# Measurements

*Cross-spectrum, Coherence and FRF*

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Course website

- ✓ Measurements on Beep

- **Problem**: **people** can be considered and modelled as mechanical systems therefore it is reasonable to think that they **interact with the mechanical/civil structures** on which they stand

- **Goal**: to **investigate** the effect of the human-structure interaction by looking at **how the dynamical characteristics** of the structure **change** when humans are on it.

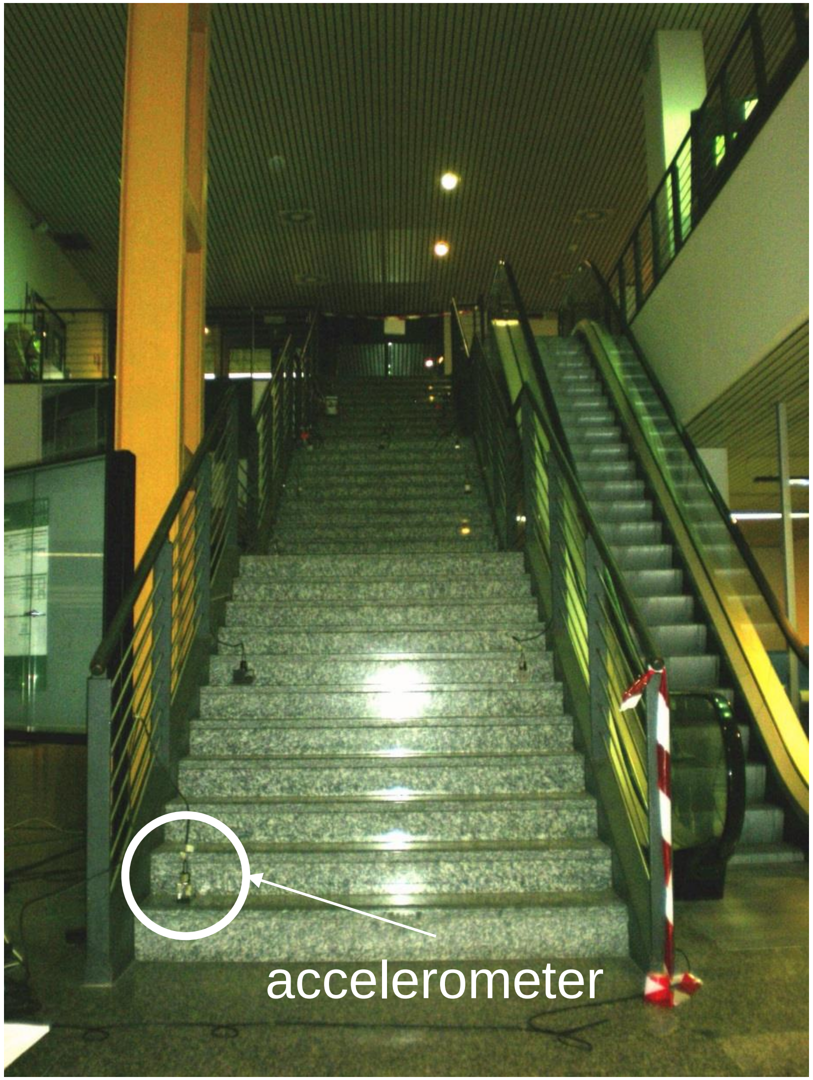
↓

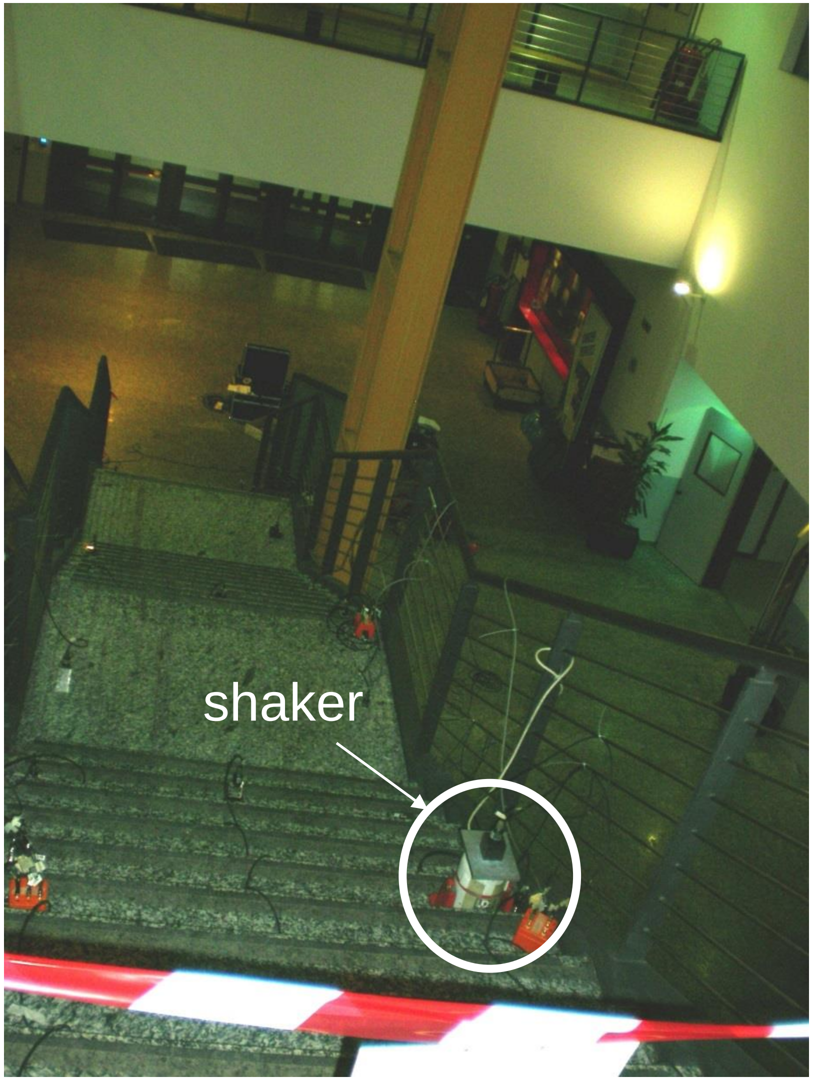
The response of the structure to a known input should be studied when people are or not on the structure itself.

As a first step the **cross-spectrum** between the input and the output of the system has to be analysed.

