

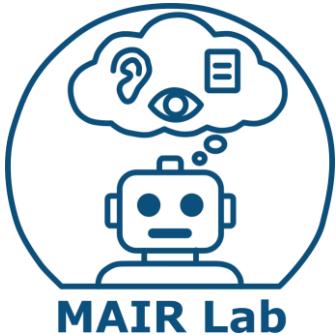
# Sound Propagation

안인규 (Inkyu An)

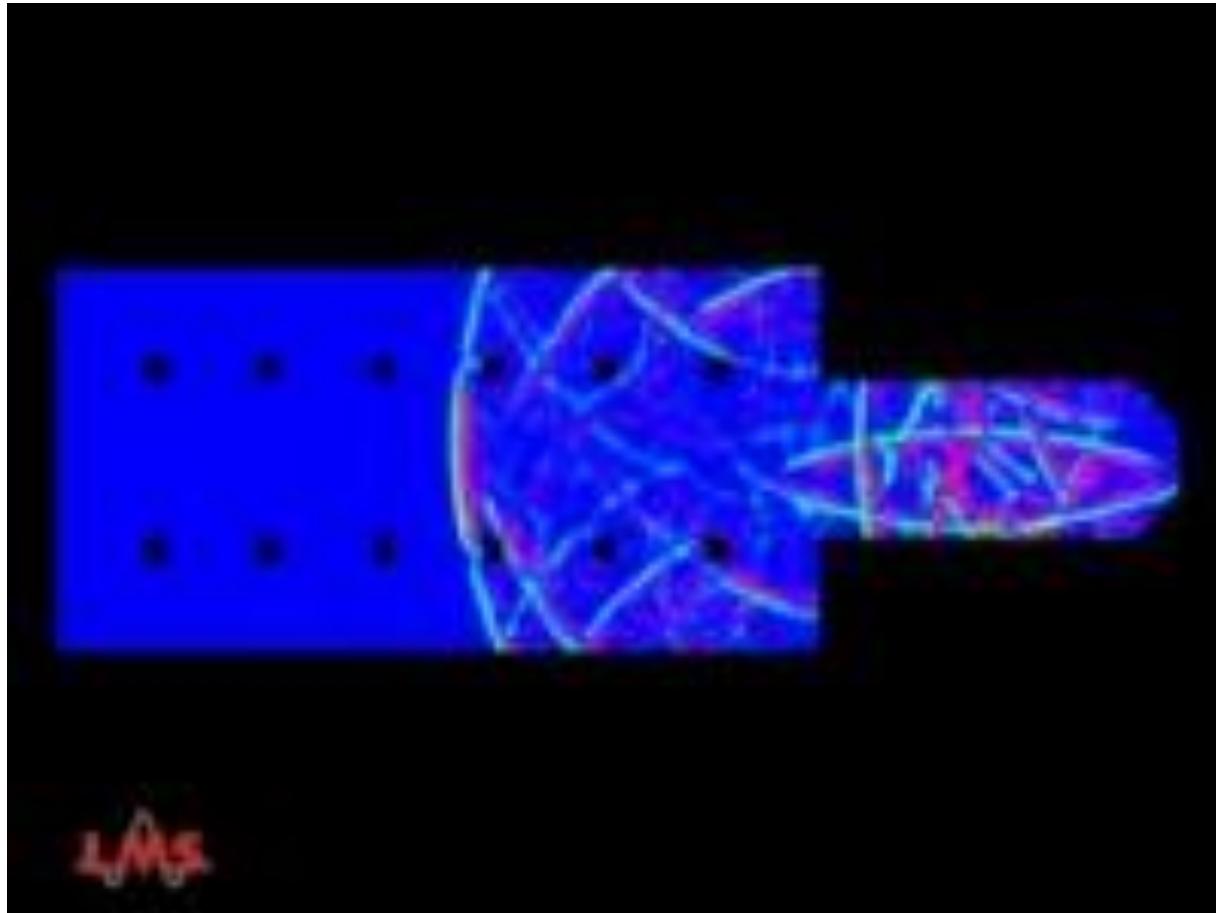
**Speech And Audio Recognition**  
**(오디오 음성인식)**

<https://mairlab-km.github.io/>

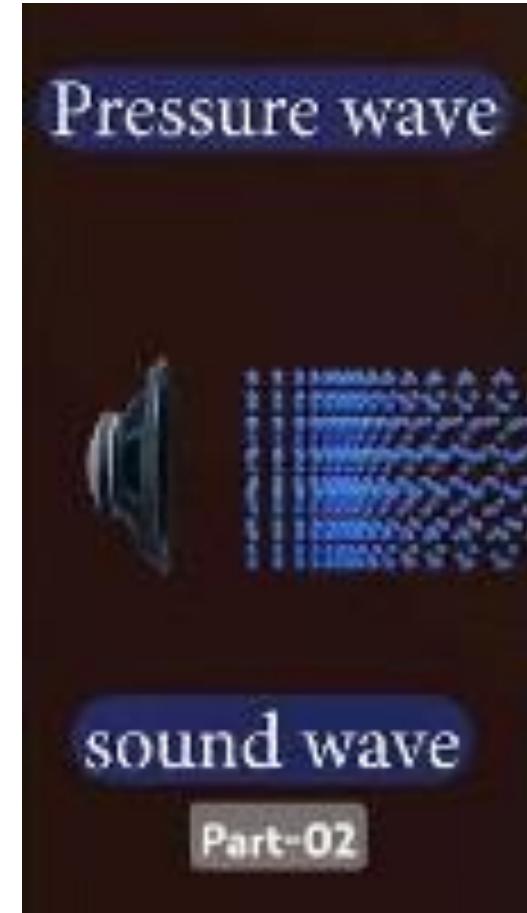
This lecture material is based on the Lecture Notes (EE837) of Prof. Jung-Woo Choi at KAIST  
and the Ph.D. dissertation of Dr. Taeyoung Kim at Samsung Electronics



# How does sound propagate?

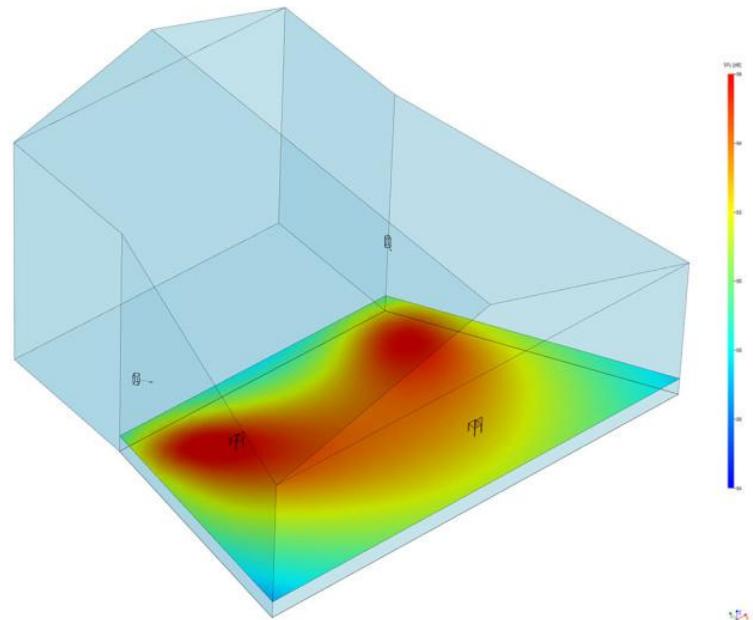


<https://www.youtube.com/watch?v=Xsx4VBEKciA>



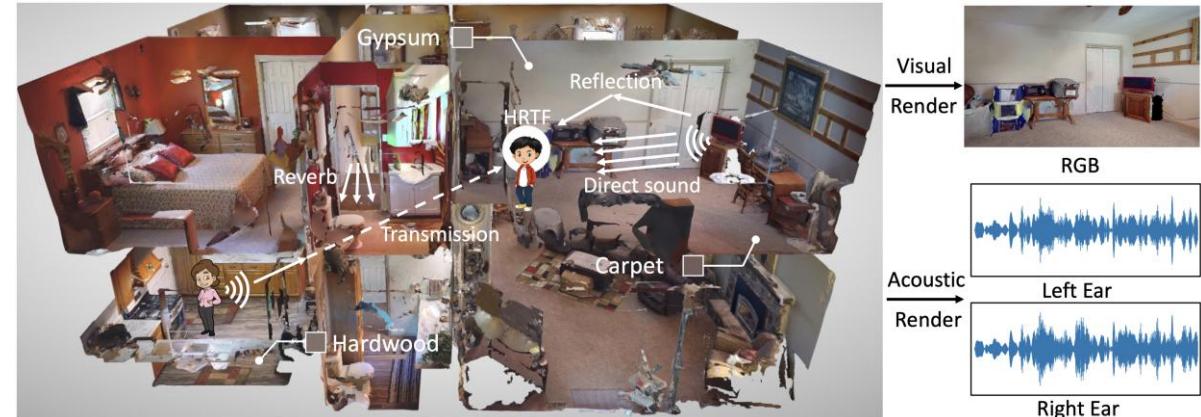
<https://www.youtube.com/shorts/GnAvquGBmnY>

