Tutorial on

Modeling and Analysis of ALOHA-based Medium Access Protocols

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Why do we use ALOHA-based MAC?

Contention-based

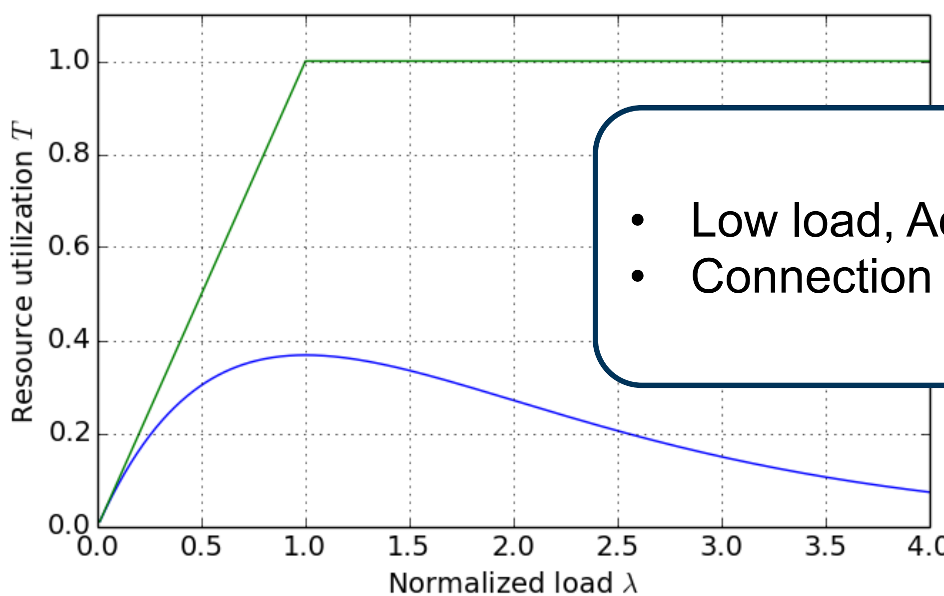
+ Low overhead
  → **No Information Exchange!**
+ Low complexity
+ Lower delay (if under-loaded)

- Low resource utilization: overprovisioning
- Non-linear degradation of the network performance (if over-loaded)

Contention-free

+ Higher resource utilization
+ Linear scaling of parameters with the load

- High overhead
  → **Information Exchange!**
- High delay (if under-loaded)



- Low load, Ad-Hoc Networks
- Connection establishment

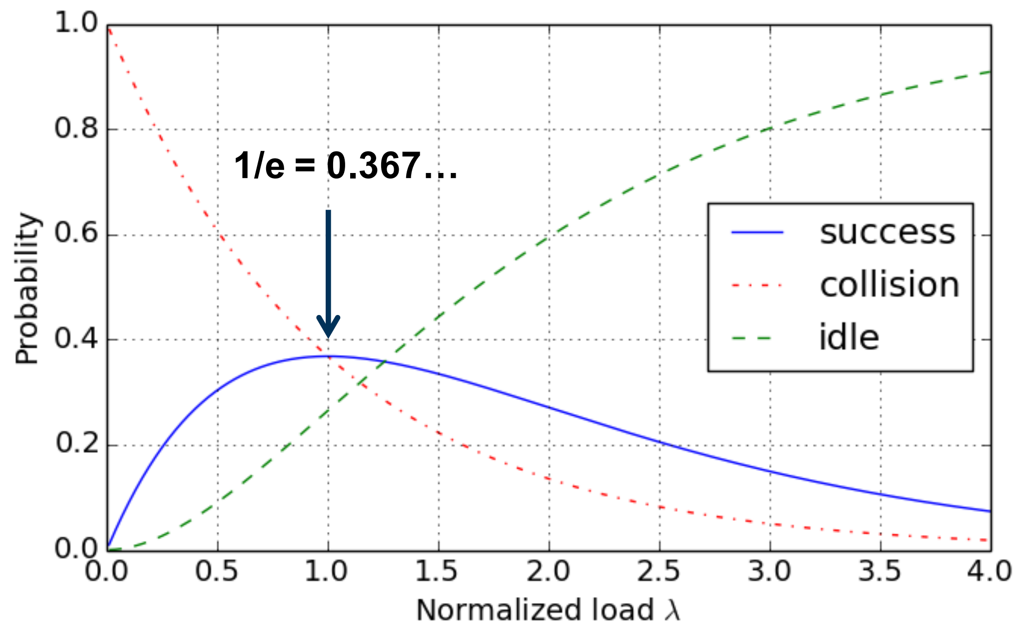
Overview

- Basic model (no back-off or retransmissions)
- Markov-Chain model (with retransmissions and bernoilli back-off)
- Performance
- Stability concerns

