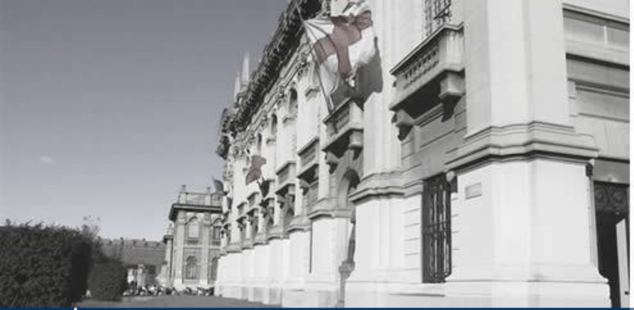

POLITECNICO DI MILANO

Mechanical System Dynamics

– Experimental modal analysis (Part 1)

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Basic concepts

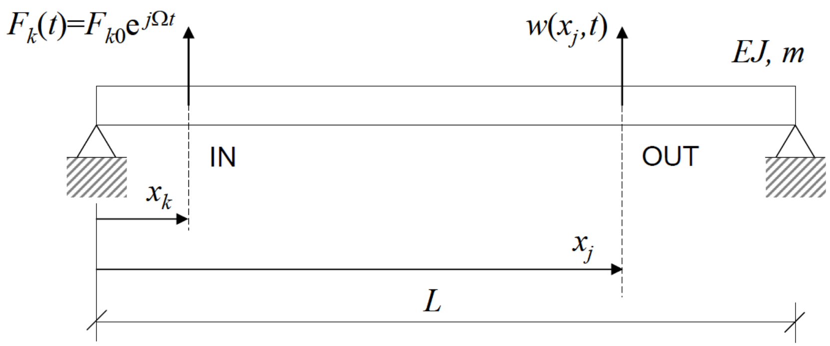
Modal analysis is the study of the natural characteristics (dynamic properties: the frequency, damping and mode shapes) of structures.

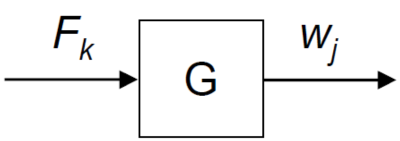
Understanding both the natural frequency and mode shape helps to design the structural systems for noise and vibration applications:

- characterisation of the dynamic properties of a mechanical system
- model validation and model updating
- vibroacoustic analysis
- structural health monitoring
- ……

Basic concepts

Frequency Response Function (FRF): an FRF (defined in frequency domain) is a measure of how much displacement, velocity, or acceleration response a structure has at an output DOF, per unit of excitation force at an input DOF.





$$G(s)=\frac{w_j(s)}{F_k(s)}$$

While the poles of the TF $G(s)$ do not depend on the choice of the positions x_k and x_j (since poles are related to the beam natural frequencies and vibration modes), the zeroes of $G(s)$ vary with these positions.

Reciprocity: $G_{jk} = G_{kj}$ FRF matrix is symmetric due to the fact that the mass, damping and stiffness matrices that describe the system are symmetric.

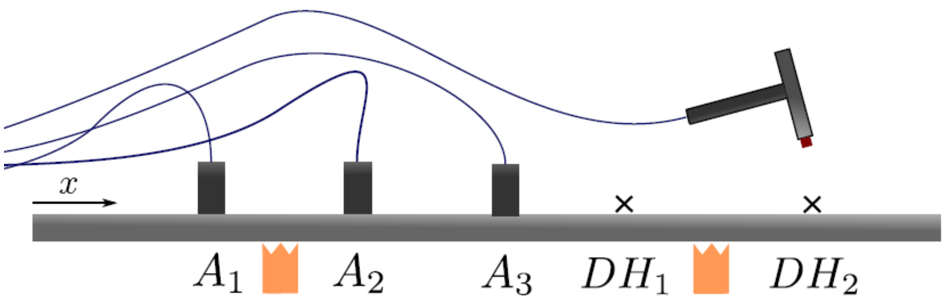
Objectives

Identify the modal parameters

(extract the information from measured FRF)

1) natural frequency
2) modal damping ratio
3) mode shape

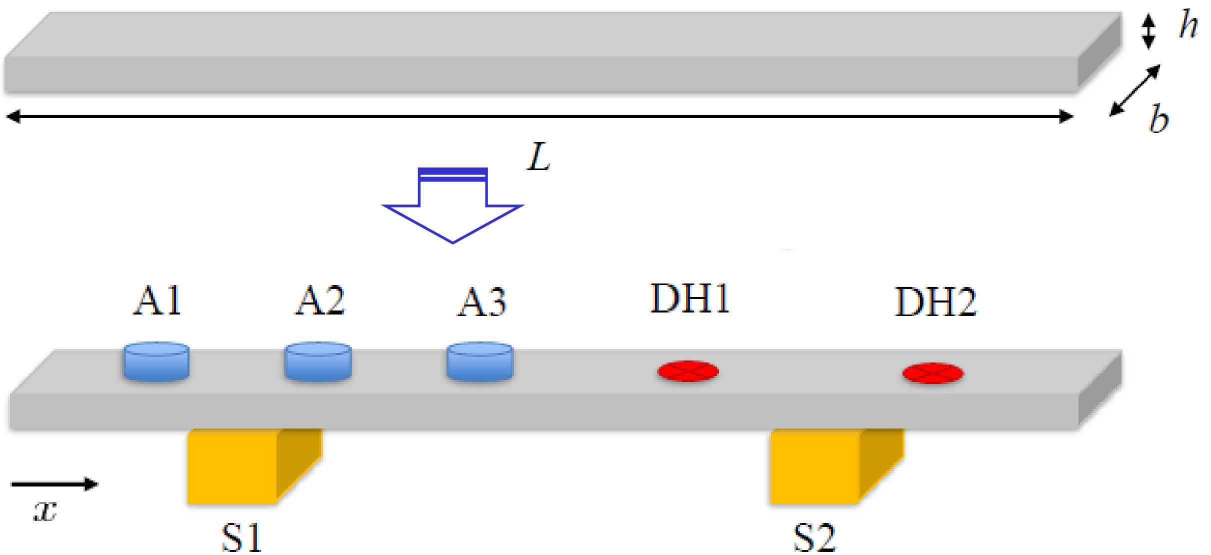
of a free-free beam by means of impact hammer test.



