PROGRAMMING IN HASKELL

Chapter 8 - Declaring Types

Type Declarations

In Haskell, a new name for an existing type can be defined using a type declaration.

```
type String = [Char]
```

String is a synonym for the type [Char].

Type declarations can be used to make other types easier to read. For example, given

```
type Pos = (Int,Int)
```

we can define:

```
origin :: Pos
origin = (0,0)

left :: Pos → Pos
left (x,y) = (x-1,y)
```

Like function definitions, type declarations can also have parameters. For example, given

```
type Pair a = (a,a)
```

we can define:

```
mult :: Pair Int → Int
mult (m,n) = m*n

copy :: a → Pair a
copy x = (x,x)
```

Type declarations can be nested:

```
type Pos = (Int,Int)

type Trans = Pos → Pos
```

However, they cannot be recursive:

```
type Tree = (Int,[Tree])
```

Data Declarations

A completely new type can be defined by specifying its values using a data declaration.

```
data Bool = False | True
```

Bool is a new type, with two new values False and True.

Note:

- The two values False and True are called the constructors for the type Bool.
- Type and constructor names must always begin with an upper-case letter.
- Data declarations are similar to context free grammars. The former specifies the values of a type, the latter the sentences of a language.

Values of new types can be used in the same ways as those of built in types. For example, given

```
data Answer = Yes | No | Unknown
```

we can define:

```
answers :: [Answer]
answers = [Yes,No,Unknown]

flip :: Answer → Answer
flip Yes     = No
flip No      = Yes
flip Unknown = Unknown
```

The constructors in a data declaration can also have parameters. For example, given

```
data Shape = Circle Float
           | Rect Float Float
```

we can define:

```
square :: Float → Shape
square n = Rect n n

area :: Shape → Float
area (Circle r) = pi * r^2
area (Rect x y) = x * y
```

Note:

- Shape has values of the form Circle r where r is a float, and Rect x y where x and y are floats.

- Circle and Rect can be viewed as functions that construct values of type Shape:

```
Circle :: Float → Shape

Rect :: Float → Float → Shape
```

Not surprisingly, data declarations themselves can also have parameters. For example, given

```
data Maybe a = Nothing | Just a
```

we can define:

```
safediv :: Int → Int → Maybe Int
safediv _ 0 = Nothing
safediv m n = Just (m `div` n)

safehead :: [a] → Maybe a
safehead [] = Nothing
safehead xs = Just (head xs)
```

Recursive Types

In Haskell, new types can be declared in terms of themselves. That is, types can be recursive.

```
data Nat = Zero | Succ Nat
```

Nat is a new type, with constructors Zero :: Nat and Succ :: Nat $\rightarrow$ Nat.

Note:

- A value of type Nat is either Zero, or of the form Succ n where n :: Nat. That is, Nat contains the following infinite sequence of values:

```
Zero

Succ Zero

Succ (Succ Zero)

⋮
```

- We can think of values of type Nat as natural numbers, where Zero represents 0, and Succ represents the successor function 1+.

- For example, the value

```
Succ (Succ (Succ Zero))
```

represents the natural number

```
1 + (1 + (1 + 0))  =  3
```

Using recursion, it is easy to define functions that convert between values of type Nat and Int:

```
nat2int :: Nat → Int
nat2int Zero     = 0
nat2int (Succ n) = 1 + nat2int n

int2nat :: Int → Nat
int2nat 0 = Zero
int2nat n = Succ (int2nat (n-1))
```

Two naturals can be added by converting them to integers, adding, and then converting back:

```
add :: Nat → Nat → Nat
add m n = int2nat (nat2int m + nat2int n)
```

However, using recursion the function add can be defined without the need for conversions:

```
add Zero     n = n
add (Succ m) n = Succ (add m n)
```

For example:

