Exercises and Solutions
Functions

Exercise 1.
For the functions

$$f = \{3 \mapsto 9, 4 \mapsto 16, 5 \mapsto 25\}$$
$$g = \{2 \mapsto 7, 3 \mapsto 16, 4 \mapsto 17\})$$

What is the value of the following

1. $g \oplus f$
2. $f^{\sim} \oplus g^{\sim}$
3. $(\{5\} \lhd f) \oplus (g \rhd \{17, 7\}$
4. $(f \cap g) \oplus (f \cup g\}$
5. $(f^{\sim} \mathbin{;} g) \oplus g$

Exercise 2.
For any two functions, f and g, in what circumstances could the following be true?

1. $f \cup g = f \oplus g$
2. $f \oplus g = g \oplus f$
3. $f \cap g = f \oplus g$
4. $f \setminus g = f \oplus g$

Exercise 3.
The following does not include functions but allows you to practice schema operations.
Given the following:

$[PERSON, MODULE]$

$$
\begin{array}{|l}
\multicolumn{1}{l}{\underline{\;ModuleReg\;}} \\
students : \mathbb{P}\, PERSON \\
degModules : \mathbb{P}\, MODULE \\
sitting : PERSON \leftrightarrow MODULE \\
\hline
\operatorname{dom} sitting \subseteq students \\
\operatorname{ran} sitting \subseteq degModules \\
\hline
\end{array}
$$

Write the following schema operations:

1. Add a student **s?** to the set of registered students.
2. Delete a student **s?** from the system (what are the conditions under which a student can be removed?)
3. Add a degree module **degM?** to the set of registered degree modules.
4. Delete a degree module **degM?** from the set of registered degree modules (what are the conditions under which a module can be removed?)
5. Add a new **'student registers for a module'** mapping. (Check preconditions).

Exercise 4.
A warehouse holds stocks of various items *carried* by a company. A computer system records the *level* of all items carried, the *withdrawal* of items from stock and the *delivery* of stock.

Occasionally, a new item will be *carried* and items will be *discontinued*, provided that their stock level is *zero*. The systems state is given as:

$[ITEM]$ the set of all items.

$$
\begin{array}{|l}
\multicolumn{1}{l}{\underline{\;Warehouse\;}} \\
carried : \mathbb{P}\, ITEM \\
level : ITEM \nrightarrow \mathbb{N} \\
\hline
\operatorname{dom} level = carried \\
\hline
\end{array}
$$

Every carried item has a level, even if it is zero.

$$\begin{array}{|l}
\textit{Withdraw} \\
\hline
\Delta Warehouse \\
i? : ITEM \\
qty? : \mathbb{N} \\
\hline
i? \in carried \\
level i? \geq qty? \\
level' = level \oplus \{i? \mapsto level(i?) + qty?\} \\
carried' = carried \\
\hline
\end{array}$$

Write schemas for the following operations:

1. Deliver a quantity ($qty?$) of item $i?$ to the warehouse (the item must be already carried). There is no upper limit on stock held.
2. Add a new item $i?$ to be carried.
3. Discontinue an item ($i?$). The item must currently be carried and have a stock-level of zero

Solutions

Solution 1.

1. $g \oplus f$

   $$\{3 \mapsto 9, 4 \mapsto 16, 5 \mapsto 25, 2 \mapsto 7\}$$

2. $f^{\sim} \oplus g^{\sim}$

   $$\{9 \mapsto 3, 16 \mapsto 3, 25 \mapsto 5, 7 \mapsto 2, 17 \mapsto 4\}$$

3. $(\{5\} \lhd f) \oplus (g \rhd \{17, 7\}$

   $$\{2 \mapsto 7, 4 \mapsto 17, 5 \mapsto 25\}$$

4. $(f \cap g) \oplus (f \cup g\}$
   This expression is invalid as $f \cup g$ is not a function.

5. $(f^{\sim} \fatsemi g) \oplus g$

   $$\{9 \mapsto 16, 16 \mapsto 17, 2 \mapsto 7, 3 \mapsto 16, 4 \mapsto 17\}$$

Solution 2.

