THRESHOLD Q-VOTER MODEL

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STANDARD THRESHOLD MODEL

In this model, we consider an integer $0 \leq q_0 \leq q$. For a network of size N, one Monte Carlo step consists of N elementary events, where one event is defined as below:

1. An agent i is selected at random.
2. Other q agents are selected from agent i neighbors and their opinions are observed.
3. If there are at least $q_0 \times q$ contrary opinions then the opinion of agent i is inverted.
4. Otherwise, the opinion of agent i can also change with probability ϵ, except if all chosen neighbors have the same opinion as agent i.

OUR MODIFICATION OF THE STANDARD THRESHOLD MODEL

Here, we consider $0 \leq q_0 \leq 1$. For a network of size N, one Monte Carlo step consists of N elementary events, where one event is defined as below:

1. An agent i is selected at random.
2. Other q agents are selected from agent i neighbors and their opinions are observed.
3. A weighted proportion q_1 of contary opinion is calculated, using the number of common neighbors with agent i as weights.
4. If $q_1 > q_0$, then the opinion of agent i is inverted.
5. Otherwise, the opinion of agent i can also change with probability ϵ, except if all chosen neighbors have the same opinion as agent i.

MODEL PARAMETERS USED IN SIMULATIONS

- We consider two types of consensus: $q_0 = 4$ for standard model or $q_0 = \frac{2}{3}$ for modified model – bizantyne consensus, and $q_0 = 3$ for standard model or $q_0 = \frac{1}{2}$ for modified model – majority.
- Each agent faces a binary choice (positive and negative opinion). Initially, the opinion spread in the network is $1:1$.
- Both models were simulated on three types of networks:

  - $RG(N, 1)$,

  - $BA(N, 8)$,

  - $WS(N, 12, 0.2)$,

  where N – network size.
- We model time to reach the consensus magnetization in time, and final concentration as function of ϵ with maximum Monte Carlo step $MCS = 10^4$, all averaged over 100 independent runs.

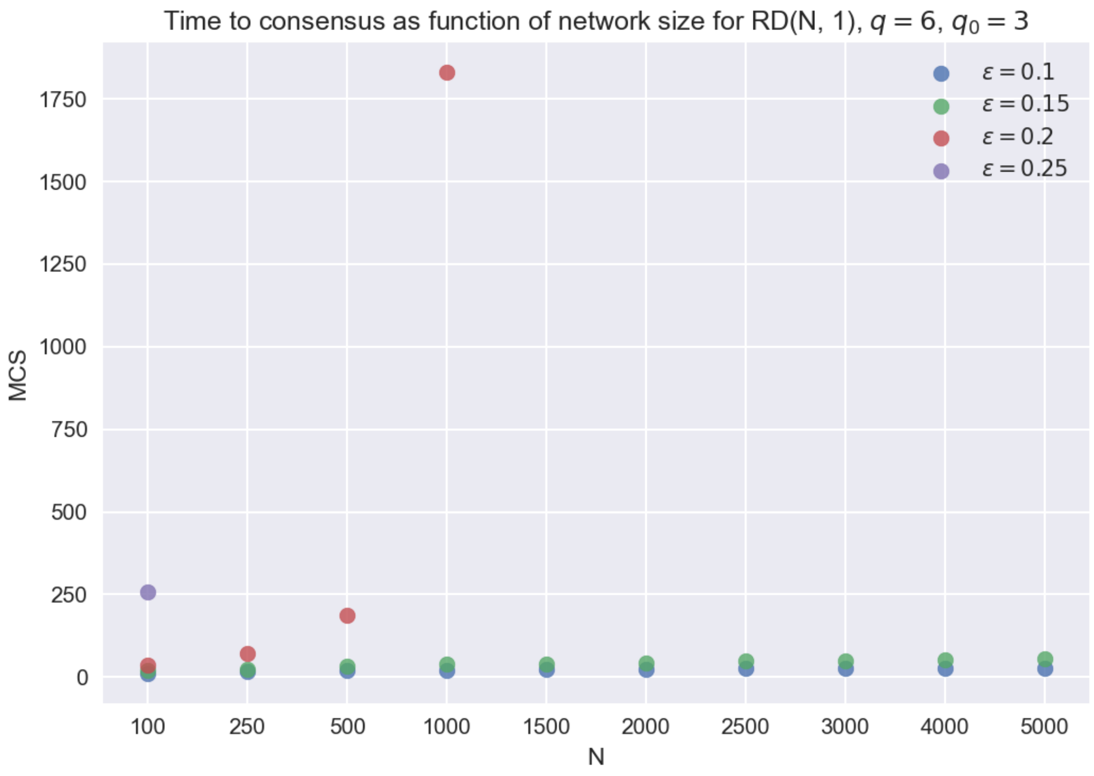
DEFINITIONS

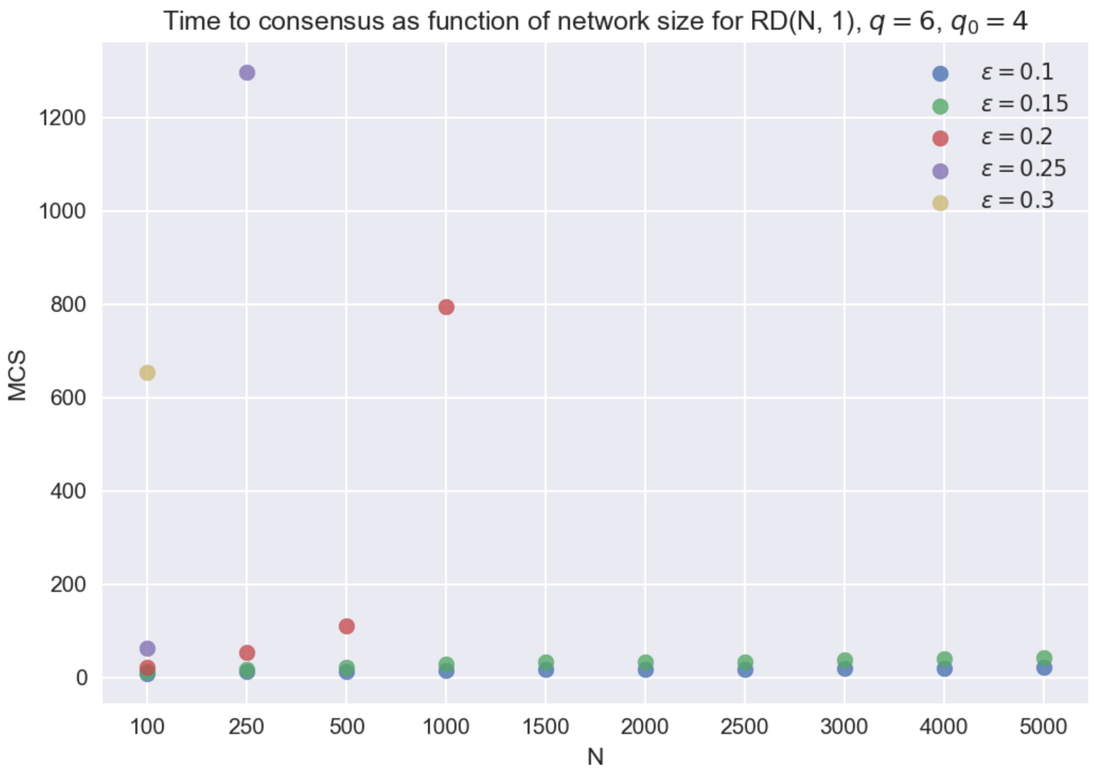
- Time to consensus – numer of Monte Carlo steps needed for one opinion to fully dominate the network.
- Magnetization – $\frac{1}{N}\sum_{i=1}^{N} S_i$, where $S_i \in \{-1, 1\}$ – opinion of agent i.
- Concentration of opinion α – $\frac{N_\alpha}{N}$, where N_α – number of agents with opinion α.

RESULTS

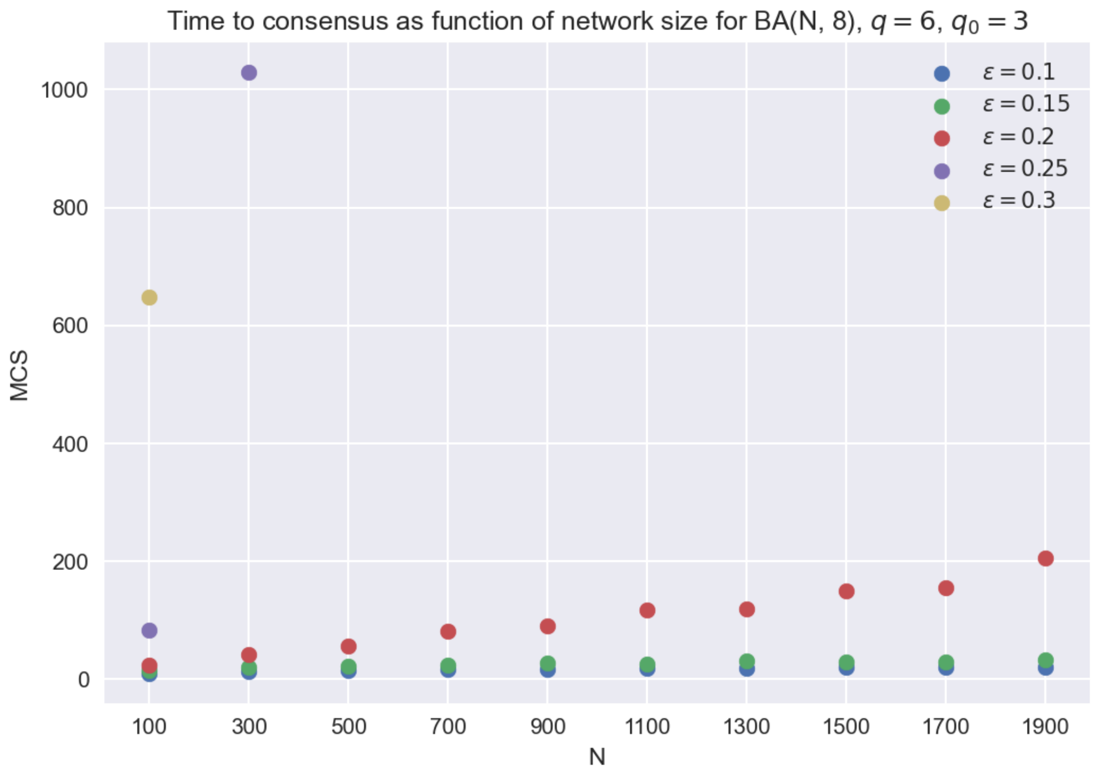
- Time to consensus
- Magnetization
- Concentration