# Test structure:
# Staircase of B12 Building

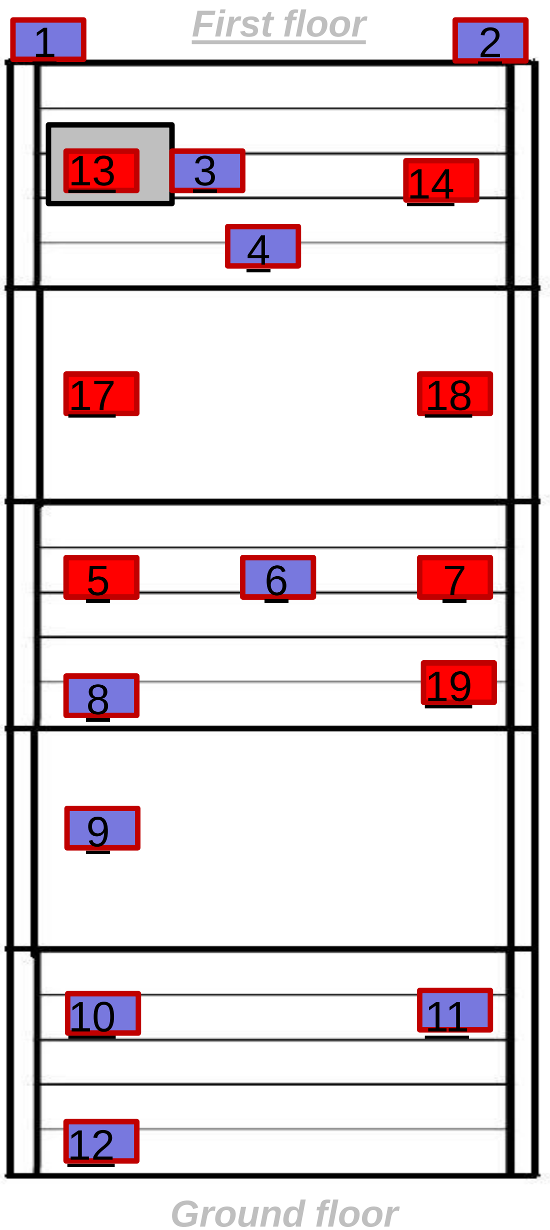




# Measurement set-up



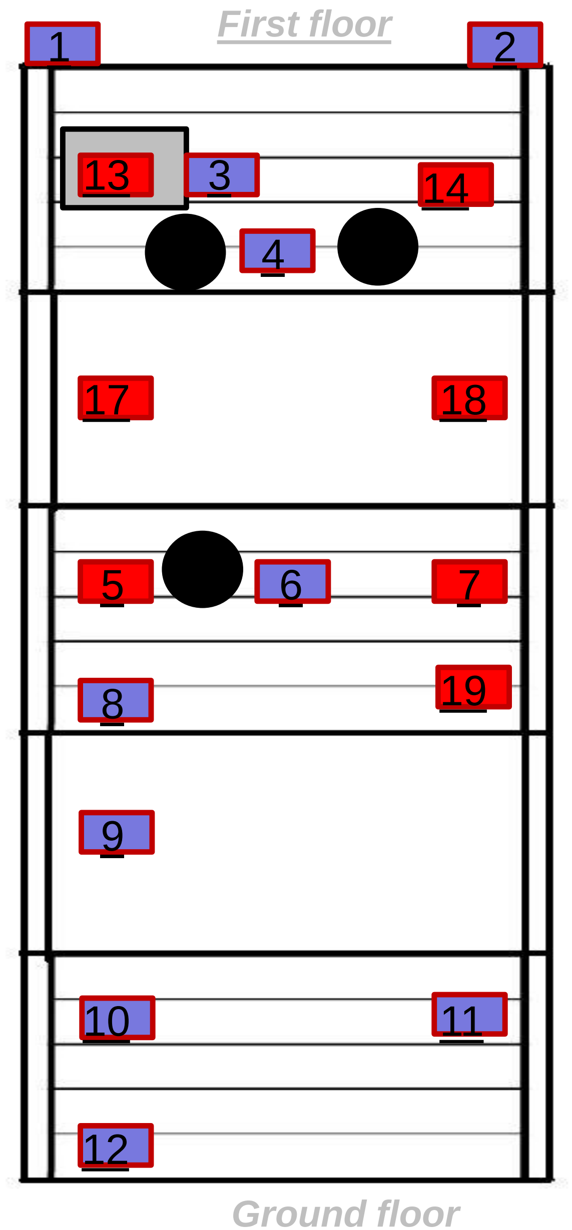



Shaker

Low sensitivity Accelerometers

High sensitivity accelerometers

Person



# Cross-spectrum

Which are the **main information** related to the cross-spectrum?

