Solution

You are provided with the dataset:

x	y
1	3
2	5
3	7

Start with $\theta_0 = 0$ and $\theta_1 = 0$. Perform one iteration of gradient descent with a learning rate $\alpha = 0.01$.

1. Calculate Predictions:

For each data point, the prediction y_{pred} is:

$y_{pred} = \theta_0 + \theta_1 \cdot x$

Using the initial values $\theta_0 = 0$ and $\theta_1 = 0$:

- For $x = 1$: $y_{pred} = 0 + 0 \cdot 1 = 0$
- For $x = 2$: $y_{pred} = 0 + 0 \cdot 2 = 0$
- For $x = 3$: $y_{pred} = 0 + 0 \cdot 3 = 0$

2. Calculate the Gradients:

The gradient for θ_0 (intercept) is:

$$\frac{\partial J}{\partial \theta_0} = \frac{1}{m} \sum_{i=1}^{m} (y_{pred}^{(i)} - y^{(i)})$$

$$= \frac{1}{3}[(0-3) + (0-5) + (0-7)]$$

$$= \frac{1}{3}[-15] = -5$$

The gradient for θ_1 (slope) is:

$$\frac{\partial J}{\partial \theta_1} = \frac{1}{m} \sum_{i=1}^{m} (y_{pred}^{(i)} - y^{(i)}) \cdot x^{(i)}$$

$$= \frac{1}{3}[(0-3) \cdot 1 + (0-5) \cdot 2 + (0-7) \cdot 3]$$

$$= \frac{1}{3}[-3 - 10 - 21] = \frac{1}{3}[-34] = -11.33$$

3. Update the Parameters:

Using the learning rate $\alpha = 0.01$:

$$\theta_0 = \theta_0 - \alpha \cdot \frac{\partial J}{\partial \theta_0}$$

$$\theta_0 = 0 - 0.01 \cdot (-5) = 0.05$$

$$\theta_1 = \theta_1 - \alpha \cdot \frac{\partial J}{\partial \theta_1}$$

$$\theta_1 = 0 - 0.01 \cdot (-11.33) = 0.1133$$

4. Calculate the Mean Squared Error (MSE):

The Mean Squared Error after the update is:

1. Calculate the new predictions with updated θ_0 and θ_1:

- For $x = 1$: $y_{pred} = 0.05 + 0.1133 \cdot 1 = 0.1633$
- For $x = 2$: $y_{pred} = 0.05 + 0.1133 \cdot 2 = 0.2766$
- For $x = 3$: $y_{pred} = 0.05 + 0.1133 \cdot 3 = 0.3899$

2. Calculate the errors:

- Error for $x = 1$: $3 - 0.1633 = 2.8367$
- Error for $x = 2$: $5 - 0.2766 = 4.7234$
- Error for $x = 3$: $7 - 0.3899 = 6.6101$

3. Calculate MSE:

$$MSE = \frac{1}{m} \sum_{i=1}^{m} (y_{pred}^{(i)} - y^{(i)})^2$$

$$= \frac{1}{3}[(2.8367)^2 + (4.7234)^2 + (6.6101)^2]$$

$$= \frac{1}{3}[8.0448 + 22.3126 + 43.6884] = \frac{74.0458}{3} = 24.6819$$