (a - b).norm():if (r1 + r2 < d + eps)return
if (r1 + d < r2 + eps)return PI * r1 * r1:
if (r2 + d < r1 + eps)return PI * r2 * r2;</pre>
double theta_1 = acos((r1 * r1 + d * d - r2 * r2) / (2 * r1)
     * d)), theta_2 = acos((r2 * r2 + d * d - r1 * r1) / (2 *
     r2 * d)):
return r1 * r1 * (theta 1 - sin(2 * theta 1) / 2.) + r2 * r2
      * (theta_2 - sin(2 * theta_2) / 2.);}
// tangent lines from point q to the circle
int tangent lines from point(PT p. double r. PT q. line &u.
     line &v){
int x = sign(dist2(p, q) - r * r); if (x < 0) return 0; //
     point in cricleif (x == 0)
{ // point on circle
u = line(q, q + rotateccw90(q - p));v = u;return 1;}
double d = dist(p, q);double l = r * r / d;double h = sqrt(r
      * r - 1 * 1); u = line(q, p + ((q - p).truncate(1) + (
     rotateccw90(g - p).truncate(h)));
v = line(q, p + ((q - p).truncate(1) + (rotatecw90(q - p).
     truncate(h))));return 2;}
// returns outer tangents line of two circles
// if inner == 1 it returns inner tangent lines
int tangents_lines_from_circle(PT c1, double r1, PT c2,
     double r2, bool inner, line &u, line &v){
if (inner)r2 = -r2;PT d = c2 - c1;double dr = r1 - r2, d2 =
     d.norm2(), h2 = d2 - dr * dr:
if (d2 == 0 || h2 < 0){assert(h2 != 0);return 0;}vector<pair</pre>
     PT, PT>> out: for (int tmp : {-1, 1}) PT v = (d * dr +
     rotateccw90(d) * sqrt(h2) * tmp) / d2;out.push_back({c1
     + v * r1, c2 + v * r2});}
u = line(out[0].first, out[0].second):
if (out.size() == 2)v = line(out[1].first, out[1].second);
     return 1 + (h2 > 0):}
// O(n^2 \log n)
// https://vjudge.net/problem/UVA-12056
struct CircleUnion
{int n;double x[2020], y[2020], r[2020];int covered[2020];
     vector<pair<double, double>> seg, cover;double arc, pol
     :inline int sign(double x) { return x < -eps ? -1 : x >
     eps; }inline int sign(double x, double y) { return
     sign(x - v): 
inline double SQ(const double x) { return x * x; }inline
     double dist(double x1, double y1, double x2, double y2)
     { return sqrt(SQ(x1 - x2) + SQ(y1 - y2)); }
inline double angle(double A, double B, double C)
\{double\ val = (SQ(A) + SQ(B) - SQ(C)) / (2 * A * B): if (val)\}
     < -1)val = -1:
if (val > +1)val = +1:return acos(val):}
```

```
void init(){n = 0;seg.clear(), cover.clear();arc = pol = 0;}
                                                           void add(double xx. double vv. double rr)
                                                           \{x[n] = xx, y[n] = yy, r[n] = rr, covered[n] = 0, n++;\}
                                                           void getarea(int i. double lef. double rig){
                                                           arc += 0.5 * r[i] * r[i] * (rig - lef - sin(rig - lef));
                                                           double x1 = x[i] + r[i] * cos(lef), v1 = v[i] + r[i] * sin(
                                                           double x2 = x[i] + r[i] * cos(rig), y2 = y[i] + r[i] * sin(
                                                               rig);pol += x1 * y2 - x2 * y1;}
                                                           double solve() {for (int i = 0: i < n: i++)
                                                           {for (int j = 0; j < i; j++){if (!sign(x[i] - x[j]) && !sign
                                                                (y[i] - y[j]) && !sign(r[i] - r[j]))
                                                           {r[i] = 0.0:break:}}
                                                           for (int i = 0; i < n; i++){for (int j = 0; j < n; j++){if (
                                                                i != j && sign(r[j] - r[i]) >= 0 && sign(dist(x[i], y[i
                                                                ], x[j], y[j]) - (r[j] - r[i])) <= 0){
                                                           covered[i] = 1:break:} for (int i = 0: i < n: i++){
                                                           if (sign(r[i]) && !covered[i]){seg.clear();
                                                           for (int j = 0; j < n; j++){if (i != j){
                                                           double d = dist(x[i], y[i], x[j], y[j]);
                                                           if (sign(d - (r[j] + r[i])) >= 0 \mid \mid sign(d - abs(r[j] - r[i]))
                                                                ])) <= 0)
                                                           {continue;}double alpha = atan2(y[i] - y[i], x[j] - x[i]);
                                                           double beta = angle(r[i], d, r[j]);pair<double, double> tmp(
                                                                alpha - beta, alpha + beta);
                                                            if (sign(tmp.first) <= 0 && sign(tmp.second) <= 0){</pre>
                                                           seg.push_back(pair<double, double>(2 * PI + tmp.first, 2 *
                                                                PI + tmp.second)):}
                                                           else if (sign(tmp.first) < 0){seg.push_back(pair<double,</pre>
                                                                double>(2 * PI + tmp.first, 2 * PI));seg.push_back(pair
                                                                <double, double>(0, tmp.second));}
                                                           else{seg.push_back(tmp);}}}
                                                           sort(seg.begin(), seg.end());double rig = 0;
                                                           for (vector<pair<double, double>>::iterator iter = seg.begin
                                                                (); iter != seg.end(); iter++){
                                                           if (sign(rig - iter->first) >= 0){rig = max(rig, iter->
                                                                second):}
                                                           else{getarea(i, rig, iter->first);rig = iter->second;}}
                                                           if (!sign(rig)){arc += r[i] * r[i] * PI;}
                                                           else{getarea(i, rig, 2 * PI);}}}return pol / 2.0 + arc;}} CU
                                                           double area_of_triangle(PT a, PT b, PT c){return fabs(cross(
                                                                b - a, c - a) * 0.5);
                                                           // -1 if strictly inside, 0 if on the polygon, 1 if strictly
                                                           int is_point_in_triangle(PT a, PT b, PT c, PT p){
                                                           if (sign(cross(b - a, c - a)) < 0)swap(b, c);
```

```
int c1 = sign(cross(b - a, p - a)): int c2 = sign(cross(c - b))
     (p - b); int c3 = sign(cross(a - c, p - c));
if (c1 < 0 || c2 < 0 || c3 < 0)return 1; if (c1 + c2 + c3 !=
    3)return 0:return -1:}
double perimeter(vector<PT> &p){
double ans = 0:int n = p.size():
for (int i = 0; i < n; i++)ans += dist(p[i], p[(i + 1) \% n])
     :return ans:}
double area(vector<PT> &p){double ans = 0;int n = p.size();
    or (int i = 0; i < n; i++)ans += cross(p[i], p[(i + 1)
     % n]):return fabs(ans) * 0.5:}
// centroid of a (possibly non-convex) polygon.
// assuming that the coordinates are listed in a clockwise
// counterclockwise fashion. Note that the centroid is often
      known as
// the "center of gravity" or "center of mass".
PT centroid(vector<PT> &p){int n = p.size();PT c(0, 0);
     double sum = 0:
for (int i = 0; i < n; i++)sum += cross(p[i], p[(i + 1) % n</pre>
    ]);double scale = 3.0 * sum;
for (int i = 0; i < n; i++){int j = (i + 1) \% n; c = c + (p[i + 1) \% n)
    ] + p[j]) * cross(p[i], p[j]);}return c / scale;}
// 0 if cw. 1 if ccw
bool get_direction(vector<PT> &p){double ans = 0;int n = p.
     size():
for (int i = 0; i < n; i++)ans += cross(p[i], p[(i + 1) % n</pre>
    1):if (sign(ans) > 0)return 1:return 0:}
// it returns a point such that the sum of distances
// from that point to all points in p is minimum
// O(n log^2 MX)
PT geometric_median(vector<PT> p){auto tot_dist = [&](PT z){
     double res = 0:
for (int i = 0; i < p.size(); i++)res += dist(p[i], z);</pre>
     return res:}:
auto findY = \lceil k \rceil (double x) {double vl = -1e5, vr = 1e5; for (
    int i = 0: i < 60: i++){
double ym1 = yl + (yr - yl) / 3;double ym2 = yr - (yr - yl)
    / 3;double d1 = tot_dist(PT(x, ym1));
double d2 = tot_dist(PT(x, ym2));if (d1 < d2)yr = ym2;</pre>
else vl = vm1:}return pair<double, double>(vl, tot dist(PT(x))
     (v1));;double x1 = -1e5, xr = 1e5;
for (int i = 0: i < 60: i++){double xm1 = xl + (xr - xl) /
double xm2 = xr - (xr - xl) / 3; double y1, d1, y2, d2; auto z
      = findY(xm1):v1 = z.first:d1 = z.second:z = findY(xm2)
v2 = z.first:d2 = z.second:
if (d1 < d2)xr = xm2:elsexl = xm1:}return {xl, findY(xl).}
     first}:}
```

```
vector<PT> convex hull(vector<PT> &p){
if (p.size() <= 1)return p;</pre>
vector<PT> v = p;sort(v.begin(), v.end());vector<PT> up, dn;
for (auto &p : v){
while (up.size() > 1 && orientation(up[up.size() - 2], up.
    back(), p) >= 0) \{up.pop back():\}
while (dn.size() > 1 && orientation(dn[dn.size() - 2], dn.
    back(), p) <= 0){dn.pop_back();}
up.push_back(p);dn.push_back(p);}
v = dn;if (v.size() > 1)v.pop_back();reverse(up.begin(), up.
     end());up.pop_back();
for (auto &p : up){v.push back(p):}
if (v.size() == 2 && v[0] == v[1])v.pop_back();return v;}
// checks if convex or not
bool is_convex(vector\PT> \&p)\{bool s[3]; s[0] = s[1] = s[2] =
     0; int n = p.size(); for (int i = 0; i < n; i++){int j =
     (i + 1) \% n; int k = (j + 1) \% n;
s[sign(cross(p[j] - p[i], p[k] - p[i])) + 1] = 1; if (s[0] &&
     s[2])return 0:}return 1:}
// -1 if strictly inside, 0 if on the polygon, 1 if strictly
     outside
// it must be strictly convex. otherwise make it strictly
     convex first
int is_point_in_convex(vector<PT> &p, const PT &x){ // O(log
     n)int n = p.size();assert(n >= 3);
int a = orientation(p[0], p[1], x), b = orientation(p[0], p[
    n - 1, x); if (a < 0 \mid | b > 0) return 1; int l = 1, r = n
     -1: while (1 + 1 < r){
int mid = 1 + r >> 1; if (orientation(p[0], p[mid], x) >= 0)1
     = mid;elser = mid;}int k = orientation(p[1], p[r], x);
    if (k \le 0) return -k;
if (1 == 1 && a == 0)return 0; if (r == n - 1 && b == 0)
    return 0:return -1:}
bool is_point_on_polygon(vector<PT> &p, const PT &z){int n =
     p.size():
for (int i = 0; i < n; i++){if (is_point_on_seg(p[i], p[(i +</pre>
     1) % n], z))return 1;}return 0;}
// returns 1e9 if the point is on the polygon
int winding_number(vector<PT> &p, const PT &z){ // O(n)
if (is_point_on_polygon(p, z))return 1e9;
int n = p.size(), ans = 0;
for (int i = 0; i < n; ++i){int j = (i + 1) \% n; bool below =
     p[i].v < z.v:
if (below != (p[j].y < z.y)){</pre>
auto orient = orientation(z, p[i], p[i]);
if (orient == 0)return 0:
if (below == (orient > 0))ans += below ? 1 : -1;}}return ans
     :}
// -1 if strictly inside, 0 if on the polygon, 1 if strictly for (int i = 1; i < n; i++){if (dot(p[1] - p[0], p[i]) <=
     outside
```

