Chapter-3: Line generation

Que-1: Geometry:

Ans: -By now we know that the smallest element is a point and that for a raster device, it is pixel.

-Pixel is the smallest addressable element of raster device. It is derived from the "Picture Element".

-The number of pixel of a display device determines the resolution of the device, and therefore higher number of pixels is preferred for graphics applications.

-Each of these pixels is associated with the coordinate of Cartesian plane.

-These coordinates identify pixel or point. To draw any image or object, some of pixels are activated by setting some intensity. In other words, the pixels turned on.

-For example, to draw a straight line segment the adjacent pixels between two points are turned on.

Que-2: Explain Geomatry of Point.

Ans:-The smallest element which can be shown on a graph is a point.

-As in the figure Point A has been positioned in the XY-plane and (x1, y1) is the position of point A.

- It implies that the A is located at x1 distance in X direction and at y1 distance in Y direction from the origin (0,0).

- Where the (x1,y1) is known as the coordinates of point A.

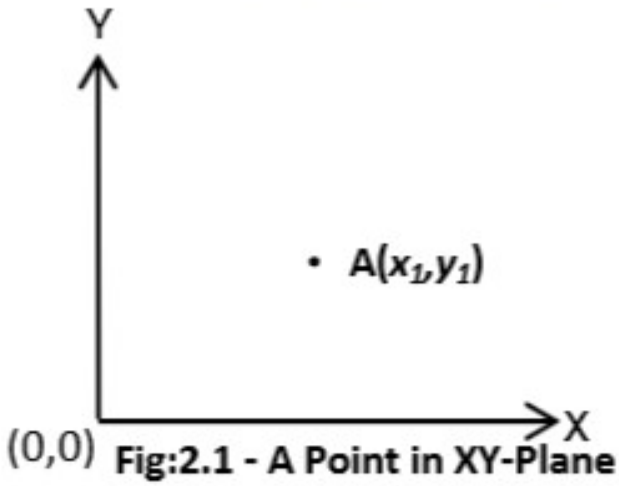


Fig:2.1 - A Point in XY-Plane

Que-3: Geometry of line. (Feb-2019, Feb-2018, Jan-2017, Feb-2016)

Ans: -Any two points specified in the Plane will define a line, which means that for defining the line we must have to specify two points. In the figure 2.2, two points P1(x1,y1) and P2(x2,y2) define a line l1.

-To define the line we need an equation. A random point P(xi,yi) is said to be on the line l1 if and only if it satisfies the equation of the line.

-The equation of a straight line can be derived with the help of slope of a straight line.

- y=mx+b is a straight line equation which is also called slope- Intercept line equation.

slope of a line: It is the rate at which an ordinate of a point on a plane changes with respect to a change in the horizontal coordinate.

-slope is denoted by m.

-slope(m) = vertical distance between two points / Horizontal distance between two points.

-let us assume two points P1(x_1,y_1) and P2 (x_2,y_2) on a line.

-For this slope is m= (y_2-y_1) / (x_2-x_1).

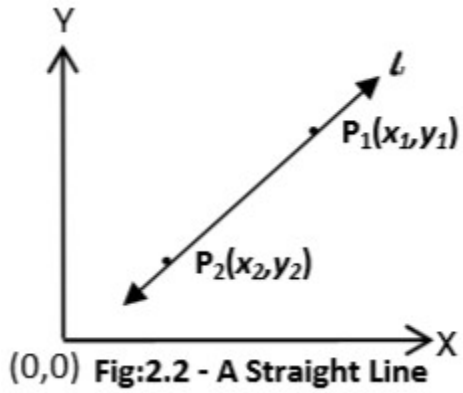
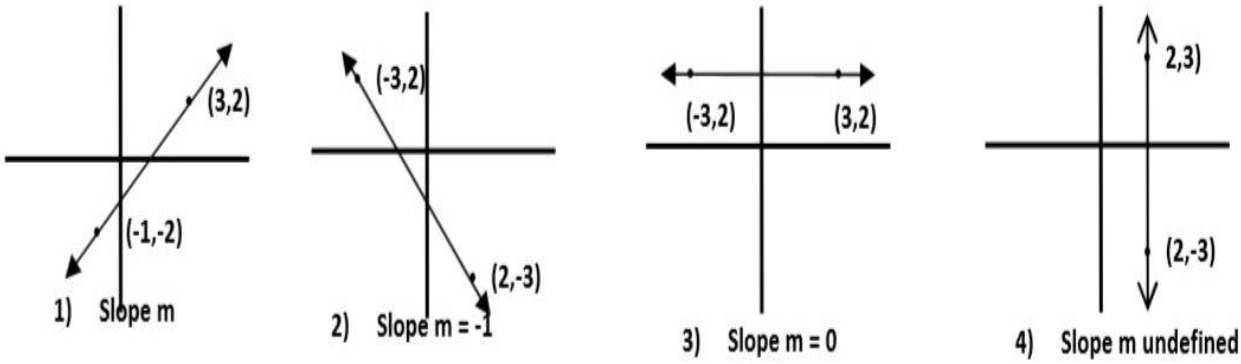


Fig:2.2 - A Straight Line

Types of Slope.(Feb-2020)

-So the slope of any line can be one of four types:

the straight line 1) with positive slope, 2) with negative slope, 3) with zero slope and 4) with undefined slope.