# To create realistic sound!



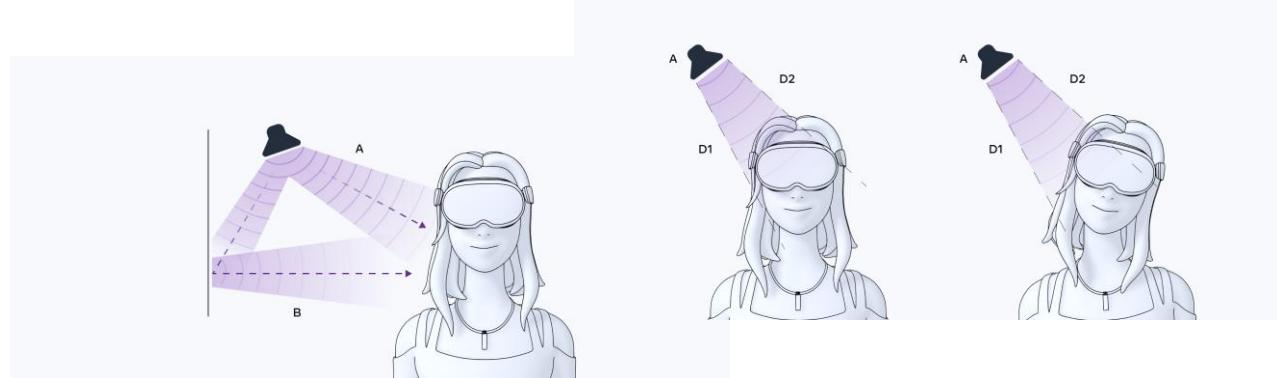
Acoustic simulation (designing Auditorium)

<https://audioplayer.com/article/acoustic-simulation-ease-5-s-acousteer-engine-redefines-design-workflows>



Sound simulation (generating synthetic data)

<https://vision.cs.utexas.edu/projects/soundspace2/>



Sound simulation (Spatial audio in VR)

[https://developers.meta.com/horizon/design/spatial\\_audio/](https://developers.meta.com/horizon/design/spatial_audio/)

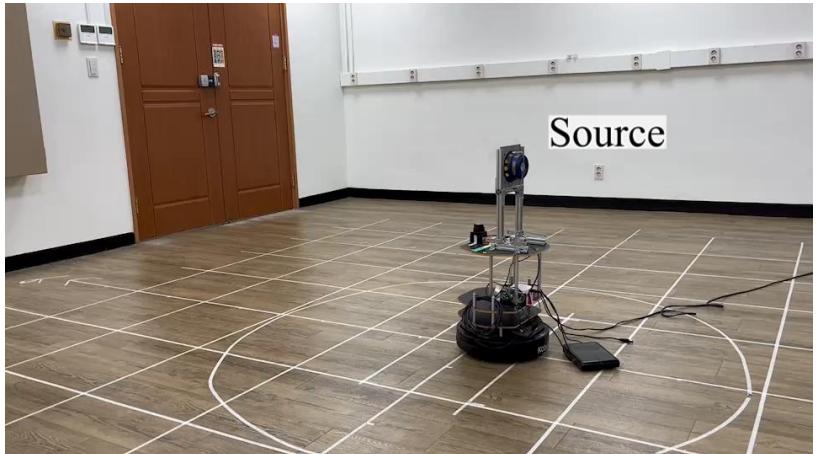
# To create realistic sound!



READY OR NOT (Game)

<https://www.youtube.com/watch?v=q267XIOBSr8&t=71s>

# To create realistic sound!



Recorded sound in the real environment

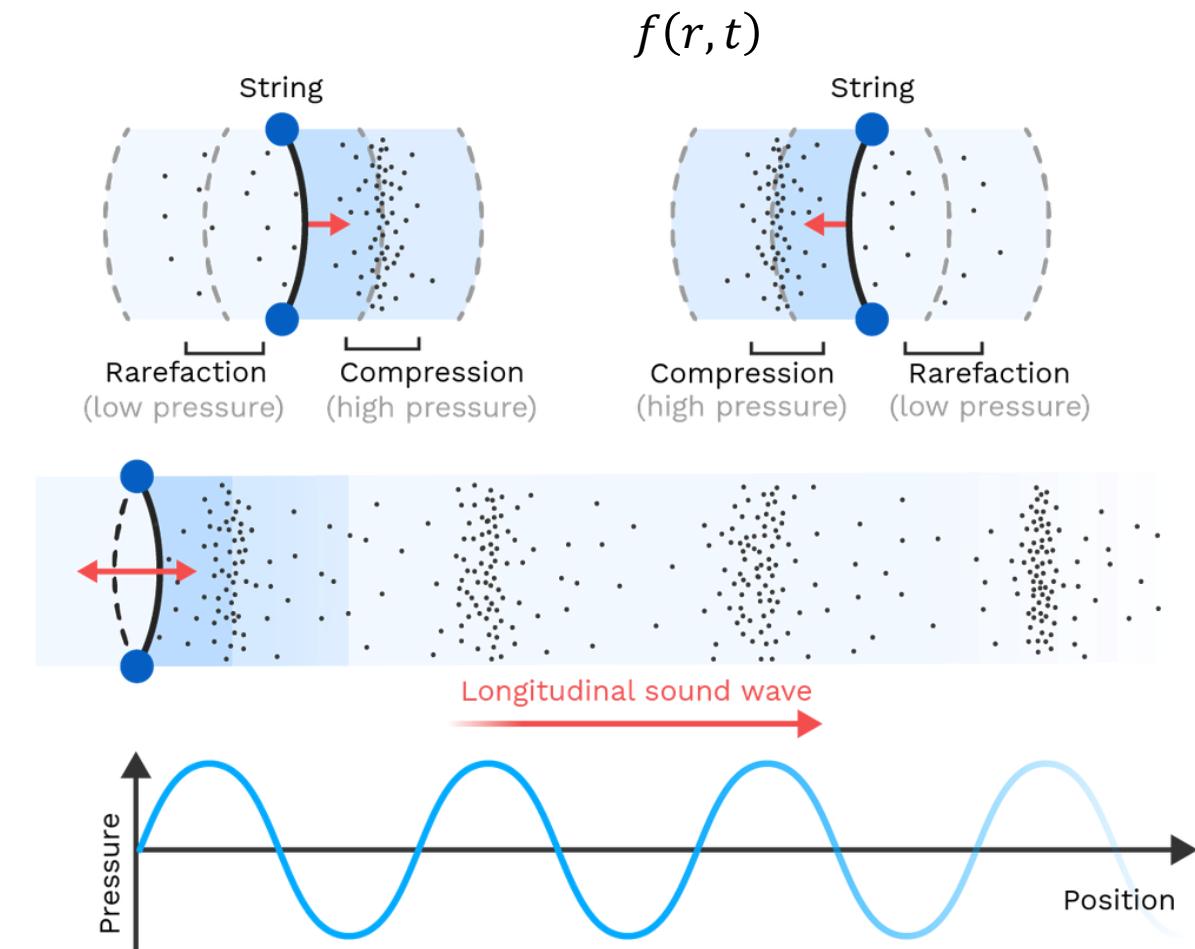
Without considering  
acoustic material

Using appropriate  
acoustic material

Generated sound in the virtual environment

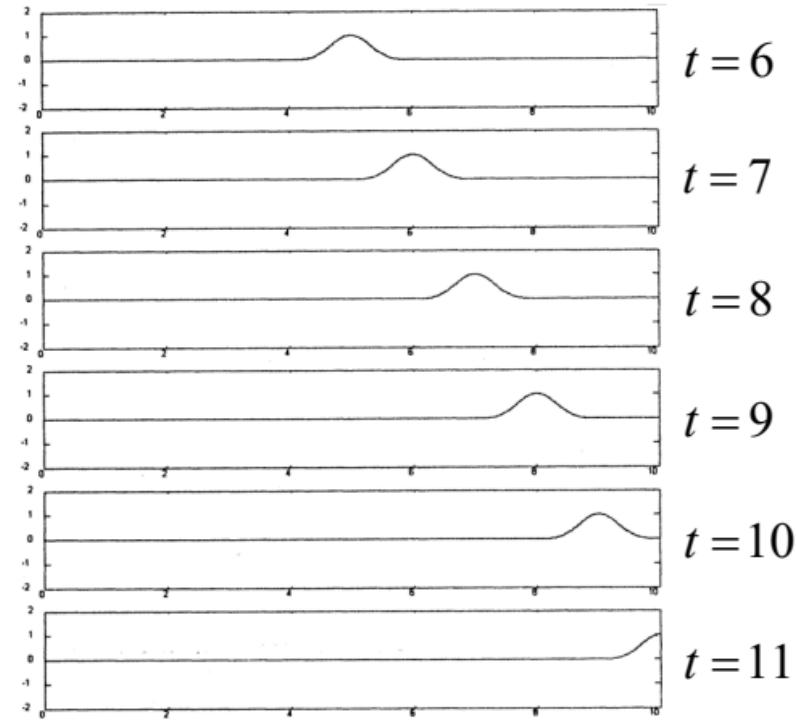
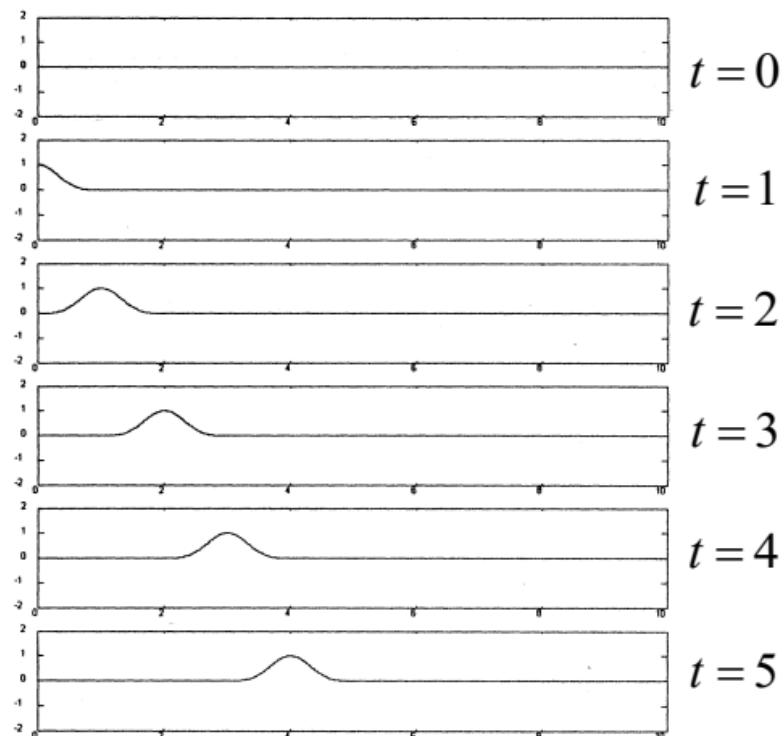
# Wave?

