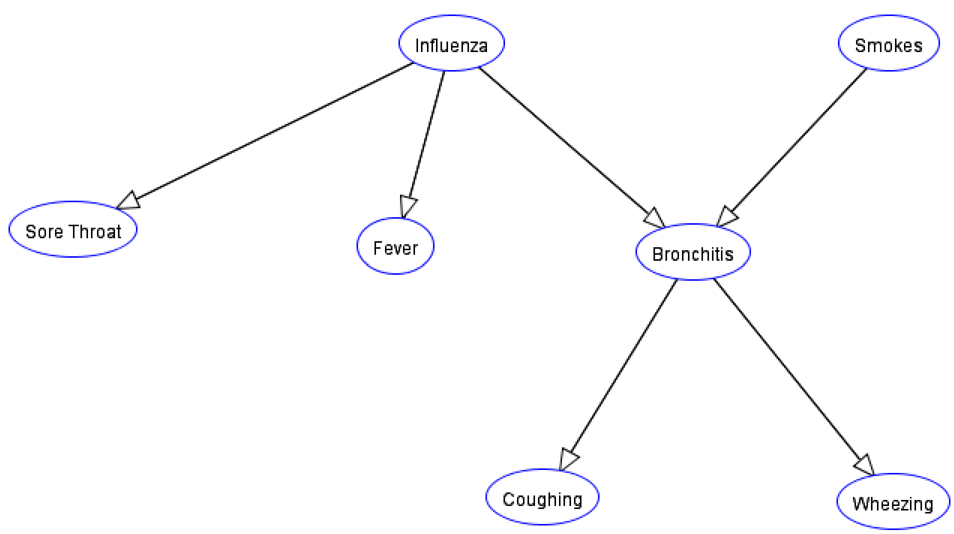
COMS W4701: Artificial Intelligence, Summer 2022

Homework 5

Instructions: Compile all solutions to the written problems on this assignment in a single PDF file. **Show your work by writing down relevant equations or expressions, and/or by explaining any logic that you are using to bypass known equations**. For coding problems, we recommend that you write, label, and comment all of your code in one Jupyter notebook file. Follow the submission instructions to submit all files to Gradescope. Please be mindful of the deadline and our late policy, as well as our course policies on academic honesty.

Problem 1: Bayesian Network (25 points)

The following Bayes net is the "Simple Diagnostic Example" from the Sample Problems of the Belief and Decision Networks tool on AIspace. All variables are binary.



(a) We can describe all guaranteed independences in the Bayes net by defining two or more *subsets* of nodes S_i, such that all nodes in S_i are independent of all nodes in S_j for $i \neq j$. For example, we can define $S_1 = \{\text{Smokes}\}$ and $S_2 = \{\text{Influenza, Sore Throat, Fever}\}$ given no observations. Do the same for *conditionally independent* nodes i) given Influenza, ii) given Bronchitis, and iii) given both Influenza and Bronchitis. Make sure your answers capture **all** guaranteed independences.

(b) Give an analytical expression in terms of the Bayes net CPTs that can be used to compute the conditional distribution Pr(Fever | +bronchitis). Then numerically solve for it using the default parameters in the applet example. You may check your answer using the applet, but you should work it out yourself and show your work.

(c) Consider using likelihood weighting to solve different queries with the indicated evidence: i) Influenza and Smokes are observed, ii) Sore Throat and Fever are observed, and iii) Sore Throat, Fever, Coughing, and Wheezing are observed. For each scenario, how many different weight values are possible among the samples?

(d) We perform Gibbs sampling and would like to resample the Bronchitis variable conditioned on the current sample $(+i, +s, +st, +f, +c, +w)$. Give a **minimal** analytical expression for the sampling distribution Pr(Bronchitis | *sample*) (or its unnormalized form). What is the maximum size of the table that has to be constructed?

(e) Numerically solve for the sampling distribution Pr(Bronchitis | *sample*) using the default parameters in the applet example. You may check your answer using the applet, but you should work it out yourself and show your work.

Supervised Learning

You will be working with the following generic dataset consisting of eight samples for the remaining problems. There are three trinary features x_1, x_2, and x_3, and two classes 0 and 1.

Sample	x_1	x_2	x_3	y
1	$+1$	-1	-1	0
2	$+1$	0	$+1$	0
3	0	0	-1	0
4	0	-1	$+1$	0
5	$+1$	$+1$	$+1$	1
6	0	$+1$	0	1
7	0	0	$+1$	1
8	-1	0	$+1$	1

Problem 2: Naive Bayes (25 points)

(a) Write a small program "training" a naive Bayes model using the provided data. You can simply hardcode the parameters since they can be computed by inspection, but we recommend that you store them neatly and also allow for smoothing to be added in a later part. Which parameters are 0 with no smoothing ($\alpha = 0$)?

(b) Identify **all** combinations of feature inputs for which our model will predict zero likelihood for both classes using the parameters learned above.