Que-4: Line Generation Algorithms.

Ans: There are 3 main line drawing algorithms:

1) **DDA Algoritm**
2) **VECGEN Algoritm**
3) **Bresenham Algorithm**

Que-5: DDA Algorithm. (Feb-2020)

Ans: Digital Differential Analyzer (DDA) algorithm is the simple line generation algorithm which is explained step by step here.

Step 1 – Get the input of two end points(X_0 ,Y_0) and (X_1 ,Y_1).

Step 2 – Calculate the difference between two end points.

dx = X_1 - X_0

dy = Y_1 - Y_0

Step 3 – Based on the calculated difference in step-2, you need to identify the number of steps to put pixel. If dx > dy, then you need more steps in x coordinate; otherwise in y coordinate.

```
if (absolute(dx) > absolute(dy))
  Steps = absolute(dx);
else
```

```
  Steps = absolute(dy);
```

Step 4 – Calculate the increment in x coordinate and y coordinate.

```
Xincrement = dx / (float) steps;

Yincrement = dy / (float) steps;
```

Step 5 – Put the pixel by successfully incrementing x and y coordinates accordingly and complete the drawing of the line.

```
for(int v=0; v < Steps; v++)

{

  x = x + Xincrement;

  y = y + Yincrement;

  putpixel(Round(x), Round(y));

}
```

Advantages of DDA:

-It is simple to implement

-It is faster than direct line equation

-We can not use multiplication method

Disdvantages of DDA:

-Round-off is time consuming

-floating point arithmatic implimentation is time consuming

-sometimes point position is not accurate

Que-6: VECGEN Algorithm. (Feb-2019)

Ans: - The concept of drawing a line or a curve, stepping along a row index (y) or a column index (x) to determine the corresponding column index or row index respectively is known as Digital Differential Analyzer (DDA).

- The line equation works well for gentle slope but when the slope is sharp the role of x and y are interchanged as well as slope m replaced by 1/m. This method is known as VECGEN method.

Step: 1 If the slope is less than or equal to 1, the unit x intervals dx=1 and compute each successive y values.

dx=1

m = dy / dx

m = (y2 - y1) / 1

m = (yk + 1 – yk) /1

yk+1 = yk + m ------ (2)

Subscript k takes integer values starting from 1,for the first point and increment by 1 until the final end point is reached.

m -> any real numbers between 0 and 1

Calculate y values must be rounded to the nearest integer

Step: 2 If the slope is greater than 1, the roles of x any y at the unit y intervals dy=1 and compute each successive y values.

dy=1

m= dy / dx

m= 1/ (x2-x1)

m = 1 / (xk+1 – xk)

xk+1 = xk + (1 / m) ------(3)

Equation 2 and Equation 3 that the lines are to be processed from left end point to the right end point.

Step: 3 If the processing is reversed, the starting point at the right

dx= -1

m= dy / dx

m = (y2 – y1) / -1

yk+1 = yk - m -------- (4)

Intervals dy=1 and compute each successive y values.

Step: 4 Here, dy= -1

m= dy / dx

m = -1 / (x2 – x1)

**m = -1 / (xk+1 – xk) **

xk+1 = xk + (1 / m) -------- (5)

Equation 2 and Equation 5 used to calculate pixel position along a line with –ve slope.

This algorithm can also be written in following form:

Step-1: Input two line end points A(xa,ya) and B(xb,yb) and start with the left endpoint A(xa,ya) and declare Δx, Δy, step, k, xinc, yinc, x, y.

Step-2: Calculate Δx = xb - xa ; Δy = yb - ya [find the difference of end points]

Step-3: If abs(Δx) > abs(Δy) then step = abs(Δx) else step = abs(Δy)

Step-4: We have xinc = Δx / steps and yinc = Δy / steps [Find total intermediate points]

Step-5: setPixel(round(xa), round(xa), 1) [Plot the first point]

Step-6: x = x + xinc; y = y + yinc ; setPixel(round(x), round(y),1) [Plot all points]

Step-7: Repeat step-6 for step times

Advantage of VECGEN:

1) It is a faster method for calculating pixel position then simple line algorithm.

