What is a Type?

A <u>type</u> is a name for a collection of related values. For example, in Haskell the basic type

Bool

contains the two logical values:

False

True

Type Errors

Applying a function to one or more arguments of the wrong type is called a <u>type error</u>.

> 1 + False error ...

1 is a number and False is a logical value, but + requires two numbers.

Types in Haskell

If evaluating an expression e would produce a value of type t, then e <u>has type</u> t, written

e :: t

Every well formed expression has a type, which can be automatically calculated at compile time using a process called type inference. All type errors are found at compile time, which makes programs <u>safer and faster</u> by removing the need for type checks at run time.

In GHCi, the :type command calculates the type of an expression, without evaluating it:

```
> not False
True
> :type not False
not False :: Bool
```

Basic Types

Haskell has a number of basic types, including:

Bool

- logical values

Char

- single characters

String

strings of characters

Int

fixed-precision integers

Integer

- arbitrary-precision integers

Float

floating-point numbers

List Types

A <u>list</u> is sequence of values of the <u>same</u> type:

```
[False,True,False] :: [Bool]
['a','b','c','d'] :: [Char]
```

In general:

[t] is the type of lists with elements of type t.

■ The type of a list says nothing about its length:

```
[False,True] :: [Bool]
[False,True,False] :: [Bool]
```

■ The type of the elements is unrestricted. For example, we can have lists of lists:

```
[['a'],['b','c']] :: [[Char]]
```

Tuple Types

A tuple is a sequence of values of different types:

```
(False,True) :: (Bool,Bool)

(False,'a',True) :: (Bool,Char,Bool)
```

In general:

(t1,t2,...,tn) is the type of n-tuples whose ith components have type ti for any i in 1...n.

■ The type of a tuple encodes its size:

```
(False,True) :: (Bool,Bool)

(False,True,False) :: (Bool,Bool,Bool)
```

The type of the components is unrestricted:

```
('a',(False,'b')) :: (Char,(Bool,Char))
(True,['a','b']) :: (Bool,[Char])
```

Function Types

A <u>function</u> is a mapping from values of one type to values of another type:

```
not :: Bool → Bool
even :: Int → Bool
```

In general:

 $t1 \rightarrow t2$ is the type of functions that map values of type t1 to values to type t2.

- The arrow \rightarrow is typed at the keyboard as ->.
- The argument and result types are unrestricted. For example, functions with multiple arguments or results are possible using lists or tuples:

```
add :: (Int,Int) → Int
add (x,y) = x+y

zeroto :: Int → [Int]
zeroto n = [0..n]
```

Curried Functions

Functions with multiple arguments are also possible by returning <u>functions</u> as <u>results</u>:

```
add' :: Int \rightarrow (Int \rightarrow Int)
add' x y = x+y
```

add' takes an integer x and returns a function <u>add' x</u>. In turn, this function takes an integer y and returns the result x+y.

add and add' produce the same final result, but add takes its two arguments at the same time, whereas add' takes them one at a time:

```
add::(Int,Int) → Int

add'::Int → (Int → Int)
```

Functions that take their arguments one at a time are called <u>curried</u> functions, celebrating the work of Haskell Curry on such functions. Functions with more than two arguments can be curried by returning nested functions:

```
mult :: Int \rightarrow (Int \rightarrow (Int \rightarrow Int))
mult x y z = x*y*z
```

mult takes an integer x and returns a function mult x, which in turn takes an integer y and returns a function mult x y, which finally takes an integer z and returns the result x*y*z.

Why is Currying Useful?

Curried functions are more flexible than functions on tuples, because useful functions can often be made by <u>partially applying</u> a curried function.

For example:

```
add' 1 ::: Int → Int

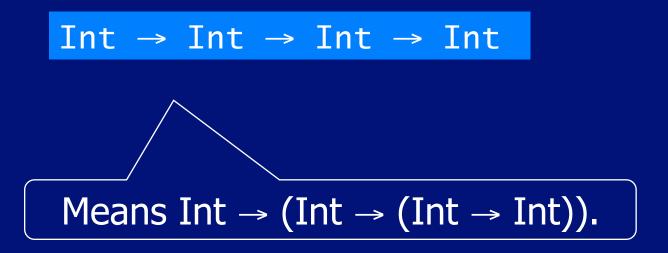
take 5 ::: [Int] → [Int]

drop 5 ::: [Int] → [Int]
```

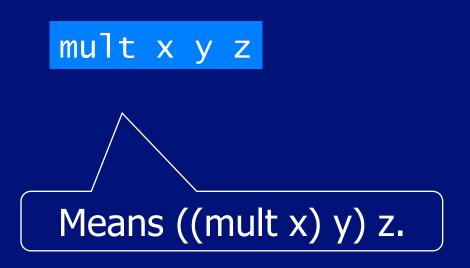
Currying Conventions

To avoid excess parentheses when using curried functions, two simple conventions are adopted:

■ The arrow \rightarrow associates to the <u>right</u>.



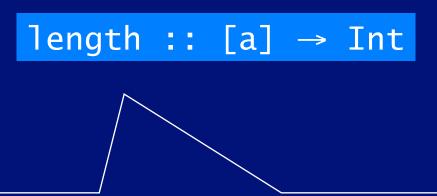
As a consequence, it is then natural for function application to associate to the <u>left</u>.



Unless tupling is explicitly required, all functions in Haskell are normally defined in curried form.

Polymorphic Functions

A function is called <u>polymorphic</u> ("of many forms") if its type contains one or more type variables.



For any type a, length takes a list of values of type a and returns an integer.

Type variables can be instantiated to different types in different circumstances:

```
> length [False,True]
2
> length [1,2,3,4]
4
a = Bool
a = Int
```

■ Type variables must begin with a lower-case letter, and are usually named a, b, c, etc.

Many of the functions defined in the standard prelude are polymorphic. For example:

```
fst :: (a,b) \rightarrow a
head :: [a] \rightarrow a
take :: Int \rightarrow [a] \rightarrow [a]
zip :: [a] \rightarrow [b] \rightarrow [(a,b)]
id :: a \rightarrow a
```

Overloaded Functions

A polymorphic function is called <u>overloaded</u> if its type contains one or more class constraints.

(+) :: Num
$$a \Rightarrow a \rightarrow a \rightarrow a$$

For any numeric type a, (+) takes two values of type a and returns a value of type a.

Constrained type variables can be instantiated to any types that satisfy the constraints:

> 1 + 2
3

> 1.0 + 2.0
3.0

$$a = Int$$

$$a = Int$$

Char is not a numeric type

Haskell has a number of type classes, including:

- Num Numeric types
- Eq Equality types
- Ord Ordered types
- For example:

(+) :: Num
$$a \Rightarrow a \rightarrow a \rightarrow a$$

(==) :: Eq $a \Rightarrow a \rightarrow a \rightarrow Bool$
(<) :: Ord $a \Rightarrow a \rightarrow a \rightarrow Bool$

$$(<)$$
 :: Ord $a \Rightarrow a \rightarrow a \rightarrow Boo$

Hints and Tips

When defining a new function in Haskell, it is useful to begin by writing down its type;

- Within a script, it is good practice to state the type of every new function defined;
- When stating the types of polymorphic functions that use numbers, equality or orderings, take care to include the necessary class constraints.