

# Sound Propagation

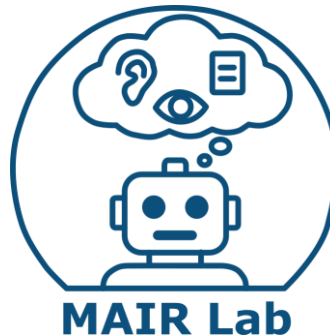
안인규 (Inkyu An)

**Speech And Audio Recognition**  
(오디오 음성인식)

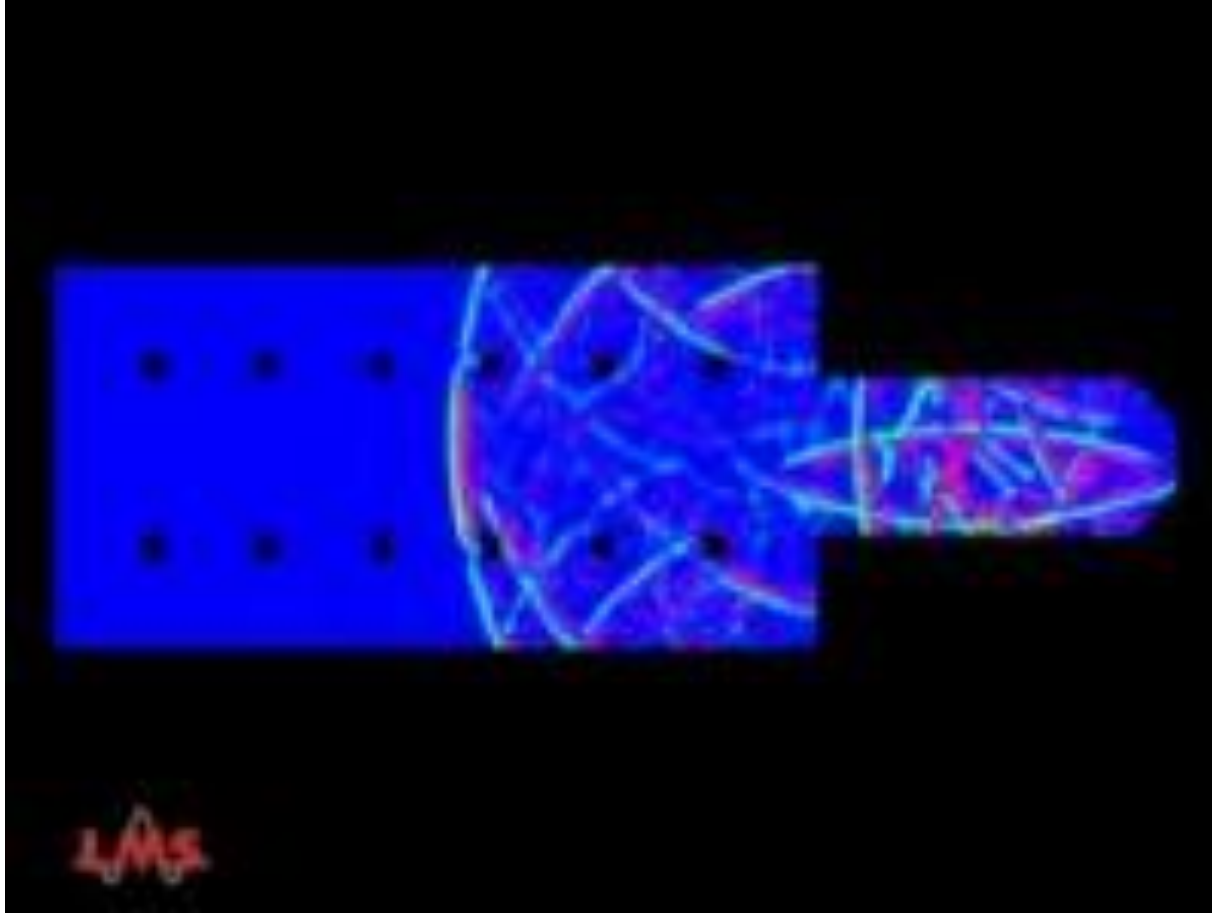
<https://mairlab-km.github.io/>



This lecture material is based on the Lecture Notes (EE837) of Prof. Jung-Woo Choi at KAIST and the Ph.D. dissertation of Dr. Taeyoung Kim at Samsung Electronics



# How does sound propagate?

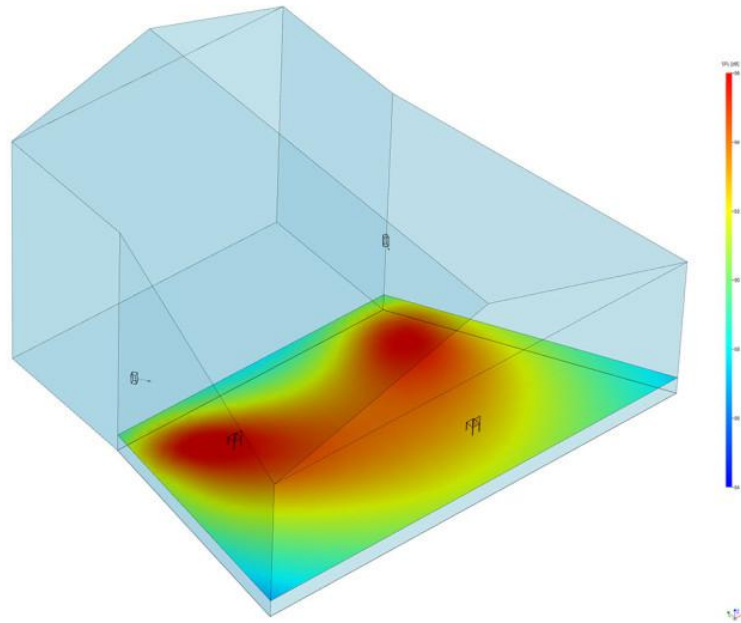


<https://www.youtube.com/watch?v=Xsx4VBEKciA>



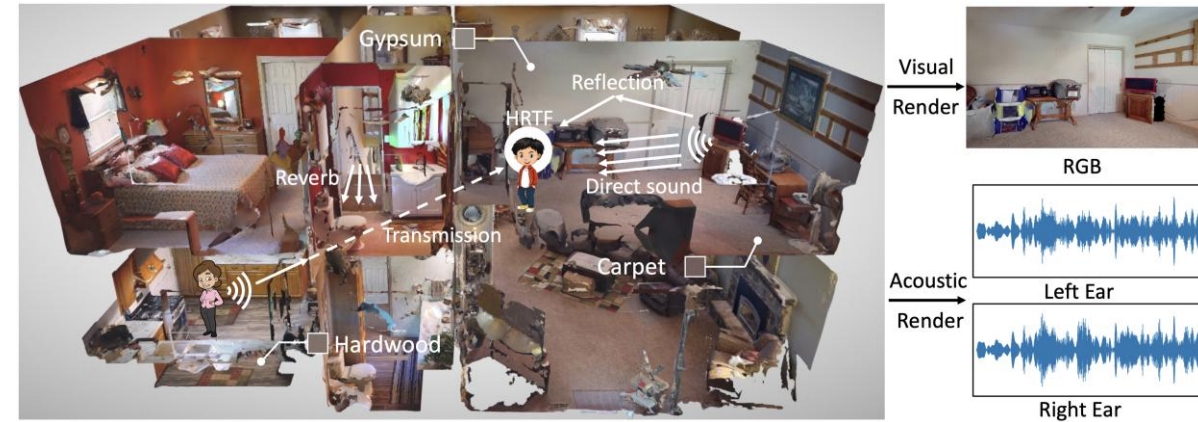
<https://www.youtube.com/shorts/GnAvquGBmnY>

# To create realistic sound!



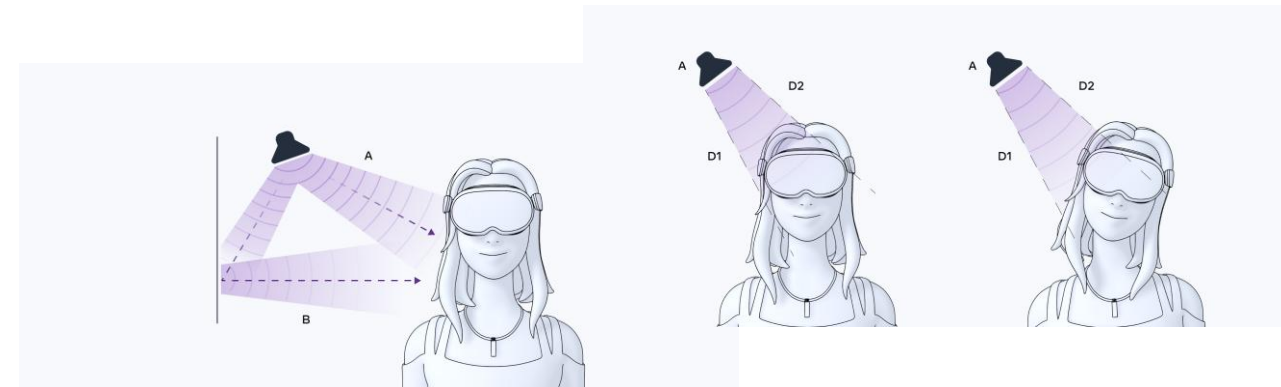
Acoustic simulation (designing Auditorium)

