POLITECNICO DI MILANO

Measurements

DFT, windowing and Leakage

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- ✓ Measurements on Beep

Let's consider a periodic signal *g(t)* with period T

For a periodic signal, the following relation holds: *g(t)=g(t+n*T)* where *T* is the period and *n* an integer number.

It is possible to demonstrate that *g(t)* can be expressed as a sum of harmonic signals (or as a sum of couples of rotating vectors with equal and opposite speed) which have equally spaced frequency ***k*f1***, where:

- *k* is an integer (including zero and negative numbers);
- *f1=1/T* represents the main harmonic component.

The k-th harmonic component can be obtained applying the Fourier transform:

$$G(f_k) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} g(t)e^{-j2\pi f_k t}dt$$

where $\boldsymbol{f_k = k*f1}$ is the frequency of the k-th harmonic component.

Discrete Fourier Transform

The signal is sampled, we know its value at constant time intervals *dt*, and is limited in time, for these reasons we describe the signal with a finite number of samples equal to ***N=fsamp*T***.

The Fourier transform has to be modified taking into account these two aspects, therefore the integral should be substituted by a summation:

$$G(kf_1) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} g(t)e^{-j2\pi k f_1 t}dt$$

$$\Downarrow$$

$$G(kf_1) = \frac{1}{N}\sum_{n=0}^{N-1} g(n)e^{-j2\pi k\frac{n}{N}}$$

The spectrum is a **complex quantity**

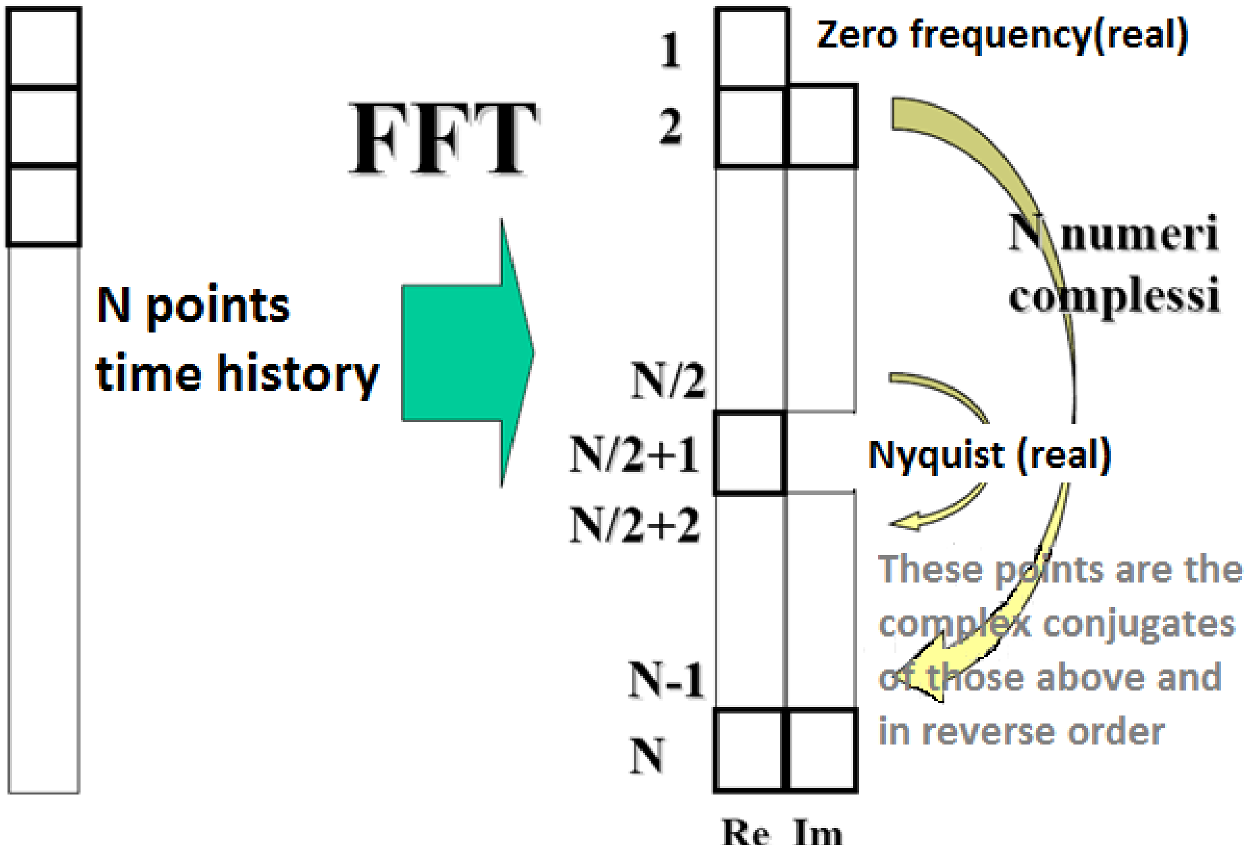
It can be represented in different ways:

- Modulus and phase
- Real and imaginary part
- Complex plane Re-Im

As function of frequency

Discrete fourier transform: FFT Matlab

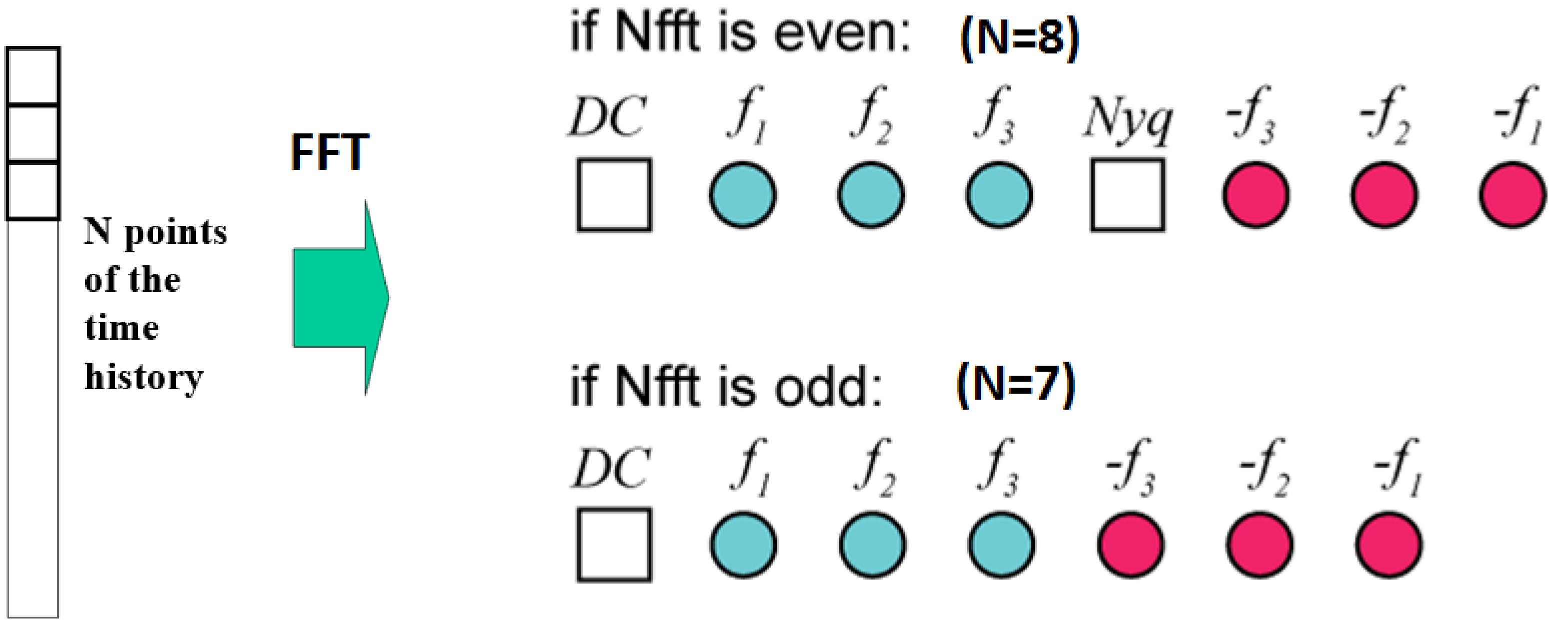
Pay attention:



when N is even(If N is odd,there is not Nyquist frequency)

Discrete fourier transform: FFT Matlab

Pay attention:



Results from a MATLAB fft. Each symbol represents an element in the vector, squares are unique (DC and Nyquist if present), circles are positive (blue) and negative (red) frequencies

Discrete fourier transform: FFT Matlab

Pay attention:

It is possible to consider just the positive frequencies but…

N even $\rightarrow$

consider (N/2+1) points

f_{max} = Nyquist

N odd $\rightarrow$

consider ((N+1)/2) points

f_{max} = Nyquist-df/2

We need to normalize in the right way

N even:

y(1)=y(1)/N

y(2:N/2)= y(2:N/2)*2/N

y(N/2+1)=y(N/2+1)/N

N odd:

y(1)=y(1)/N

y(2:(N+1)/2)= y(2:(N+1)/2)*2/N

Fourier Transform: the Leakage error

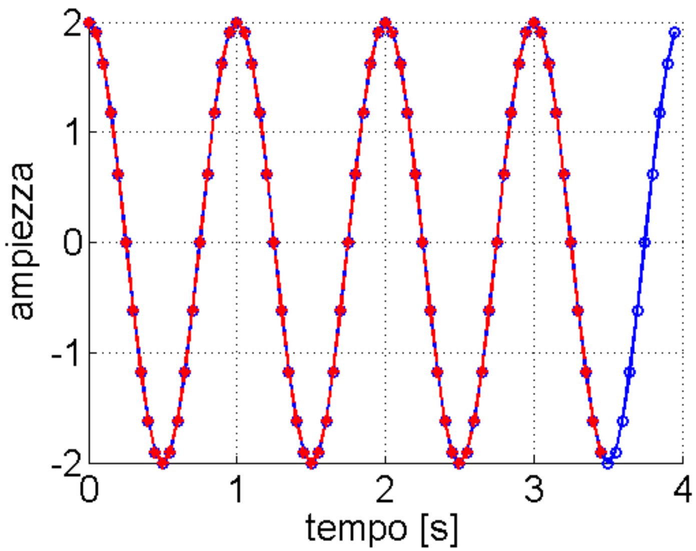
$$G(kf_1) = \frac{1}{N}\sum_{n=0}^{N-1} g(n)e^{-j2\pi k\frac{n}{N}}$$

The Fourier transform considers the signal periodic. Therefore the period of the signal g(n) for the Fourier Transform is the observation interval.

If our signal is not periodic in the observation interval, its real frequency will not be described by the Fourier transform. The frequency resolution will not allow to identify the fundamental frequency of the signal.

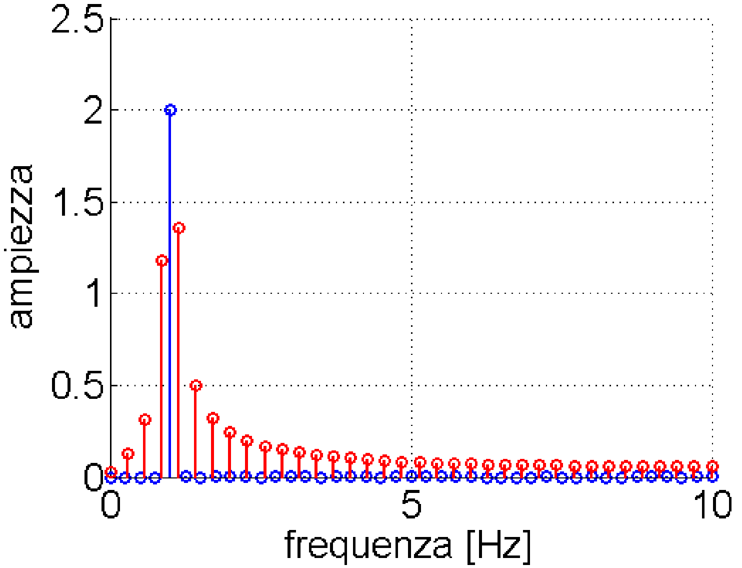
The signal representation in the frequency domain in terms of amplitude and phase will be affected by the leakage error.