Slotted ALOHA modeling

Basic model

- **Poisson** arrivals with rate **λ**
- **Three states:** Success (1), Idle (0), Collisions (e)
- **Throughput T** – ratio of successful receptions (1) to the total number of slots (1+0+e) **= resource utilization**



Probability of exactly one transmission in a slot is (success probability):

$$P[k=1] = \frac{\lambda^k e^{-\lambda}}{k!} = \boxed{\lambda e^{-\lambda} \quad = T}$$

$$P[k=0] = \frac{\lambda^k e^{-\lambda}}{k!} = e^{-\lambda} \quad = Idle$$

$$P[k>1] = 1 - P[k=0] - P[k=1] \ = Collision$$

How large is the delay? **No answer!**

In real applications: retransmissions!

Modeling preliminaries [Ber92]

1. Poisson arrivals
2. **Three states:** Success (1), Idle (0), Collisions (e)
3. Immediate feedback about the state

4. Unlimited retransmission: backlog
   - ➢ Could be a retransmission limit
5. No buffering – only one backlogged packet from the node
   - ➢ With buffering
   - ➢ Infinite number of sources

6. Back-off after collision is governed by a Bernoulli trial

→delay is a geometric random variable

[Ber92] Bertsekas, Dimitri, and Robert Gallager. "Data networks. 1992." PrenticeHall, Englewood Cliffs, NJ (1992).

Markov Chain Model (1): Formulation

- Total number of nodes: *m*
- Number of backlogged nodes: *n*
- Total arrival rate λ
- Arrival rate per node λ/m

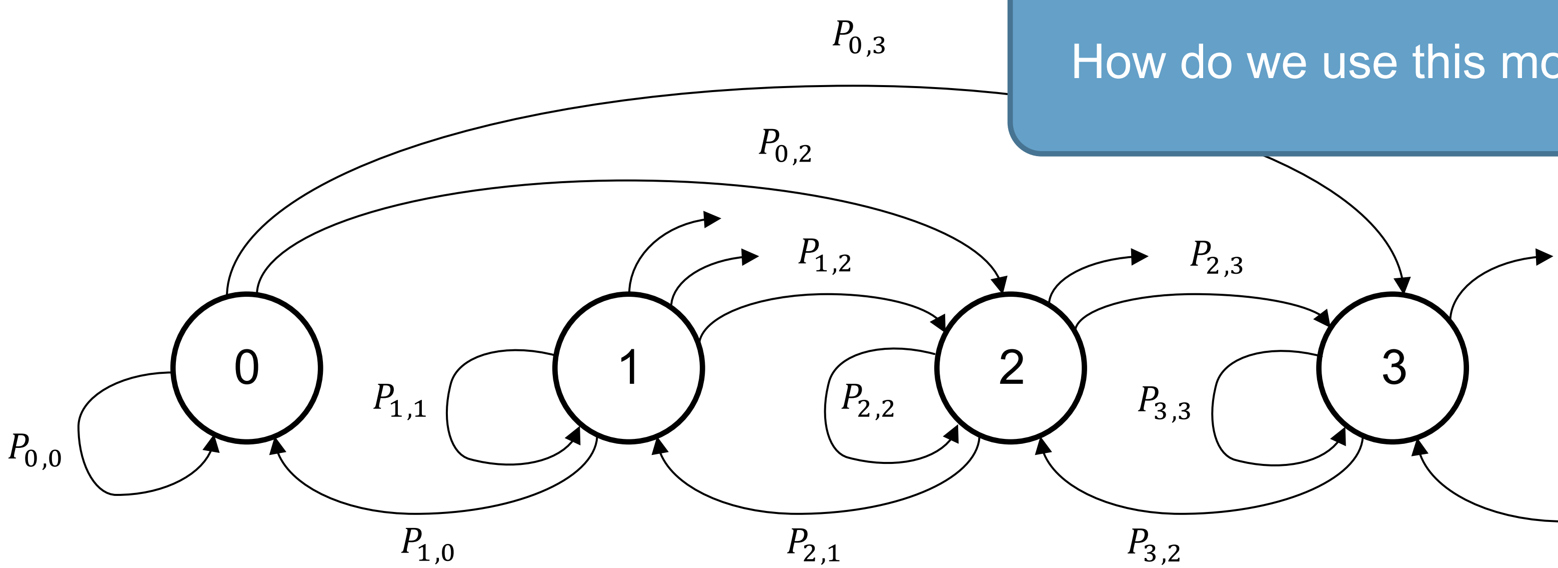
- Probability of transmission from **unbacklogged** nodes: $q_a = 1 - e^{-\frac{\lambda}{m}}$
- Probability of transmission from **backlogged** nodes: q_r

