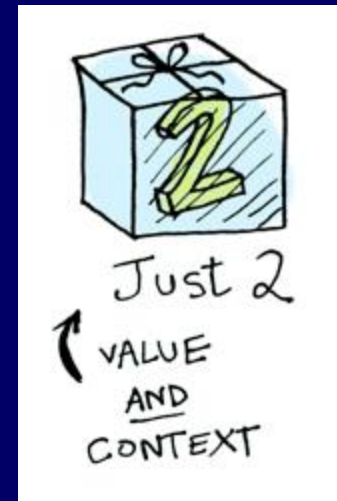
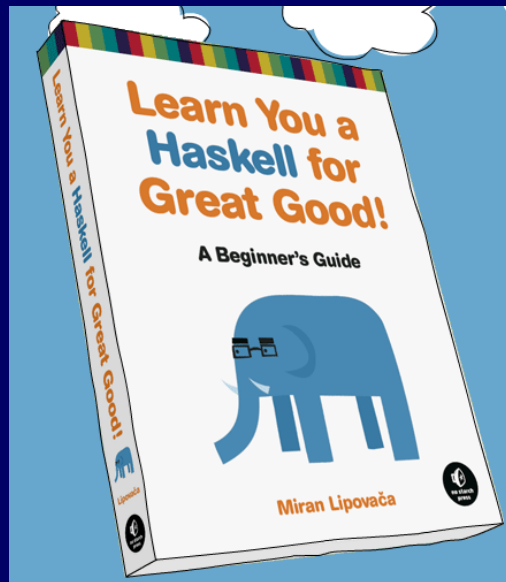
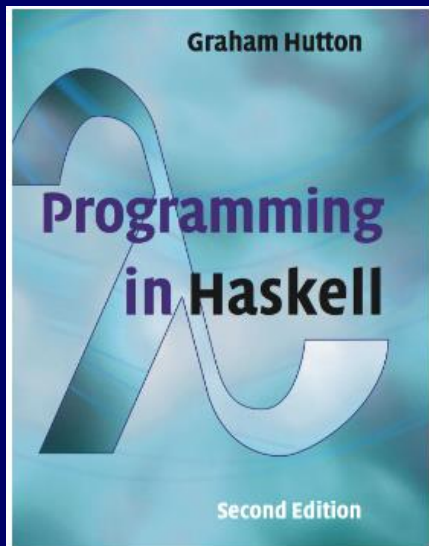


PROGRAMMING IN HASKELL

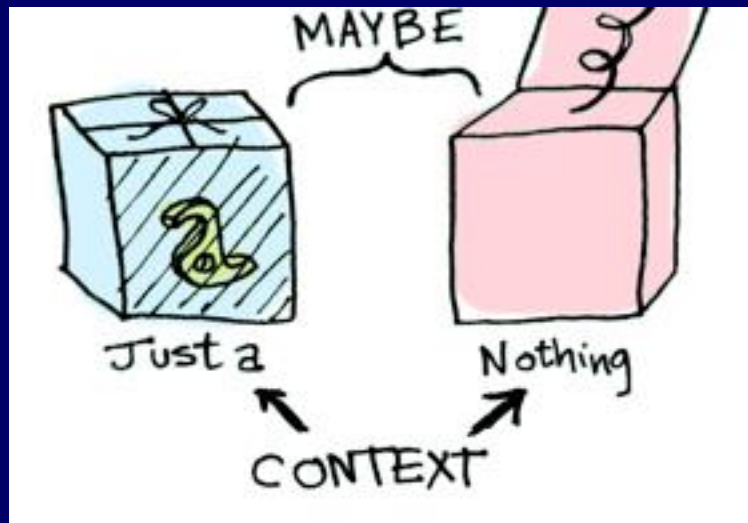
Monads



Based on lecture notes by Graham Hutton,
the book “Learn You a Haskell for Great Good”,
pictures from Aditya Bhargava

