Study of lunar surface through analysis of LRO data with Hapke model

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We have analysed the LRO LAMP data for the wavelength range of 130-140 nm with Hapke model. In this report we present the fit method of the Hapke model for the LRO data. The standard Hapke function which relates the reflectance to the incidence, emission and phase angle has been modified with certain approximation. This modification has allowed us to fit only two parameters, single scattering albedo(w) and the asymmetric factor(b). We have found asymmetric factor (b) value to be close to the earlier studies.

1 Instroduction

The Moon has always fascinated scientists and astro-enthusiasts. It being the closest astronomical body from the earth has always gained much attention than other astronomical objects in terms of exploration. With the development of modern technologies in terms of imaging and remote sensing, many new discoveries are made. The data collected from this sensors help us understand what the Moon is made of and what its surface is like on a very tiny scale.

At the forefront of this endeavor is the Lunar Reconnaissance Orbiter (LRO) Lyman Alpha Mapping Project (LAMP) instrument. This instrument have a wide range of data collection capability with major focus on the ultraviolet(UV) and far ultraviolet(FUV) wavelength.

This instrument facilitates the analysis of lunar surface composition and also unveils the subtle effects of space weathering – a phenomenon crucial for understanding the dynamic interactions between the lunar surface and its cosmic environment. This instrument can go to the level of micro surface properties enabling the researchers for detailed exploration of the Moon surface.

Moreover, the comprehensive nature of LAMP data presents a unique opportunity to probe into the hydration of the lunar surface. By examining subtle spectral signatures indicative of water molecules, researchers can unravel the mysteries surrounding lunar water abundance and distribution.

In this report we present the study on the LRO data for the wavelength band of 130-140 nm. The aim was to fit the reduced reflectance with an empirical model called Hapke Model which relates the reflectance with incidence angle of light from the source(sun or star), emission angle at which it reaches the satellite(LRO) and phase angle (sum of incidence and emission angles). Our goal was to find two quantities with the Hapke Model namely, *average single-scattering albedo* and *asymmetric factor*. This values are important for further radiative correction to study the surface of the Moon.

2 Hapke Model

The reflectsnce relation given by Hapke is the following[1, 2, 3, 4],

$$r = K\frac{w}{4\pi}\frac{\mu_{0e}}{\mu_{0e}+\mu_e}[\{1+B_{s0}(g)B_s(g)\}p(g)+H(\mu_{0e})H(\mu_e)-1][1+B_{c0}B_c(g)]S(i_e,e_e,\phi) \tag{1}$$

where K is porosity factor, w is the single-scattering albedo, μ_{0e} is effective cosine incident angle, μ_e is effective cosine emission angles, B_{S0} is amplitude of Shadow Hiding Opposition Effect (SHOE), $B_S(g)$ is SHOE function, $p(g)$ is phase function, H is multiple scattering function, B_{C0} is amplitude of Coherent Back scatter Opposition Effect (CBOE), $B_C(g)$ is CBOE function, and $S(i_e, e_e, \phi)$ is shadowing function.

These parameters refer to how light scatters off the particles in the lunar soil and the overall structure of the soil itself, including details like the size of the particles, how empty space is distributed within it, and how rough its surface appears when viewed up close.

Some simplification to this model can be made as following. Remote sensing cannot accurately determine the porosity factor, K, although recent studies indicate that it's approximately 1.3 for the top layer of lunar soil. In this study, we assume K to be 1 to maintain consistency with earlier photo metric research.

The roughness shadowing function S which is dependent on $\bar{\theta}_p$, a correction factor to the Bidirectional Reflectance Diffusion Function(BRDF) of a smooth surface, where $\bar{\theta}_p$ is defined as the mean subpixel slope angle with which photo metric roughness is represented. In our case the satellite is *nadir looking* and thus the values of phase angle is about 25∼60 degrees. It is known from the previous studies that the value of $\bar{\theta}_p$ has maximum effect when phase angle is at the terminator. Thus, with these approximations we treat the surface of the sun to be horizontal and $S = 1$.

Following the same assumption as above we also approximate the $\mu_{0e} = \mu_0$ and $\mu_e = \mu$. This will have some bias on our fitting parameters but for now we continue with this approximations.