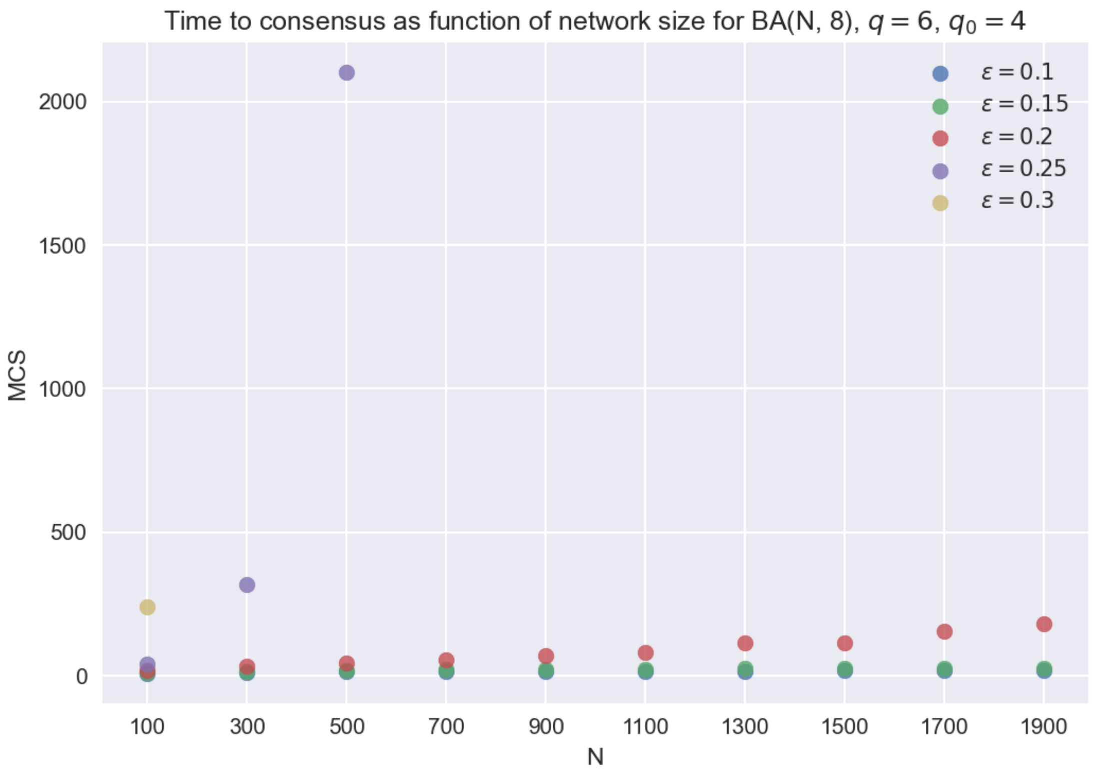
TIME TO CONSENSUS



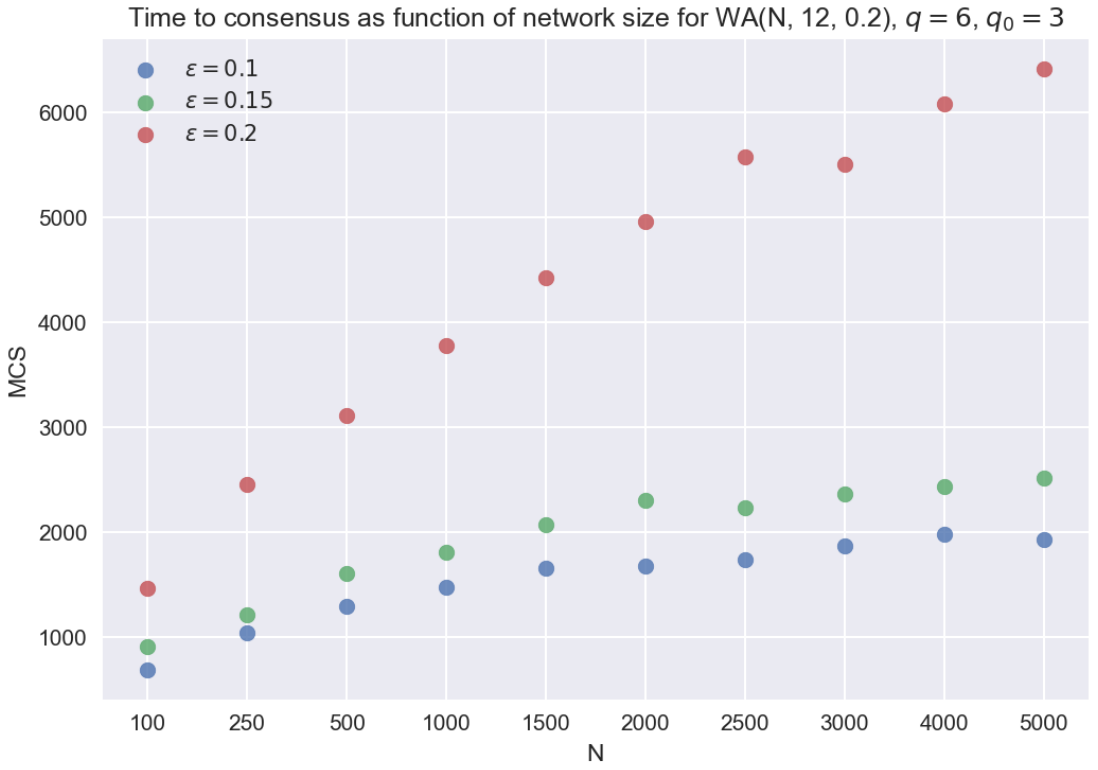


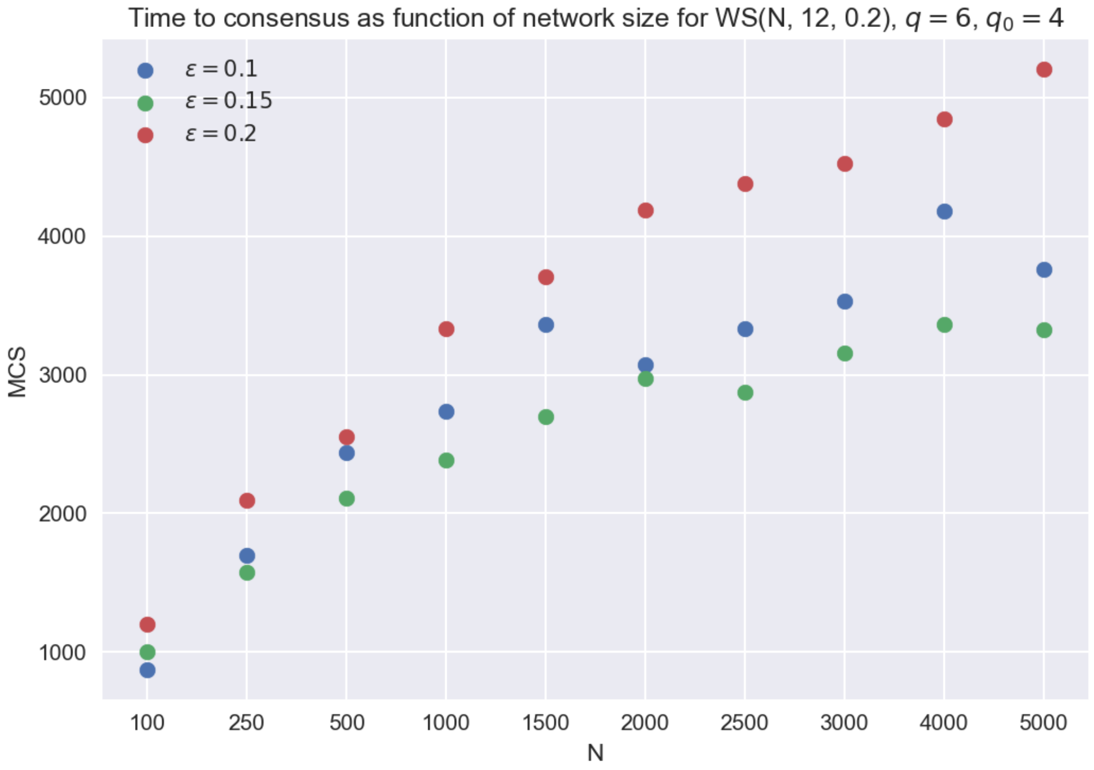
TIME TO CONSENSUS





TIME TO CONSENSUS

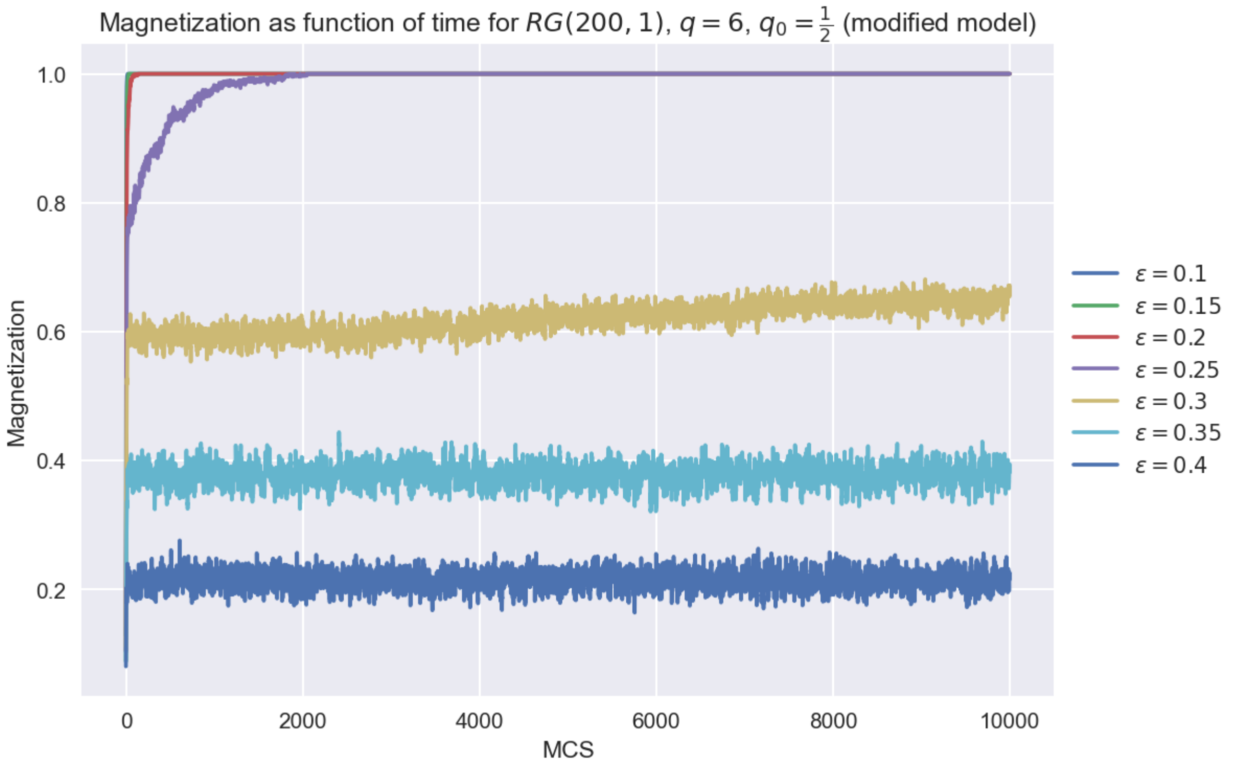




TIME TO CONSENSUS – CONCLUSIONS

- For each network, there exists a critical value of ϵ, below which (and included) the time to consensus grows with root of N.
- Above the critical ϵ, we observe that the relationship between time to consensus and network size becomes exponential.
- The model needs more time to reach the consensus for majority consensus than for byzantine consensus.
- In general, WS graph (which resembles the real social networks most accurately) needs more time to reach the consensus.