<https://audioxpress.com/article/acoustic-simulation-ease-5-s-acousteer-engine-redefines-design-workflows>



Sound simulation (generating synthetic data)

<https://vision.cs.utexas.edu/projects/soundspace2/>



Sound simulation (Spatial audio in VR)

[https://developers.meta.com/horizon/design/spatial\\_audio/](https://developers.meta.com/horizon/design/spatial_audio/)

# To create realistic sound!



READY OR NOT (Game)

<https://www.youtube.com/watch?v=q267XIOBSr8&t=71s>

# To create realistic sound!



Recorded sound in the real environment

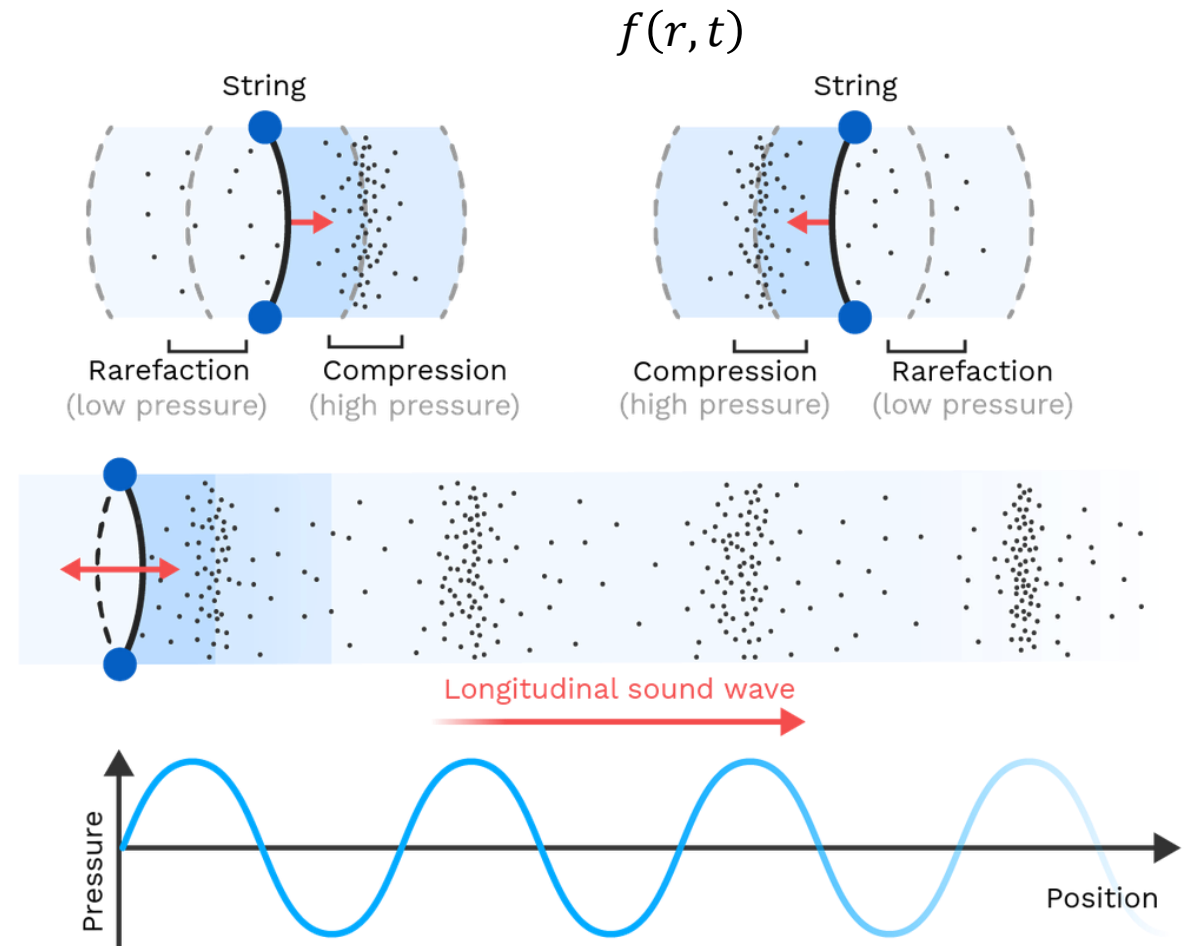
Without considering  
acoustic material

Generated sound in the virtual environment

Using appropriate  
acoustic material

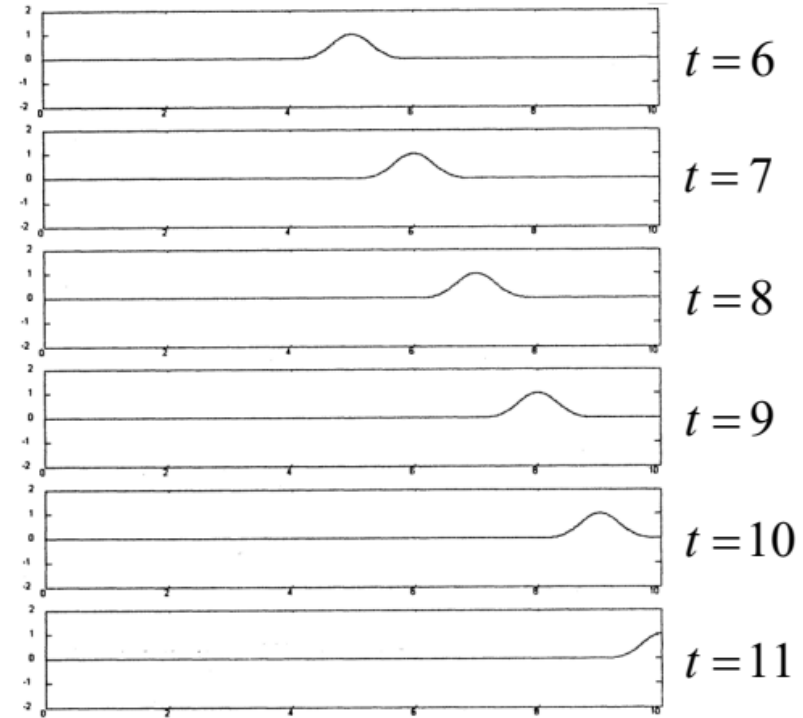
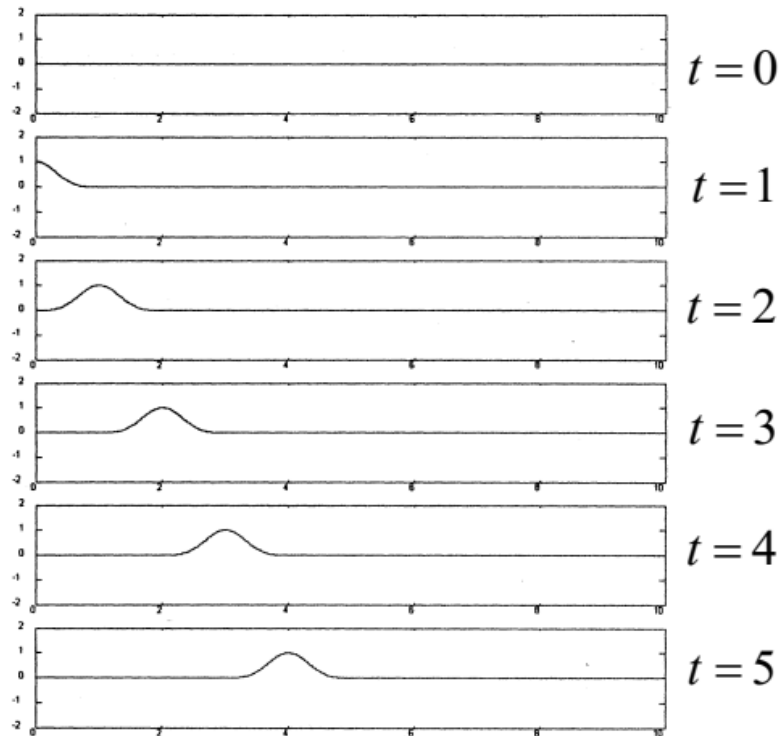
# Wave?

- What is wave?
  - Propagation of disturbances through medium in space and time
- Acoustic properties to consider
  - Acoustic pressure
  - Particle velocity
  - Acoustic density
  - Sound power and intensity
  - ...



