Chapter-5: Geometric Transformation

Que-1: Basic Transformations.

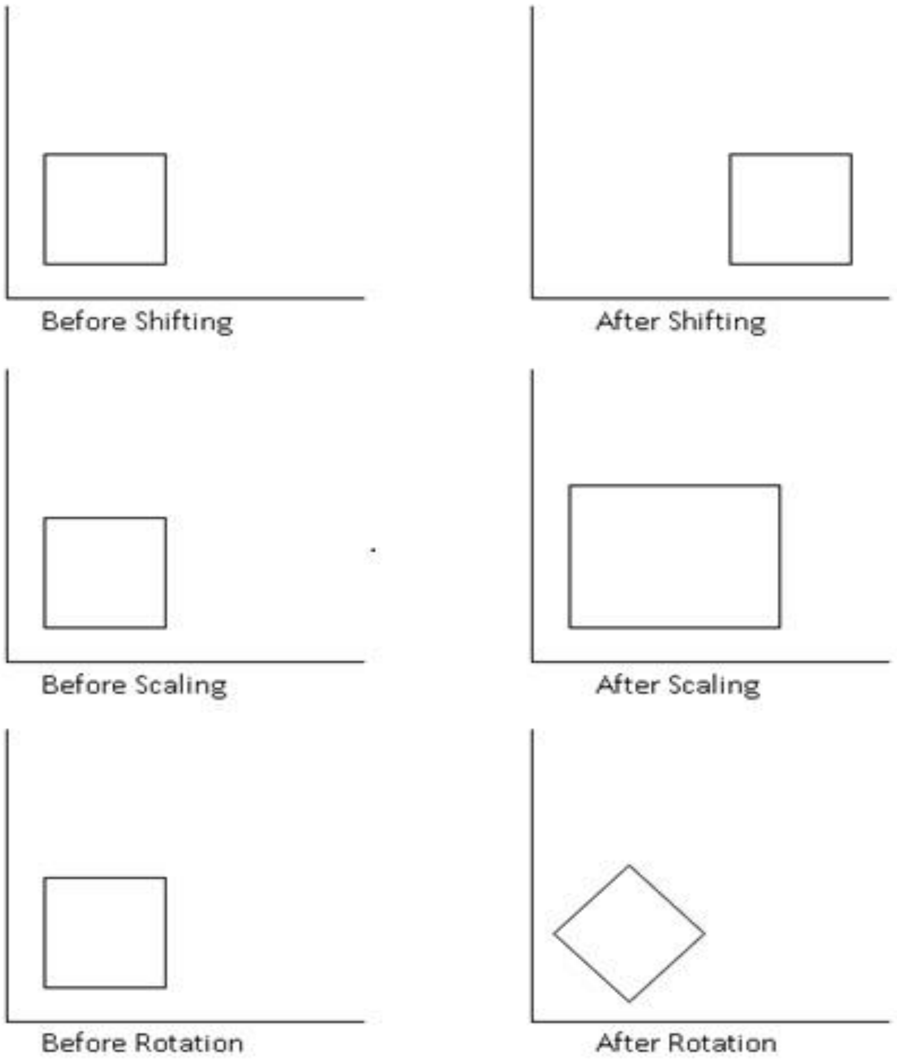
Ans: -There are three basic actions or movements, moving, scaling, and rotating which are widely used in the graphics applications.

-These movements are performed through some basic geometry.

-Since these transformations are performed using geometry, they are also known as geometric transformations.

-There are three basic transformations:

1) **Translation (Shifting)**
2) **Scaling**
3) **Rotation**



Que-2: Explain Translation in Detail. (Feb-2020)

- Translating an object means moving (shifting) an object from one place to another.

-Translation of an object is possible in any one of the following three directions.

1) Translation of an object in the horizontal direction that is translation parallel to x-axis.

Suppose we want to translate a point (x,y) in the horizontal direction by Tx units, then this movement can be obtained by simply adding Tx units to x-coordinates.

$$x' = x + Tx$$

$$y' = y$$

2) Translation of an object in the vertical direction that is translation parallel to y-axis.

Suppose we want to translate a point (x,y) in the vertical direction by Ty units, then this movement can be obtained by simply adding Ty units to y-coordinates.

$$x' = x$$

$$y' = y + Ty$$

3) Translation of an object in the horizontal and the vertical directions simultaneously.