Experimental test set-up

Test object: an aluminium beam with the following parameters

Parameter	symbol	unit	value
Lenght	L	mm	1200
Thickness	h	mm	8
Width	b	mm	40
Density	ρ	kg/m^3	2700
Young's Modulus	E	GPa	68

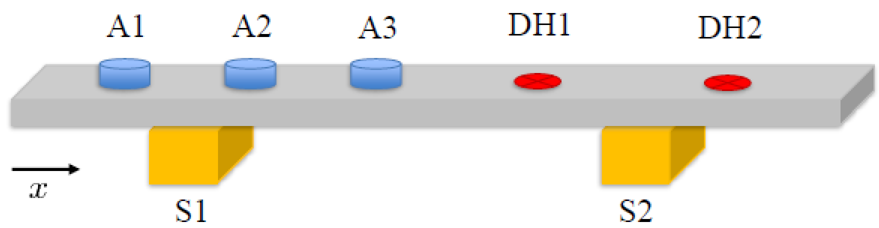


flexible supports ≈ free-free BC

Experimental test set-up

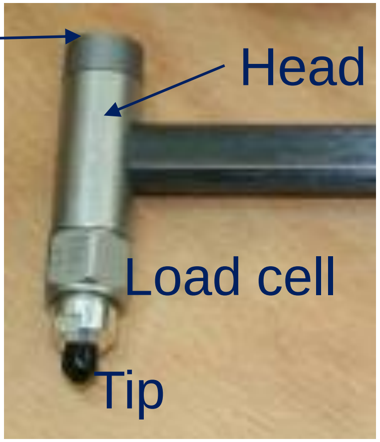
Excitation devices: dynamometric hammer

Parameter	symbol	x [mm]	Transducer	Sensitivity
Dynamometric Hammer	DH1	815	Piezo	2.17 mV/N
Dynamometric Hammer	DH2	1065	Piezo	2.17 mV/N



Experimental test set-up

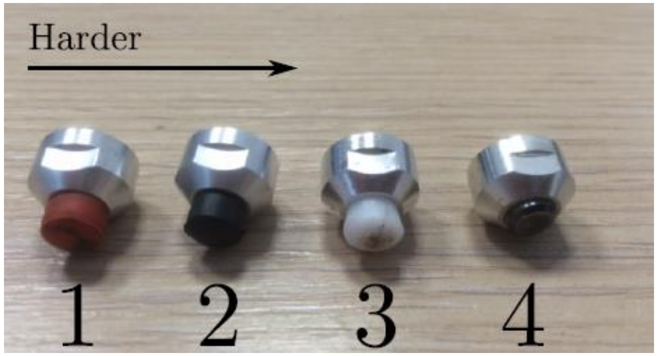
Excitation devices: dynamometric hammer



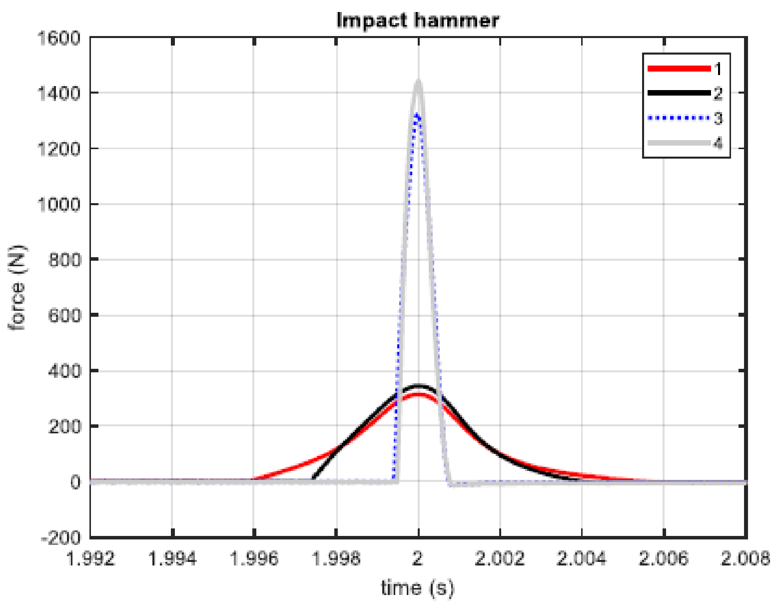
Since the force is an **impulse**, the amplitude level of the energy applied to the structure is a function of the **mass** and the **velocity** of the hammer. This is due to the concept of linear momentum. It is difficult though to control the velocity of the hammer, so **the force level is usually controlled by varying the mass**.