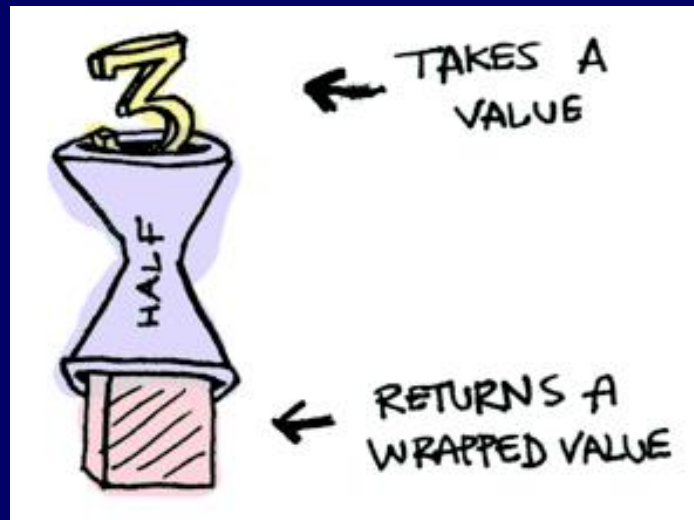
Monads

We look at Maybe to illustrate



Monads

Suppose half is a function that only works on even numbers:



```
half x = if even x
        then Just (x `div` 2)
        else Nothing
```

Monads

What if we feed it a wrapped value?



We need to use `>>=` to shove our wrapped value into the function. Here's a photo of `>>=`:



Monads

How does it work?

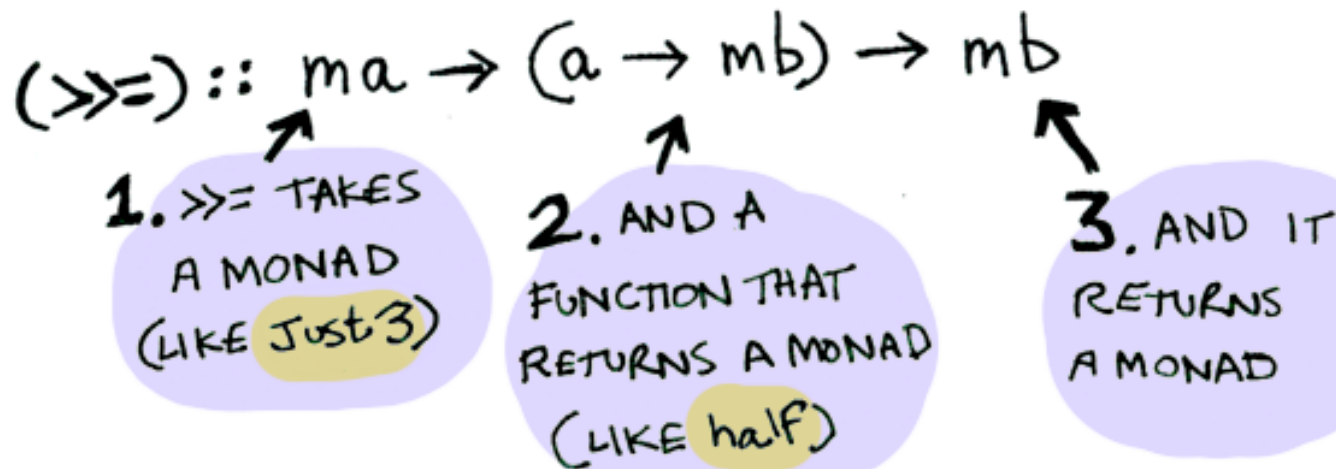
```
> Just 3 >>= half Nothing  
> Just 4 >>= half Just 2  
> Nothing >>= half Nothing
```

Monads

What's happening inside? Monad is another typeclass. Here's a partial definition:

```
class Monad m where  
(>>=) :: m a -> (a -> m b) -> m b
```

Where $>>=$ is:



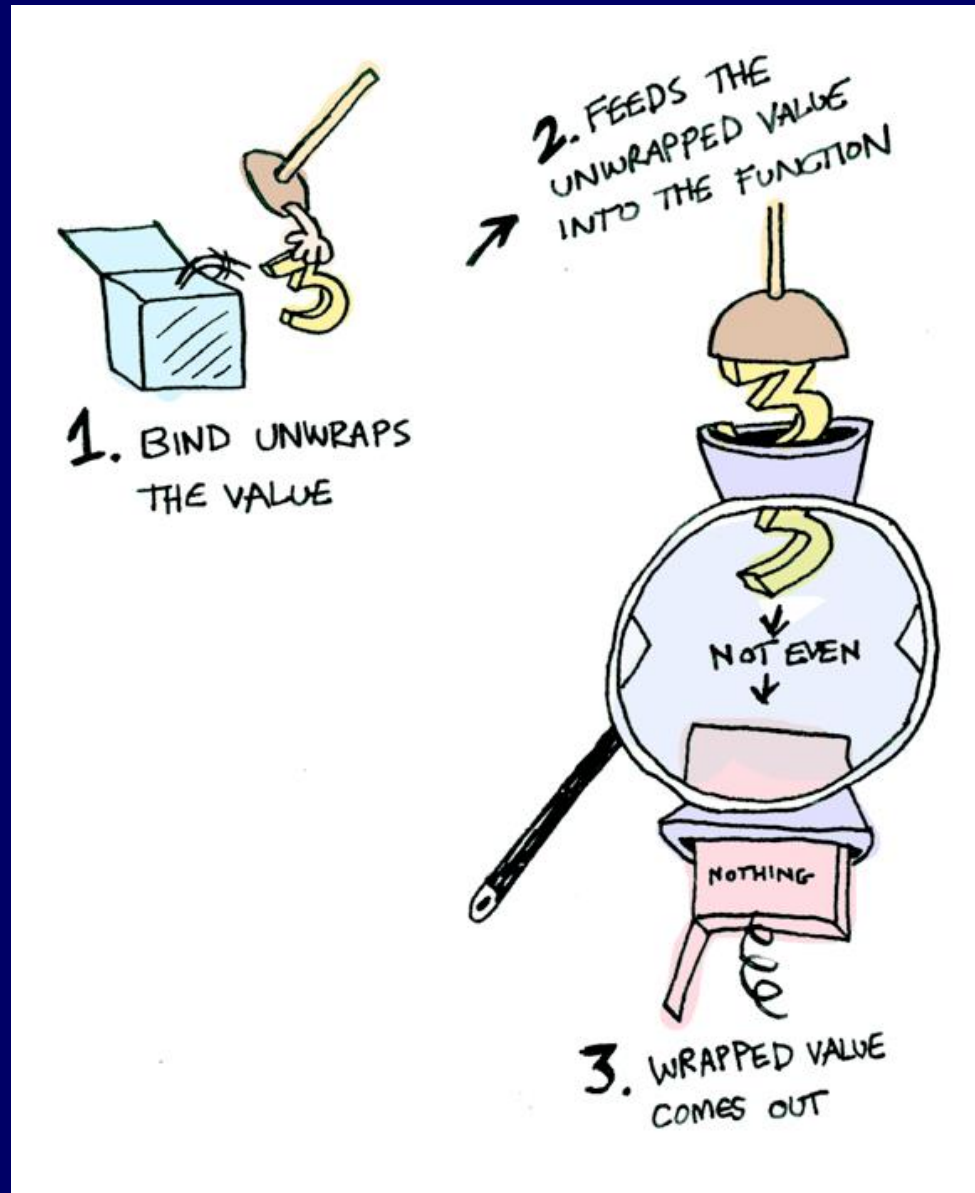
Monads

So Maybe is a Monad:

```
instance Monad Maybe where  
  Nothing >>= func = Nothing  
  Just val >>= func = func val
```

