Robotika in računalniško zaznavanje (RRZ)

Regije

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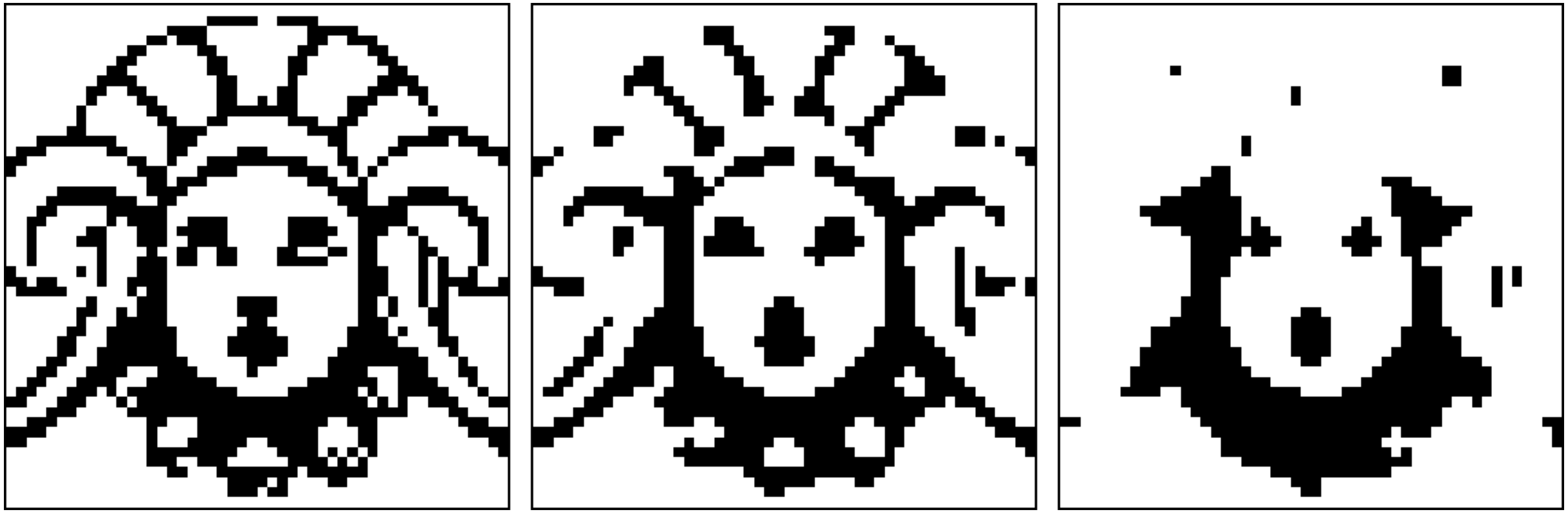
Literatura: W. Burger, M. J. Burge (2008).
Digital Image Processing, poglavja 10, 11

v1.0

Morfološki filtri

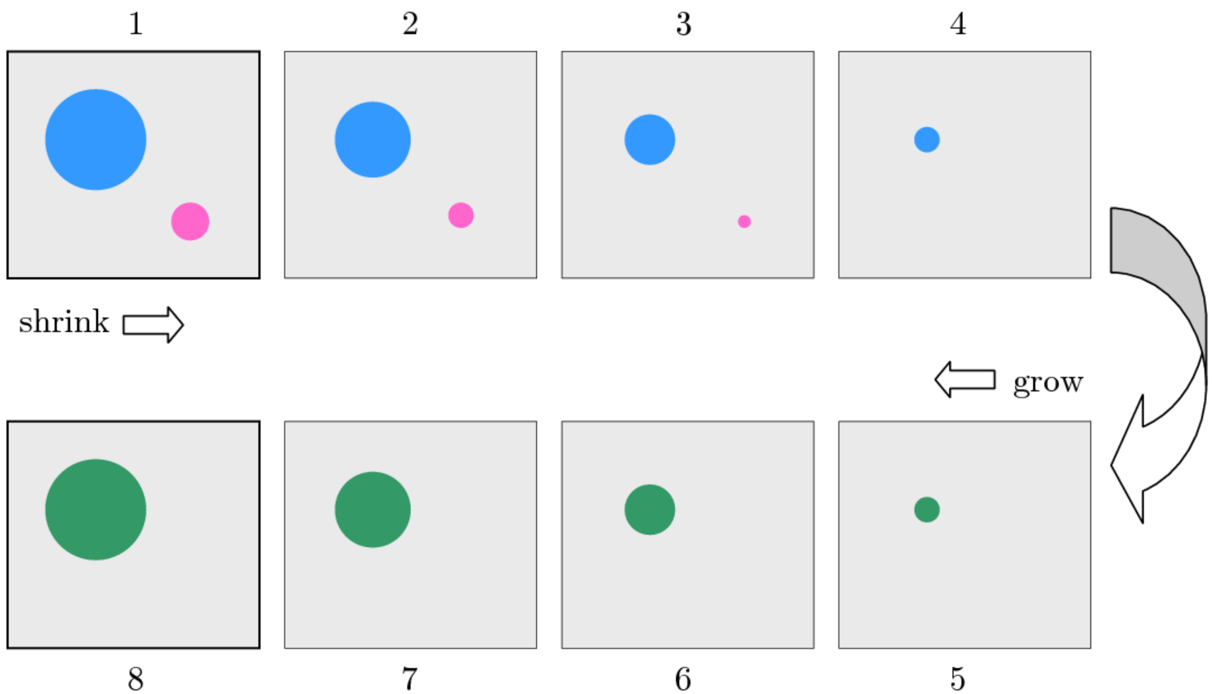
- Linearni filtri ne spreminjajo topologije slike
- Medianin filter deloma spremeni strukturo slike:



- Morfološki filtri so namenjeni ravno spreminjanju lokalne strukture slike
  - Odstranjevanje majhnih elementov slike
  - Polnjenje lukenj
  - Iskanje obrisov

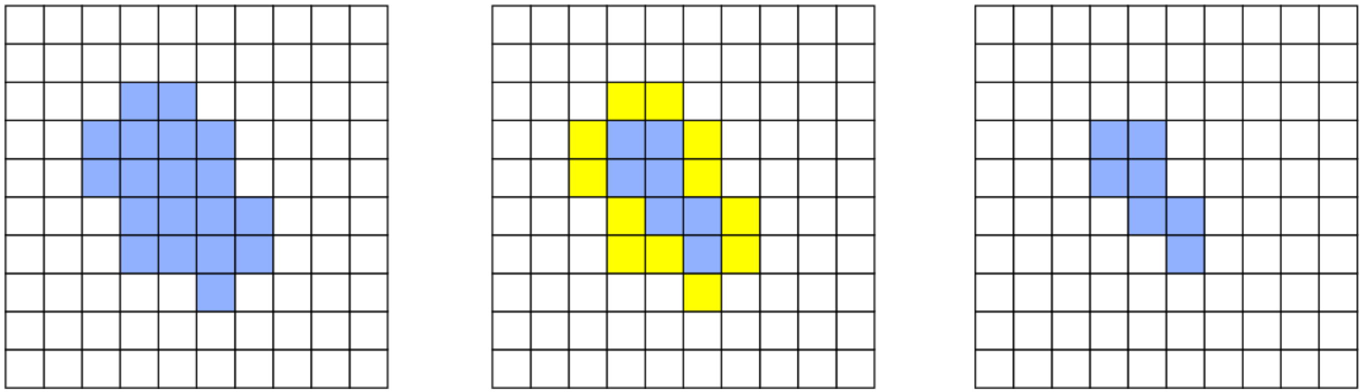
Krčenje in rast

- Odstranjevanje majhnih elementov na sliki
- Algoritem:
  1. Vse strukture skrči, tako da se odstranijo zunanji deli struktur
     - Na ta način se izgubijo majhne strukture
  2. Preostale struture naj spet zrastejo – se dodajajo zunanje plasti
     - Dosežejo približno enako velikost in obliko kot na začetku

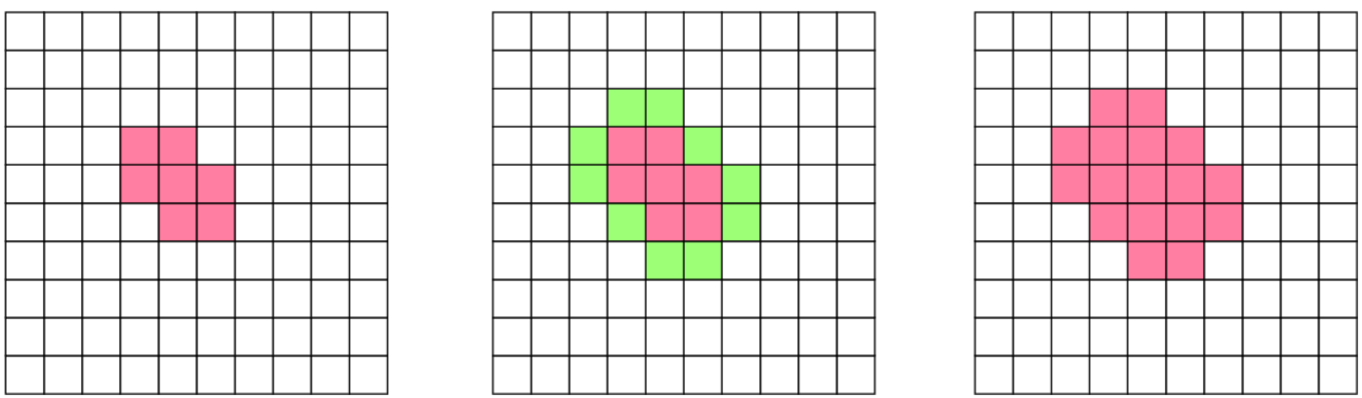


Krčenje in rast

- Krčenje
  - Odstrani se zunanja plast slikovnih elementov

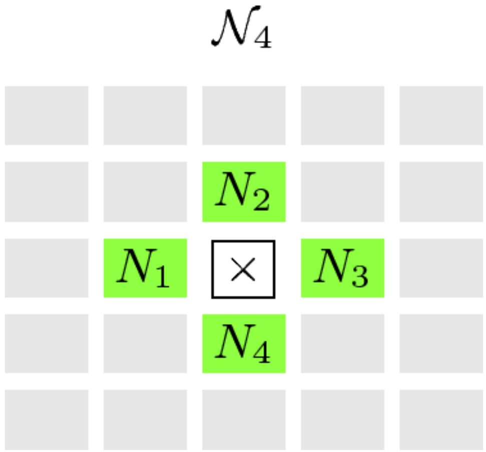


- Rast
  - doda se zunanja plast slikovnih elementov



Sosednost slikovnih elementov

- 4-sosednost



- 8-sosednost

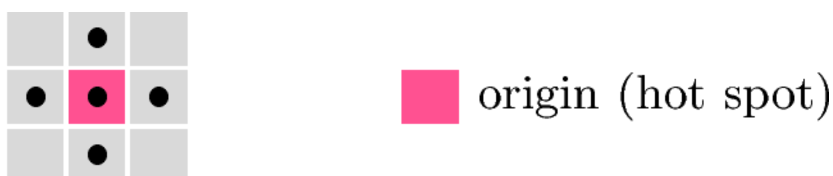
Osnovne morfološke operacije

- Krčenje in Rast (Shrinking in Growing)
- Erozija in Širitev (Erosion, Dilation)
  - Bolj splošne
  - Definirane s strukturnim elementom

- Strukturni element
  - Definira lastnosti morfološkega filtra
  - Matrika binarnih vrednosti

$$H(i,j) \in \{0,1\}$$

$H =$

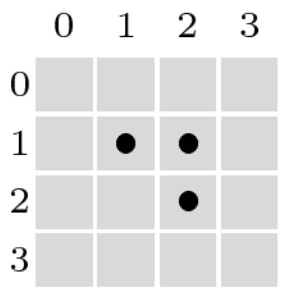


Množice točk

- Morfološke operacije ponavadi izvajamo na binarnih slikah
  $$I(u,v) \in \{0,1\}$$
- Binarne slike in strukturni element lahko predstavimo z množico točk (parov koordinat $\boldsymbol{p} = (u,v)$):

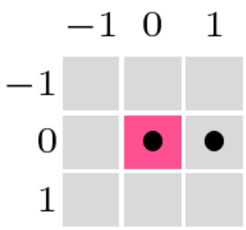
$$\mathcal{Q}_I = \{\boldsymbol{p} \mid I(\boldsymbol{p}) = 1\}$$

I



$$I \equiv \mathcal{Q}_I = \{(1,1),(2,1),(2,2)\}$$

H



$$H \equiv \mathcal{Q}_H = \{(0,0),(1,0)\}$$

Osnovne binarne operacije

- Invertiranje - komplementarna množica

$$\mathcal{Q}_{\bar{I}} = \bar{\mathcal{Q}}_I = \{\boldsymbol{p} \in \mathbb{Z}^2 \mid \boldsymbol{p} \notin \mathcal{Q}_I\}$$

- ALI – unija

$$\mathcal{Q}_{I_1 \vee I_2} = \mathcal{Q}_{I_1} \cup \mathcal{Q}_{I_2}$$

- Translacija – prištevanje

$$I_{\boldsymbol{d}} \equiv \big\{(\boldsymbol{p} + \boldsymbol{d}) \mid \boldsymbol{p} \in I\big\}$$

- Zrcaljenje - negacija

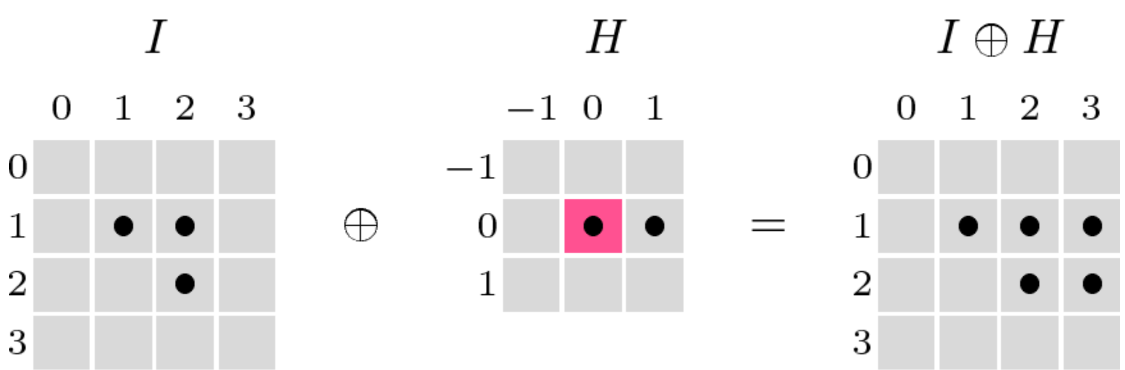
$$H^* \equiv \{-\boldsymbol{p} \mid \boldsymbol{p} \in H\}$$

Širitev

- Iz Rasti - strukturni element se razmnoži na vsakem slikovnem elementu ospredja

$$I \oplus H \equiv \{(\boldsymbol{p} + \boldsymbol{q}) \mid \text{for some } \boldsymbol{p} \in I \text{ and } \boldsymbol{q} \in H\}$$

$$I \oplus H \equiv \bigcup_{\boldsymbol{p} \in I} H_{\boldsymbol{p}} = \bigcup_{\boldsymbol{q} \in H} I_{\boldsymbol{q}}$$



$$I \equiv \{(1,1), (2,1), (2,2)\}, \;\; H \equiv \{(\mathbf{0},\mathbf{0}), (\mathbf{1},\mathbf{0})\}$$