- What is wave?
  - Propagation of disturbances through medium in space and time
- Acoustic properties to consider
  - Acoustic pressure
  - Particle velocity
  - Acoustic density
  - Sound power and intensity
  - ...



# Simple Wave Propagation

- Wave propagation on a string
  - What propagating is an 'event', not a medium itself



# Simple Wave Propagation

- Wave propagation on string

- Non-dispersive wave: 파동의 속도가 주파수에 의존하지 않는 파동 (즉, 주파수 성분이 같은 속도로 이동)

- Right-going wave (waveform)

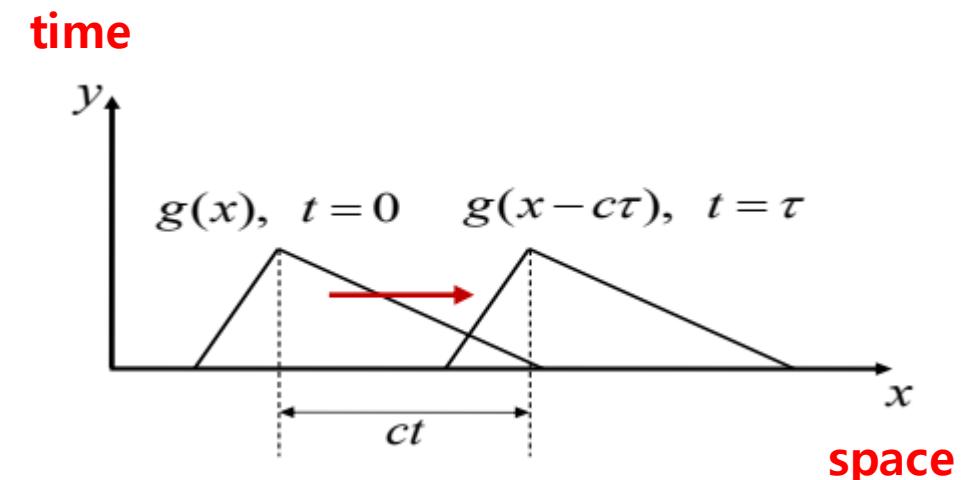
- $y(x, t) = g(x - ct) = g\left(-c\left(t - \frac{x}{c}\right)\right)$

- Left-going wave (waveform)

- $y(x, t) = h(x + ct)$

- General form

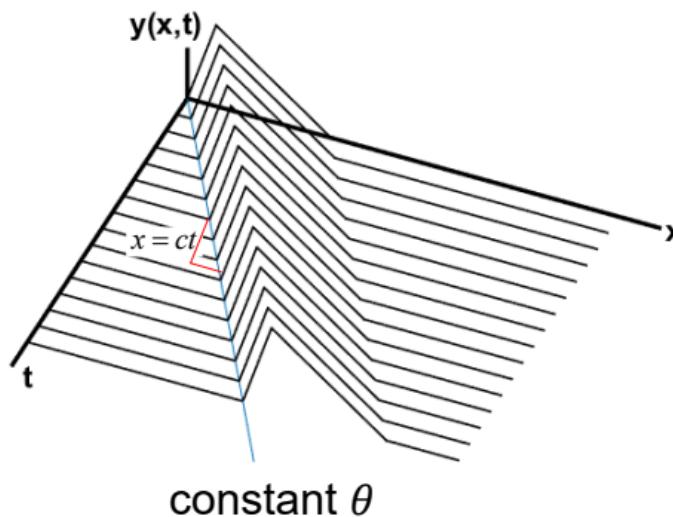
- $y(x, t) = g(x - ct) + h(x + ct)$



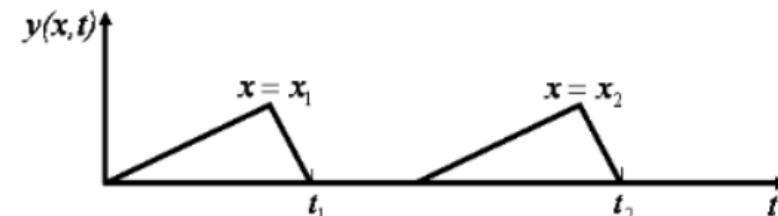
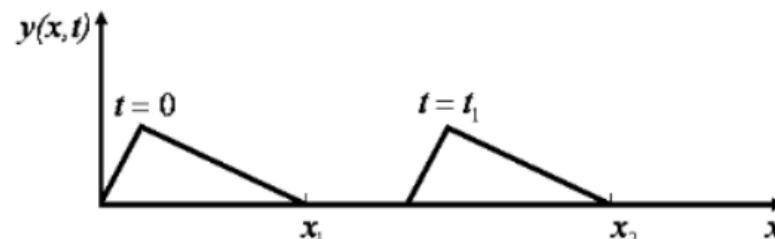
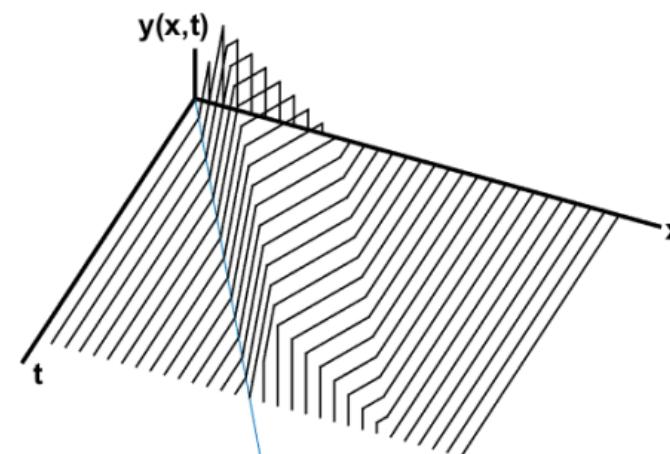
# Simple Wave Propagation

- Wave observed in space-time domain

Snapshot at a fixed time

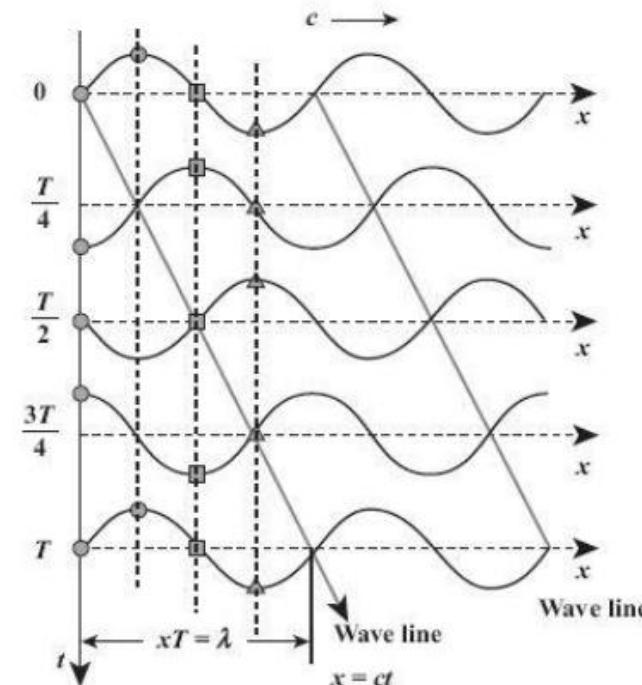
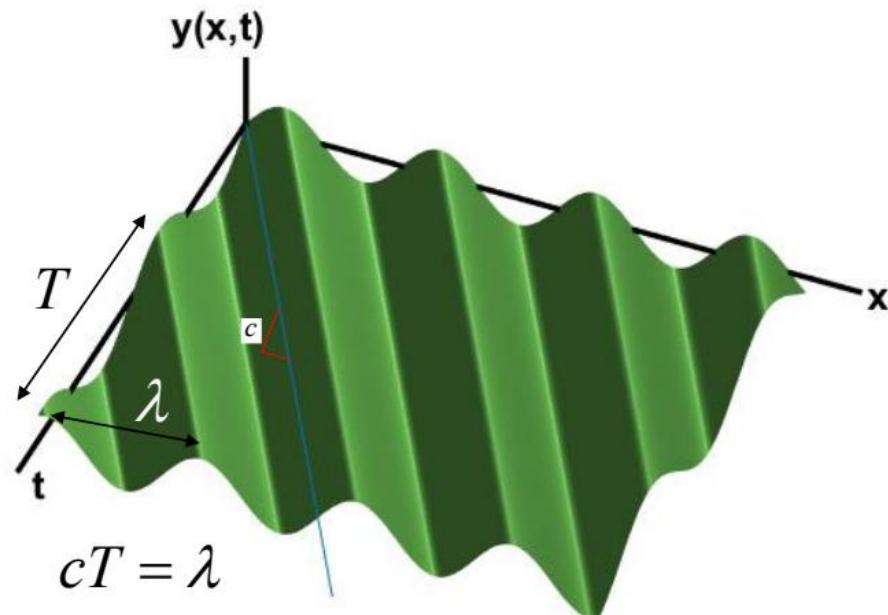