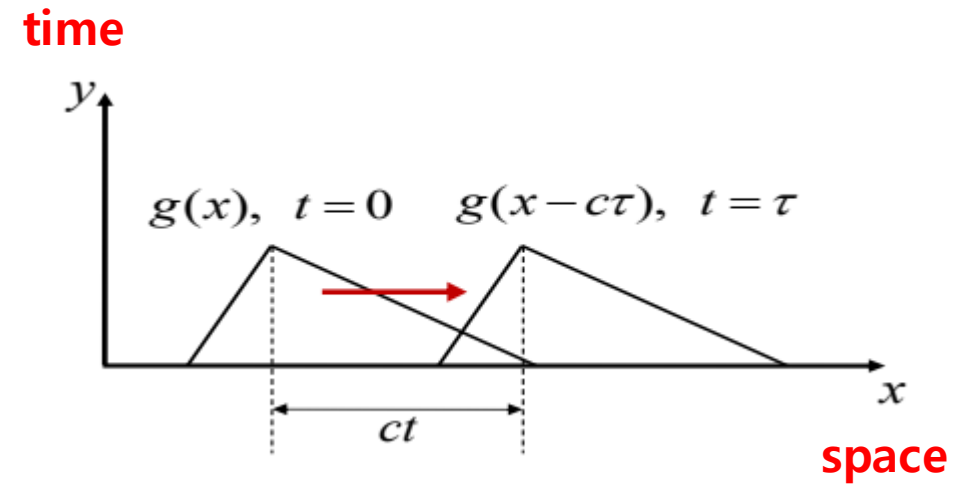
# Simple Wave Propagation

- Wave propagation on a string
  - What propagating is an 'event', not a medium itself



# Simple Wave Propagation

- Wave propagation on string
  - Non-dispersive wave: 파동의 속도가 주파수에 의존하지 않는 파동 (즉, 주파수 성분이 같은 속도로 이동)
  - Right-going wave (waveform)
    - $y(x, t) = g(x - ct) = g\left(-c\left(t - \frac{x}{c}\right)\right)$
  - Left-going wave (waveform)
    - $y(x, t) = h(x + ct)$
  - General form
    - $y(x, t) = g(x - ct) + h(x + ct)$

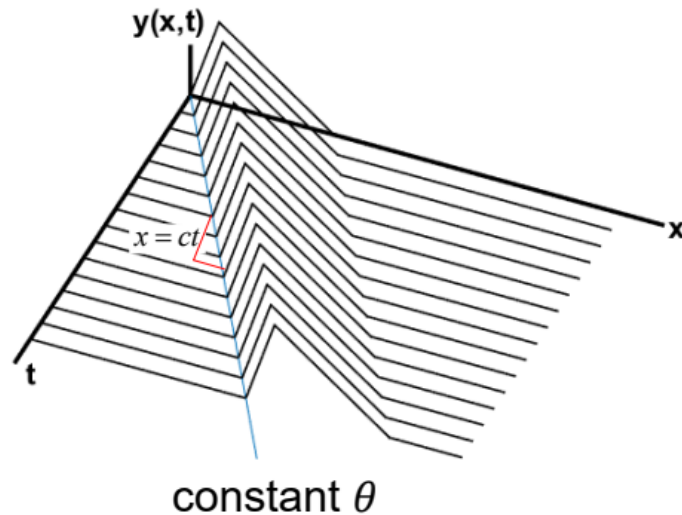




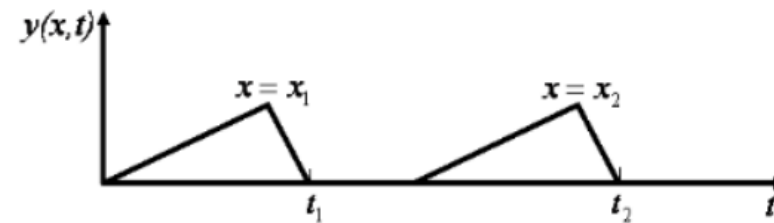
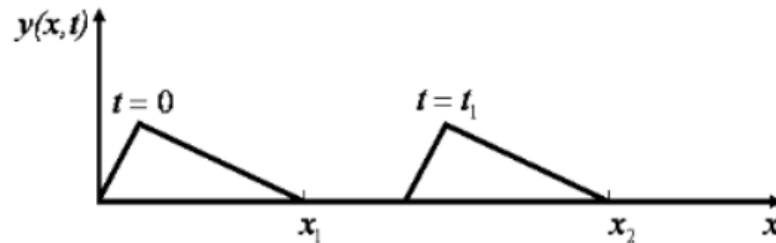
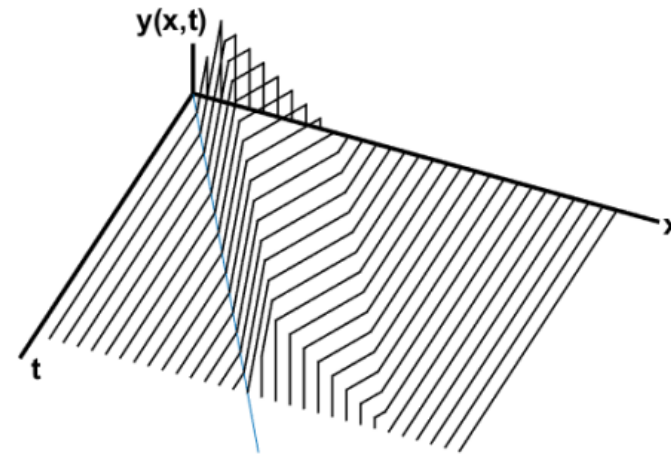
# Simple Wave Propagation

- Wave observed in space-time domain

Snapshot at a fixed time

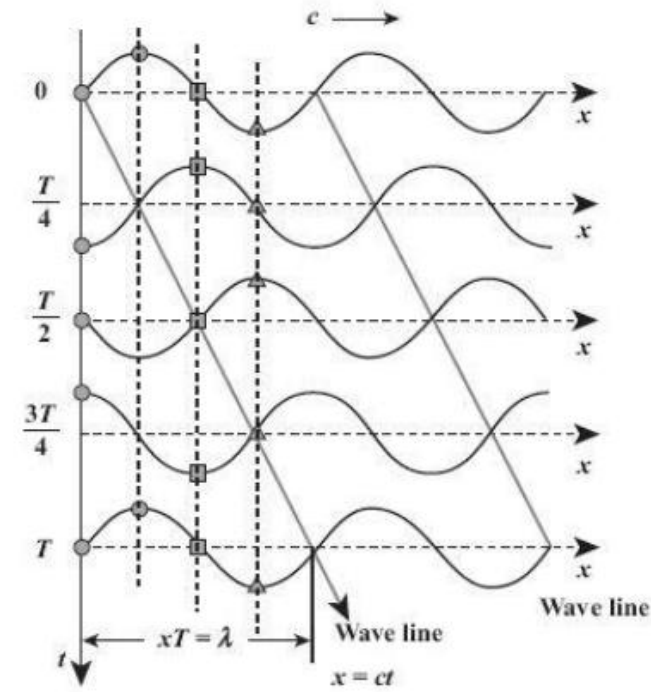
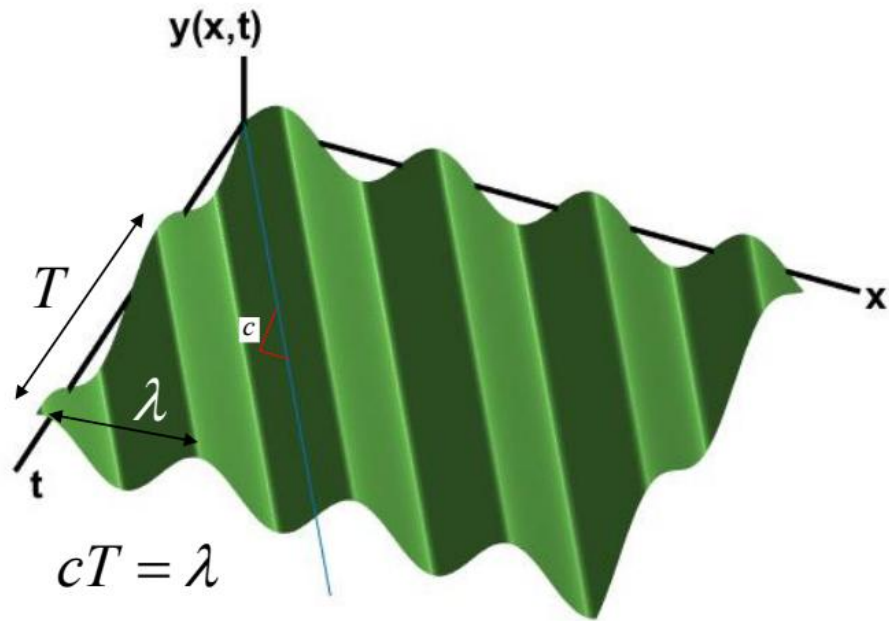