-Suppose We translate a two-dimensional point by adding translation factors (Tx, Ty) to the original coordinate position (x, y) to move the point to a new position (x', y').

$$x' = x + Tx$$

$$y' = y + Ty$$

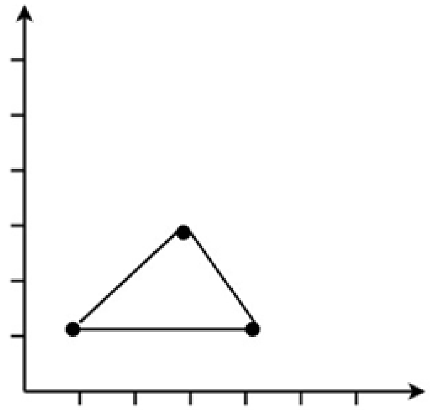
-These followers as to write the two-dimensional translation equation in the matrix form.

$$(x',y') = (x+Tx\ ,\ y+Ty)$$

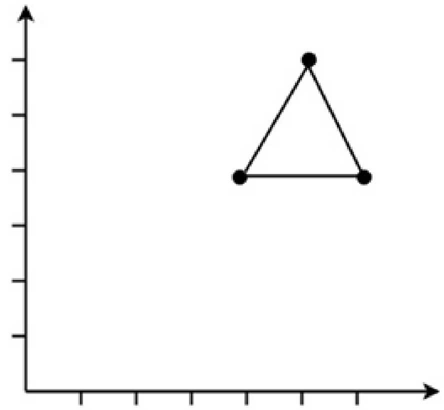
$$[\,x' \quad y'\,] = [\,x \quad y\,] + [\,Tx \quad Ty\,]$$

$$P' = P + T$$

- **Where P' is 1x2 row matrix of coordinates of the translated point.**
- **P is 1x2 row matrix of coordinates of the original point.**
- **T is 1x2 row matrix of Translation factor.**



Original Position



Polygon after Translation

-Translation is performed on each and every vertex of an object to translate the entire object.

-Translation of other object like circles and ellipses is performed by translating their parameters.

Que-3: Explain Scaling Transformation in Detail. (Feb-2018, Jan-2017, Feb-2016)

- A scaling transformation alters a size of the object.

- This operation can be carried out for polygon by multiplying coordinates value (x, y) of which vertex by scaling factors (Sx, Sy) to produce the transform coordinate (x', y').

$$x' = x \cdot Sx$$

$$y' = y \cdot Sy$$

- Scaling factor Sx scales in the x direction while sy scales in the y direction.

- The transformation can be also return in the matrix form.

$$(x',y') = (x.Sx\ ,\ y.Sy)$$

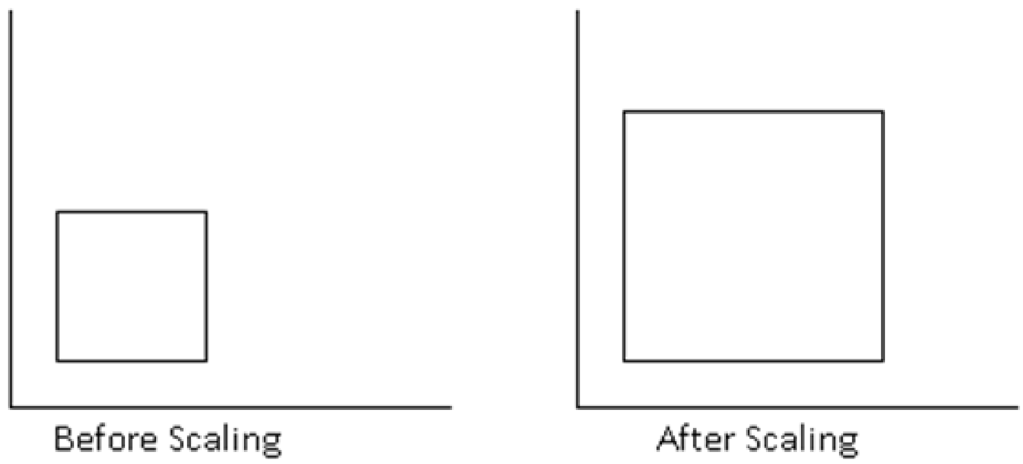
$$[\,x' \quad y'\,] = [\,x \quad y\,] * \begin{bmatrix} Sx & 0 \\ 0 & Sy \end{bmatrix}$$

$$P'=P.S$$

- Where P' is 1x2 row matrix of coordinates of the scaled point.