The cross-spectrum indicates the **correlation level** of two signals in a given frequency range

$$G_{xy}(f_k) = \frac{1}{N} * \sum_{i=1}^{N} (conj(X(f_k)) * Y(f_k))$$

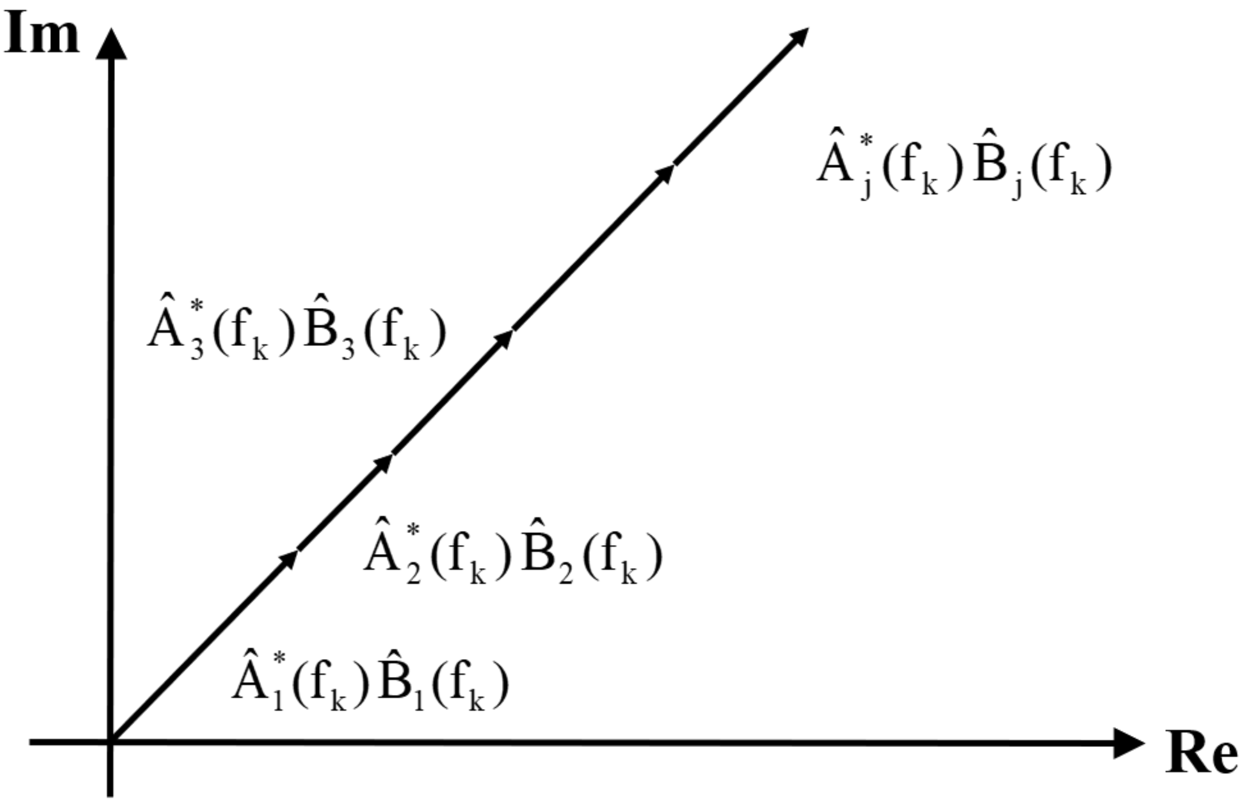
The **average process** allows to increase statistical reliability of the result

