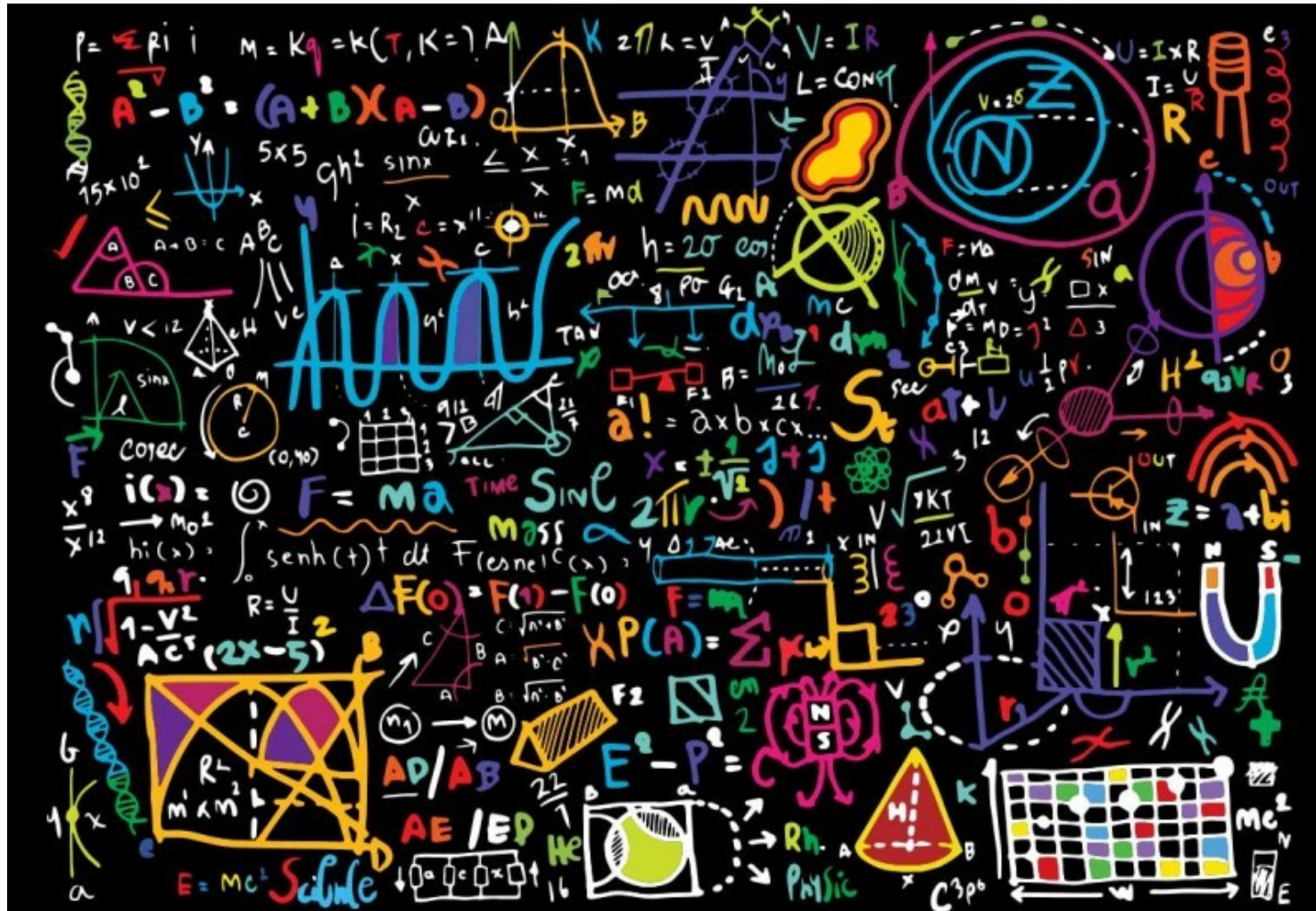
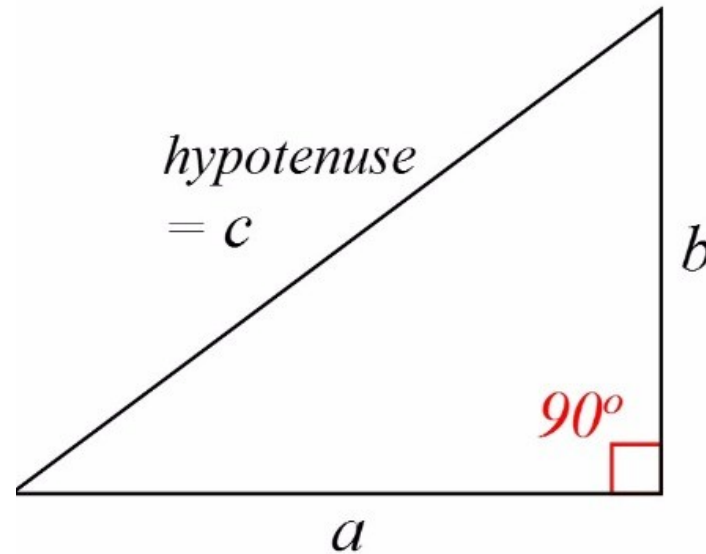