For any two functions, f and g, in what circumstances could the following be true?

1. $f \cup g = f \oplus g$
   When $disjoint\langle \mathrm{dom}\ \ f, \mathrm{dom}\ \ g\rangle$

2. $f \oplus g = g \oplus f$
   When $\mathrm{dom}\ \ f \cap \mathrm{dom}\, g = \varnothing or f = g$

3. $f \cap g = f \oplus g$
   When f = g

4. $f \setminus g = f \oplus g$
   When $g = \varnothing$

Solution 3.

1. Add a student **s?** to the set of registered students.

AddStudent

$\Delta ModuleReg$
$s? : PERSON$

$s? \notin students$

$students' = students \cup \{s?\}$
$degModules' = degModules$
$sitting' = sitting$

2. Delete a student **s?** from the system (what are the conditions under which a student can be removed?)
   We will only remove a student if they are not currently **sitting** on any module

$$
\begin{array}{|l}
\hline
\textit{DeleteStudent} \\
\Delta ModuleReg \\
s? : PERSON \\
\hline
s? \in students \\
s? \notin \operatorname{dom} sitting \\
\\
students' = students \setminus \{s?\} \\
degModules' = degModules \\
sitting' = sitting \\
\hline
\end{array}
$$

3. Add a degree module **degM?** to the set of registered degree modules.

$$
\begin{array}{|l}
\hline
\textit{AddModule} \\
\Delta ModuleReg \\
degM? : MODULE \\
\hline
degM? \notin degModules \\
\\
students' = students \\
degModules' = degModules \cup \{degM?\} \\
sitting' = sitting \\
\hline
\end{array}
$$

4. Delete a degree module **degM?** from the set of registered degree modules (what are the conditions under which a module can be removed?)
   As before, module deleted when no student registered on module

$$
\begin{array}{|l}
\hline
\textit{DeleteModule} \\
\Delta ModuleReg \\
degM? : MODULE \\
\hline
degM? \in degModules \\
degM? \notin \operatorname{ran} sitting \\
\\
students' = students \\
degModules' = degModules \setminus \{degM?\} \\
sitting' = sitting \\
\hline
\end{array}
$$

5. Add a new **'student registers for a module'** mapping. (Check preconditions).

$$
\begin{array}{|l}
\hline
\textit{RegForModule} \\
\Delta ModuleReg \\
m? : MODULE \\
s? : PERSON \\
\hline
m? \in degModules \\
s? \in students \\
s? \mapsto m? \notin sitting \\
\\
students' = students \\
degModules' = degModules \\
sitting' = sitting \cup \{s? \mapsto m?\} \\
\hline
\end{array}
$$

Solution 4.

1. Deliver a quantity ($qty?$) of item $i?$ to the warehouse (the item must be already carried). There is no upper limit on stock held.

$$
\begin{array}{|l}
\hline
\textit{DeliverIItem} \\
\Delta Warehouse \\
qty? : \mathbb{N}_1 \\
i? : ITEM \\
\hline
i? \in carried \\
\\
level' = level \oplus \{i? \mapsto (level(i?) + qty?)\} \\
carried' = carried \\
\hline
\end{array}
$$

2. Add a new item $i?$ to be carried.

$$
\begin{array}{|l}
\hline
\textit{AddNewItem} \\
\Delta Warehouse \\
i? : ITEM \\
\hline
i? \notin carried \\
\\
level' = level \\
carried' = carried \cup \{i?\} \\
\hline
\end{array}
$$

3. Discontinue an item ($i?$). The item must currently be carried and have a stock-level of zero

$$
\begin{array}{l}
\textit{DiscontinueItem} \\
\hline
\Delta Warehouse i? : ITEM \\
\hline
i? \in carried \\
level(i?) = 0 \\
\\
level' = \{i?\} \lhd level \\
carried' = carried \setminus \{i?\}
\end{array}
$$