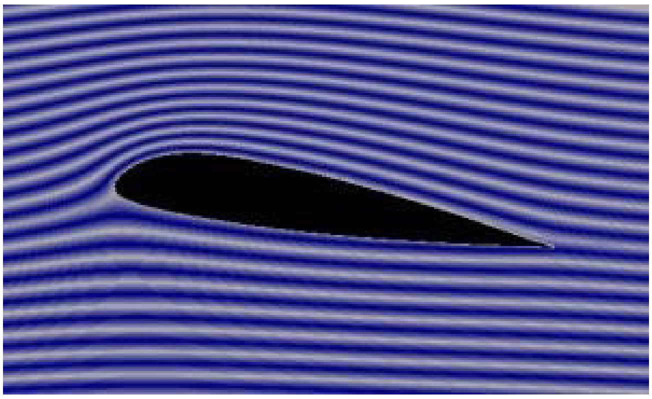
Streamlines and The Equation of Continuity

Streamlines:

When looking at airflow diagrams one will notice that they all have these lines that trace an estemated path for the direction the air is flowing. If you were to take a fluid particle which is a tiny fluid element and trace it's path you would be tracing out a 'streamlline'. So a streamline is the path traced out by a fluid particle.

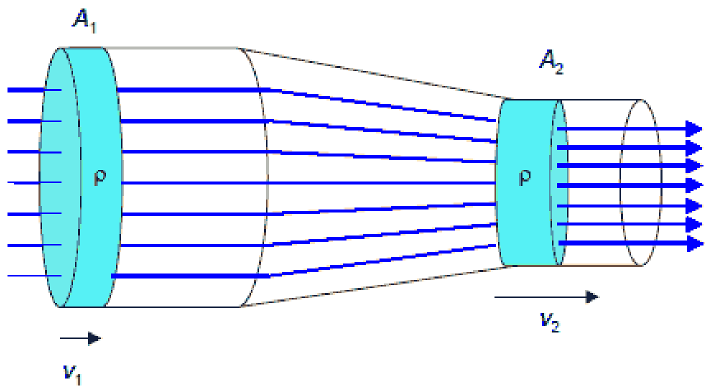


In the above diagram you can see where the name 'streamline' comes from because they are quite litraly lines that represent streams of fluids. They are mostly used for looking at how air moves along different air-foils because they are quite useless for stationary objects. Note that the velocity vector of a fluid particle at any instant is always tangent to the the streamline that it folows.

Streamlines will never cross each other because if they did, a fluid particle coming to their intersection would have two different velocities simultaneusly, and that doesn't happen in the real world and it would be impossible to caculate the velocity at that point.

In flows like the one over the car we can make an imagineary tube over the object that has the same shape or boundary at the streamlines. This kind of thins can act like a pipe because the any fluid particle in it cannot escape or else that would mean that the stream lines would need to intersect which as said before cannot happen. Doing this makes it easier to analyze the motion of the fluid because we can look at the cross sectional area as we would a pipe that is the same shape of the streamline and we could use equations in the same way as well.

Volume Flow Rate and the Equation of Continuity:



In the above diagram we show the crosssection area of 'the tube'. Given the velocity of the fluid at point and the cross sectional area at that point we need to find the volume of the fluid that passes through that crossectional area over a given interval of time. To do this we need to define a law which is ; **the volume of fluid that passes through the cross sectioinal area at A_1 will be equal to the volume of fluid that passes through A_2 over a given interval of time.**

So to calculate the volume over the time interval we can think of it as the volume of a cylinder with the crosssectional area being its crosssectional area = **A** and the length being the product of the velocity of the fluid and the time it travled = **v(dt).** So the volume of this cylinder would be its crossectional area into its length. $\mathbf{V = A_1 v_1 (dt)}$. Now because the volume is the same at $\mathbf{A_2}$ we can say that $\mathbf{A_1 v_1 (dt) = A_2 v_2 (dt)}$ **which becomes** $\mathbf{A_1 v_1 = A_2 v_2}$. This means that the rate of flow of some volume (**R**) ramains **constant.** And we can fainaly conclude that:

$$\mathbf{R = Av = some\ constant}$$

This is our **Equation of Continuity**. It is really an expression for conservation of mass and it tells us a little more then you might think. It tells us that the velocity of a fluid particle increases as the crossectional area of the 'tube' decreases $\mathbf{v = 1/A}$ or it is inverely proportional to the area. From the equation we can derive the mass flow rate by multiplying it by density. The last thing that the equation can show us which is something important for lift is that streamlines get denser as the velocity of a fluid particle in it increase as a result of a smaller crossectional area and you can see this happening in air flow diagrams.