2) It eliminates continues multiplication in equation "Y=mx+b" so that appropriate increments are applied on x or y.

Disadvantage of VECGEN:

1) Round off error in successive additions can cause the pixel positions to drift away from the true line path for long line segments.

2) The rounding operations and floating point arithmetic are still time consuming.

Que-7: Explain Bresenham's Line Drawing Algorithm. (Feb-2019, Feb-2018, Jan-2017, Feb-2016)

Ans: -The line generating algorithm developed by Bresenham is accurate and faster; which converts scan lines using incremental integer calculations that can be used to draw not

only lines but also used to display circles and other curves.

-As we can see in fig:2.4 last turned on pixel is (2,b) and the problem is which pixel should be turned on next,(3,c) or (3,d)?

-According to the Bresenham's algorithm the pixel closest to the line to be drawn will be chosen. For this he has defined a decision parameter, so let understand what is this decision parameter?

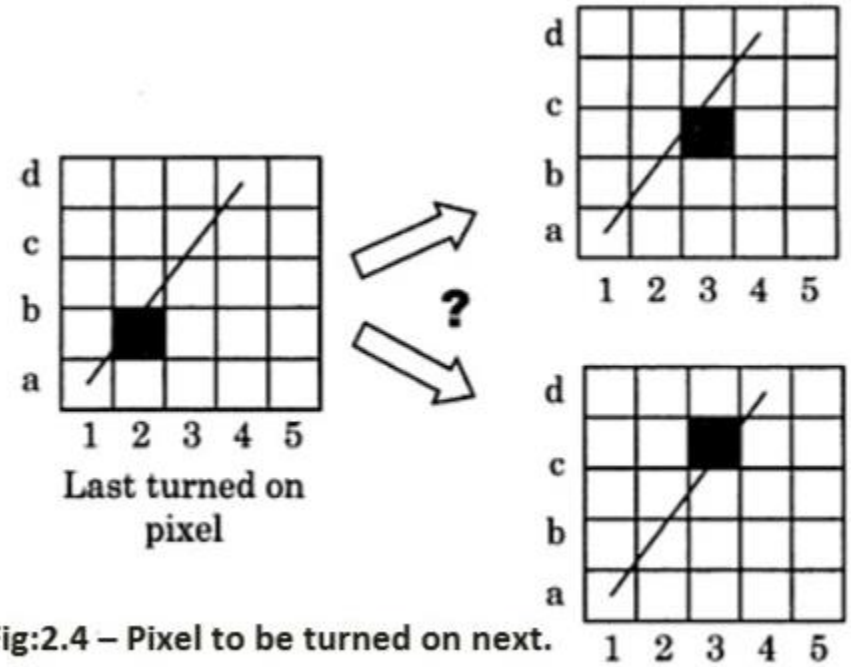


Fig:2.4 – Pixel to be turned on next.

-Let us assume that we are drawing a line y = mx+b that passes through a point (x0,y0). Here 0 < m < 1. Let us also assume that the last pixel turned on is (xk,yk) and the decision to be made is for the next step that is for the vertical distance xk+1 as in fig:2.5.

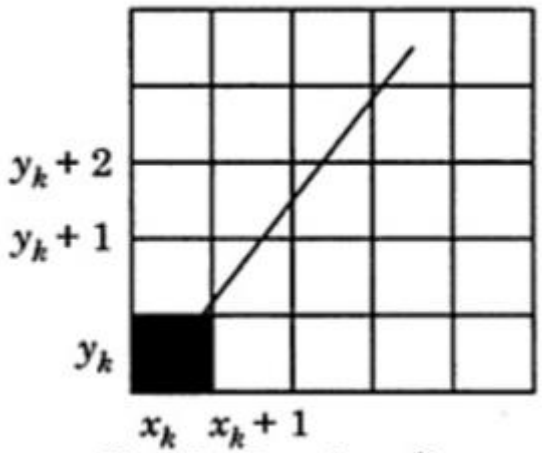


Fig:2.5 – Drawing a line.

-Now let us assume a vertical row of pixels which passes through horizontal distance xk+1.

-There are three vertical points, (xk+1, yk), (xk+1, y) and (xk+1, yk+1), fall on this assumed line.

-In addition the assumed distance between (xk+1, yk) and (xk+1, y) is d1 and distance between (xk+1, y) and (xk+1, yk+1) is d2 in figure.