```
int is point in polygon(vector<PT> &p. const PT &z){ // O(n)
int k = winding_number(p, z); return k == 1e9 ? 0 : k == 0 ?
     1: -1:}
// id of the vertex having maximum dot product with z
// polygon must need to be convex
// top - upper right vertex
// for minimum dot product negate z and return -dot(z, p[id
int extreme_vertex(vector<PT> &p, const PT &z, const int top
     ){ // O(\log n)
int n = p.size():if (n == 1)return 0:
double ans = dot(p[0], z): int id = 0:
if (dot(p[top], z) > ans)ans = dot(p[top], z), id = top;
int 1 = 1, r = top - 1;
while (1 < r){int mid = 1 + r >> 1;
if (dot(p[mid + 1], z) >= dot(p[mid], z))1 = mid + 1;
else r = mid:}
if (dot(p[1], z) > ans)ans = dot(p[1], z), id = 1;1 = top +
     1. r = n - 1: while (1 < r)
int mid = 1 + r >> 1;if (dot(p[(mid + 1) % n], z) >= dot(p[
     mid], z))1 = mid + 1;elser = mid;}1 %= n;
if (dot(p[1], z) > ans)ans = dot(p[1], z), id = 1; return id
// maximum distance from any point on the perimeter to
     another point on the perimeter
double diameter(vector<PT> &p){int n = (int)p.size();if (n
     == 1)return 0;if (n == 2)return dist(p[0], p[1]);double
      ans = 0:int i = 0, i = 1:
while (i < n) {
while (cross(p[(i + 1) % n] - p[i], p[(j + 1) % n] - p[j])
     >= 0){ans = max(ans, dist2(p[i], p[j])); j = (j + 1) % n}
ans = max(ans, dist2(p[i], p[j]));i++;}return sqrt(ans);}
// minimum distance between two parallel lines (non
     necessarily axis parallel)
// such that the polygon can be put between the lines
double width(vector<PT> &p){int n = (int)p.size();if (n <=</pre>
     2)return 0;double ans = inf;int i = 0, j = 1;while (i <
      n){
while (cross(p[(i + 1) % n] - p[i], p[(j + 1) % n] - p[j])
     >= 0)i = (i + 1) \% n : ans = min(ans.
     dist_from_point_to_line(p[i], p[(i + 1) % n], p[j]));i
     ++: return ans: }
// minimum perimeter
double minimum_enclosing_rectangle(vector<PT> &p){int n = p.
if (n <= 2)return perimeter(p); int mndot = 0; double tmp =
     dot(p[1] - p[0], p[0]);
     tmp)\{tmp = dot(p[1] - p[0], p[i]); mndot = i;\}\}
```

```
double ans = inf:int i = 0, i = 1, mxdot = 1:
while (i < n){PT cur = p[(i + 1) \% n] - p[i]:while (cross(
    cur, p[(j + 1) \% n] - p[j]) >= 0)j = (j + 1) \% n; while
    (dot(p[(mxdot + 1) \% n], cur) >= dot(p[mxdot], cur))
    mxdot = (mxdot + 1) \% n;
while (dot(p[(mndot + 1) % n], cur) <= dot(p[mndot], cur))</pre>
    mndot = (mndot + 1) \% n; ans = min(ans, 2.0 * ((dot(p[
    mxdot], cur) / cur.norm() - dot(p[mndot], cur) / cur.
    norm()) + dist_from_point_to_line(p[i], p[(i + 1) % n],
     p[j])));i++;}return ans:}
// given n points, find the minimum enclosing circle of the
// call convex_hull() before this for faster solution
// expected O(n)
circle minimum_enclosing_circle(vector<PT> &p){
    random_shuffle(p.begin(), p.end());int n = p.size();
    circle c(p[0], 0):
for (int i = 1; i < n; i++){if (sign(dist(c.p, p[i]) - c.r)
    > 0){c = circle(p[i], 0):
for (int i = 0: i < i: i++){</pre>
if (sign(dist(c.p, p[i]) - c.r) > 0){
c = circle((p[i] + p[j]) / 2, dist(p[i], p[j]) / 2);
for (int k = 0; k < j; k++){if (sign(dist(c.p, p[k]) - c.r)
    > 0){c = circle(p[i], p[j], p[k]);}}}}return c;}
// returns a vector with the vertices of a polygon with
    everything
// to the left of the line going from a to b cut away.
vector<PT> cut(vector<PT> &p. PT a. PT b){vector<PT> ans:int
     n = (int)p.size();
for (int i = 0; i < n; i++){double c1 = cross(b - a, p[i] -
    a); double c2 = cross(b - a, p[(i + 1) \% n] - a);
if (sign(c1) \ge 0)ans.push_back(p[i]); if (sign(c1 * c2) < 0)
    {if (!is_parallel(p[i], p[(i + 1) % n], a, b)){PT tmp;
    line_line_intersection(p[i], p[(i + 1) % n], a, b, tmp)
    ;ans.push_back(tmp);}}}return ans;}
// not necessarily convex, boundary is included in the
    intersection
// returns total intersected length
// it returns the sum of the lengths of the portions of the
    line that are inside the polygon
double polygon line intersection(vector<PT> p. PT a. PT b){
int n = p.size();p.push_back(p[0]);line l = line(a, b);
    double ans = 0.0:vector<pair<double, int>> vec:for (int
     i = 0; i < n; i++){int s1 = orientation(a, b, p[i]);
    int s2 = orientation(a, b, p[i + 1]);
if (s1 == s2)continue: line t = line(p[i], p[i + 1]): PT inter
     = (t.v * 1.c - 1.v * t.c) / cross(1.v, t.v); double tmp
     = dot(inter, 1.v):int f:
if (s1 > s2)f = s1 && s2 ? 2 : 1:
```

```
tmp - eps : tmp + eps), f)); // keep eps very small
    like 1e-12}
sort(vec.begin(), vec.end()): for (int i = 0, i = 0; i + 1 < 0)
     (int)vec.size(); i++){j += vec[i].second;
if (i)ans += vec[i + 1].first - vec[i].first: // if this
    portion is inside the polygon// else ans = 0; // if we
    want the maximum intersected length which is totally
     inside the polygon, uncomment this and take the maximum
     of ans}
ans = ans / sqrt(dot(1.v, 1.v));p.pop_back();return ans;}
// given a convex polygon p, and a line ab and the top
     vertex of the polygon
// returns the intersection of the line with the polygon
// it returns the indices of the edges of the polygon that
     are intersected by the line
// so if it returns i, then the line intersects the edge (p[
    i], p[(i + 1) % n])
array<int. 2> convex line intersection(vector<PT> &p. PT a.
    PT b, int top){int end_a = extreme_vertex(p, (a - b).
    perp(), top); int end_b = extreme_vertex(p, (b - a).perp
    (), top):
auto cmp_l = [&](int i){ return orientation(a, p[i], b); };
    if (cmp_1(end_a) < 0 \mid cmp_1(end_b) > 0) return \{-1,
     -1}; // no intersectionarray<int, 2> res;for (int i =
    0; i < 2; i++){int lo = end_b, hi = end_a, n = p.size()
while ((lo + 1) \% n != hi) \{ int m = ((lo + hi + (lo < hi ? 0 + hi) + (lo < hi) ? 0 \} \}
    (n) / 2) % n; (cmp_1(m) = cmp_1(end_b) ? lo : hi) = m
     :}res[i] = (lo + !cmp l(hi)) % n:
swap(end_a, end_b);}
if (res[0] == res[1])return {res[0], -1}; // touches the
    vertex res[0]if (!cmp_l(res[0]) && !cmp_l(res[1]))
switch ((res[0] - res[1] + (int)p.size() + 1) % p.size()){
     case 0:return {res[0], res[0]}; // touches the edge (
    res[0], res[0] + 1)case 2:return {res[1], res[1]}: //
    touches the edge (res[1], res[1] + 1)}
return res; // intersects the edges (res[0], res[0] + 1) and
     (res[1], res[1] + 1)
pair<PT, int> point poly tangent(vector<PT> &p. PT Q, int
    dir, int 1, int r){
while (r - 1 > 1){int mid = (1 + r) >> 1:bool pvs =
    orientation(Q, p[mid], p[mid - 1]) != -dir;
bool nxt = orientation(Q, p[mid], p[mid + 1]) != -dir;
if (pvs && nxt)return {p[mid], mid}:
if (!(pvs || nxt)){auto p1 = point_poly_tangent(p, Q, dir,
    mid + 1, r):
auto p2 = point_poly_tangent(p, Q, dir, 1, mid - 1);
return orientation(Q, p1.first, p2.first) == dir ? p1 : p2;}
```

```
else f = s1 && s2 ? -2 : -1; vec.push back(make pair((f > 0 ? | if (!pvs){if (orientation(Q.p[mid],p[]]) == dir)r = mid -
                                                               else if (orientation(Q, p[1], p[r]) == dir)r = mid - 1;elsel
                                                                     = mid + 1:
                                                               if (!nxt){if (orientation(Q, p[mid], p[1]) == dir)1 = mid +
                                                               else if (orientation(Q, p[1], p[r]) == dir)r = mid - 1;
                                                               else l = mid + 1:}
                                                               pair<PT, int> ret = {p[1], 1};
                                                               for (int i = 1 + 1; i \le r; i++)ret = orientation(Q, ret.
                                                                    first, p[i]) != dir ? make pair(p[i], i) : ret:
                                                               return ret:}
                                                               // (ccw, cw) tangents from a point that is outside this
                                                                    convex polygon
                                                               // returns indexes of the points
                                                               // ccw means the tangent from Q to that point is in the same
                                                                     direction as the polygon ccw direction
                                                               pair<int, int> tangents_from_point_to_polygon(vector<PT> &p,
                                                                     PT Q) {int ccw = point_poly_tangent(p, Q, 1, 0, (int)p.
                                                                    size() - 1).second:int cw = point polv tangent(p, Q.
                                                                    -1, 0, (int)p.size() - 1).second; return make_pair(ccw,
                                                                    CW):}
                                                               // minimum distance from a point to a convex polygon
                                                               // it assumes point lie strictly outside the polygon
                                                               double dist_from_point_to_polygon(vector<PT> &p, PT z){
                                                                    double ans = inf;int n = p.size();if (n <= 3){for (int</pre>
                                                                    i = 0; i < n; i++)ans = min(ans, dist_from_point_to_seg</pre>
                                                                    (p[i], p[(i + 1) % n], z)):return ans:}
                                                               auto [r, 1] = tangents_from_point_to_polygon(p, z);if (1 > r
                                                                    )r += n:
                                                               while (1 < r){int mid = (1 + r) >> 1; double left = dist2(p[
                                                                    mid \% n], z), right = dist2(p[(mid + 1) \% n], z);ans =
                                                                    min({ans, left, right});if (left < right)r = mid;elsel</pre>
                                                                    = mid + 1;
                                                               ans = sqrt(ans); ans = min(ans, dist_from_point_to_seg(p[1 %
                                                                    n], p[(1 + 1) \% n], z)); ans = min(ans).
                                                                    dist\_from\_point\_to\_seg(p[1 \% n], p[(1 - 1 + n) \% n], z)
                                                                    ):return ans:}
                                                               // minimum distance from convex polygon p to line ab
                                                               // returns 0 is it intersects with the polygon
                                                               // top - upper right vertex
                                                               double dist_from_polygon_to_line(vector<PT> &p, PT a, PT b,
                                                                    int top){ // O(log n)
                                                               PT orth = (b - a).perp();
                                                               if (orientation(a, b, p[0]) > 0) orth = (a - b).perp();
                                                               int id = extreme vertex(p, orth, top):if (dot(p[id] - a,
                                                                    orth) > 0)return 0.0;// if orth and a are in the same
                                                                    half of the line, then poly and line intersects
                                                               return dist_from_point_to_line(a, b, p[id]); // does not
```

intersect}