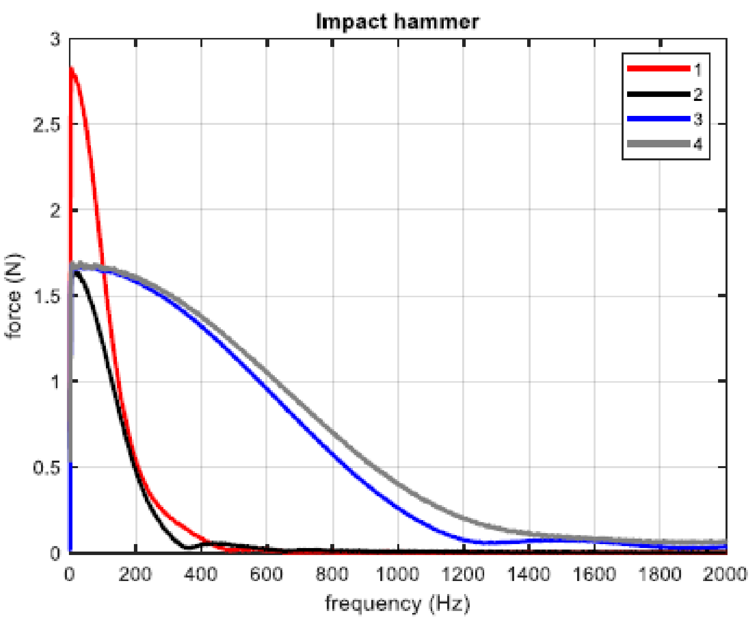
Experimental test set-up

The frequency content of the energy applied to the structure is a function of the stiffness of the contacting surfaces and, to a lesser extent, the mass of the hammer. It is not feasible to change the stiffness of the test object, therefore **the frequency content is controlled by varying the stiffness of the hammer tip**.



Tip no.2 (intermediate hardness) is selected for this experiment

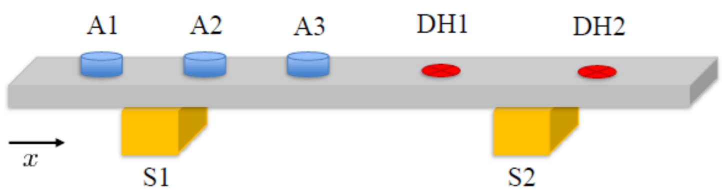


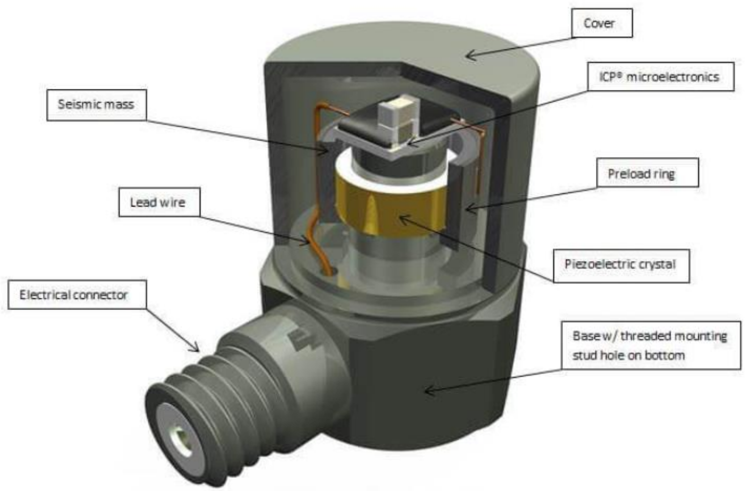


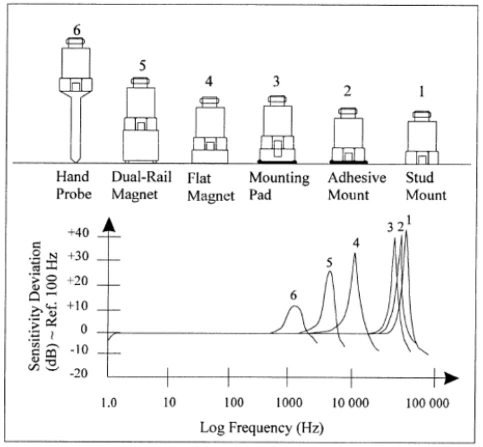
Experimental test set-up

Excitation devices: accelerometer

Parameter	symbol	x [mm]	Transducer	Sensitivity
Accelerometer	A1	105	Piezo	100 mV/g
Accelerometer	A2	415	Piezo	100 mV/g
Accelerometer	A3	600	Piezo	100 mV/g