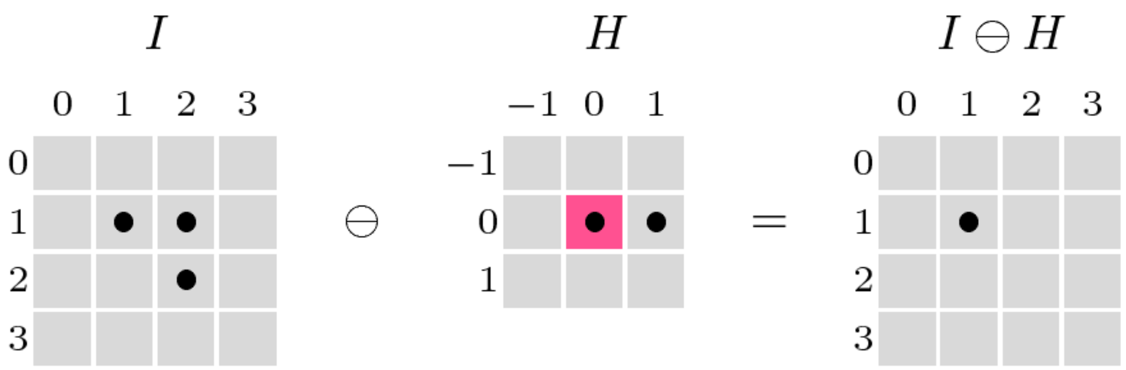
$$\begin{aligned} I \oplus H \;\; \equiv \;\; \{\; & (1,1) + (\mathbf{0},\mathbf{0}),\; (1,1) + (\mathbf{1},\mathbf{0}), \\ & (2,1) + (\mathbf{0},\mathbf{0}),\; (2,1) + (\mathbf{1},\mathbf{0}), \\ & (2,2) + (\mathbf{0},\mathbf{0}),\; (2,2) + (\mathbf{1},\mathbf{0}) \;\} \end{aligned}$$

Erozija

- Iz Krčenja - kvazi-inverz Širitve

$$I \ominus H \equiv \{\boldsymbol{p} \in \mathbb{Z}^2 \mid (\boldsymbol{p} + \boldsymbol{q}) \in I, \text{ for every } \boldsymbol{q} \in H\}$$

$$I \ominus H \equiv \{\boldsymbol{p} \in \mathbb{Z}^2 \mid H_{\boldsymbol{p}} \subseteq I\}$$



$$I \equiv \{(1,1),(2,1),(2,2)\},\ H \equiv \{(\mathbf{0},\mathbf{0}),(\mathbf{1},\mathbf{0})\}$$

$$I \ominus H \equiv \{\,(1,1)\,\} \text{ because}$$

$$(1,1) + (\mathbf{0},\mathbf{0}) = (1,1) \in I \quad \textbf{and} \quad (1,1) + (\mathbf{1},\mathbf{0}) = (2,1) \in I$$

Lastnosti Širitve in Erozije

- Komutativnost

$$I \oplus H = H \oplus I$$

$$I \ominus H \neq H \ominus I$$

- Asociativnost

$$(I_1 \oplus I_2) \oplus I_3 = I_1 \oplus (I_2 \oplus I_3)$$

$$H_{\text{big}} = H_1 \oplus H_2 \oplus \ldots \oplus H_K$$

$$I \oplus H_{\text{big}} = (\ldots((I \oplus H_1) \oplus H_2) \oplus \ldots \oplus H_K)$$

$$(I_1 \ominus I_2) \ominus I_3 = I_1 \ominus (I_2 \oplus I_3)$$

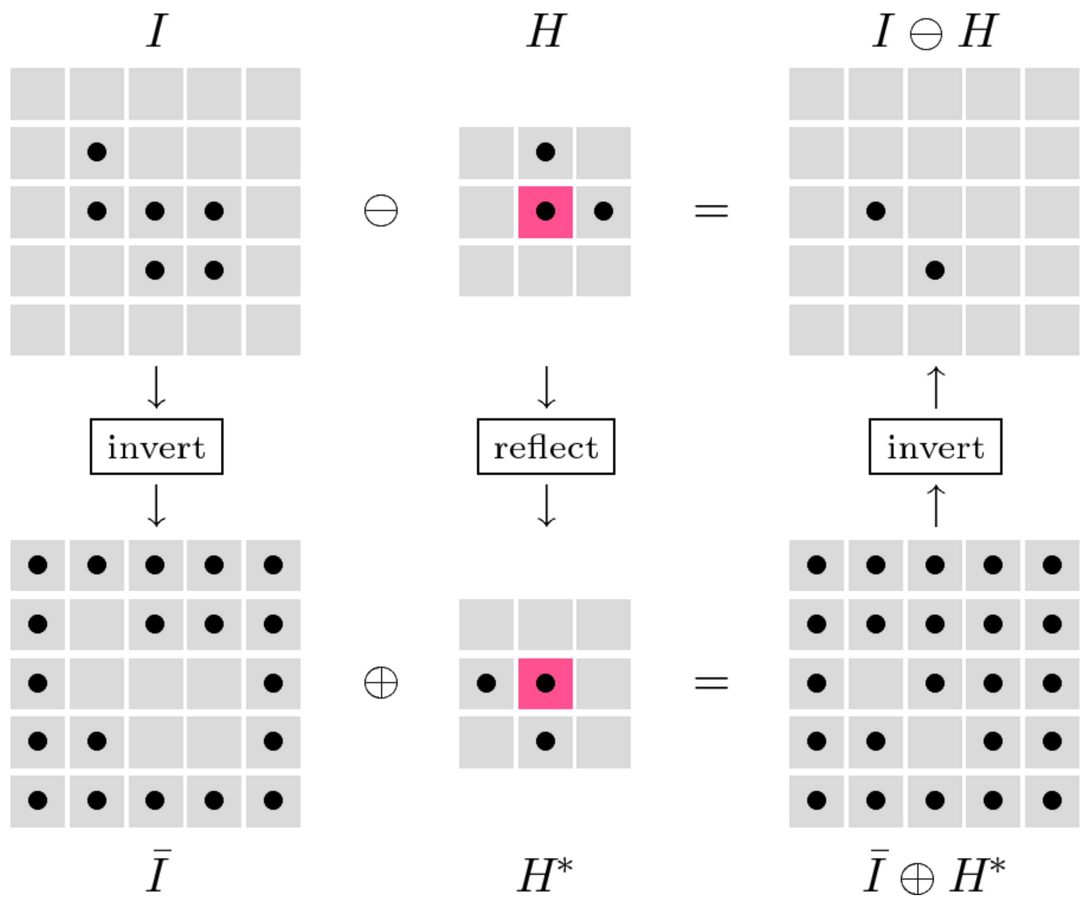
- Nevtralni element

$$I \oplus \delta = \delta \oplus I = I, \qquad \text{with } \delta \equiv \{(0,0)\}$$

Dualnost Širitve in Erozije

- Širitev ospredja je enaka Eroziji ozadja in obratno

$$I \oplus H \;\equiv\; \overline{(\bar{I} \ominus H^*)} \qquad I \ominus H \;\equiv\; \overline{(\bar{I} \oplus H^*)}$$



Algoritem

DILATE (I, H)

I: binary image of size $w \times h$
H: binary structuring element defined over region $\mathcal{R}_H$
Returns the dilated image $I' = I \oplus H$

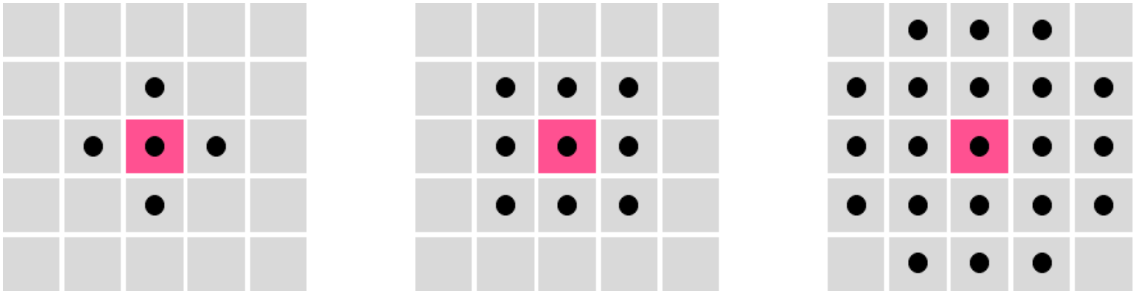
$I' \leftarrow$ new binary image of size $w \times h$
$I'(u, v) \leftarrow 0$, for all (u, v) $\qquad \triangleright\ I' \leftarrow \varnothing$
for all $(i, j) \in \mathcal{R}_H$ **do** $\qquad \triangleright\ (i, j) = \boldsymbol{q}$
  if $H(i, j) = 1$ **then** $\qquad \triangleright\ \boldsymbol{q} \in H$
    MERGE THE SHIFTED $I_{\boldsymbol{q}}$ WITH I': $\qquad \triangleright\ I' \leftarrow I' \cup I_{\boldsymbol{q}}$
    for $u \leftarrow 0 \ldots (w-1)$ **do**
      for $v \leftarrow 0 \ldots (h-1)$ **do** $\qquad \triangleright\ (u, v) = \boldsymbol{p}$
        if $I(u, v) = 1$ **then** $\qquad \triangleright\ \boldsymbol{p} \in I$
          $I'(u+i, v+j) \leftarrow 1$ $\qquad \triangleright\ I' \leftarrow I' \cup (\boldsymbol{p}+\boldsymbol{q})$
return I'.

ERODE (I, H)

$\bar{I} \leftarrow$ INVERT(I) $\qquad \triangleright\ \bar{I} \leftarrow \neg I$
$H^* \leftarrow$ REFLECT(H)
return INVERT(DILATE$(\bar{I}, H^*)$). $\qquad \triangleright\ I \ominus H = \overline{(\bar{I} \oplus H^*)}$

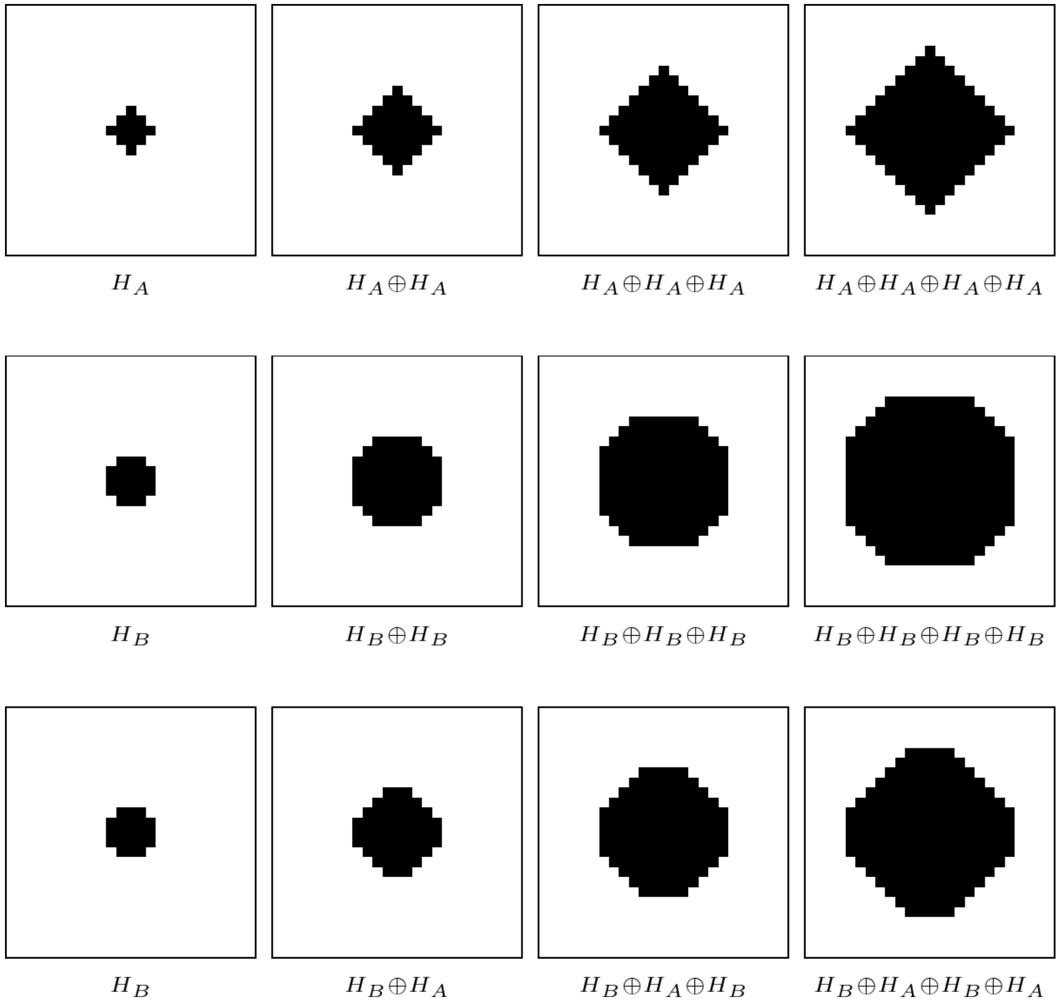
Morfološki filtri

- Morfološki filter je točno definiran s
  - Tipom operacije
  - Strukturnim elementom
- Ponavadi se uporabljajo okrogli strukturni elementi (izotropičen filter)
  - Pri Širitvi strukturni element z radijem *r* doda *r* slikovnih elementov plasti okrog ospredja
  - Pri Eroziji strukturni element z radijem *r* zbriše *r* plasti slikovnih elementov z ospredja