MAGNETIZATION



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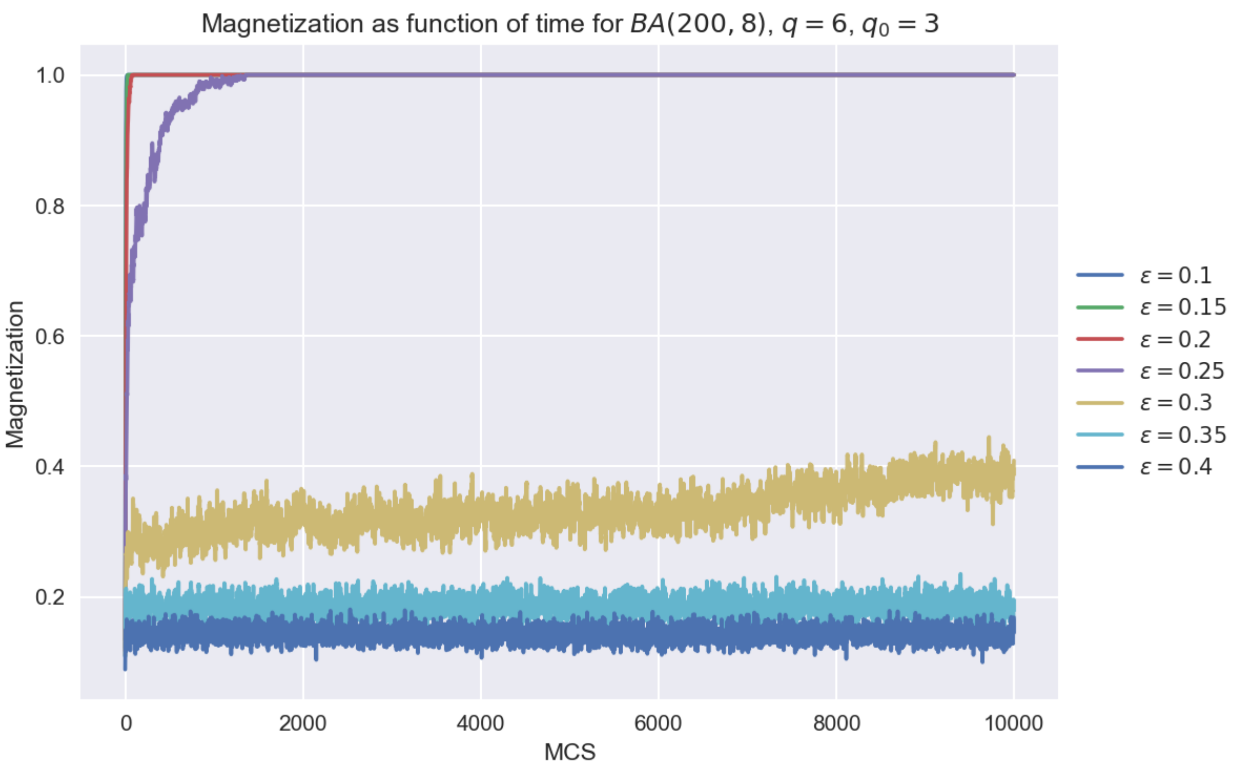

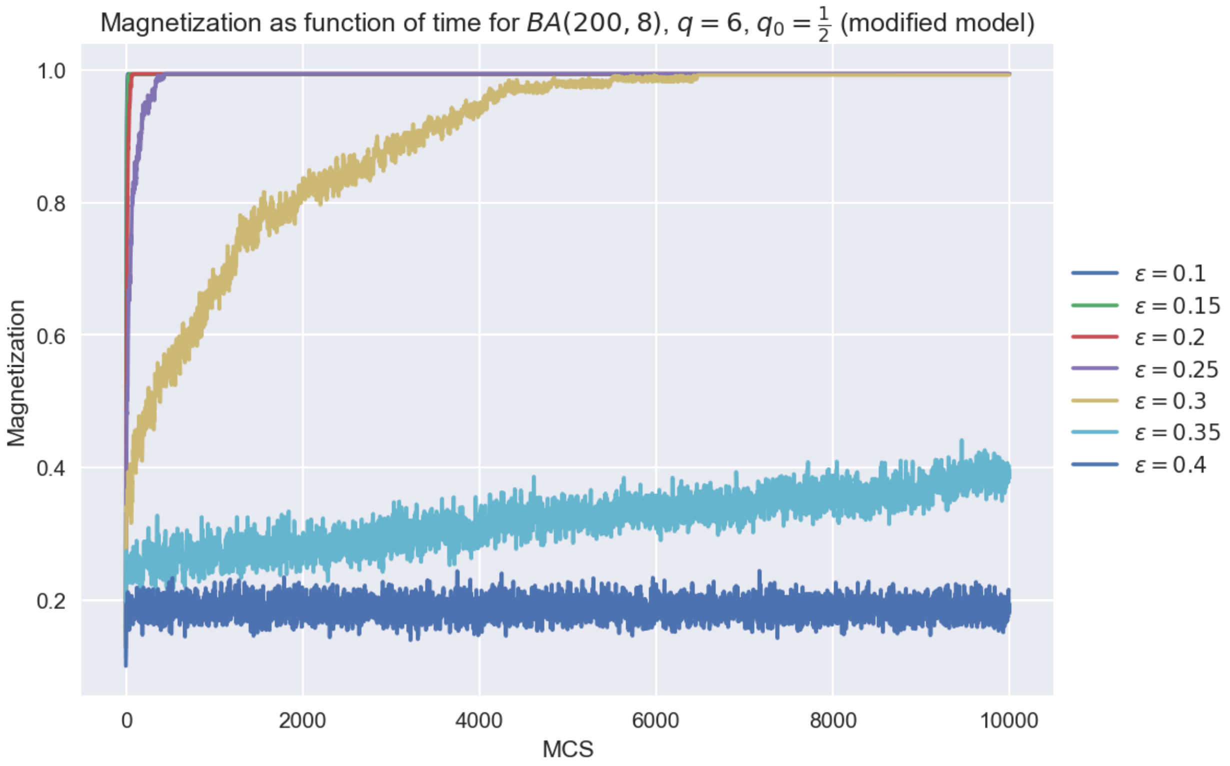

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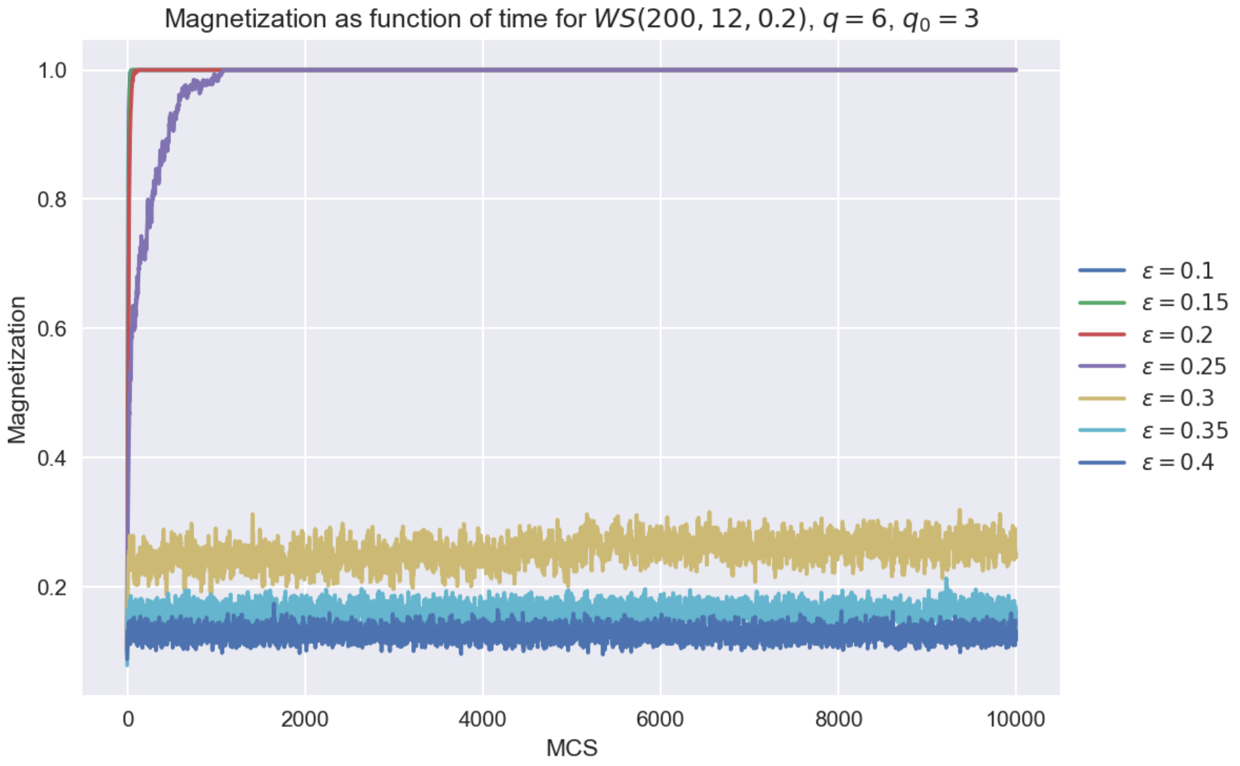