- P is 1x2 row matrix of coordinates of the original point.

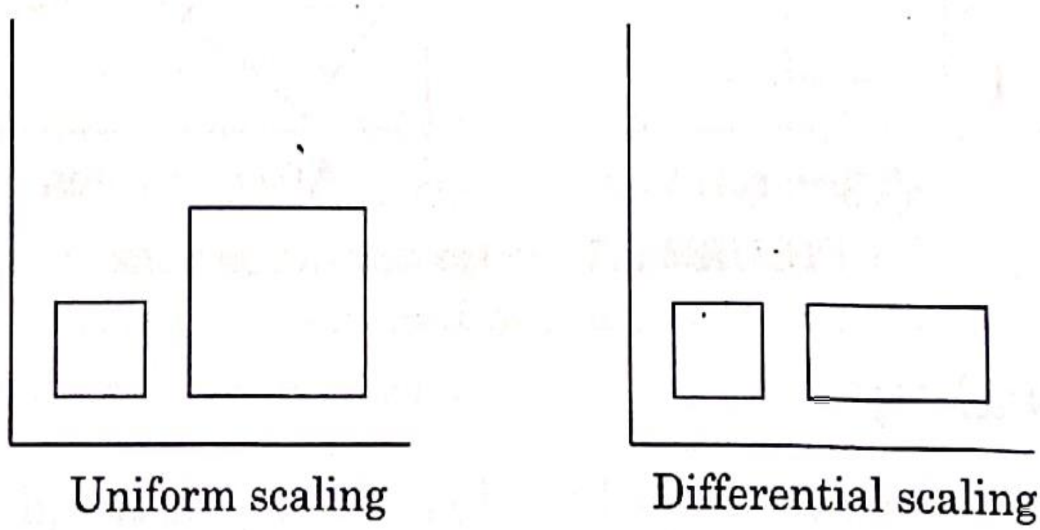
- S is 2 × 2 scaling factor matrix.



- Any positive number can be assign to the scaling factors Sx and Sy values less than 1 reduce the size of object values greater than 1 produce in enlargement.

- Specifying a value of 1 for both Sx and Sy lives the size of object unchanged.

- When Sx and Sy are assign the same value, Then it is known as uniform scaling (Sx=Sy) otherwise Differential scaling (Sx≠Sy).



- This matrix will work with the objects define by straight lines.

-But objects like circles and ellipses will not work with this matrix.

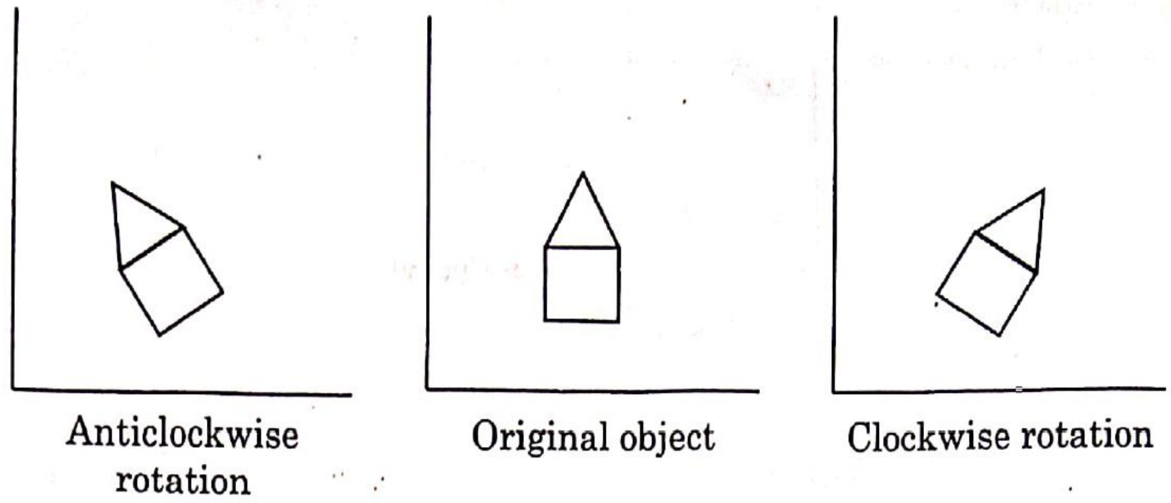
-For scaling this type of objects, the scaling is performed on their parameter like radius, major and minor axis.

