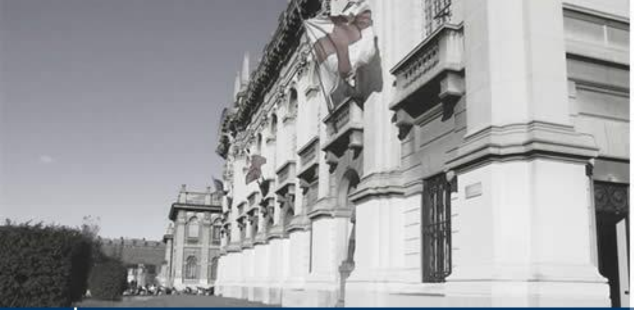

POLITECNICO DI MILANO

Mechanical System Dynamics

– Experimental modal analysis (Part 2)

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Objectives

Compute the natural frequencies and the mode shapes of the free-free beam already considered in Part 1, by means of the analytical model of a slender beam.

Compare the results with the experimental ones obtained in Part 1.

Contents:

- Data of the tested structure (geometry and material properties)
- Analytical solution for free-free beam (to be computed)

Tested structure: beam

Free-free beam

Standing wave solution

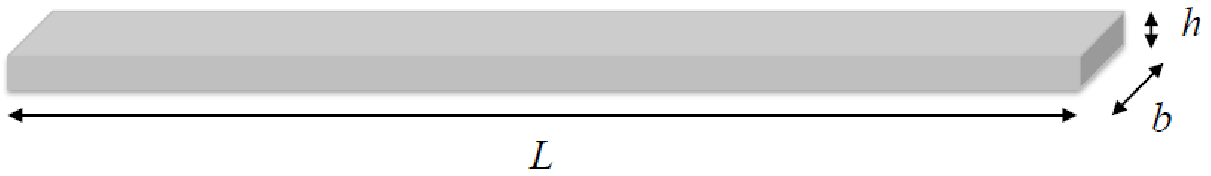
$$w(x,t) = (A\cos\gamma x + B\sin\gamma x + C\cosh\gamma x + D\sinh\gamma x)\cos(\omega t + \varphi)$$

$$where\ \gamma^4 = \frac{m\omega^2}{EJ}$$

$$m, E, J, A, B, C, D\ ?$$

Tested structure: beam

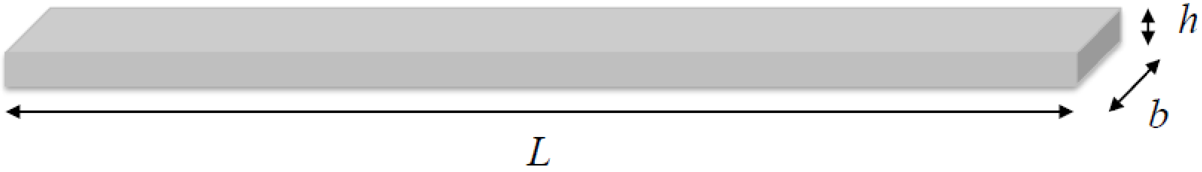
Test object: an aluminium beam with the following parameters



Parameter	symbol	unit	value
Lenght	L	mm	1200
Thickness	h	mm	8
Width	b	mm	40
Density	ρ	$\mathrm{kg/m^3}$	2700
Young's Modulus	E	GPa	68

Tested structure: beam

Aluminum beam with rectangular constant cross-section



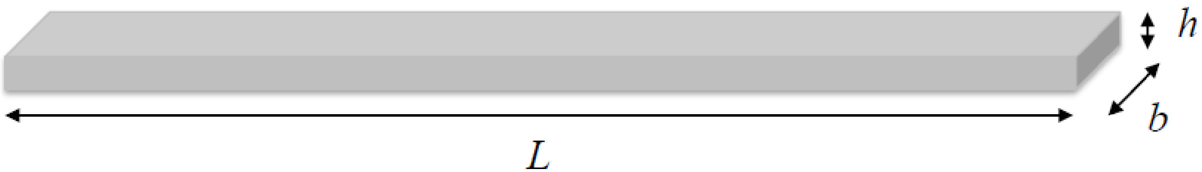
$$m = \rho b h$$
$$x = [0, L]$$

The moment of inertia of a rectangle with respect to an axis passing through its centroid, is given by the following expression:

$$J = \frac{bh^3}{12}$$

where *b* is the rectangle width, and specifically its dimension parallel to the axis, and *h* is the height (more specifically, the dimension perpendicular to the axis).