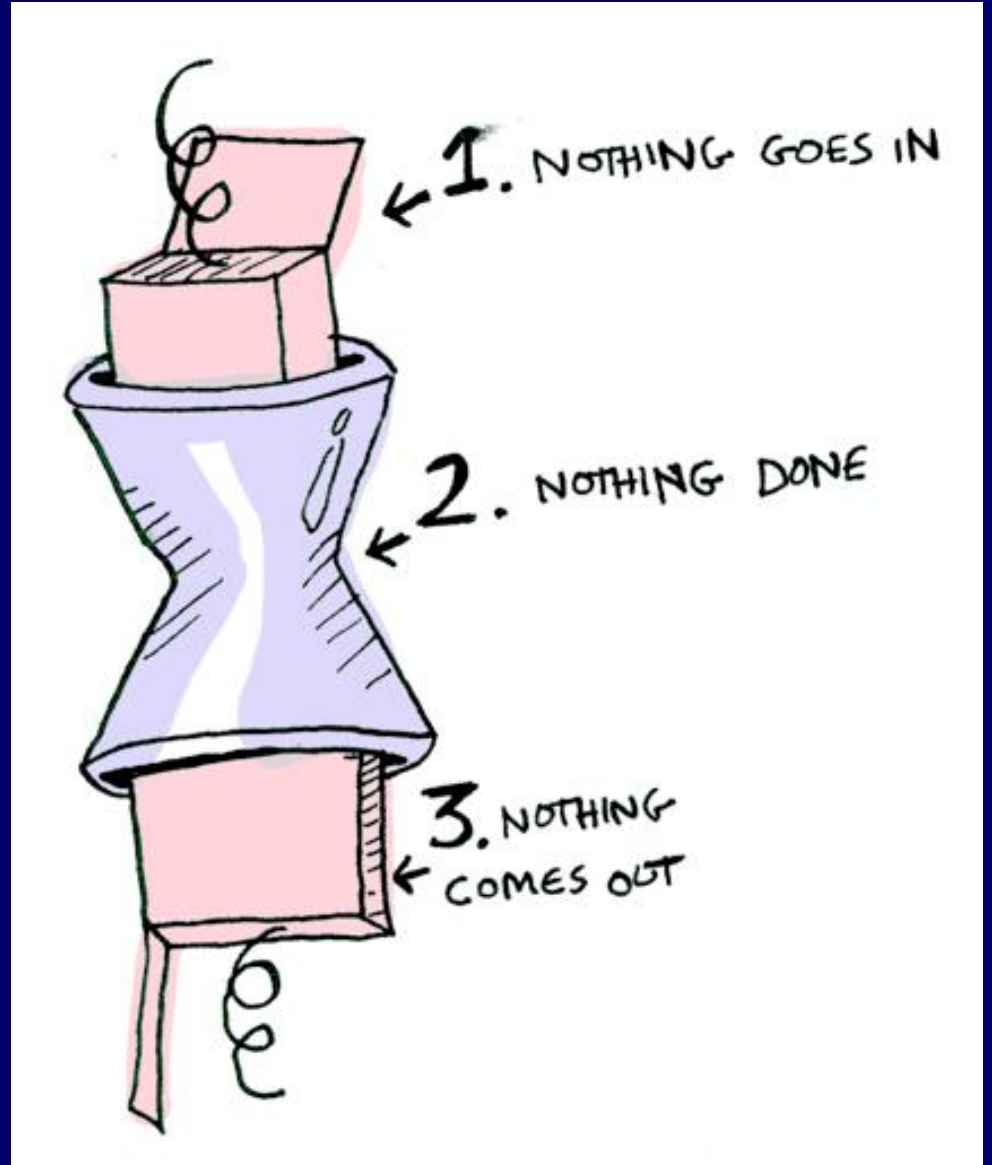

Monads

Here it is
in action
with
a Just 3!



Monads

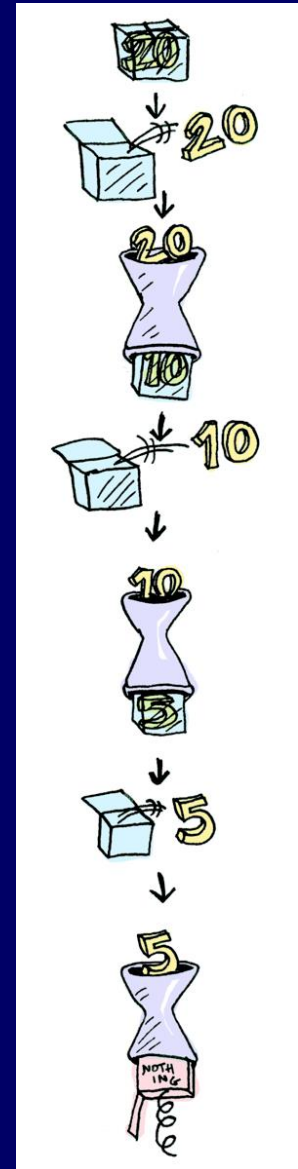
And if you pass
in a Nothing it's
even simpler:



Monads

You can also chain calls:

```
> Just 20 >>= half >>= half >>= half  
Nothing
```



What is a Monad? – Computerphile

Graham Hutton
Computer Science at the University of
Nottingham



Monads

A monad is defined by 3 things:

- A type constructor m
- A function $return$
- An operator $(\gg=)$ – read as “bind”

Originally, these were introduced for IO, but they can be much more powerful (and difficult to understand).

The function and operator are methods of the Monad typeclass and have types themselves, which are required to obey 3 laws (more later):

```
return :: a -> m a
(>>=)  :: m a -> (a -> m b) -> m b
```

Monads example: Maybe

```
return :: a -> Maybe a
return x = Just x

(>>=) :: Maybe a -> (a -> Maybe b) -> Maybe b
m >>= g = case m of
    Nothing -> Nothing
    Just x   -> g x
```

- Maybe is the monad, and return brings a value into it by wrapping it with Just.
- As for (>>=), it takes a `m :: Maybe a` value and a `g :: a -> Maybe b` function. If `m` is `Nothing`, there is nothing to do and the result is `Nothing`. Otherwise, in the `Just x` case, `g` is applied to `x`, the underlying value wrapped in `Just`, to give a `Maybe b` result.
- To sum it all up, if there is an underlying value of type `a` in `m`, we apply `g` to it, which brings the underlying value back into the `Maybe` monad.

Monads: motivation

Continuing this example:

Imagine a family database with two functions, which look up the parents of a person:

```
father :: Person -> Maybe Person  
mother :: Person -> Maybe Person
```

The Maybe is there in case our database is missing some information.

We then might want to build other functions, to look up grandparents, etc (next slide)

Monads: motivation

Now, let's find grandparents:

```
maternalGrandfather :: Person -> Maybe Person
maternalGrandfather p =
  case mother p of
    Nothing -> Nothing
    Just mom -> father mom
```

But there's a better way! Maybe is a monad, so can do:

```
maternalGrandfather p = mother p >>= father
```


Monads:

Even more drastic:

```
bothGrandfathers :: Person -> Maybe (Person, Person)
bothGrandfathers p =
  case father p of
    Nothing -> Nothing
    Just dad ->
      case father dad of
        Nothing -> Nothing
        Just gf1 ->                                -- found first
grandfather
      case mother p of
        Nothing -> Nothing
        Just mom ->
          case father mom of
            Nothing -> Nothing
            Just gf2 ->                                -- found second
grandfather
          Just (gf1, gf2)
```

Becomes:

```
bothGrandfathers p =
  father p >>=
    (\dad -> father dad >>=
      (\gf1 -> mother p >>=    -- gf1 is only used in the
final return
        (\mom -> father mom >>=
          (\gf2 -> return (gf1,gf2) ))))
```

Monads: nuts and bolts

Monad the typeclass lives in `Control.Monad`, and has the following methods:

```
class Applicative m => Monad m where
    return :: a -> m a
    (>>=)  :: m a -> (a -> m b) -> m b

    (>>)   :: m a -> m b -> m b
    fail   :: String -> m a
```

- Aside from `return` and `bind`, there are two additional methods, `(>>)` and `fail`.
 - Both of them have default implementations, and so you don't need to provide them when writing an instance.

Monads: nuts and bolts

Monad is a subclass of Applicative, which is a subclass of Functor. So these are also functors!

Note: ($>>$) and fail both have default implementations, and so you don't need to provide them when writing an instance.