Recording at a fixed location



# Simple Wave Propagation

- Harmonically propagating wave
  - Right-going wave (waveform)
    - $y(x, t) = y(x - ct) = Y\cos(k(x - ct) + \phi_0)$



The distance across which a wave travels for a period  $T$  with a propagation speed  $c$  will be a wavelength  $\lambda$

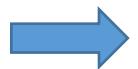
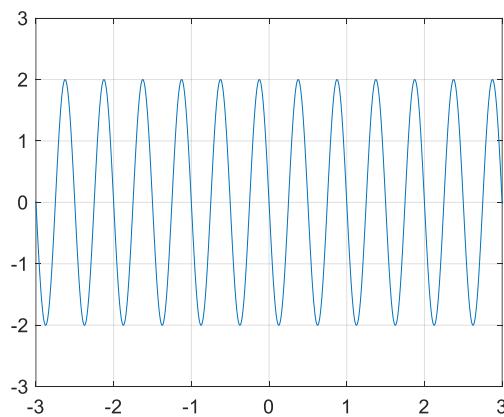
# Fourier Series

- Any absolutely integrable periodic function  $x(t)$  with period  $P$  can be represented as

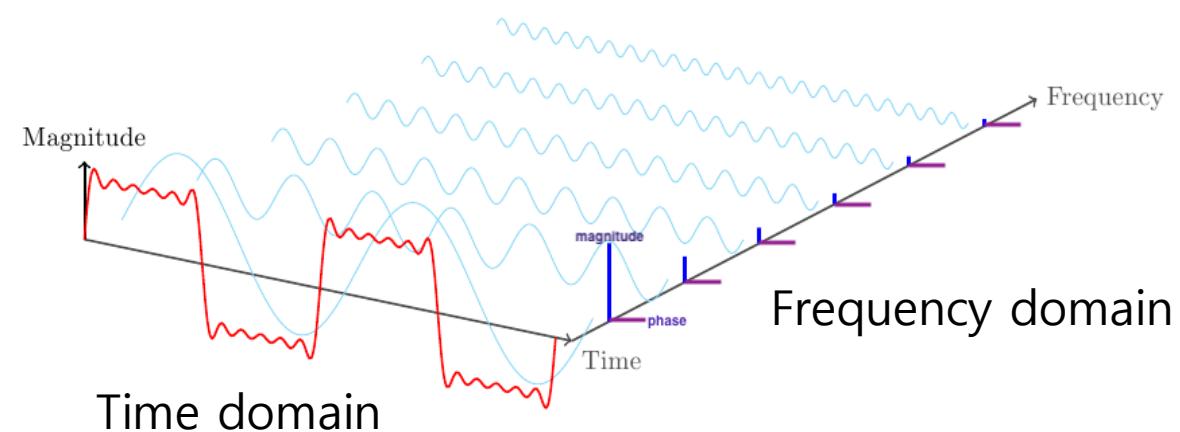
$$x(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left(2\pi \frac{n}{P} t - \phi_n\right)$$

Amplitude      Phase  
Frequency

- What is the amplitude and the phase?



- Amplitude?
- Phase?



# Simple Wave Propagation

- Harmonically propagating wave
  - Right-going wave (waveform)
    - $y(x, t) = y(x - ct) = Y\cos(k(x - ct) + \phi_0)$
  - Frequency: number of temporal oscillation per second
    - Cyclic frequency (Hz):  $f$
    - Radian frequency (radian/s):  $\omega = 2\pi f$
    - $y(x, t) = Y\cos(kx - \omega t)$
  - Wavenumber: spatial oscillation per meter
    - Spatial frequency: radian/m
  - Period: time taken for single oscillation ( $T$ )
  - Wavelength: space taken for single oscillation ( $\lambda$ )

# Simple Wave Propagation

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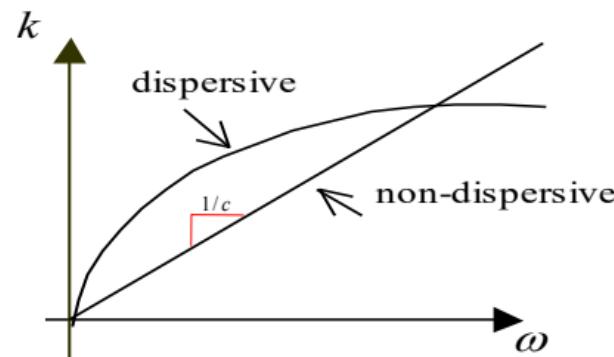
## <Space-time relations>

- Frequency-time relation:
  - $f = \frac{1}{T}$
  - $\omega = 2\pi f = \frac{2\pi}{T}$
- Spatial frequency-space:
  - $k = \frac{2\pi}{\lambda}$
- Space-time:
  - $k = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{cT}$

Space-time variation is not independent in waves!

# Simple Wave Propagation

- Space-time coupling
  - Dispersion relation
    - **Non-dispersive wave: sound speed is independent of frequency**  $\rightarrow \omega = ck$
    - Dispersive wave: sound speed varies with frequency



- Why the space-time coupling occurs?
  - Medium's behavior (dynamics): 파동이 지나가는 매질 (medium)의 물리적 특성이  $\omega - k$ 의 관계를 결정
  - 공기는 선형관계 (즉, non-dispersive)

$\rightarrow$  **Wave equation!**

# Wave Equation

- Wave equation:

- 파동 현상을 수학적으로 기술하는 편미분 방정식
- One-dimensional wave equation (space가 1-D)

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}$$

↑                      ↑  
Spatial              Time

- Wave equation describes the space-time coupling relation
- Second-order derivatives?
  - (Roughly) spatial curvature is related to temporal curvature

# Wave Equation

- Wave equation:

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↑                    ↑  
Spatial              Time



- Wave equation describes the space-time coupling relation
- Second-order derivatives?
  - (Roughly) spatial curvature is related to temporal curvature

Sound pressure

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

 1D → 2D

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

# Solutions of Wave Equation

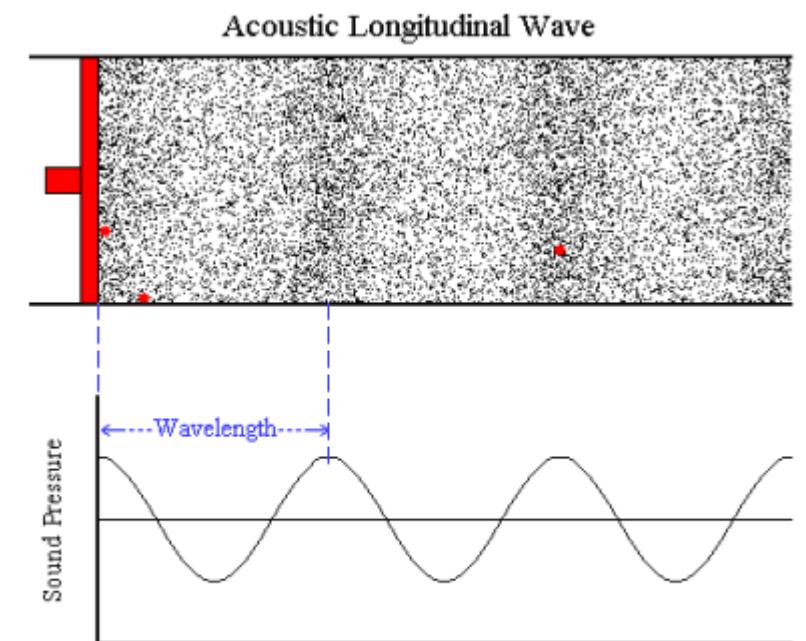
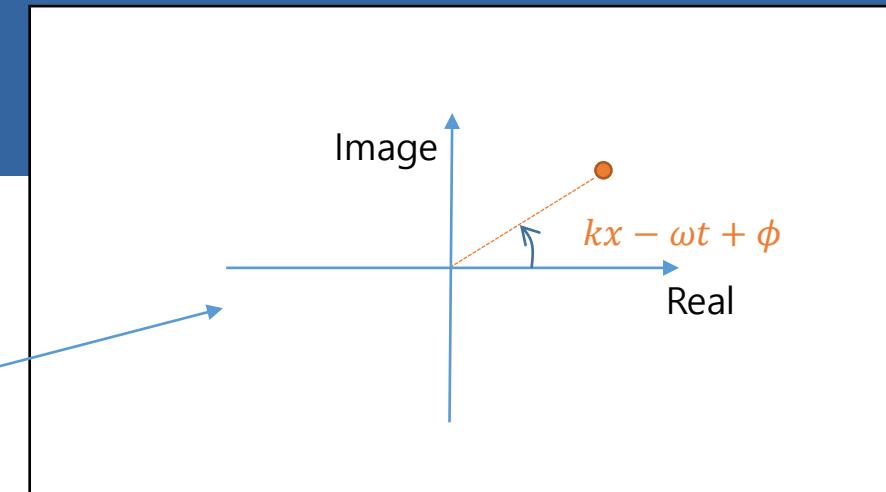
- Single frequency component of a **plane wave**

