COMS W4701: Artificial Intelligence, Summer 2022

Homework 4

Instructions: Compile all solutions to the written problems on this assignment in a single PDF file. **Show your work by writing down relevant equations or expressions, and/or by explaining any logic that you are using to bypass known equations**. Coding solutions may be directly implemented in the provided Python file(s). **Do not modify the filename or any code outside of the indicated sections.** When ready, follow the submission instructions to submit all files to Gradescope. Please be mindful of the deadline and our late policy, as well as our course policies on academic honesty.

Problem 1: Robot Localization (30 points)

A robot is wandering around a room with some obstacles, labeled as # in the grid below. It can occupy any of the free cells labeled with a letter, but we are uncertain about its true location and thus keep a belief distribution over its current location. At each timestep it moves from its current cell to a neighboring free cell in one of the four cardinal directions with uniform probability; it cannot stay in the same cell. For example, from A the robot can move to either B or C with probability $\frac{1}{2}$, while from D it can move to B, C, E, or F, each with probability $\frac{1}{4}$.

A	B	#
C	D	E
#	F	#

The robot may also make an observation after each transition, returning what it sees in a random cardinal direction. Possibilities include observing #, "wall", or "empty" (for a free cell). For example, in B the robot observes "wall", # (each with probability $\frac{1}{4}$), or "empty" (probability $\frac{1}{2}$).

(a) Suppose the robot wanders around forever without making any observations. What is the stationary distribution π over the robot's predicted location? Hint: You can use `numpy.linalg.eig` in Python. The first return value is a 1D array of eigenvalues; the second return value is a 2D array, where each column is a corresponding eigenvector. Remember that eigenvectors may not sum to 1 by default.

(b) Now suppose that we know that the robot is in state D; i.e., $\Pr(X_0 = D) = 1$. Starting from this state, the robot makes one transition and observes $e_1 = \#$. What is the updated belief distribution $\Pr(X_1 \mid e_1)$?

(c) The robot makes a second transition and observes $e_2 =$ "empty". What is the updated belief distribution $\Pr(X_2 \mid e_1, e_2)$?

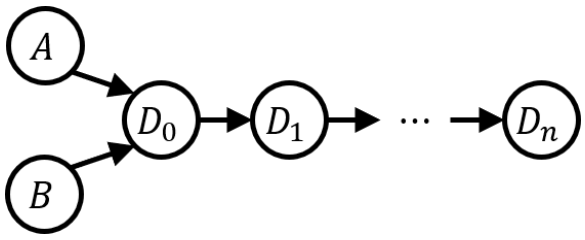
(d) Compute the joint distribution $\Pr(X_1, X_2 \mid e_1, e_2)$. You do not need to explicitly list the values that have probability 0. What is the most likely state sequence(s)?

(e) Compute $\mathbf{b} = \Pr(e_2 \mid X_1)$. Briefly explain what this quantity represents.

(f) Compute the smoothed distribution $\Pr(X_1 \mid e_1, e_2)$ by multiplying $\mathbf{f} = \Pr(X_1 \mid e_1)$ with $\mathbf{b} = \Pr(e_2 \mid X_1)$ and normalizing the result. Confirm that the distribution is the same as that obtained from marginalization of $\Pr(X_1, X_2 \mid e_1, e_2)$.

Problem 2: Descendants of Effects (20 points)

We will investigate the absence of conditional independence guarantees between two random variables when an arbitrary descendant of a common effect is observed. We will consider the simple case of a causal chain of descendants:



Suppose that all random variables are binary. The marginal distributions of A and B are both uniform $(0.5, 0.5)$, and the CPTs of the common effect D_0 and its descendants are as follows:

A	B	$\Pr(+d_0 \mid A, B)$
$+a$	$+b$	1.0
$+a$	$-b$	0.5
$-a$	$+b$	0.5
$-a$	$-b$	0.0

D_{i-1}	$\Pr(+d_i \mid D_{i-1})$
$+d_{i-1}$	1.0
$-d_{i-1}$	0.0

(a) Give an analytical expression for the joint distribution $\Pr(D_0, D_1, \cdots, D_n)$. Your expression should only contain CPTs from the Bayes net parameters. What is the size of the full joint distribution, and how many entries are nonzero?

(b) Suppose we observe $D_n = +d_n$. Numerically compute the CPT $\Pr(+d_n|D_0)$. Please show how you can solve for it using the joint distribution in (a), even if you do not actually use it.

(c) Let's turn our attention to A and B. Give a **minimal** analytical expression for $\Pr(A, B, D_0, +d_n)$. Your expression should only contain CPTs from the Bayes net parameters or the CPT you found in part (b) above.

(d) Lastly, compute $\Pr(A, B \mid +d_n)$. Show that A and B are not independent conditioned on D_n.

Problem 3: Part-of-Speech Tagging

In this problem you will explore part-of-speech (POS) tagging, a standard task in natural language processing. The goal is to identify parts of speech and related labels for each word in a given corpus. Hidden Markov models are well suited for this problem, with parts of speech being hidden states and the words themselves being observations. We will be using data from the English EWT

treebank from Universal Dependencies, which uses 17 POS tags. We are providing clean versions of training and test data for you. The data format is such that each line contains a word and associated tag, and an empty line signifies the end of a sentence. Feel free to open the files in a text editor to get an idea.

The provided Python file contains a couple of functions that can be used to read a data file (you do not need to call these yourself). The global variable `POS` contains a list of all possible parts of speech. You will be filling in the remaining functions in the file and running the code in main where instructed.