Recording at a fixed location



# Simple Wave Propagation

- Harmonically propagating wave
  - Right-going wave (waveform)
    - $y(x, t) = y(x - ct) = Y\cos(k(x - ct) + \phi_0)$



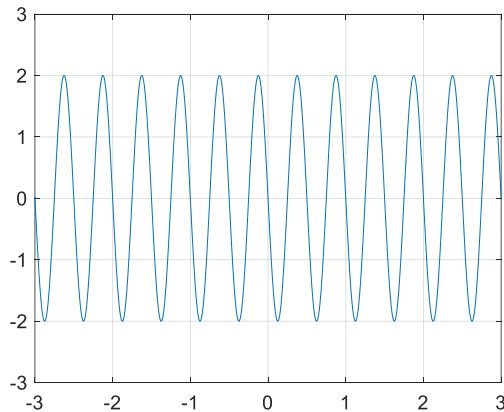
The distance across which a wave travels for a period  $T$  with a propagation speed  $c$  will be a wavelength  $\lambda$

# Fourier Series

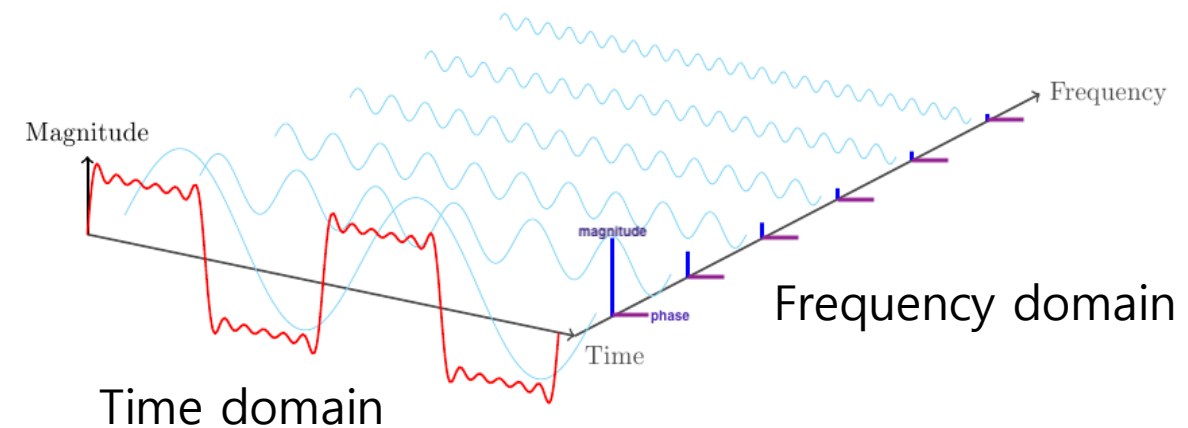
- Any absolutely integrable periodic function  $x(t)$  with period  $P$  can be represented as

$$x(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} \overbrace{A_n}^{\text{Amplitude}} \cos\left(2\pi \underbrace{\frac{n}{P}t}_{\text{Frequency}} - \underbrace{\phi_n}_{\text{Phase}}\right)$$

- What is the amplitude and the phase?



- Amplitude?
- Phase?



# Simple Wave Propagation

- Harmonically propagating wave
  - Right-going wave (waveform)
    - $y(x, t) = y(x - ct) = Y \cos(k(x - ct) + \phi_0)$
  - Frequency: number of temporal oscillation per second
    - Cyclic frequency (Hz):  $f$
    - Radian frequency (radian/s):  $\omega = 2\pi f$
    - $y(x, t) = Y \cos(kx - \omega t)$
  - Wavenumber: spatial oscillation per meter
    - Spatial frequency: radian/m
  - Period: time taken for single oscillation ( $T$ )
  - Wavelength: space taken for single oscillation ( $\lambda$ )

# Simple Wave Propagation

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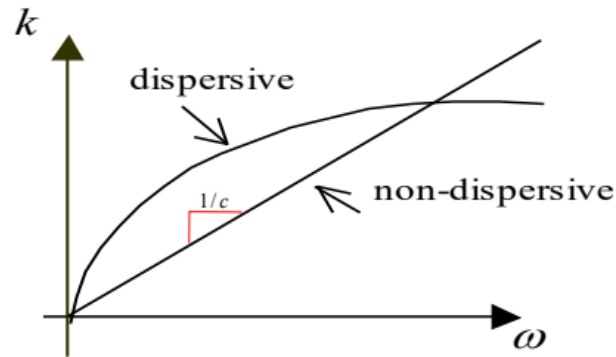
## <Space-time relations>

- Frequency-time relation:
  - $f = \frac{1}{T}$
  - $\omega = 2\pi f = \frac{2\pi}{T}$
- Spatial frequency-space:
  - $k = \frac{2\pi}{\lambda}$
- Space-time:
  - $k = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{cT}$

**Space-time variation is not independent in waves!**

# Simple Wave Propagation

- Space-time coupling
  - Dispersion relation
    - **Non-dispersive wave: sound speed is independent of frequency**  $\Rightarrow \omega = ck$
    - Dispersive wave: sound speed varies with frequency




- Why the space-time coupling occurs?
  - Medium's behavior (dynamics): 파동이 지나가는 매질 (medium)의 물리적 특성이  $\omega - k$ 의 관계를 결정
  - 공기는 선형관계 (즉, non-dispersive)

$\Rightarrow$  **Wave equation!**

# Wave Equation

- Wave equation:
  - 파동 현상을 수학적으로 기술하는 편미분 방정식
  - One-dimensional wave equation (space가 1-D)

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}$$

  
↑                      ↑  
Spatial              Time

- Wave equation describes the space-time coupling relation
- Second-order derivatives?
  - (Roughly) spatial curvature is related to temporal curvature

# Wave Equation

- Wave equation:

- 파동 현상을 수학적으로 기술하는 편미분 방정식
- One-dimensional wave equation (space가 1-D)

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}$$



Spatial



Time



Sound pressure

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$



1D → 2D

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

- Wave equation describes the space-time coupling relation
- Second-order derivatives?
  - (Roughly) spatial curvature is related to temporal curvature



# Solutions of Wave Equation

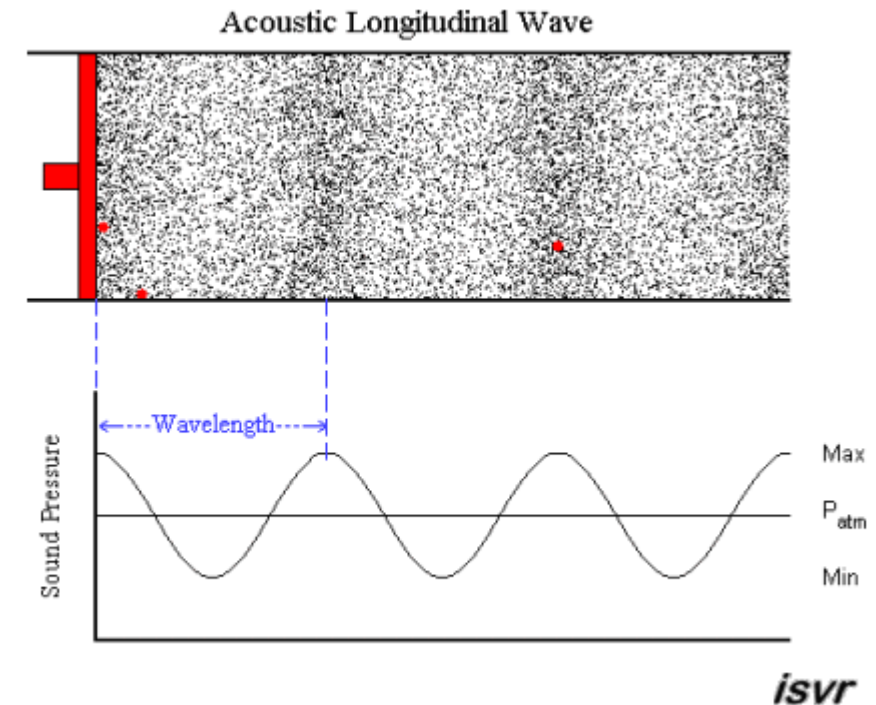
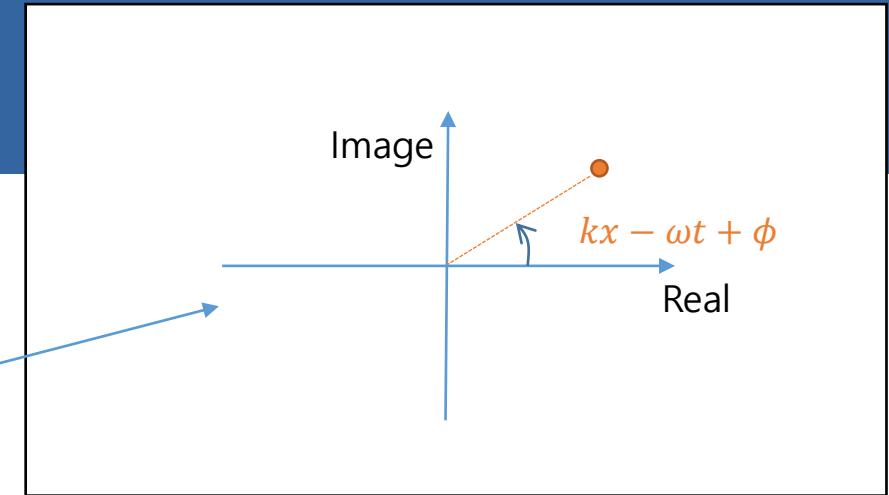
- Single frequency component of **a plane wave**

$$A = |A|e^{i\phi}$$

- Complex pressure:  $p(x, t) = Ae^{i(kx - \omega t)}$

- What we observe in reality is only the real part!