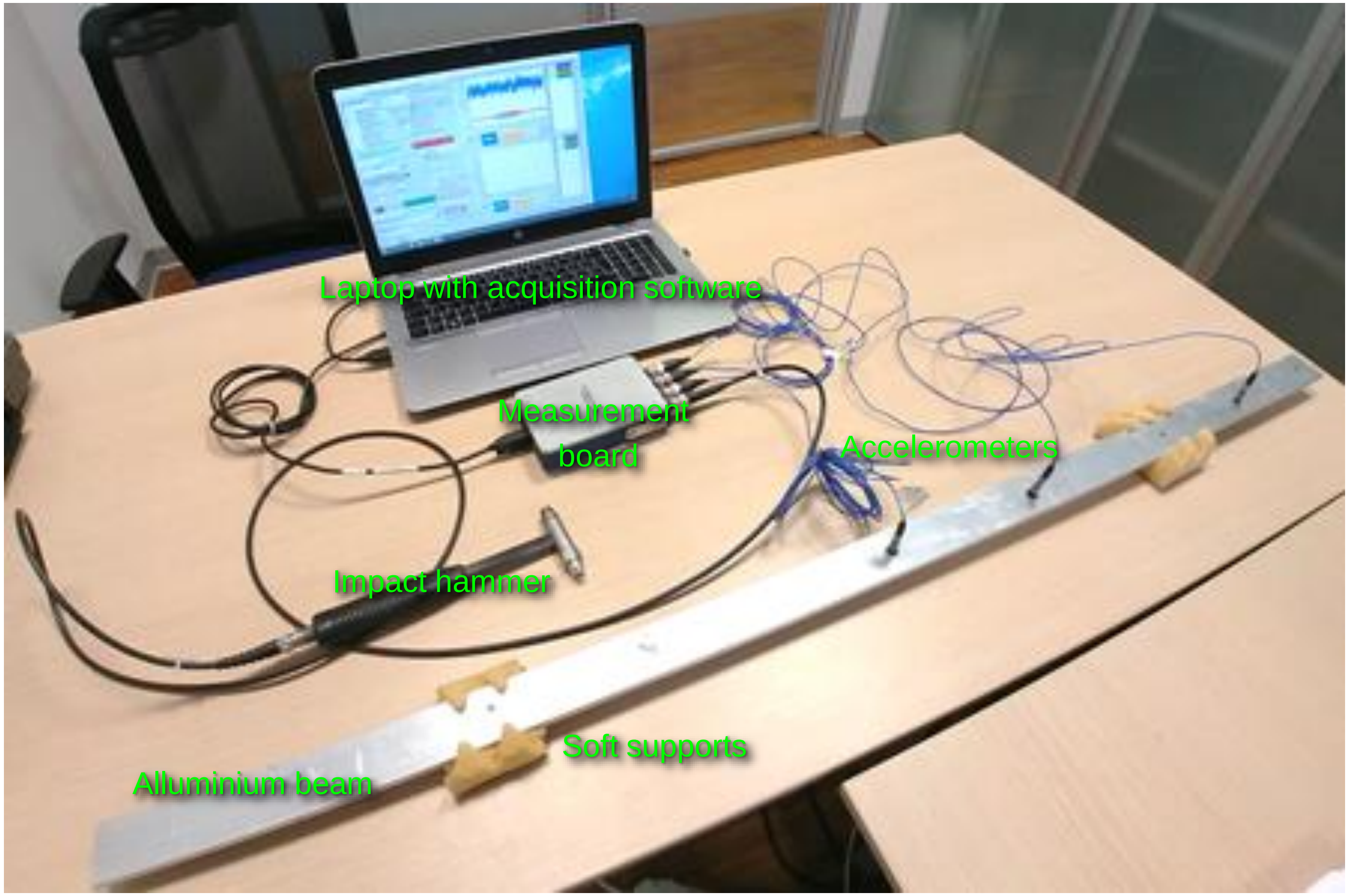


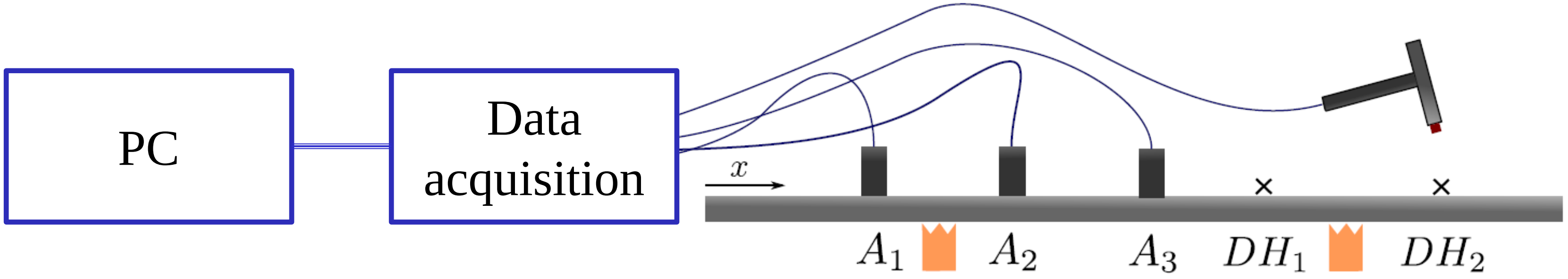


Performance	ENGLISH	SI
Sensitivity(± 10 %)	100 mV/g	10.2 mV/(m/s²)
Measurement Range	± 50 g pk	± 491 m/s² pk

Physical	
Sensing Element	Ceramic
Sensing Geometry	Shear
Housing Material	Titanium
Sealing	Welded Hermetic
Size (Hex x Height)	9/32 in x 18.5 mm
Weight	2.0 gm
Electrical Connector	10-32 Coaxial Jack
Electrical Connection Position	Top
Mounting Thread	5-40 Male
Mounting Torque	90 to 135 N-cm

Experimental test set-up





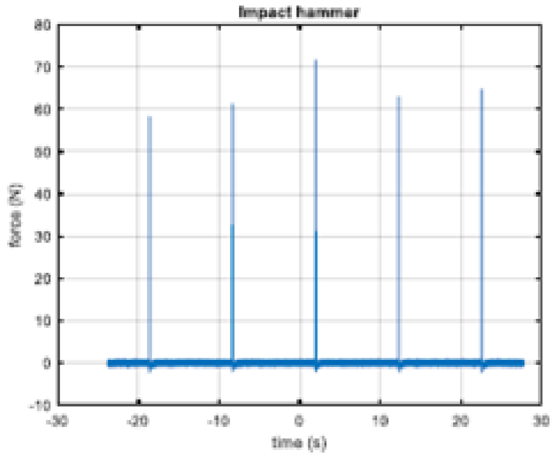
Data processing

Impact testing has two potential signal processing problems associated with it.