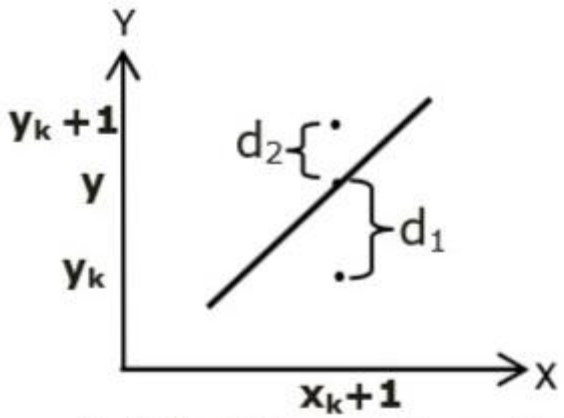


Fig:2.6 – Distance between Points

Bresenham's Line-Drawing Algorithm for |m| < 1

Step 1: Input the two line endpoints (x1,y1) and (x2,y2).

Step 2: Calculate constants dx=x2-x1, dy=y2-y1, 2dx, 2dy, and 2dy – 2dx,

Step 3: obtain the starting value for the decision parameter as Pk = 2dy - dx

Step 4: perform the following test:

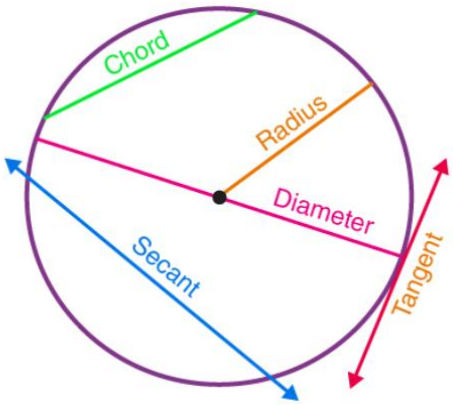
If Pk < 0, the next point to plot is (xk +1, yk) and Pk +1 = Pk + 2dy

Otherwise, the next point to plot is (xk +1, yk +1)and Pk +1 = Pk + 2dy - 2dx

Step 5: Repeat step 4 dx no. of times.

Que-8: Geomatry of Circle.

Ans:- The circle is defined as a set of all points that are placed at a fixed given distance from a given centre point.



Some of the important terminologies used in the circle are as follows:

Terms	Description
Circumference	The boundary of the circle is known as the circumference
Radius	The line from the centre "O" of the circle to the circumference of the circle is called the radius and it is denoted by "r"
Diameter	The line that passes through the centre of the circle and touches the two points on the circumference is called the diameter and it is denoted by the symbol "D"
Arc	Arc is the part of the circumference where the largest arc is called the major arc and the smaller one is called the minor arc
Chord	The straight line that joins any two points in a circle is called a chord

Circle Formulas:

Equation of Circle, $x^2 + y^2 = \text{radius}^2$

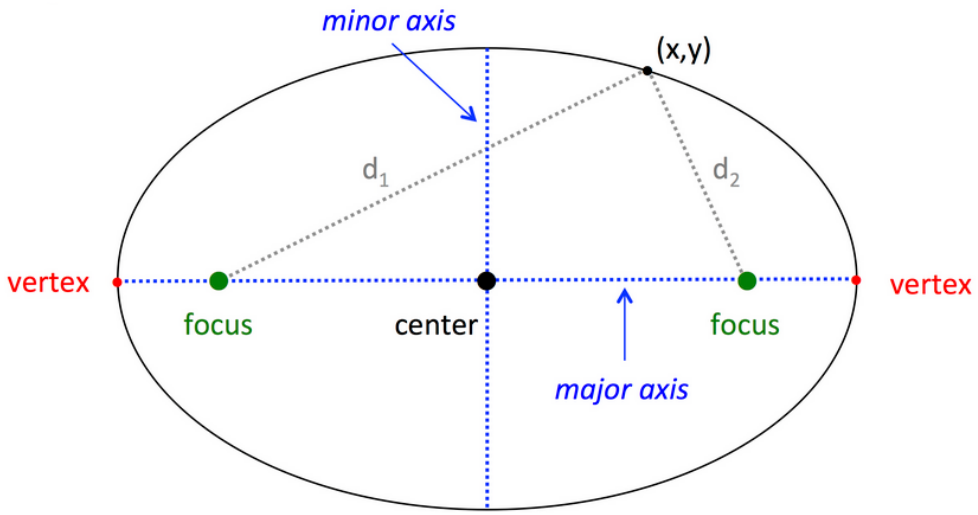
Area of a circle, $A = \pi r^2$

The circumference of a circle = $2\pi r$

Diameter = 2 x Radius

Que-9: Geomatry of Ellipse.