3.1: Supervised Learning (12 points)

Your first task is to learn the three sets of HMM parameters from the training data. The initial distribution $\Pr(X_0)$ will be stored in a 1D array of size 17. The transition probabilities will be stored in a 2D array of size 17×17. The observation probabilities will be stored in a dictionary, where each key is a word and the value is a 1D array (size 17) of probabilities $\Pr(\text{word} \mid \text{POS})$. These probabilities should follow the same order as in the `POS` list.

Implement `learn_model` so that it compiles and returns these three structures. The `data` input is a list of sentences, each of which is a list of `(word, POS)` pairs. Your method should iterate over each sentence in the training data, counting the POS appearances in the first word, the number of POS to POS transitions, and the number of POS to word observations. Treat each sentence independently; do not count transitions between different sentences.

Be sure to correctly normalize all of these distributions. Make sure that the quantities $\sum_i \Pr(X_0)_i$, $\sum_i \Pr(X_{t+1} = i \mid X_t = j)$, and $\sum_k \Pr(E_t = k \mid X_t)$ are all equal to 1.

3.2: Viterbi Algorithm (12 points)

The next task is to implement the Viterbi algorithm to predict the most likely sequence of states given a sequence of observations. We will break this into two pieces: `viterbi_forward` and `viterbi_backward`. `viterbi_forward` takes in four parameters: initial distribution `X0` (1D array), transition matrix `Tprobs` (2D array), observation probabilities `Oprobs` (dictionary), and observation sequence `obs` (list of word observations) of length `T`.

`viterbi_forward` should compute and return two quantities: $\max_{x_1,...,x_{T-1}} \Pr(x_1, ..., x_{T-1}, X_T, e_{1:T})$ as a 1D array of size 17, and a `T` $\times 17$ 2D array of pointer indices. Row `i` of the pointer array should contain $\text{argmax}_{x_{i-1}} \Pr(x_1, ..., x_{i-1}, X_i, e_{1:i})$. For simplicity, pointers will be the *indices* of the POS in the `POS` array rather than strings. Note that it is possible for an observation e to not exist in the `Oprobs` dictionary if it was not present when the model was trained. If this occurs, you may simply take $\Pr(e \mid x) = 1$ for all POS x; this is equivalent to skipping the observation step.

`viterbi_backward` takes the two quantities returned by `viterbi_forward` as parameters. It should start with the most likely POS according to $\max_{x_1,...,x_{T-1}} \Pr(x_1, ..., x_{T-1}, X_T, e_{1:T})$ and then follow the pointers backward to reconstruct the entire POS sequence $\text{argmax}_{x_1,...,x_T} \Pr(x_1, ..., x_T \mid e_{1:T})$. The returned object should be a list of POS (strings) from state 1 to state `T`. Note that the predicted state for X_0 should not be included, so your list should be length `T`.

3.3: Model Evaluation (8 points)

Your Viterbi implementation can now be used for prediction. `evaluate_viterbi` takes in the HMM parameters, along with a `data` list in the same format as in `learn_model`. Complete the function so that it runs Viterbi on each sentence separately on the data set. Then compare all returned POS predictions with the true POS in `data`. Compute and return the **accuracy rate** as the proportion of correct predictions (this should be a number between 0 and 1). After implementing this function, run the associated code in main and answer the following questions.

(a) Report the accuracies of your Viterbi implementation on each data set. Why is the accuracy on the test data set lower than that of the training set?

(b) Why can we not expect 100% accuracy on the training set, despite defining the HMM parameters to maximize the likelihood on the training data set?

3.4: Forward and Backward Algorithms (12 points)

Next, implement the the forward, backward, and forward-backward algorithms (ideally fewer than 5 lines of code each). `forward` takes in the same four parameters as `viterbi_forward` above, and it computes and returns $\Pr(X_k, e_{1:k})$ (no normalization). `backward` also takes in four parameters, dropping `X0` but newly including the state index `k`. It should compute and return $\Pr(e_{k+1:T} \mid X_k)$. Note that `k` follows Python indexing; in other words, `k=0` corresponds to X_1 and `backward` should return $\Pr(e_{2:T} \mid X_1)$. Again, to deal with the scenario in which a word is not in the observation model, you should explicitly check when this occurs and if so simply use $\Pr(e \mid x) = 1$.

Once you have both `forward` and `backward`, it should be straightforward to call these to implement `forward_backward`, which computes the smoothed state distribution $\Pr(X_k \mid e_{1:T})$. Note that the call to `forward` should only use the observations up to (and including) the `k`th one. Don't forget to normalize the smoothed distribution.

3.5: Inference Comparisons (6 points)

Now that you have all of these inference algorithms implemented, come up with a short English phrase `p`, ideally around 10 words or fewer. You should then identify a word `w` within this phrase, such that (i) the most likely POS of `w` according to the result of the forward algorithm on the first portion of `p` up to word `w`, and (ii) the most likely POS of `w` according to the result of the forward-backward algorithm on the entirety of `p`, **are different**.

Leave the code that you used to obtain these observations in the main function of the code file. In your writeup, give the phrase that you used and the word that you evaluated. Indicate the POS returned by each of the results as described above. Give a brief explanation about why each of the methods returns something different.

Submission

You should have one PDF document containing your solutions for problems 1-2, as well as your responses to 3.3 and 3.5. You should also have a completed Python file implementing problem 3; make sure that all provided function headers and the filename are unchanged. Submit the document and `.py` code file to the respective assignment bins on Gradescope. For full credit, you must tag your pages for each given problem on the former.