(c) Compute the distribution $\Pr(Y|x_1, x_2, x_3)$ for each training sample. Generate two bar charts, one showing $\Pr(+y)$ for each sample (i.e., there should be eight separate bars), and one showing $\Pr(-y)$ for each sample.

(d) Retrain the naive Bayes parameters using Laplace smoothing with $\alpha = 1$. Repeat part (c) with the new smoothed parameters. Briefly compare and contrast your observations about the distributions with and without smoothing.

(e) For either model, on which two training samples would we feel the most **uncertain** about our predictions? Briefly explain, referencing the specific feature combinations of those samples.

Problem 3: Decision Tree (20 points)

Suppose we train a decision tree model using the provided training data.

(a) Show all information gain computations that would be considered when learning the model. You do not have to show all of the numerical calculations, but please show the expressions that would be used. Sketch the resulting decision tree (ties may be broken arbitrarily).

(b) Now consider training a "random" forest of three decision trees, where each tree is trained on all data using the three different subsets of two features, i.e., (x_1, x_2) only, (x_1, x_3) only, and (x_2, x_3) only. Sketch each of the learned trees (no need to show calculations). What is the training accuracy of each tree individually? What is the highest attainable training accuracy of the forest if we classify using a majority vote?

Problem 4: Linear Classifier (15 points)

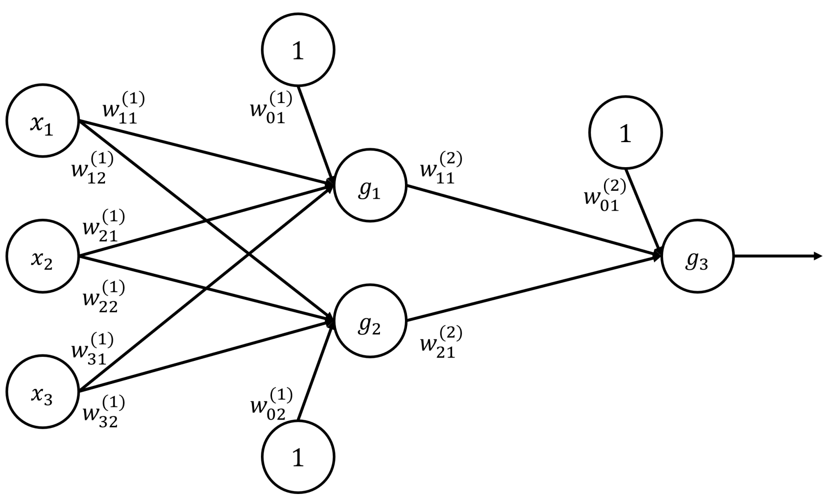
Suppose we train a linear classifier with a hard threshold activation function using the perceptron algorithm on the provided training data. We predict $y = 0$ for $h_{\mathbf{w}} < 0$ and $y = 1$ for $h_{\mathbf{w}} > 0$. We will look at the implication of the tiebreaker decision being assigned to each class.

(a) Write a small program implementing perceptron, starting with weights $\mathbf{w} = (0, 0, 0, 0)$ and using learning rate $\alpha = 1$. Learn two models, one in which the tiebreaker $h_{\mathbf{w}} = 0$ predicts $y = 0$, and one in which it predicts $y = 1$. Track the weights over each iteration until convergence. Generate two plots, one for each model, of the four weight components (you can plot them as four separate lines in one graph). Remember that convergence is achieved only when the classifier is correct on a full pass through the training data.

(b) Experiment with increasing and decreasing the learning rate α for each model. How does α affect the final learned weights and convergence speed?

(c) Compute and plot the sigmoid values of each of the training data using the two learned models (i.e., sample on x-axis and value on y-axis, one plot for each model). Which data predictions do we feel the most uncertain about, and what are their associated probabilities?

Problem 5: Neural Network (15 points)



Consider training a simple neural network model for classification of the training data set. We will be using the network shown above, with a three-unit input layer, two-unit hidden layer, and one-unit output layer. We will initially refer to the activation functions generically as g_1, g_2, and g_3. Each takes in a bias component in addition to the outputs from the previous layer.

(a) In terms of the inputs, weights, and activation functions, write expressions for each of the two hidden layer outputs as well as the output of the output layer.

(b) Suppose that g_1 and g_2 are the ReLU activation functions and g_3 is the sigmoid activation function. All weights in the network are currently equal to 1. What is the result of the forward pass on sample 1?

(c) We use the squared loss function $L(h_{\mathbf{w}}) = \frac{1}{2}(y - \hat{y})^2$. Write analytical expressions for $\frac{\partial L}{\partial w}$, for each of $w = w_{01}^{(1)}$, $w = w_{11}^{(1)}$, $w = w_{01}^{(2)}$, $w = w_{11}^{(2)}$, in terms of the network parameters and outputs (do not leave partial derivatives in your expressions).

Submission

You should have one PDF document containing all solutions, responses, and plots. You should also have a Python file or notebook containing all code that you wrote for each of the problems. Submit the document and code file to the respective assignment bins on Gradescope. For full credit, you must tag your pages for each given problem on the former.