# Effect of the average process on the cross-spectrum evaluation



**Complete correlation** between the two signals:

The **phase shift** between the signals **doesn't change** with varying the considered time history

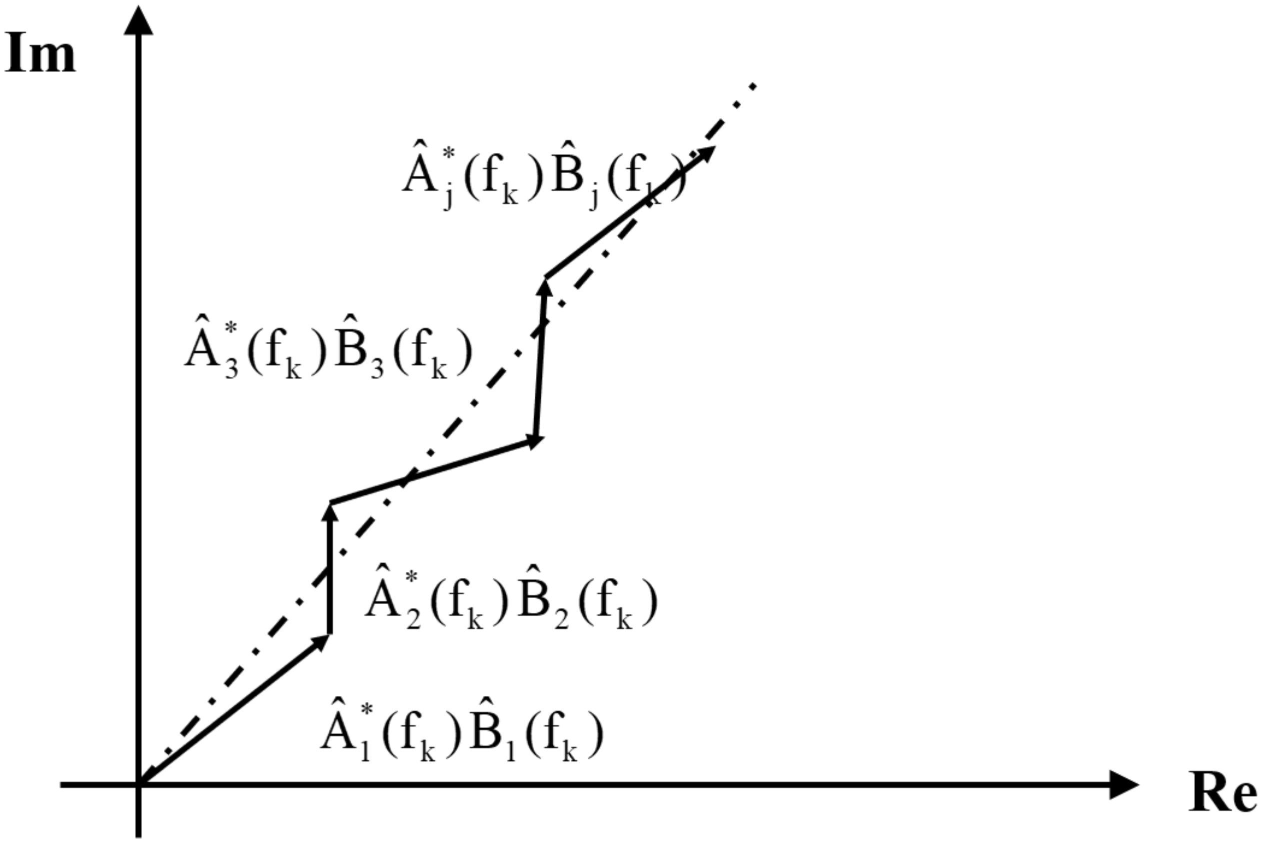


# Effect of the average process on the cross-spectrum evaluation



**Partial** correlation between the two signals:

The **phase shift** between the signals **change** with varying the considered time history