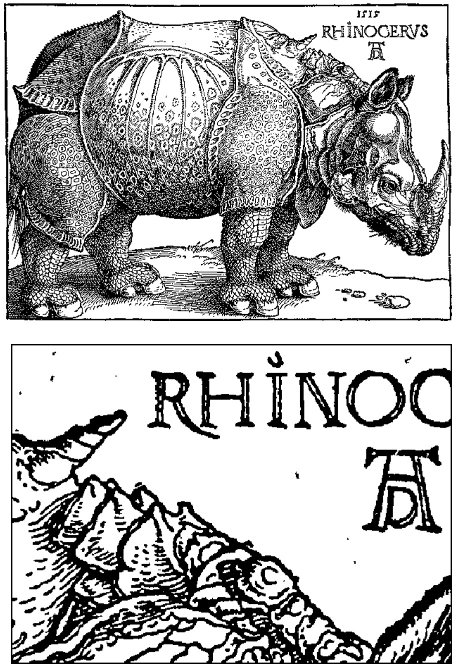


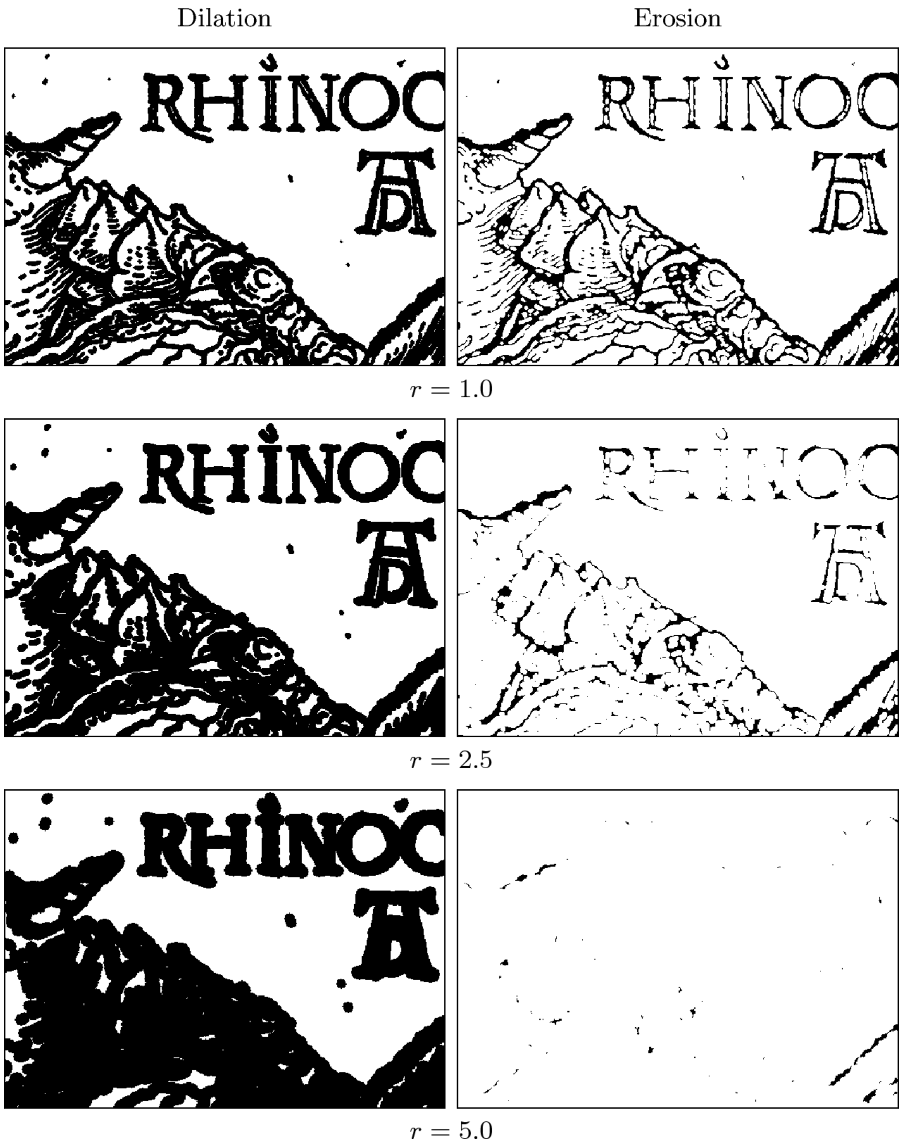
Iterativnost filtrov

- Aproksimacija izotropičnih filtrov z ustreznim vrstnim redom manjših filtrov

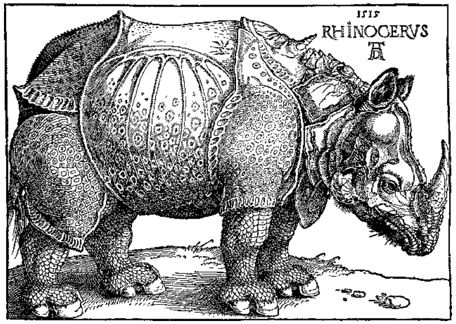


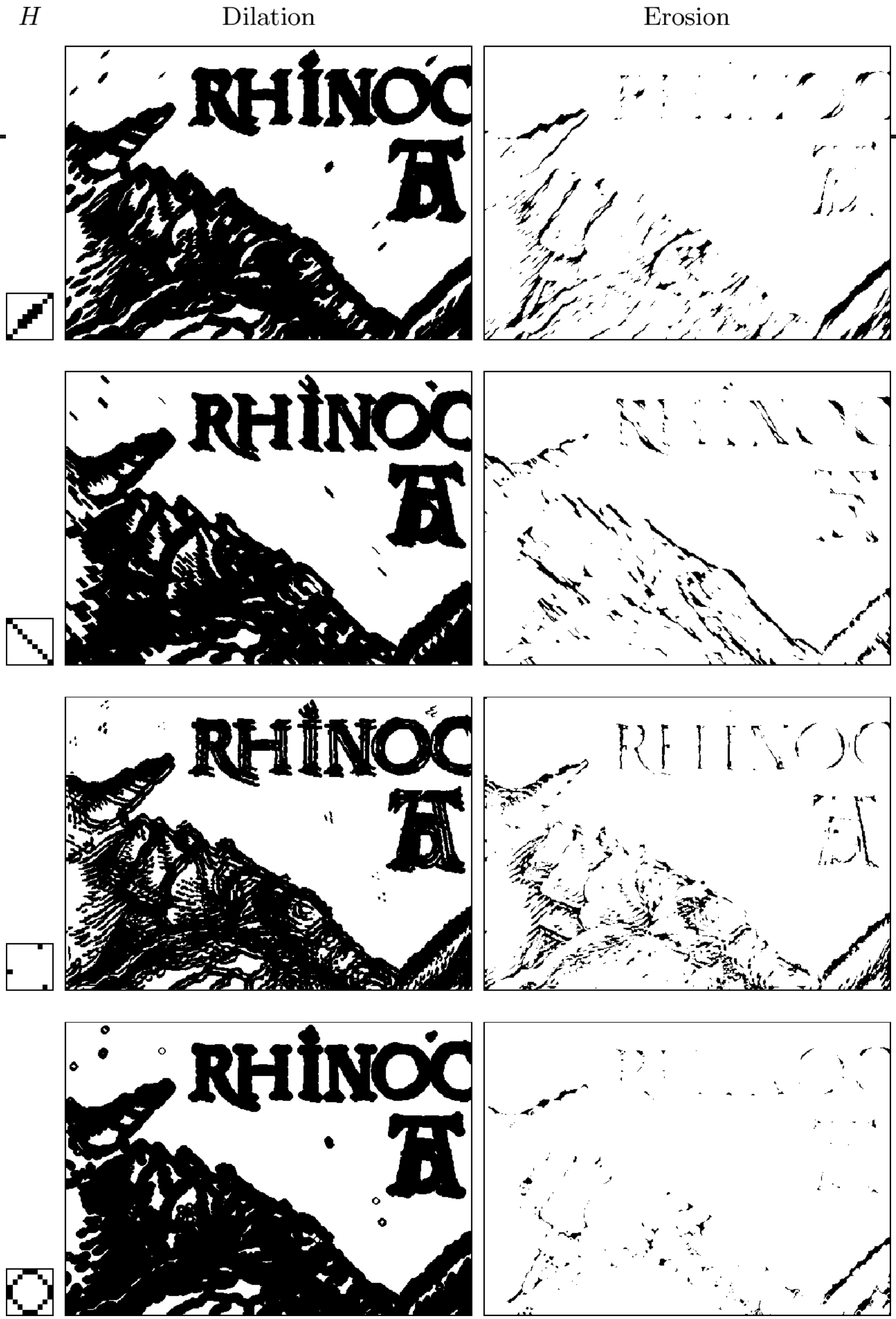
Primer





Primer





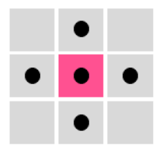
Obrisi

- Določanje mejnih slikovnih elementov
  - Presek med originalno in invertirano „erodirano“ sliko

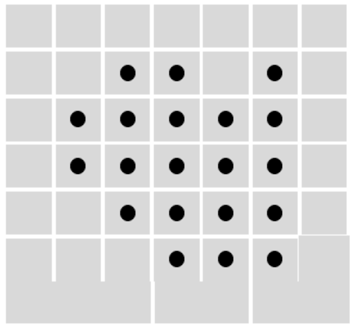
$$I' = I \ominus H_n$$

$$B = I \cap \overline{I'} = I \cap \overline{(I \ominus H_n)}$$

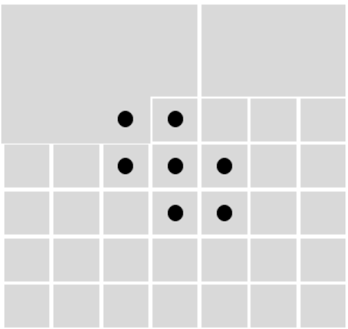
$H_n =$



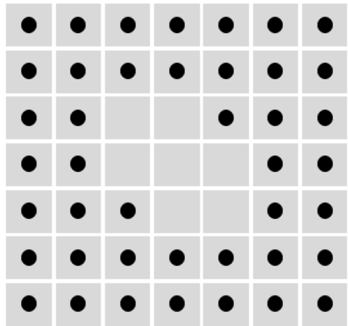
I



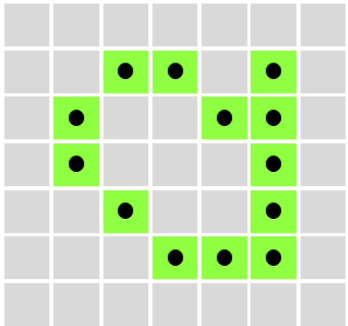
$I \ominus H_n$



$\overline{I \ominus H_n}$



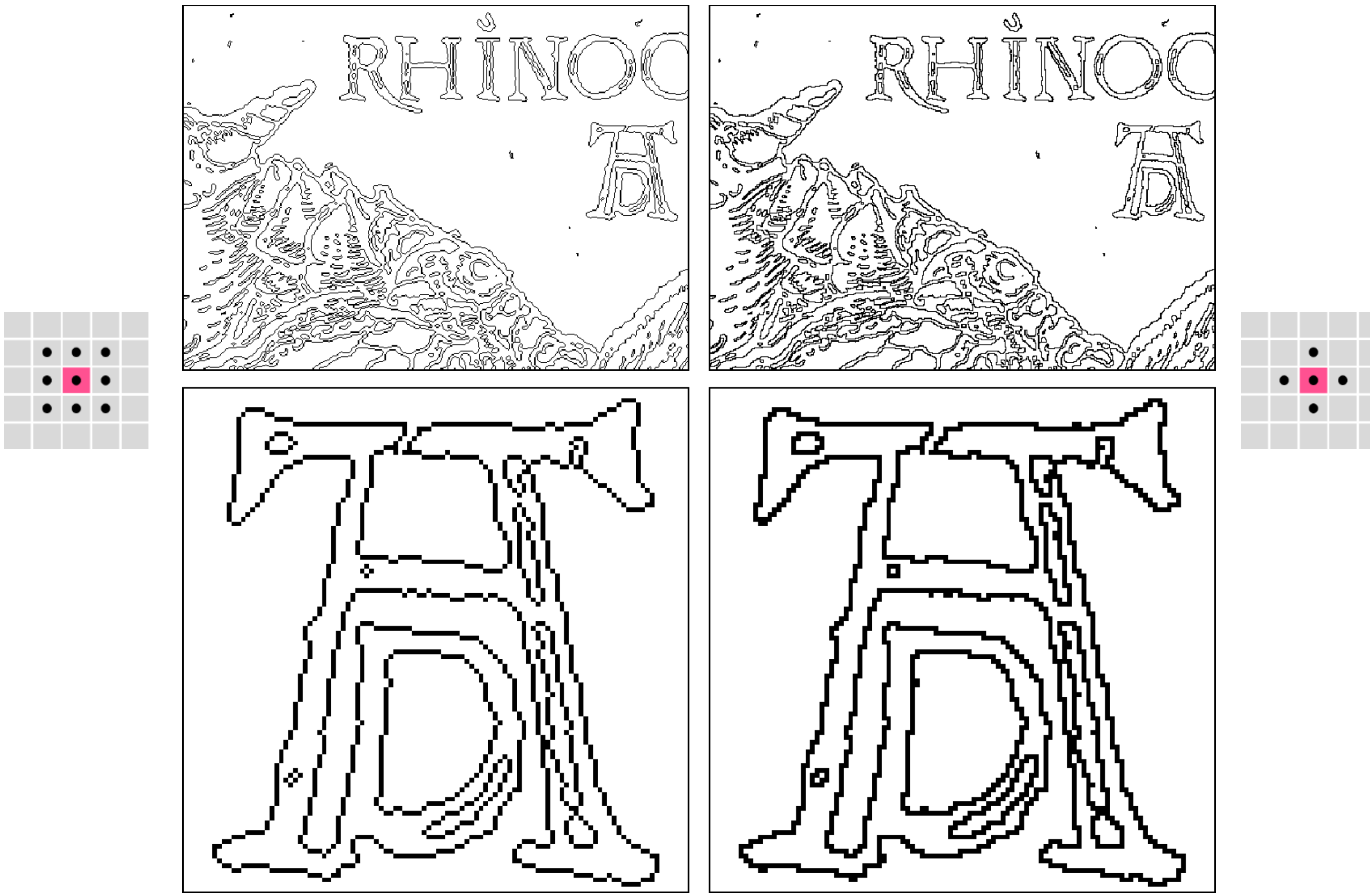
$B = I \cap \overline{I \ominus H_n}$



  - oz XOR med originalno in „erodirano“ sliko

Primer

- 8-sosedni strukturni element -> 4-sosedni obris
- 4-sosedni strukturni element -> 8-sosedni obris



Kompozitni operatorji

- Odprtje (opening):
  - Erozija, nato Širitev z istim strukturnim elementom
  - Za odstranjevanje majhnih elementov

