Avoiding Social Disappointment in Elections

Paper #204

ABSTRACT

Mechanism design is concerned with settings where a policy maker (or social planner) faces the problem of aggregating the announced preferences of multiple agents into a collective (or social), system-wide decision. One of the most important ways for aggregating preference that has been used in multi-agent systems is election. In an election, the aim is to select the candidate who reflects the common will of the whole society. Despite the importance of this subject, in the real-world situations, under special circumstances, the result of the election is completely an antithesis of the purpose of those who execute it or the election leads to the dissatisfaction of a large amount of people and in some cases causes polarization in societies. For analyzing these situations, we introduce notions called social frustration and social disappointment and we show that which voting rules can prevent each of them in elections. In addition, we propose new protocols to prevent social disappointment in elections. A version of the impossibility theorem is proved regarding social disappointment in elections, showing that there is no voting rule for four or more candidates that simultaneously satisfies avoiding social disappointment and Condorcet winner criteria. We have also empirically compared our protocols with seven well-known voting protocols and we observed that our protocols are capable of preventing social disappointment, and also more robust against manipulations.

KEYWORDS

mechanism design; social choice theory; voting procedures; impossibility theorem; social disappointment; manipulation

1 BASIC DEFINITIONS AND CONCEPTS

In this paper we consider elections with more than two candidates. In order to define some central concepts such as ballot, profile, voting rule, and so on, we need to consider the following basic definitions and notations.

Preference Relations. In set theory $|A|$ denotes the number of elements in the finite set A. Any subset R of A × A is a binary relation on A, and in this case we write "aRb" to indicate that (a, b) ∈ R, and we write "¬(aRb)" to indicate that (a, b) ∉ R. The binary relations we are most concerned with satisfy one or more of the following properties.

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Definition 1.1. A binary relation R on a set A is:

$$
\begin{array}{lll}
reflexive & if & \forall x \in A, xRx \\
symmetric & if & \forall x, y \in A, \text{ if } xRy \text{ then } yRx \\
asymmetric & if & \forall x, y \in A, \text{ if } xRy \text{ then } \neg(yRx) \\
antisymmetric & if & \forall x, y \in A, \text{ if } xRy \text{ and } yRx \text{ then } x = y \\
transitive & if & \forall x, y, z \in A, \text{ if } xRy \text{ and } yRz \text{ then } xRz \\
complete & if & \forall x, y \in A, \text{ either } xRy \text{ or } yRx.
\end{array}
$$

Also, a binary relation R on a set A is a *weak ordering* (of A) if it is transitive and complete and a *linear ordering* (of A) if it is also antisymmetric. If R is a weak ordering of A, then the derived relations of *strict preference* P and *indifference* I are arrived at by asserting that xPy iff ¬(yRx) and xIy iff xRy and yRx.

Definition 1.2. If A is a finite non-empty set (which we think of as the set of alternatives (candidates) from which the voters are choosing), then an A-ballot is a weak ordering of A. If, additionally, n is a positive integer (where we think of N $=\{1, \ldots, n\}$ as being the set of voters), then an (A, n)-profile is an n-tuple of A-ballots. Similarly, a linear A-ballot is a linear ordering of A, and a linear (A, n)-profile is an n-tuple of linear A-ballots.

The following definition collects some additional ballot-theoretic notation we will need.

Definition 1.3. Suppose P is a linear (A, n)-profile, X is a set of alternatives (that is, X ⊆ A), and i is a voter (that is, i ∈ N). Then:

$$
\begin{array}{lll}
top_i(P) = x & iff & \forall x \in A, xR_iy \\
max_i(X, P) = x & iff & x \in X, \forall y \in X : xR_iy \\
min_i(X, P) = x & iff & x \in X, \forall y \in X : yR_ix
\end{array}
$$

Here we just define Hare and Coombs procedures formally.

The Hare procedure and the Coombs procedure are special cases of the general idea of repeatedly using a single procedure to break ties among winners.