```
// minimum distance from a convex polygon to another convex
// the polygon doesnot overlap or touch
// tested in https://toph.co/p/the-wall
double dist_from_polygon_to_polygon(vector<PT> &p1, vector<</pre>
    PT > &p2) \{ // O(n log n) \}
double ans = inf;
for (int i = 0; i < p1.size(); i++){ans = min(ans,</pre>
     dist_from_point_to_polygon(p2, p1[i]));}
for (int i = 0; i < p2.size(); i++){ans = min(ans,</pre>
     dist from point to polygon(p1, p2[i]));}
return ans:}
// maximum distance from a convex polygon to another convex
double maximum_dist_from_polygon_to_polygon(vector<PT> &u,
     vector < PT > &v) { // O(n) }
int n = (int)u.size(), m = (int)v.size():double ans = 0;
if (n < 3 | 1 | m < 3)
for (int i = 0: i < n: i++){</pre>
for (int i = 0; i < m; i++)ans = max(ans, dist2(u[i], v[i]))
     ;}return sqrt(ans);}
if (u[0].x > v[0].x)swap(n, m), swap(u, v); int i = 0, j = 0,
      step = n + m + 10;
while (j + 1 < m \&\& v[j].x < v[j + 1].x)j++;
while (step--)\{if (cross(u[(i + 1) % n] - u[i], v[(j + 1) % n] - u[i], v[(j + 1) % n]\}\}
    m] - v[j]) >= 0)j = (j + 1) \% m; elsei = (i + 1) % n; ans
     = max(ans, dist2(u[i], v[j]));}
return sqrt(ans):}
// calculates the area of the union of n polygons (not
    necessarily convex).
// the points within each polygon must be given in CCW order
// complexity: O(N^2), where N is the total number of points
double rat(PT a, PT b, PT p){return !sign(a.x - b.x) ? (p.y
    -a.v) / (b.v - a.v) : (p.x - a.x) / (b.x - a.x):
double polygon_union(vector<vector<PT>> &p){
int n = p.size();double ans = 0;
for (int i = 0; i < n; ++i){</pre>
for (int v = 0: v < (int)p[i].size(): ++v){
PT a = p[i][v], b = p[i][(v + 1) % p[i].size()]:vector<pair<
     double, int>> segs;segs.emplace_back(0, 0), segs.
     emplace back(1, 0):
for (int j = 0; j < n; ++j){
if (i != j){for (size_t u = 0; u < p[j].size(); ++u){</pre>
PT c = p[i][u], d = p[i][(u + 1) \% p[i].size()]:int sc =
     sign(cross(b - a, c - a)), sd = sign(cross(b - a, d - a))
     )):if (!sc && !sd){
if (sign(dot(b - a, d - c)) > 0 \&\& i > j){segs.emplace_back(}
     rat(a, b, c), 1), segs.emplace_back(rat(a, b, d), -1)
```

```
:}}else{
double sa = cross(d - c, a - c), sb = cross(d - c, b - c);
if (sc >= 0 && sd < 0)segs.emplace_back(sa / (sa - sb), 1);</pre>
     else if (sc < 0 && sd >= 0)segs.emplace back(sa / (sa -
      sb), -1);}}}}
sort(segs.begin(), segs.end()):double pre = min(max(segs[0].
    first, 0.0), 1.0), now, sum = 0; int cnt = segs[0].
     second:
for (int j = 1; j < segs.size(); ++j){now = min(max(segs[j].</pre>
    first, 0.0), 1.0); if (!cnt)sum += now - pre; cnt += segs
     [i].second:pre = now:}ans += cross(a, b) * sum:}}return
     ans * 0.5:
// contains all points p such that: cross(b - a, p - a) >= 0
struct HP{PT a, b;HP() {}HP(PT a, PT b) : a(a), b(b) {}HP(
     const HP &rhs) : a(rhs.a), b(rhs.b) {}
int operator<(const HP &rhs) const{PT p = b - a;PT q = rhs.b</pre>
int fp = (p.y < 0 \mid | (p.y == 0 \&\& p.x < 0)); int fq = (q.y < 0)
    0 \mid \mid (q.y == 0 \&\& q.x < 0));
if (fp != fq)return fp == 0;if (cross(p, q))return cross(p,
    q) > 0; return cross(p, rhs.b - a) < 0; PT
    line_line_intersection(PT a, PT b, PT c, PT d)
{b = b - a; d = c - d; c = c - a; return a + b * cross(c, d)}
     cross(b, d):}
PT intersection(const HP &v){return line_line_intersection(a
     , b, v.a, v.b);}};
int check(HP a, HP b, HP c){return cross(a.b - a.a, b.
    intersection(c) - a.a) > -eps: //-eps to include
     polygons of zero area (straight lines, points)}
// consider half-plane of counter-clockwise side of each
// if lines are not bounded add infinity rectangle
// returns a convex polygon, a point can occur multiple
    times though
// complexity: O(n log(n))
vector<PT> half plane intersection(vector<HP> h){sort(h.
    begin(), h.end());vector<HP> tmp;
for (int i = 0; i < h.size(); i++){if (!i || cross(h[i].b -</pre>
    h[i].a, h[i-1].b-h[i-1].a) {tmp.push_back(h[i])
     : }}
h = tmp: vector < HP > q(h.size() + 10): int ah = 0, qe = 0
for (int i = 0; i < h.size(); i++){</pre>
while (qe - qh > 1 \&\& ! check(h[i], q[qe - 2], q[qe - 1]))qe
while (qe - qh > 1 && !check(h[i], q[qh], q[qh + 1]))qh++;q[
    qe++] = h[i];}
while (qe - qh > 2 \&\& ! check(q[qh], q[qe - 2], q[qe - 1]))qe
while (qe - qh > 2 \&\& ! check(q[qe - 1], q[qh], q[qh + 1]))qh
```

```
vector<HP> res:
for (int i = qh; i < qe; i++)res.push_back(q[i]);vector<PT>
if (res.size() > 2){for (int i = 0: i < res.size(): i++){</pre>
     hull.push_back(res[i].intersection(res[(i + 1) % ((int)
     res.size())])):}}
return hull:}
// rotate the polygon such that the (bottom, left)-most
     point is at the first position
void reorder_polygon(vector<PT> &p){int pos = 0;for (int i =
      1; i < p.size(); i++){if (p[i].y < p[pos].y || (sign(p
     [i].v - p[pos].v) == 0 && p[i].x < p[pos].x))pos = i;}
     rotate(p.begin(), p.begin() + pos, p.end());}
// a and b are convex polygons
// returns a convex hull of their minkowski sum
// min(a.size(), b.size()) >= 2
// https://cp-algorithms.com/geometry/minkowski.html
vector<PT> minkowski_sum(vector<PT> a, vector<PT> b){
reorder polygon(a):reorder polygon(b):
int n = a.size(), m = b.size(); int i = 0, i = 0; a.push back(
     a[0]);a.push_back(a[1]);b.push_back(b[0]);b.push_back(b
     [1]):vector<PT> c:
while (i < n \mid j < m)\{c.push\_back(a[i] + b[j]);double p =
     cross(a[i + 1] - a[i], b[j + 1] - b[j]); if (sign(p) >=
     0)++i;if (sign(p) <= 0)++j;}
return c:}
// returns the area of the intersection of the circle with
     center c and radius r
// and the triangle formed by the points c, a, b
double _triangle_circle_intersection(PT c, double r, PT a,
     PT b){
double sd1 = dist2(c, a), sd2 = dist2(c, b);
if (sd1 > sd2)swap(a, b), swap(sd1, sd2):double sd = dist2(a
     , b); double d1 = sqrtl(sd1), d2 = sqrtl(sd2), d = sqrt(sd2)
     sd):double x = abs(sd2 - sd - sd1) / (2 * d):double h =
      sartl(sd1 - x * x):
if (r \ge d2)return h * d / 2; double area = 0;
if (sd + sd1 < sd2)\{if (r < d1)area = r * r * (acos(h / d2))\}
     -a\cos(h/d1)) / 2;else{area = r * r * (acos(h/d2) -
      acos(h / r)) / 2;double y = sqrtl(r * r - h * h);area
     += h * (v - x) / 2:}
else{if (r < h)area = r * r * (acos(h / d2) + acos(h / d1))
     / 2:else{area += r * r * (acos(h / d2) - acos(h / r)) / }
      2; double y = sqrtl(r * r - h * h); area += h * y / 2; if
      (r < d1){area += r * r * (acos(h / d1) - acos(h / r))}
     / 2:area += h * v / 2:}elsearea += h * x / 2:}}
return area;}
// intersection between a simple polygon and a circle
double polygon_circle_intersection(vector<PT> &v, PT p,
     double r){int n = v.size();double ans = 0.00;PT org =
```

```
{0, 0}:
for (int i = 0; i < n; i++){int x = orientation(p, v[i], v[(
    i + 1) % n]);if (x == 0)continue;double area =
    _triangle_circle_intersection(org, r, v[i] - p, v[(i +
    1) \% n] - p); if (x < 0)ans -= area; elseans += area;}
return abs(ans):}
// find a circle of radius r that contains as many points as
     possible
// O(n^2 \log n):
double maximum_circle_cover(vector<PT> p, double r, circle &
    c) {int n = p.size():int ans = 0:int id = 0:double th =
for (int i = 0; i < n; ++i){
// maximum circle cover when the circle goes through this
vector<pair<double, int>> events = {{-PI, +1}, {PI, -1}};
for (int j = 0; j < n; ++j){if (j == i)continue;double d =
    dist(p[i], p[j]);if (d > r * 2)continue;
double dir = (p[j] - p[i]).arg();double ang = acos(d / 2 / r
    ):double st = dir - ang. ed = dir + ang:
if (st > PI)st -= PI * 2;if (st <= -PI)st += PI * 2;if (ed >
     PI)ed -= PI * 2:if (ed <= -PI)ed += PI * 2:
events.push_back({st - eps, +1}); // take care of precisions
events.push_back({ed, -1}); if (st > ed){events.push_back({-
    PI, +1}); events.push_back({+PI, -1});}}sort(events.
    begin(), events.end()); int cnt = 0; for (auto &&e :
    events){cnt += e.second:if (cnt > ans){ans = cnt:id = i
     ;th = e.first;}}PT w = PT(p[id].x + r * cos(th), p[id]
    l.v + r * sin(th)):c = circle(w, r): // best circle
return ans;}
// radius of the maximum inscribed circle in a convex
double maximum_inscribed_circle(vector<PT> p){int n = p.size
     (); if (n \le 2) return 0; double 1 = 0, r = 20000;
while (r - 1 > eps)\{double mid = (1 + r) * 0.5; vector < HP > h;
    const int L = 1e9;h.push_back(HP(PT(-L, -L), PT(L, -L))
    );h.push_back(HP(PT(L, -L), PT(L, L)));h.push_back(HP(
    PT(L, L), PT(-L, L)));h.push_back(HP(PT(-L, L), PT(-L,
    -L))); for (int i = 0; i < n; i++) {PT z = (p[(i + 1) % n
    ] - p[i]).perp():z = z.truncate(mid):PT v = p[i] + z. q
     = p[(i + 1) \% n] + z;h.push_back(HP(p[i] + z, p[(i +
    1) % n] + z)): \rector < PT > nw = half plane intersection(
    h); if (!nw.empty())1 = mid; elser = mid;}
return 1;}
// ear decomposition, O(n^3) but faster
vector<vector<PT>> triangulate(vector<PT> p){
vector<vector<PT>> v:
while (p.size() >= 3){
```