$$p(x, t) = \text{Re}\{Ae^{i(kx - \omega t)}\} = |A| \cos(kx - \omega t + \phi)$$



# Solutions of Wave Equation

- Single frequency component of **a plane wave**
  - Does Complex pressure satisfy Wave equation?

$$p(x, t) = Ae^{i(kx - \omega t)}$$

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

$$\begin{aligned}\frac{\partial^2 p}{\partial x^2} &= \frac{\partial^2}{\partial x^2} (Ae^{i(kx - \omega t)}) \\ &= (jk)(jk)Ae^{i(kx - \omega t)} \\ &= -k^2 Ae^{i(kx - \omega t)} \\ &= -k^2 p(x, t)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 p}{\partial t^2} &= \frac{\partial^2}{\partial t^2} (Ae^{i(kx - \omega t)}) \\ &= (-j\omega)(-j\omega)Ae^{i(kx - \omega t)} \\ &= -\omega^2 Ae^{i(kx - \omega t)} \\ &= -\omega^2 p(x, t)\end{aligned}$$

$$\begin{aligned}-k^2 &= -\frac{\omega^2}{c^2} \\ \Rightarrow k &= \frac{\omega}{c}\end{aligned}$$

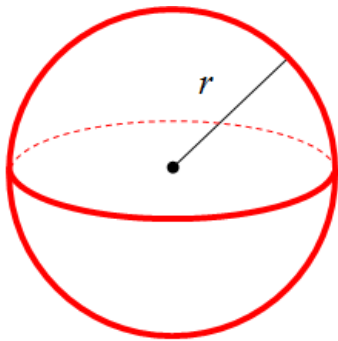
$$\begin{aligned}k &= \frac{2\pi}{\lambda} \\ \omega &= 2\pi f\end{aligned}$$

# Solutions of Wave Equation

- Single frequency component of a **spherical wave**

- Spherical coordinate,  $p(\vec{r}, t) = p(r, \theta, \phi, t)$

- Complex pressure:  $p(\vec{r}, t) = \frac{A}{r} e^{i(kr - \omega t)}$



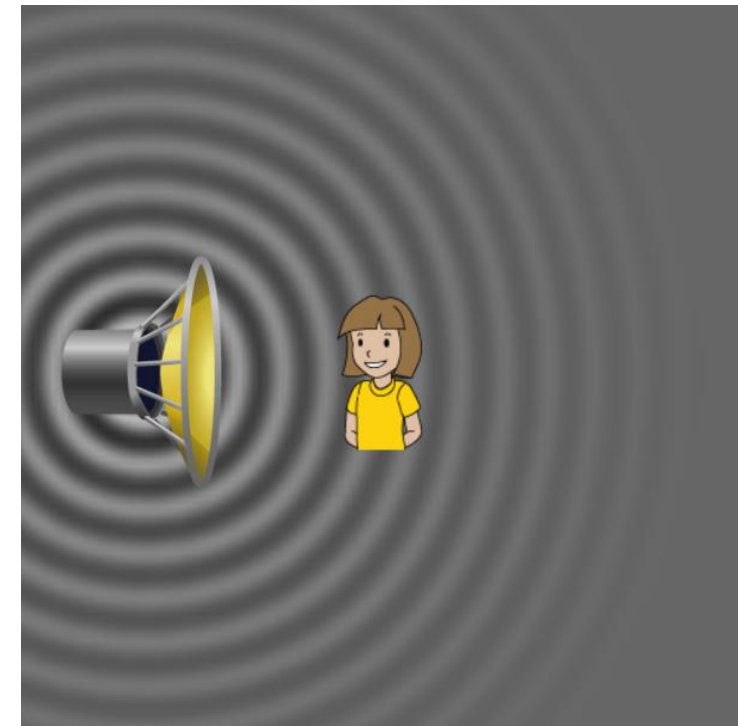
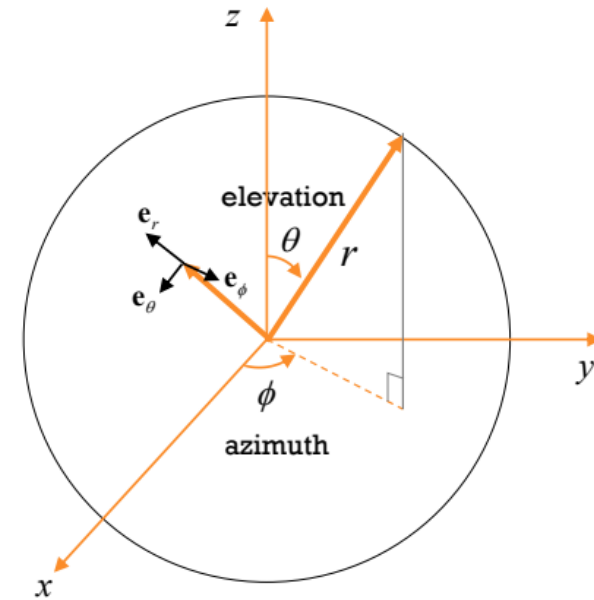
area of a sphere:  $4\pi r^2$

→ Spherical wave의 r이 증가하여도,  
→ 에너지보존 법칙에 의해 구 표면의  
에너지가 항상 같아야 한다.