Analytical solution of the free-free beam



$$w(x,t) = (A\cos\gamma x + B\sin\gamma x + C\cosh\gamma x + D\sinh\gamma x)\cos(\omega t + \varphi)$$

$$where\ \gamma^4 = \frac{m\omega^2}{EJ}$$

Boundary conditions (4 BCs)

1) ..
2) ..
3) ..
4) ..

Matrix formulation

$$[H(\omega)]\,\underline{X} = \underline{0} \qquad \underline{X} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}$$

Analytical solution of the free-free beam

Solution of the characteristic equation $\det\left[H(\omega)\right] = 0 \;\rightarrow\; \omega_i$

Mode shape computation: $\omega_i \rightarrow \left[H(\omega_i)\right] \underline{X}^{(i)} = \underline{0}$

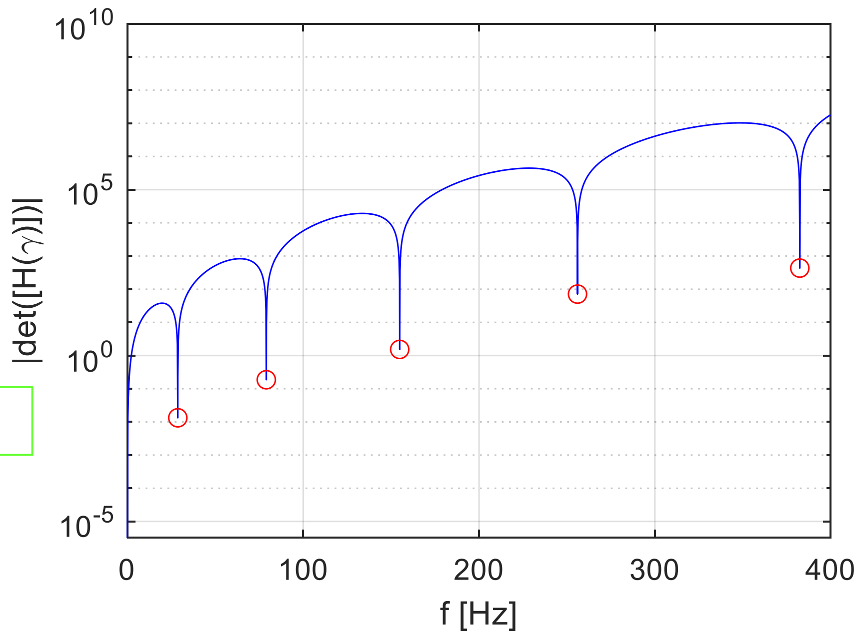
arbitrary chose $A_1^{(i)}$ = 1, use ω*i* obtained above to evaluate the following equation

$$\begin{bmatrix} \text{1st row} \\ \underline{N}(\omega_i) \quad [\hat{H}(\omega_i)] \end{bmatrix} \begin{bmatrix} 1 \\ \underline{\hat{X}}^{(i)} \end{bmatrix} = \underline{0}$$

$$\underline{\hat{X}}^{(i)} = -\left[\hat{H}(\omega_i)\right]^{-1} \underline{N}(\omega_i)$$

In Matlab `A^(-1)` is equivalent to `inv(A)`

$$\underline{X}^{(i)} = \begin{bmatrix} A^{(i)} \\ B^{(i)} \\ C^{(i)} \\ D^{(i)} \end{bmatrix} = \begin{bmatrix} 1 \\ \underline{\hat{X}}^{(i)} \end{bmatrix}$$

