**MASTER OF FINANCIAL ENGINEERING**
**UCLA Anderson School**
**Credit Risk**
**Prof. Holger Kraft**
**Problem Set 2**
**Due: 12 Oct 2012, 8am**

**Problem 3 (Jump Probabilities)**

(a) Consider a standard Poisson process $N_t^c$ with constant intensity $\lambda = 0.01$. Compute the probabilities that $P(N_8^c = n)$ for $n = 0, ..., 5$.

(b) Now, assume a Cox process $N_t^s$ with stochastic intensity that is either modeled by a Vasicek process with dynamics

$$d\lambda_t = \kappa_v(\theta_v - \lambda_t)\,dt + \sigma_v\,dW_t,$$

or by a CIR process with dynamics

$$d\lambda_t = \kappa_c(\theta_c - \lambda_t)\,dt + \sigma_c\sqrt{\lambda_t}\,dW_t.$$

Simulate the paths of both intensities on the interval $[0, 10]$. The parameters are given by $\kappa_v = \kappa_c = 0.8$, $\theta_v = \theta_c = 0.03$, $\sigma_v = 0.02$, and $\sigma_c = 0.15$. Use $\lambda_0 = 0.01$ as initial value.

(c) What are the unconditional jump probabilities for a Cox process? Write down the formula given in the lecture.

(d) Use this general formula to simulate the probabilities $P(N_1^s = 2)$ for both intensity models via Monte Carlo simulation. Use the parameters from part (b).

**Problem 4 (Bond Pricing)** Unless otherwise stated, use the parameters from Problem 3 part (b). All calculations have to be done for a CIR model.

(a) Consider a Cox process with stochastic intensity $\lambda$. For affine models, it is known that

$$\mathrm{E}[e^{-\int_0^t \lambda_u\,du}] = e^{A(t)-B(t)\lambda_0}.$$

Implement the functions $A(t)$ and $B(t)$.

(b) Calculate the spreads of zero coupon bonds for maturities $T = 1, ..., 10$. Assume recovery of par with $R = 0.5$ and a risk-free interest rate of $r = 0.05$.

(c) What do you observe when $T \to 0$? Explain.

(d) Compute the fair prices of defaultable coupon bonds with recovery of par and $R = 0.5$ for maturities $T = 1, ..., 10$. Assume that coupons of $c = 0.03$ are payed quarterly. Please disregard the accrued payments.

(e) How do the prices change when you include the accrued payments? (optional question)