```
n-1: i-1: int nxt = i == n-1?0: i+1: int
     ori = orientation(p[i], p[pre], p[nxt]); if (ori < 0){</pre>
     int ok = 1;for (int j = 0; j < n; j++){if (j == i || j</pre>
     == pre || j == nxt)continue; if (is_point_in_triangle(p[
     i], p[pre], p[nxt], p[i]) < 1){ok = 0:break;}}if (ok){v
     .push_back({p[pre], p[i], p[nxt]});p.erase(p.begin() +
     i):break:}}}
return v;}
struct star{
int n: // number of sides of the star
double r; // radius of the circumcircle
star(int _n, double _r){n = _n;r = _r;}
double area(){double theta = PI / n;double s = 2 * r * sin(
     theta); double R = 0.5 * s / tan(theta); double a = 0.5 *
     n * s * R:double a2 = 0.25 * s * s / tan(1.5 * theta):
    return a - n * a2:}}:
// given a list of lengths of the sides of a polygon in
     counterclockwise order
// returns the maximum area of a non-degenerate polygon that
      can be formed using those lengths
double get_maximum_polygon_area_for_given_lengths(vector
     double> v){
if (v.size() < 3){return 0:}
int m = 0:double sum = 0:
for (int i = 0; i < v.size(); i++){if (v[i] > v[m]){m = i;}}
     sum += v[i]:}
if (sign(v[m] - (sum - v[m])) >= 0){return 0: // no non-
     degenerate polygon is possible}
// the polygon should be a circular polygon
// that is all points are on the circumference of a circle
double 1 = v[m] / 2, r = 1e6; // fix it correctlyint it =
     60:
auto ang = [](double x, double r) { // x = length of the }
     chord. r = radius of the circle
return 2 * asin((x / 2) / r):}:
auto calc = [=](double r){double sum = 0;for (auto x : v){
     sum += ang(x, r);}return sum; };
// compute the radius of the circle
while (it--)\{double\ mid = (l+r) / 2; if\ (calc(mid) <= 2 *
    PI){r = mid:}else{l = mid:}}
if (calc(r) \le 2 * PI - eps) \{ // the center of the circle is
      outside the polygon
auto calc2 = [&](double r){double sum = 0:for (int i = 0: i
     \langle v.size(); i++\rangle \{double x = v[i]; double th = ang(x, r);
     if (i != m){sum += th:}else{sum += 2 * PI - th:}}return
     sum:  }: 1 = v[m] / 2:r = 1e6: it = 60: while (it--){
```

## 5 Graph

## **5.1** Flow

```
lli n. m: vector<set<lli>> roads: vector<vector<lli>> adim:
     vector<lli> parents;
lli bfs(lli s. lli t){
fill(parents.begin(), parents.end(), -1); parents[s] = -2;
queue<pair<lli, lli>> q; q.push({s, LLONG_MAX});
while (!q.empty()){ lli ind = q.front().first; lli flow = q.
     front().second; q.pop();
for (lli cind: roads[ind]){ if (parents[cind] == -1 && adjm[
     ind][cind] > 0){
 parents[cind] = ind; if (cind == t) return min(flow, adjm[
      indl[cind]):
 q.push({cind, min(adjm[ind][cind], flow)});}}} return 0; }
lli getFlow(){
lli totflow = 0, nflow:
while (nflow = bfs(0, n - 1)) \{ totflow += nflow; lli ind = n \}
while (ind != 0){ adim[ind][parents[ind]] += nflow:
 adjm[parents[ind]][ind] -= nflow; ind = parents[ind];
}}
return totflow:
```

## 6 NumberTheory

#### 6.1 CRT

```
using T = __int128;
// ax + by = __gcd(a, b)
// returns __gcd(a, b)
```

#### 6.2 NTT shortened

```
const int N = 1 \ll 20;
const int mod = 998244353:
const int root = 3;
int lim, rev[N], w[N], wn[N], inv_lim;
void reduce(int &x) { x = (x + mod) \% mod: }
int POW(int x, int y, int ans = 1) {
for (; y; y >>= 1, x = (long long) x * x \% mod) if (y & 1)
     ans = (long long) ans * x % mod;
return ans:}
void precompute(int len) {
 \lim_{n \to \infty} = \min[0] = 1; int s = -1; while (\lim_{n \to \infty} < \lim_{n \to \infty} < < 1,
 for (int i = 0: i < lim: ++i) rev[i] = rev[i >> 1] >> 1 |
      (i & 1) << s:
 const int g = POW(root, (mod - 1) / lim): inv lim = POW(
      lim. mod - 2):
 for (int i = 1; i < lim; ++i) wn[i] = (long long) wn[i -</pre>
      1] * g % mod:
void ntt(vector<int> &a. int tvp) {
 for (int i = 0: i < \lim_{i \to +i} if (i < rev[i]) swap(a[i], a
      [rev[i]]):
 for (int i = 1; i < lim; i <<= 1) { for (int j = 0, t =
      \lim / i / 2; j < i; ++j) w[j] = wn[j * t];
   for (int j = 0; j < \lim_{j \to \infty} j += i << 1) { for (int k = 0; k
       const int x = a[k + j], y = (long long) a[k + j + i]
            * w[k] % mod:
       reduce(a[k + j] += y - mod), reduce(a[k + j + i] = x
            - v); } }}
```

```
if (!typ) { reverse(a.begin() + 1, a.begin() + lim);
    for (int i = 0; i < lim; ++i) a[i] = (long long) a[i] *
        inv_lim % mod;}
}
vector<int> multiply(vector<int> &f, vector<int> &g) {
    if (f.empty() or g.empty()) return {};
    int n = (int)f.size() + (int)g.size() - 1;
    if (n == 1) return {(int)((long long) f[0] * g[0] % mod)};
    precompute(n); vector<int> a = f, b = g; a.resize(lim); b.
        resize(lim);
    ntt(a, 1), ntt(b, 1); for (int i = 0; i < lim; ++i) a[i] =
        (long long) a[i] * b[i] % mod;
    ntt(a, 0); a.resize(n + 1); return a;
}</pre>
```

## 6.3 NTT with any prime mod short

```
const int N = 3e5 + 9, mod = 998244353;
struct base {
 double x, y;
 base() { x = y = 0; }
 base(double x, double y): x(x), y(y) { }
inline base operator + (base a. base b) { return base(a.x +
    b.x, a.v + b.v); }
inline base operator - (base a, base b) { return base(a.x -
    b.x. a.v - b.v): }
inline base operator * (base a, base b) { return base(a.x *
    b.x - a.y * b.y, a.x * b.y + a.y * b.x); }
inline base coni(base a) { return base(a.x. -a.v): }
int lim = 1; vector<base> roots = {{0, 0}, {1, 0}}; vector<</pre>
     int > rev = \{0, 1\}:
const double PI = acosl(- 1.0);
void ensure_base(int p) {
 if(p <= lim) return: rev.resize(1 << p):</pre>
  for(int i = 0; i < (1 << p); i++) rev[i] = (rev[i >> 1] >>
        1) + ((i \& 1) << (p - 1));
 roots.resize(1 << p); while(lim < p) { double angle = 2 *</pre>
      PI / (1 << (lim + 1)):
    for(int i = 1 << (lim - 1); i < (1 << lim); i++) {</pre>
     roots[i << 1] = roots[i]; double angle_i = angle * (2 *</pre>
           i + 1 - (1 << lim));
     roots[(i << 1) + 1] = base(cos(angle i), sin(angle i));
           } lim++:
 }
}
void fft(vector<base> &a, int n = -1) {
```

```
if(n == -1) n = a.size(): assert((n & (n - 1)) == 0): int
    zeros = builtin ctz(n):
ensure_base(zeros); int shift = lim - zeros;
for(int i = 0; i < n; i++) if(i < (rev[i] >> shift)) swap(a[
    i], a[rev[i] >> shift]);
for(int k = 1: k < n: k <<= 1) { for(int i = 0: i < n: i +=
for(int j = 0; j < k; j++) { base z = a[i + j + k] * roots[j]
     + k]; a[i + j + k] = a[i + j] - z;
a[i + j] = a[i + j] + z; \}
//eq = 0: 4 FFTs in total
//eq = 1: 3 FFTs in total
vector<int> multiply(vector<int> &a. vector<int> &b. int eq
int need = a.size() + b.size() - 1; int p = 0; while((1 << p
    ) < need) p++:
ensure_base(p); int sz = 1 << p; vector<base> A, B; if(sz >
     (int)A.size()) A.resize(sz):
for(int i = 0: i < (int)a.size(): i++) {int x = (a[i] \% mod
    + mod) % mod;
 A[i] = base(x & ((1 << 15) - 1), x >> 15); }
fill(A.begin() + a.size(), A.begin() + sz, base{0, 0});
fft(A, sz); if(sz > (int)B.size()) B.resize(sz);
if(eq) copy(A.begin(), A.begin() + sz, B.begin());
else { for(int i = 0; i < (int)b.size(); i++) {</pre>
 int x = (b[i] % mod + mod) % mod; B[i] = base(x & ((1 <<</pre>
     15) - 1), x >> 15); }
 fill(B.begin() + b.size(), B.begin() + sz, base{0, 0}); fft
     (B. sz):}
double ratio = 0.25 / sz;
base r2(0, -1), r3(ratio, 0), r4(0, -ratio), r5(0, 1);
for(int i = 0; i <= (sz >> 1); i++) { int j = (sz - i) & (sz
 base a1 = (A[i] + conj(A[j])), a2 = (A[i] - conj(A[j])) *
 base b1 = (B[i] + conj(B[j])) * r3, b2 = (B[i] - conj(B[j])
     ) * r4:
 if(i != j) { base c1 = (A[i] + coni(A[i])), c2 = (A[i] - coni(A[i]))
     coni(A[i])) * r2:
 base d1 = (B[i] + coni(B[i])) * r3, d2 = (B[i] - coni(B[i]))
      1)) * r4:
 A[i] = c1 * d1 + c2 * d2 * r5; B[i] = c1 * d2 + c2 * d1;
 A[i] = a1 * b1 + a2 * b2 * r5; B[i] = a1 * b2 + a2 * b1; 
 fft(A, sz); fft(B, sz); vector<int> res(need);
 for(int i = 0: i < need: i++) { long long aa = A[i].x +</pre>
   long long bb = B[i].x + 0.5; long long cc = A[i].y + 0.5;
  res[i] = (aa + ((bb \% mod) << 15) + ((cc \% mod) << 30))%
```

## 6.4 bigmod

```
11 bigmod(11 a, 11 b, 11 n) {11 res = 1;
if (b == 0) return 1; a = a % n; if (a == 0)
return 0; while (b > 0) {if (b % 2)res = (res * a) % n;
b = b / 2; a = (a * a) % n;} return res;}
```

#### 6.5 extended euclid

```
#include <bits/stdc++.h>
using namespace std;int x, y;
int gcdExtended(int a, int b, int *x, int *y) {
   if (b==0) {*x = 1;*y = 0;return a;}
   int x1, y1;int gcd = gcdExtended(b,a%b,&x1,&y1);
   *x =y1;*y =x1-y1*(a/b);return gcd;}
   int main() { int a = 50, b = 10;
      cout<<"gcd "<<gcdExtended(a, b, &x, &y)<<endl;;
      cout<<x<<" " <<y<<endl;return 0;}</pre>
```

#### 6.6 fft