Suppose that A is a set of alternatives, n is a positive integer, and V is a voting rule defined not for just (A, n), but for (A', n) for every $A' \subseteq$ A. Now, for every (A, n) profile P, we can consider the sequence $\langle W_1, \ldots, W_{|A|}\rangle$, where $W_1 = V(P), W_2 = V(P|_{W_1}), W_3 = V(P|_{W_2})$, etc. Notice that

(i) $A \supseteq W_1 \supseteq W_2 \supseteq \cdots \supseteq W_{|A|}$, and
(ii) if $W_j = W_{j+1}$, then $W_{j+1} = \cdots = W_{|A|}$.

Definition 1.4. One repeatedly deletes the alternative or the alternatives with the fewest first-place votes, with the last group of alternatives to be deleted tied for the win. More precisely, V is the Hare voting rule (also called "the Hare system" or "the Hare procedure") if $V = V_H^*$ where $V_H(P)$ is the set of all alternatives except those with the fewest first-place votes in P (and all tie if all have the same number of first-place votes).

Definition 1.5. One repeatedly deletes the alternative or the alternatives with the most last-place votes, with the last group of alternatives to be deleted tied for the win. More precisely, V is the Coombs voting rule (also called "the Coombs procedure") if V =

V_C^* where V_C(P) is the set of all alternatives except those with the most last-place votes in P (and all tie if all have the same number of first-place votes).

2 SOCIAL FRUSTRATION AND SOCIAL DISAPPOINTMENT IN VOTING SYSTEMS

In order to illustrate the key concepts SF and SD, consider the following example[1].

Example 2.1. Consider the following situation in which there are four Dutchmen, three Germans, and two Frenchmen who have to decide on which drink to be served for lunch (only a single drink will be served to all).

Voters 1-4	Voters 5-8	Voters 9 and 10
Milk	Beer	Wine
Wine	Wine	Beer
Beer	Milk	Milk

Now, which drink should be served based on these individuals' preferences? Milk could be chosen since it has the most agents ranking it first. Milk is the winner according to the plurality rule, which only considers how often each alternative is ranked in the first place. However, the majority of agents will be dissatisfied with this choice as they prefer any other drink to Milk. For such an occasion in terms of social choice theory one can say that the Condorcet Loser-in this example milk-is the social choice and this, on its own, is one of the undesirable situations in the social choice theory. In other words, social frustration has occurred.

Now to look at it from another perspective, Milk is the alternative which is at the bottom of more than half of the voters' preferences lists i.e., social choice is an alternative with the least social support and has the most social dissatisfaction or to be even more serious has the most social resentment. In other words, social disappointment has occurred.

2.1 Our voting procedures to avoid social disappointment: The Least Unpopular (LU) and the Least Unpopular Reselection (LUR)

First, we show that plurality, Borda, Condorcet, Copeland, Seq. Pairs., and Hare do not satisfy SDC.

Proposition 2.2. *Plurality, Borda, Condorcet, Copeland, Seq. Pairs., and Hare method do not satisfy SDC.*

Proof. • **Plurality**: see Example 2.1 that shows plurality rule violates SDC.

- **Borda**: see Proposition 3.1 that shows Borda rule failes to satisfy SDC.
- **Copeland, Condorcet, and Seq. Pairs.**: Consider the following profile (each column shows a ballot of each voter):

d	d	d	c	b	b
a	a	c	a	c	c
b	b	a	b	a	a
c	c	b	d	d	d

The alternative 'd' is the unique social choice when the Copeland and CondorcetâĂŹs method is used. Although the alternative 'd' is a social choice, it is at the bottom of half of the individual preference lists and so social disappointment has taken place. Also, suppose that alphabetic ordering of the alternatives is the agenda. So, $\{c, d\}$ is the social choice and SDC is violated where Seq. Pairs. is used for voting.

- **Hare**: Consider the three alternatives 'a', 'b', and 'c' and the following sequence of ten preference lists grouped into voting blocks of size four, three, and one:

Voters 1-4	Voters 5-7	Voters 8-10
a	c	b
b	b	c
c	a	a

The alternatives 'a' is the social choice set when the Hare system is used. Although 'a' is the social choice, it is at the bottom of more than half of the individual preference lists and so social disappointment has taken place.

