

# Exercises

## The Lambda Calculus

### Exercise 1

Keeping in mind both alpha equivalence and how multiple heads are nested, choose an answer that is equivalent to the listed lambda term.

1.  $\lambda xy.xz$ 
  - (a)  $\lambda xz.xz$
  - (b)  $\lambda mn.mz$
  - (c)  $\lambda z(\lambda x.xz)$
2.  $\lambda xy.xxy$ 
  - (a)  $\lambda mn.mnp$
  - (b)  $\lambda x.(\lambda y.xy)$
  - (c)  $\lambda a(\lambda b.aab)$
3.  $\lambda xyz.zx$ 
  - (a)  $\lambda x.(\lambda y.(\lambda z))$
  - (b)  $\lambda tos.st$
  - (c)  $\lambda mnp.mn$

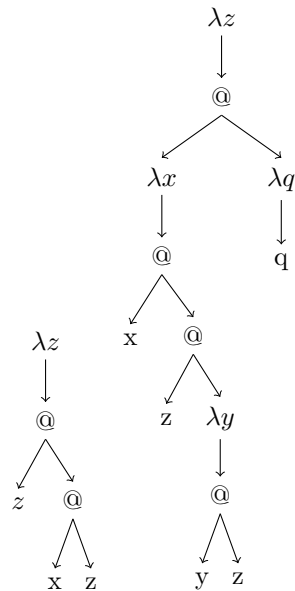
### Exercise 2

Write out lambda calculus trees for the following expressions.

1.  $\lambda x.\lambda y.xy$
2.  $\lambda x.\lambda y.xy.\lambda z.yz$
3.  $\lambda x.(\lambda y.xy)(\lambda z.yz)$

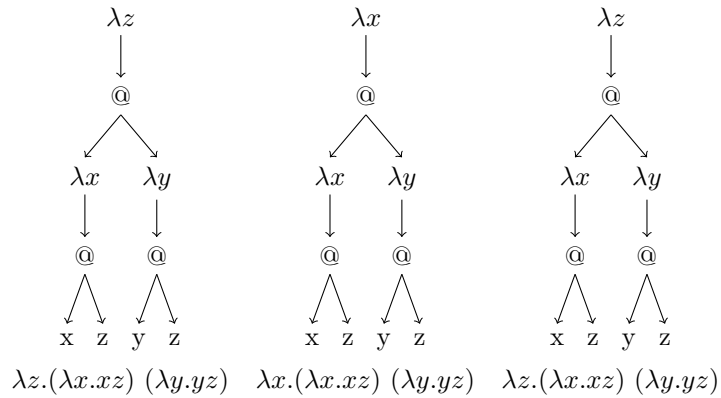
### Exercise 3

Write out the equivalent lambda calculus expression for the following trees.



#### Exercise 4

Find the free variables. What are the free variables? To which lambdas are bound variables bound?



#### Exercise 5

Using  $\beta$  reduction etc., rewrite these expressions in normal form.

1.  $(\lambda x.x)y$
2.  $(\lambda x.xz)(\lambda y.y)$
3.  $(\lambda x.x(\lambda x.y))(\lambda z.z)$
4.  $(\lambda x.(\lambda y.x))y(\lambda z.z)$

5.  $(\lambda x.xx)(\lambda x.xx)$

### Exercise 6

Rewrite, using Haskell and evaluate the following:

1.  $(\lambda x.x)2$
2.  $(\lambda x.(x * 2))4$
3.  $(\lambda x.(\lambda y.x * y))3\ 4$
4.  $(\lambda x.\lambda y.(if\ x < y\ then\ -1\ else\ if\ x == y\ then\ 0\ else\ 1))\ 3\ 4$   
(**Note:** Use of if inside the lambda expression. )

### Exercise 7

Write a lambda version of the following functions:

1. **abs**: which takes an Integer and returns the non-negative value.  
e.g.  $abs\ -1 = 1$ ,  $abs\ 4 = 4$ .
2. **mymax**: which takes two numbers and returns the larger of the two
3. **mymin**: which takes two numbers and returns the smaller of the two