$$\text{Complex pressure: } p(x, t) = A e^{i(kx - \omega t)}$$

$$A = |A| e^{i\phi}$$

- What we observe in reality is only the real part!

$$p(x, t) = \operatorname{Re}\{A e^{i(kx - \omega t)}\} = |A| \cos(kx - \omega t + \phi)$$



# Solutions of Wave Equation

- Single frequency component of a **plane wave**
  - Does Complex pressure satisfy Wave equation?

$$p(x, t) = A e^{i(kx - \omega t)}$$

$$\frac{\partial^2 p}{\partial x^2} \quad \mp \quad \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

$$\begin{aligned}\frac{\partial^2 p}{\partial x^2} &= \frac{\partial^2}{\partial x^2} (A e^{i(kx - \omega t)}) \\&= (jk)(jk) A e^{i(kx - \omega t)} \\&= -k^2 A e^{i(kx - \omega t)} \\&= -k^2 p(x, t)\end{aligned}$$

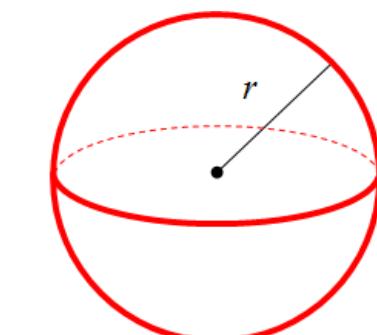
$$\begin{aligned}\frac{\partial^2 p}{\partial t^2} &= \frac{\partial^2}{\partial t^2} (A e^{i(kx - \omega t)}) \\&= (-j\omega)(-j\omega) A e^{i(kx - \omega t)} \\&= -\omega^2 A e^{i(kx - \omega t)} \\&= -\omega^2 p(x, t)\end{aligned}$$

$$\begin{aligned}-k^2 &= -\frac{\omega^2}{c^2} \\ \Rightarrow k &= \frac{\omega}{c}\end{aligned}$$

$$\begin{aligned}k &= \frac{2\pi}{\lambda} \\ \omega &= 2\pi f\end{aligned}$$

# Solutions of Wave Equation

- Single frequency component of **a spherical wave**
  - Spherical coordinate,  $p(\vec{r}, t) = p(r, \theta, \phi, t)$

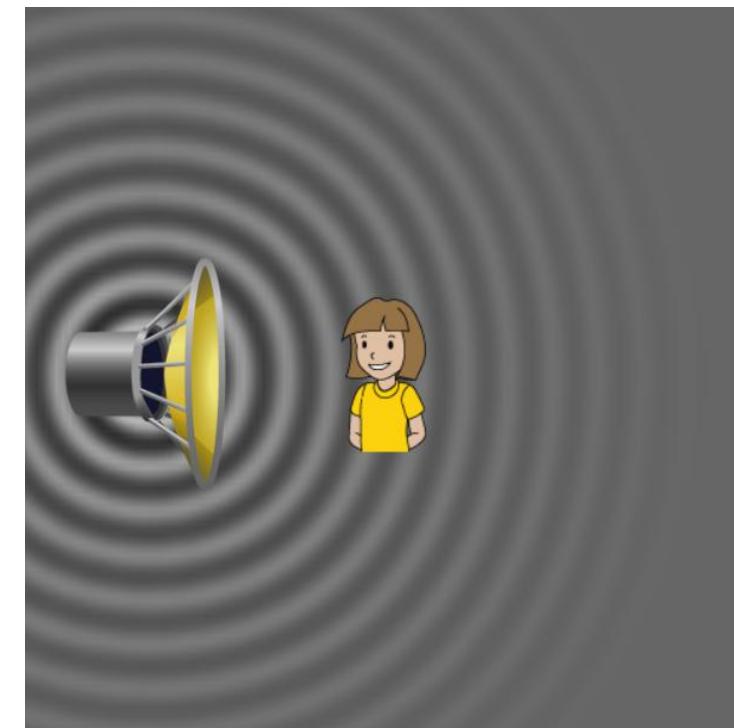
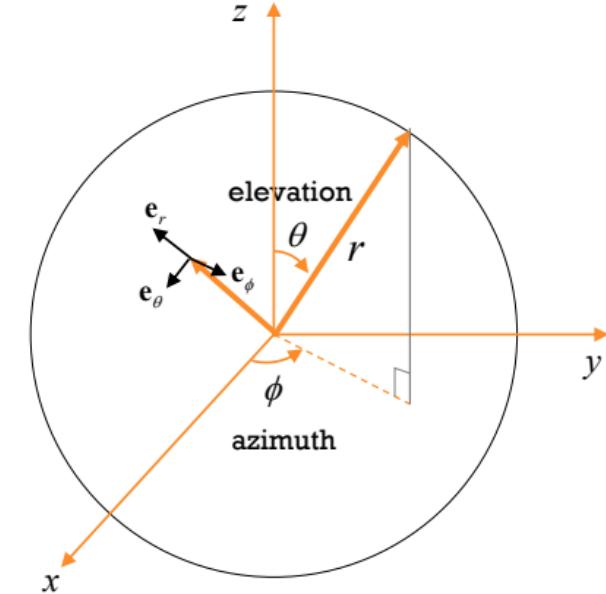


area of a sphere:  $4\pi r^2$   
→ Spherical wave의  $r$ 이 증가하여도,  
→ 에너지보존 법칙에 의해 구 표면의  
에너지가 항상 같아야 한다.

