Naïve Bayes Classifier

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Naïve Bayes Classifier

- Naive Bayes is a probabilistic classifier based on Bayes' Theorem, assuming independence among features. It's often used for classification tasks such as spam filtering, sentiment analysis, and medical diagnosis.

Understanding Bayes' Theorem

- Bayes' Theorem calculates the probability of a hypothesis given some evidence:

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}$$

- Where:
  - P(H|E) is the **posterior probability** (probability of the hypothesis H given the evidence E).
  - P(E|H) is the **likelihood** (probability of observing E given H).
  - P(H) is the **prior probability** (probability of the hypothesis before seeing the evidence).
  - P(E) is the **evidence** or **marginal likelihood** (probability of the evidence across all hypotheses).

Types of Naïve Bayes' Classifier

There are several variations of the Naive Bayes classifier depending on the type of data:

1. **Gaussian Naive Bayes**: For continuous features, we assume they follow a normal distribution.
2. **Multinomial Naive Bayes**: Suitable for discrete data, especially for document classification tasks where features represent word frequencies.
3. **Bernoulli Naive Bayes**: For binary/Boolean data, where features represent binary outcomes (e.g., presence/absence of a word).

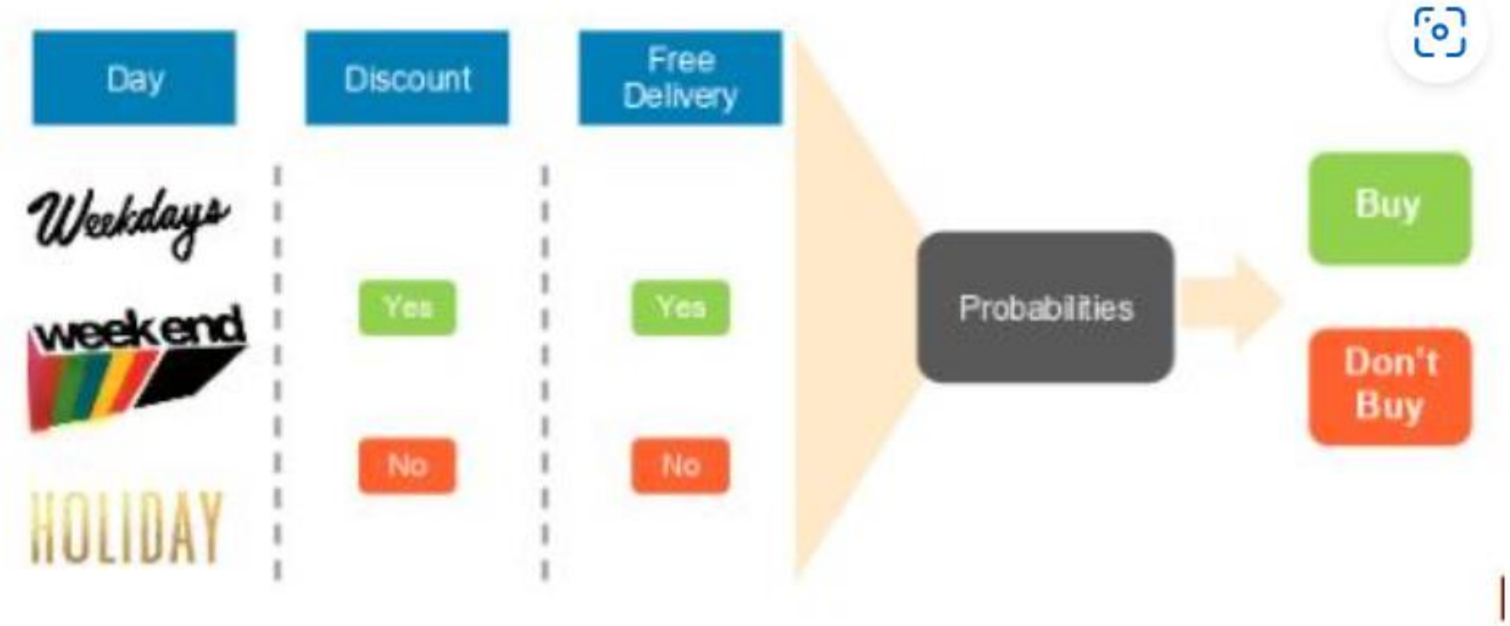
Training the model

To train the Naive Bayes model:

1. **Calculate Prior Probabilities**
2. **Calculate the likelihoods**

In **Gaussian Naive Bayes**, the likelihood is computed using the probability density function of a normal distribution.
In **Multinomial Naive Bayes**, the likelihood is based on feature counts (e.g., word frequencies).

Multinomial Naïve Bayes' Classifier



Source: Naive Bayes Classifier | Simplilearn
Dataset Link: Naive_Bayes_Dataset - Google Sheets

Calculating the Prior Probabilities

Frequency Table		Buy	
		Yes	No
Discount	Yes	19	1
	No	5	5

Frequency Table		Buy	
		Yes	No
Free Delivery	Yes	21	2
	No	3	4

Frequency Table		Buy	
		Yes	No
Day	Weekday	9	2
	Weekend	7	1
	Holiday	8	3

Calculating the Likelihood

Likelihood Table		Buy: Yes	Buy: No	
Day	Weekday	9/24	2/6	11/30
	Weekend	7/24	1/6	8/30
	Holiday	8/24	3/6	11/30
		24/30	6/30	

Frequency Table		Buy: Yes	Buy: No	
Discount	Yes	19/24	1/6	20/30
	No	5/24	5/6	10/30
		24/30	6/30	

Frequency Table		Buy: Yes	Buy: No	
Free Delivery	Yes	21/24	2/6	23/30
	No	3/24	4/6	7/30
		24/30	6/30	

Example

- Consider a combination of the following factors where E equals:
  - Day = Holiday
  - Discount = Yes
  - Free Delivery = Yes

Probability that the customer will not purchase: H=No Buy

- P(H|E)=P(No Buy| Discount= Yes, Free Delivery=Yes, Day=Holiday)

Probability that customer will buy. H=Buy

- P(H|E)=P(Buy| Discount= Yes, Free Delivery=Yes, Day=Holiday)

Results

- From the calculations, we find that:
  - Probability of purchase = 0.986
  - Probability of no purchase = 0.178
- Finally, we have a conditional probability of purchase on this day.
- Next, normalize these probabilities to get the likelihood of the events:
  - Sum of probabilities = 0.986 + 0.178 = 1.164
    - Likelihood of purchase = 0.986 / 1.164 = 84.71 percent
    - Likelihood of no purchase = 0.178 / 1.164 = 15.29 percent
- Result: As 84.71 percent is greater than 15.29 percent, we can conclude that an average customer will buy on holiday with a discount and free delivery.

Gaussian Naïve Bayes

- Gaussian Naive Bayes assumes that features follow a normal (Gaussian) distribution.
- Let’s take a sample dataset

Feature 1	Feature 2	Class
6.0	3.0	0
2.0	2.0	0
3.0	1.0	0
8.0	3.0	1
7.0	2.0	1
9.0	3.0	1

Example

- We want to classify a new data point **(Feature 1 = 4.0, Feature 2 = 2.0)**

Calculate the prior probabilities

The prior probability $P(Class = 0)$ and $P(Class = 1)$ can be calculated as the proportion of each class in the dataset.

- $P(Class = 0)$: There are 3 samples in Class 0 and 6 total samples.

$$P(Class = 0) = \frac{3}{6} = 0.5$$

- $P(Class = 1)$: Similarly, there are 3 samples in Class 1.

$$P(Class = 1) = \frac{3}{6} = 0.5$$

Calculate the mean and variance

- Mean

$$\text{Mean}(\mu) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- Standard Deviation

$$\text{Standard Deviation}(\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2}$$

- Variance

$$\text{Variance}(\sigma^2) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

Calculate mean and variance (For Class 0)

- For each class, we calculate the **mean** and **variance** of each feature, assuming Gaussian (normal) distribution.