($>>$) sequences two monadic actions when the second action does not involve the result of the first, which is a common scenario for monads such as IO. Simple example:

```
printSomethingTwice :: String -> IO ()  
printSomethingTwice str = putStrLn str >> putStrLn str
```

Monads: back to that example

In fact, that grandfather example can be made even better if we use the `do` notation with braces and semi-colons.

This will look more like imperative code that we're used to! (Like IOs.)

Here, `father` and `mother` are functions that might fail to produce results, i.e. raise an exception, and when that happens, the whole `do`-block will fail, i.e. terminate with an exception.

```
bothGrandfathers p = do {  
    dad <- father p;  
    gf1 <- father dad;  
    mom <- mother p;  
    gf2 <- father mom;  
    return (gf1, gf2);  
}
```

Monad laws

Monads are required to obey 3 laws:

```
m >>= return      = m                -- right unit
return x >>= f      = f x              -- left unit

(m >>= f) >>= g     = m >>= (\x -> f x >>= g) -- associativity
```

Monads originally come from a branch of mathematics called Category Theory.

(Fortunately, it is entirely unnecessary to understand category theory in order to understand and use monads in Haskell.)

Monadic Programming

1. Having designed a Monad (i.e. 'I need a Monad with this shape), we define an instance of a Monad. This involves essentially writing what
 1. `result` and
 2. `>>=` mean in this Monad
1. We can now write programs using `>>=`, `return` etc. allowing us to delineate effectful programming
2. Having written the Monad, we can now use the `do` notation which will include the failure / effectual side-effects.

Note : we can write the programs using 2. or 3. above. `do` is easier to read !

Monadic Programming

Having designed a Monad (i.e. 'I need a Monad with this shape), we define an instance of a Monad. This involves essentially writing what

1. `result` and
2. `>>=` mean in this Monad

1. For the purposes of this module, we concentrate on the instance of Monad. However, in order to write the full code, we need to remember that

1. Monad is an Applicative
2. Applicative is a Functor

So we need to write the instances down the chain

We look at MyMaybe (you may remember this from Maybe!)

MyMaybe from scratch

```
data MyMaybe a = MyJust a | Nuttin deriving (Eq, Show)
```

```
instance Functor MyMaybe where  
  fmap f (MyJust x) = MyJust (f x)  
  fmap _ _ = Nuttin
```

```
instance Applicative MyMaybe where  
  pure = MyJust  
  Nuttin <*> _ = Nuttin  
  (MyJust f) <*> something = fmap f something
```

```
instance Monad MyMaybe where  
  return = MyJust  
  Nuttin >>= _ = Nuttin  
  MyJust x >>= f = f x
```


Using MyMaybe

```
-- use this to test this with ghci  
mayf :: Num a => a -> MyMaybe a  
mayf x = MyJust (x+1)
```

Call as :

```
*Main> MyJust 4 >>= mayf  
MyJust 5  
*Main> Nuttin >>= mayf  
Nuttin  
*Main> █
```

Desugaring do blocks from Graham Hutton's Computerphile video.

```
data Expr = Val Int | Div Expr Expr

safediv :: Int -> Int -> Maybe Int
safediv n m = if m == 0 then
                Nothing
            else
                Just (n `div` m)

-- First attempt
eval :: Expr -> Maybe Int
eval (Val n) = Just n
eval (Div x y) = case eval x of
                    Nothing -> Nothing
                    Just n -> case eval y of
                                Nothing -> Nothing
                                Just m -> safediv n m
```

Desugaring do blocks

```
-- Second attempt using >>= binding
eval :: Expr -> Maybe Int
eval (Val n) = return n
eval (Div x y) =      eval x >>= (\n ->
                                eval y >>= (\m ->
                                safediv n m ))
```

```
-- Third attempt using do block
eval :: Expr -> Maybe Int
eval (Val n)      =      return n
eval (Div x y) = do n <- eval x
                  m <- eval y
                  safediv n m
```

Desugaring do blocks

Bind Operator

do x <- foo bar	===	foo >>= (\x -> bar)
--------------------	-----	---------------------

Sequence Operator

do act1 act2	===	act1 >> act2
-----------------	-----	--------------