→ Sound intensity:  $I = \frac{P}{A}$

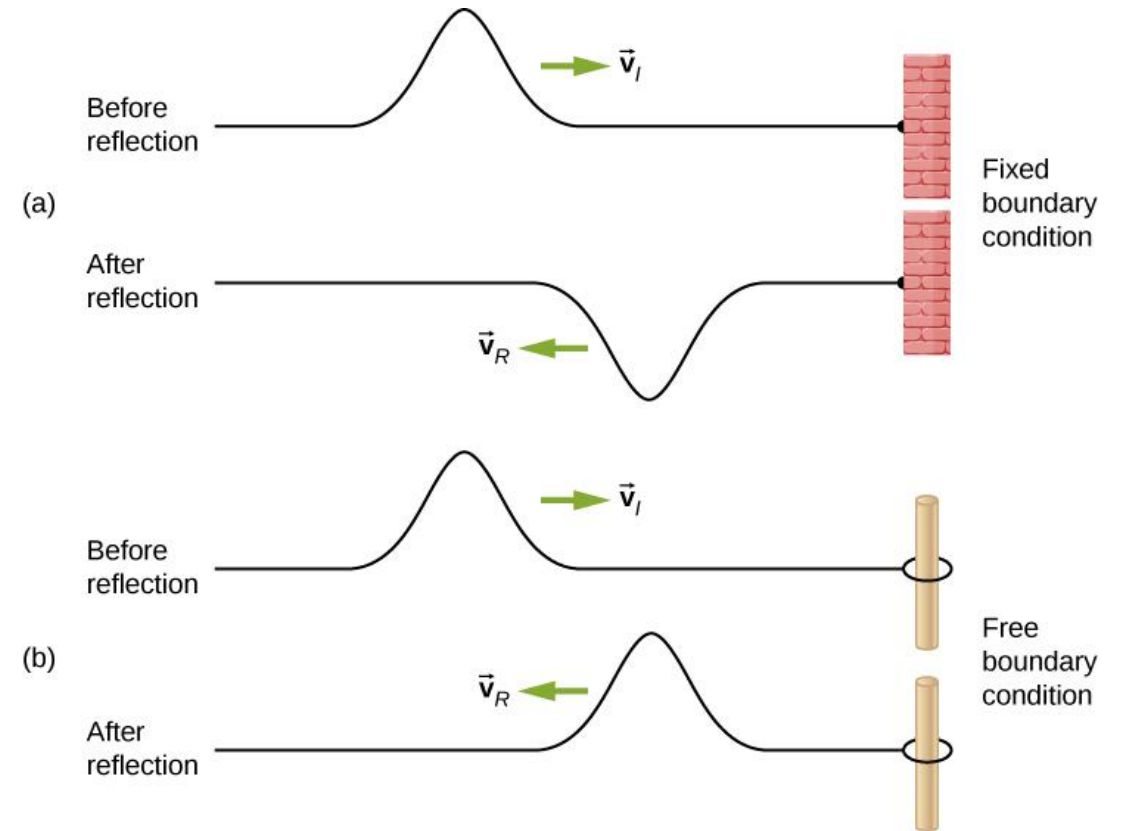
→ Sound power  $P$

$$\approx \frac{1}{T} \int_0^T [p(\vec{r}, t)]^2 dt$$



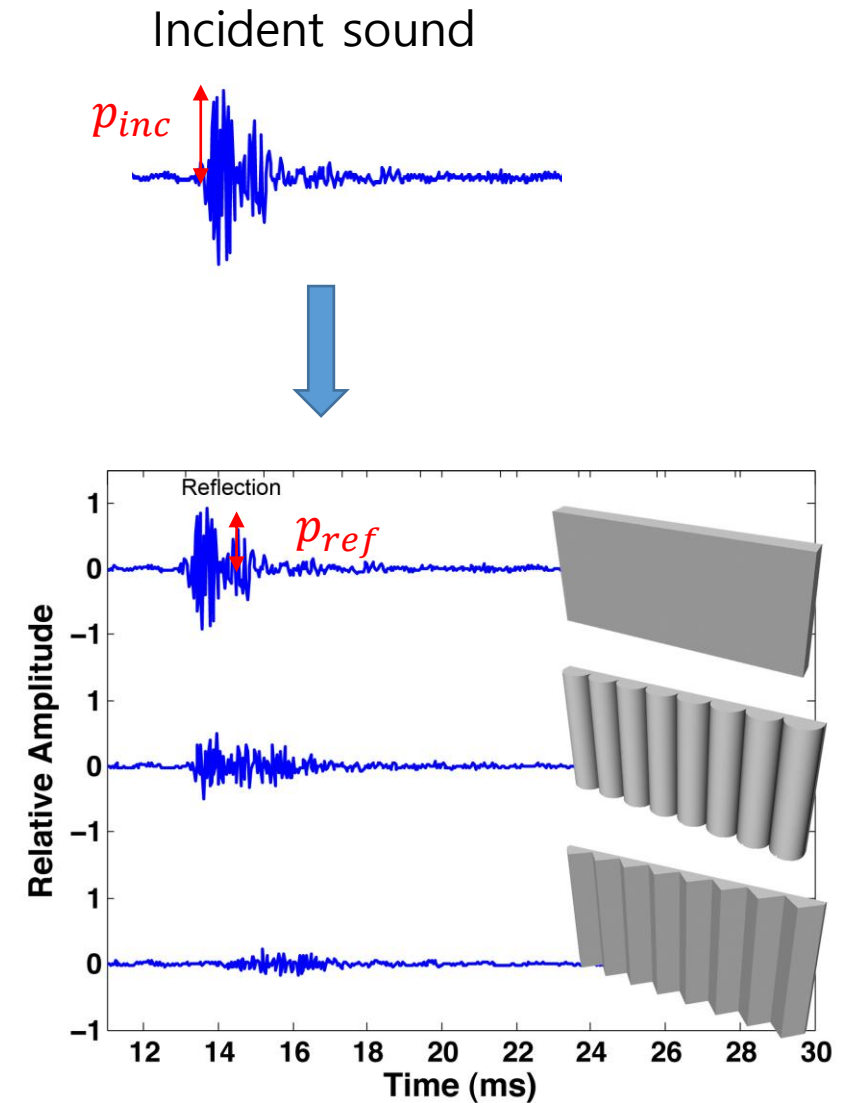
# Reflection

- If reflection occurs, what happens?
  - If the wall is a rigid body, It should be a fixed boundary condition



# Reflection

- If reflection occurs, what happens?
  - If the wall is a rigid body, It should be a fixed boundary condition
- How will the sound intensity change?
  - Reflection coefficient:  $R_p = \frac{p_{ref}}{p_{inc}}$
  - Reflection coefficient는 material마다 다르다!



# Reflection

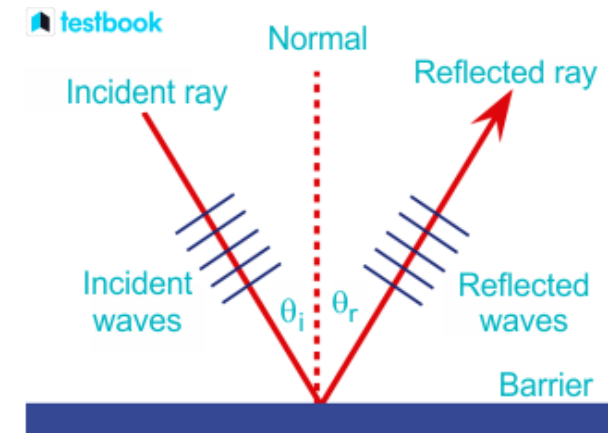
- If reflection occurs, what happens?
  - If the wall is a rigid body, It should be a fixed boundary condition

- How will the sound intensity change?

- Reflection coefficient:  $R_p = \frac{p_{ref}}{p_{inc}}$

입사각에 따라 Reflection coefficient가 달라진다.

- What happens if the angle of incidence is not 90 degrees? → **Specular reflection**



Reflection law

# Reflection

- If reflection occurs, what happens?
  - If the wall is a rigid body, It should be a fixed boundary condition

- How will the sound intensity change?

- Reflection coefficient:  $R_p = \frac{p_{ref}}{p_{inc}}$

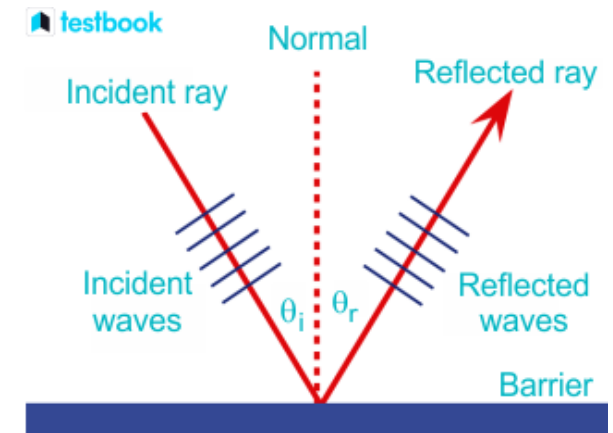
$$p_{inc}(x, t) = Ae^{i(kx - \omega t)}$$



$$p_{ref}(x, t) = R_p \cdot p_{inc}(-x, t) \cdot e^{i\pi}$$

입사각에 따라 Reflection coefficient가 달라진다.

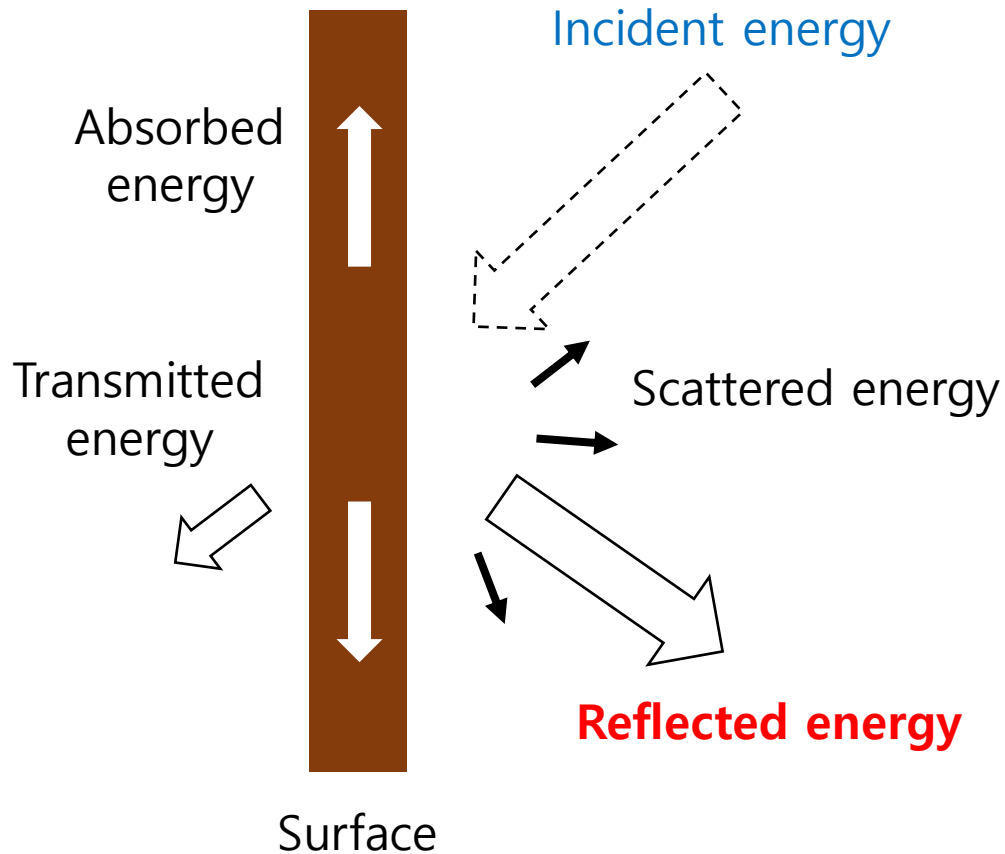
- What happens if the angle of incidence is not 90 degrees? → **Specular reflection**