Ans: An ellipse is the locus of all those points in a plane such that the sum of their distances from two fixed points in the plane, is constant. The fixed points are known as the foci points (singular focus), which are surrounded by the curve.



The foci(focus) are two fixed points equidistant from the center of the ellipse.

Ellipse is defined by its two-axis along x and y-axis:

1)Major axis: The major axis is the longest diameter of the ellipse, going through the center from one end to the other, at the broad part of the ellipse. Half of major axis is called semi-major axis

2)Minor Axis: the minor axis is the shortest diameter of ellipse, crossing through the centre at the narrowest part. Half of minor axis is called semi-minor axis.

Equation of Ellipse:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

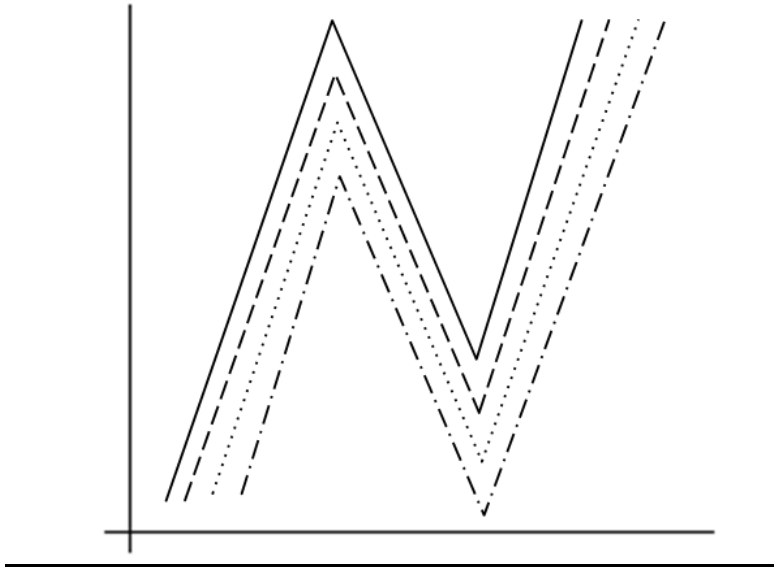
Que-10: Explain Line Styles.(Feb-2020)

Ans:- Generally, solid lines, dashed lines, dotted lines and thick lines are used as different line styles in graphics applications.

- Dashed lines and dotted lines are generated by just a minor modification in our standard DDA algorithms.

- A dashed line can be generated by inter-dash spaces between the dashes. Generally, dashes and inter-dash spaces are equal in length whereas a dotted line can be generated by very short length dashes.

- By using different characters in this method, different line styles can be created.



Que-11: Expalin Thick Lines.

Ans: The thick line primitive is more interesting. A line or a line segment generated by the standard DDA algorithm is a single pixel width. Let us assume a line with a gentle slope, that is, a slope less than 1. If you need such a line segment with a thickness of three pixels, you need to put there a vertical span of three pixels for each value of . Thus, a line with any width , can be generated just by putting a vertical span of (w/2) pixels on both the sides of the central line as shown in fig:2.7

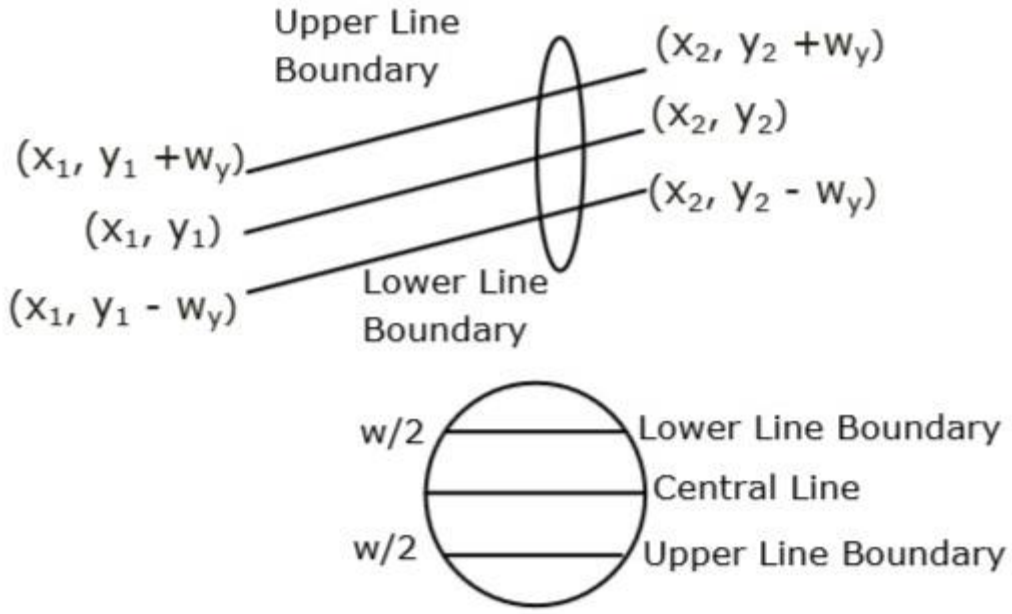


Fig. 2.7- Thick Line Generation

Let p1(x1,y1) and p2(x2,y2) be the end points of a line segment with width w , which is to be drawn. Here two parallel line segments will be drawn from a central line(x1,y1) and

(x2,y2) at a distance(w/2).