```
base wn = base(cos(ang), (inv ? 1. : -1.) * sin(ang)), w; for | 11 find_all_solutions(11 a, 11 b, 11 c, 11 minx, 11 maxx, 11 | // No of terms in the Recurrence Relation.
     (int i = 0, i, k: i < n: i += m) {
for(w = base(1, 0), j = i, k = i + 1; j < k; ++j, w = w * wn
base t = w * p[i + 1]; p[i + 1] = p[i] - t; p[i] = p[i] + t
     : }}}
if(inv) for(int i = 0; i < n; ++i) p[i].a /= n, p[i].b /= n</pre>
     ;}
vector<long long> multiply(vector<int> &a, vector<int> &b) {
int n = a.size(), m = b.size(), t = n + m - 1, sz = 1; while()
     sz < t) sz <<= 1:
vector < base > x(sz), v(sz), z(sz) : for(int i = 0 : i < sz: ++i
    ) {
x[i] = i < (int)a.size() ? base(a[i], 0) : base(0, 0);
v[i] = i < (int)b.size() ? base(b[i], 0) : base(0, 0);}
fft(x), fft(y); for(int i = 0; i < sz; ++i) z[i] = x[i] * y[i]
    l:fft(z, 1):
vector<long long> ret(sz);for(int i = 0; i < sz; ++i) ret[i]</pre>
      = (long long) round(z[i].a):
// while((int)ret.size() > 1 && ret.back() == 0) ret.
    pop_back();
return ret;}
```

## linear diphantine equation

```
#include<bits/stdc++.h> using namespace std;using ll = long
     long:
ll extended_euclid(ll a, ll b, ll &x, ll &y) {
11 xx = y = 0; 11 yy = x = 1; while (b) {
11 q = a / b; 11 t = b; b = a % b; a = t;
t = xx: xx = x - q * xx: x = t:
t = yy; yy = y - q * yy; y = t;}return a;}
// a*x+b*y=c. returns valid x and y if possible.
// all solutions are of the form (x0 + k * b / g, y0 - k * b | 6.8 linear sieve
bool find any solution (11 a. 11 b. 11 c. 11 &x0. 11 &v0. 11
      &g) {
if (a == 0 and b == 0) {if (c) return false:
x0 = y0 = g = 0; return true;}
g = extended_euclid (abs(a), abs(b), x0, y0); if (c % g != 0)
      return false:
x0 *= c / g; y0 *= c / g; if (a < 0) x0 *= -1; if (b < 0) y0 *=
return true: \void shift solution(ll &x. ll &v. ll a. ll b.
    11 cnt) {
x += cnt * b; y -= cnt * a; 
// returns the number of solutions where x is in the range[
    minx, maxx] and y is in the range[miny, maxy]
```

```
miny, ll maxy) {ll x, y, g;
if (find_any_solution(a, b, c, x, y, g) == 0) return 0;if (a
                == 0 \text{ and } b == 0) {
 assert(c == 0);return 1LL * (maxx - minx + 1) * (maxy - miny
if (a == 0) {return (maxx - minx + 1) * (miny <= c / b and c</pre>
               / b <= maxv):}
               / a <= maxx);}
 a \neq g, b \neq g: 11 sign a = a > 0? +1: -1:11 sign b = b > 0
               ? +1 : -1:
 shift_solution(x, y, a, b, (minx - x) / b);if (x < minx)</pre>
             shift_solution(x, y, a, b, sign_b);
 if (x > maxx) return 0;ll lx1 = x;shift_solution(x, y, a, b,
                 (\max - x) / b);
 if (x > maxx) shift_solution (x, y, a, b, -sign_b);ll rx1 =
             x; shift_solution(x, y, a, b, -(miny - y) / a);
 if (v < minv) shift_solution (x, y, a, b, -sign_a);if (y >
            maxv) return 0:
11 1x2 = x; shift_solution(x, y, a, b, -(maxy - y) / a); if (y)
               > maxy) shift_solution(x, y, a, b, sign_a);
 11 \text{ rx2} = x; \text{if } (1x2 > rx2) \text{ swap } (1x2, rx2); 11 1x = max(1x1, rx2); 11 1x = m
            1x2):11 rx = min(rx1, rx2):
 if (lx > rx) return 0; return (rx - lx) / abs(b) + 1;}
 int32_t main() { ios_base::sync_with_stdio(0);cin.tie(0);int
               t. cs = 0: cin >> t:
 while (t--) {11 a, b, c, x1, x2, y1, y2; cin >> a >> b >> c
            >> x1 >> x2 >> y1 >> y2;
 cout << "Case " << ++cs << ": " << find all solutions(a, b,
             -c, x1, x2, y1, y2) << '\n';}
 return 0;}// https://lightoj.com/problem/solutions-to-an-
             equation
```

```
const 11 N = 1e7 + 7;bool isPrime[N];vector < 11 > p;
void lin_sieve () {ll i; for (i = 2; i < N; i++) {</pre>
if (!isPrime[i]) p.push_back (i);for (ll j : p) {
if (i * j >= N) break; isPrime[i * j] = 1; if (i % j == 0)
    break: }}}
```

## 6.9 mat expo

```
mt19937 mt_rand(chrono::high_resolution_clock::now().
    time since epoch().count()):
const int mod = 1e9 + 7;// const int N = 5e5 + 6;
```

```
const int N = 4;const long long M = 1000000007;
                                                              // Multiplies two matrices A and B and stores the result in
                                                              void multiply (long long A[N][N], long long B[N][N]){long
                                                                   long R[N][N]:
                                                              // Multiply A and B and store result in R.
                                                               for (int i = 0; i < N; i++){for (int j = 0; j < N; j++){
if (b == 0) {return (maxy - miny + 1) * (minx <= c / a and c | R[i][j] = 0; for (int k = 0; k < N; k++){R[i][j] = (R[i][j] +
                                                                    A[i][k] * B[k][i]) % M;}}
                                                               // Copy contents of R in A.
                                                               for (int i = 0: i < N: i++){for (int i = 0: i < N: i++){A[i
                                                                   ][i] = R[i][i];}}}
                                                               // Raise matrix A to the power of n in O(log n).
                                                              void power_matrix (long long A[N][N], ll n){long long B[N][N
                                                                   ];// B = Identity Matrix.
                                                               for (int i = 0: i < N: i++){for (int i = 0: i < N: i++){B[i
                                                                   ][i] = A[i][i]; \}
                                                               // A = \bar{A} * A ^ (n - 1).
                                                               n = n - 1: while (n > 0) {// If n is odd, A = A * B.
                                                              if (n \& 1) multiply (A, B); // B = B * B.
                                                               multiply (B, B); // n = n / 2.
                                                               n = n \gg 1;}// A = Coefficient Matrix, B = Base Matrix.
                                                               // It returns the nth term of the recurrence relation formed
                                                                    from A and B in O(\log n).
                                                              long long solve_recurrence (long long A[N][N], long long B[N
                                                                   [] [1]. ll n){//Base Cases.
                                                               if (n < N) return B[N - 1 - n][0]; // A = A ^ <math>(n - N + 1).
                                                               power_matrix (A, n - N + 1);long long result = 0;for (int i
                                                                   = 0: i < N: i++)
                                                               result = (result + A[0][i] * B[i][0]) % M; return result;}
                                                               void Solve(){
                                                               /*
                                                               The recurrence relation used here is: -
                                                               R(n) = 2 * R(n-1) + R(n-2) + 3 * R(n-3) + 3.
                                                               Base Cases: R(0) = 1, R(1) = 2, R(2) = 3.
                                                               // Forming the Coefficient Matrix
                                                              0}, {0, 0, 0, 1}};
                                                               //Forming the Base Matrix
                                                               long long B[N][1] = \{\{3\}, \{2\}, \{1\}, \{3\}\}; 11 \text{ n;cin} >> \text{n;if} (n)
                                                                    <= 2){
                                                               cout << n + 1 << endl;return ;}long long R_n =</pre>
                                                                   solve_recurrence (A, B, n + 1);
                                                               cout << R n << endl:}</pre>
```

#### 6.10 mobius

```
void mobius() {mob[1] = 1;for (int i = 2; i < N; i++){
mob[i]--;for (int j = i + i; j < N; j += i) {mob[j] -= mob[i
      ];}}</pre>
```

#### 6.11 ncr for mod

```
11 fact[200008];11 bigmod(l1 b, l1 p) {if(p == 0) return 1;
11 h = bigmod(b, p/2);h = h * h % mod;if(p&1) h = h * b %
    mod;
return h;}11 ncr(l1 n, l1 r) {if(n<r)return 0;
return fact[n] * bigmod(fact[r] * fact[n-r] % mod, mod - 2)
    % mod;}
void Fact(){fact[0] = 1;for(int i=1; i<=200002; i++)
fact[i] = fact[i-1] * i % mod;}</pre>
```

#### 6.12 ntt

```
const int N = 1 << 20:const int mod = 998244353:const int</pre>
int lim, rev[N], w[N], wn[N], inv_lim; void reduce(int &x) {
     x = (x + mod) \% mod: 
int POW(int x, int y, int ans = 1) {for (; y; y \gg 1, x = (
     long long) x * x % mod)
if (y & 1) ans = (long long) ans * x % mod; return ans; }void
     precompute(int len) {
lim = wn[0] = 1; int s = -1; while (lim < len) lim <<= 1, ++s
for (int i = 0: i < \lim +i) rev[i] = rev[i >> 1] >> 1 | (i
const int g = POW(root, (mod - 1) / lim);inv_lim = POW(lim,
for (int i = 1; i < \lim_{i \to +i} wn[i] = (long long) wn[i - 1]
    * g % mod;}
void ntt(vector<int> &a. int tvp) {
for (int i = 0; i < lim; ++i) if (i < rev[i]) swap(a[i], a[</pre>
for (int i = 1: i < lim: i <<= 1) {for (int i = 0, t = lim /
     i / 2: i < i: ++i)
w[j] = wn[j * t]; for (int j = 0; j < lim; j += i << 1) {for}
     (int k = 0; k < i; ++k) {
const int x = a[k + j], y = (long long) a[k + j + i] * w[k]
reduce(a[k + j] += y - mod), reduce(a[k + j + i] = x - y)
     ;}}}if (!typ) {
reverse(a.begin() + 1, a.begin() + lim); for (int i = 0; i <</pre>
    lim: ++i)
```

```
a[i] = (long long) a[i] * inv_lim % mod;}}
vector<int> multiply(vector<int> &f, vector<int> &g) {
   if (f.empty() or g.empty()) return {}; int n = (int)f.size()
        + (int)g.size() - 1;
   if (n == 1) return {(int)((long long) f[0] * g[0] % mod)};
        precompute(n);
vector<int> a = f, b = g;a.resize(lim); b.resize(lim);
ntt(a, 1), ntt(b, 1); for (int i = 0; i < lim; ++i)
a[i] = (long long) a[i] * b[i] % mod;ntt(a, 0);a.resize(n +
        1); return a;}</pre>
```

## 6.13 phi

## 6.14 pollard rho

```
using 11 = long long;namespace PollardRho {
mt19937 rnd(chrono::steady_clock::now().time_since_epoch().
 const int P = 1e6 + 9;11 seq[P];int primes[P], spf[P];
 inline 11 add mod(11 x, 11 v, 11 m) {return (x += v) < m ? x
 inline 11 mul_mod(11 x, 11 y, 11 m) {11 res = __int128(x) *
 return res;// 11 res = x * y - (11)((long double)x * y / m +
       0.5) * m:
 // return res < 0 ? res + m : res:
 }inline 11 pow_mod(11 x, 11 n, 11 m) {11 res = 1 % m;
for (; n; n >>= 1) {if (n & 1) res = mul_mod(res, x, m);
 x = mul_mod(x, x, m);}return res;}
 // O(it * (logn)^3), it = number of rounds performed
 inline bool miller rabin(ll n) {if (n <= 2 | | (n & 1 ^ 1))</pre>
     return (n == 2);
 if (n < P) return spf[n] == n; ll c, d, s = 0, r = n - 1; for
      (: !(r \& 1): r >>= 1, s++) {}
 // each iteration is a round
```