→ Sound intensity:  $I = \frac{P}{A}$

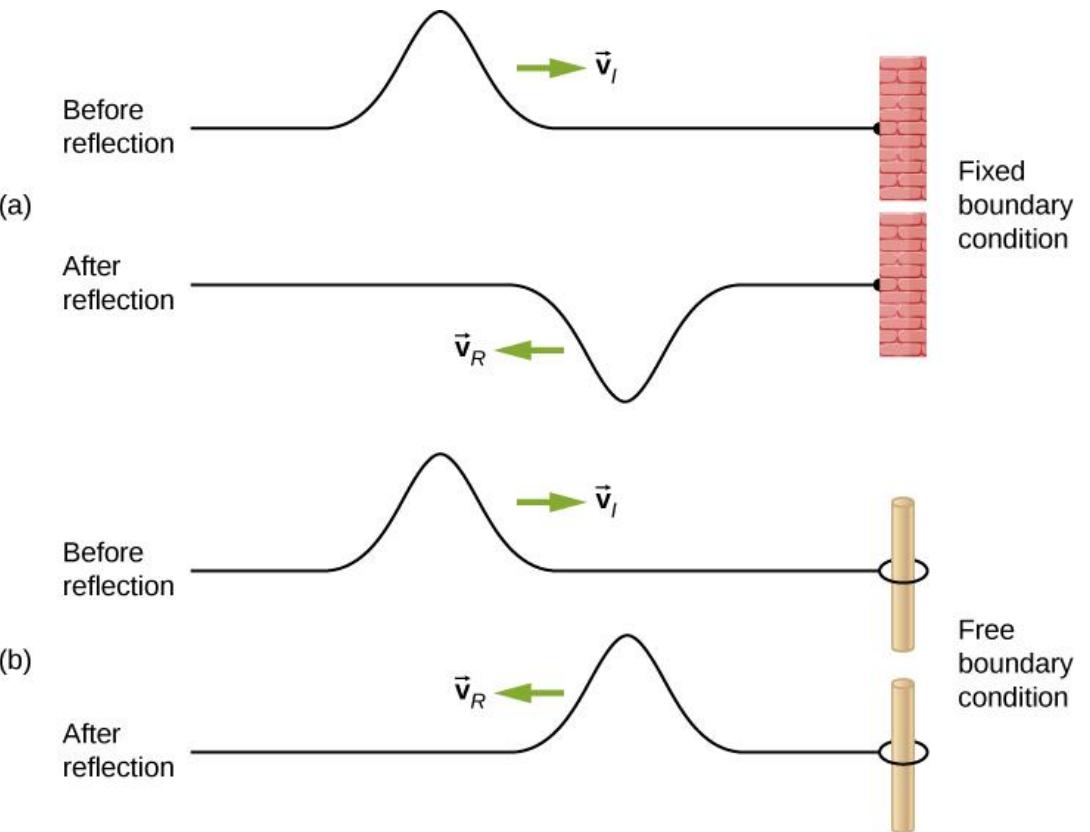
→ Sound power  $P$

$$\approx \frac{1}{T} \int_0^T [p(\vec{r}, t)]^2 dt$$



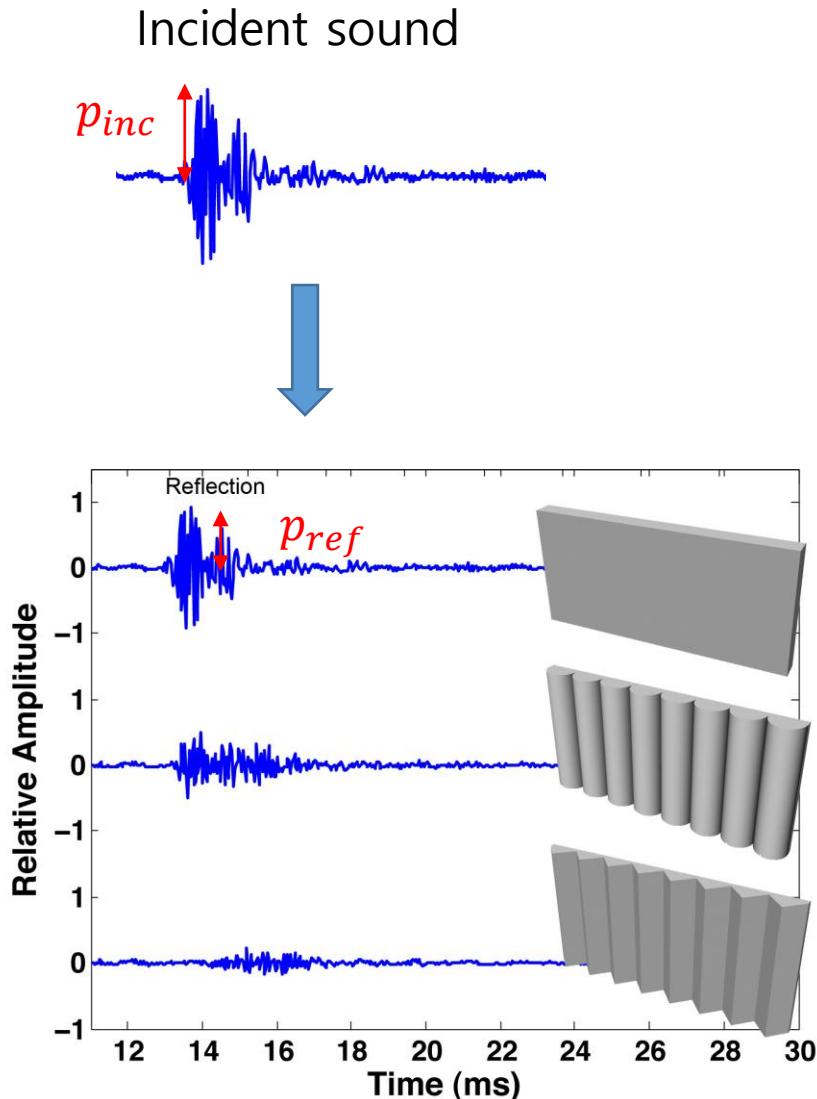
# Reflection

- If reflection occurs, what happens?
  - If the wall is a rigid body, It should be a fixed boundary condition



# Reflection

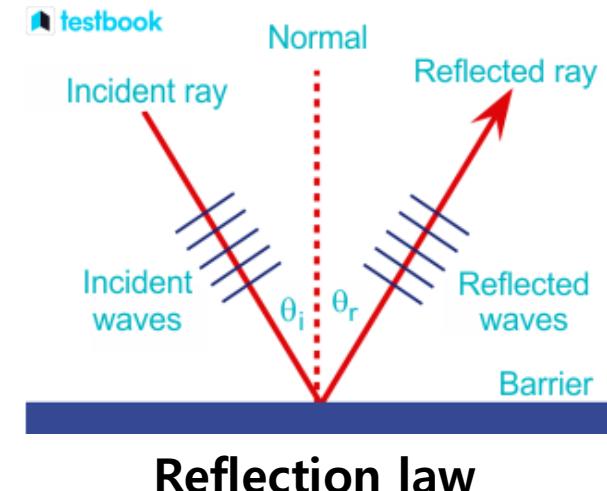
- If reflection occurs, what happens?
  - If the wall is a rigid body, It should be a fixed boundary condition
  - How will the sound intensity change?
    - Reflection coefficient:  $R_p = \frac{p_{ref}}{p_{inc}}$
    - Reflection coefficient는 material마다 다르다!



# Reflection

- If reflection occurs, what happens?
  - If the wall is a rigid body, It should be a fixed boundary condition
  - How will the sound intensity change?
    - Reflection coefficient:  $R_p = \frac{p_{ref}}{p_{inc}}$
  - What happens if the angle of incidence is not 90 degrees? → **Specular reflection**

입사각에 따라 Reflection coefficient가 달라진다.



# Reflection

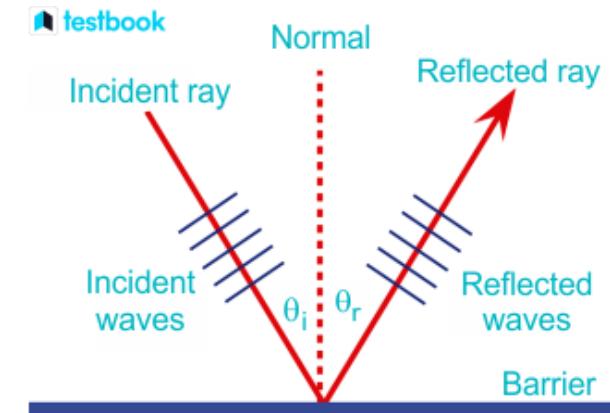
- If reflection occurs, what happens?
  - If the wall is a rigid body, It should be a fixed boundary condition
  - How will the sound intensity change?
    - Reflection coefficient:  $R_p = \frac{p_{ref}}{p_{inc}}$
  - What happens if the angle of incidence is not 90 degrees? → **Specular reflection**

$$p_{inc}(x, t) = Ae^{i(kx - \omega t)}$$

↓

$$p_{ref}(x, t) = R_p \cdot p_{inc}(-x, t) \cdot e^{i\pi}$$

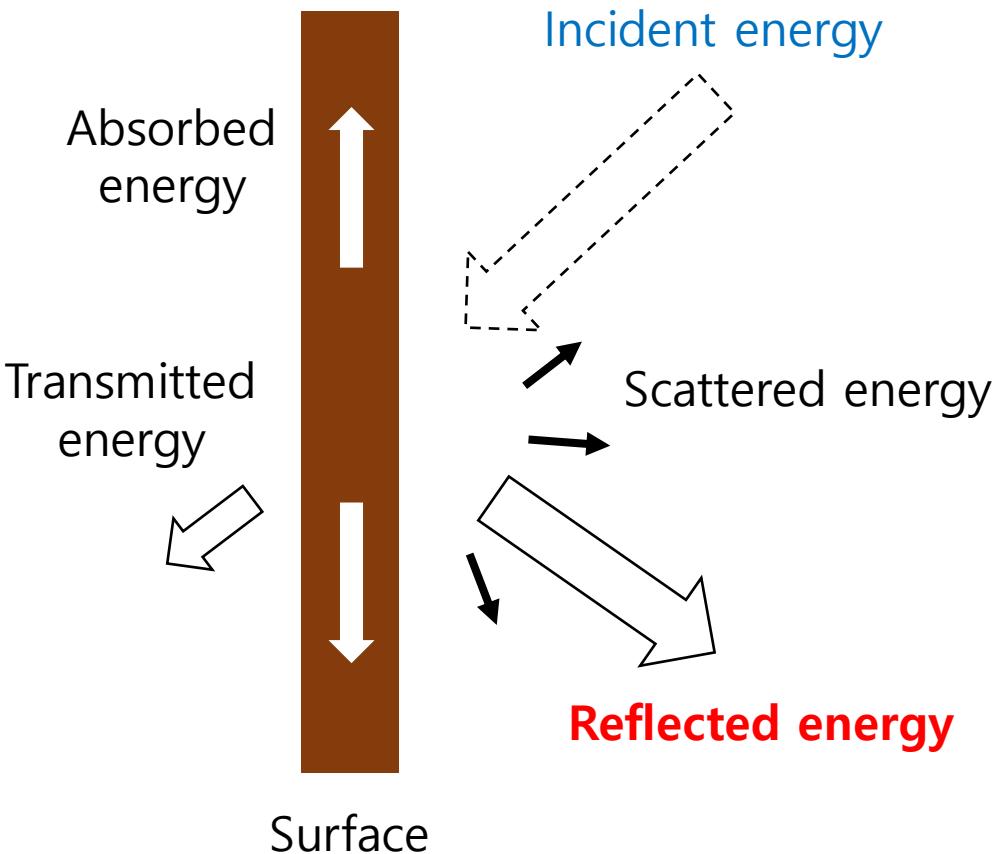
입사각에 따라 Reflection coefficient가 달라진다.



