Deep Learning

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1 Introduction

2 A Brief Overview of Tensors

You are likely familiar with scalars, vectors, and matrices. These can be thought of as analagous data structures in zero, one, and two-dimensions, respectively. When generalizing to N dimensions, we refer to these collectively as tensors. A scalar is a zero-order tensor, a vector is a first-order tensor, and a matrix is a second-order tensor. A third-order tensor can be visualized as a stack of matrices. A fourth-order tensor would then be a vector of third order tensors. A fifth-order tensor is a matrix of third-order tensors... and so on.

2.1 Tensor Products

Tensor additon and subtraction are self-explanatory if matrix addition and subtraction are understood. The same cannot be said for tensor products. Below is an overview of tensor products necessary for the decompositions that will be presented in the next section.

2.1.1 Outer Product $\circ$

A tensor $T^{(N)}$ can be expressed as a product of N vectors. This is called the outer product (denoted $\circ$).

$$T^{(N)} = u_1 \circ u_2 \circ \cdots \circ u_N$$

2.1.2 Kronecker Product $\otimes$

The Konecker product of two matrices A and B, their Kronecker product is a matrix of the products of each element in A and the entire matrix B.

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \dots & a_{1n}B \\ a_{21}B & a_{22}B & \dots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & am2B & \dots & a_{mn}B \end{bmatrix}$$

2.1.3 Khatri-Rao Product $\odot$

The Khatri-Rao product of two matrices A and B, each with the same number of columns, is a matrix composed of the Kronecker products of the columns in matrix A and the columns in matrix B with the same indices.

$$A^{:\times n} \odot B^{:\times n} = \begin{bmatrix} a_{:,1} \otimes b_{:,1} & a_{:,2} \otimes b_{:,2} & \dots & a_{:,n} \otimes b_{:,n} \end{bmatrix}$$

2.1.4 Hadamard Product $*$

The Hadamard product of two matrices A and B of the same dimensions is a matrix formed of the products of the elements in A and B with the same indices.

$$A^{m\times n} * B^{m\times n} = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} & \dots & a_{1n}b_{1n} \\ a_{21}b_{21} & a_{22}b_{22} & \dots & a_{2n}b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}b_{m1} & a_{m2}b_{m2} & \dots & a_{mn}b_{mn} \end{bmatrix}$$

2.2 Tensor Decompositions

2.2.1 CP Decomposition

The canonical polyadic, or CP Decomposition, decomposes a tensor into vectors.

$$T = \sum_{r-1}^{R} u_r^{(1)} \circ u_r^{(2)} \circ \dots u_r^{(R)}$$

2.2.2 Tucker Decomposition

The Tucker decomposition decomposes a tensor into a core tensor and factored matrices.

$$T = C \prod_{n=1}^{N} A^n$$

,

where C is the core tensor.

2.2.3 Tensor Train

The tensor train decomposition decomposes a tensor into a product of third-order tensors. It is used when a tensor is too large for the CP decomposition to be practical.

3 Selecting A Network Architecture

4 Convolutional Neural Networks

4.1 What Is Convolution?

4.2 Image Classification Example

5 Recurrent Neural Networks

6 Generative Models

6.1 Generative Adversarial Neural Networks

6.2 Variational Autoencoders

6.3 Quantization