The opposition effect (OE) have contributions from both $CBOE$ and $SHOE$, which are dominant only when the phase angle is small. In the previous studies the OE has been set to 0 for fitting single-scattering albedo w and asymmetric factor b.

With all this approximations we have the modified bidirectional reflectance relation,

$$r = \frac{w}{4\pi}\frac{\mu_0}{\mu_0+\mu}[p(g)+H(\mu_0)H(\mu)-1] \tag{2}$$

The bidirectional reflectance is divided by π and the Lommel-Seeliger function, LS, which serves as a standard component in the radiative transfer equation for theoretical photo metric functions of particulate media.

$$LS = \frac{\mu_0}{\mu_0+\mu} \tag{3}$$

Which gives us the reduced reflantance relation,

$$r_{reduced} = \frac{w}{4}[p(g) + H(\mu_0)H_\mu - 1] \tag{4}$$

Where $p(g)$ is the phase function and g is the phase angle and μ_0 is the cosine of the incident angle and μ is the cosine of the emission angle. The $p(g)$ and $H(\mu_0)$ and $H(\mu)$ is given by,

$$p(g) = \frac{1 - b^2}{(1 + 2b\cos(g) + b^2)^{3/2}} \tag{5}$$

Where the only parameter to be found is b and the the function H is given by,

$$H(x) = \frac{1 + 2x}{1 + 2x\sqrt{1 - w}} \tag{6}$$

Where x can be μ_0 and μ. This gives the final form of the empirical relation that we want to fit with our data.

3 Data processing

The satellite data provides us information about the weighted photon count, the latitude and longitude covered for that weighted photon count. We find the area from the latitude and longitude and the solid angle as well. The weighted photon count is divided by the area and the solid angle and the exposure time as well to get the value of the *brightness*. The data we have from the lunar surface is form the day time. We find the solar irradiance from another satellite SORLAR SOLSTICE. The ratio of the brightness and the irradiance gives us the reflectance $I/F(r)$. The $I/F(r)$ is further divide by π and LS function to get the reduced reflectance. The phase angle is taken as the sum of incidence and emission angle which are in degrees. Finally we have binned the data with 25 to 90 degrees with step of 1 for incidence, emission and phase angle. The value of reduced reflectance for a particular bin of all the 3 angles we look into central tendencies. The first choice is mean and we have also taken median.

The wavelength that we are considering for the study is 130-140 nm. The value of reflectance can not be greater than 1. There are lot of noises with the data close to one. We procede with noise reduction to get the final data set which is then binned and fitted. The data analysis is done with Pandas library of python [5]. The data is open source for this project.

3.1 Fitting Method

The algorithm which we have used for fitting the data is Levenberg-Marquardt[6] from the `scipy` library of Python. This is a very popular optimization function in `scipy`[7]. The overview of the algorithm is as follows,

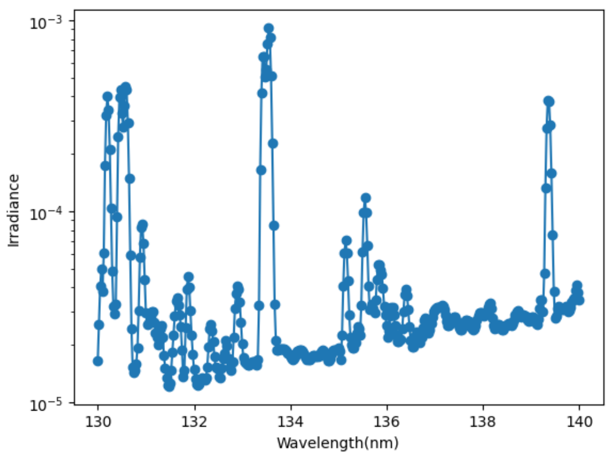
1. **Initialization**: The algorithm starts with an initial guess for the parameters that define the model. In our case we have guessed it with plotting the function and finding the values which are close to the values visually.

2. **Objective Function**: With this initial guesses a function value is found which gives the difference between the predicted values from the fitting function and the observed data. This is typically the squares of the residuals.