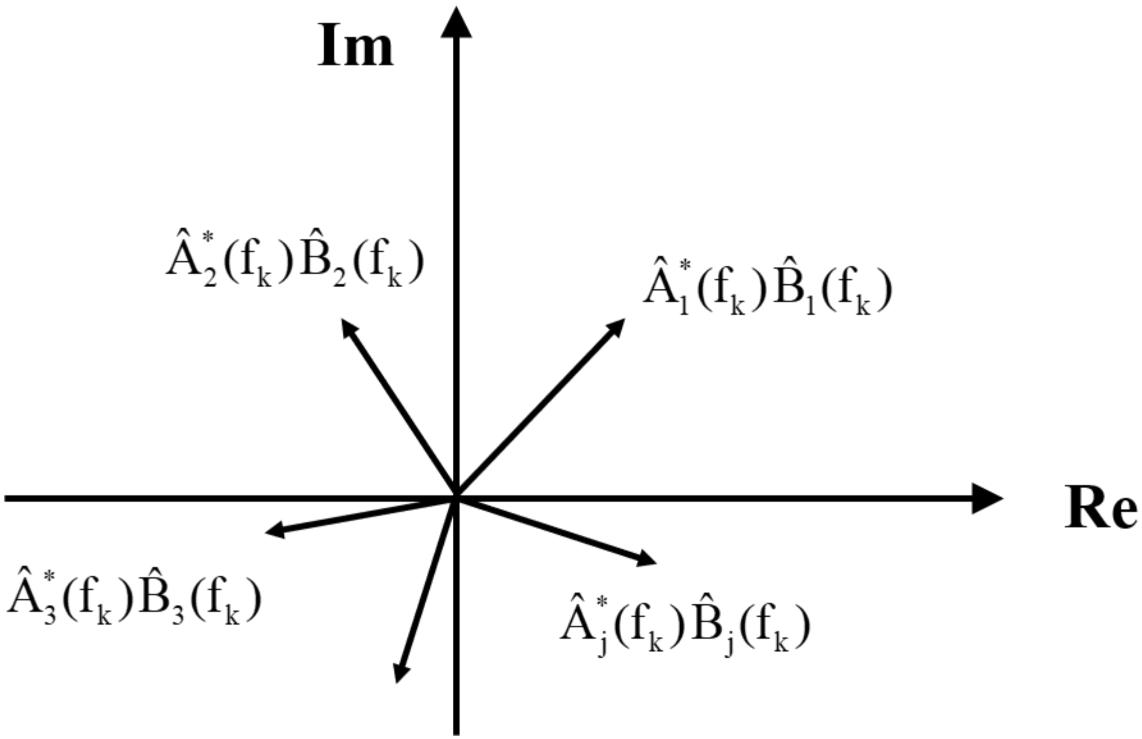


# Effect of the average process on the cross-spectrum evaluation



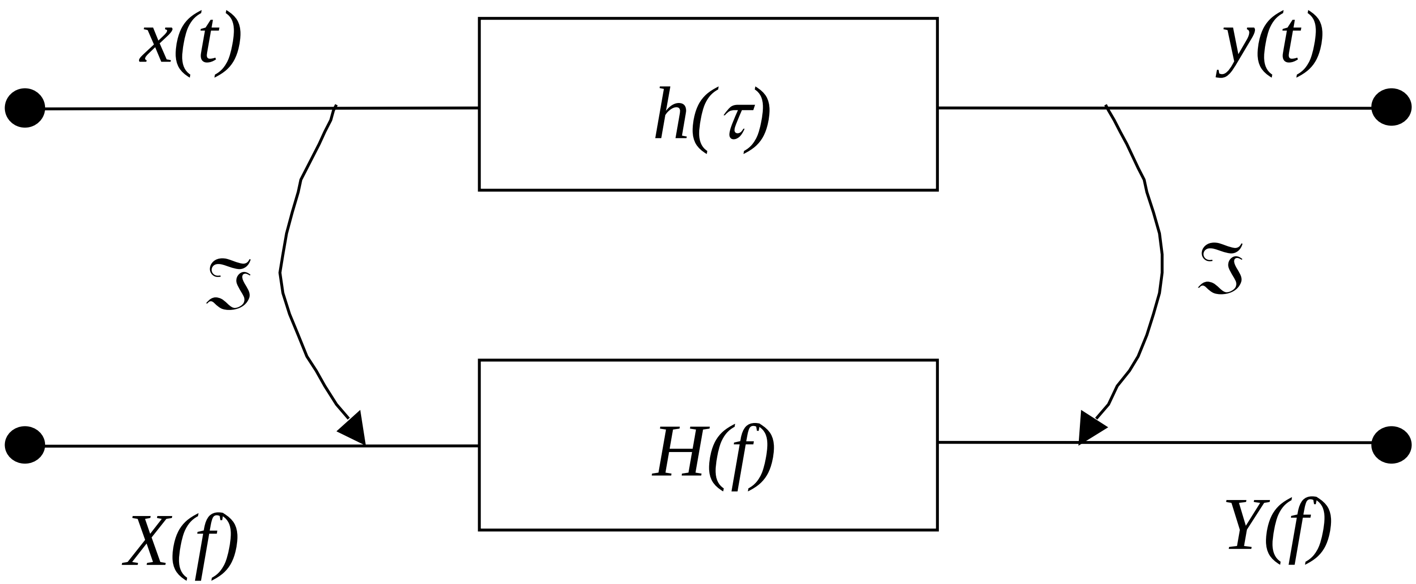
**Lack of correlation** between the two signals:

The **phase shift changes randomly** with varying the considered time history ($n_d \to \infty$ $S_{AB} \to 0$)



# Transfer Functions

For any linear time-invariant system, **the transfer function** is defined as




$$y(t) = \int_0^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

Convolution in the time domain

Product in the frequency domain

$$\boxed{Y(f) = X(f) \cdot H(f)}$$

Frequency Responce Function of the system

$$\boxed{Y(f) = X(f) \cdot H(f)}$$

$$X^*(f)\, Y(f) = H(f)\; X^*(f)\; X(f)$$

$$G_{xy}(f) = H(f)\; G_{xx}(f)$$

$$H(f) = \frac{Y(f)}{X(f)} \equiv \frac{G_{xy}(f)}{G_{xx}(f)} = H_1(f)$$

**H1 estimates**

- ✓ is not affected by the uncorrelated noise on the output measurement;
- ✓ underestimate the frequency responce function in the case of noise on the input measurement

# FRF $H_2$ estimates



$$\boxed{Y(f) = X(f) \cdot H(f)}$$

$$Y^*(f)\,Y(f) = H(f)\;\;Y^*(f)\;\;X(f)$$

$$G_{yy}(f) = H\;(f)\,G_{yx}(f)$$

$$H(f) = \frac{Y(f)}{X(f)} \equiv \frac{G_{yy}(f)}{G_{yx}(f)} = H_2(f)$$

**H2 estimate**

- ✓ Is not affected by the uncorrelated noise on the input measurement;
- ✓ Overestimates the transfer function when ther is noise on the output measurement

Coherence:

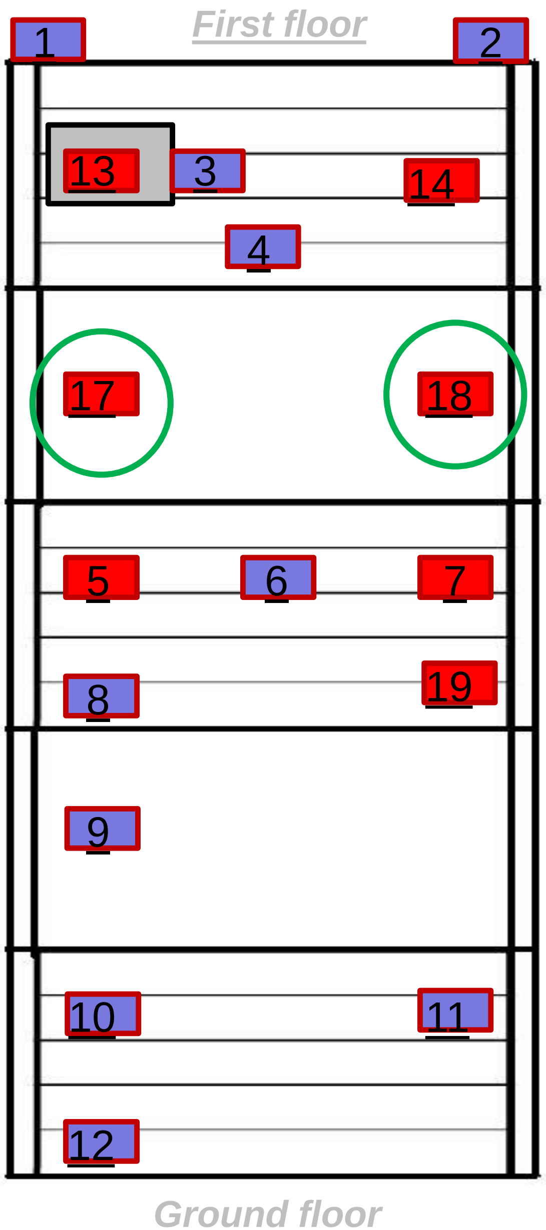
$$\gamma^2(f) = \frac{H_1(f)}{H_2(f)} = \frac{S_{xy}(f)^2}{S_{xx}(f)S_{yy}(f)}$$

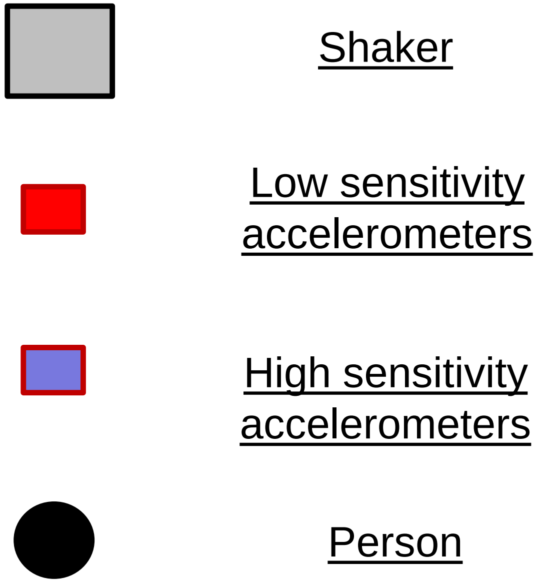
The coherence function drops below unity if:

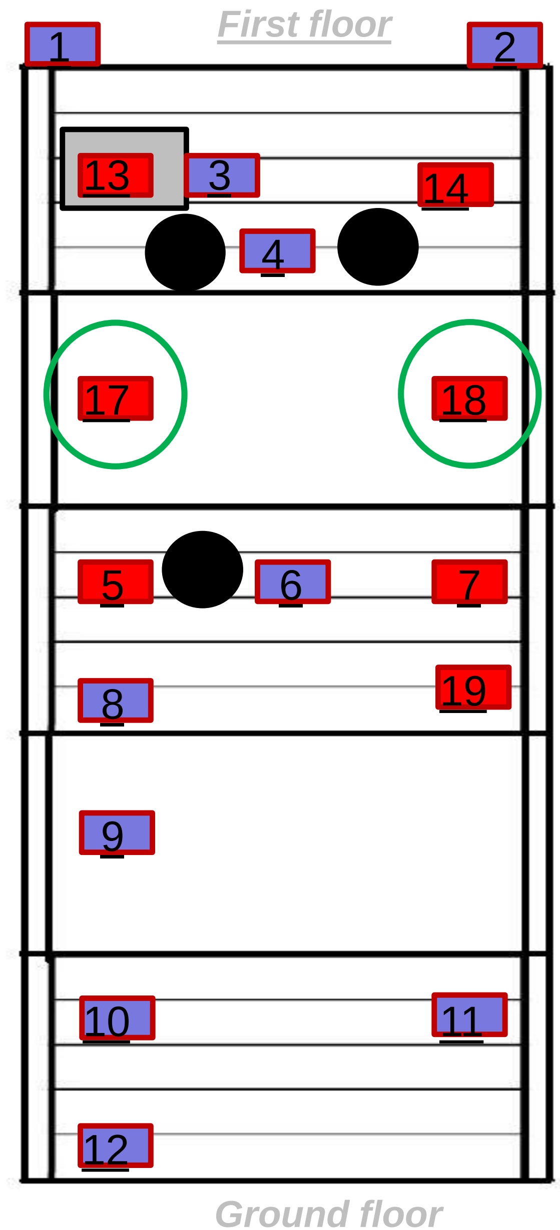
- Noise on the output or on the input signals;
- There are leakage measurement errors not reduced by windowing
- The system H(f) is nonlinear or not time invariant.
- There are non-measured inputs affecting the output.

