

Exercises and Solutions

Sets, Types and Variables

Exercise 1.

Given the following sets:

$$R = \{a, e, i, o, u\}$$

$$S = \{a, o, u\}$$

$$T = \{i, e\}$$

$$V = \{a, e\}$$

Write down the sets as specified below:

1. $R \cup S$
2. $R \cap S$
3. $R \setminus S$
4. $S \setminus T$
5. $\bigcup \{ S, V \}$
6. $\bigcup \{ R, T, V \}$
7. $\bigcap \{ R, S, V \}$
8. $\bigcap \{ R, S, T, \{ \} \}$

Exercise 2.

How many elements are in the following set :

$$\{ \{ \} \}$$

Exercise 3.

Construct the powersets of the following sets. State how many elements are in these powersets.

1. $R = \{ \}$
2. $S = \{ a \}$
3. $T = \{ a, b \}$
4. $V = \{ a, b, c \}$

Exercise 4.

Using the following sets:

$[PERSON]$ of all people,

$prog : \mathbb{P} PERSON$ of people who are programmers
 $code : \mathbb{P} PERSON$ of people who write code
 $spec : \mathbb{P} PERSON$ of people who write specifications
 $read : \mathbb{P} PERSON$ of people who read specifications

Express the following rules using set notation:

1. All specifiers read specifications.
2. Some programmers write specifications.
3. All programmers who write code read specifications.
4. Only one programmer writes specifications
5. No more than 10 programmers write code.

Solutions

Solution 1.

1. $R \cup S$
 $\{a, e, i, o, u\}$
2. $R \cap S$
 $\{a, o, u\}$
3. $R \setminus S$
 $\{e, i\}$
4. $S \setminus T$
 $\{a, o, u\}$
5. $\bigcup \{S, V\}$
 $\{a, o, u, e\}$
6. $\bigcup \{R, T, V\}$
 $\{a, e, i, o, u\}$
7. $\bigcap \{R, S, V\}$
 $\{a\}$
8. $\bigcap \{R, S, T, \{\ \ \} \}$
 $\{\ \ \} \text{ or } \emptyset$

Solution 2.

How many elements are in the following set :

$\{ \{ \ \ \} \}$
1

Solution 3.

1. $R = \{ \ \ \}$
 $\mathbb{P} R = \emptyset \text{ or } \{ \ \ \}$
2. $S = \{ a \}$
 $\mathbb{P} S = \{ \{ a \}, \emptyset \}$
3. $T = \{ a, b \}$
 $\mathbb{P} T = \{ \{ a \}, \{ b \}, \{ a, b \}, \emptyset \}$
4. $V = \{ a, b, c \}$
 $\mathbb{P} V = \{ \{ a \}, \{ b \}, \{ c \}, \{ a, b \}, \{ b, c \}, \{ a, c \}, \{ a, b, c \}, \emptyset \}$

Solution 4.

1. All specifiers read specifications.
 $\text{read} \subseteq \text{spec}$
2. Some programmers write specifications.
 $(\text{prog} \cap \text{spec}) \neq \emptyset$
3. All programmers who write code read specifications.
 $(\text{prog} \cup \text{spec}) \subseteq \text{read}$
4. Only one programmer writes specifications.
 $\#(\text{prog} \cap \text{spec}) = 1$
5. No more than 10 programmers write code.
 $\#(\text{prog} \cap \text{code}) \leq 10$