```
for (int i = 0; primes[i] < n && primes[i] < 32; i++) {c =
    pow_mod(primes[i], r, n);
for (int j = 0; j < s; j++) {d = mul_mod(c, c, n); if (d == 1</pre>
      && c != 1 && c != (n - 1)) return false:
c = d;}if (c != 1) return false;}return true;} void init() {
    int cnt = 0:
for (int i = 2; i < P; i++) { if (!spf[i]) primes[cnt++] =</pre>
    spf[i] = i:
for (int j = 0, k; (k = i * primes[j]) < P; j++) { spf[k] =</pre>
    primes[j];
if (spf[i] == spf[k]) break;}}}// returns O(n^{(1/4)})
ll pollard rho(ll n) {while (1) {ll x = rnd() \% n, v = x, c}
    = rnd() % n, u = 1, v, t = 0;
11 *px = seq, *py = seq; while (1) {*py++ = y = add_mod(
    mul_mod(y, y, n), c, n);
*py++ = y = add_mod(mul_mod(y, y, n), c, n); if ((x = *px++)
    == v) break:v = u:
u = mul_mod(u, abs(y - x), n); if (!u) return __gcd(v, n);
if (++t == 32) {t = 0:if ((u = gcd(u, n)) > 1 && u < n)
    return u:}}
if (t && (u = _{-gcd(u, n)}) > 1 && u < n) return u;}}
vector<ll> factorize(ll n) {if (n == 1) return vector <ll>()
    ;if (miller_rabin(n)) return vector<ll> {n};
vector <ll> v, w; while (n > 1 && n < P) { v.push_back(spf[n</pre>
    ]);n /= spf[n];}
if (n >= P) { ll x = pollard_rho(n); v = factorize(x); w =
    factorize(n / x):
v.insert(v.end(), w.begin(), w.end());}return v;}}
int32_t main() { ios_base::sync_with_stdio(0); cin.tie(0);
    PollardRho::init():
int t; cin >> t; while (t--) { ll n; cin >> n; auto f =
    PollardRho::factorize(n);
sort(f.begin(), f.end()): cout << f.size() << ''; for (auto</pre>
     x: f) cout << x << ' '; cout << '\n';}return 0;}
// https://judge.yosupo.jp/problem/factorize
```

#### 6.15 power tower

```
const int N = 1e5 + 9; using ll = long long; map<ll, ll> mp;
ll phi(ll n) { if (mp.count(n)) return mp[n]; ll ans = n, m =
    n;
for (ll i = 2; i * i <= m; i++) { if (m % i == 0) {while (m
    % i == 0) m /= i;
ans = ans / i * (i - 1); } if (m > 1) ans = ans / m * (m - 1)
    ; return mp[n] = ans; }
inline ll MOD(ll x, ll m) { if (x < m) return x; return x % m
    + m; }
ll power(ll n, ll k, ll mod) { ll ans = MOD(1, mod); while (
    k) {</pre>
```

#### 6.16 sieve all

```
vector<bool>prime(N, true);vector<int>vec ;
void seive() { prime[0] = false:prime[1] = false :
for (int i = 2; i * i < N; i++) {if (prime[i]) {</pre>
for (int j = i * i; j < N; j += i) {prime[j] = false ;}}}</pre>
for (int i = 2; i < N; i++) if (prime[i])vec.push_back(i);}</pre>
void pro() { int n; cin >> n ; int ans = 1 ;
for (auto it : vec) { if (it * it > n)break ;
if (n % it == 0) { int cnt = 1 : while (n % it == 0) {
n \neq it : cnt++ : ans *= cnt : } if (n > 1) ans *= 2 :
cout << ans - 1 << endl;}/// Segmented Sieve</pre>
void pro() { int n, m:cin >> n >> m: bool ara[m - n + 1] :
memset(ara, true, sizeof(ara)); for (auto it : sve){ if (it
    * it > m)break:
int fmpl = (n + it - 1) / it; fmpl *= it ;int strt = max(
    fmpl. it * it):
for (int j = strt; j <= m; j += it) { ara[j - n] = false ;}}</pre>
if (n == 1)ara[0] = false ; for (int i = n; i <= m; i++)if (
    ara[i - n]) cout << i << endl:}
```

#### 6.17 totient

```
#include<bits/stdc++.h>using namespace std ;
const double pi = 2 * acos (0.0) ;const int N=5000006 ;
const int INF=INT_MAX;const int mod=1000000007 ;
vector<int>phi(N,0) ;void totient_seive(){
for(int i=1; i<N; i++)phi[i]=i;for(int i=2; i<N; i++){
   if(phi[i]==i) { for(int j=i; j<N; j+=i) {
    phi[j]= ( phi[j] - (phi[j]/i) ) ;}}}}
//FOR ANY SINGLE NUMBER ___ CALCULTAING THE VALUE OF PHI
        USING SQRT COMPLEXITY</pre>
```

```
const int N=1000009 :const int INF=INT MAX :const int mod
     =1000000007 :
int sqrt_phi(int n){ int ans=n ; for(int i=2; i*i<=n; i++){</pre>
if(n\%i==0){while(n\%i==0)n/=i :ans-=(ans/i) :}}
if(n>1) ans-=(ans/n) ;return ans ;}
//CALCULATING PHI VALUE USING SUM OF PHI....
//SUM OF TOTALENT VALUE FOR ALL DIVISOR OF N IS EQUAL TO N
// PHI(10)+PHI(5)+PHI(2)+PHI(1)
//= 4 + 4 + 1 + 1
// 10
const int N=10000007 :const int INF=INT MAX:const int mod
     =1000000007 :
vector<int>phi(N,0) ;void calcphi(){phi[0]=0 ;phi[1]=1 ;
for(int i=2; i<N; i++) phi[i]=i-1;</pre>
//(loop er modde 1 divisor hisabe ani nai....tai 1 er
     contribution bad)
for(int i=2; i<N; i++){ for(int j=2*i; j<N; j+=i) {phi[j]-=</pre>
     phi[i] ;}}}
```

## 7 String

#### 7.1 Aho

```
#include<bits/stdc++.h>
using namespace std; const int N = 1e5 + 9; //credit: Alpha_Q
struct AC { int N, P; const int A = 26; vector <vector <int</pre>
     >> next:
vector <int> link, out_link; vector <vector <int>> out;
AC(): N(0), P(0) {node();} int node() { next.emplace_back(A,
link.emplace back(0): out link.emplace back(0):
out.emplace_back(0); return N++;} inline int get (char c) {
     return c - 'a':
} int add_pattern (const string T) {int u = 0; for (auto c :
if (!next[u][get(c)]) next[u][get(c)] = node(): u = next[u][
     get(c)];}
out[u].push back(P):return P++: \rangle void compute() \{ queue < int >
for (q.push(0); !q.empty();) {int u = q.front(); q.pop();
for (int c = 0; c < A; ++c) { int v = next[u][c];</pre>
if (!v) next[u][c] = next[link[u]][c];else {link[v] = u ?
     next[link[u]][c] : 0;
out link[v] = out[link[v]].emptv() ? out link[link[v]] :
     link[v];q.push(v);}}}
int advance (int u, char c) { while (u && !next[u][get(c)])
     u = link[u]:
u = next[u][get(c)]; return u;}};
```

```
int32 t main() { ios base::svnc with stdio(0):
cin.tie(0); auto st = clock(); int t, cs = 0; cin >> t;
while (t--) { int n; cin >> n; vector<string> v;
for (int i = 0: i < n: i++) { string s: cin >> s:
v.push_back(s); } sort(v.begin(), v.end());
v.erase(unique(v.begin(), v.end()), v.end()):AC aho:
vector<int> len(n + 3, 0); for (auto s: v) {len[aho.
    add_pattern(s)] = s.size();}
aho.compute();string s; cin >> s;n = s.size();
vector\langle int \rangle dp(n, n + 10); int u = 0; for (int i = 0; i < n; i
char c = s[i]:u = aho.advance(u, c):for (int v = u: v: v =
    aho.out_link[v]) {
for (auto p : aho.out[v]) { dp[i] = min(dp[i], (i - len[p]
    >= 0 ? dp[i - len[p]] : 0) + 1);}}
cout << "Case " << ++cs << ": "; if (dp[n - 1] == n + 10) {
    cout << "impossible\n":}</pre>
else { cout << dp[n - 1] << '\n'; \}}cout << 1.0 * (clock() -
    st) / 1000 << '\n':return 0:}
```

## 7.2 String matching using bitset

```
#include<bits/stdc++.h>
using namespace std; const int N = 1e5 + 9;
vector<int> v;bitset<N>bs[26], oc;
int main() {int i, j, k, n, q, l, r; string s, p;
cin >> s; for(i = 0; s[i]; i++) bs[s[i] - 'a'][i] = 1;
cin >> q;while(q--) {cin >> p;oc.set();
for(i = 0: p[i]: i++) oc &= (bs[p[i] - 'a'] >> i):
cout << oc.count() << endl; // number of occurences</pre>
int ans = N. sz = p.size():int pos = oc. Find first():
v.push_back(pos);pos = oc._Find_next(pos);while(pos < N) {</pre>
v.push_back(pos);pos = oc._Find_next(pos);}
for(auto x : v) cout << x << ', ': // position of occurences</pre>
cout << endl; v.clear(); cin >> 1 >> r; // number of
    occurences from 1 to r.where 1 and r is 1-indexed
if(sz > r - 1 + 1) cout << 0 << endl:else cout <math><< (oc >> (1 + 1))
     -1), count() - (oc >> (r - sz + 1)), count() << endl:
}return 0:}
```

## 7.3 Trie

```
struct node {bool endmark;node* next[26 + 1];node(){
  endmark = false;for (int i = 0; i < 26; i++)next[i] = NULL;}
} * root;void insert(char* str, int len){node* curr = root;
  for (int i = 0; i < len; i++) {int id = str[i] - 'a';
  if (curr->next[id] == NULL)curr->next[id] = new node();
```

## 7.4 kmp

```
// returns the longest proper prefix array of pattern p
// where lps[i]=longest proper prefix which is also suffix
    of p[0...i]
vector<int> build_lps(string p) {int sz = p.size();vector<</pre>
lps.assign(sz + 1, 0); int j = 0; lps[0] = 0; for (int i = 1; i)
     < sz: i++) {
while (j \ge 0 \&\& p[i] != p[j]) \{if (j \ge 1) j = lps[j-1];
    else j = -1;
i++:lps[i] = i:}return lps:}vector<int>ans:
// returns matches in vector ans in 0-indexed
void kmp(vector<int> lps, string s, string p) {int psz = p.
    size(), sz = s.size();
int j = 0; for (int i = 0; i < sz; i++) {while (j >= 0 && p[j
    ] != s[i])
if (j \ge 1) j = lps[j - 1]; else j = -1; j++; if (j == psz) {
j = lps[j - 1];// pattern found in string s at position i-
ans.push_back(i - psz + 1);}
// after each loop we have j=longest common suffix of s[0..i
    ] which is also prefix of p}}
```

## 7.5 manacher

```
struct Manacher {vector<int> p[2];
// p[1][i] = (max odd length palindrome centered at i) / 2 [
    floor division]
// p[0][i] = same for even, it considers the right center
```

## 7.6 palindrome<sub>h</sub> ashing