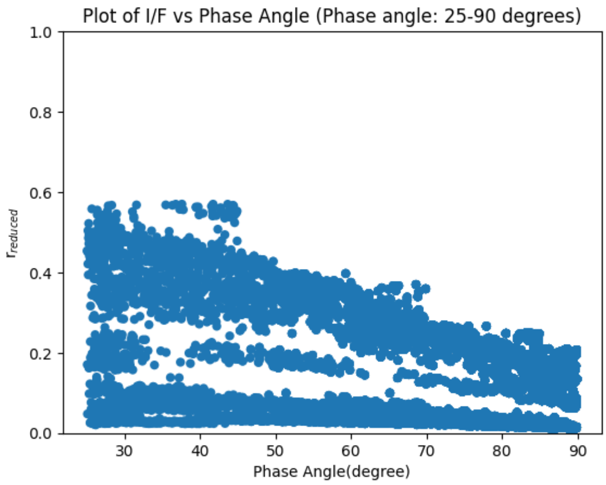
3. **Jacobian Matrix**: The algorithm computes the Jacobian matrix, which represents the partial derivatives of the objective function with respect to each parameter. The Jacobian matrix provides information about the sensitivity of the objective function to changes in the parameters.
4. **Update Step**: The algorithm calculates an update step for the parameters using a combination of the Gauss-Newton method and a damping factor with gradient descent.
5. **Damping Factor**: The damping factor, often denoted as lambda (λ), controls the trade-off between the Gauss-Newton step and the gradient descent step. It is adjusted dynamically during the optimization process to ensure stability and convergence.
6. **Step Acceptance**: The algorithm evaluates the proposed parameter update and decides whether to accept it based on the improvement in the objective function. If the proposed update improves the fit, it is accepted; otherwise, the damping factor is adjusted, and a new update step is computed.
7. **Convergence Criteria**: The algorithm terminates when one of the following convergence criteria is met: the change in the objective function is small enough, the change in the parameters is small enough, or the maximum number of iterations is reached.
8. **Output**: Once convergence is achieved, the algorithm returns the estimated parameters that best fit the model to the data.

4 Results

The solar irradiance for the wavelength range of our interest is given in the figure-1a. The observed data is shown in the figure-1b.

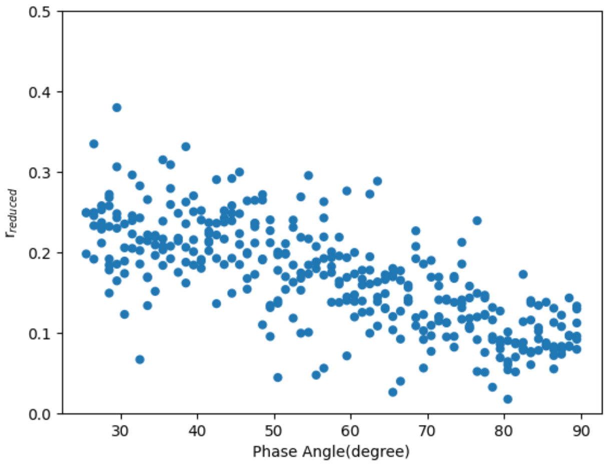


(a) Solar irradiance plot over the whole wavelength range.

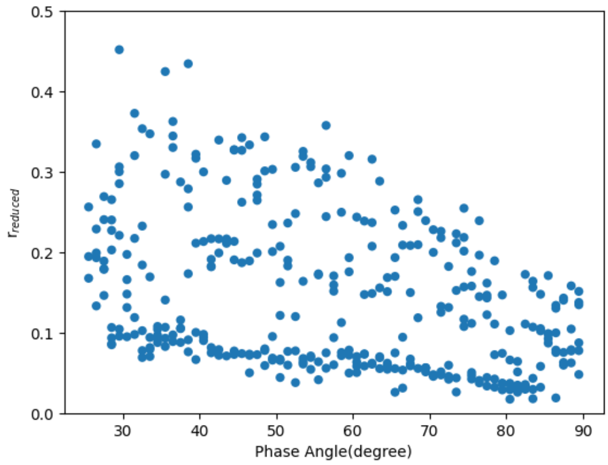


(b) Observed data of reduced reflectance before binning.