# Maths and Physics



# Quick review exercises

# Pythagoras' theorem

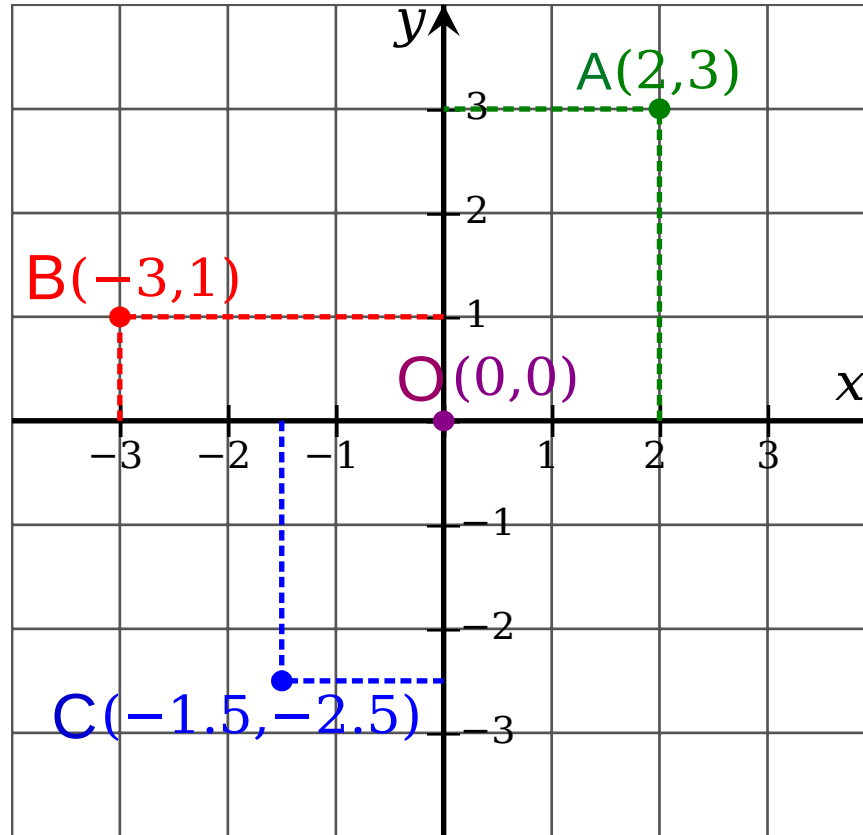


$$c^2 = a^2 + b^2$$

The square on the hypotenuse is equal to the sums of the squares on the other two sides.