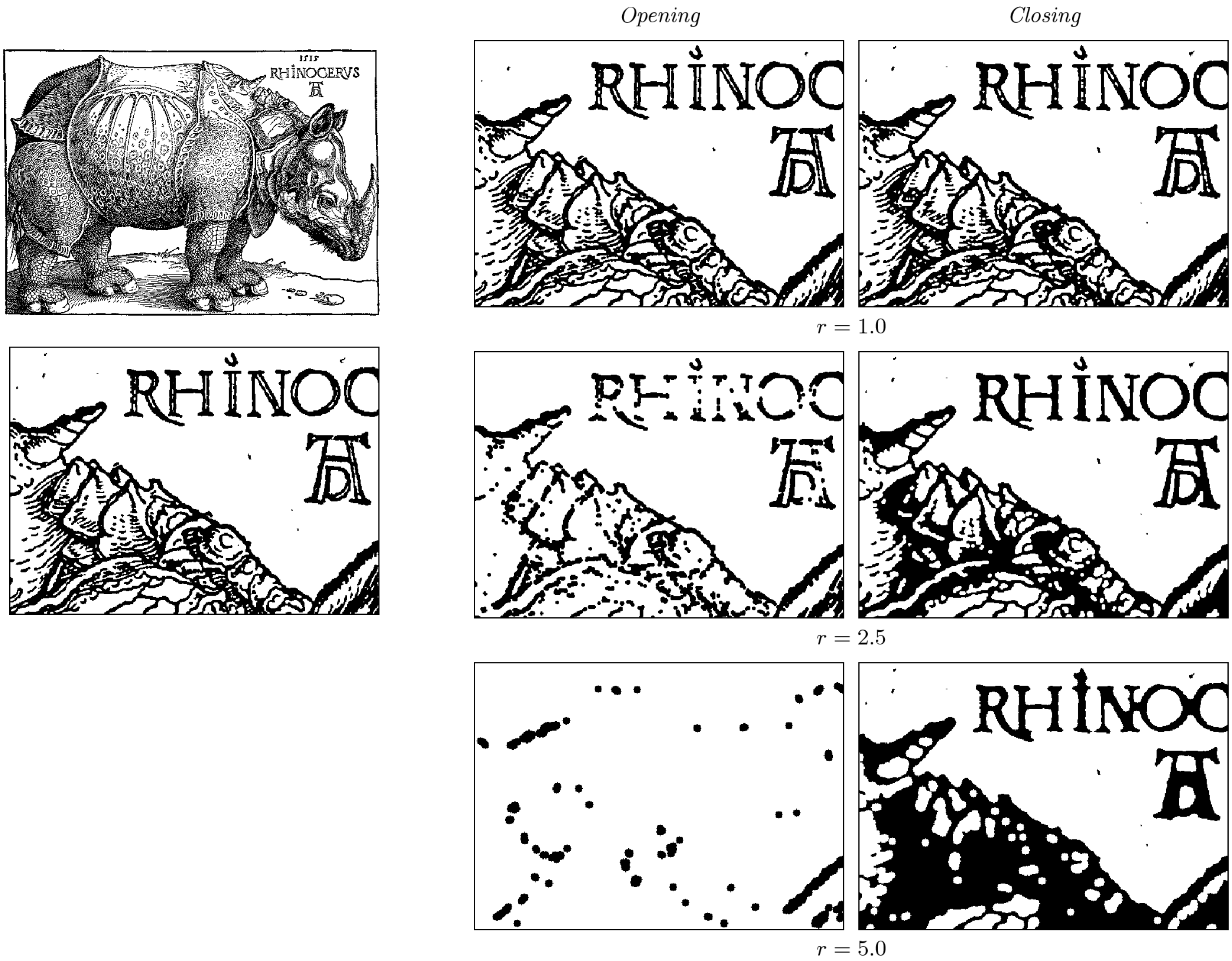
$$I \circ H = (I \ominus H) \oplus H$$

- Zaprtje (closing)
  - Širitev, nato erozija
  - Za polnjenje lukenj

$$I \bullet H = (I \oplus H) \ominus H$$

- Operaciji Odprtje in Zaprtje sta
  - Idempotentni: $$I \circ H = (I \circ H) \circ H = ((I \circ H) \circ H) \circ H = \ldots$$ $$I \bullet H = (I \bullet H) \bullet H = ((I \bullet H) \bullet H) \bullet H = \ldots$$
  - Dualni: $$I \circ H = \overline{(\bar{I} \bullet H)} \quad \text{and} \quad I \bullet H = \overline{(\bar{I} \circ H)}$$

Primer Odprtja in Zaprtja



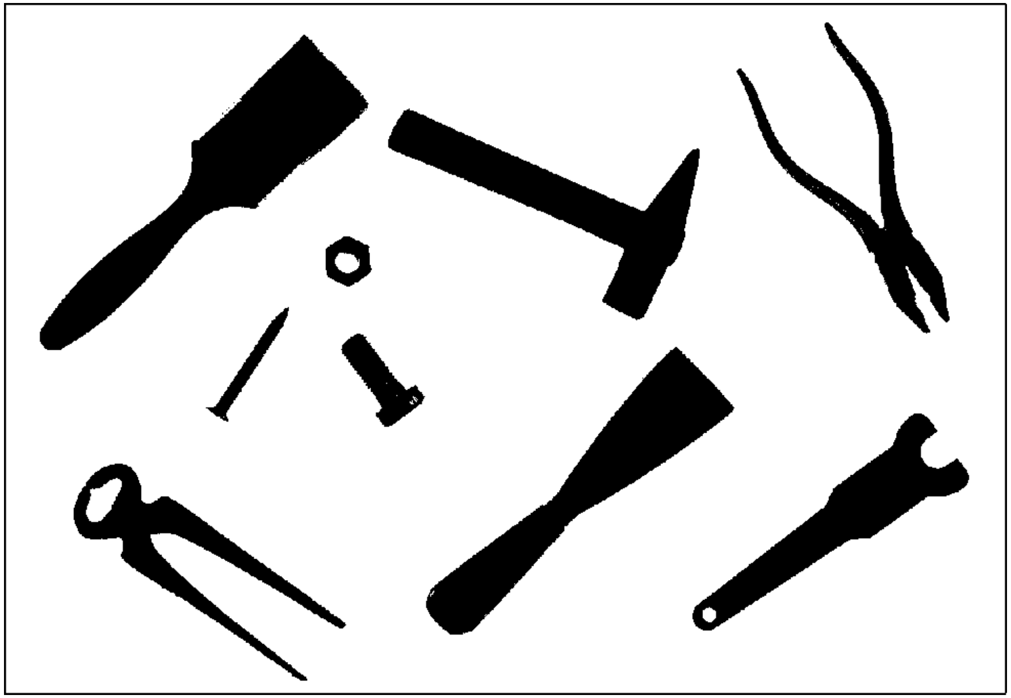
Morfološki operatorji na sivinskih slikah

- Širitev in Erozija sta tudi posplošeni za uporabo na sivinskih slikah

  - Primer:

Regije v binarnih slikah

- Ospredje (foreground)
- Ozadje (background)

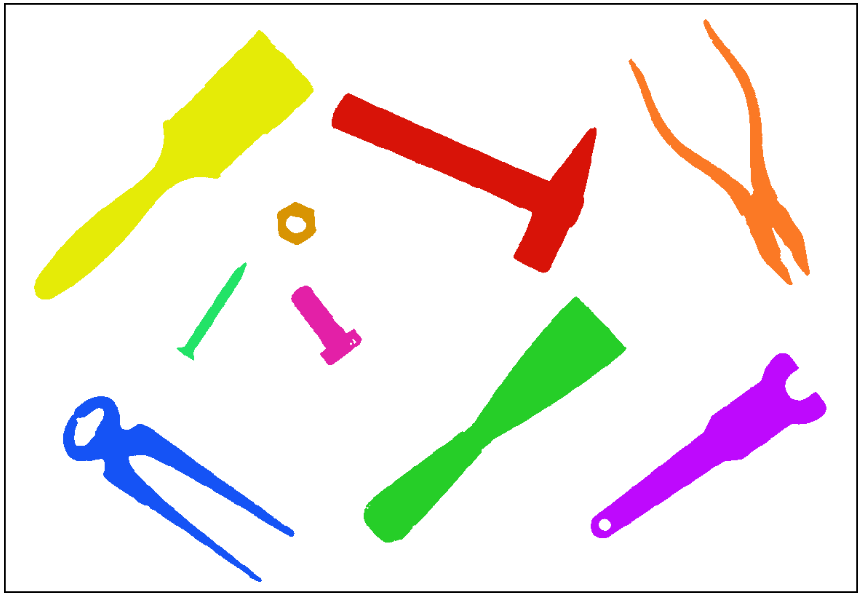


- Število in vrsta predmetov na binarnih slikah

Iskanje regij na sliki

- Labeliranje (barvanje) regij
  - Kateri slikovni elementi pripadajo kateri regiji?
  - Koliko regij je na sliki?
  - Kje so regije locirane?

$$I(u,v) = \begin{cases} 0 & \textit{background} \text{ pixel} \\ 1 & \textit{foreground} \text{ pixel} \\ 2, 3, \ldots & \text{region } \textit{label}. \end{cases}$$



- Iskanje povezanih slikovnih elementov
  - 4-sosednost
  - 8-sosednost
- Več algoritmov:
  - Poplavljanje (flood filling)
  - Zaporedno označevanje regij (Sequential region marking)
  - Kombinacija obeh

Barvanje regij s poplavljanjem

- Enostaven algoritem:
  1. Poišči en neoznačen slikovni element ospredja
  2. Označi vse sosednje slikovne elemente ospredja in tako naprej
- Kot poplavljanje pokrajine

- Različni načini poplavljanja:
  1. Rekurzivno
     - zelo prostorosko zahtevno
  2. Iterativno z uporabo sklada (najprej v globino)
  3. Iterativno z uporabo vrste (najprej v širino)
     - najbolj priporočljivo

Rekurzivni algoritem

```
RegionLabeling(I)
        I: binary image (0 = background, 1 = foreground)
        The image I is labeled (destructively modified) and returned.
    Initialize m ← 2 (the value of the next label to be assigned).
    Iterate over all image coordinates (u, v).
        if I(u, v) = 1 then
            FloodFill(I, u, v, m)        ▷ use any of the 3 versions below
            m ← m + 1.
    return the labeled image I.
```

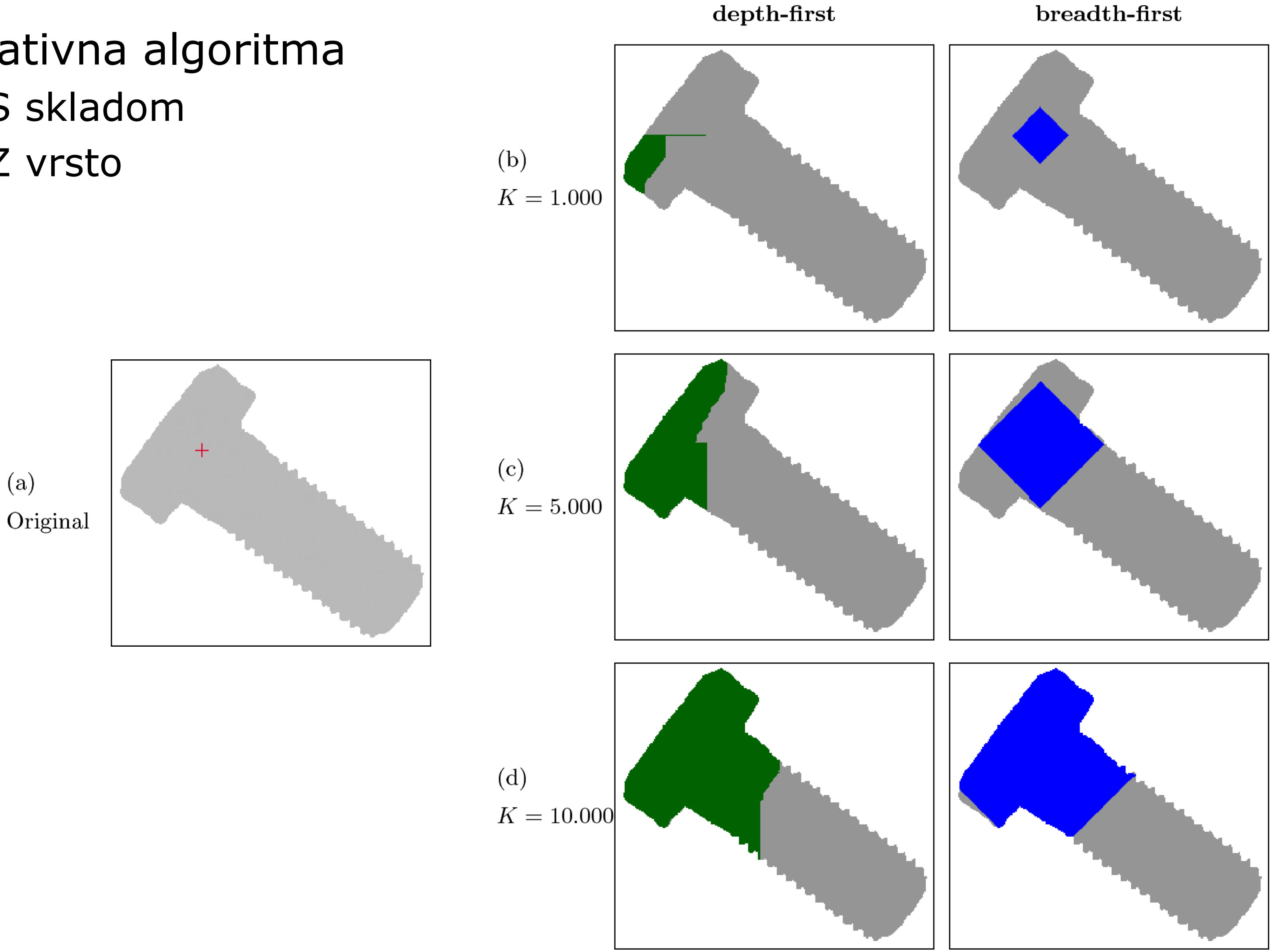
```
FloodFill(I, u, v, label)                          ▷ Recursive Version
    if coordinate (u, v) is within image boundaries and I(u, v) = 1 then
        Set I(u, v) ← label
        FloodFill(I, u+1, v, label)
        FloodFill(I, u, v+1, label)
        FloodFill(I, u, v−1, label)
        FloodFill(I, u−1, v, label)
    return.
```

Iterativna algoritma

```
FloodFill(I, u, v, label)                                   ▷ Depth-First Version
    Create an empty stack S
    Put the seed coordinate ⟨u, v⟩ onto the stack: Push(S, ⟨u, v⟩)
    while S is not empty do
        Get the next coordinate from the top of the stack:
            ⟨x, y⟩ ← Pop(S)
        if coordinate (x, y) is within image boundaries and I(x, y) = 1
            then
            Set I(x, y) ← label
            Push(S, ⟨x+1, y⟩)
            Push(S, ⟨x, y+1⟩)
            Push(S, ⟨x, y−1⟩)
            Push(S, ⟨x−1, y⟩)
    return.

FloodFill(I, u, v, label)                                   ▷ Breadth-First Version
    Create an empty queue Q
    Insert the seed coordinate ⟨u, v⟩ into the queue: Enqueue(Q, ⟨u, v⟩)
    while Q is not empty do
        Get the next coordinate from the front of the queue:
            ⟨x, y⟩ ← Dequeue(Q)
        if coordinate ⟨x, y⟩ is within image boundaries and I(x, y) = 1
            then
            Set I(x, y) ← label
            Enqueue(Q, ⟨x+1, y⟩)
            Enqueue(Q, ⟨x, y+1⟩)
            Enqueue(Q, ⟨x, y−1⟩)
            Enqueue(Q, ⟨x−1, y⟩)
    return.
```

Primer

- Iterativna algoritma
  - S skladom
  - Z vrsto



Zaporedno označevanje regij

- Sliko sprocesiramo zaporedno od zgornjega levega do spodnjega desnega vogala v dveh korakih
- Korak 1:
  - Če je trenutni slikovni element del ospredja preverimo del njegovih sosedov

$$\mathcal{N}_4(u,v) = \begin{array}{|c|c|c|} \hline & N_2 & \\ \hline N_1 & \times & \\ \hline & & \\ \hline \end{array} \quad \text{or} \quad \mathcal{N}_8(u,v) = \begin{array}{|c|c|c|} \hline N_2 & N_3 & N_4 \\ \hline N_1 & \times & \\ \hline & & \\ \hline \end{array}$$

    - Če so vsi sosedi del ozadja, inicializiramo novo labelo
    - Če imajo vsi ospredni sosedi enako labelo jo priredimo tudi trenutnemu slikovnemu elementu
    - Če imajo različne labele, si zapomnimo to kot konflikt
      - Gradimo grafe med seboj povezanih label
- Korak 2:
  - Razrešimo konflikte: poiščemo povezane komponente v grafih konfliktov in popravimo labelo ustreznih slikovnih elementov