As shown in fig:, the coordinates of end points of the upper and lower line boundaries are $[(x_1,y_1 + w_y), (x_2,y_2 + w_y)]$ and $[(x_1,y_1 - w_y), (x_2,y_2 - w_y)]$ respectively. Where the w_y is as below:

$$w_y = \frac{w-1}{2}$$

In the above equation, w-1 is taken instead of w. the reason being, on both sides the line boundaries are drawn which are of 1 pixel in length. Therefore, from both the sides half of the pixel width will be deducted to get the exact width w. for a line segment with steep slope, that is slope greater than 1, the role of x and y in equation () will be interchanged. Thus, for steep slope, the value of w_y will be

$$\boldsymbol{w_y = \frac{w-1}{2} \cdot \frac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}{|x_2 - x_1|}} \quad \text{..... (4)}$$

In the above equation, $w-1$ is taken instead of w. The reason is we are considering the boundaries which are drawn on both the sides are of 1 pixel in length. So from both side 1 deducted to get its original width w. When the slope is steep that is greater than 1, the role of x and y in the above equation will get change and the equation would be written as follows:

$$\boldsymbol{w_x = \frac{w-1}{2} \cdot \frac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}{|y_2 - y_1|}} \quad \text{..... (5)}$$

In this case, pixels with horizontal span are activated simultaneously.

Que-12: Explain Thick Line Segments.

Ans:- When two thick line segments are connected with each other, they create problems at the joining end points.

- It creates an angle at the joining end. Joint of two such lines is shown in the figure below:

Fig.2.9 – Joint of two thick line segments

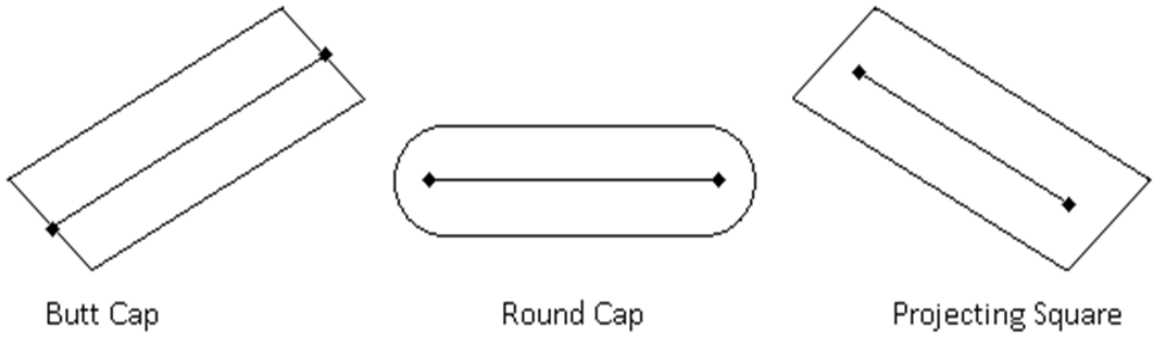
- The discreteness that occurs while joining two thick lines can be removed from the poly-lines by adding some additional portion to the joint as some specific types of joining process.

Que-13: Explain Line Caps. (Feb-2020, Feb-2018, Jan-2017, Feb-2016)

Ans:- When we construct the thick lines the issue is generated for its ends whether it should be vertical or horizontal.

- To clear this issue line caps can be used.

- There are three basic techniques or caps which are used in graphics applications: 1) butt cap, 2) round cap, and 3) projecting square cap.



1)Butt Cap: In the butt cap the end positions of the thick lines are adjusted in such a way that the line segment appears with square ends. The ends of such thick lines are perpendicular to the original line segment.

2) Round Cap:In the round cap, the upper and lower line boundaries are joined by a circular arch with a semi-circle of the diameter of the width of the thick line segment.

3) Projecting square Cap:In the projecting square cap, the portion of line segment is just extended to give a line segment an effect of square.

