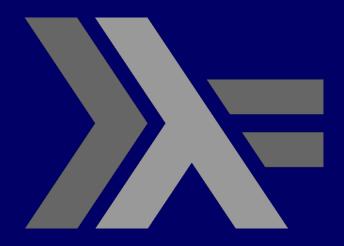# PROGRAMMING IN HASKELL

Chapter 6 - Recursive Functions

Introduction

As we have seen, many functions can naturally be defined in terms of other functions.

```
fac :: Int → Int
fac n = product [1..n]
```

fac maps any integer n to the product of the integers between 1 and n.

Expressions are <u>evaluated</u> by a stepwise process of applying functions to their arguments.

For example:

`fac 4`

=

`product [1..4]`

=

`product [1,2,3,4]`

=

`1*2*3*4`

=

`24`

Recursive Functions

In Haskell, functions can also be defined in terms of themselves. Such functions are called <u>recursive</u>.

```
fac 0 = 1
fac n = n * fac (n-1)
```

fac maps 0 to 1, and any other integer to the product of itself and the factorial of its predecessor.

For example:

```
    fac 3
=
    3 * fac 2
=
    3 * (2 * fac 1)
=
    3 * (2 * (1 * fac 0))
=
    3 * (2 * (1 * 1))
=
    3 * (2 * 1)
=
    3 * 2
=
    6
```

Note:

- ❑ fac 0 = 1 is appropriate because 1 is the identity for multiplication: 1*x = x = x*1.

- ❑ The recursive definition <u>diverges</u> on integers < 0 because the base case is never reached:

```
> fac (-1)

*** Exception: stack overflow
```

Why is Recursion Useful?

❑ Some functions, such as factorial, are <u>simpler</u> to define in terms of other functions.

❑ As we shall see, however, many functions can <u>naturally</u> be defined in terms of themselves.

❑ Properties of functions defined using recursion can be proved using the simple but powerful mathematical technique of <u>induction</u>.

Recursion on Lists

Recursion is not restricted to numbers, but can also be used to define functions on lists.

```
product :: Num a ⇒ [a] → a
product []     = 1
product (n:ns) = n * product ns
```

product maps the empty list to 1, and any non-empty list to its head multiplied by the product of its tail.

For example:

```
    product [2,3,4]
=
    2 * product [3,4]
=
    2 * (3 * product [4])
=
    2 * (3 * (4 * product []))
=
    2 * (3 * (4 * 1))
=
    24
```

Using the same pattern of recursion as in product we can define the length function on lists.

```
length :: [a] → Int
length []      = 0
length (_:xs) = 1 + length xs
```

length maps the empty list to 0, and any non-empty list to the successor of the length of its tail.

For example:

```
length [1,2,3]
=
1 + length [2,3]
=
1 + (1 + length [3])
=
1 + (1 + (1 + length []))
=
1 + (1 + (1 + 0))
=
3
```

Using a similar pattern of recursion we can define the <u>reverse</u> function on lists.

```
reverse :: [a] → [a]
reverse []     = []
reverse (x:xs) = reverse xs ++ [x]
```

reverse maps the empty list to the empty list, and any non-empty list to the reverse of its tail appended to its head.

For example:

```
reverse [1,2,3]
=
reverse [2,3] ++ [1]
=
(reverse [3] ++ [2]) ++ [1]
=
((reverse [] ++ [3]) ++ [2]) ++ [1]
=
(([] ++ [3]) ++ [2]) ++ [1]
=
[3,2,1]
```

Multiple Arguments

Functions with more than one argument can also be defined using recursion. For example:

❑ Zipping the elements of two lists:

```
zip :: [a] → [b] → [(a,b)]
zip []      _       = []
zip _       []      = []
zip (x:xs) (y:ys) = (x,y) : zip xs ys
```

❑ Remove the first n elements from a list:

```
drop :: Int → [a] → [a]
drop 0 xs      = xs
drop _ []      = []
drop n (_:xs) = drop (n-1) xs
```

❑ Appending two lists:

```
(++) :: [a] → [a] → [a]
[]       ++ ys = ys
(x:xs) ++ ys = x : (xs ++ ys)
```

Quicksort

The <u>quicksort</u> algorithm for sorting a list of values can be specified by the following two rules:

❑ The empty list is already sorted;

❑ Non-empty lists can be sorted by sorting the tail values $\leq$ the head, sorting the tail values $>$ the head, and then appending the resulting lists on either side of the head value.

Using recursion, this specification can be translated directly into an implementation:

```
qsort :: Ord a ⇒ [a] → [a]
qsort []      = []
qsort (x:xs) =
    qsort smaller ++ [x] ++ qsort larger
    where
        smaller = [a | a ← xs, a ≤ x]
        larger  = [b | b ← xs, b > x]
```

Note:

❑ This is probably the <u>simplest</u> implementation of quicksort in any programming language!

For example (abbreviating qsort as q):

q [3,2,4,1,5]

q [2,1] ++ [3] ++ q [4,5]

q [1] ++ [2] ++ q []   q [] ++ [4] ++ q [5]

[1]   []   []   [5]