Fourier Transform: the Leakage error



Number of period sampled: not an integer

The energy of the signal is scattered around its real frequency



Fourier Transform: the Leakage error

Problem

For a given signal it is not always possible to identify a period and therefore sample it for an integer number of periods

Leakage

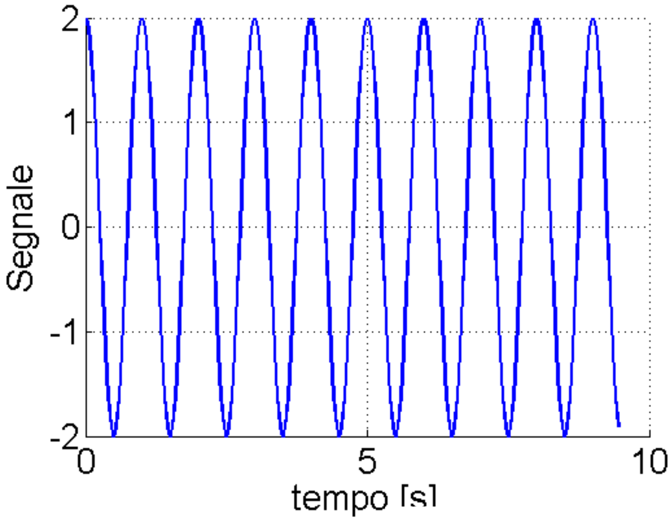
Problem mitigation

It is possible to use windows different from the rectangular one

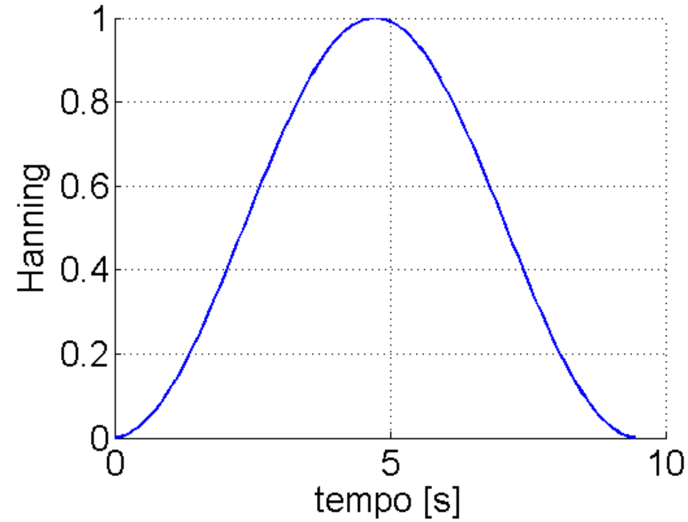
Each window modifies the signal and its spectrum in a different way

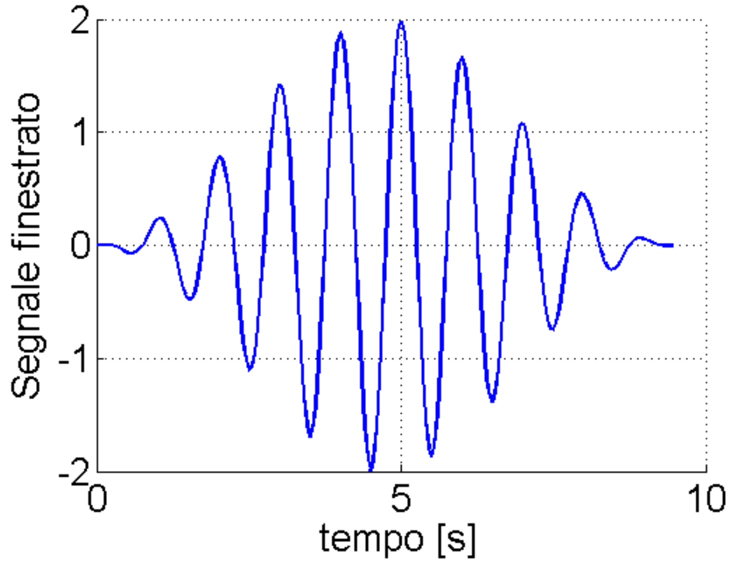
The choice of the best window depends on the signal under analysis and on the goals of the measurement

Windows


Segnale
2
1
0
-1
-2
0
5
10
tempo [s]

X


Hanning
1
0.8
0.6
0.4
0.2
0
0
5
10
tempo [s]


Segnale finestrato
2
1
0
-1
-2
0
5
10
tempo [s]

Laboratory – First part (Fourier transform)

