

Exercises

Relations

Exercise 1.

Given the following sets:

$$\begin{aligned}A &= \{1\} \\ B &= \{2, 3\}\end{aligned}$$

Write down the sets:

1. $A \times B$
2. $\mathbb{P}(A) \times B$
3. $\mathbb{P}(A \times B)$
4. $(A \times B) \times (A \times B)$

Exercise 2.

Given the relation

$$\begin{aligned}R &= \{1 \mapsto 1, 2 \mapsto 4, 3 \mapsto 9, 4 \mapsto 16, 5 \mapsto 25\} \\ &\text{and the set} \\ S &= \{1, 4, 5\}\end{aligned}$$

Simplify the value of each of the following expressions :

1. $S \triangleleft R$
2. $R \triangleright S$
3. $(S \triangleleft R) \triangleright S$
4. $(R \triangleright \text{dom } R) \sim \triangleright S$

Exercise 3.

Citing Papers

Given the following:

$$\begin{aligned}&[PAPER] \\ &\textit{cites} : PAPER \leftrightarrow PAPER \\ &\text{and that} \\ &(\textit{paper1}, \textit{paper2}) \in \textit{cites} \text{ has the meaning that paper1 cites paper2.}\end{aligned}$$

Write down the following:

1. Write a Z expression for the set of all papers cited directly or indirectly by paper x.
2. Write a Z expression for the set of all papers which cite other papers (directly or indirectly) but themselves are not cited (directly or indirectly).
3. Write a Z expression which states that if any paper cites another (directly or indirectly) the second one must not cite the first (directly or indirectly).
4. Write a Z expression for the number of papers cited directly by paper x.
5. Write a Z expression for the number of papers cited directly or indirectly by paper x.

Exercise 4.

Family Relations

Given

$[PERSON]$

and $parent : PERSON \leftrightarrow PERSON$

$male, female : \mathbb{P} PERSON$

and that

$(abe, homer) \in parent$ has the meaning that abe is homers parent.

1. Write a Z expression for each of the following:
 - (a) The parents of person x.
 - (b) The grandparents of person x.
 - (c) The grandchildren of person x.
 - (d) The descendants of person x.
 - (e) The siblings of person x.
 - (f) The aunts of person x.
2. Give a Z expression for the set of all people in the database who have no relatives in the database.
3. Write an invariant to say that no person can have more than 2 parents.