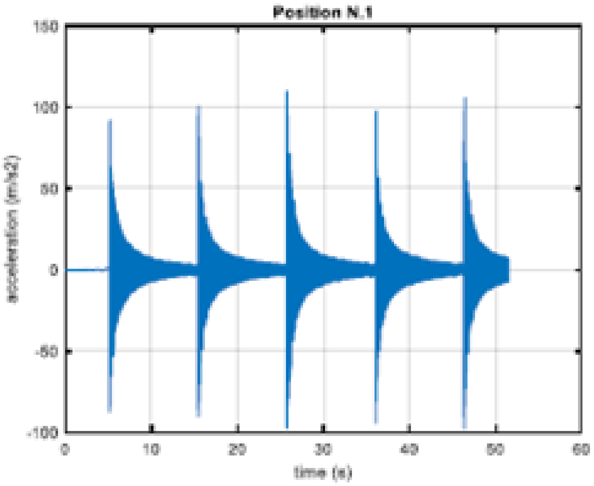
- **noise** – can be present in either the force or response signal as a result of a long time record.
- **leakage** – can be present in the response signal as a result of a short time record.

Compensation for both these problems can be accomplished with windowing techniques.

Input force



Output acceleration



Data processing

1) Measurements are performed so as to collect a data set of N pairs of sampled time histories for the input force F_k and the output vibration x_j (the length of each of the 2N time histories is indicated with T_0). This requires N test repetitions.
2) If needed, a Hanning (or other) window is used to minimize spectral leakage
3) Discrete Fourier Transform is applied to all signals, thus obtaining 2N discrete spectra F_{ki} and X_{ji} with fundamental frequency $\omega_0=2\pi/T_0$
4) PSD (Power Spectral Density - real) and CSD (Cross-Spectral Density - complex) functions are computed, according to the following formulae:

$$G_{XX}(m\omega_0)=\frac{1}{N}\sum_{i=1}^{N}\frac{X_{ji}(m\omega_0)X_{ji}^*(m\omega_0)}{2\omega_0} \qquad G_{FF}(m\omega_0)=\frac{1}{N}\sum_{i=1}^{N}\frac{F_{ki}(m\omega_0)F_{ki}^*(m\omega_0)}{2\omega_0}$$

$$G_{XF}(m\omega_0)=\frac{1}{N}\sum_{i=1}^{N}\frac{X_{ji}(m\omega_0)F_{ki}^*(m\omega_0)}{2\omega_0}$$

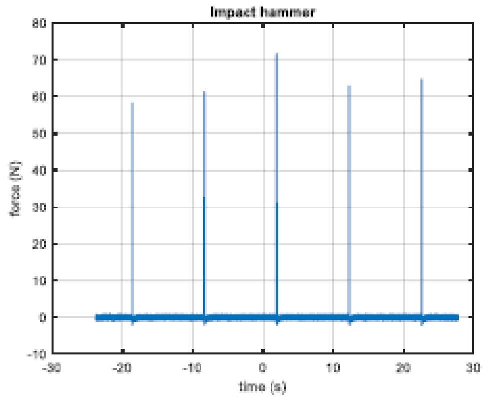
5) Finally the FRF G_{jk}^{EXP} and the coherence function γ_{jk}^2 are estimated:

$$G_{jk}^{EXP}(m\omega_0)=\frac{X_j(m\omega_0)}{F_k(m\omega_0)}=\frac{G_{XF}(m\omega_0)}{G_{FF}(m\omega_0)} \qquad \gamma_{jk}^2(m\omega_0)=\frac{\left|G_{XF}(m\omega_0)\right|^2}{G_{XX}(m\omega_0)G_{FF}(m\omega_0)}$$

