

Exercises and Solutions

Sequences

Exercise 1.

Simplify the following expressions:

1. $\langle 1, 2, 3 \rangle \cap \langle \rangle$
2. $\text{dom}\langle a, b, c \rangle$
3. $\text{ran}\langle 1, 1, 2 \rangle$
4. $\{a \mapsto 2, b \mapsto 3, c \mapsto 1\} \sim$
5. $\text{dom}(\langle 1, 2 \rangle \cap \langle 3, 4 \rangle)$
6. $\{1\} \triangleleft (\text{tail}(\langle a, b, c \rangle))$
7. $\text{dom}(\text{front}(\langle 1, 3, 5, 7 \rangle)) \sim$
8. $\text{head}(\text{tail}(\text{tail}(\langle 1, 7, 9, 2, 2 \rangle \cap \langle 2, 4, 5 \rangle)))$
9. $\text{last}(\text{tail}(\langle \langle \rangle, \langle 1 \rangle, \langle 1, 2 \rangle, \langle 1, 2, 3 \rangle, \langle 1, 2, 3, 4 \rangle \rangle)) \cap \langle 1, 2 \rangle$
10. $\text{squash}(\text{3..5} \triangleleft \langle a, b, c, d, e, f \rangle)$
11. $\text{squash}(\text{dom}(\langle a, b, c \rangle) \triangleleft (\text{rev}(\langle 2, 3, 4, 6, 8 \rangle)))$

Exercise 2.

Given a sequence of natural numbers :

$s : \text{seq } \mathbb{N}$

Write down a predicate that is true the numbers in the sequence are in ascending order (i.e. increasing from left to right).

Exercise 3.

Write a Z expression for the number of occurrences of a natural number n in a sequence of natural numbers s .

Exercise 4.

Write a Z predicate which states that a given sequence of characters s is a palindrome, i.e. it spells the same backwards as forwards.

Exercise 5.

Given the following

$[CHAR]$ *thesetofallcharacters*
 $s : \text{seq } CHAR$

Write an expression for s' where s' has the following value: (s' should be a sequence)

1. s with elements at positions 2 to 4 (inclusive) deleted.
2. s with all occurrences of the character f deleted.
3. s with the sequence $\langle a, b, c \rangle$ inserted after position 3. (You may assume that $\#s$ is at least 3).

Solutions

Solution 1.

1. $\langle 1, 2, 3 \rangle \cap \langle \rangle$
 $\langle 1, 2, 3 \rangle$
2. $\text{dom}\langle a, b, c \rangle$
 $\{1, 2, 3\}$
3. $\text{ran}\langle 1, 1, 2 \rangle$
 $\{1, 2\}$
4. $\{a \mapsto 2, b \mapsto 3, c \mapsto 1\} \sim$
 $\langle c, a, b \rangle$
5. $\text{dom}(\langle 1, 2 \rangle \cap \langle 3, 4 \rangle)$
 $\{1, 2, 3, 4\}$
6. $\{1\} \triangleleft (\text{tail}(\langle a, b, c \rangle))$
 $\{1 \mapsto b\}$
7. $\text{dom}(\text{front}(\langle 1, 3, 5, 7 \rangle)) \sim$
 $\{1, 3, 5\}$
8. $\text{head}(\text{tail}(\text{tail}(\langle 1, 7, 9, 2, 2 \rangle \cap \langle 2, 4, 5 \rangle)))$
 1
9. $\text{last}(\text{tail}(\langle \langle \rangle, \langle 1 \rangle, \langle 1, 2 \rangle, \langle 1, 2, 3 \rangle, \langle 1, 2, 3, 4 \rangle \rangle)) \cap \langle 1, 2 \rangle$
 $\langle 1, 2, 3, 4, 1, 2 \rangle$
10. $\text{squash}(3..5 \triangleleft \langle a, b, c, d, e, f \rangle)$
 $\langle c, d, e, f \rangle$
11. $\text{squash}(\text{dom}(\langle a, b, c \rangle) \triangleleft (\text{rev}(\langle 2, 3, 4, 6, 8 \rangle)))$
 $\langle 3, 2 \rangle$

Solution 2.

$$\forall i : \text{dom } s \bullet s(i) \leq s(i+1)$$

Solution 4.

$$s = \text{rev}(s)$$

Solution 5.

1. $s' = \text{squash}(2..4 \triangleleft s)$
2. $s' = \text{squash}(s \triangleright \{f\})$
3. $s' = 1..3 \triangleleft s \cap \langle a, b, c \rangle \cap \text{squash}(4.. \# s \triangleleft s)$