□

In this section, we introduce two new voting protocols that satisfy monotonicity and also prevents SD in voting systems.

Definition 2.3. (*The Least Unpopular (LU) procedure*) The social choice(s) in the least unpopular procedure (LU) is (are) the alternative(s) that appear(s) less than the others at the bottom of individual preference lists. More precisely, V is the LU procedure if V(P)=$\{x \in A | lp(x)$ is minimum$\}$, where $lp(x) = \{i \in N | \min_i(A, P) = x\}$.

Proposition 2.4. *The Least Unpopular procedure does not satisfy the CLC, CWC, IIA, and Pareto criteria.*

Proof. Consider the four alternatives 'a', 'b', 'c', and 'd' and the following profile:

Voters 1 and 2	Voter 3	Voter 4
a	c	d
b	a	a
c	b	b
d	d	c

The alternatives 'a' and 'b' are the social choices when the Least Unpopular procedure is used. Thus, the alternative 'b' is in the set of social choices even though everyone prefers 'a' to 'b'. This shows that the Pareto criterion fails. Now consider the three alternatives 'a', 'b', 'c' and the following profile:

[1] We have adapted this example from [1].

Voters 1 and 2	Voter 3
a	b
b	c
c	a

The alternative 'b' is the social choice when the Least Unpopular procedure is used. However, 'a' is clearly the Condorcet's winner, defeating each of the other alternatives in one-on-one competitions. Since the Condorcet's winner is not the social choice in this situation, it is clear that the Least Unpopular procedure does not satisfy the Condorcet's winner criterion. In other words, the alternative 'b' is a non-winner. Now suppose that voter 3 changes his or her list by interchanging the alternatives 'a' and 'c'. The lists then become:

Voters 1 and 2	Voter 3
a	b
b	a
c	c

Notice that the alternative 'b' is still above 'a' in the third voter's list. However, the Least Unpopular procedure now has 'a' and 'b' tied as the winner. Thus, although no one changed his or her preference regarding the alternatives 'a' and 'b', the alternative 'a' changed position from being a non-winner to being a winner. This shows that the independence of irrelevant alternatives fails in the Least Unpopular procedure.

Now, consider the following profile: Obviously, a is the Con-

32 voters	38 voters	10 voters
b	c	b
a	a	c
c	b	a

dorcet loser but LU protocol considers a as the winner of the social selection. This shows that LU does not satisfies CLC. □

Proposition 2.5. *The Least Unpopular procedure satisfies the SDC, AAW, and Monotonicity criterion.*

Proof. Since $|A|$ i.e., the number of candidates is a finite number, so V(P)=$\{x \in A | lp(x)$ is minimum$\} \neq \emptyset$, which means LU satisfies AAW criterion.

Assume that alternative a appears at the bottom of at least half of the individual preference lists. Since $|A| \geq 3$, so $a \notin \{x \in A | lp(x)$ is minimum$\}$ i.e., $a \notin V(P)$, which means LU satisfies SDC.

LU satisfies monotonicity because raising candidate a up on some preference lists can never increase the number of last-place votes that a receives, nor can it decrease the number of last-place votes that any other candidate receives. Hence, if a wins the election before such a change, she wins afterwards as well. □

Definition 2.6. (*The Least Unpopular Reselection (LUR) Procedure*) One repeatedly removes the alternative or the alternatives except those with the least last-place votes, with the last group of alternatives to be removed was tied for the win. More precisely, V is the least unpopular reselection (LUR) if V = V^*_{LUR}, where V_{LUR}(P) is the set of all alternatives with the least last-place votes in P (and all tie if all have the same number of first-place votes).

Proposition 2.7. *LUR protocol satisfies the AAW, Monotonicity, Pareto criteria, and SDC, but does not satisfy the CWC, CLC, and IIA criteria.*

Proof. If candidate a is ahead of candidate b on every preference list, candidate a has no last-place votes and therefore b suffers elimination in the at most last round. Hence LUR method satisfies Pareto.