```
#include <bits/stdc++.h>
 using namespace std;
 vector<vector<long long>> HASH, REV_HASH, POW;
 1000000000 + 9};
 #define lim 1000006 string text, pattern; void init(){
 POW = vector<vector<long long>>(2, vector<long long>(lim));
POW[0][0] = POW[1][0] = 1; for (int b = 0; b < 2; b++)
 for (int j = 1; j < lim; j++)POW[b][j] = (POW[b][j - 1] *</pre>
     BASE[b]) % MOD[b]; return;
}void initHash(string str){int len = str.size();HASH[0][0] =
      HASH[1][0] = 0:
for (int b = 0; b < 2; b++)for (int i = 1; i <= len; i++)
HASH[b][i] = (HASH[b][i-1] * BASE[b] + (str[i-1] - 'a' +
REV_HASH[0][len + 1] = REV_HASH[1][len + 1] = 0; for (int b = 0)
      0: b < 2: b++)
for (int i = len; i; i--)REV_HASH[b][i] = (REV_HASH[b][i +
     1] * BASE[b] + (str[i - 1] - a' + 1)) % MOD[b];
 return: }long long getHash(int left, int right, int hsh){int
     len = (right - left + 1);
long long ret = (HASH[hsh][right] - HASH[hsh][left - 1] *
     POW[hsh][len]) % MOD[hsh]:
 if (ret < 0)ret += MOD[hsh]:return ret:}</pre>
 pair<long long, long long> getHash(int left, int right){long
      long hsh0 = getHash(left, right, 0);
 long long hsh1 = getHash(left, right, 1);return {hsh0, hsh1
 long long getRevHash(int left, int right, int hsh){int len =
      (right - left + 1);
long long ret = (REV_HASH[hsh][left] - REV_HASH[hsh][right +
      1] * POW[hsh][len]) % MOD[hsh];
```

```
if (ret < 0)ret += MOD[hsh]; return ret; pair<long long, long
    long> getRevHash(int left, int right){
long long hsh0 = getRevHash(left, right, 0); long long hsh1 =
    getRevHash(left, right, 1);
return {hsh0, hsh1}; bool palindrome(int l, int r){return
    getHash(l, r) == getRevHash(l, r);}
void solve(){string s = "aaabbabbaaac"; HASH = vector<vector<
    long long>>(2, vector<long long>(s.size() + 5));
REV_HASH = vector<vector<long long>>(2, vector<long long) size() + 5)); initHash(s);
cout << (palindrome(1, s.size())? "YES\n" : "NO\n"); cout << (palindrome(1, s.size() - 1)? "YES\n" : "NO\n");
return; int32_t main(){ios_base::sync_with_stdio(0); cin.tie
    (0); init(); solve(); return 0;}</pre>
```

## 7.7 pallindromic tree

```
// s = "#" + s:
struct PaliTree{ #define sz 26 struct node{ int
lng; int link; int next[sz]; int occ; node(int _
lng){ lng = _lng; link = 0; occ = 0; memset(next
.-1.sizeof(next)): } }: vector<node> tree: strin
g s; int cur; PaliTree(){ tree.push_back(node(-
1)): //img
tree.push back(node(0)): //root
cur = 1; } void clear(){ tree.clear(); tree.push
back(node(-1)): //img
tree.push_back(node(0)); //root
cur = 1; } int get_id(char c){return c-'a';} int
get_link(int now,int i){ char c = s[i]; while(1
){ if(now == 0 or (i-1-tree[now].lng > 0 and s[i -1-tree[now
    ].lng] == c)) break; now = tree[now].
link: } int id = get id(c): return (tree[now].ne
xt[id] == -1)?now:tree[now].next[id]; } void add
(int i) { char c = s[i]: int id = get id(c): whil
e(1){ if(cur == 0 or (i-1-tree[cur].lng > 0 and
s[i-1-tree[cur].lng] == c)) break; cur = tree[cu
r].link: f(cur == 0 \text{ and } s[i] == s[i-1]) cur =
1; if(tree[cur].next[id] == -1){ node tmp(tree[
cur].lng+2); if(tmp.lng == 1) tmp.link = 1; else
tmp.link = get_link(tree[cur].link,i); tree.pus
h_back(tmp); tree[cur].next[id] = tree.size()-1;
} cur = tree[cur].next[id]; tree[cur].occ++; }
void calc(){ for(int i = tree.size()-1; i > 1; i --) tree[
    tree[i].link].occ += tree[i].occ; } };
```

## 7.8 suffix array occurrence of substr in own for (int i = 1, 1 = 0, r = 0; i < n; ++i) fif (i <= r)z[i] = string

```
int Table[N][20], a[N]; void Build(vector<int>lcp){
int n = lcp.size();for (int i = 1; i <= n; i++)</pre>
Table[i][0] = lcp[i - 1]; for (int k = 1; k < 20; k++){
for (int i = 1; i + (1 << k) - 1 <= n; i++)
Table[i][k] = min(Table[i][k-1], Table[i+(1 << (k-1))]
    ][k - 1]):}}
int Query(int 1, int r){1++, r++; int k = log_2(r - 1 + 1);
return min(Table[1][k], Table[r - (1 << k) + 1][k]);}</pre>
pair<int, int>FindRight(int low, int high, int val) // Find
    maximum R such that lcp(low, low+1...)>Val and return
    lcp(low, R)
\{int \ l = low, \ r = high, \ mid: int \ ans = low - 1 : while (1 <= r)
mid = (1 + r) / 2; if (Query(low, mid) > val){ans = mid, 1 =
else r = mid - 1; if (ans == low - 1) return \{low, -1\};
else return {ans + 1, Query(low, ans)};}
void Solve(){string s;cin >> s;SuffixArray ehhe(s);
11 n = s.size();vector<int>p = ehhe.sa;vector<int>lcp;
lcp = ehhe.lcp:Build(lcp):11 ans = 0:
for (int i = 0; i < n; i++){int high = n - 1; int pans = i
    ? lcp[i - 1] : 0:
int len = n - p[i]; while (pans < len){</pre>
pair<int, int> pt = FindRight(i, high, pans); // pt = {
    maximum r such that lcp(i, r)>val, lcp(i, r)}
int right = pt.f;ll templ = right - i + 1;
if (pt.f == i) pt.s = len;ll contr = (pt.s - pans);
ans += (contr * (templ * (templ))); // len of contr occurs
    templ times
high = pt.f;pans = pt.s;}}cout << ans << endl;}
// Problem link : https://codeforces.com/contest/802/problem
    /I
```

## z algo

```
// An element Z[i] of Z array stores length of the longest
// starting from str[i] which is also a prefix of str[0..n
// The first entry of Z array is meaning less as complete
    string is always prefix of itself.
// Here Z[0]=0.
vector<int> z_function(string s) {int n = (int) s.length();
    vector<int> z(n):
```

```
min (r - i + 1, z[i - 1]):
while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]]) ++z[i];
if (i + z[i] - 1 > r)l = i, r = i + z[i] - 1; return z;
```

## Tree

## 8.1 Articulation bridge

```
vector<int>node[10003];int lowtime[10003],intime[10003],vis
    [10003];
vector<pair<int.int>>edge:int timer:
void dfs(int p, int parent){intime[p]=lowtime[p]=timer;timer
vis[p]=1;for(int child:node[p]){if(child==parent)continue;
if(vis[child]){lowtime[p]=min(lowtime[p], intime[child]); //
     node - child is a back edge
}else{// node - child is a forward edge
dfs(child,p);if(lowtime[child] > intime[p])edge.push_back({p
     . child}):
lowtime[p] = min(lowtime[p] , lowtime[child]);}}
```

## Articulation point

```
vector<int>node[10003];int lowtime[10003],intime[10003],vis
    Γ10003]:
set<int>cut_vertex; int timer;
void IS CUTPOINT(int x){cut vertex.insert(x):}
void dfs(int p, int parent){intime[p]=lowtime[p]=timer;
timer++;int children=0;vis[p]=1;for(int child:node[p]){
if(child==parent) continue:if(vis[child]){
lowtime[p]=min(lowtime[p], intime[child]); // node - child
    is a back edge
}else{// node - child is a forward edge
dfs(child,p);if(lowtime[child] >= intime[p] && parent!=-1)
IS_CUTPOINT(p);lowtime[p] = min(lowtime[p] , lowtime[child])
children++;}}if(parent == -1 && children>1)//for root
IS CUTPOINT(p):}
```

## 8.3 Dijkstra

```
priority_queue< pair<11, 11>, vector<pair<11 , 11> >,
    greater<pair<11 , 11 > > > pq;
```

```
int parrent[100003]:void dijsktra(ll p){parrent[1] = 1:pg.
    push({0, p}):
dis[p] = 0;ans.push_back(p);while (!pq.empty()){
11 curr_node = pq.top().second; ll curr_dis = pq.top().first;
pg.pop();for ( pair<11 , 11> child : node[curr_node]){
if (child.second + curr dis < dis[child.first]){</pre>
parrent[child.first] = curr_node;dis[child.first] = child.
     second + curr dis:
pq.push({dis[child.first], child.first});}}}
```

#### 8.4 Eular tour

```
vector<int>node[N];int Intime[N], Outtime[N], Level[N] , a[N
int timer = 1;int n, q;void EulerTour(int p, int par, int d)
Intime[p] = timer++: Level[p] = d: for (auto i : node[p]){if
     (i == par)
continue;EulerTour(i, p, Level[p] + 1);} Outtime[p] = timer;
//if timer++ then intime[u] to intime[v] can be find path
}struct BIT {} T[2];void Solve(){ cin >> n >> q;
for (int i = 1 ; i <= n ; i++) cin >> a[i];
for (int i = 1; i < n; i++){ int u, v; cin >> u >> v;
node[u].push_back(v); node[v].push_back(u);}EulerTour(1, 0,
for (int i = 1 ; i <= n ; i++){T[Level[i] % 2].upd(Intime[i</pre>
    l. Intime[i]. a[i]):
T[!(Level[i] % 2)].upd(Intime[i], Intime[i], 0);}while (q--)
    { int type;
cin >> type; if (type == 1){ ll x, val; cin >> x >> val;}
T[Level[x] % 2].upd(Intime[x], Outtime[x]-1, val);
T[!(Level[x] % 2)].upd(Intime[x], Outtime[x]-1, -val);} else
cin >> x: cout << T[Level[x] % 2].guerv(Intime[x]. Intime[x</pre>
    1) << endl:}}}</pre>
```

## 8.5 Floyd Warshall

```
11 vis[504][504]; void warshall(11 n){
for (int k = 1; k \le n; k++){for (int i = 1; i \le n; i++)
{for (int j = 1; j \le n; j++){
dis[i][j] = min(dis[i][j], dis[i][k] + dis[k][j]);}}}
```

## 8.6 HLD(update on edge)