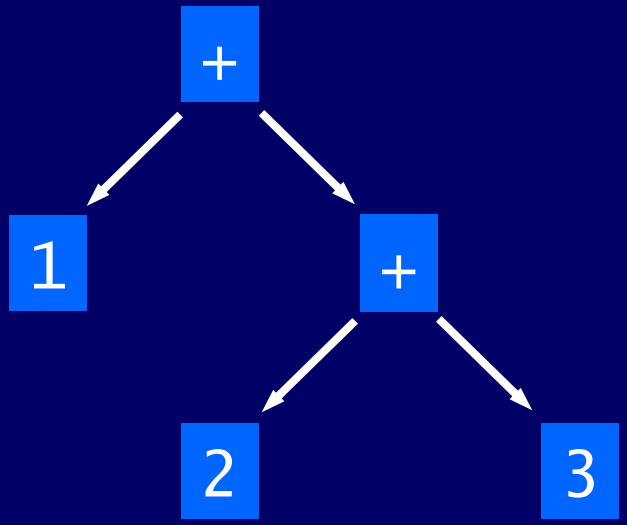
```
  add (Succ (Succ Zero)) (Succ Zero)
=
  Succ (add (Succ Zero) (Succ Zero))
=
  Succ (Succ (add Zero (Succ Zero))
=
  Succ (Succ (Succ Zero))
```

Note:

- The recursive definition for add corresponds to the laws 0+n = n and (1+m)+n = 1+(m+n).

Arithmetic Expressions

Consider a simple form of expressions built up from integers using addition and multiplication.



Using recursion, a suitable new type to represent such expressions can be declared by:

```
data Expr = Val Int
          | Add Expr Expr
```

For example, the expression on the previous slide would be represented as follows:

```
Add (Val 1) Add (Val 2) (Val 3))
```

Using recursion, it is now easy to define functions that process expressions. For example:

```
size :: Expr → Int
size (Val n)   = 1
size (Add x y) = size x + size y

eval :: Expr → Int
eval (Val n)   = n
eval (Add x y) = eval x + eval y
```

Note:

- The two constructors have types:

```
Val :: Int → Expr
Add :: Expr → Expr → Expr
```

```
data Expr = Val Int
          | Add Expr Expr
```

```
Val :: Int → Expr
Add :: Expr → Expr → Expr
```

```
value :: Expr → Int
value (Val v )   = n
value (Add x y)  = value x + value y
```

For example the expression 1 + (2 + 3) is evaluated as follows:

```
Add (Val 1) Add (Val 2) (Val 3))
```

```
value (Add (Val 1) Add (Val 2) (Val 3))
= {applying value}
value (Val 1) + value (Add (Val 2) (Val 3))
= {applying the first value and  then second value}
  1 + value (Val 2) + value (Val 3)
= {applying values }
  1 + 2 + 3
```

Improving our expression evaluator

In this example, the order of evaluation of the expression was determined by Haskell (left to right).

We now look at encoding the controlling of which step should come next. We call this an

abstract machine

We use a control stack for the abstract machine which contains the list of operations to be performed by the machine after the current evaluation has been completed.

```
type Cont = [Op]

data Op = EVAL Expr | ADD Int
```

We now define a function that evaluates an expression in the context of a control stack

We now define a function that evaluates an expression in the context of a control stack

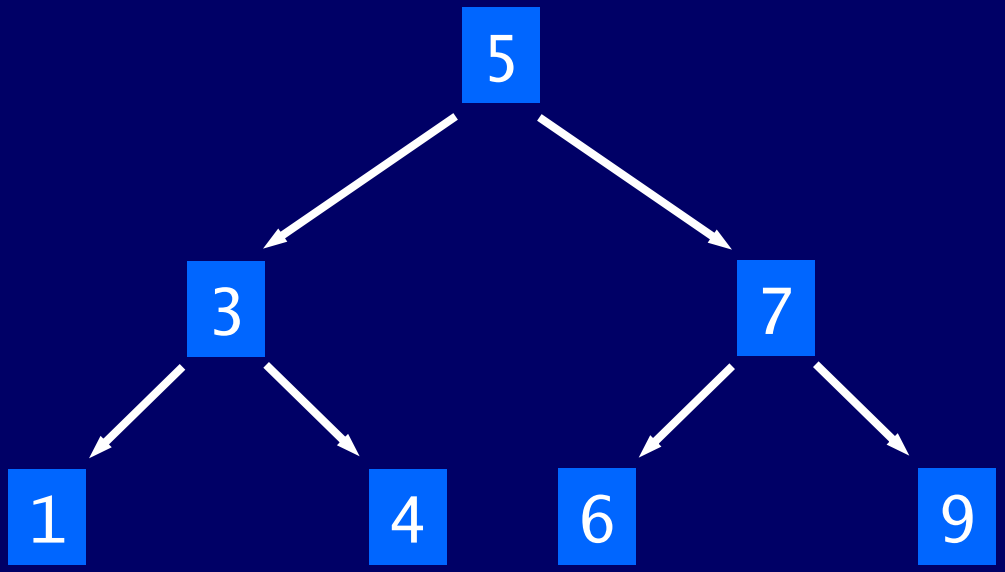
```
eval :: Expr -> Cont -> Int
eval (Val n)     c = exec c n
eval (Add x y)   c = exec x (EVAL y:c)
```

```
exec :: Cont -> Int -> Int
exec []              n  = n
exec (EVAL y : c)    n  = eval y (ADD n: c)
exec (ADD n: c)      m  = exec c (n + m)
```

```
value :: Expr-> Int
value e = eval e []
```

Binary Trees

In computing, it is often useful to store data in a two-way branching structure or binary tree.