Primer

(a)

0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	0	1	1	0	1	0
0	1	1	1	1	1	1	0	0	1	0	0	1	0
0	0	0	0	1	0	1	0	0	0	0	0	1	0
0	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	1	1	1	1	1	1	1	1	1	0
0	1	1	0	0	0	1	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0

0	Background
1	Foreground

(b) only background neighbors

0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	0	1	1	0	1	0
0	1	1	1	1	1	1	0	0	1	0	0	1	0
0	0	0	0	1	0	1	0	0	0	0	0	1	0
0	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	1	1	1	1	1	1	1	1	1	0
0	1	1	0	0	0	1	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0

new label (**2**)

0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	2	1	0	0	1	1	0	1	0
0	1	1	1	1	1	1	0	0	1	0	0	1	0
0	0	0	0	1	0	1	0	0	0	0	0	1	0
0	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	1	1	1	1	1	1	1	1	1	0
0	1	1	0	0	0	1	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0

Primer

(c) exactly one neighbor label

0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	2	1	0	0	1	1	0	1	0
0	1	1	1	1	1	1	0	0	1	0	0	1	0
0	0	0	0	1	0	1	0	0	0	0	0	1	0
0	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	1	1	1	1	1	1	1	1	1	0
0	1	1	0	0	0	1	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0

neighbor label is propagated

0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	2	2	0	0	1	1	0	1	0
0	1	1	1	1	1	1	0	0	1	0	0	1	0
0	0	0	0	1	0	1	0	0	0	0	0	1	0
0	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	1	1	1	1	1	1	1	1	1	0
0	1	1	0	0	0	1	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0

(d) two different neighbor labels

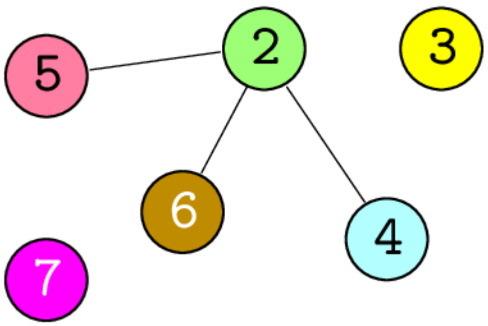
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	2	2	0	0	3	3	0	4	0
0	5	5	5	1	1	1	0	0	1	0	0	1	0
0	0	0	0	1	0	1	0	0	0	0	0	1	0
0	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	1	1	1	1	1	1	1	1	1	0
0	1	1	0	0	0	1	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0

one of the labels (**2**) is propagated

0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	2	2	0	0	3	3	0	4	0
0	5	5	5	2	1	1	0	0	1	0	0	1	0
0	0	0	0	1	0	1	0	0	0	0	0	1	0
0	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	1	1	1	1	1	1	1	1	1	0
0	1	1	0	0	0	1	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0

Primer

0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	2	2	0	0	3	3	0	4	0
0	5	5	5	2	2	2	0	0	3	0	0	4	0
0	0	0	0	2	0	2	0	0	0	0	0	4	0
0	6	6	2	2	2	2	2	2	2	2	2	2	0
0	0	0	0	2	2	2	2	2	2	2	2	2	0
0	7	7	0	0	0	2	0	2	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0



0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	2	2	0	0	3	3	0	2	0
0	2	2	2	2	2	2	0	0	3	0	0	2	0
0	0	0	0	2	0	2	0	0	0	0	0	2	0
0	2	2	2	2	2	2	2	2	2	2	2	2	0
0	0	0	0	2	2	2	2	2	2	2	2	2	0
0	7	7	0	0	0	2	0	2	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0

Algoritem – korak 1

SEQUENTIALLABELING(I)

I: binary image ($0 =$ *background*, $1 =$ *foreground*)
The image I is labeled (destructively modified) and returned.

PASS 1—ASSIGN INITIAL LABELS:

Initialize $m \leftarrow 2$ (the value of the next label to be assigned).
Create an empty set $\mathcal{C}$ to hold the collisions: $\mathcal{C} \leftarrow \{\}$.
for $v \leftarrow 0 \ldots H-1$ **do** ▷ $H =$ height of image I
  for $u \leftarrow 0 \ldots W-1$ **do** ▷ $W =$ width of image I
    if $I(u,v) = 1$ **then** do one of:
      if all neighbors of (u,v) are background pixels (all $n_i = 0$) **then**
        $I(u,v) \leftarrow m$.
        $m \leftarrow m + 1$.
      else if exactly *one* of the neighbors has a label value $n_k > 1$ **then**
        set $I(u,v) \leftarrow n_k$
      else if *several* neighbors of (u,v) have label values $n_j > 1$ **then**
        Select one of them as the new label:
          $I(u,v) \leftarrow k \in \{n_j\}$.
        for all other neighbors of $u,v)$ with label values $n_i > 1$ and $n_i \neq k$ **do**
          Create a new label collision $\boldsymbol{c}_i = \langle n_i, k \rangle$.
          Record the collision: $\mathcal{C} \leftarrow \mathcal{C} \cup \{\boldsymbol{c}_i\}$.

Remark: The image I now contains label values $0, 2, \ldots m-1$.

Algoritem – korak 2

PASS 2—RESOLVE LABEL COLLISIONS:

Let $\mathcal{L} = \{2, 3, \ldots m-1\}$ be the set of preliminary region labels.

Create a partitioning of $\mathcal{L}$ as a *vector of sets*, one set for each label value: $\mathcal{R} \leftarrow [\mathcal{R}_2, \mathcal{R}_3, \ldots, \mathcal{R}_{m-1}] = [\{2\}, \{3\}, \{4\}, \ldots, \{m-1\}]$, so $\mathcal{R}_i = \{i\}$ for all $i \in \mathcal{L}$.

for all collisions $\langle a, b \rangle \in \mathcal{C}$ **do**

    Find in $\mathcal{R}$ the sets $\mathcal{R}_a$, $\mathcal{R}_b$ containing the labels a, b, resp.:

        $\mathcal{R}_a \leftarrow$ the set that currently contains label a

        $\mathcal{R}_b \leftarrow$ the set that currently contains label b

    if $\mathcal{R}_a \neq \mathcal{R}_b$ (a and b are contained in different sets) **then**

        Merge sets $\mathcal{R}_a$ and $\mathcal{R}_b$ by moving all elements of $\mathcal{R}_b$ to $\mathcal{R}_a$:

            $\mathcal{R}_a \leftarrow \mathcal{R}_a \cup \mathcal{R}_b$

            $\mathcal{R}_b \leftarrow \{\}$

Remark: All *equivalent* label values (i. e., all labels of pixels in the same region) are now contained in the same set $\mathcal{R}_i$ within $\mathcal{R}$.

PASS 3—RELABEL THE IMAGE:

Iterate through all image pixels (u, v):

    if $I(u, v) > 1$ **then**

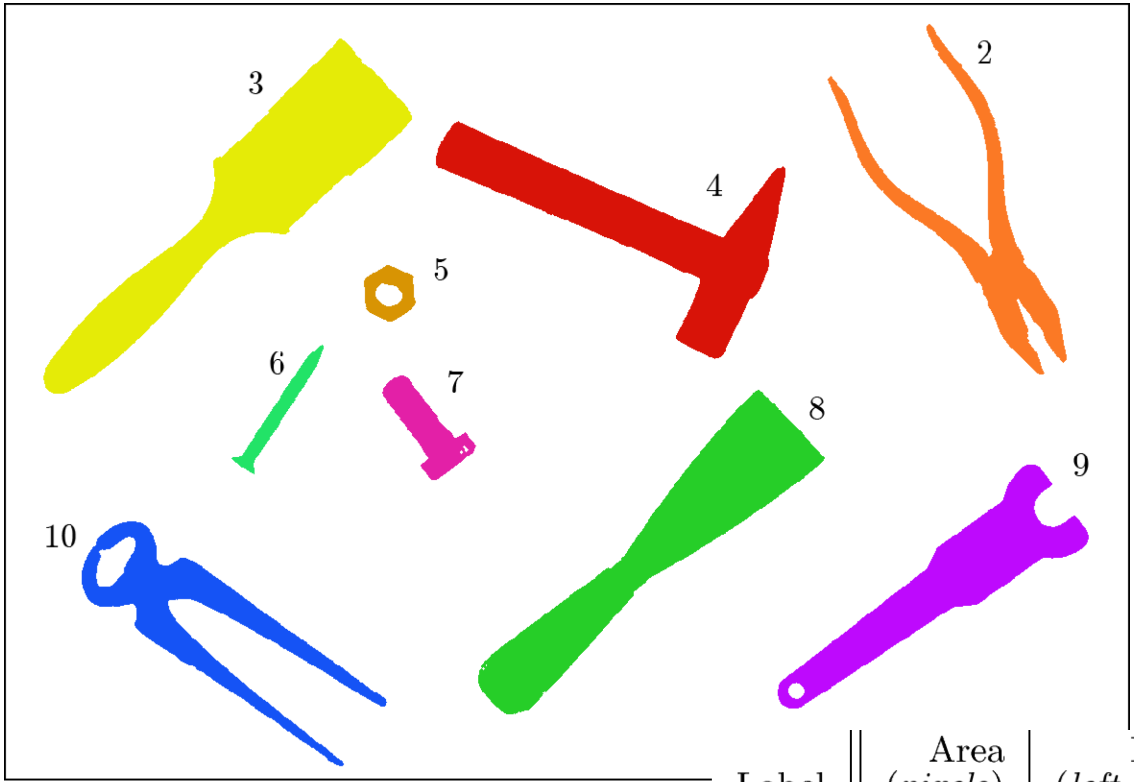
        Find the set $\mathcal{R}_i$ in $\mathcal{R}$ that contains label $I(u, v)$.

        Choose one unique representative element k from the set $\mathcal{R}_i$ (e. g., the minimum value, $k = \min(\mathcal{R}_i)$).

        Replace the image label: $I(u, v) \leftarrow k$.

return the labeled image I.

Primer

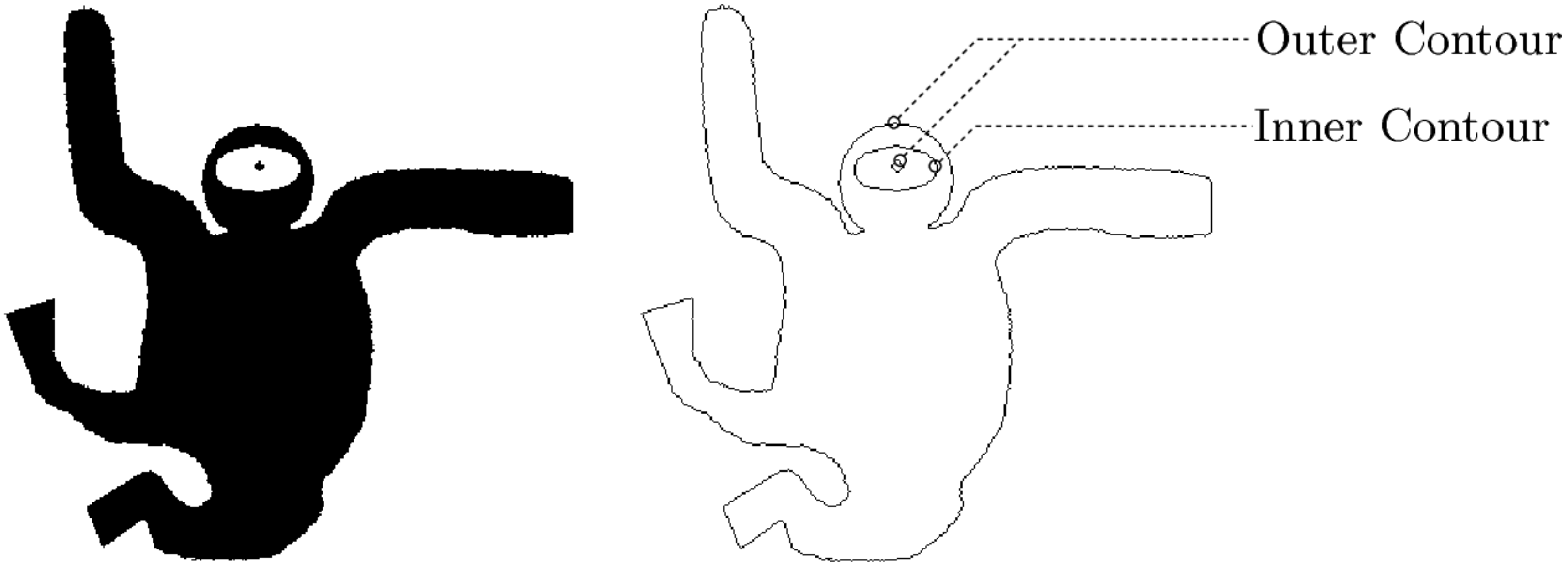


Label	Area (*pixels*)	Bounding Box (*left, top, right, bottom*)	Center (x_c, y_c)
2	14978	(887, 21, 1144, 399)	(1049.7, 242.8)
3	36156	(40, 37, 438, 419)	(261.9, 209.5)
4	25904	(464, 126, 841, 382)	(680.6, 240.6)
5	2024	(387, 281, 442, 341)	(414.2, 310.6)
6	2293	(244, 367, 342, 506)	(294.4, 439.0)
7	4394	(406, 400, 507, 512)	(454.1, 457.3)
8	29777	(510, 416, 883, 765)	(704.9, 583.9)
9	20724	(833, 497, 1168, 759)	(1016.0, 624.1)
10	16566	(82, 558, 411, 821)	(208.7, 661.6)

Obrisi regij

- Morfološki operatorji označijo slikovne elemente na obrisu
- Potrebujemo tudi urejeno zaporedje robnih slikovnih elementov