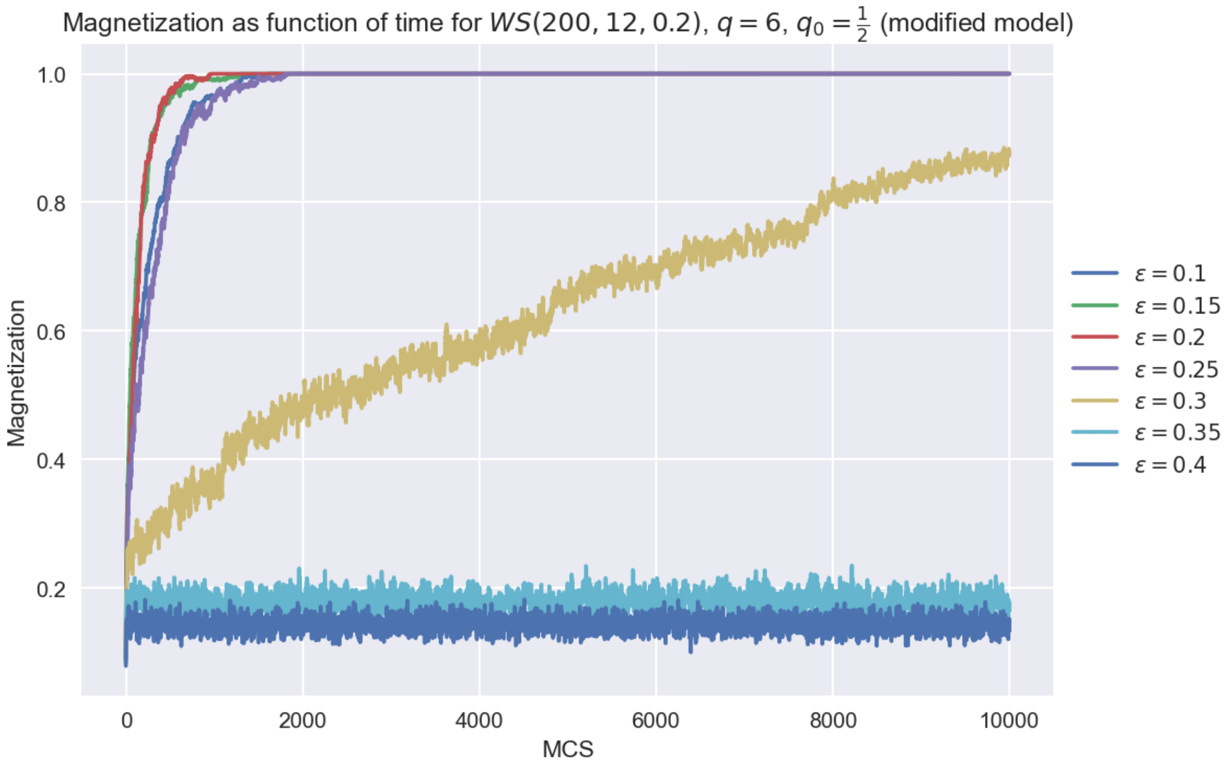

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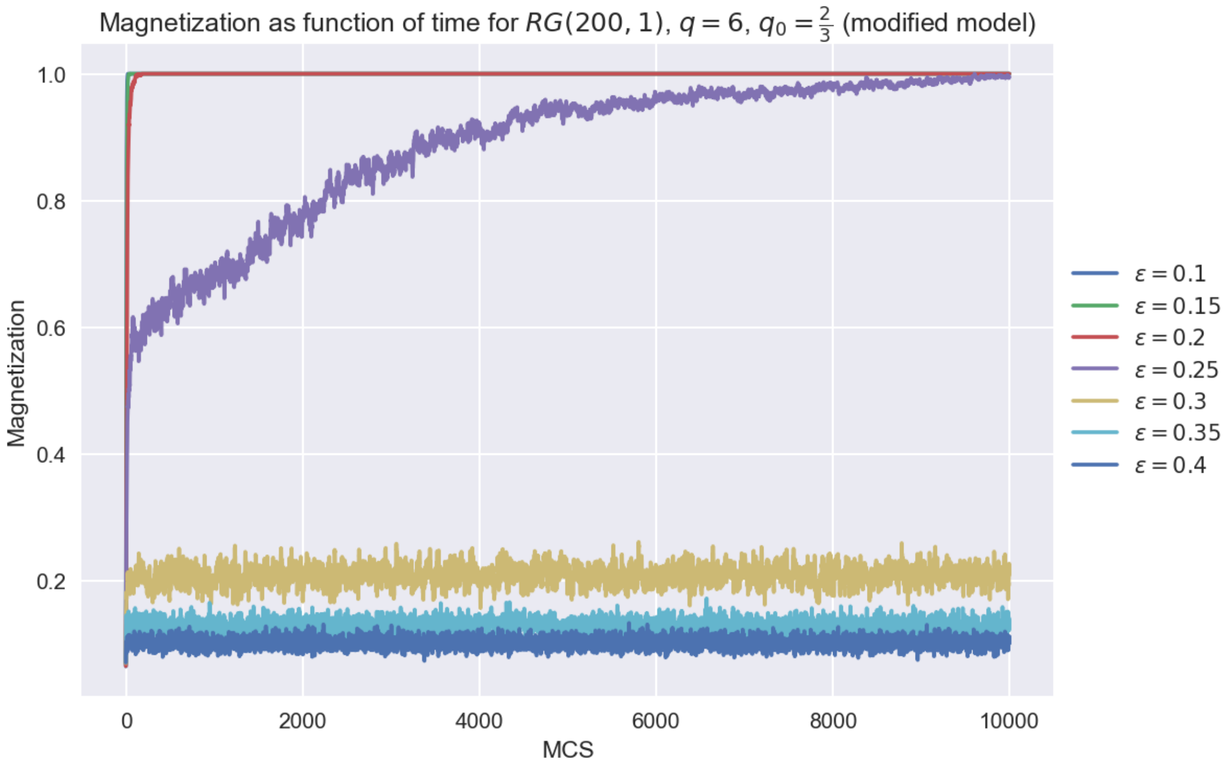

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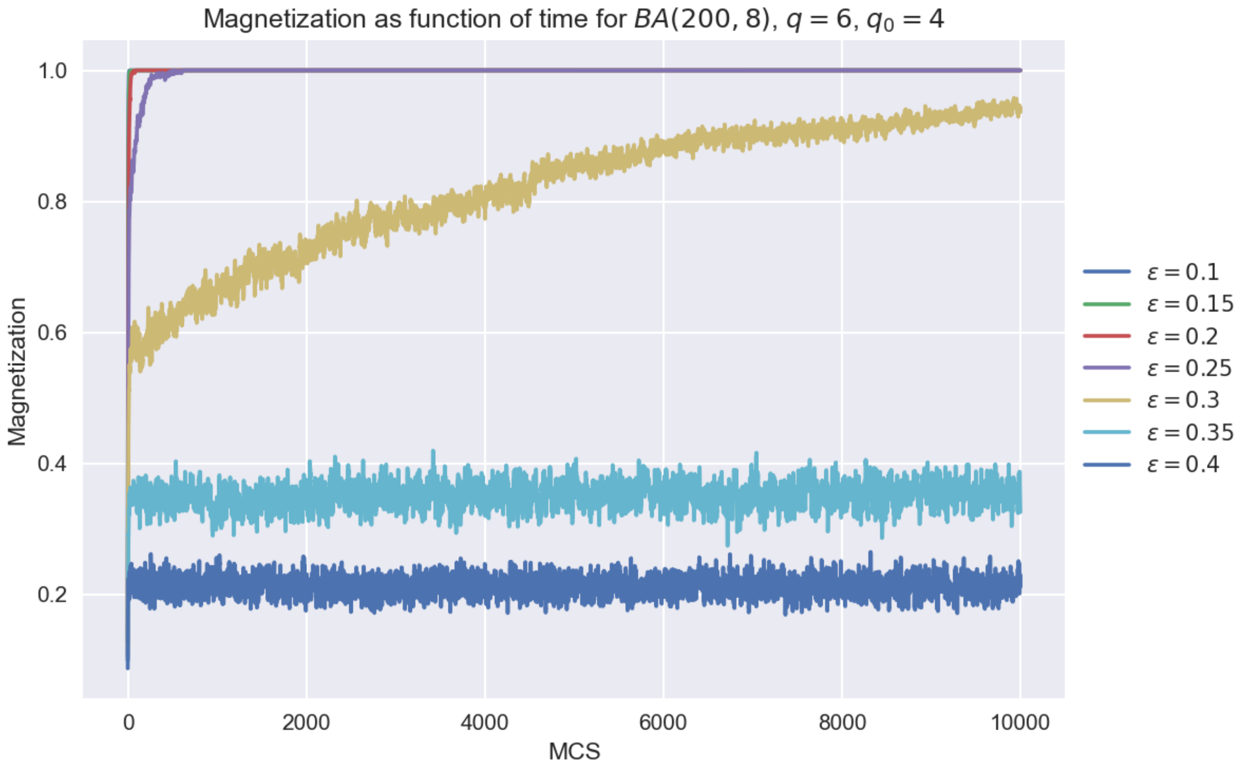