Que-4: Explain Rotation in Detail. (Feb-2020, Feb-2019, Feb-2018, Feb-2016)

-Rotation moves an object on a circular path about an origin.

-This transformation requires two major parameters, direction of rotation and angle of rotation.

-Any given object can be rotated either in clockwise direction or in anti-clockwise direction by a given angle θ.



- Let us take a point P(x, y) to be rotated in anti-clockwise direction.

- the point P'(x', y') is a rotated point, which is the result of anti-clockwise rotation performed on point P(x, y) with θ angle relative to origin(0,0).

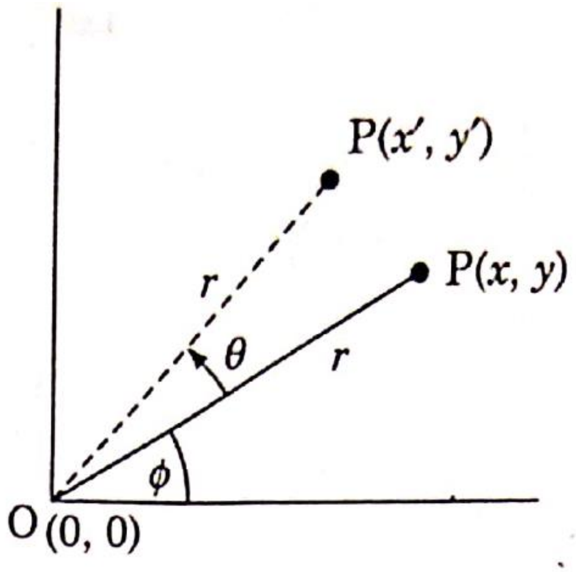
- ϕ is an angle of point P from X-axis.

- According to simple trigonometric principles

$x=r \cos \phi$ and $y= r \sin \phi$

- where r is the distance of point P from the origin(0,0).

- Following figure shows anticlockwise rotation of point P.



- **$x' = r\cos(\theta + \phi)$**

$x' = r\cos\theta\ \cos\phi - r\sin\theta\ \sin\phi$

Substitute $r\cos\phi = x$ we get,

$x' = x\cos\theta - y\sin\theta$

- **similarly, $y' = r\sin(\theta + \phi)$**

$y' = r\sin\theta\ \cos\phi + r\cos\theta\ \sin\phi$

Substitute $r\sin\phi = y$ we get,

$y' = x\sin\theta + y\cos\theta$

-Anticlockwise-Rotation Matrix is given by,

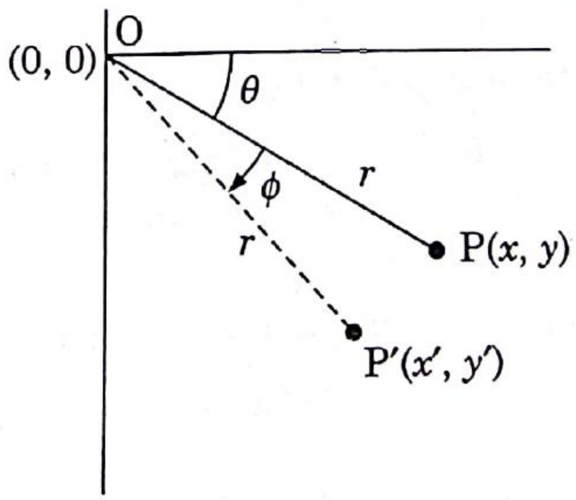
$$[x' \quad y'] = [x \quad y] * \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

P'=P*R.

-Similarly, we can derive the equation to rotate a point in clockwise direction by angle θ about origin (0, 0).

