

Exercises

Functions

Exercise 1.

For the functions

$$\begin{aligned}f &= \{3 \mapsto 9, 4 \mapsto 16, 5 \mapsto 25\} \\g &= \{2 \mapsto 7, 3 \mapsto 16, 4 \mapsto 17\}\end{aligned}$$

What is the value of the following

1. $g \oplus f$
2. $f \sim \oplus g \sim$
3. $(\{5\} \triangleleft f) \oplus (g \triangleright \{17, 7\})$
4. $(f \cap g) \oplus (f \cup g)$
5. $(f \sim \circ g) \oplus g$

Exercise 2.

For any two functions, f and g , in what circumstances could the following be true?

1. $f \cup g = f \oplus g$
2. $f \oplus g = g \oplus f$
3. $f \cap g = f \oplus g$
4. $f \setminus g = f \oplus g$

Exercise 3.

The following does not include functions but allows you to practice schema operations.

Given the following:

$[PERSON, MODULE]$

<i>ModuleReg</i>	
<i>students</i> : $\mathbb{P} PERSON$	
<i>degModules</i> : $\mathbb{P} MODULE$	
<i>sitting</i> : $PERSON \leftrightarrow MODULE$	
$\text{dom } sitting \subseteq students$	
$\text{ran } sitting \subseteq degModules$	

Write the following schema operations:

1. Add a student **s?** to the set of registered students.
2. Delete a student **s?** from the system (what are the conditions under which a student can be removed?)
3. Add a degree module **degM?** to the set of registered degree modules.
4. Delete a degree module **degM?** from the set of registered degree modules (what are the conditions under which a module can be removed?)
5. Add a new ‘**student registers for a module**’ mapping. (Check pre-conditions).

Exercise 4.

A warehouse holds stocks of various items *carried* by a company. A computer system records the *level* of all items carried, the *withdrawal* of items from stock and the *delivery* of stock.

Occasionally, a new item will be *carried* and items will be *discontinued*, provided that their stock level is *zero*. The systems state is given as:

$[ITEM]$ the set of all items.

<i>Warehouse</i>	
<i>carried</i> : $\mathbb{P} ITEM$	
<i>level</i> : $ITEM \rightarrow \mathbb{N}$	
$\text{dom } level = carried$	

Every carried item has a level, even if it is zero.

<i>Withdraw</i>	
$\Delta Warehouse$	
$i? : ITEM$	
$qty? : \mathbb{N}$	
$i? \in carried$	
$level i? \geq qty?$	
$level' = level \oplus \{i? \mapsto level(i?) - qty?\}$	
$carried' = carried$	

Write schemas for the following operations:

1. Deliver a quantity ($qty?$) of item $i?$ to the warehouse (the item must be already carried). There is no upper limit on stock held.
2. Add a new item $i?$ to be carried.
3. Discontinue an item ($i?$). The item must currently be carried and have a stock-level of zero