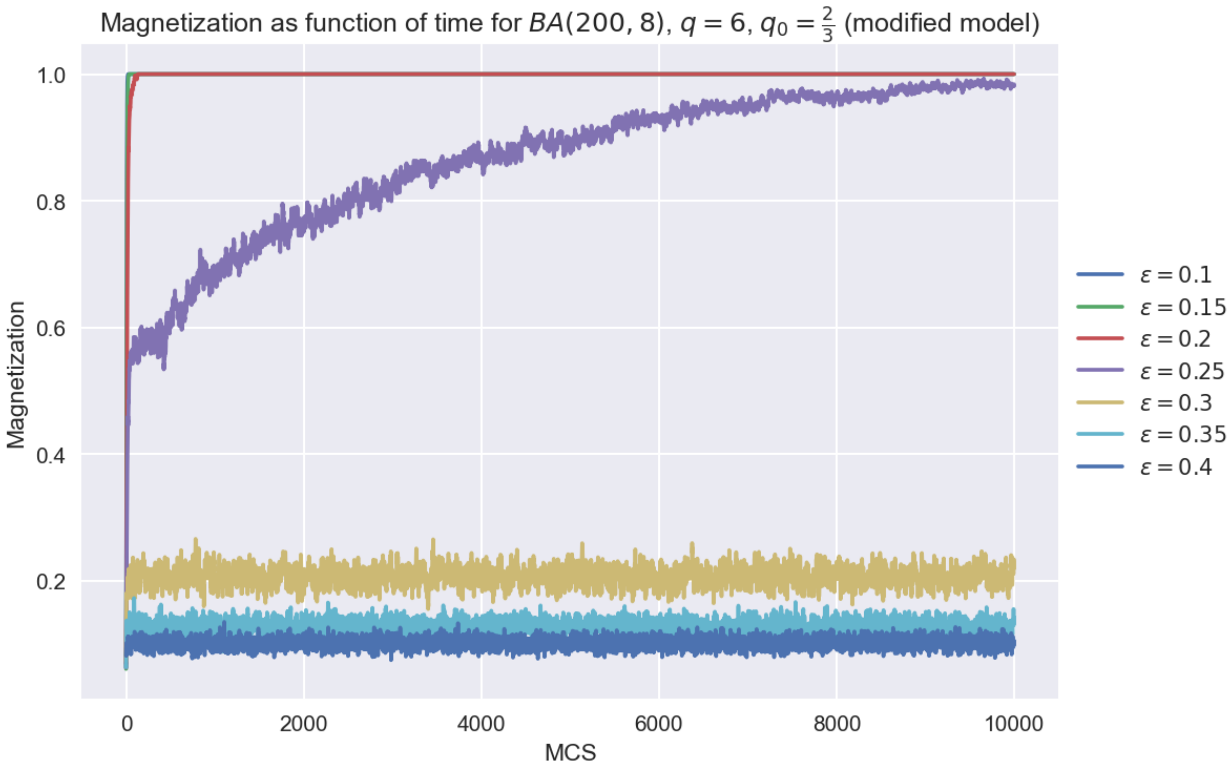

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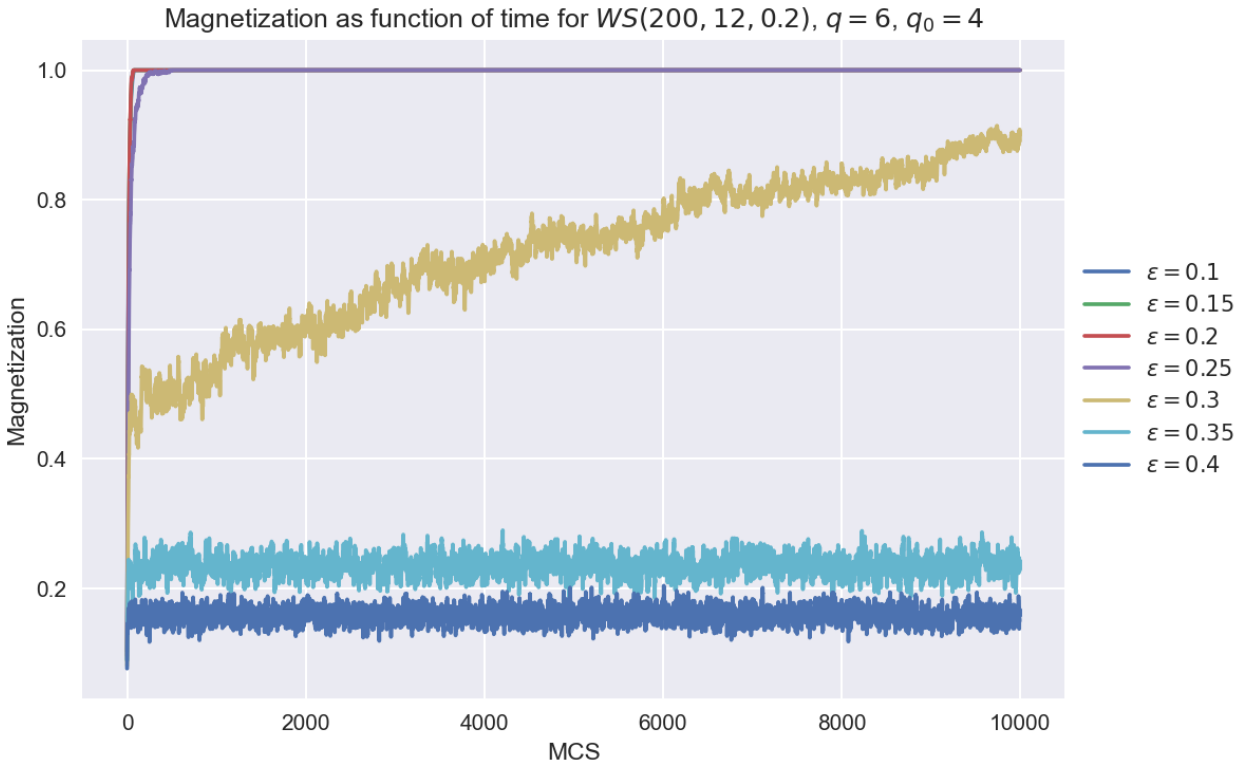
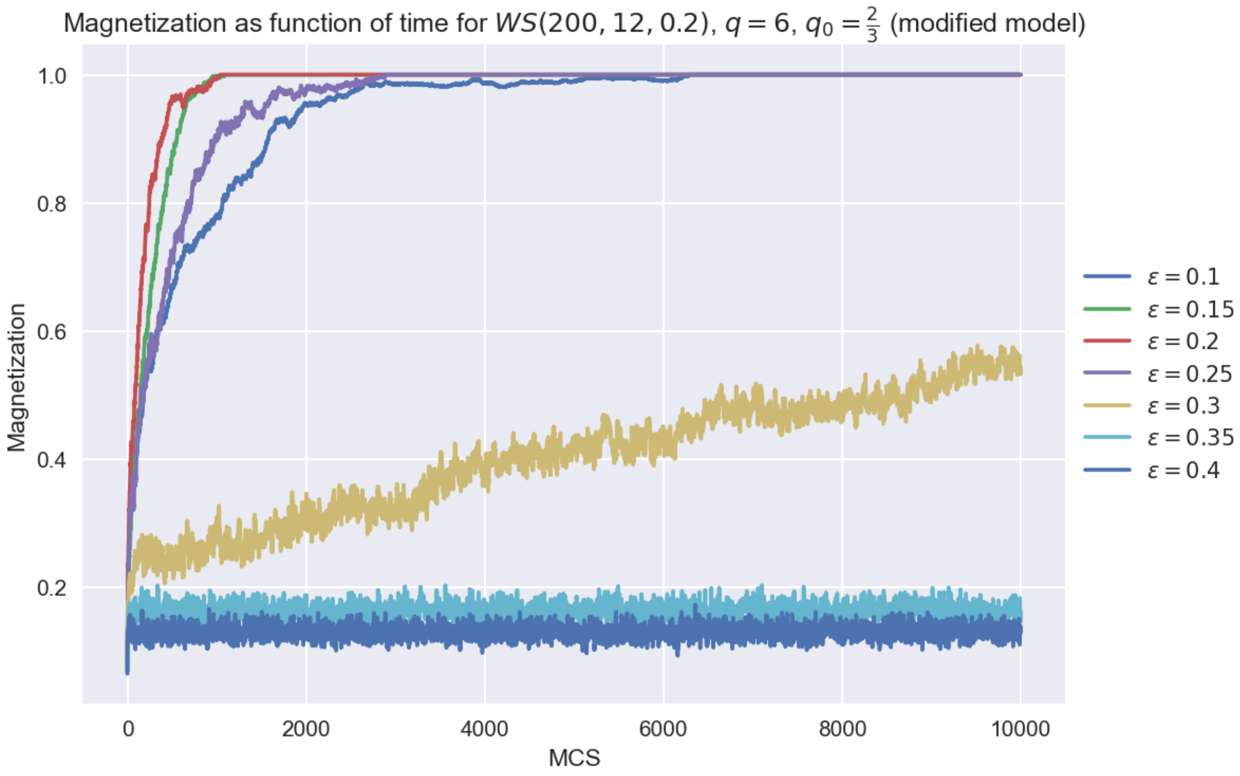
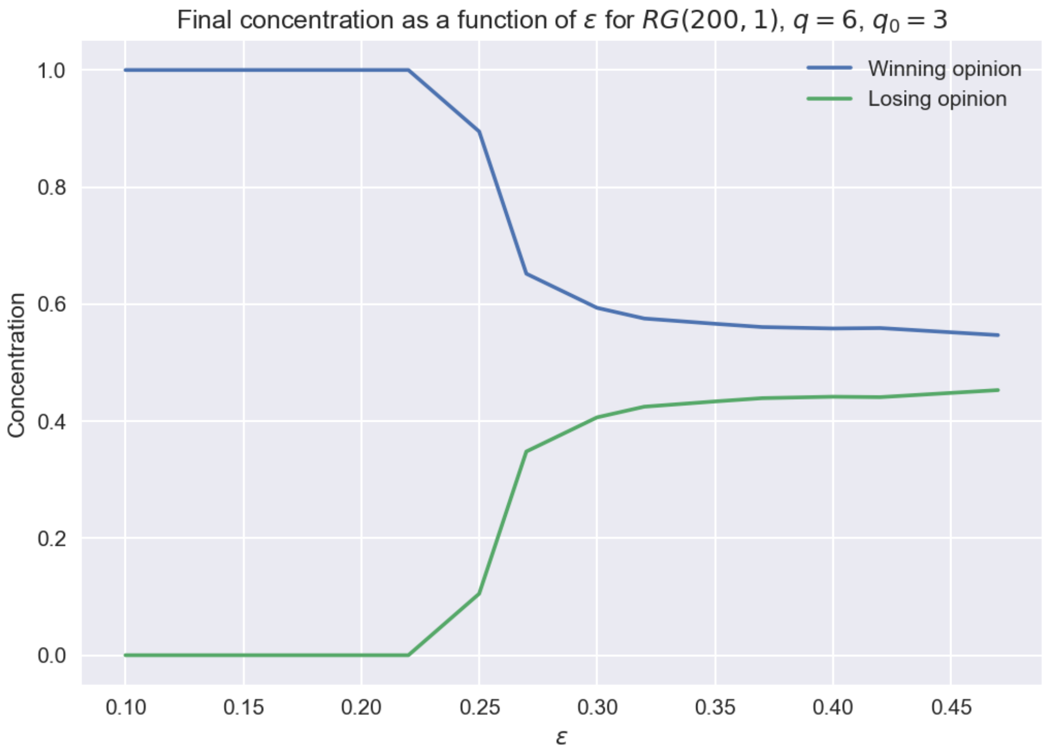