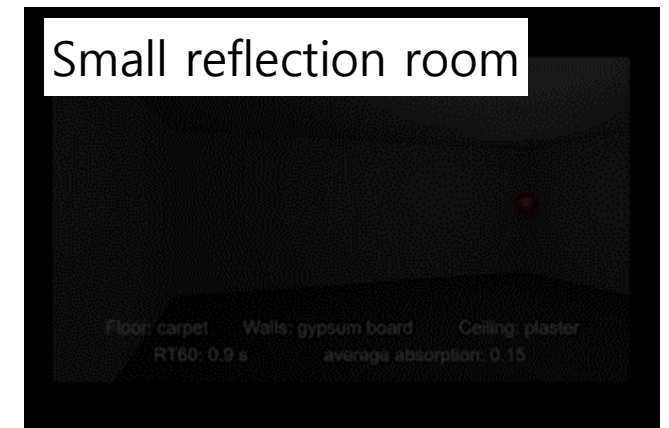
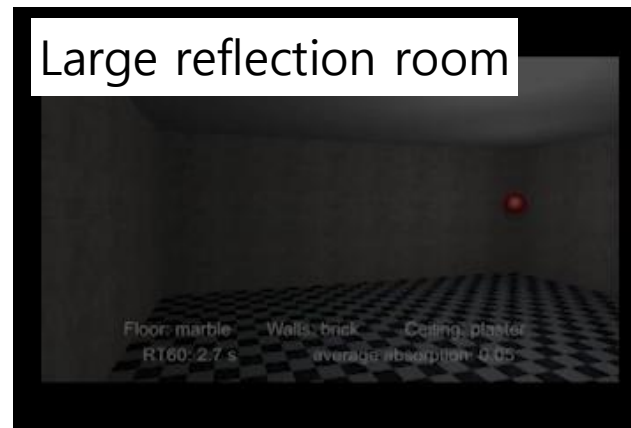
Reflection law

# Acoustic Materials

- The properties to determine how incident sound interacts with the surface.
  - The acoustic material depends on the frequency of the sound.



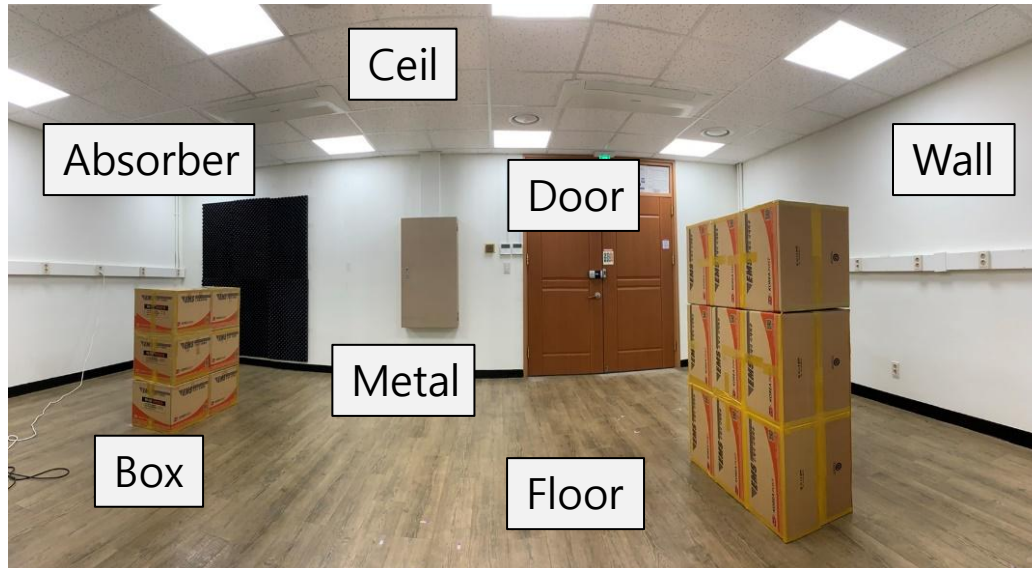
Sound comparison between different acoustic material (e.g. reflection)



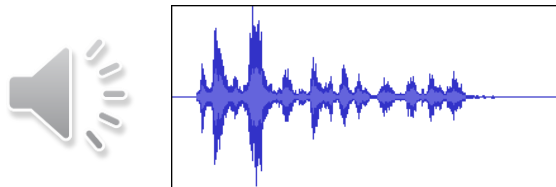
Videos from "Acoustic Classification and Optimization for Multi-Modal Rendering of Real-World Scenes, TVCG 18"



# Acoustic Materials

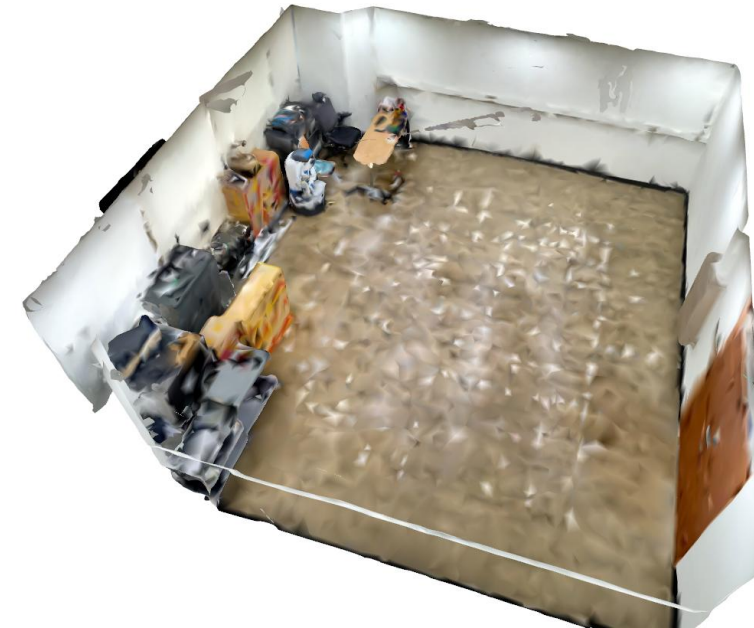


Real world indoor scene



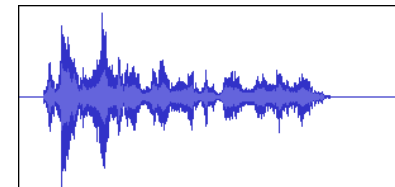
Recorded sound in real world

Estimating  
Acoustic  
Materials

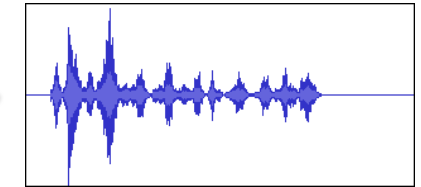


Virtual world indoor scene

Utilizing  
Sound  
Simulator  
(Habitat2.0)