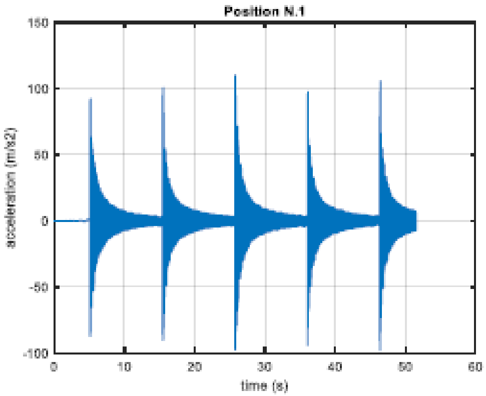
Data processing

Time histories

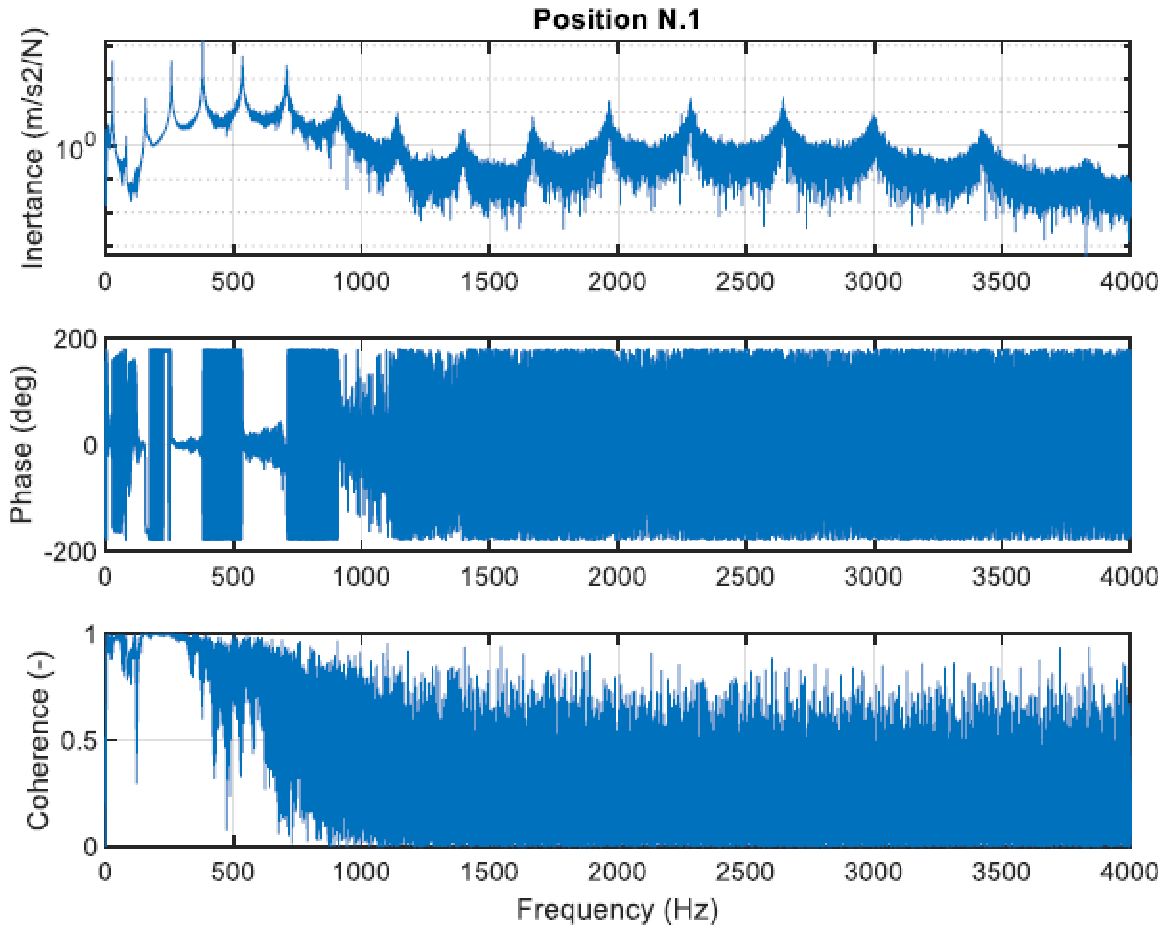
INPUT FORCE



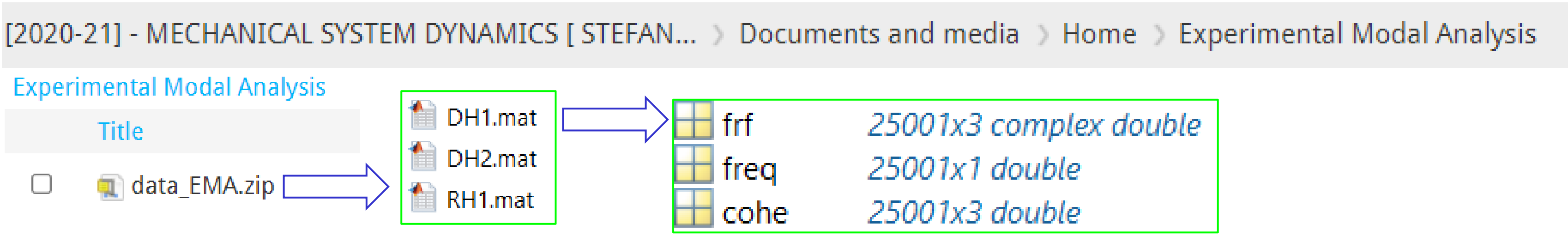
OUTPUT ACCELERATION



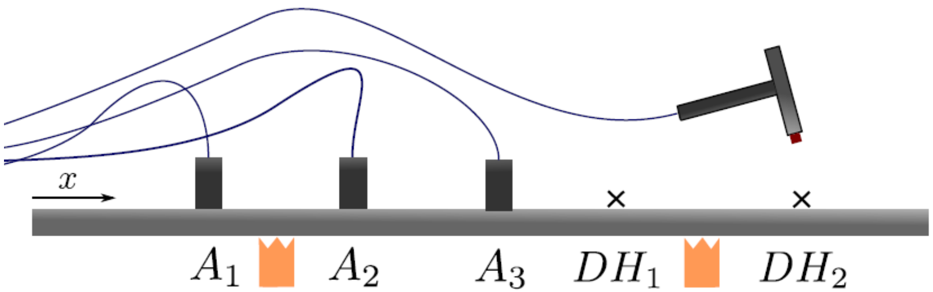
Frequency Response Functions



Data available on Beep



- *DH1.mat* Hammer in DH1 position
- *DH2.mat* Hammer in DH2 position
- *RH1.mat* A1 and DH1 position interchanged (**reciprocity** against *DH1.mat*)



Variables contained in the files:

-*frf* FRF(complex), collected by columns(A1, A2, A3)

-*freq* frequency vector (df=0.02 Hz)

-*cohe* coherence function, collected by columns(A1, A2, A3)