# Coordinate system & vectors

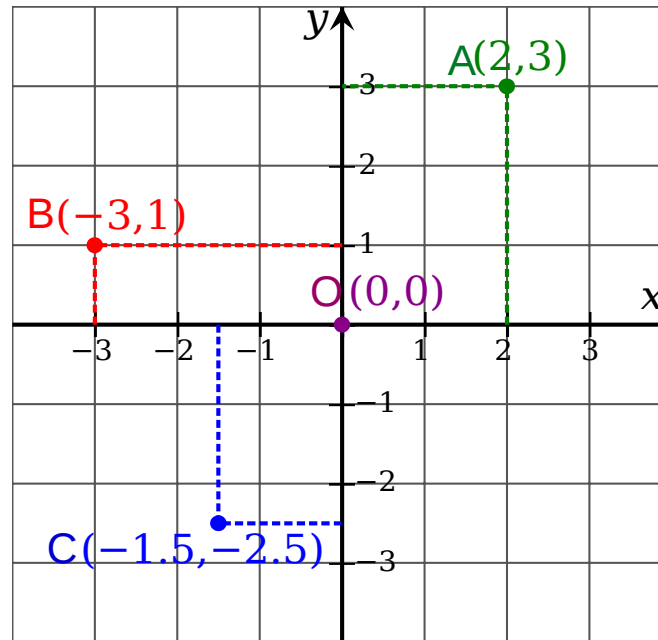


$$\vec{AO} \quad \vec{BC} \quad \vec{AC}$$

Compute for each of those vectors:

- the length of each vector.
- a unit vector in the same direction
- a perpendicular unit vector (norm)

# Coordinate system & vectors 2



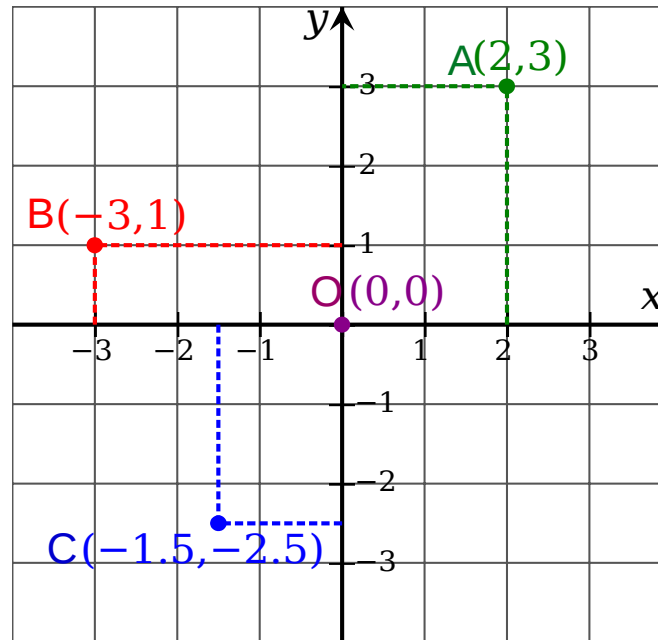
Compute :