-Here we are rotating point P(x, y) in opposite direction of that we did in the anticlockwise rotation. In this case angle will be θ.

- Following figure shows clockwise rotation of point P.



-Clockwise-Rotation Matrix is given by,

[x’ y’]=[x y]* $\begin{bmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{bmatrix}$

[x’ y’]=[x y]* $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$

P’=P*R.

Que-5: Explain Homogeneous coordinates in Detail.

-The general form of the basic transformation based on following equation.

P’=PG1+G2

-where P’ is a transformed matrix, P is matrix of point to be transformed, G1 is a 2x2 matrix of transformation factors and G2 is another 1x2 row matrix of translation factor.

-Here G1= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ **; a, b, c and d are constants which assume different values for different transformations.**

-For example, the values of a, b, c and d are Sx, 0, 0, and Sy respectively for scaling transformation and cos θ, sin θ, -sinθ and cosθ respectively for rotation in anticlockwise direction by angle θ.

-G2= [Tx, Ty]; Tx, Ty are translation factors.

-Matrix multiplication and addition is not possible for a matrix of order 1x2 and order 3x3.

-so the coordinate matrices are also to be expanded by one more column.

-by adding newly element to the matrix is called homogeneous coordinates.

-If we want to do first scaling, then rotation and then translation. Then this process is very time consuming.

-Instead of this, if we have a single matrix which performs all this three transformations simultaneously using a single transformation matrix then it is given as below.

-Let us rewrite all the matrices with homogeneous rows and columns.

-Coordinate matrix [x y 1]

-Scale Transformation Matrix $\begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix}$

-Rotation Matrices are $\begin{bmatrix} \cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **and** $\begin{bmatrix} \cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

-Translation Transformation Matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Tx & Ty & 1 \end{bmatrix}$

-And generalized transformation Matrix will be $\begin{bmatrix} a & b & 0 \\ c & d & 0 \\ Tx & Ty & 1 \end{bmatrix}$

-where a, b, c, and d constants will take different values for scaling and rotation transformations, depending on the transformation on hand.

Some Other Transformations

- **Some package provides few additional transformations which are useful in certain applications. Two such transformations are reflection and shear.**

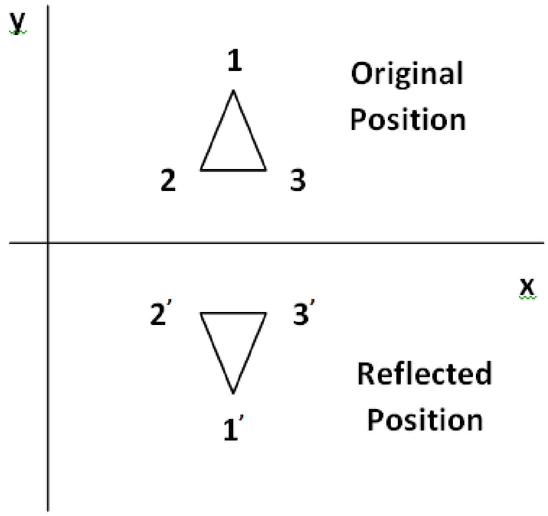
Que-6: Explain Reflection in Detail. (Feb-2019)

-A reflection is a transformation that produces a mirror image of an object.

-The mirror image for a two –dimensional reflection is generated relative to an axis of reflection by rotating the object 180^{o} about the reflection axis.

-Reflection gives image based on position of axis of reflection. Transformation matrix for few positions are discussed here.

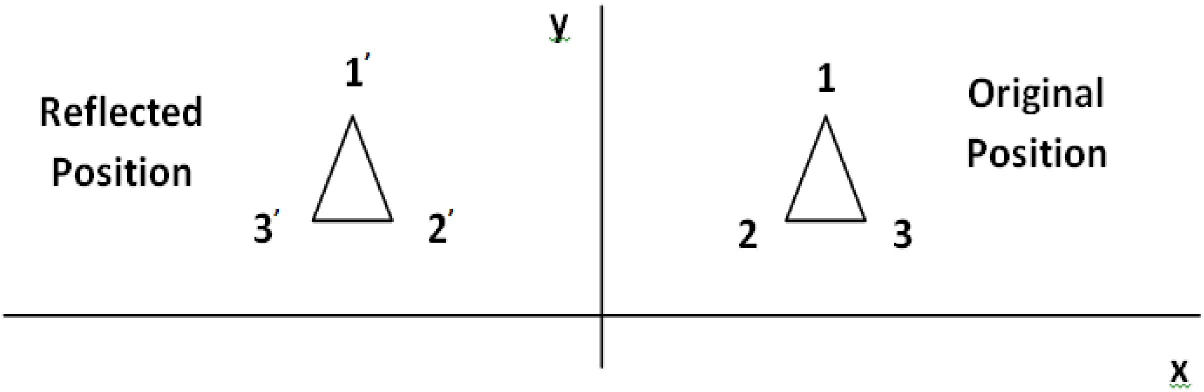
Transformation matrix for reflection about the line $y = 0$, $the\ x\ axis$.