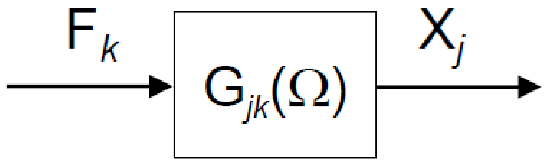
Simplified single-mode identification

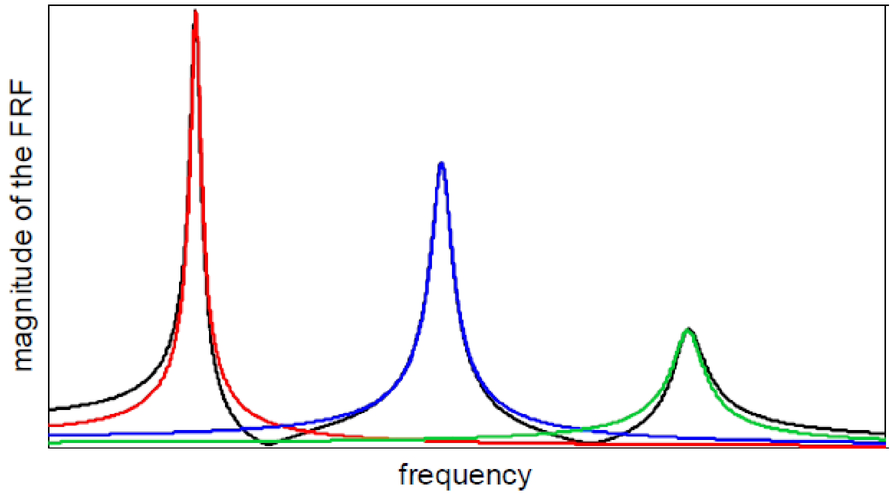
This approach is valid under the hypothesis

- the system is light damped
- the natural frequencies of the system are well separated (i.e. limited modal overlap).

If this is the case, in a relatively narrow band centred on the resonance peak, the contribution of the non-resonant modes can be neglected.

The beam under investigation is the case.

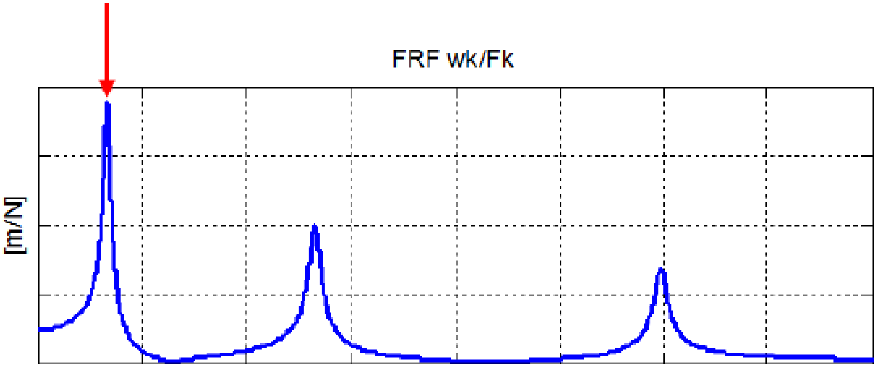




The FRF of the system is the linear combination of the FRFs of n SDOF.

Simplified single-mode identification

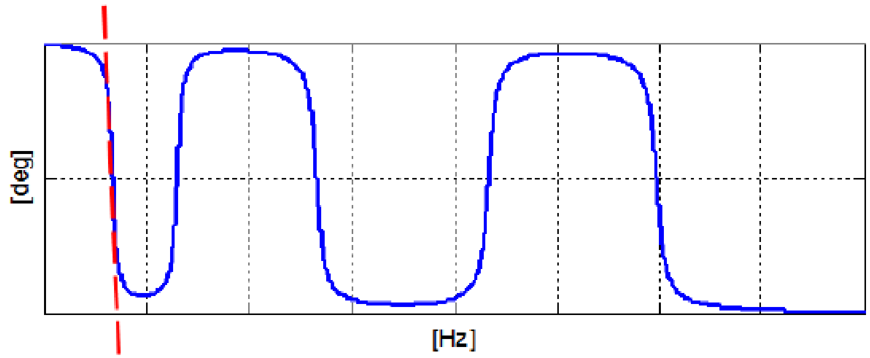
1) Natural frequencies can be directly identified from the position of the resonance peaks in the experimental FRF.



In the vicinity of the i-th resonance peak:

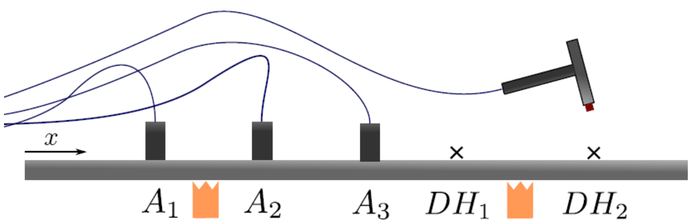
$$G_{jk}(\Omega) \cong \frac{X_j^{(i)} X_k^{(i)}}{-\Omega^2 m_i + j\Omega c_i + k_i}$$

2) The damping ratio ξ_i of the i-th mode can be identified from the **slope of the phase diagram**, in correspondence with ω_i



$$\xi_i = -\frac{1}{\omega_i \cdot \left. \dfrac{\partial \Phi_{jk}}{\partial \Omega} \right|_{\Omega=\omega_i}}$$