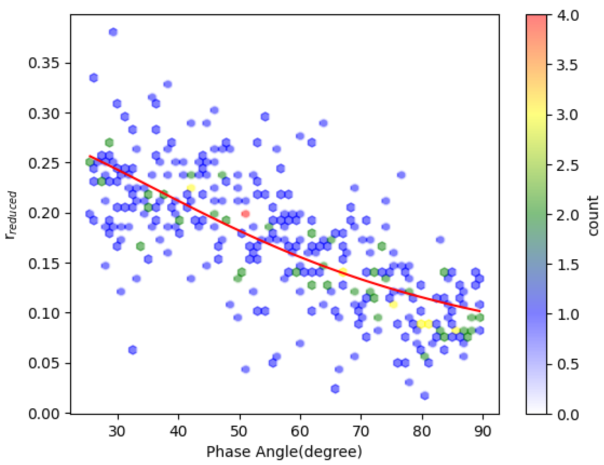
We have two choices to bin the reduced reflectance value, one is with taking their mean for the binned reflectance and the other is median.



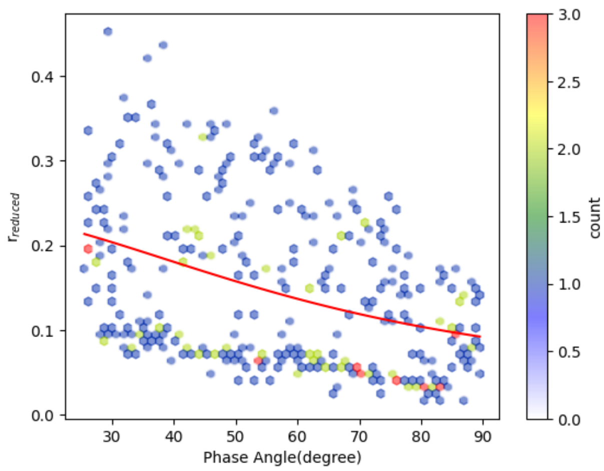
(a) Red. reflectance binned with mean value.



(b) Red. reflectance binned with median value.



(a) Red. reflectance binned with mean value.



(b) Red. reflectance binned with median value.

In the figure-2a the binned data with mean of reduced reflenctance is shown. In figure-2b, the binned data for the reduced reflectance is shown median. We find that the mean values are very close with respect to the different incidence and emission angle. However the value for the median case are very much distributed with respect to the incidence and emission angle. The figure-3a and figure-3b shows the fitted curve for the variation of phase angle with reduced reflectance. The fit parameters are listed below. For the reduced reflectance which is binned as mean we have,

Asymmetry Factor(b)= -0.290 $\pm$ 0.011
Average Single-Scattering Albedo(w)= 0.249 $\pm$ 0.009

Similarly the case when median is taken in to consideration we have,

Asymmetry Factor(b)= -0.267 $\pm$ 0.025
Average Single-Scattering Albedo(w)= 0.232 $\pm$ 0.019

The fitting is in four dimension, with three variables and r data to fit. For the incidence, emission and the phase angle the mid position of the bin is considered. The fitted line is with the constant emission and incidence angle and varying phase angle. Thus, the fitted line doesn't exactly illustrate the fitted curve which will be curve in 4D. The error in the values are the standard deviation of the parameters obtained from the diagonal of the covarinace matrix.

5 Summary and conclusion and outlook

In this we have found the reflectance fitting parameters for the Hapke Model with LRO LAMP data. We have considered two types of central tendencies for estimating the binned values of reduced reflectance. When compared with the particular result of the Moon surface which has been reported in a paper, we found that the asymmetric factor (b) value is close to the value which has been found in previous studies [1]. It is to be noted that the literature dealt with a particular area of the Moon called highland mare with this wavelength. Whereas we have covered the whole latitude -75 to 75 degrees but the longitude is for a small region. There is a large deviation in the single scattering albedo value. It must be looked into with analysing more data and further looking at the steps that has been implemented.

The outlook and future direction of the study are following:

- Study to be performed with more data set.
- One can propose some other modified form of the function which might fit the data well.
- Further possible correction to the data can be made for better results.
- If the function can be modified and more data is aggregated one can look to find the more fitting parameters defined in the new modified model equation to better understand the surface features.

References

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