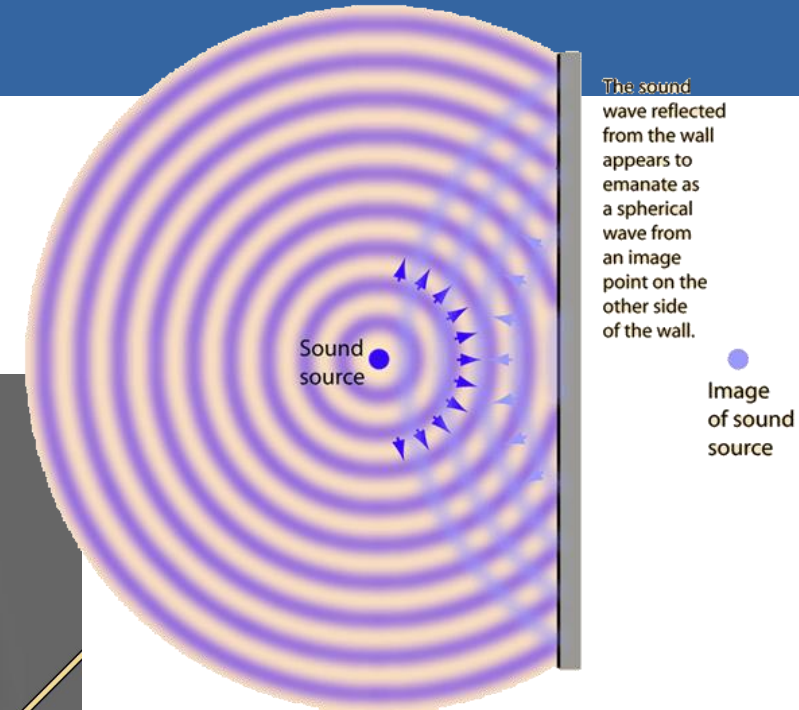
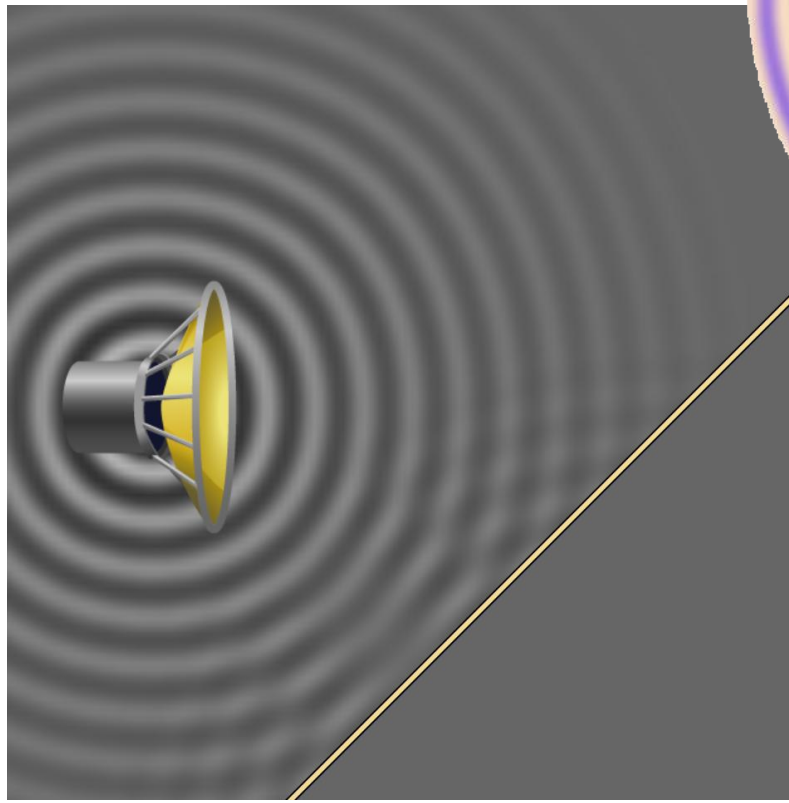
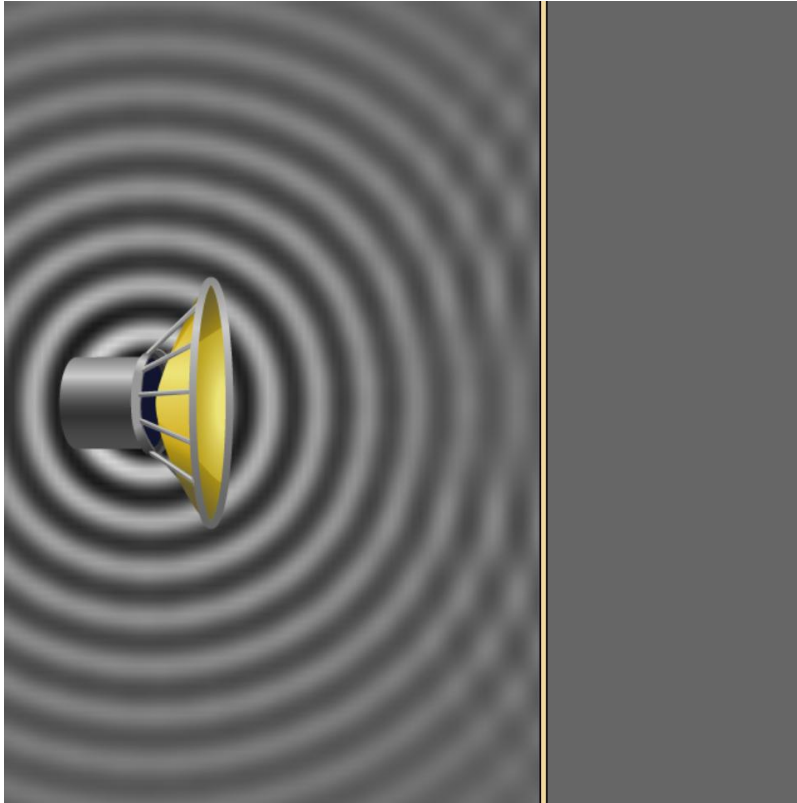
Generated sound without  
considering acoustic material



Generated sound using  
appropriate acoustic material

# Reflection

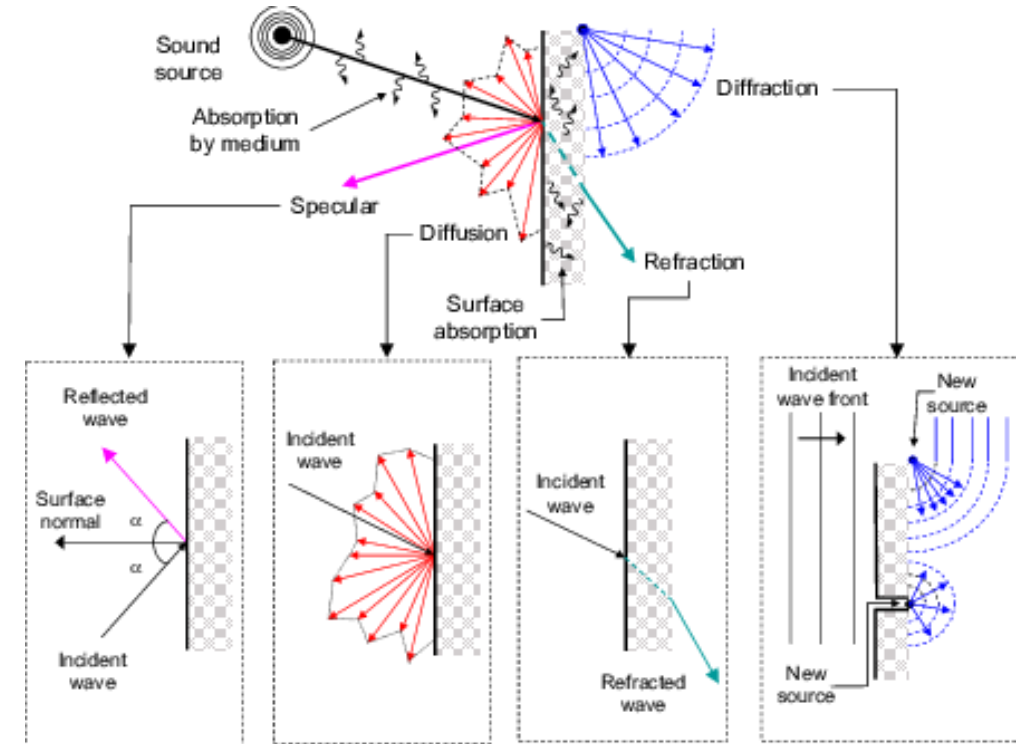
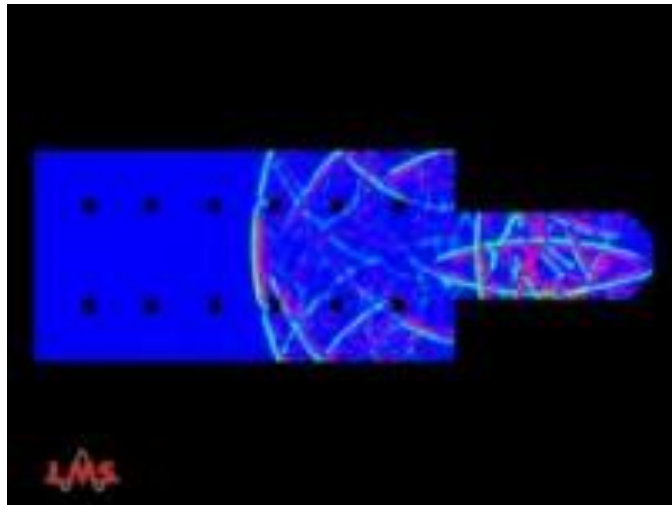
- If reflection occurs, what happens?
  - Spherical wave → **Imaginary source**  
**(Specular reflection only)**



# Reflection

- If reflection occurs, what happens?
  - If the wall is a rigid body, It should be a fixed boundary condition

소리가 물체에 충돌할 때, specular reflection (정반사) 뿐만 아니라, **Diffuse, refraction, diffraction** 등이 함께 발생한다.



이를 정확하게 Simulation하는 것은 매우 시간이 많이 걸린다.

# Geometrical Acoustics

- The easiest way to model a realistic sound is by recording.
  - Difficult to simulate the various sound effects.
- Recently, a method using “**Sound propagation**” has been used.
  - Sound simulator needs **3-D geometry information** and **acoustic material properties** for realistic sound generation.