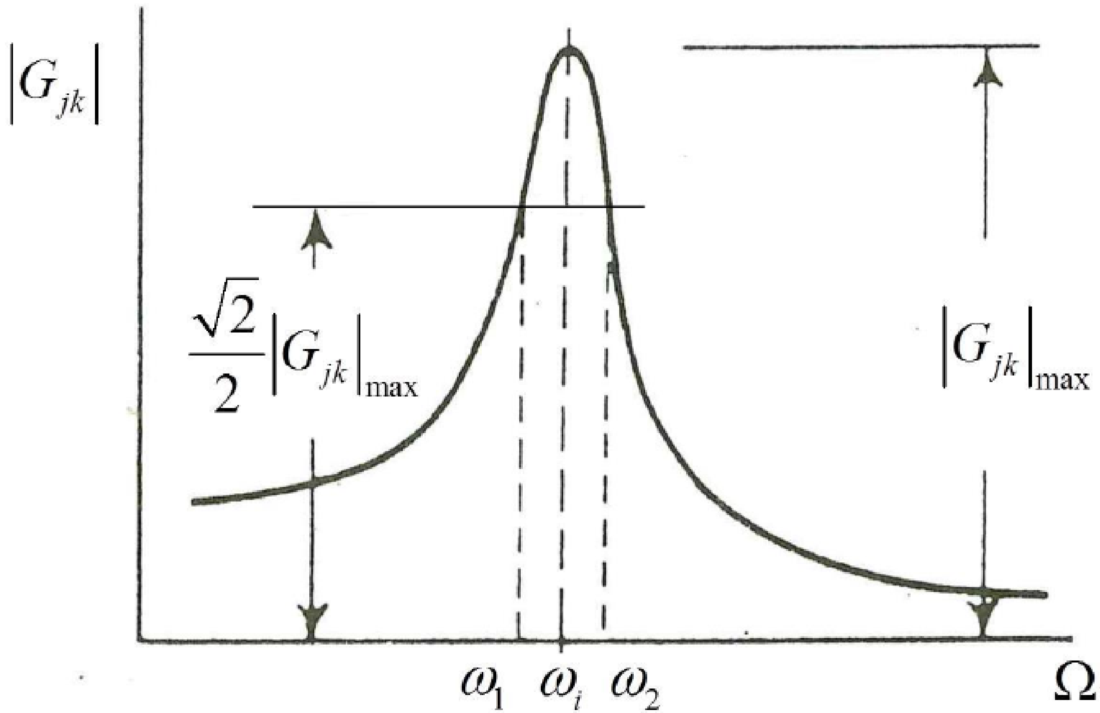
$j = \mathrm{A1, A2, A3}$
$k = \mathrm{DH1, DH2}$



Simplified single-mode identification

2) The damping ratio ξ_i of the *i*-th mode

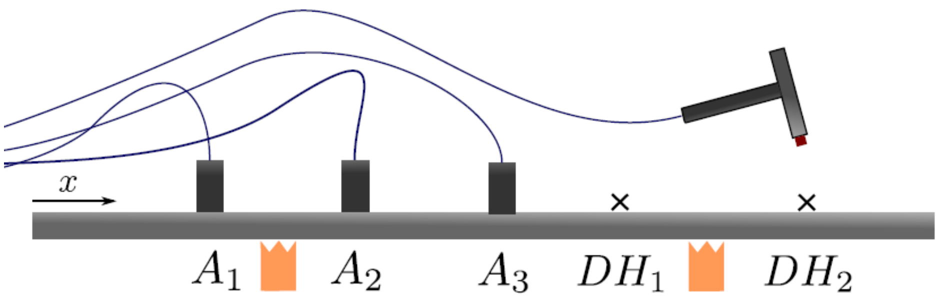
As an alternative, the damping ratio ξ_i of the *i*-th mode can be identified according to the **formula of the half-power points**:

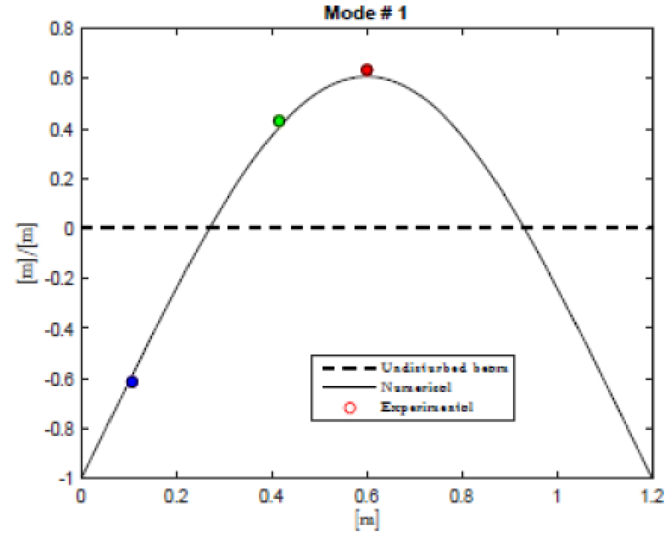


$$\xi_i = \frac{\omega_2^2 - \omega_1^2}{4\omega_i^2}$$

Simplified single-mode identification

3) The mode shapes $\underline{X}^{(i)}$ are identified from the imaginary part of the experimental FRFs, evaluated for variable output node j, in correspondence with $\Omega=\omega_i$





$$G_{jk}(\omega_i) \cong \frac{X_j^{(i)} X_k^{(i)}}{j\omega_i c_i} = \underbrace{\left(-j\frac{X_k^{(i)}}{\omega_i c_i}\right)} X_j^{(i)}$$

$$j = \text{A1, A2, A3}$$
$$k = \text{DH1, DH2}$$

this quantity is constant for assigned input position k and mode i, i.e. it is independent of the output node j

Assignment (Part 1/2)

Single-mode identification (up to **5-th** mode)

1) Identification of the **natural frequencies**
2) Identification of the damping ratio by the “**half-power points**” method
3) Identification of the damping ratio by the “**slope of the phase diagram**” method

Prepare a short report including the identification results for **at least one** test configuration among DH1, DH2, and RH1.

Collect the results in table form (for each accelerometer A1, A2, A3). Short comments on the results.