- This transformation keeps x values are same, but flips (Change the sign) y values of coordinate positions.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

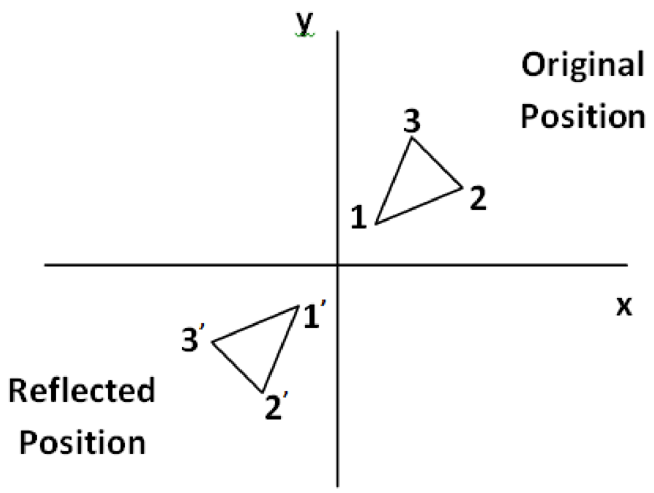
Transformation matrix for reflection about the line $x = 0$, $the\ y\ axis$.



- This transformation keeps y values are same, but flips (Change the sign) x values of coordinate positions.

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

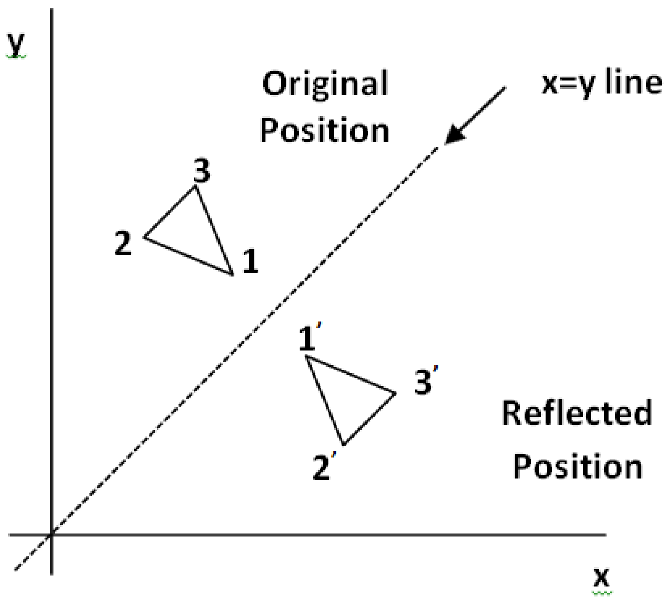
Transformation matrix for reflection about the $Origin$.



- This transformation flips (Change the sign) x and y both values of coordinate positions.