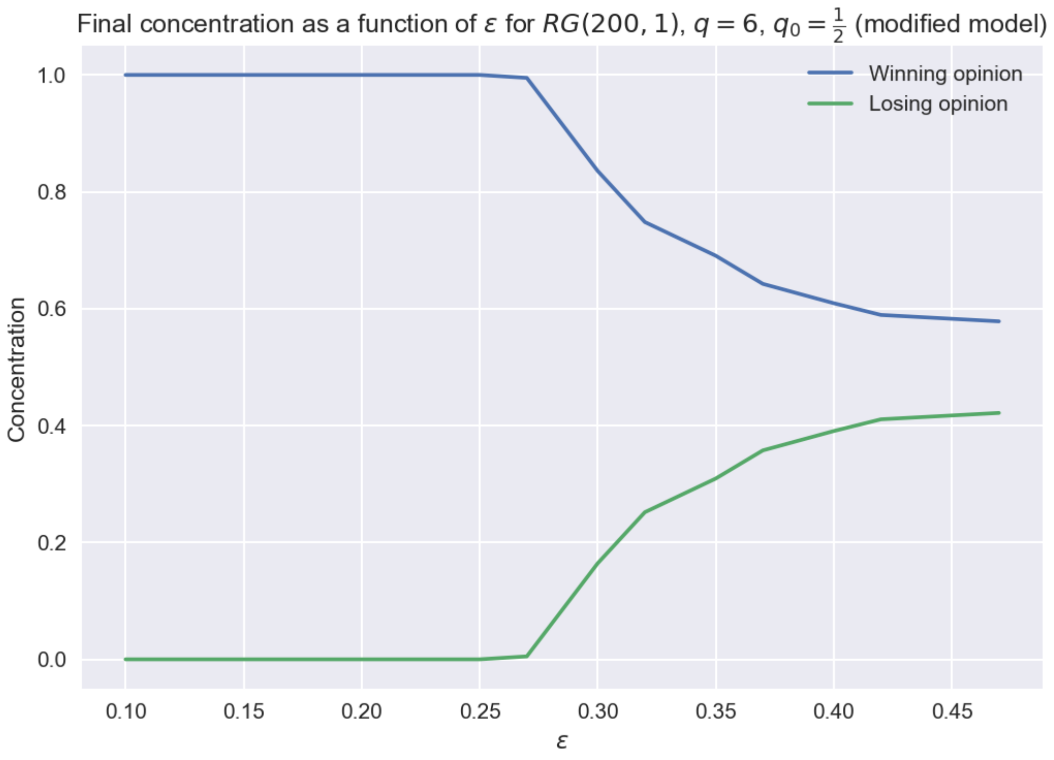


MAGNETIZATION – CONCLUSIONS

- Even if consensus is not reached, the system finds some stationary state.
- In few examples we did not observe any stable state, but we can expect that it would be evetually reached after passing the max step limit.
- In general, our modification to the standard model makes it harder for the system to stabilize and reach the consensus.
- The least stable of networks is WS, probably due to its very irregular structure.

CONCENTRATION





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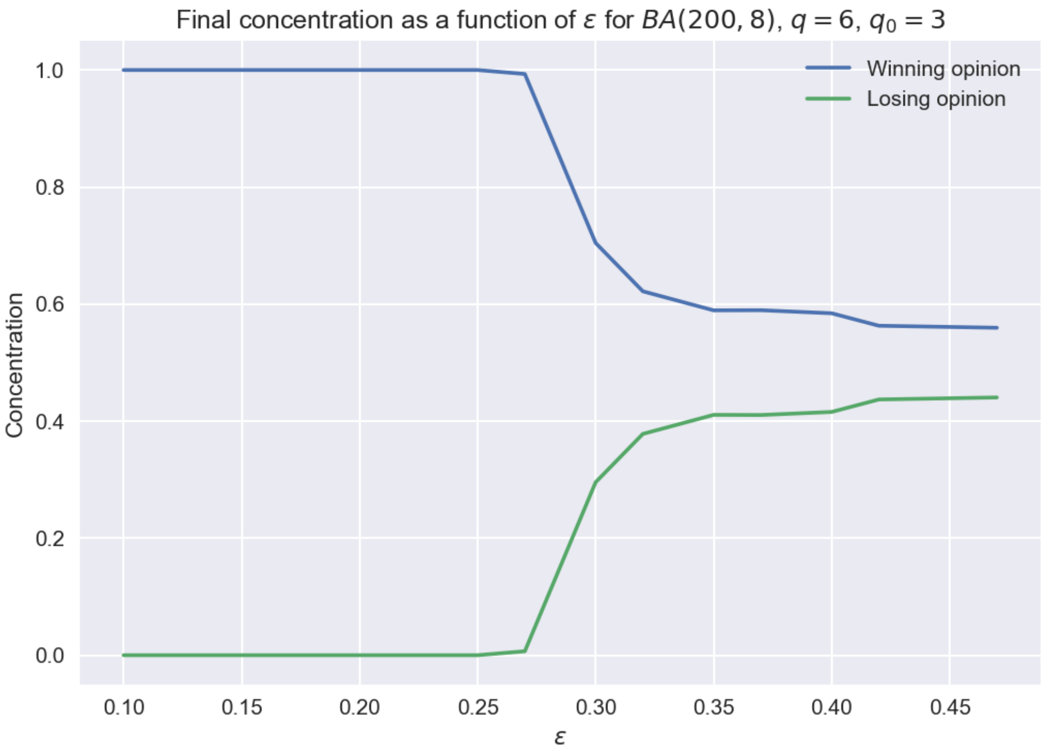

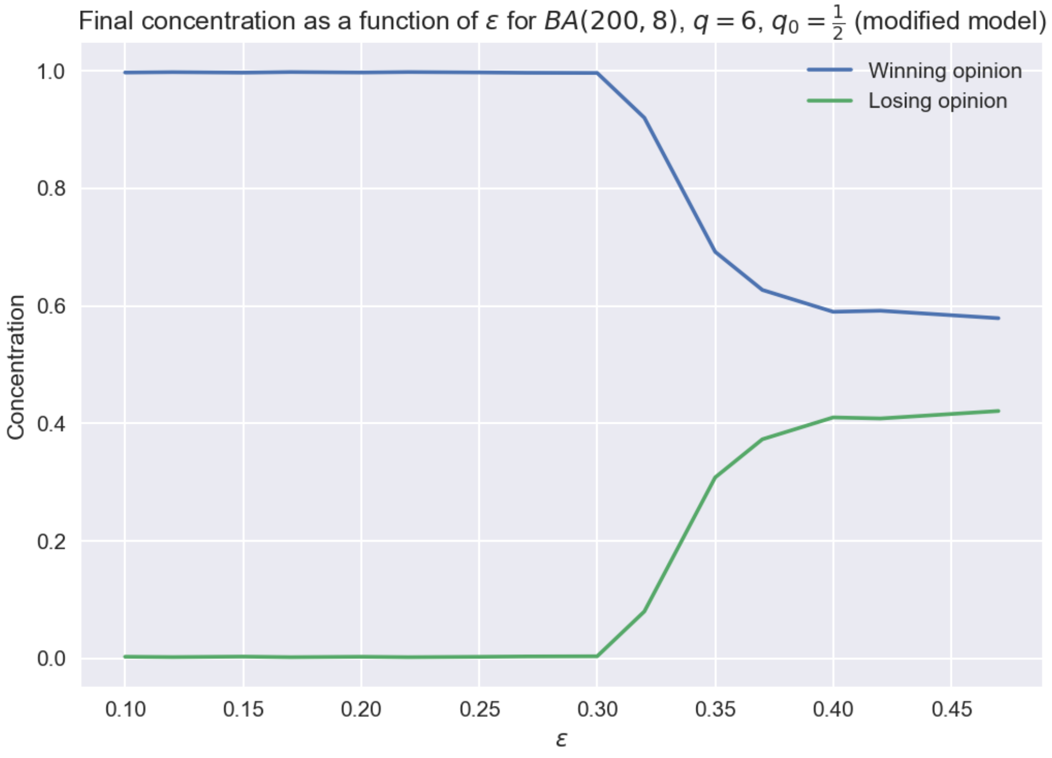

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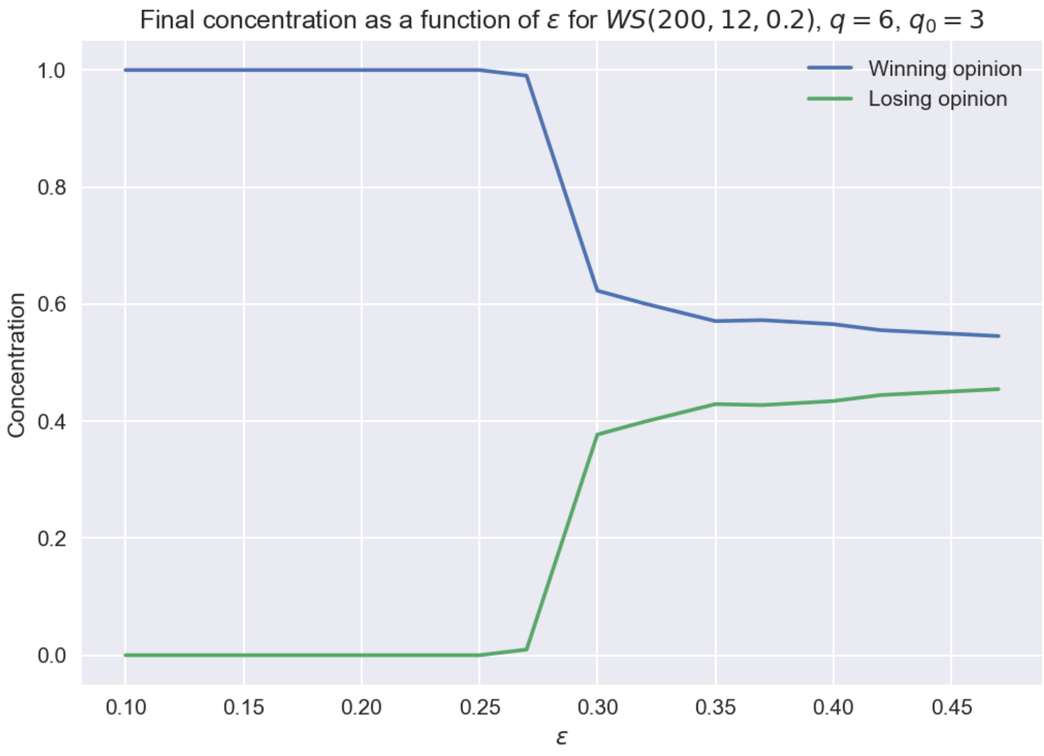