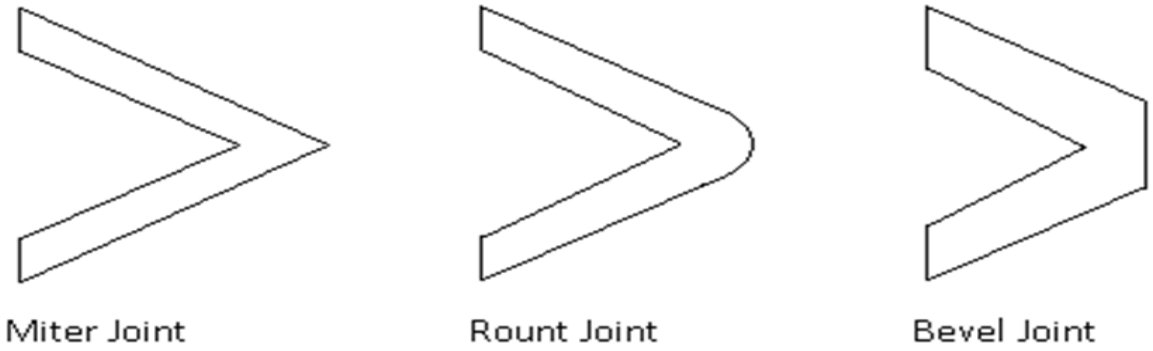
Que-14 : Explain Thick line joints. (Feb-2018, Jan-2017, Feb-2016)

Ans:-There are three types of Thick line joints:

1) Miter Joint

2) Round Joint

3) Bevel Joint.



1) Miter Joint: The miter joint is one in which the exterior boundaries of two thick line segments are extended up to their intersection point – the point where these two line segments meet each other, if they are extended.

2) Round Joint: In the round joint, the exterior boundaries are connected with a circular arch with a diameter of total thickness of two thick lines, which are joined.

3) Bevel Joint: The bevel joint is one where butt cap is used to draw line segment and the little triangle that is created at the joint is filled with color of these two line segments.

Que-15: Expalin Pen Styles.

Ans:- There are many graphics applications, which make use of different pens and brushes of different shapes.

- Different color is also used with different shapes of pens and brushes. Some of the brushes and pens are shown in the following figure.

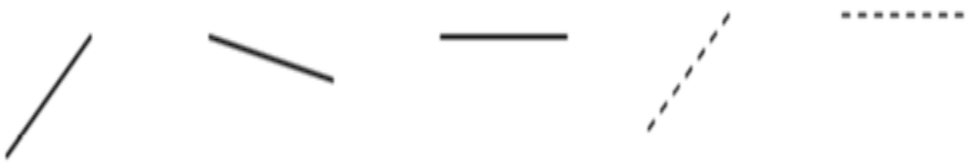


Fig.2.11 – Different Shapes of Brushes

Que-16: Anti-Aliasing of Line.

Ans:-When a line segment of an image is enlarged, one can notice this dimension as Jaggy boundaries of it as shown in figure.

-This type of effect is an error in line segment of image. This error is called aliasing. (jagged line are known as aliasing).

-There are many aliasing error that are observed in practice. Most common aliasing effects are jagged profile, disappeared or improper line details, and disintegrating texture.

-Jagged profile or jagged line is the most probable and known effect.

-The process of removing aliasing error is called anti-aliasing. In other words, the process of making image boundary smoother, is called anti-aliasing process.

Methods of Anti-aliasing: There are Two general approaches:

1) Super-sampling:

-samples at higher resolution, then filters down the resulting image

-Sometimes called post-filtering

-The prevalent form of anti-aliasing in hardware

-It is expensive method.

2) Area sampling:

-sample primitives with a box (or Gaussian, or whatever) rather than spikes

-Requires primitives that have area (lines with width) -Sometimes referred to as pre-filtering

a) Unweighted area sampling:

Fill pixels according to the proportion of their square covered by the line

b) Weighed area sampling:

weight the contribution according to where in the square the primitive falls

Short Questions

Que-1: What is line segment? How to find a slope of line segment?(Feb-2020, Feb-2018)

Ans: -Any two points specified in the Plane will define a line, which means that for defining the line we must have to specify two points.

-y=mx+b is a straight line equation.

slope of a line: It is the rate at which an ordinate of a point on a plane changes with respect to a change in the horizontal coordinate.

-slope is denoted by m.

-slope(m) = vertical distance between two points / Horizontal distance between two points.

-slope (m)= $(y_2-y_1) / (x_2-x_1)$.

Que-2: Explain slope of a line segment.(Jan-2017)

Ans: - slope of a line is the rate at which an ordinate of a point on a plane changes with respect to a change in the horizontal coordinate.

-slope is denoted by m.

-slope(m) = vertical distance between two points / Horizontal distance between two points.