```
11 n, q;vector<11>node[N];11 a[N];
struct SegmentTree { vector<11> tree;
vector<ll> lazv: vector<ll> aa: SegmentTree() {
tree.resize(4 * N); lazy.resize(4 * N); aa.resize(4 * N);}
void build(ll node, ll b, ll e) { if (b == e) {
tree[node] = 0; lazy[node] = -1;return;}
11 \text{ mid} = (b + e) \gg 1; build(2 * node, b, mid);
build(2 * node + 1, mid + 1, e): lazv[node] = -1:
tree[node] = tree[2 * node] + tree[2 * node + 1];}
void push(ll node, ll b, ll e) { tree[node] = (e - b + 1) *
     lazv[node]:
if (b != e) { lazy[2 * node] = lazy[2 * node + 1] = lazy[
lazy[node] = -1;} void update(ll node, ll b, ll e, ll l, ll
     r, 11 x) {
if (lazv[node] != -1) push(node, b, e); if (l > e || r < b)
     return:
if (1 \le b \&\& r \ge e) \{ tree[node] = (e - b + 1) * x;
if (b != e) { lazy[2 * node] = lazy[2 * node + 1] = x; }
lazy[node] = -1; return; } 11 mid = (b + e) >> 1;
update(2 * node, b, mid, l, r, x): update(2 * node + 1, mid
     + 1. e. l. r. x):
tree[node] = tree[2 * node] + tree[2 * node + 1];}
11 query(11 node, 11 b, 11 e, 11 1, 11 r) { if (lazy[node]
     ! = -1)
push(node, b, e); if (1 > e \mid \mid r < b) return 0; if (1 <= b)
    && r \ge e) {return tree[node];}ll mid = (b + e) >> 1:
   return query(2 * node, b, mid, 1, r) + query(2 * node +
        1. mid + 1. e. l. r):
};SegmentTree st;ll par[N][LG + 1], dep[N], sz[N];void dfs(
     int u. int p = 0){
par[u][0] = p; dep[u] = dep[p] + 1; sz[u] = 1; for (int i = 1;
    i <= LG: i++){
par[u][i] = par[par[u][i - 1]][i - 1];}for (auto v : node[u]
    1){
if (v == p) continue;dfs(v, u);sz[u] += sz[v];}}
int lca(int u, int v){
 // ache already}
int intime[N]. head[N]:int timer = 1:map<11. 11>alledge[N]:
void decompose(int p, int parent, int Head_node){intime[p] =
      timer++:
head[p] = Head_node;st.update(1, 1, n, intime[p], intime[p],
      alledge[parent][p]);
int heavysize = -1. heavychild = -1:for (auto i : node[p]){
     if (i != parent){
if (sz[i] > heavysize)heavysize = sz[i], heavychild = i;}}if
      (heavychild == -1)
return ; decompose(heavychild, p, Head_node);for (auto i :
     node[p]){
```

```
if (i == heavychild | | i == parent)continue:decompose(i, p.
11 sumpath(int u, int v){11 ans = 0;
//cout << "here " << u << " " << v << endl:
if (u == v) return 0; while (head[u] != head[v]){ if (dep[
    head[u]] > dep[head[v]])
swap(u, v); ans += st.query(1, 1, n, intime[head[v]], intime
     [v]);v = par[head[v]][0];}
if (dep[u] > dep[v])swap(u, v);if (u != v)ans += st.query(1,
     1, n, intime[u] + 1, intime[v]);
return ans:}void reset(int n){for (int i = 0 : i <= n : i++)</pre>
intime[i] = head[i] = dep[i] = sz[i] = 0;a[i] = 0;node[i].
timer = 1;alledge[i].clear();}}void Solve(){cin >> n://reset
st.build(1, 1, n):vector<pair<int, int> > edge:for (int i =
    1 : i < n : i++)
ll u. v. w:cin >> u >> v >> w:alledge[u][v] = w:alledge[v][u
    ] = w:node[u].push back(v):
node[v].push_back(u);edge.push_back({u, v});}dfs(1);
    decompose(1, 0, 1);int q;
cin >> q; while (q--){int type; cin >> type; if (type == 1){
11 id, x;cin >> id >> x;id--;int p = edge[id].f;int q = edge
     [id].s:
if (dep[p] > dep[q]) swap(p, q);st.update(1, 1, n, intime[q
    ], intime[q], x);
alledge[p][q] = x;alledge[q][p] = x}else{int u, v;cin >> u
int 1 = lca(u, v):if (u == v){cout << 0 << endl:continue:}</pre>
if (1 == u \mid | 1 == v) \{ if (dep[u] > dep[v]) swap(u, v); cout
     << sumpath(u, v) << endl;}
else{cout << sumpath(1, u) + sumpath(1, v) << endl:}}}}</pre>
```

## 8.7 HLD(update on node)

```
int n,q;vector<int>node[N];int a[N];
struct SegmentTree {
   vector<int>tree;vector<int>lazy;vector<int>aa;
   SegmentTree() {tree.resize(4*N);lazy.resize(4*N);aa.
        resize(4*N);}
   void build(int node,int b,int e) {if(b==e){tree[node]=0;
        lazy[node]=-1;return;}int mid=(b+e)>>1;build(2*node,b,mid);build(2*node+1,mid+1,e);lazy[node]=-1;tree[node]=max(tree[2*node],tree[2*node+1]);}
   void push(int node,int b,int e) {tree[node]=lazy[node];if
        (b!=e){lazy[2*node]=lazy[2*node+1]=lazy[node];}lazy[node]=-1;}
```

```
void update(int node.int b.int e.int l.int r.int x) {if(
        lazv[node]!=-1)push(node.b.e):if(l>e||r<b)return:if(</pre>
        1 \le b \& r \ge 0 {tree[node] = x; if (b!=e) {lazy[2*node] = lazy
        [2*node+1]=x:}lazv[node]=-1:return:}int mid=(b+e)
        >>1;update(2*node,b,mid,l,r,x);update(2*node+1,mid
        +1.e.l.r.x):tree[node]=max(tree[2*node].tree[2*node
        +1]):}
   int query(int node,int b,int e,int l,int r) {if(lazy[node
        ]!=-1)push(node,b,e); if(l>e||r<b)return 0; if(l<=b\&\&r
        >=e){return tree[node];}int mid=(b+e)>>1;return max(
        querv(2*node.b.mid.l.r).querv(2*node+1.mid+1.e.l.r))
SegmentTree st;int par[N][LG+1],dep[N],sz[N];
void dfs(int u,int p=0){par[u][0]=p;dep[u]=dep[p]+1;sz[u]=1;
    for(int i=1;i<=LG;i++){par[u][i]=par[par[u][i-1]][i</pre>
    -1]:}for(auto v:node[u]){if(v==p)continue:dfs(v.u):sz[u
    ]+=sz[v]:}}
int lca(int u,int v){if(dep[u] < dep[v])swap(u,v);for(int k=LG</pre>
    ;k>=0;k--)if(dep[par[u][k]]>=dep[v])u=par[u][k];if(u==v
    )return u;for(int k=LG;k>=0;k--)if(par[u][k]!=par[v][k
    ])u=par[u][k],v=par[v][k];return par[u][0];}
int intime[N],head[N];int timer=1;
void decompose(int p,int parent,int Head_node){intime[p]=
    timer++;head[p]=Head_node;st.update(1,1,n,intime[p],
    intime[p],a[p]);int heavysize=-1,heavychild=-1;for(auto
     i:node[p]){if(i!=parent){if(sz[i]>heavysize)heavysize=
    sz[i].heavychild=i:}}if(heavychild==-1)return:decompose
    (heavychild,p,Head_node);for(auto i:node[p]){if(i==
    heavychild||i==parent)continue:decompose(i.p.i):}}
int maxnode(int u,int v){int ans=0; while(head[u]!=head[v]){
    if(dep[head[u]]>dep[head[v]])swap(u,v);ans=max(ans,st.
    query(1,1,n,intime[head[v]],intime[v]));v=par[head[v
    ]][0]; }if(dep[u]>dep[v])swap(u,v); ans=max(ans,st.query
    (1,1,n,intime[u],intime[v]));return ans;}
void Solve(){cin>>n>>g:for(int i=1:i<=n:i++){cin>>a[i]:}st.
    build(1.1.n):for(int i=1:i<n:i++){int u.v:cin>>u>>v:
    node[u].push_back(v);node[v].push_back(u);}dfs(1);
    decompose(1,0,1); while(q--){int type; cin>>type; if(type
    ==1){int u,x;cin>>u>>x;st.update(1,1,n,intime[u],intime
    [u].x):}else{int u.v:cin>>v:int l=lca(u.v):cout<<max</pre>
    (maxnode(1,u),maxnode(1,v))<<" ";}}}
```

## 8.8 Inverse Graph

```
void bfs(int u){queue<int>que ;que.push(u) ;wh_cmpnnt[u] =
    cmpnnt ;while(!que.empty()){auto u = que.front() ;que.
    pop();vector<int>restricted;for(auto v: graph[u]){if(!
    wh_cmpnnt[v]){restricted.push_back(v);adj[v] = 1 ;}}for
```

```
(auto v: not_visited){if(v == u || adj[v]) continue ;
que.push(v) ;wh_cmpnnt[v] = cmpnnt ;}for(auto v:
restricted)adj[v] = 0 ;not_visited = restricted ;}}
```

#### 8.9 LCA

```
const int N = 3e5 + 9, LG = 18; vector < int > g[N]; int par[N][
     LG + 1], dep[N], sz[N];
void dfs(int u, int p = 0) \{par[u][0] = p; dep[u] = dep[p] +
    1;sz[u] = 1;
for (int i = 1; i <= LG; i++) par[u][i] = par[par[u][i -</pre>
    1]][i - 1];
for (auto v: g[u]) if (v != p) {dfs(v, u);sz[u] += sz[v];}}
int lca(int u. int v) {
if (dep[u] < dep[v]) swap(u, v);</pre>
for (int k = LG; k \ge 0; k--) if (dep[par[u][k]] \ge dep[v])
    u = par[u][k]:
if (u == v) return u;
for (int k = LG; k >= 0; k--) if (par[u][k] != par[v][k]) u
     = par[u][k], v = par[v][k]; return par[u][0];}
int kth(int u, int k) {assert(k >= 0);
for (int i = 0; i <= LG; i++) if (k & (1 << i)) u = par[u][i</pre>
    ];return u;}
int dist(int u, int v) {
int l = lca(u, v); return dep[u] + dep[v] - (dep[l] << 1);
//kth node from u to v, Oth node is u
int go(int u, int v, int k) {
int 1 = lca(u, v); int d = dep[u] + dep[v] - (dep[1] << 1);
     assert(k <= d); if (dep[l] + k <= dep[u]) return kth(u,
    k);k \rightarrow dep[u] - dep[l];return kth(v, dep[v] - dep[l] -
     k):}
int32_t main() {int n; cin >> n; for (int i = 1; i < n; i++)
     {int u, v; cin >> u >> v;g[u].push_back(v);g[v].
    push_back(u);}dfs(1);int q; cin >> q;while (q--) {int u
     , v; cin >> u >> v; cout << dist(u, v) << '\n';}return
    0;}
```

## 8.10 $dfs_t ree$

#### 8.11 strongly connected components

```
vector<int>node[10000], transpose_node[10000];
int visit[10000], out_time[10000], in_time[10000];
vector<int>order;//by out_time we can sort by out_time but
    stack/vector reduce complexity
vector<int>SCC;int timer;void dfs(int x) {
visit[x] = 1:in time[x] = ++timer:for (auto i : node[x])
if (!visit[i]) dfs(i); out_time[x] = ++timer;
order.push_back(x); // all the child of this node (X)
    already visited
}void dfs_for_scc(int x) //This dfs for find scc
{visit[x] = 1:SCC.push back(x):for (auto child :
    transpose node[x])
if (!visit[child]) dfs_for_scc(child);}while (m--){
cin >> a >> b;node[a].push_back(b);transpose_node[b].
//to find scc we need to run dfs is transpose graph of main
}for (int i = 0; i <= n; i++)if (visit[i] == 0) dfs(i);</pre>
for (int i = 0 ; i <= n ; i++) visit[i] = 0;</pre>
cout << "here is node list by order of out time\n":</pre>
for (int i = n - 1; i \ge 0; i--)
cout << order[i] << " out time is -> " << out time[order[i]]</pre>
      << endl:
for (int i = n - 1; i >= 0; i--) {if (visit[order[i]] ==
    0) //order[i] is by largest outime
{SCC.clear(); //previous SCC cleard
dfs_for_scc(order[i]); // for finding scc we run dfs in
    transpose graph
cout << "Strongly Connected Components are \n";for (auto</pre>
    child : SCC)
cout << child << " ";cout << endl;}}</pre>
```

## 9 u blank

#### 9.1 blank

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