$$\vec{AC} - \vec{AO}$$

$$\vec{AC} \cdot \vec{AO}$$

$$\vec{BO} \times \vec{AO}$$

# Coordinate system & vectors 3

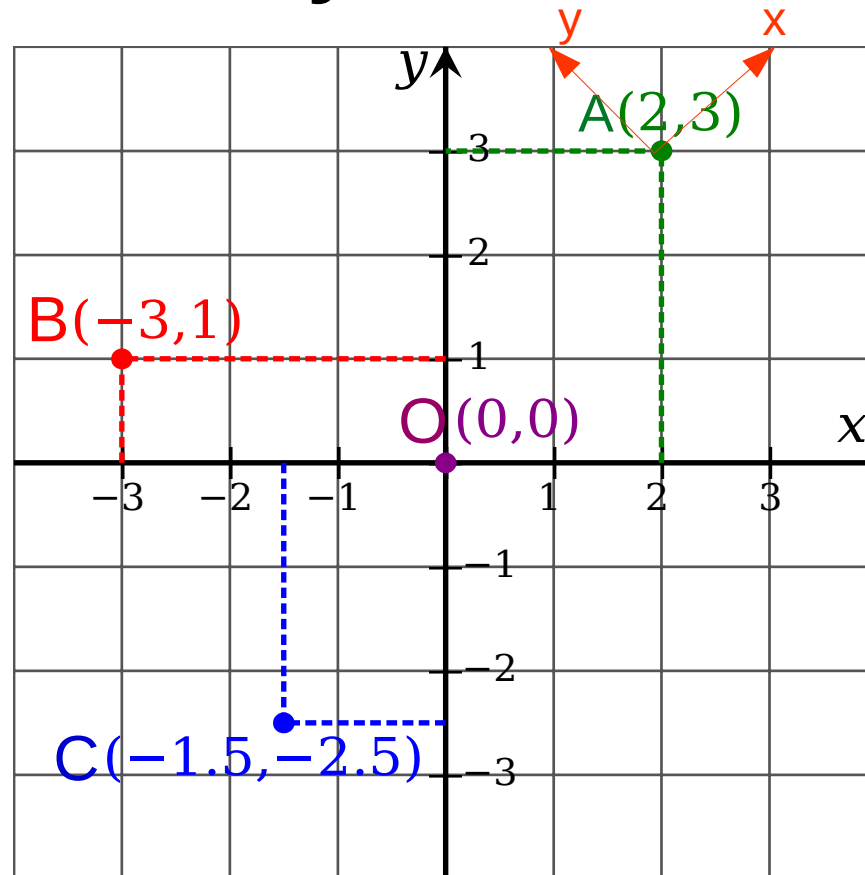


Compute the angle between:

$\vec{AC}$  and  $\vec{AO}$

$\vec{BO}$  and  $\vec{AC}$

# Coordinate system & vectors 4



Compute coordinates of  $A$ ,  $B$ ,  $C$ ,  $O$  in the orange coordinate frame

# Matrices

Compute the following multiplication:

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & 0.5 & 0 \\ 3 & 2 & -2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

# Matrices

Write the following equation in a system of equations and solve it.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ -1 \end{bmatrix}$$



# Derivatives

What is the derivative of function  $f(t)$  with respect to  $t$ ?

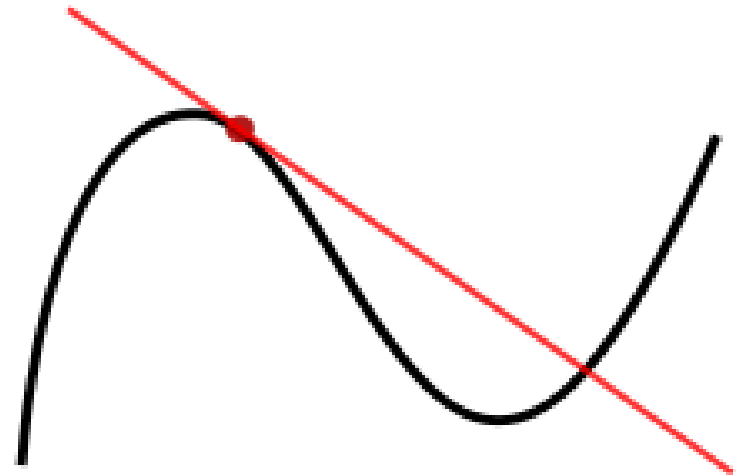
Compute  $f'(t)$

1)  $f(t) = 3t$

2)  $f(t) = t^2$

3)  $f(t) = \sin(t)$

4)  $f(t) = 3t + 5t^2 + 6\sin(t)$



# Integrals

What is the integral of function  $f(t)$  with respect to  $t$ ?