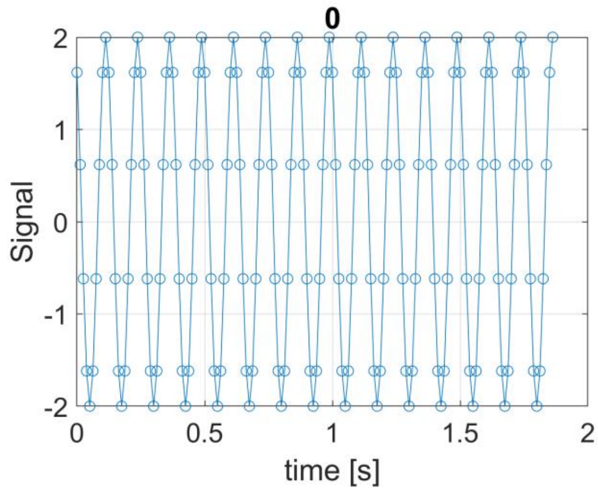
1. Load the data «*Lab_5_Data_first_part*»
2. Represent the spectrum (fft) in the following ways:
   - Modulus and phase (abs, angle)
   - Real and imaginary part (real, imag)
   - Complex plane Re-Im
3. Try to answer the following questions:

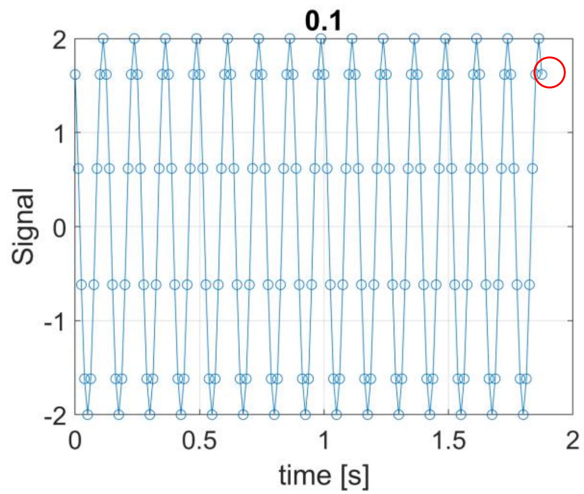
$$G(kf_1) = \frac{1}{N}\sum_{n=0}^{N-1} g(n)e^{-j2\pi k\frac{n}{N}}$$

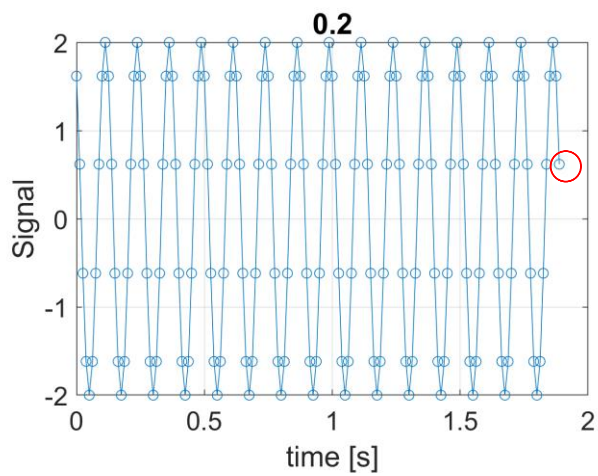
- If a signal is sampled with a sampling frequency fc, which are the frequencies that can be represented in its spectrum as rotating vectors?
- How can I choose the **interval of values** that ***k*** can assume?
- What happens if I multiply the right number of k by 4?

Laboratory – Second part (Windowing)

The data for this part is composed by 11 files containing the same periodic signal with 10 samples per period. For each subsequent file, a sample is added, as in the figures below.







… and others

Laboratory – Second part (Windowing)

1. For each of the given signals in «*Lab_5_Data_second_part.zip*», calculate the Discrete Fourier Transform and plot its modulus (consider just the positive frequencies).

2. For each spectrum find the frequency component with the maximum modulus. Plot all the maximum values as function of the cycle part ΔT_s ($T = T_{signal} N + \Delta T_s$) . <u>How the amplitudes change? And the frequencies?</u>

3. For each of the given signals apply the <u>**Hanning**</u> and the <u>**Flat-top**</u> window. Calculate the DFT and plot its modulus (consider just the positive frequencies). (hanning, window(@flattopwin,N))

4. For each spectrum find the frequency component with the maximum modulus. Plot all the maximum values of the spectra as function of the cycle part ΔT_s.

MATLAB: fft, hanning, stem

Laboratory – Second part (Windowing)

5. Compare the results between each other and with the ones obtained with the rectangular window

6. Try to answer the following questions:

   - What happens to the spectra in these cases?
   - Is it possible to identify a correction factor that allows to estimate the correct amplitude
   - Which is the best solution if I want to estimate the frequency?