-slope (m)= $(y_2-y_1) / (x_2-x_1)$.

Que-3: Define intercept of line. How to find the intercept of line?(Feb-2019)

Ans: -The intercepts of a graph are points at which the graph crosses the axes.

-The *x*-intercept is the point at which the graph crosses the *x*-axis. At this point, the *y*-coordinate is zero.

-The *y*-intercept is the point at which the graph crosses the *y*-axis. At this point, the *x*-coordinate is zero.

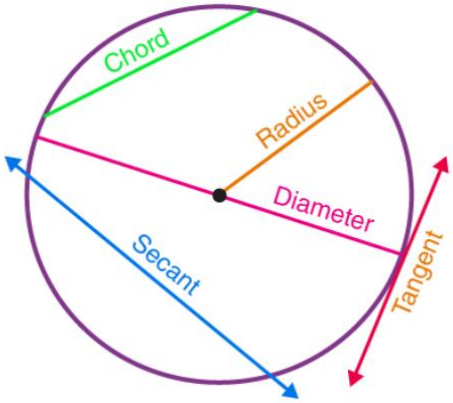
-To determine the *x*-intercept, we set *y* equal to zero and solve for *x*. Similarly, to determine the *y*-intercept, we set *x* equal to zero and solve for *y*.

Que-4: Define Circle and chord of a circle.(Feb-2018)

Ans: -The circle is defined as a set of all points that are placed at a fixed given distance from a given centre point.

-Chord: The straight line that joins any two points in a circle is called a chord

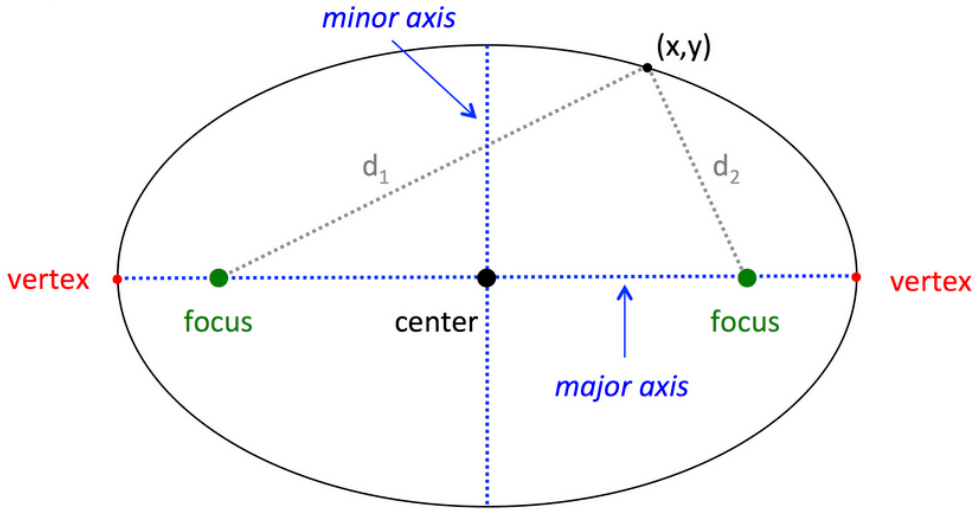
-Equation of Circle, $x^2 + y^2 = radius^2$



Que-5:What is an ellipse?(Feb-2019, Feb-2016)

Ans:- An ellipse is the locus of all those points in a plane such that the sum of their distances from two fixed points in the plane, is constant. The fixed points are known as the foci points (singular focus), which are surrounded by the curve.

- The foci(focus) are two fixed points equidistant from the center of the ellipse.



Que-6:Explain major axis and minor axis of ellipse.(Feb-2019, Jan-2017)

Ans: Ellipse is defined by its two-axis along x and y-axis:

1) Major axis: The major axis is the longest diameter of the ellipse, going through the center from one end to the other, at the broad part of the ellipse. Half of major axis is called semi-major axis

2) Minor Axis: the minor axis is the shortest diameter of ellipse, crossing through the centre at the narrowest part. Half of minor axis is called semi-minor axis.

Que-7:What is aliasing? How will you remove it?(Feb-2020, Jan-2017, Feb-2016)

Ans: - When a line segment of an image is enlarged, one can notice this dimension as Jaggy boundaries of it as shown in figure.

-This type of effect is an error in line segment of image. This error is called aliasing. (jagged line are known as aliasing).

-The process of removing aliasing error is called anti-aliasing. In other words, the process of making image boundary smoother, is called anti-aliasing process.

-Methods of Anti-aliasing:

-There are Two general approaches for anti-aliasing:

1) Super-sampling

2) Area sampling