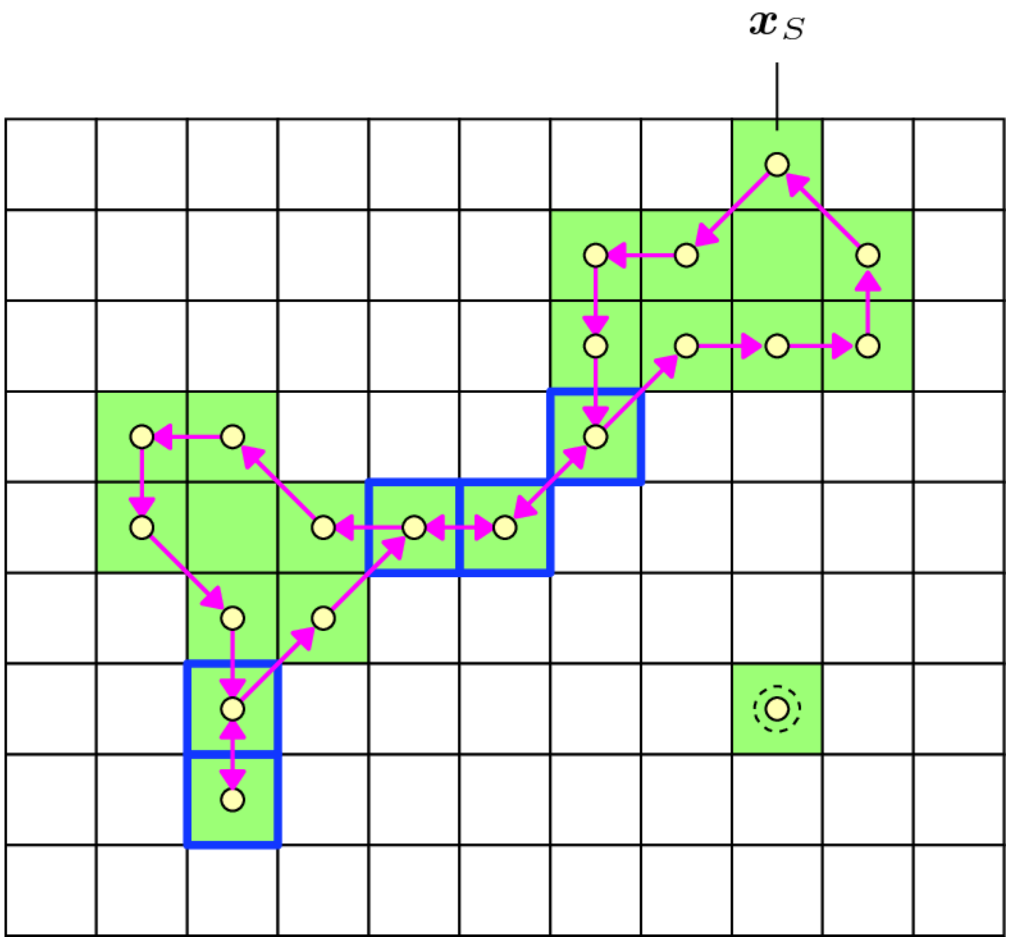
- Zunanji obris in notranji obris



Iskanje obrisa

1. Poišči povezano regijo
2. Poišči rob regije in mu sledi naokrog

- Težave:
  - Notranji obrisi
  - Deli regij debeline en sl. element
  - Izolirani s- elementi

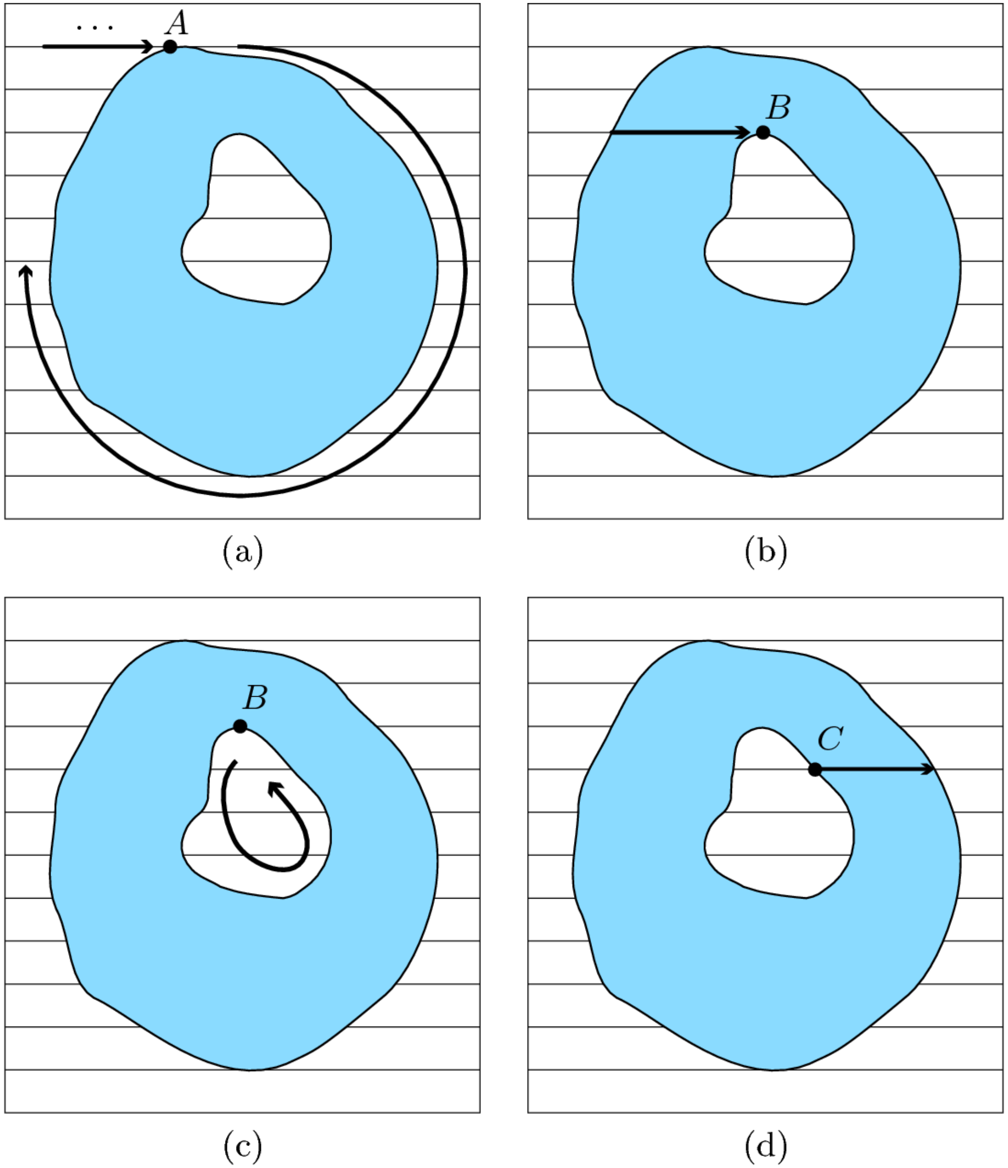


Kombinirano iskanje regij in obrisov

- Hkrati označi regije in poišče obrise

- Algoritem preišče celotno sliko od zgornjega levega do spodnjega desnega slikovnega elementa
- V nekem slikovnem elementu so možni tri primeri:
  - Primer A: BG->neoznačen FG => zunanji obris
    - Dodeli novo oznako regiji
    - Prepotuj celoten zunanji obris in ustrezno označi sl. elemente
    - Sosednje BG sl. Elemente označi z -1
  - Primer B: FG->neoznačen BG => notranji obris
    - Prepotuj celoten notranji obris in ustrezno označi sl. elemente
    - Mejne BG sl. Elemente označi z -1
  - Primer C: označen FG
    - Prenesi oznako na sl. elemente na desni

Kombinirano iskanje regij in obrisov



Algoritem

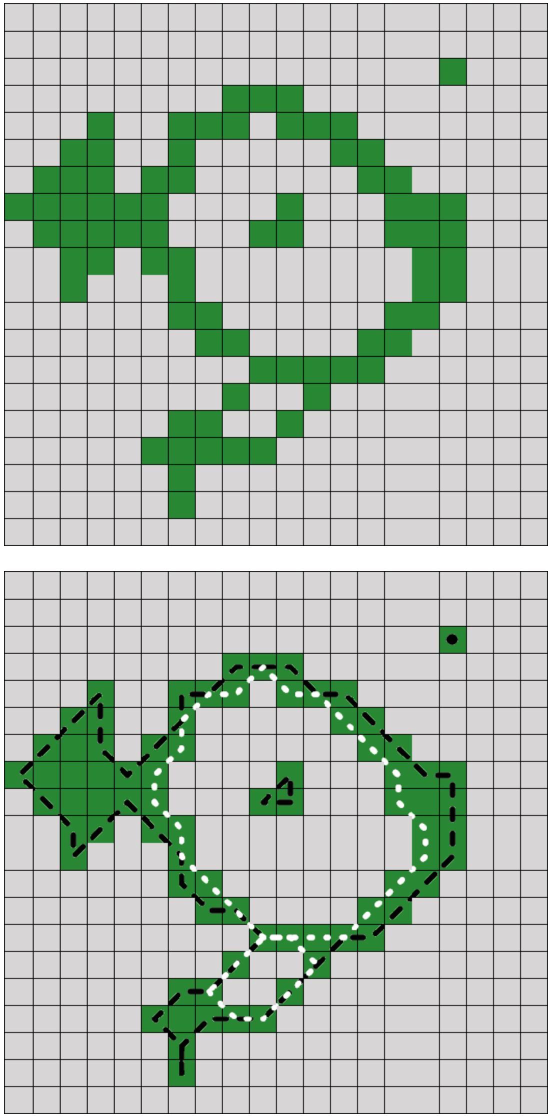
```
CombinedContourLabeling (I)
      I: binary image
      Returns a set of contours and a label map (labeled image).
   Create an empty set of contours: C ← {}
   Create a label map LM of the same size as I and initialize:
   for all (u, v) do
       LM(u, v) ← 0                                   ▷ label map LM
   R ← 0                                              ▷ region counter R

   Scan the image from left to right and top to bottom:
   for v ← 0 ... N−1 do
       L_k ← 0                                        ▷ current label L_k
       for u ← 0 ... M−1 do
           if I(u, v) is a foreground pixel then
               if (L_k ≠ 0) then                      ▷ continue existing region
                   LM(u, v) ← L
               else
                   L_k ← LM(u, v)
                   if (L_k = 0) then                  ▷ hit new outer contour
                       R ← R + 1
                       L_k ← R
                       x_S ← (u, v)
                       c_outer ← TraceContour(x_S, 0, L_k, I, LM)
                       C ← C ∪ {c_outer}              ▷ collect new contour
                       LM(u, v) ← L_k
           else                                       ▷ I(u, v) is a background pixel
               if (L ≠ 0) then
                   if (LM(u, v) = 0) then             ▷ hit new inner contour
                       x_S ← (u−1, v)
                       c_inner ← TraceContour(x_S, 1, L_k, I, LM)
                       C ← C ∪ {c_inner}              ▷ collect new contour
                   L ← 0
   return (C, LM).                  ▷ return the set of contours and the label map
```

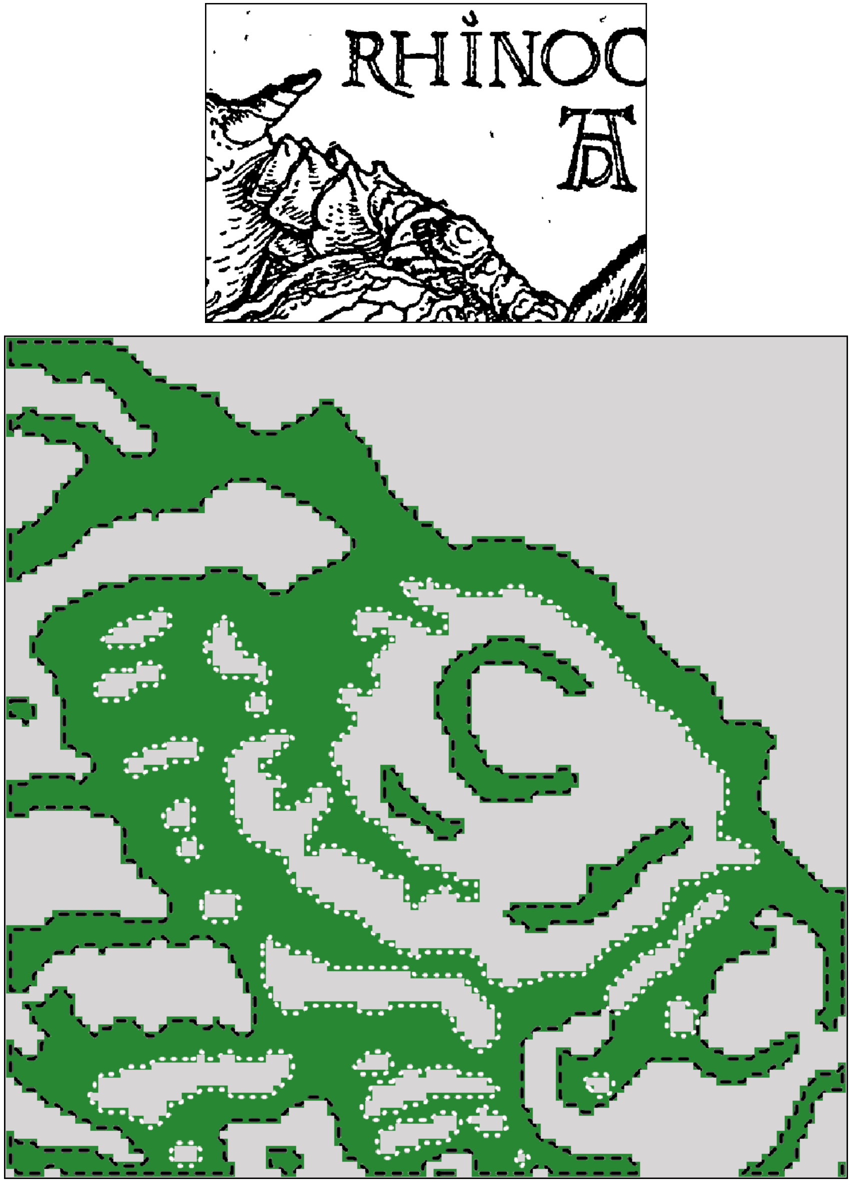
```
TraceContour(x_S, d_S, L_k, I, LM)
      x_S: start position, d_S: initial search direction,
      L_c: label for this contour
      I: original image, LM: label map.
      Traces and returns the contour starting at x_S.
   (x_T, d_next) ← FindNextPoint(x_S, d_S, I, LM)
   c ← [x_T]                              ▷ create a contour starting with x_T
   x_p ← x_S                              ▷ previous position x_p = (u_p, v_p)
   x_c ← x_T                              ▷ current position x_c = (u_c, v_c)
   done ← (x_S ≡ x_T)                     ▷ isolated pixel?
   while (¬done) do
       LM(u_c, v_c) ← L_c
       d_search ← (d_next + 6) mod 8
       (x_n, d_next) ← FindNextPoint(x_c, d_search, I, LM)
       x_p ← x_c
       x_c ← x_n
       done ← (x_p ≡ x_S ∧ x_c ≡ x_T)      ▷ back at start point?
       if (¬done) then
           Append(c, x_n)                 ▷ add point x_n to contour c
   return c.                              ▷ return this contour

FindNextPoint(x_c, d, I, LM)
      x_c: start point, d: search direction,
      I: original image, LM: label map.
   for i ← 0 ... 6 do                     ▷ search in 7 directions
       x' ← x_c + Delta(d)                ▷ x' = (u', v')
       if I(u', v') is a background pixel then
           LM(u', v') ← −1                ▷ mark background as visited (−1)
           d ← (d + 1) mod 8
       else                               ▷ found a nonbackground pixel at x'
           return (x', d)
   return (x_c, d).                       ▷ found no next point, return start point
```