$$Q_a(i,n) = \binom{m-n}{i}(1-q_a)^{m-n-i} q_a^i$$

$$Q_r(i,n) = \binom{n}{i}(1-q_r)^{n-i} q_r^i$$

$$P_{n,n+i} = \begin{cases} Q_a(i,n) & 2 \le i \le m-n \\ Q_a(1,n)[1-Q_r(0,n)] & i = 1 \\ Q_a(1,n)Q_r(0,n) + Q_a(0,n)[1-Q_r(1,n)] & i = 0 \\ Q_a(0,n)Q_r(1,n) & i = -1 \end{cases}$$

How do we use this model?



Markov Chain Model (2): Performance

- Steady-state probabilities:

At most one success at a time!

$$p_n = \sum_{i=0}^{n+1} p_i P_{i,n}$$

Another state

Transition probability

$$p_{n+1} = \frac{1}{P_{n+1,n}} \left(p_n (1 - P_{n,n}) - \sum_{i=0}^{n-1} p_i P_{i,n} \right)$$

- Delay:

$E[\mathrm{n}]$

$$E[D] = \frac{E[n]}{\lambda}$$

$$\boldsymbol{pP} = \boldsymbol{p}$$

- Throughput T = expected success probability:

$$P_{succ}(n) = Q_a(1,n) Q_r(0,n) + Q_a(0,n) Q_r(1,n)$$

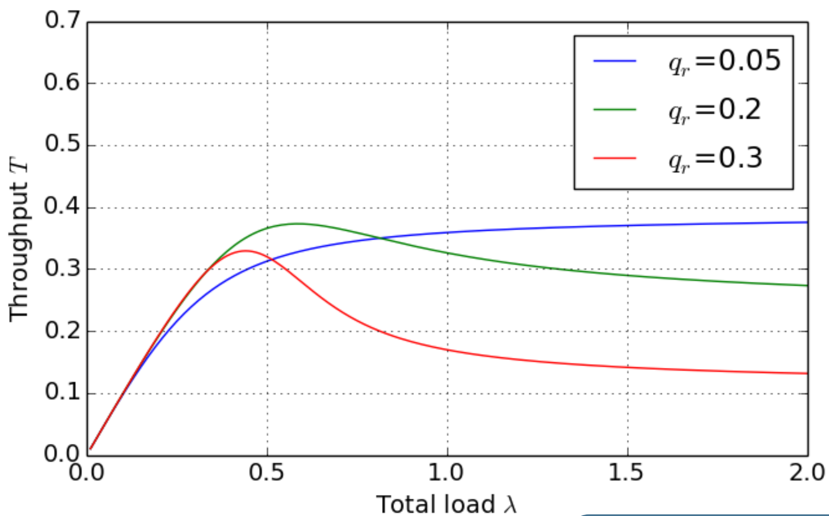
$$T = \sum_{n=0}^{m} p_n P_{succ}(n)$$

Markov Chain Model (3): Example

- For $m=10$, $q_r = 0.2$
- Compute transition matrix for every λ
- Compute state probabilities

```
Load ---> 1.0
['0.75 ', '0.00 ', '0.18 ', '0.05 ', '0.01 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ']
['0.12 ', '0.55 ', '0.12 ', '0.16 ', '0.04 ', '0.01 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ']
['0.00 ', '0.19 ', '0.45 ', '0.19 ', '0.14 ', '0.03 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ']