# Test set-up




First floor
1
2
13
3
14
4
17
18
5
6
7
8
19
9
10
11
12
Ground floor


Shaker
Low sensitivity accelerometers
High sensitivity accelerometers
Person


First floor
1
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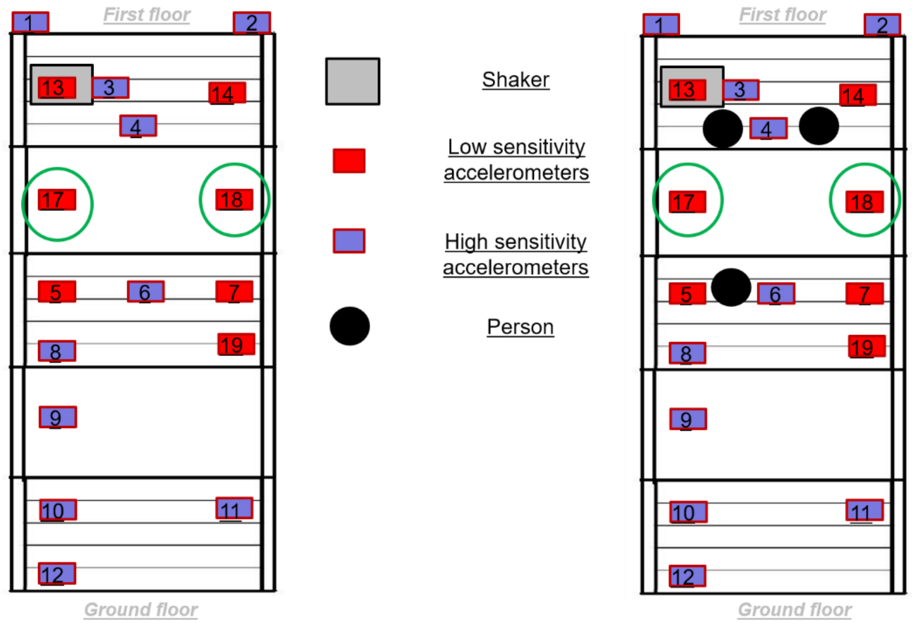
Ground floor

# Lab 7




The data sets are related to 2 different tests on the staircase:

- «Lab7_staircase_empty»: no people on the staircase
- «Lab7_staircase_people»: 3 people on the staircase

Each file contains the sampling frequency (fsamp) and the time histories of:

- The forcing term (random input) in Data(:,1)
- Acceleration measured at point 17 in Data(:,2)
- Acceleration measured at point 18 in Data(:,3)

# Lab 7

1. Evaluate, for both the cases (with and without people):
   - ✓ Power-spectra of the input and the output (both accelerometer 17 and 18), averaged using time histories of 30 s and 60 s
   - ✓ Cross-spectrum averaged using time histories of 30 s and 60 s.

   For both points use the autocross function in Matlab.

   Is the structure response correlated to the input?

   Is the information given by the **cross-spectrum enough** to answer the previous question?

   Otherwise **what should we look at**?
2. Evaluate the Frequency response function between the input and the output as the **Ratio of the Fourier transforms** (H=Y/X) of the output and input
3. Evaluate the Frequency response function between the input and the output as **H1**, using time windows of **30 s** and **60 s**
4. Evaluate the Frequency response function between the input and the output as **H2**, using time windows of **30 s** and **60 s**

Which is the correct way to represent the actual FRF of the system?

Is it possible to identify which one of the proposed methods to calculate the FRF is the best one to represent the actual FRF of the system under analysis?