$\textsc{Delta}(d) = (\Delta x, \Delta y)$, with

d	0	1	2	3	4	5	6	7
Δx	1	1	0	−1	−1	−1	0	1
Δy	0	1	1	1	0	−1	−1	−1

Primer



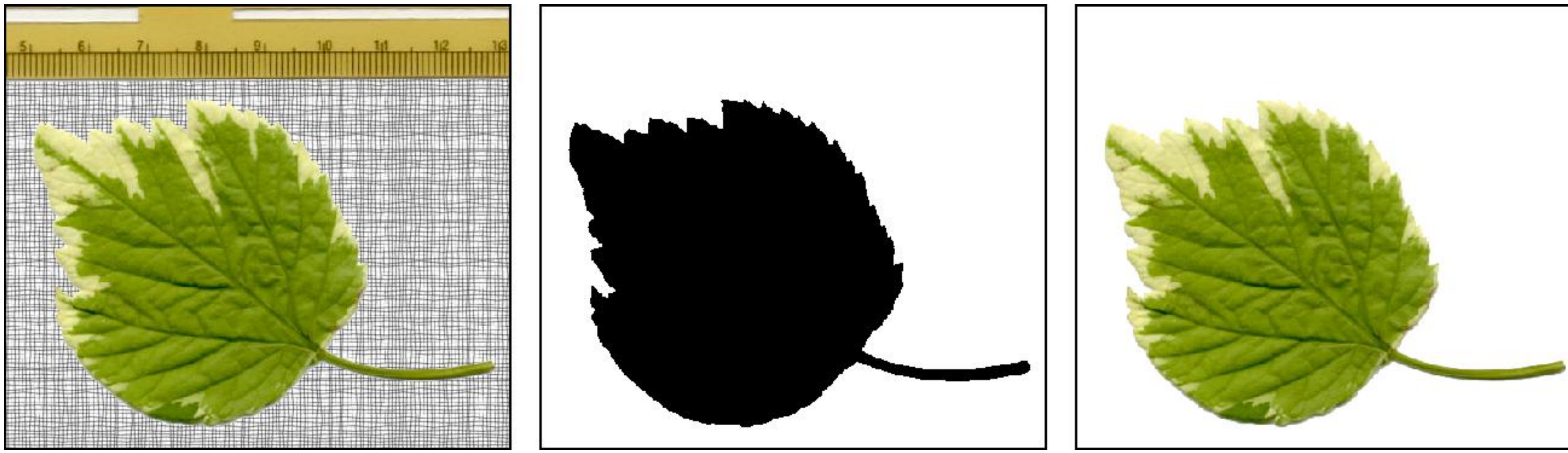


Predstavitve slikovnih regij

- Predstavitev z matriko
- Run Length Encoding (RLE)
- Absolutna verižna koda
- Diferenčna verižna koda

Predstavitev z matriko

- Najbolj klasična predstavitev
- Binarna maska določa regijo:



- Predstavitev ni odvisna vsebine slike
- Pogosto neučinkovita (prostorsko potratna)

Run Length Encoding (RLE)

- Kodiranje zaporedij slikovnih elementov ospredja s tremi parametri:

$$Run_i = \langle \mathsf{row}_i, \mathsf{column}_i, \mathsf{length}_i \rangle$$

- Primer:

Bitmap

	0	1	2	3	4	5	6	7	8
0									
1			×	×	×	×	×	×	
2									
3					×	×	×	×	
4		×	×	×		×	×	×	
5	×	×	×	×	×	×	×	×	×
6									

$\longrightarrow$

RLE

$\langle \mathsf{row}, \mathsf{column}, \mathsf{length} \rangle$

$\langle 1, 2, 6 \rangle$
$\langle 3, 4, 4 \rangle$
$\langle 4, 1, 3 \rangle$
$\langle 4, 5, 3 \rangle$
$\langle 5, 0, 9 \rangle$

- Prvi parameter lahko spustimo, če je vrstica ista
- Enostavna brezizgubna metoda za kompresijo
- Že zelo dolgo, veliko uporabljana (faks, algoritmi za kompresijo slik, itd.)

Absolutna verižna koda

- Absolute Chain Code, tudi Freemanove kode
- Kodiranje obrisa regije

$$\boldsymbol{c}_{\mathcal{R}} = [\boldsymbol{x}_0, \boldsymbol{x}_1, \ldots \boldsymbol{x}_{M-1}] \text{ with } \boldsymbol{x}_i = \langle u_i, v_i \rangle$$

- Začnemo v nekem robnem slikovnem elementu in gremo po obrisu, pri čemer vsak korak zakodiramo:

$$\boldsymbol{c}'_{\mathcal{R}} = [c'_0, c'_1, \ldots c'_{M-1}]$$

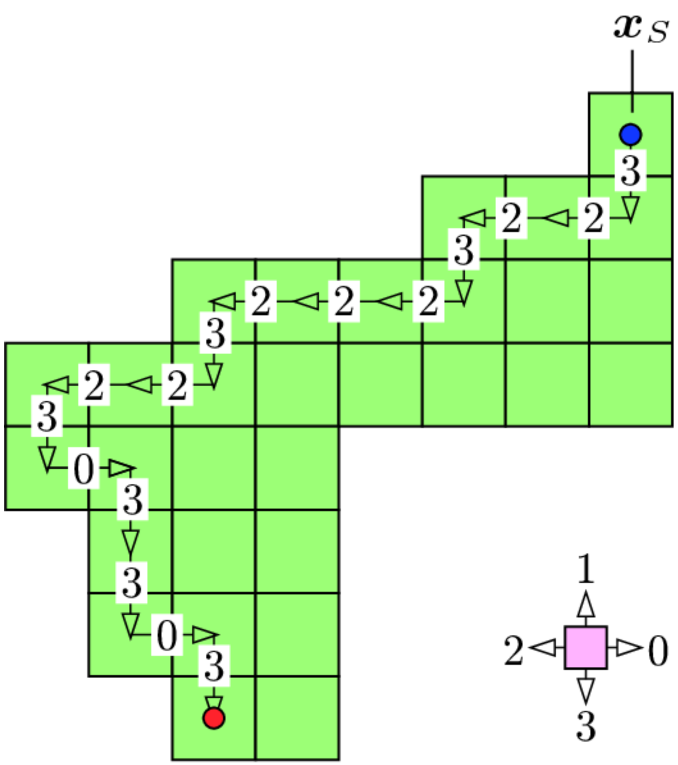
$$c'_i = \textsc{Code}(\Delta u_i, \Delta v_i)$$

$$(\Delta u_i, \Delta v_i) = \begin{cases} (u_{i+1} - u_i, v_{i+1} - v_i) & \text{for } 0 \leq i < M-1 \\ (u_0 - u_i, v_0 - v_i) & \text{for } i = M-1, \end{cases}$$

Δu	1	1	0	-1	-1	-1	0	1
Δv	0	1	1	1	0	-1	-1	-1
$\textsc{Code}(\Delta u, \Delta v)$	0	1	2	3	4	5	6	7

Primer

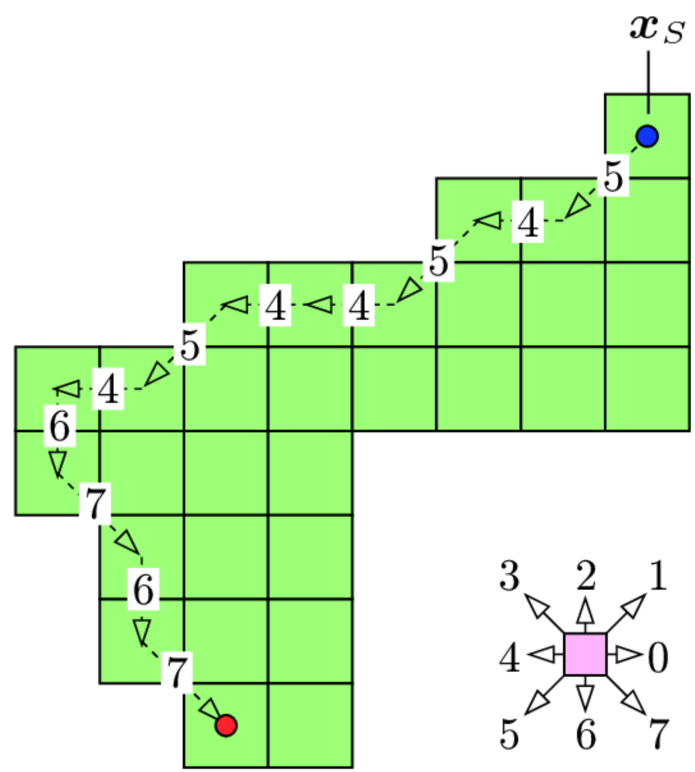
- Lahko upoštevamo 4- ali 8-sosednost



4-Chain Code

3223222322303303...111

$length = 28$



8-Chain Code

54544546767...222

$length = 16 + 6\sqrt{2} \approx 24{,}5$

- Koda je odvisna od začetne točke in orientacije obrisa

Diferenčna verižna koda

- Differential chain code
- Namesto kodiranja razlike pozicij, kodiramo razlike smeri

$$c_i'' = \begin{cases} (c_{i+1}' - c_i') \bmod 8 & \text{for } \; 0 \leq i < M-1 \\ (c_0' - c_i') \bmod 8 & \text{for } \; i = M-1 \end{cases}$$

- Primer:

$$\boldsymbol{c}_{\mathcal{R}}' = [5, 4, 5, 4, 4, 5, 4, 6, 7, 6, 7, \ldots 2, 2, 2]$$
$$\boldsymbol{c}_{\mathcal{R}}'' = [7, 1, 7, 0, 1, 7, 2, 1, 7, 1, 1, \ldots 0, 0, 3]$$

- Sedaj lahko regijo zarotiramo za 90 stopinj, pa se koda ne spremeni
- Še vedno pa je koda odvisna od začetne pozicije

Števila oblik

- Shape numbers
- Verižne kode moramo poravnati, da imajo enako začetno točko, potem jih lahko primerjamo
- Elemente kode interpretiramo kot števke v bazi b (8 ali 4):

$$\textsc{Val}(\boldsymbol{c}''_{\mathcal{R}}) = c''_0 \cdot b^0 + c''_1 \cdot b^1 + \ \ldots \ = \sum_{i=0}^{M-1} c''_i \cdot b^{i-1}$$

- Poiščemo maksimum in kodo zamaknemo za *k* mest

$$k_{\max} = \arg \max_{0 \le k < M} \textsc{Val}(\boldsymbol{c}''_{\mathcal{R}} \rhd k)$$

- Ni potrebno dejansko tega računati, lahko samo sortiramo
- Primer:

$$\boldsymbol{c}''_{\mathcal{R}} = [\,0, 1, 3, 2, \ldots 9, 3, 7, 4\,]$$

$$\boldsymbol{c}''_{\mathcal{R}} \rhd 2 = [\,7, 4, 0, 1, 3, 2, \ldots 9, 3\,]$$

- Zdaj koda ni več odvisna od začetnega položaja, lahko je regija rotirana za 90 stopinj
- Še vedno pa predstavitev ni invariantna na poljubne rotacije in skaliranje
  - Rešitev: Fourierjevi deskriptorji

Lastnosti binarnih regij

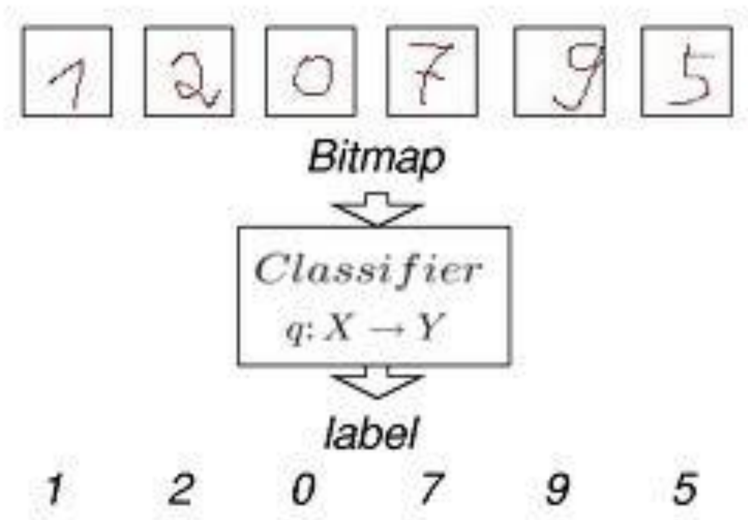
- Kako opisati sliko
  - Matrika števil: zaporedje slikovnih elementov
  - „Pretežno bel krog na pretežno zeleni podlagi.“
  - „Nogometna žoga na travi.“

- Zelo težko je priti do semantičnega opisa slike
- Lažje posamezne regije na sliki opišemo z enostavnimi lastnostmi

Značilnice

- Ko imamo regije, jih opišemo z značilnicami
- Na osnovi vektorjev značilnic lahko regije primerjamo med seboj
  - Lahko iščemo podobnosti med regijami
  - Lahko merimo in preverjamo dimenzije in oblike

- Primer: OCR:



Geometrične značilnice

- Geometrične značilnice definiramo za binarno regijo

$$\mathcal{R} = \{\boldsymbol{x}_0, \boldsymbol{x}_1 \ldots \boldsymbol{x}_{N-1}\} = \{(u_0, v_0), (u_1, v_1) \ldots (u_{N-1}, v_{N-1})\}$$

- Obseg (Perimeter)
- Površina (Area)
- Kompaktnost oz. okroglost (Compactness and roundness)
- Obsegajoč pravokotnik (Bounding box)
- Konveksna ovojnica (Convex hull)

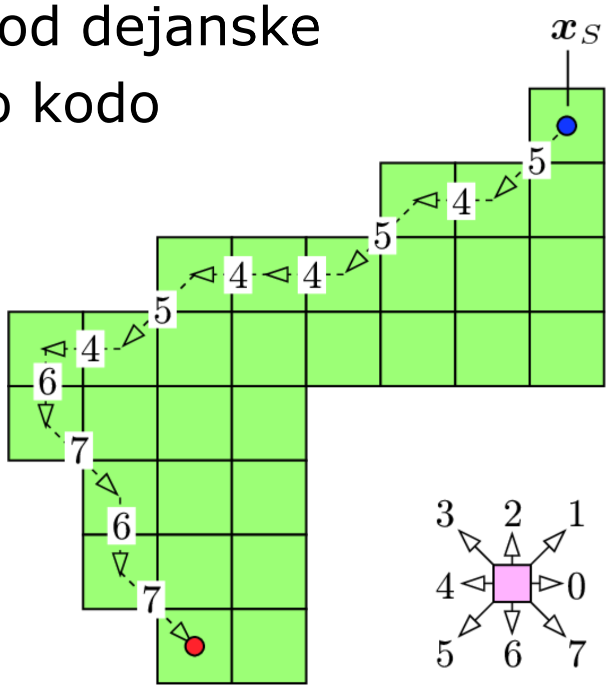
Obseg

- Perimeter
- Dolžina zunanjega obsega povezane regije
- V primeru 4-sosednosti bo dolžina večja od dejanske
- Ponavadi uporabljamo 8-sosedno Verižno kodo

$$\boldsymbol{c}'_{\mathcal{R}} = [c'_0, c'_1, \ldots c'_{M-1}]$$

$$\text{Perimeter}(\mathcal{R}) = \sum_{i=0}^{M-1} \text{length}(c'_i)$$

$$\text{with} \quad \text{length}(c) = \begin{cases} 1 & \text{for} \quad c = 0,2,4,6 \\ \sqrt{2} & \text{for} \quad c = 1,3,5,7 \end{cases}$$



8-Chain Code

54544546767...222

length = 16 + $6\sqrt{2} \approx$ 24,5

- To nam vrne nekoliko prevelik obseg
  - Zaradi diskretizacije poševnih ravnih črt
  - Normaliziramo:

$$P(\mathcal{R}) \approx \text{Perimeter}_{\text{corr}}(\mathcal{R}) = 0.95 \cdot \text{Perimeter}(\mathcal{R})$$