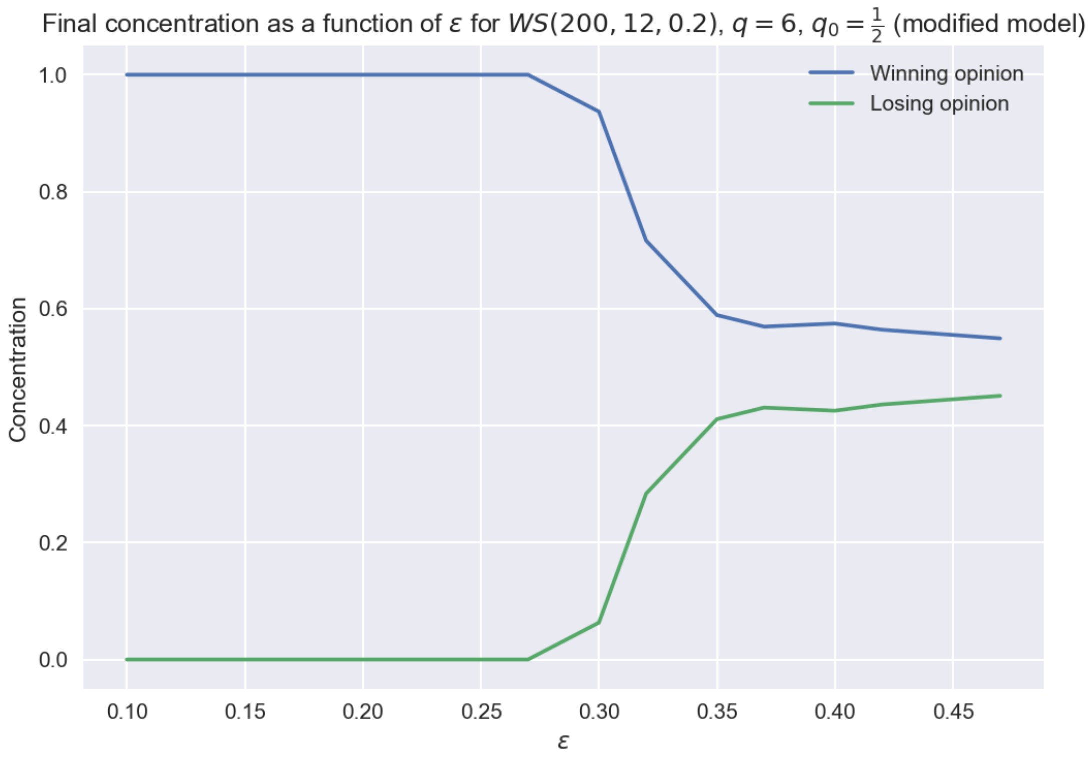

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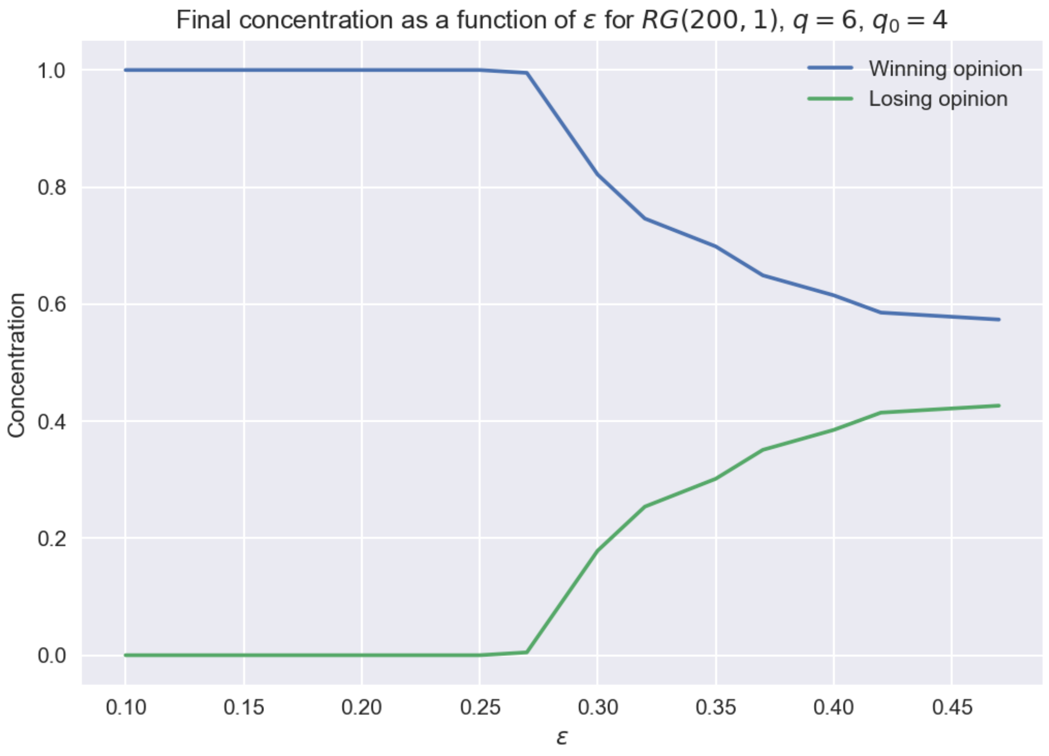

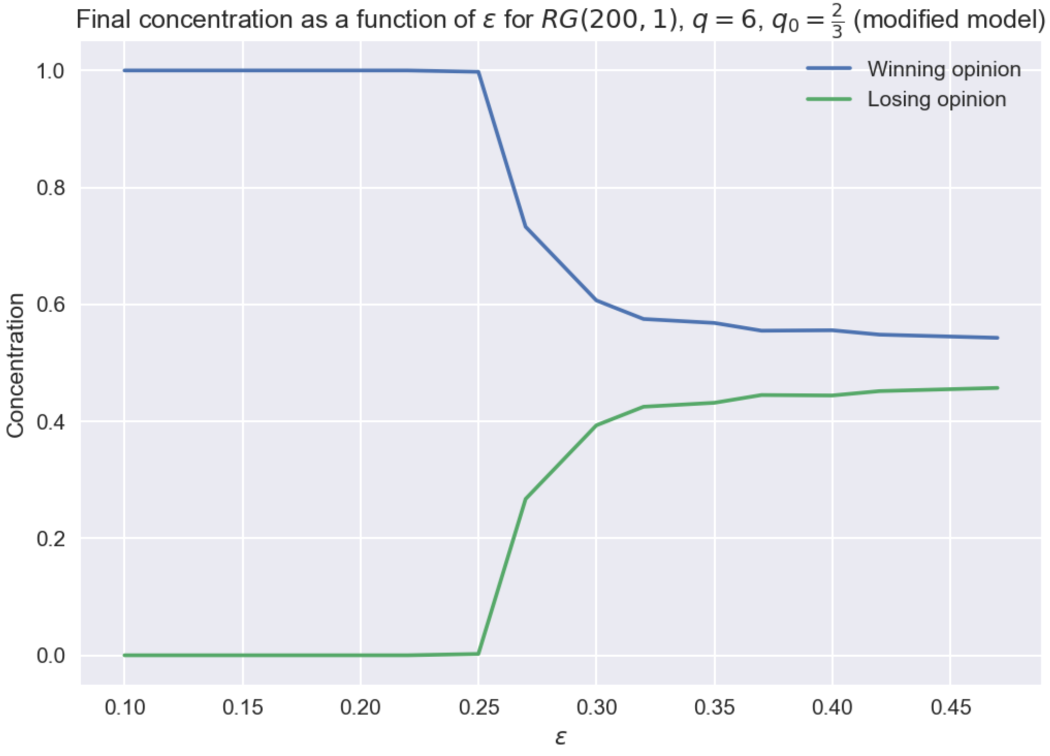

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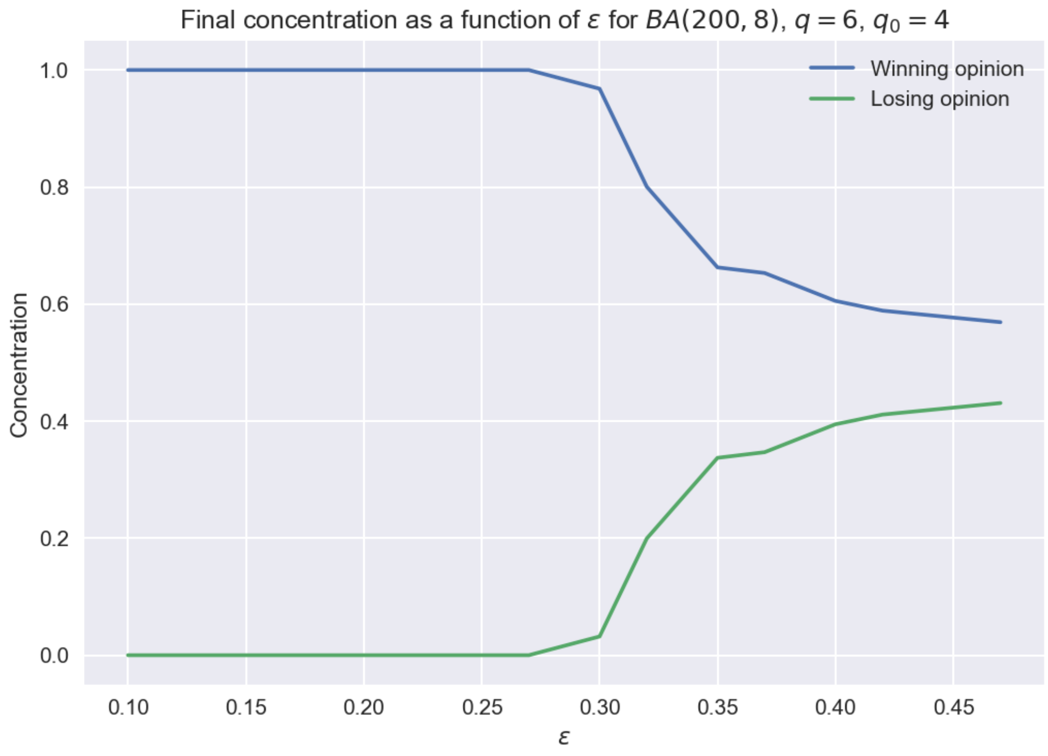