The rest of proof is the same as proof of Proposition 2.4 and 2.5. □

3 EXPERIMENTAL RESULTS FOR SOCIAL DISAPPOINTMENT IN VOTING PROCEDURES

Our experimental results (see Figure 1) show that Borda and Copeland violate SDC in fairly small number of cases, indicating that social disappointment happens for these methods just in rare cases. The following propositions support this observation for Borda rule theoretically.

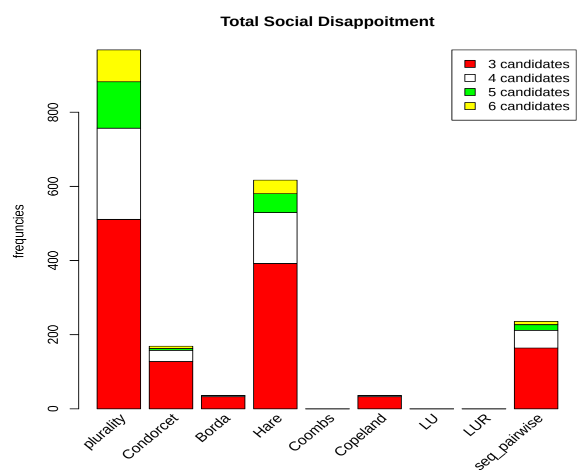


Figure 1: Performances of voting procedures: Number of occurrence of social disappointment in different elections has been shown with different colors based on the number of candidates. Social disappointment does not happen for Coombs, LU, and LUR. When the number of candidates increases, the number of social disappointment occurrence in elections decreases. The performance of Borda and Copeland rules are acceptable in this regard.

Proposition 3.1. *The Borda count rule can always prevent SD in voting except in one case. In this case the social choice set will consist of all the alternatives.*

Proof. Consider the three alternatives 'a', 'b', and 'c' and the following sequences of two preference lists:

Voters 1 and 2	Voters 3 and 4
a	c
b	b
c	a

The alternatives 'a', 'b' and 'c' are the social choice when the Borda count procedure is used. Although 'a' is the social choice (also 'c'), it is at the bottom of half of individual preference lists and so social disappointment has taken place.

Note that there are 'k' candidates ($k \geq 3$) and 'n' voters ($n \geq 3$). The total sum of scores in Borda count rule is equal to:

$$n((k-1)+(k-2)+\ldots+2+1+0)=\frac{n(k-1)k}{2}$$

Now consider that 'n' is an odd number, without loss of generality, and also consider that x_1 is a social choice and there is SD in voting, so x_1 must be at least the last preference in $\frac{n+1}{2}$ of individual preferences lists. Now consider the most optimistic possibility that in $\frac{n-1}{2}$ of the remaning lists x_1 is at the top. Thus Borda score for the alternative x_1 equals: $\frac{(n-1)(k-1)}{2}$. Now if this amount is subtracted from the whole Borda score it gives:

$$\frac{k(k-1)n}{2}-\frac{(k-1)(n-1)}{2}=\frac{(k-1)(kn-k+1)}{2}$$

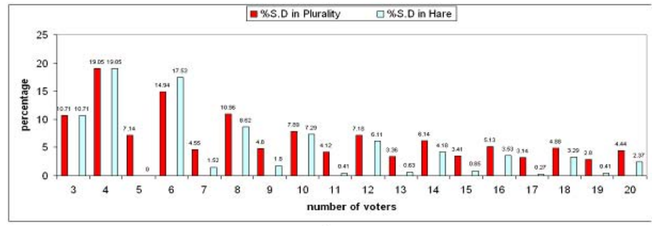
Now if, in the most optimistic possibility, the remaining score is again shared among the other $k-1$ candidates equally, the amount of Borda score for every other candidate is $\frac{(k-1)n+1}{2}$ which clearly is more than Borda score for x_1, and this is against x_1 being the social choice. In the end if the number of voters is an odd number and voting is done according to Borda count rule, SD will definitely not occur.