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

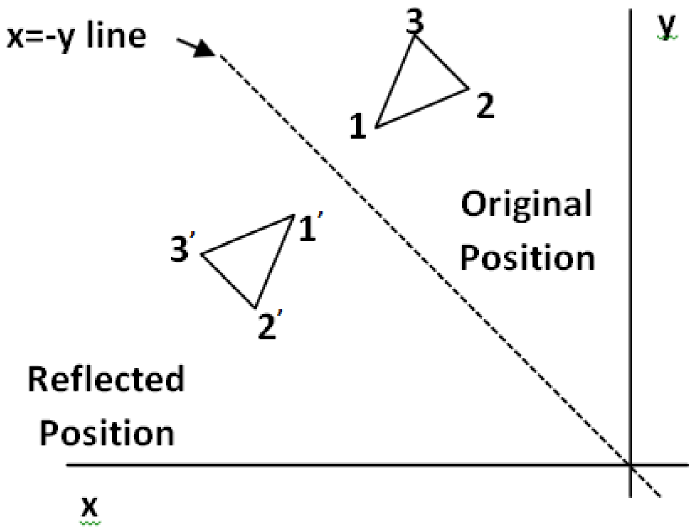
<u>Transformation matrix for reflection about the line x = y .</u>



- This transformation interchange x and y values of coordinate positions.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transformation matrix for reflection about the line x = −y .



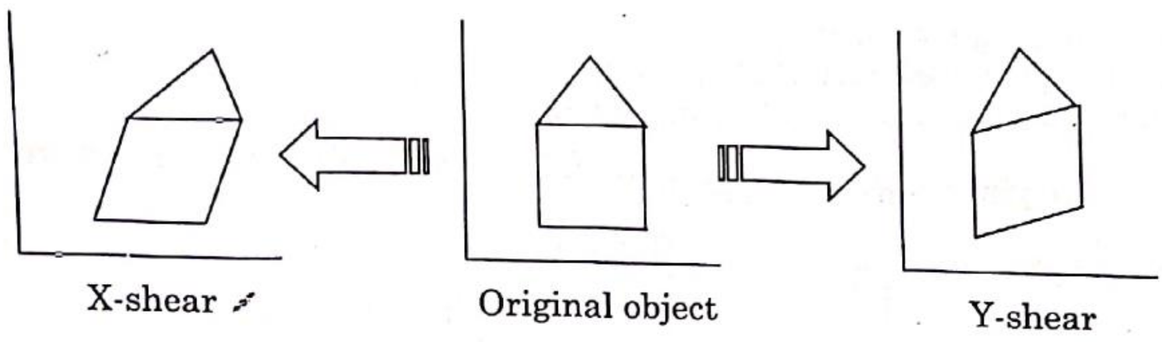
- This transformation interchange x and y values of coordinate positions.

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Que-7: Explain Shearing in Detail. (Feb-2019, Feb-2018, Jan-2017)

- A transformation that distorts the shape of an object such that the transformed shape appears as if the object were composed of internal layers that had been caused to slide over each other is called shear.

-Two common shearing transformations are those that shift coordinate x values and those that shift y values.



- Shear relative to $x - axis$ that is y = 0 line can be produced by following equation:

x' = $x + sh_x \cdot$, here $s$hx is a x-shear factor

y' = y

-Transformation matrix for x-shear is:

$$\begin{bmatrix} 1 & 0 & 0 \\ Shx & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Shear relative to y – $axis$ that is x = 0 line can be produced by following equation:

$\boldsymbol{x'} = \boldsymbol{x}$

$\boldsymbol{y'}$ **= x . shy+ y, here $s$hy is a y-shear factor**

-Transformation matrix for y-shear is:

$$\begin{bmatrix} 1 & Shy & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

-The generalized shearing Matrix is given by,

$$\begin{bmatrix} 1 & Shy & 0 \\ Shx & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

-Here it will interesting to show that shearing can also be obtained by Scaling and Rotation.

- Scaling matrix and anticlockwise rotation matrix is given below.

$$\begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \textbf{and} \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

-We want to derive shearing matrix as a result of composite transformation of scaling and rotation. That is given by,

$$\begin{bmatrix} 1 & Shy & 0 \\ Shx & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & Shy & 0 \\ Shx & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} Sx.cos\theta & Sx.sin\theta & 0 \\ -Sy.sin\theta & Sy.cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

-therefore $Sx.cos\theta$=1 and $Sy.cos\theta$=1

-therefore Sx= 1/ $cos\theta$ and Sy= 1/$cos\theta$

Substituting it in above matrix we get,

$$\begin{bmatrix} 1 & Shy & 0 \\ Shx & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} cos\theta/cos\theta & sin\theta/cos\theta & 0 \\ -sin\theta/cos\theta & cos\theta/cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & Shy & 0 \\ Shx & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & tan\theta & 0 \\ -tan\theta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Short Questions

Que-1: What is transformation? List out various types of transformations. (Feb-2019)

-In most of the graphics applications, objects or images are required to be altered.