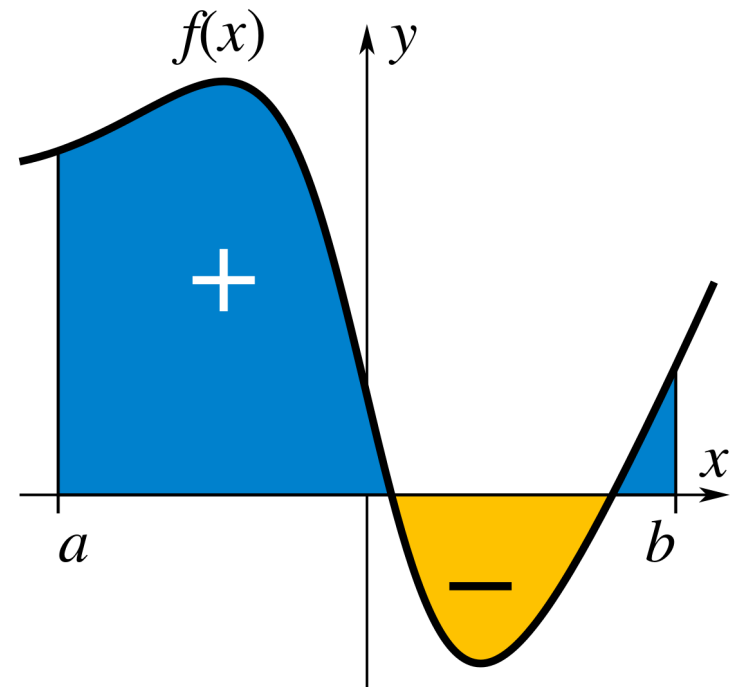
Compute  $\int_0^T f(t) dt$

1)  $f(t) = 3t$

2)  $f(t) = t^2$

3)  $f(t) = \sin(t)$

4)  $f(t) = 3t + 5t^2 + 6\sin(t)$



# Linear dynamics

What is the relation between position  $x(t)$ , linear velocity  $v(t)$  and linear acceleration  $a(t)$ ?

How much time does a rock need to hit the ground of a well 50 m deep? Reminder: gravity accelerates objects at  $9.81 \text{ m/s}^2$ .



# Angular dynamics

What is the relation between orientation  $\theta(t)$ , angular velocity  $\omega(t)$  and angular acceleration  $\alpha(t)$ ?

How much time does a carousel need to make a full rotation if it turns at  $\omega(t) = 15^\circ/\text{s}$  ?

What if  $\omega(t) = -3\pi/4 \text{ rad/s}$  ?



# Probabilities

What does  $p(x) = 1$  mean?

What is  $p(head)$  in a head or tail throw?

What does this mean?

$$p(cancer \mid \text{smoking cigarette}) > p(cancer)$$

If  $p(y) = 0.3$  and  $p(x \mid y) = 0.4$ , what is  $p(x, y)$ ?

If  $p(x) = 0.1$ , what is  $p(y \mid x)$  ?

# Probabilities

$$E[X] = \sum_{x \in X} x p(x)$$

What is the expected value of a coin toss, if *tails* = 0 and *heads* = 1?

What if *tails* = 12 and *heads* = 16?

What about the value of a thrown dice?

What is the expected value of a random number taken from 0 to 5?

# Probabilities

$$E[X] = \sum_{x \in X} x p(x)$$

Law of large numbers:

If  $\overline{X}_n = \frac{1}{n} (X_1 + X_2 + \dots + X_n)$  is the average of the samples,

$$\overline{X}_n \rightarrow E[X] \quad \text{when} \quad n \rightarrow \infty$$

# Probabilities

Exercise that we did :

$$L * l = n$$

What did you notice?

the *higher* the  $n$ , the *closer* the average was to exactly 2.5

Experiment:

Run the code 1 000 000 times

For 5 categories of  $n$ , compute the average difference  $\Delta$ :  $|2.5 - x|$ :

$n < 4$	$\Delta = 0.85$
$4 < n < 10$	$\Delta = 0.47$
$10 < n < 25$	$\Delta = 0.29$
$25 < n < 50$	$\Delta = 0.19$
$50 < n < 100$	$\Delta = 0.14$



# Probabilities

Experiment:

Run the code 1 000 000 times

For 5 categories of  $n$ , compute the average difference  $\Delta$ :  $|2.5 - A|$ :

$n < 4$	$\Delta = 0.85$
$4 < n < 10$	$\Delta = 0.47$
$10 < n < 25$	$\Delta = 0.29$
$25 < n < 50$	$\Delta = 0.19$
$50 < n < 100$	$\Delta = 0.14$

The larger the  $n$ , the lower the difference of  $A$  from  $E[X]$ !