['0.00 ', '0.00 ', '0.22 ', '0.40 ', '0.24 ', '0.12 ', '0.02 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ']
['0.00 ', '0.00 ', '0.00 ', '0.23 ', '0.41 ', '0.26 ', '0.09 ', '0.01 ', '0.00 ', '0.00 ', '0.00 ']
['0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.22 ', '0.44 ', '0.27 ', '0.07 ', '0.01 ', '0.00 ', '0.00 ']
['0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.20 ', '0.50 ', '0.25 ', '0.04 ', '0.00 ', '0.00 ']
['0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.18 ', '0.58 ', '0.21 ', '0.02 ', '0.00 ']
['0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.16 ', '0.67 ', '0.16 ', '0.01 ']
['0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.14 ', '0.77 ', '0.09 ']
['0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.00 ', '0.12 ', '0.88 ']
```

['0.00', '0.00', '0.01', '0.04', '0.09', '0.17', '0.23', '0.23', '0.15', '0.06', '0.01']



$q_r n$>>1
Steady-state is not enough to analyse!

Alternative: game-theoretic analysis using Nash-equilibrium for finding optimal qr [MKe01]

Exponential Back-off: [Jeo95]

Source code: https://github.com/mvilgelm/saloha

Markov Chain Model (4): Equilibrium

- Expected change in the backlog over time

$$D_n = (m-n)q_a - P_{succ}$$

new arrivals

expected number of successes

- Attempt rate *G(n):* total number of transmissions (from backlogged and unbacklogges)

$$G(n) = (m-n)q_a + nq_r$$

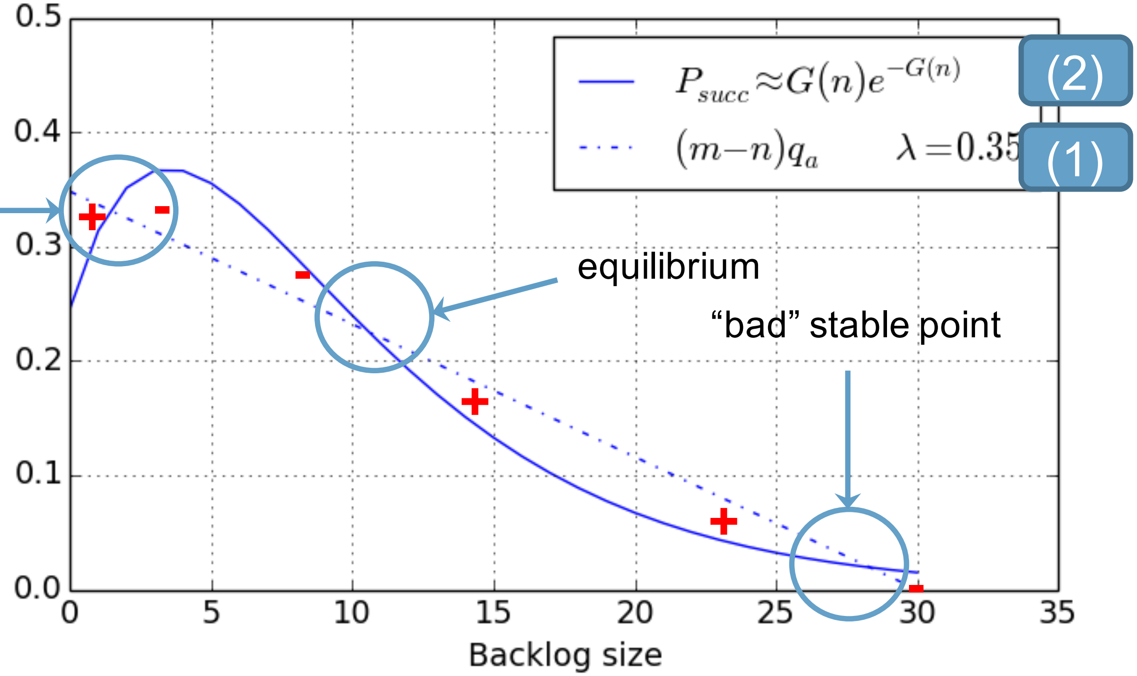
- Throughput:

$$P_{succ} \approx G(n)e^{-G(n)}$$
$$(for\ small\ q_a\ and\ q_r)$$

"good" stable point

(1) Arrival rate into the system

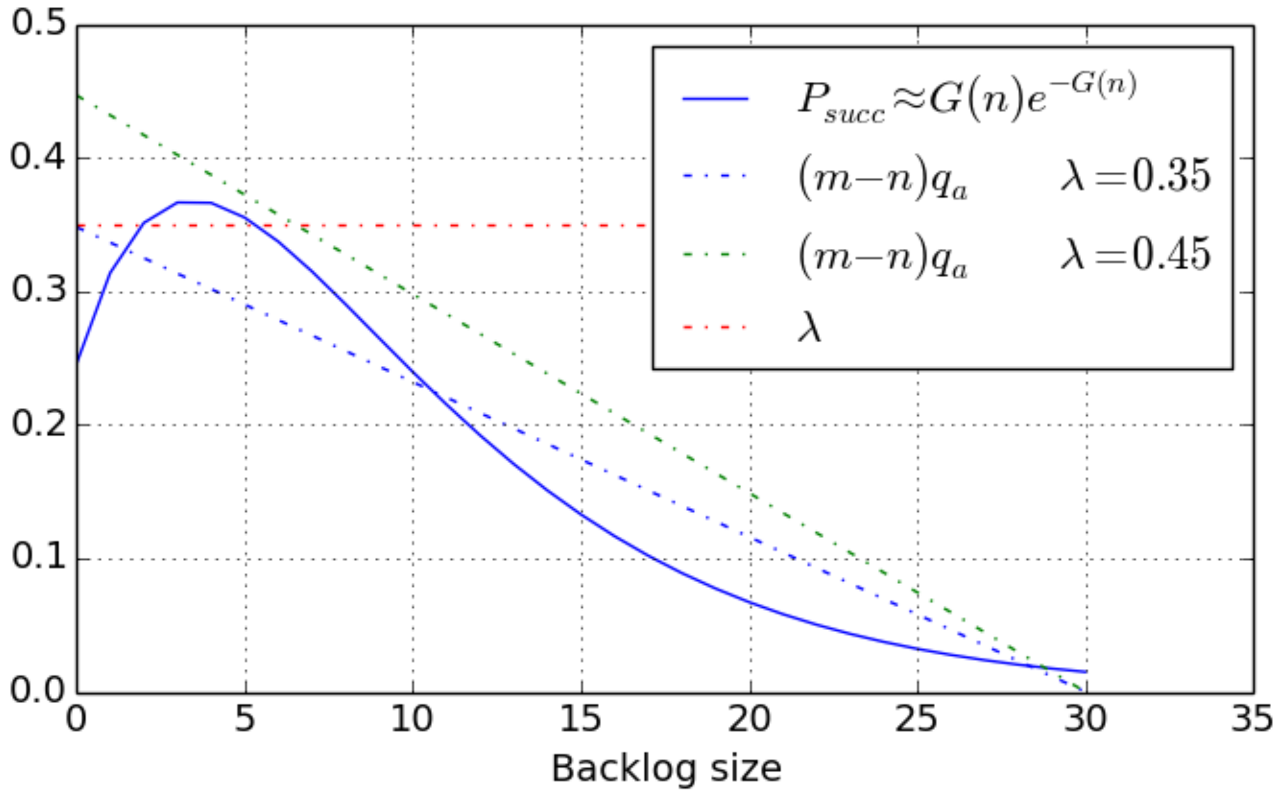
(2) Departure rate from the system



λ=0.35, m=30, q_r=0.2

Markov Chain Model (5): More on Stability

Arrival rate effects

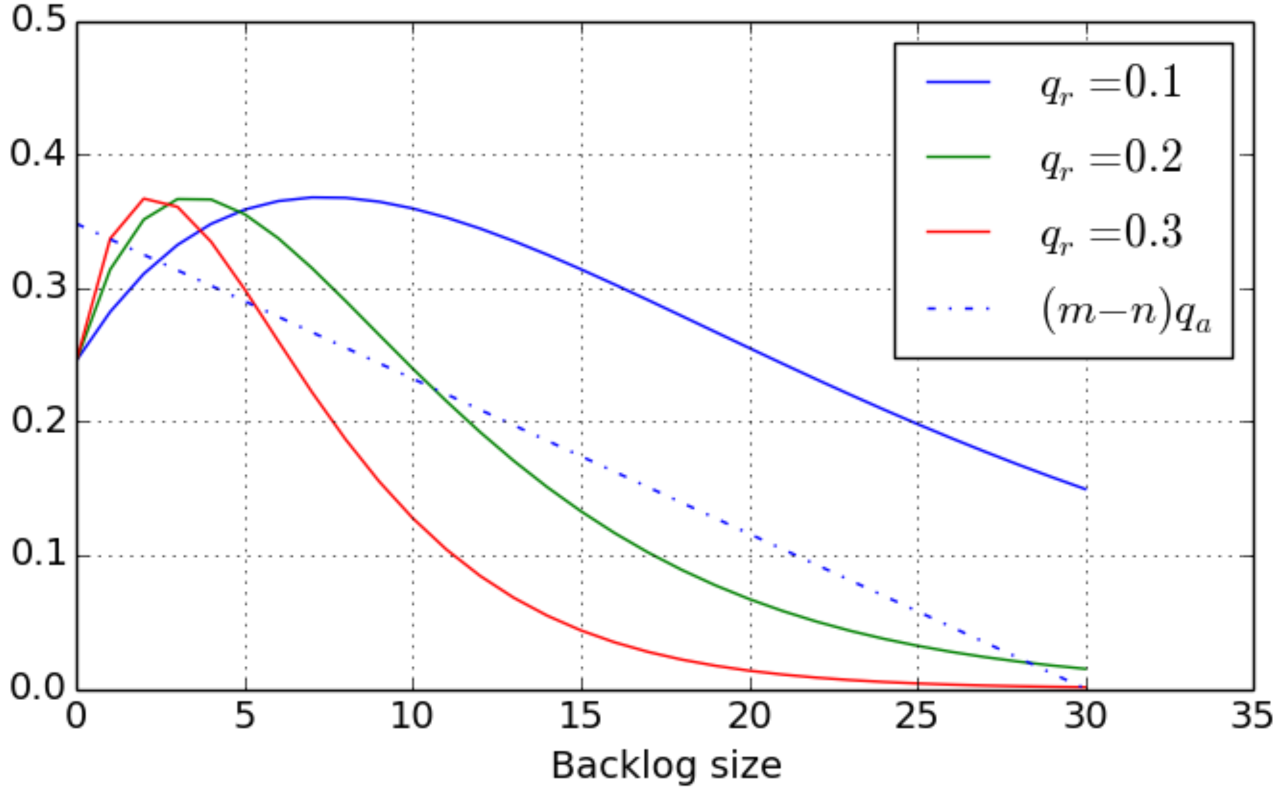


Infinite sources: unstable!

λ=0.35, m=30, q_r=0.2

Markov Chain Model (6): Even More on Stability

Retransmission probability = back-off length



Proper back-off choice leads to stable behavior

Delay-Stability trade-off

λ=0.35, m=30, q_r=0.2

Slotted ALOHA: stabilization options

Many options: [Tsy85], [HVL83], [Riv85]

Basic principles:

- Backlog on arrival

← Similar to Access Barring in LTE!

- Dynamic choice of retransmission probability
  - Once the backlog is high → reduce the retransmissions

References

[Ber92] Bertsekas, Dimitri, and Robert Gallager. "Data networks. 1992." PrenticeHall, Englewood Cliffs, NJ (1992)
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[Jeo95] Jeong, D.G. and Jeon, W.S., 1995. Performance of an exponential backoff scheme for slotted-ALOHA protocol in local wireless environment. Vehicular Technology, IEEE Transactions on, 44(3), pp.470-479.
[Tya13] Tyagi et. al, Impact of Retransmission Limit on Preamble Contention in LTE-Advanced Network, 2013

Questions?