For Class 0:

- **Feature 1:**

$$\mu_{0,1} = \frac{6.0 + 2.0 + 3.0}{3} = 3.67$$

$$\sigma^2_{0,1} = \frac{(6.0 - 3.67)^2 + (2.0 - 3.67)^2 + (3.0 - 3.67)^2}{3} = 2.89$$

- **Feature 2:**

$$\mu_{0,2} = \frac{3.0 + 2.0 + 1.0}{3} = 2.0$$

$$\sigma^2_{0,2} = \frac{(3.0 - 2.0)^2 + (2.0 - 2.0)^2 + (1.0 - 2.0)^2}{3} = 0.67$$

Calculate mean and variance (For Class 1)

For Class 1:

- Feature 1:

$$\mu_{1,1} = \frac{8.0 + 7.0 + 9.0}{3} = 8.0$$

$$\sigma^2_{1,1} = \frac{(8.0 - 8.0)^2 + (7.0 - 8.0)^2 + (9.0 - 8.0)^2}{3} = 0.67$$

- Feature 2:

$$\mu_{1,2} = \frac{3.0 + 2.0 + 3.0}{3} = 2.67$$

$$\sigma^2_{1,2} = \frac{(3.0 - 2.67)^2 + (2.0 - 2.67)^2 + (3.0 - 2.67)^2}{3} = 0.22$$

Compute the likelihood

- For a new data point (4.0,2.0), we need to calculate the likelihood $P(x_i|Class_k)$ using the Gaussian probability density function:

$$P(x_i|Class_k) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right)$$

For class 0

For Class 0:

- **Feature 1:**

$$P(4.0|Class = 0) = \frac{1}{\sqrt{2\pi \cdot 2.89}} \exp\left(-\frac{(4.0 - 3.67)^2}{2 \cdot 2.89}\right)$$

$$= \frac{1}{2.68} \exp\left(-\frac{0.11}{5.78}\right) = 0.15$$

- **Feature 2:**

$$P(2.0|Class = 0) = \frac{1}{\sqrt{2\pi \cdot 0.67}} \exp\left(-\frac{(2.0 - 2.0)^2}{2 \cdot 0.67}\right)$$

$$= \frac{1}{2.05} \exp(0) = 0.49$$

For class 1

For Class 1:

- **Feature 1:**

$$P(4.0|Class = 1) = \frac{1}{\sqrt{2\pi \cdot 0.67}} \exp\left(-\frac{(4.0 - 8.0)^2}{2 \cdot 0.67}\right)$$

$$= \frac{1}{2.05} \exp\left(-\frac{16.0}{1.34}\right) \approx 4.95 \times 10^{-6}$$

- **Feature 2:**

$$P(2.0|Class = 1) = \frac{1}{\sqrt{2\pi \cdot 0.22}} \exp\left(-\frac{(2.0 - 2.67)^2}{2 \cdot 0.22}\right)$$

$$= \frac{1}{1.18} \exp\left(-\frac{0.45}{0.44}\right) \approx 0.27$$

Calculate the posterior probabilities

Now calculate the posterior probabilities for each class by multiplying the prior with the likelihoods.

- **For Class 0**:

$$P(Class = 0|4.0, 2.0) \propto P(Class = 0) \cdot P(4.0|Class = 0) \cdot P(2.0|Class = 0)$$

$$= 0.5 \cdot 0.15 \cdot 0.49 = 0.03675$$

- **For Class 1**:

$$P(Class = 1|4.0, 2.0) \propto P(Class = 1) \cdot P(4.0|Class = 1) \cdot P(2.0|Class = 1)$$

$$= 0.5 \cdot 4.95 \times 10^{-6} \cdot 0.27 \approx 6.68 \times 10^{-7}$$

Results

Since $P(Class = 0|4.0, 2.0)$ is larger than $P(Class = 1|4.0, 2.0)$, we predict **Class 0** for the new data point $(4.0, 2.0)$.

Class activity

- When to use Naïve bayes classifier?
- When not to use Naïve bayes classifier?