Now consider that 'n' is an even number, without loss of generality and also consider that x_1 is a social choice and there is SD in voting. So, in the most optimistic possibility, Borda score for x_1 is equal to: $\frac{n(k-1)}{2}$. Now, if this amount is subtracted from the whole Borda score gives:

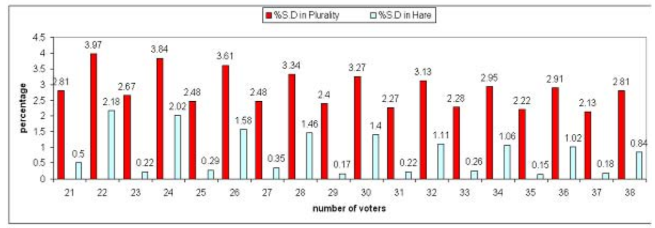
$$\frac{k(k-1)n}{2}-\frac{(k-1)n}{2}=\frac{(k-1)n(k-1)}{2}$$

If, in the most optimistic possibility, the remaining score again shared among the other $k-1$ candidates equally, the amount of Borda score for every other candidate would be $\frac{(k-1)n}{2}$ which is clearly equal to the Borda score for x_1 and thus the social choice set consists of all the candidates. Otherwise, if the remaining score is shared among every other candidate, x_1 can no longer be the social choice according to Borda count rule. The reason is that there is at least one candidate that has a score higher than that of x_1. □

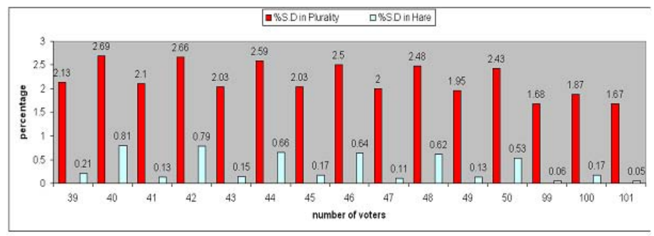
Based on our experiments, the chance of social disappointment is low in elections with large number of voters, where the number of candidates in the election is fixed. Experimental results of plurality and Hare system for *three* candidates and $3 \leq n \leq 101$ voters has been shown in Figure 2.



1. a



1. b



1. c

Figure 2: Percentage of the SD in the plurality rule and the Hare system with 3 candidates in elections.

4 EXPERIMENTAL RESULTS FOR RESISTANCE AGAINST MANIPULATION IN VOTING

Constructive Control by Adding/Deleting Voters' Ballots (second scenario): For this case, we removed randomly 20 percent of voters' ballots and then replaced all of them with a fixed random ballot. As shown in Figure 3, LU and LUR are more robust against manipulation in this constructive control manipulation scenario compared to other procedures. No meaningful difference can be seen relating to the performance of other seven protocols in this scenario. Figure 3 shows that the number of affected elections in this scenario is independent of the number of candidates.

REFERENCES

[1] Felix Brandt, Vincent Conitzer, and Ulle Endriss. 2012. Computational Social Choice. In *Multiagent Systems* (2nd ed.), Gerhard Weiss (Ed.). MIT Press, Cambridge, Massachusetts, London, England, Chapter 6, 213–283.

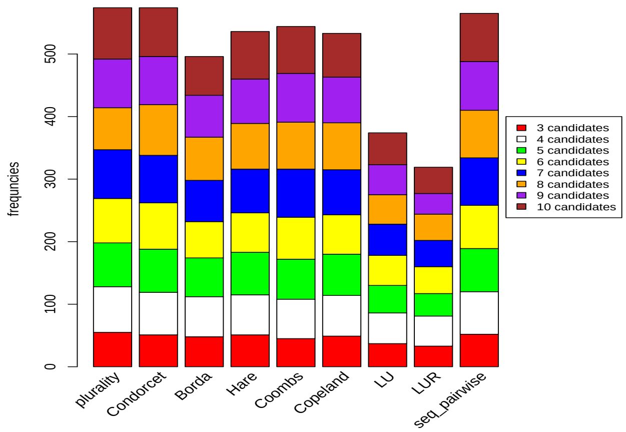


Figure 3: Performances of voting procedures against manipulation in the scenario of constructive control by deleting and replacing 20 percent of voters' ballots. LU and LUR present better performance than other methods, and also other procedures perform as bad as plurality rule in this scenario. Number of affected elections has been shown with different colors corresponding to the number of candidates.