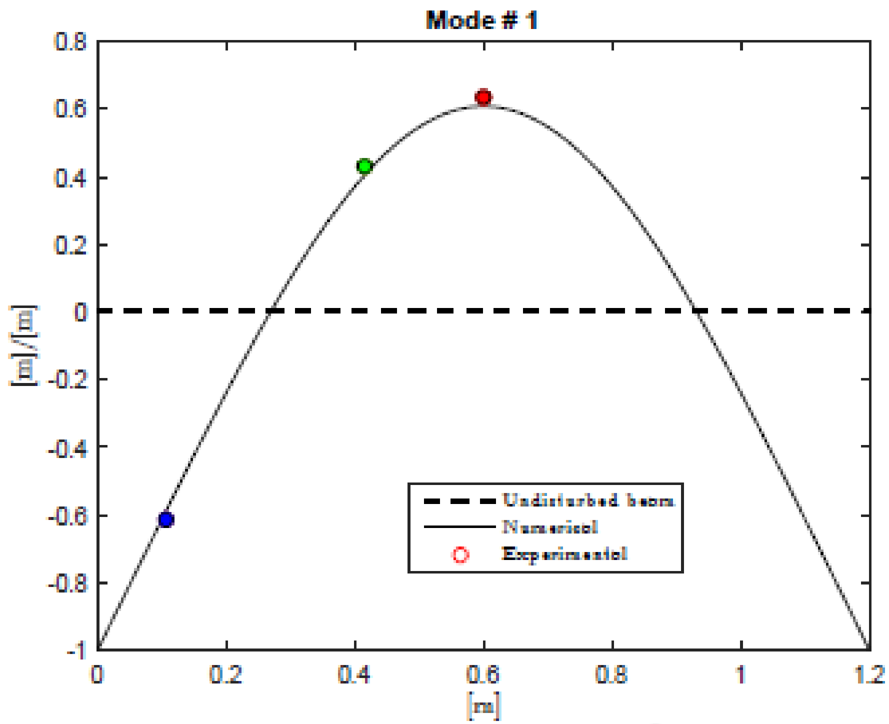


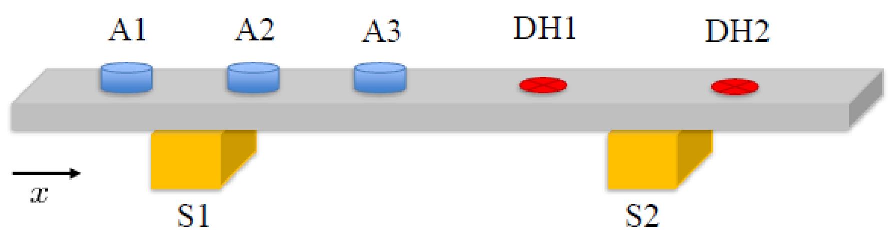
$\omega = 2\pi f$

Plot the analytical mode shapes with the associated natural frequencies

Analytical solution of the free-free beam

Comparison with the experimental results (a common normalization is recommended for the visualization of the analytical and experimental comparison)





symbol	x [mm]
A1	105
A2	415
A3	600

Assignment (Part 2/2)

1) Identification of the natural frequencies and mode shapes by the **analytical solution**
2) Comparison of the **Analytical** *vs* **Experimental modes (**first 5 modes**)**

Briefly describe the procedure followed for computing the natural frequencies and mode shapes and plot a diagram showing the comparison between the experimental and computed mode shapes.

Assignment (Part 1/2)

Single-mode identification (up to **5-th** mode)

1) Identification of the **natural frequencies**
2) Identification of the damping ratio by the "**half-power points**" method
3) Identification of the damping ratio by the "**slope of the phase diagram**" method

Prepare a short report including the identification results for **at least one** test configuration among DH1, DH2, and RH1.

Collect the results in table form (for each accelerometer A1, A2, A3).

Short comments on the results.

One single report for Part 1 + Part 2.