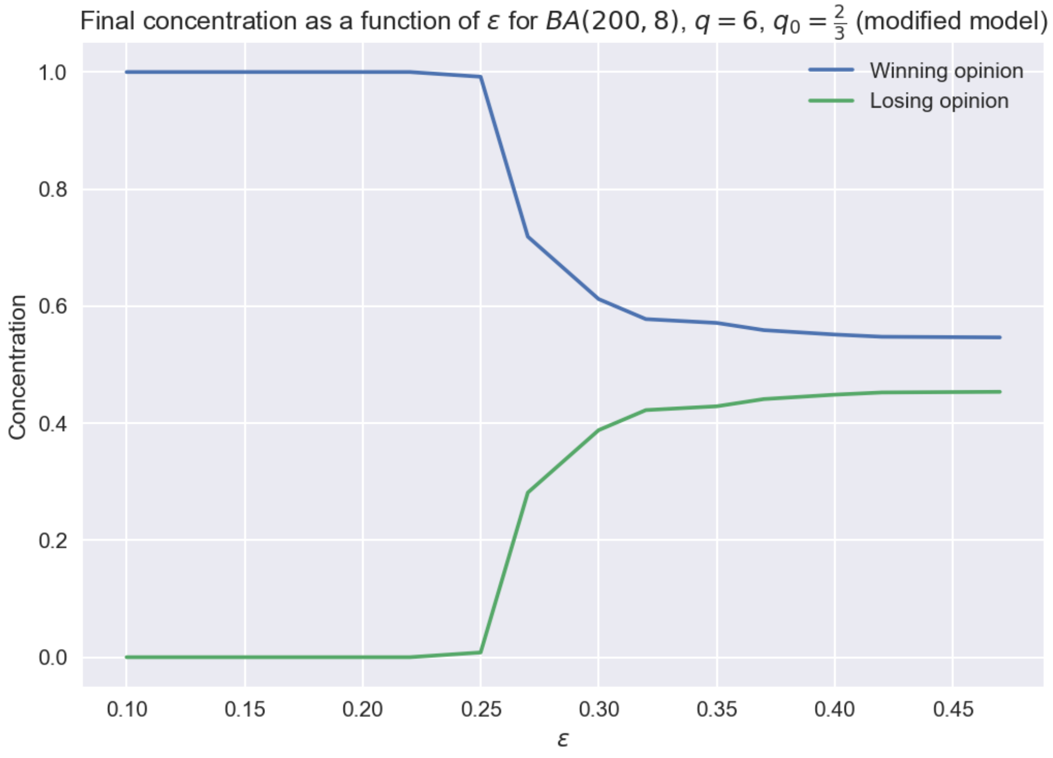

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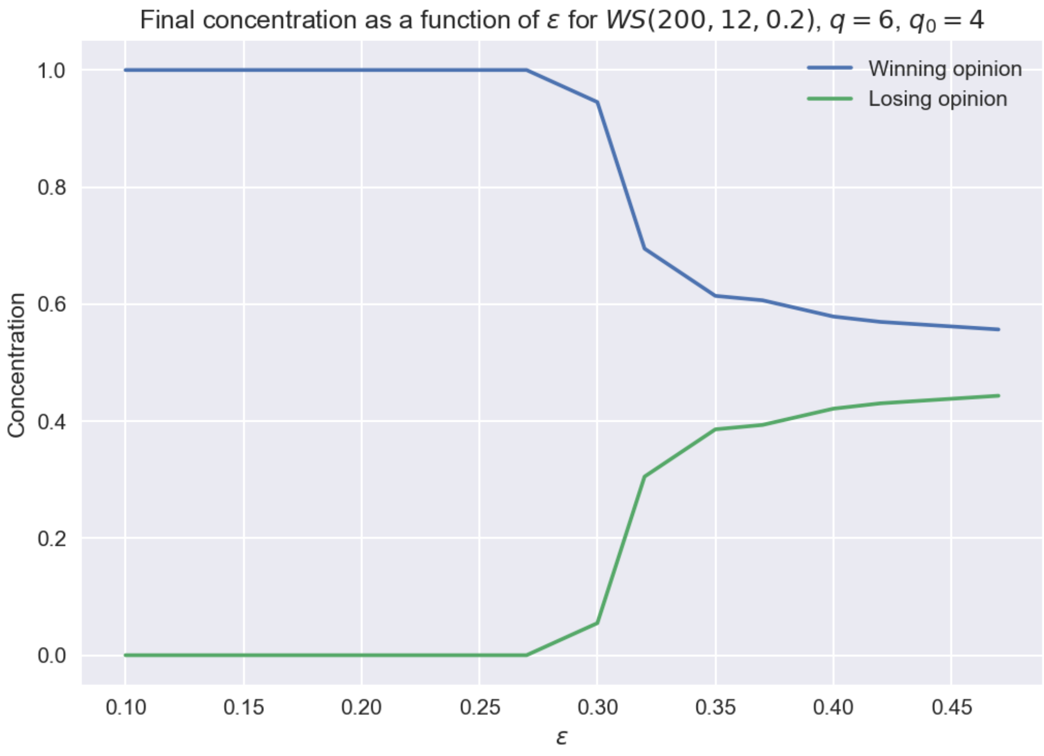
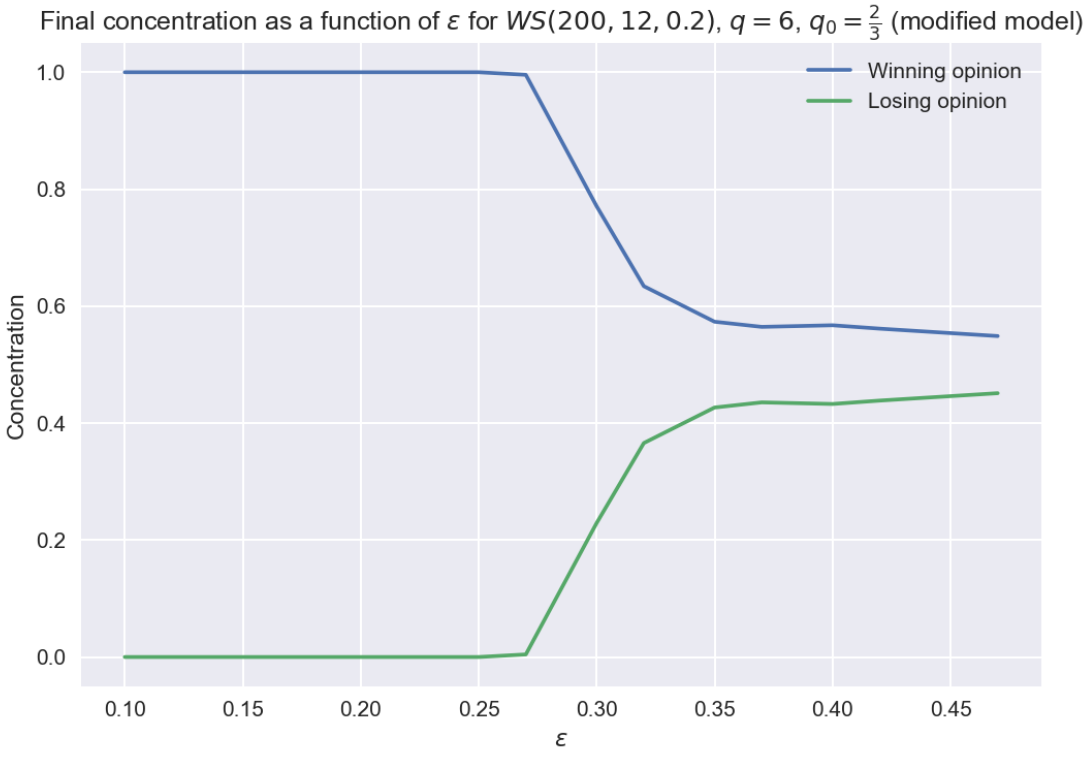
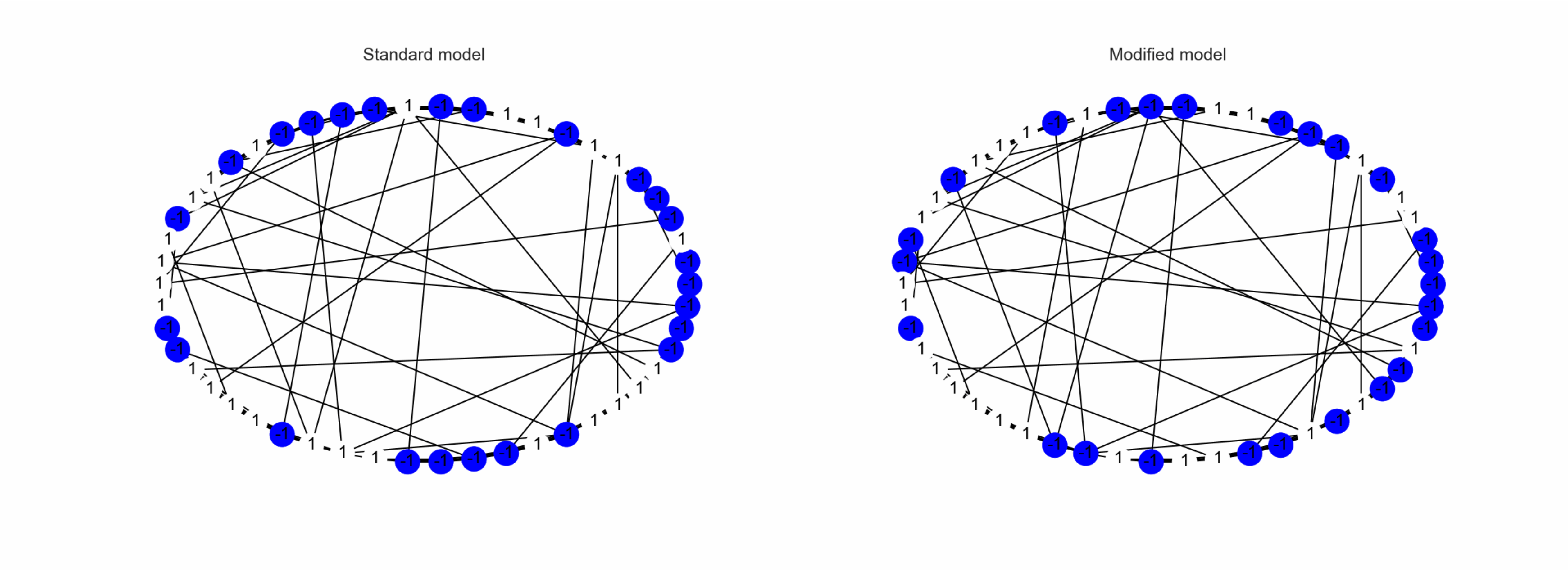




CONCENTRATION – CONCLUSIONS

- For small values of ϵ we always observe an absolute domination of the winning opinion.
- The decrease (with respect to ϵ) of the difference between winning and losing opinion proportion is slower for bigger values of q_0.
- We see that for bigger values of ϵ, the system remains very close to the initial state with opinions proportion $1:1$.
- Our modified model starts to converge to the initial state faster (for smaller values of ϵ) than the standard model.

VISUALISATION OF BOTH MODELS ON WS(50, 6, 0)



THANK YOU

CONTRIBUTION TO THE PROJECT:

- IDEAS, COMPUTATION – ALL
- PRESENTATION – ADA AND PAWEŁ
- SPEAKING – KASIA

BIBLIOGRAPHY

1. Allan R. Vieira, Celia Anteneodo (2018) *Threshold q-voter model*
2. Bartłomiej Nowak, Bartosz Stoń, Katarzyna Sznajd-Weron (2021) *Discontinuous phase transitions in the multi-state noisy q-voter model: quenched vs annealed disorder*