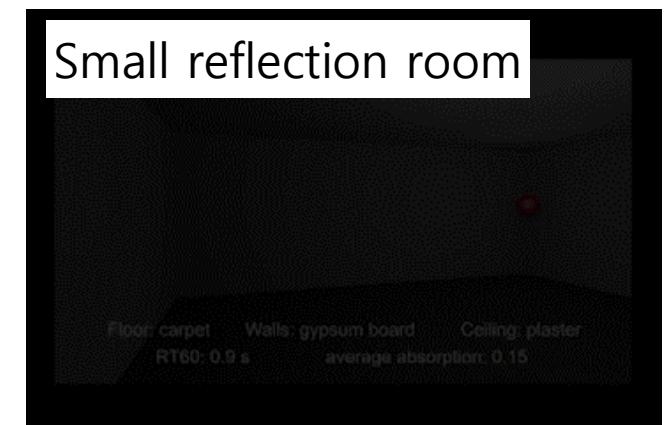
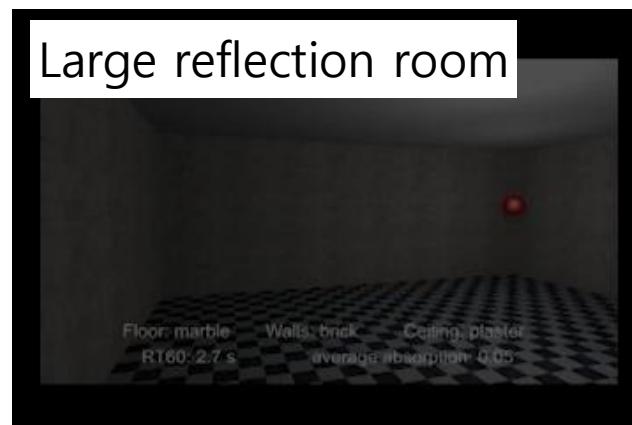
Reflection law

# Acoustic Materials

- The properties to determine how incident sound interacts with the surface.
  - The acoustic material depends on the frequency of the sound.

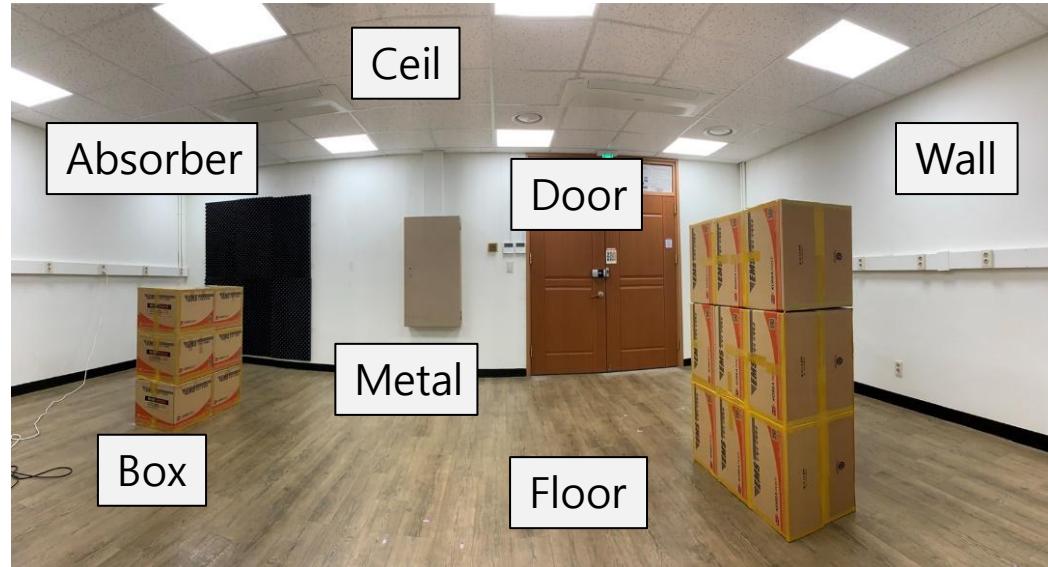


Sound comparison between different acoustic material (e.g. reflection)

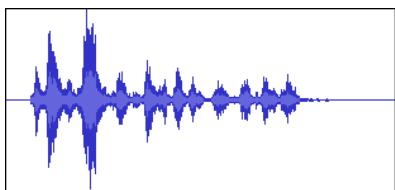


Videos from "Acoustic Classification and Optimization for Multi-Modal Rendering of Real-World Scenes, TVCG 18"

# Acoustic Materials



Real world indoor scene



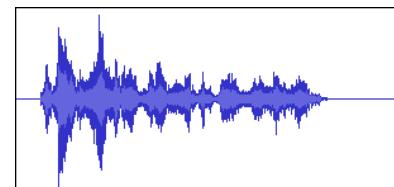
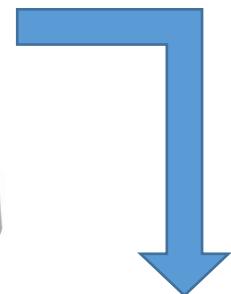
Recorded sound in real world

Estimating  
Acoustic  
Materials

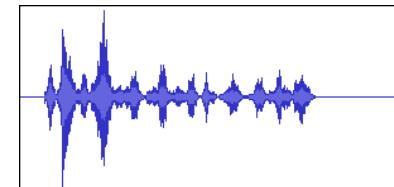


Virtual world indoor scene

Utilizing  
Sound  
Simulator  
**(Habitat2.0)**



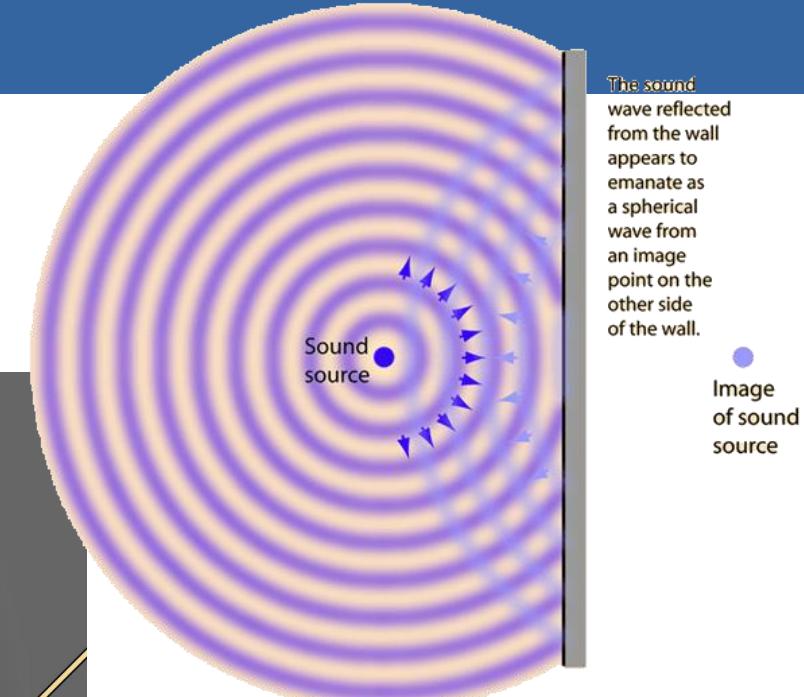
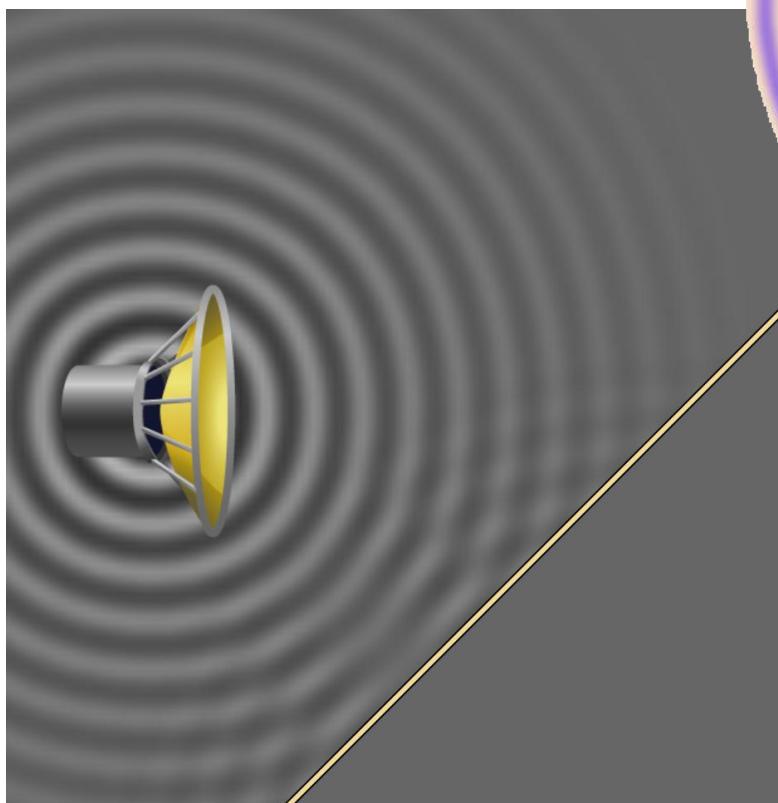
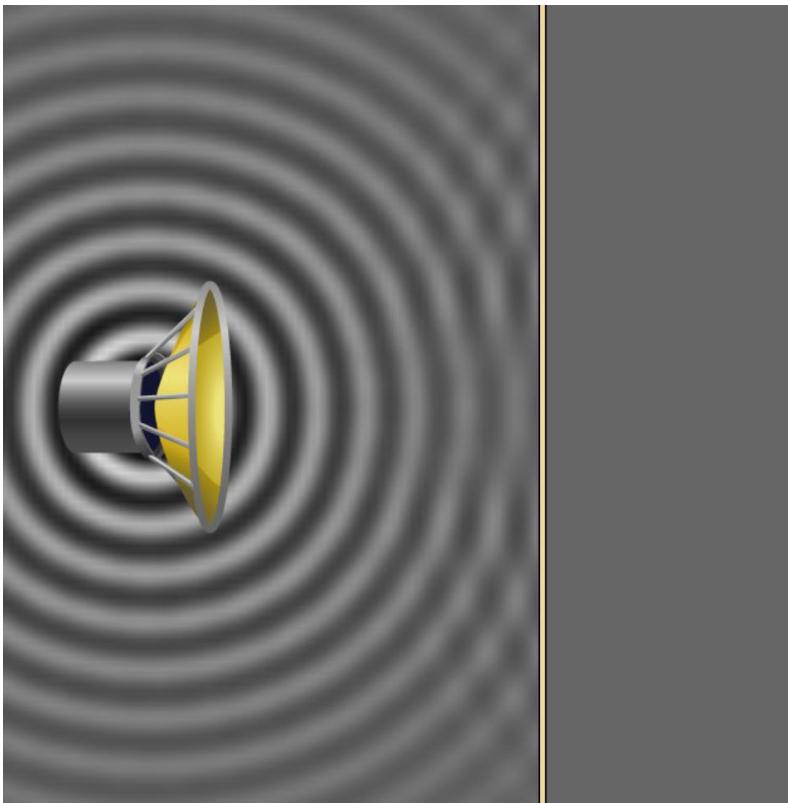
Generated sound without  
considering acoustic material



