Linear Algebra

Assignment. 2

Submitted by:

Maira Anjum

Registration No:

FA20-BCS-033

Submitted to:

Sir Umair Umar

(Question)

Define determinants and its properties with examples?

Definition:-

The determinant of matrix is a special scalar value that can be computed from square matrix.

It is often denoted by det(A) or |A|.

* Properties of Determinant with examples:-

(i)

Determinant of identity matrix is always 1.

$$\det(I) = 1$$

Example:-

$$I = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\det(I) = 1\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$$

$$= 1(1-0) - 0 + 0$$

$$= 1$$

(ii)

If matrix A has rows / columns of zeros then

$$\det(A) = 0$$

Example:-

$$A = \begin{vmatrix} 4 & 1 \\ 0 & 0 \end{vmatrix}$$

$$|A| = (4 \times 0) - (1 \times 0)$$

$$= 0 - 0$$

$$= 0$$

(iii)

Let A be the matrix then

$$\det(A) = \det(A^t)$$

Example:-

$$|A| = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$$

$$= 10 - 12$$

$$= -2$$

$$|A^t| = \begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$$

$$|A^t| = 10 - 12$$

$$= -2$$

So

$$|A| = |A^t|$$

$$-2 = -2$$

(iv)

If matrix A has two identical rows and columns then determinant will be 0.

$$\det(A) = 0$$

Example:-

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}$$

$$= 2 - 2$$

$$= 0$$

(V)

Let A be square matrix, if any 2 rows or columns interchanged sign of determinant changes

$$|A| = -|A|$$

Example:-

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$|A| = 4 - 3$$

$$= 1$$

Now exchange rows

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

$$|A| = 3 - 4$$

$$= -1$$

So

$$|A| = -|A|$$

Scanned with CamScanner

(vi)

Let A be the matrix then

$$\det(kA) = k^n \det(A)$$

Examples:-

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$\det(A) = (4 \times 1) - (3 \times 2)$$
$$= 4 - 6$$
$$= -2$$

To find det(2A) Multiply A by 2

$2A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$

$$\det(2A) = 16 - 24$$
$$= -8$$

$$\det(2A) = 2^2 \det(A)$$
$$= 4(-2)$$
$$= -8$$

So
$$\det(kA) = k^n \det(A)$$

(vii)

If we have matrix A and B both of which are square then

$$\det(AB) = \det(A)\det(B)$$

Example:-

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 2 \\ 4 & 2 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$$

$$= 1-6$$
$$= -5$$

$$\det(B) = \begin{vmatrix} 2 & 2 \\ 4 & 2 \end{vmatrix}$$

$$= 4-8$$
$$= -4$$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 2 + 2\times 4 & 1\times 2 + 2\times 2 \\ 3\times 2 + 1\times 4 & 3\times 2 + 1\times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+8 & 2+4 \\ 6+4 & 6+2 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 6 \\ 10 & 8 \end{bmatrix}$$

Scanned with CamScanner

$$\det(AB) = 10 \times 8 - 6 \times 10$$
$$= 80 - 60$$
$$= 20$$

$$\det(AB) = \det(A)\det(B)$$
$$= (-5)(-4)$$
$$= 20$$

(viii)

If we have matrix A and B both of which are square then

$$\det(A+B) \neq \det(A) + \det(B)$$

Example:-

$$A = \begin{bmatrix} 10 & -6 \\ -3 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 5 & -6 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 11 & -4 \\ 2 & -7 \end{bmatrix}$$

$$\det(A) = 10 \times -1 - (-6)(-3)$$
$$= -28$$

$$\det(B) = (1)(-6) - (2)(5)$$
$$= -16$$

$$\det(A+B) = 11 \times -7 - (-4) \times (-2)$$
$$= 69$$

$$\det(A+B) \neq \det(A) + \det(B)$$
$$69 \neq -28 - 16$$
$$69 \neq -44$$