Using recursion, a suitable new type to represent such binary trees can be declared by:

```
data Tree a = Leaf a
            | Node (Tree a) a (Tree a)
```

For example, the tree on the previous slide would be represented as follows:

```
t :: Tree Int
t = Node (Node (Leaf 1) 3 (Leaf 4)) 5
         (Node (Leaf 6) 7 (Leaf 9))
```

We can now define a function that decides if a given value occurs in a binary tree:

```
occurs :: Ord a ⇒ a → Tree a → Bool
occurs x (Leaf y)       = x == y
occurs x (Node l y r) = x == y
                        || occurs x l
                        || occurs x r
```

But… in the worst case, when the value does not occur, this function traverses the entire tree.

Now consider the function flatten that returns the list of all the values contained in a tree:

```
flatten :: Tree a → [a]
flatten (Leaf x)     = [x]
flatten (Node l x r) = flatten l
                       ++ [x]
                       ++ flatten r
```

A tree is a search tree if it flattens to a list that is ordered. Our example tree is a search tree, as it flattens to the ordered list [1,3,4,5,6,7,9].

Search trees have the important property that when trying to find a value in a tree we can always decide which of the two sub-trees it may occur in:

```
occurs x (Leaf y)                = x == y
occurs x (Node l y r) | x == y = True
                      | x < y  = occurs x l
                      | x > y  = occurs x r
```

This new definition is more efficient, because it only traverses one path down the tree.

Typeclasses in Haskell

- a means of defining the behaviour associated with a type separately from that type's definition.

- There are a number of typeclasses already defined in Haskell's base package.

- We look at
    - Eq
    - Ord
    - Num
    - Show (IO - later)
    - Read (IO later)

Typeclasses in Haskell - Eq

- All basic datatypes from Prelude except for functions and IO have instances of Eq.
- If a type instantiates Eq it means that we know how to compare two values for *value* or *structural* equality.

Typeclasses in Haskell - Eq

Required methods

```
(==) :: Eq a => a -a-> Boolean
(/=) :: Eq a => a -a-> Boolean
```

Typeclasses in Haskell - Ord

- If a type instantiates **Ord** it means that we know a “natural” ordering of values of that type.

Typeclasses in Haskell - Ord

Uses a custom algebraic data type:

```
data Ordering = LT | EQ | GT
```

Required methods:

compare :: Ord a => a -> a -> Ordering or

(**<=**) **::** **Ord** a **=>** a **->** a **->** Boolean

(the standard's default **compare** method uses

(**<=**) in its implementation)

Typeclasses in Haskell - Ord

Defines

```
compare :: Ord a => a-> a -> Ordering
(<) :: Ord a => a -a-> Boolean
(>=) :: Ord a => a -a-> Boolean
(<=) :: Ord a => a -a-> Boolean
(>) :: Ord a => a -a-> Boolean
max :: Ord a => a-> a -> a
min :: Ord a => a-> a -> a
```

Typeclasses in Haskell - Num

- The most general class for number types
- This class contains both
  - integral types **Int**, **Integer**, Word32 etc.) and
  - fractional types (**Double**, **Rational**, also complex numbers etc.)

Typeclasses in Haskell - Num

```
Defines
fromInteger :: Num a=> Integer ->
(Convert an integer to the general Num
type)
(+) :: Num a => a -> a-> a
(-) :: Num a => a -> a-> a
(*) :: Num a => a -> a-> a
negate :: Num a => a -> a-> a
abs :: Num a => a -> a-> a
signum :: Num a => a -> a-> a
```

Derived Instances

- When new types are declared, usually appropriate to make them instances of
  - Eq
  - Ord
  - Show
  - Read

  Using the deriving mechanism

Derived Instances

```
Data Bool = False | True
            deriving (Eq, Ord, Show, Read)
```

```
> False == False
True
```

```
> False < False
True
```