Mechanical System Dynamics

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Assignment

In-plane Finite Element Model of the supporting structure of an overhead contact line

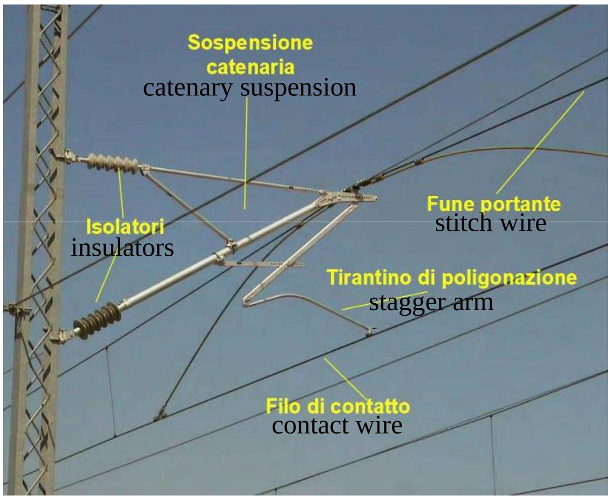


Figure 1 – Supporting structure of an overhead contact line: physical system

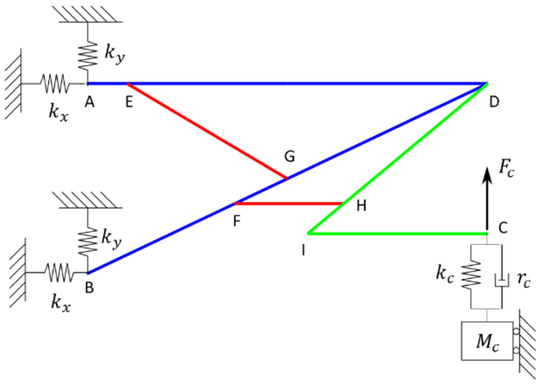
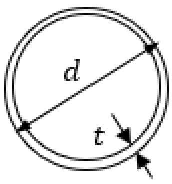


Figure 2 – Supporting structure of an overhead contact line: structural scheme

The system shown in Figure 1 is the supporting structure of the overhead contact line of a high speed railway line. Figure 2 represents a simplified in-plane scheme of the structure, suitable for the FE discretization. All beams are made of steel ($E = 2.06 \cdot 10^{11}$ N/m^2, $\rho = 7800$ kg/m^3) and have tubular cross section. The structure is connected to its pole in points A and B by means of springs acting in horizontal and vertical direction. A simplified way to account for the dynamic interaction with the catenary consists in a lumped mass elastically suspended in point C. The following tables summarize the geometric data and the main properties of the structure.

Point	x [m]	y [m]		Point	x [m]	y [m]
A	0	1.9		F	1.473	0.7
B	0	0		G	2	0.95
C	4	0.4		H	2.56	0.7
D	4	1.9		I	2.2	0.4
E	0.4	1.9				

Lumped springs	
k_x	5e4 N/m
k_y	5e4 N/m
Suspended mass	
k_c	1e5 N/m
r_c	50 Ns/m
M_c	5 kg



Beam section	d [mm]	t [mm]
Blue	60	2
Green	40	1.5
Red	25	1.5

Structural damping is introduced according to the Rayleigh damping model: $[C] = \alpha[M] + \beta[K]$, with $\alpha = 1.5$ s^{-1} and $\beta = 9 \cdot 10^{-5}$ s.

For the structure shown in Figure 2, work out the following items and prepare a short report summarizing the solution obtained.

1. Define a FE model of the structure in the 0 – 20 Hz frequency range (check that for each element k, $\frac{\omega_{1,k}}{\Omega_{max}} \geq 1.5$).
2. Compute the structure's natural frequencies and vibration modes up the 4th one. Plot the mode shapes with the indication of the associated natural frequencies.
3. Calculate the frequency response functions which relate the input force (vertical force in C) to the following outputs.

   a) Horizontal displacement and horizontal acceleration of node A, vertical displacement and vertical acceleration of node D. Plot the corresponding magnitude and phase diagrams.

   b) Shear force, bending moment and axial force evaluated in the midpoint of the EG beam. Plot the corresponding magnitude and phase diagrams.

   Assume the input force to vary in the 0 – 20 Hz frequency range and set the frequency resolution to 0.01 Hz.
4. Using the modal superposition approach and considering the structure's first two modes, calculate the frequency response functions which relate the same input force of question 3 (vertical force applied in node C) to the horizontal displacement and horizontal acceleration of node A, and to the vertical displacement and vertical acceleration of node D. Plot the corresponding magnitude and phase diagrams superimposed to those obtained in question 3a. Point out the differences and comment the results.
5. For the same excitation condition of question 3 (vertical force applied in node C), plot the time history (time length 5 s with resolution of 0.01 s) of the vertical displacement of node D. Relying on the superposition principle, consider the input force composed by the two harmonic contributions hereafter reported.

   a) $A_1 = 300$ N $\quad f_1 = 1$ Hz $\quad \varphi_1 = 0$

   b) $A_2 = 50$ N $\quad f_2 = 5$ Hz $\quad \varphi_2 = 0$
6. Optional question

   As shown in Figure 3, replace the clamp joints in nodes E, G, F and H with pin joints (note that AD is a single beam; the same for BD and DI). Find out a solution for modelling the four pin joints and repeat the tasks given in questions 1, 2 and 3b. Point out the differences and comment the results.

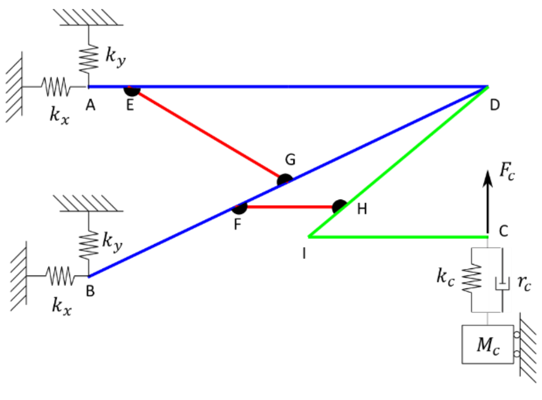


Figure 3 – Supporting structure of an overhead contact line: structural scheme with inner pin joints