-To add reality, many times these objects are needed to be moved, rotated, enlarged or reduced.

-This alteration or manipulation processes of objects is called transformation.

-There are three basic transformations:

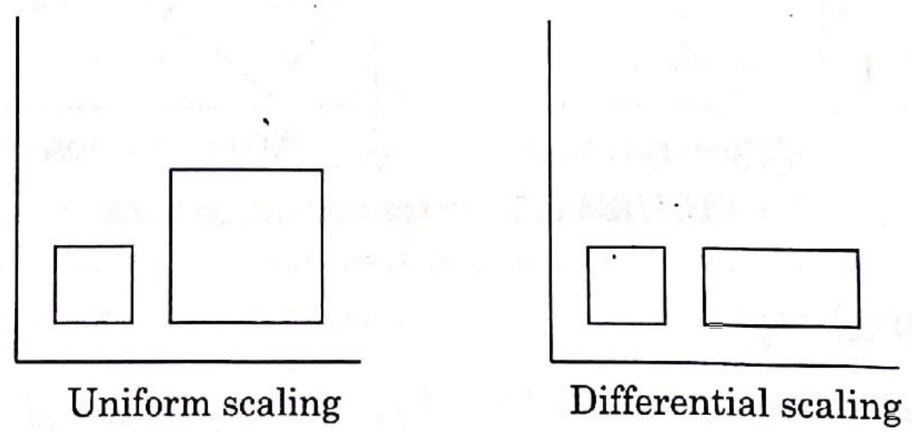
1) **Translation (Shifting)**
2) **Scaling**
3) **Rotation**

Que-2: What is Sacling? Differentiate between Uniform and Diffrential scaling. (Feb-2018)

- A scaling transformation alters a size of the object.

- This operation can be carried out for polygon by multiplying coordinates value (x, y) of which vertex by scaling factors (Sx, Sy) to produce the transform coordinate (x', y').

- When Sx and Sy are assign the same value, Then it is known as uniform scaling (Sx=Sy) otherwise Differential scaling (Sx≠Sy).



Que-3: Give the transformation matrix used to move an object from its original place. (Jan-2017)

- Translating an object means moving (shifting) an object from one place to another.

-Translation matrix is given by,

[x'   y'] = [x   y] + [Tx Ty]

P' = P + T

- **Where P' is 1x2 row matrix of coordinates of the translated point.**
- **P is 1x2 row matrix of coordinates of the original point.**
- **T is 1x2 row matrix of Translation factor.**

Que-4: What is Homogeneous coordinates? Explain with example.(Feb-2020)

-Matrix multiplication and addition is not possible for a matrix of order 1x2 and order 3x3.

-so the coordinate matrices are also to be expanded by one more column.

-by adding newly element to the matrix is called homogeneous coordinates.

-e.g. $\begin{bmatrix} Sx & 0 \\ 0 & Sy \end{bmatrix}$ **is 2x2 matrix, and by aading one more column to this matrix is homogeneous coordinates.** $\begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Que-5: Give matrix to get reflection about the line y=x and y=-x.(Jan-2017)

-Reflection matrix about line y=x is given by,

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

-Reflection matrix about line y=-x is given by,

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Que-6: Define Reflection.(Feb-2018)

-A reflection is a transformation that produces a mirror image of an object.

-The mirror image for a two –dimensional reflection is generated relative to an axis of reflection by rotating the object 180° about the reflection axis.

Que-7: What is transformation? Write shearing transformation matrix.(Feb-2016)

-In most of the graphics applications, objects or images are required to be altered.

-To add reality, many times these objects are needed to be moved, rotated, enlarged or reduced.

-This alteration or manipulation processes of objects is called transformation.

-Shearing matrix is given by,

$$\begin{bmatrix} 1 & Shy & 0 \\ Shx & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Que-8: What is shearing? List types of shearing.(Feb-2020)

- A transformation that distorts the shape of an object such that the transformed shape appears as if the object were composed of internal layers that had been caused to slide over each other is called shear.

-Two common shearing transformations are:

1) X-shear

2) y-Shear