Generated sound using  
appropriate acoustic material

# Reflection

- If reflection occurs, what happens?
  - Spherical wave → **Imaginary source**  
**(Specular reflection only)**

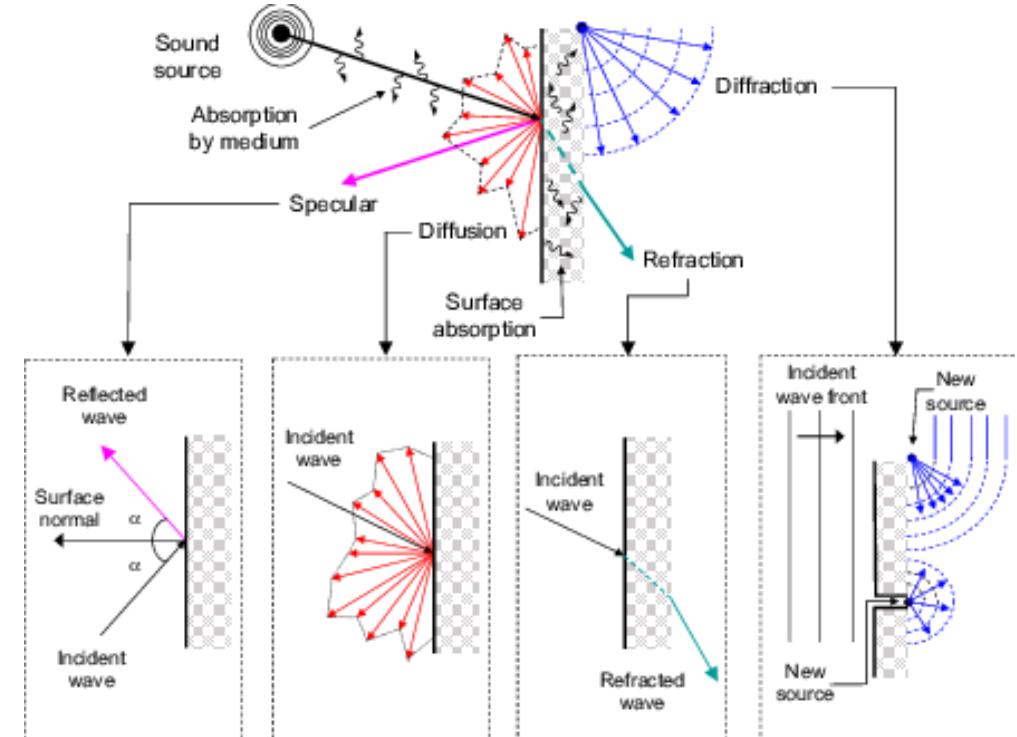
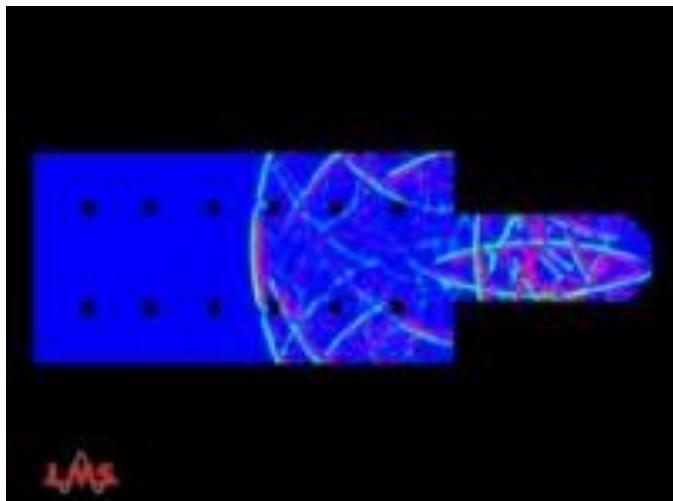


# Reflection

- If reflection occurs, what happens?

- If the wall is a rigid body, It should be a fixed boundary condition

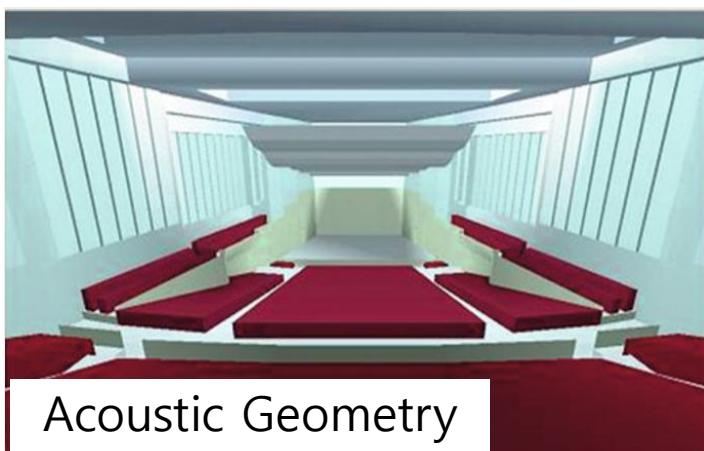
소리가 물체에 충돌할 때, specular reflection (정반사) 뿐만 아니라, Diffuse, refraction, diffraction 등이 함께 발생한다.



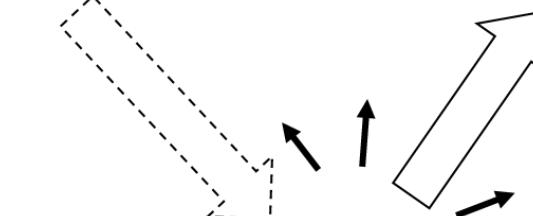
이를 정확하게 Simulation하는 것은  
매우 시간이 많이 걸린다.

# Geometrical Acoustics

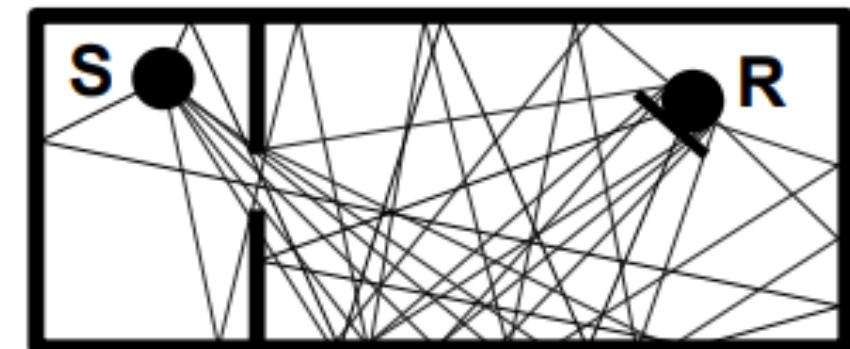
- The easiest way to model a realistic sound is by recording.
  - Difficult to simulate the various sound effects.
- Recently, a method using “**Sound propagation**” has been used.
  - Sound simulator needs **3-D geometry information** and **acoustic material properties** for realistic sound generation.



+



Acoustic Material →



Sound propagation paths