Površina

- Area
- Število slikovnih elementov v regiji:

$$A(\mathcal{R}) = |\mathcal{R}| = N.$$

- Lahko tudi ocenimo iz obrisa
  - če regija nima lukenj
  - Uporabimo zapis v Verižni kodi:

$$A(\mathcal{R}) \approx \frac{1}{2} \cdot \left| \sum_{i=0}^{M-1} \left(u_i \cdot v_{(i+1) \bmod M} - u_{(i+1) \bmod M} \cdot v_i \right) \right|$$

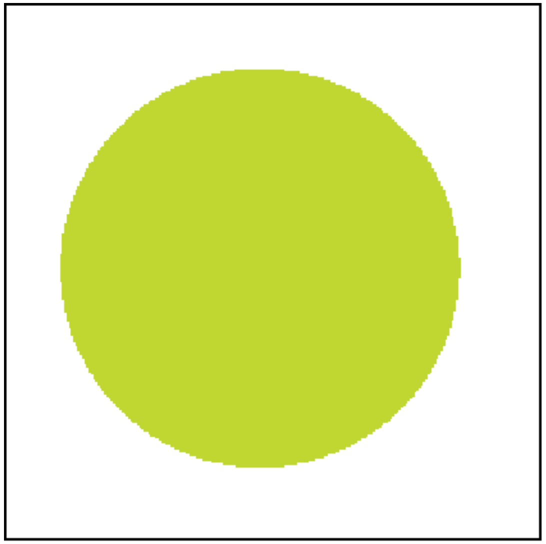
- Tako obseg kot površina sta
  - Neodvisna od translacije in rotacije
  - Odvisna od skale (velikosti, oddaljenosti)

Kompaktnost oz. okroglost

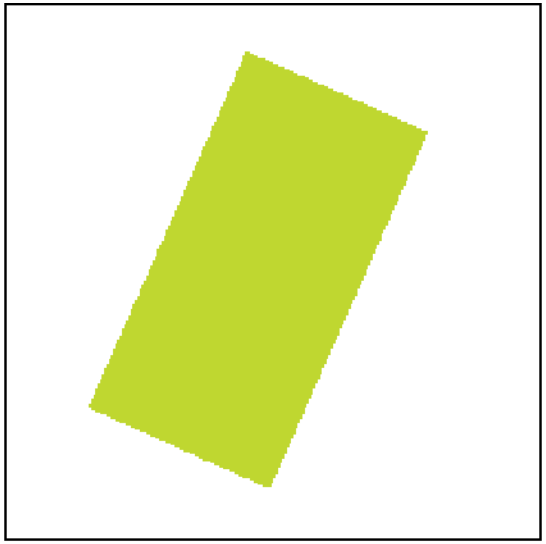
- Compactness and circularity
- Meri kako kompaktna oz. okrogla je regija

$$\text{Circularity}(\mathcal{R}) = 4\pi \cdot \frac{A(\mathcal{R})}{P^2(\mathcal{R})}$$

- Invariantna na translacijo, rotacijo in skalo
- Primer:



1.001



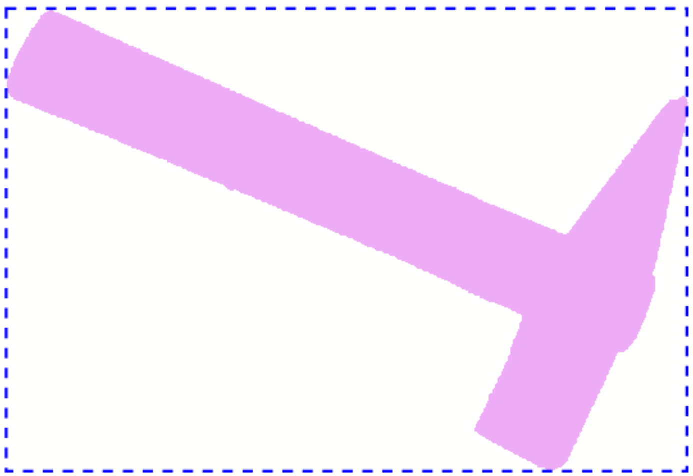
0.672

0.086

Obsegajoč pravokotnik

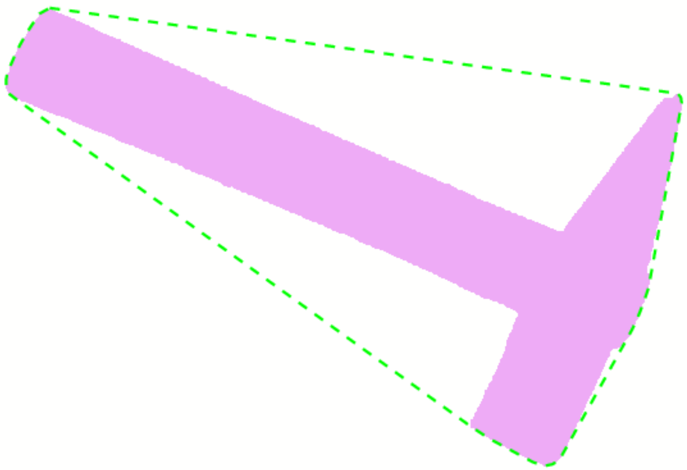
- Bounding box
- Najmanjši pravokotnik z osmi vzporednimi s koordinatnimi osmi, ki vsebuje celotno regijo

$$\mathsf{BoundingBox}(\mathcal{R}) = \langle u_{\min}, u_{\max}, v_{\min}, v_{\max} \rangle$$



Konveksna ovojnica

- Convex hull
- Najmnjši poligon, ki vsebuje vse elemente regije
- Računamo jo lahko z algoritmom QuickHull (kompleksnost O(NH))



- Konveksnost = dolžina konveksne ovojnice / obseg regije
- Gostota = površina regije / površina konveksne ovojnice
- Premer = razdalja me dvema maksimalno oddaljenima točkama na konveksni ovojnici

Statistične značilnice oblike

- Elemente regije obravnavamo kot točke točke porazdeljne v 2D prostoru

- Centroid
- Momanti
- Centralni momenti
- Normalizirani centralni momenti

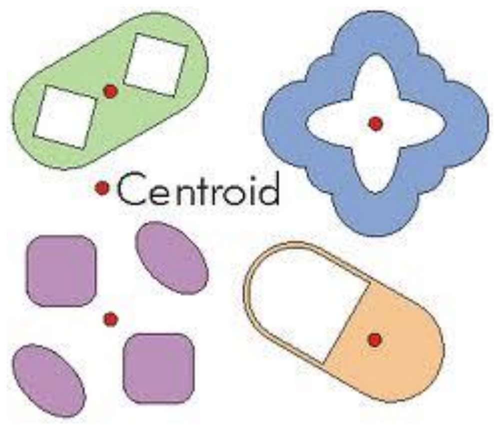
- Geometrične lastnosti, ki temeljijo na momentih
  - Orientacija
  - Ekscentričnost
  - Invariantni momenti

Centroid

- Težišče regije
- Aritmetična vsota koordinat v smereh x in y

$$\bar{x} = \frac{1}{|\mathcal{R}|} \cdot \sum_{(u,v)\in\mathcal{R}} u \qquad \text{and} \qquad \bar{y} = \frac{1}{|\mathcal{R}|} \cdot \sum_{(u,v)\in\mathcal{R}} v$$

- Primeri:



Momenti

- Splošni statistični koncept:

$$m_{pq} = \sum_{(u,v)\in\mathcal{R}} I(u,v) \cdot u^p v^q$$

- Za binarne slike:

$$m_{pq} = \sum_{(u,v)\in\mathcal{R}} u^p v^q$$

- Površina:

$$A(\mathcal{R}) = |\mathcal{R}| = \sum_{(u,v)\in\mathcal{R}} 1 = \sum_{(u,v)\in\mathcal{R}} u^0 v^0 = m_{00}(\mathcal{R})$$

- Centroid:

$$\bar{x} = \frac{1}{|\mathcal{R}|} \cdot \sum_{(u,v)\in\mathcal{R}} u^1 v^0 = \frac{m_{10}(\mathcal{R})}{m_{00}(\mathcal{R})} \qquad \bar{y} = \frac{1}{|\mathcal{R}|} \cdot \sum_{(u,v)\in\mathcal{R}} u^0 v^1 = \frac{m_{01}(\mathcal{R})}{m_{00}(\mathcal{R})}$$

Centralni momenti

- Vzamemo centroid za referenčno točko – središče koordinatnega sistema

$$\mu_{pq}(\mathcal{R}) = \sum_{(u,v)\in\mathcal{R}} I(u,v) \cdot (u-\bar{x})^p \cdot (v-\bar{y})^q$$

- Za binarne slike (regije):

$$\mu_{pq}(\mathcal{R}) = \sum_{(u,v)\in\mathcal{R}} (u-\bar{x})^p \cdot (v-\bar{y})^q$$

- Momenti tako niso več odvisni od položaja regije na sliki

- Normalizirani centralni momenti:
  - Normalizirati moramo za faktor $s^{(p+q+2)}$

$$\bar{\mu}_{pq}(\mathcal{R}) = \mu_{pq} \cdot \left(\frac{1}{\mu_{00}(\mathcal{R})}\right)^{(p+q+2)/2}$$

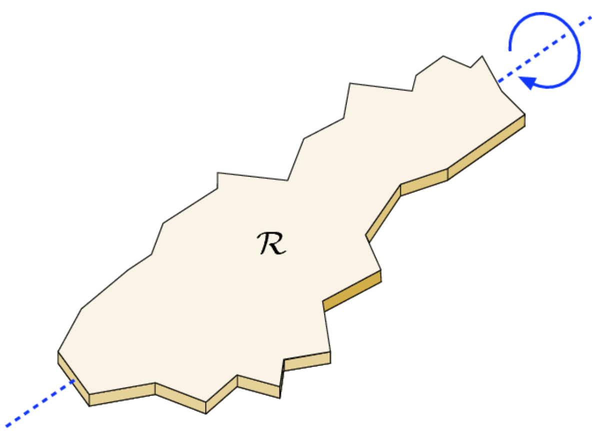
  - Centralni momenti so tako invariantni tudi na skalo
  - Primerni za klasifikacijo regij

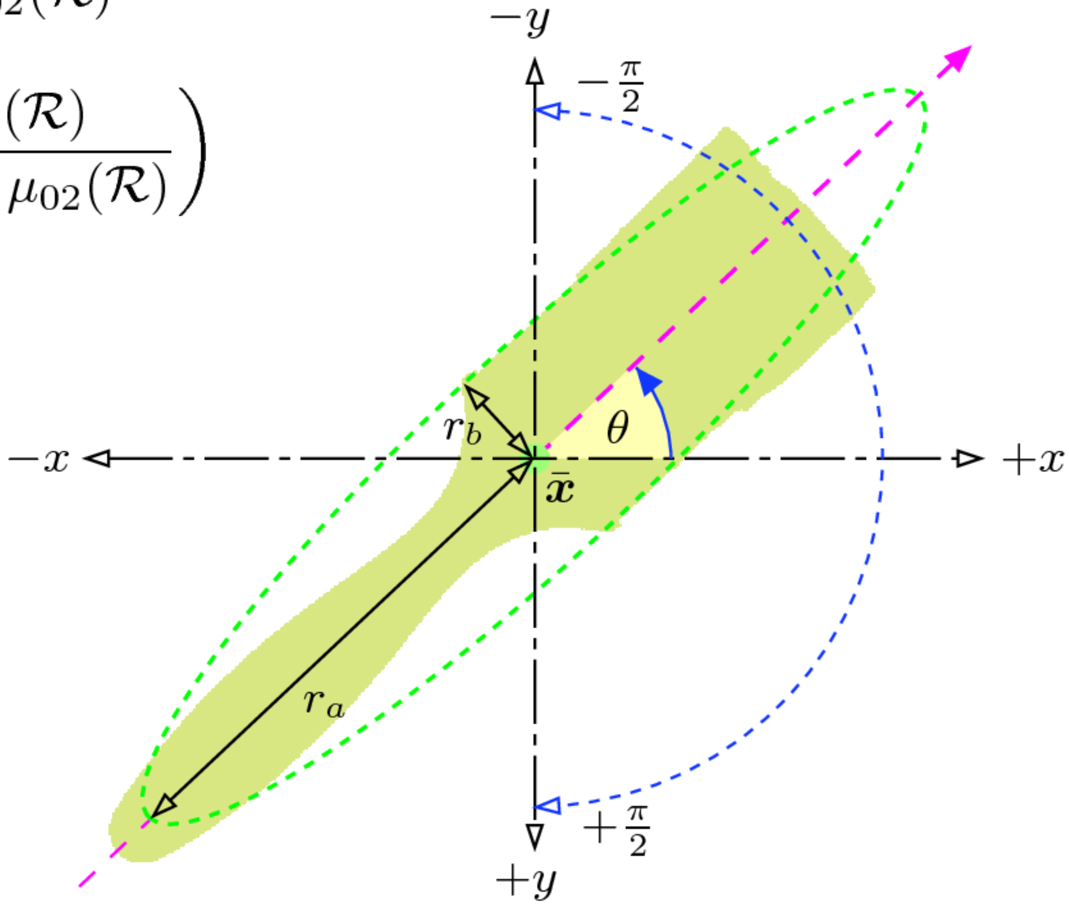
Orientacija

- Smer glavne osi

$$\tan(2\,\theta_{\mathcal{R}}) = \frac{2 \cdot \mu_{11}(\mathcal{R})}{\mu_{20}(\mathcal{R}) - \mu_{02}(\mathcal{R})}$$

$$\theta_{\mathcal{R}} = \frac{1}{2}\tan^{-1}\left(\frac{2 \cdot \mu_{11}(\mathcal{R})}{\mu_{20}(\mathcal{R}) - \mu_{02}(\mathcal{R})}\right)$$





Orientacija

- Vizualizacija vektorja smeri orientacije:

$$\boldsymbol{x} = \bar{\boldsymbol{x}} + \lambda \cdot \boldsymbol{x}_d = \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} + \lambda \cdot \begin{pmatrix} \cos(\theta_{\mathcal{R}}) \\ \sin(\theta_{\mathcal{R}}) \end{pmatrix}$$

$$\tan(2\theta_{\mathcal{R}}) = \frac{2 \cdot \mu_{11}(\mathcal{R})}{\mu_{20}(\mathcal{R}) - \mu_{02}(\mathcal{R})} = \frac{A}{B} = \frac{\sin(2\theta_{\mathcal{R}})}{\cos(2\theta_{\mathcal{R}})}$$

$$x_d = \cos(\theta_{\mathcal{R}}) = \begin{cases} 0 & \text{for} \quad A = B = 0 \\ \left[\frac{1}{2}\left(1 + \frac{B}{\sqrt{A^2+B^2}}\right)\right]^{\frac{1}{2}} & \text{otherwise,} \end{cases}$$

$$y_d = \sin(\theta_{\mathcal{R}}) = \begin{cases} 0 & \text{for} \quad A = B = 0 \\ \left[\frac{1}{2}\left(1 - \frac{B}{\sqrt{A^2+B^2}}\right)\right]^{\frac{1}{2}} & \text{for} \quad A \geq 0 \\ -\left[\frac{1}{2}\left(1 - \frac{b}{\sqrt{A^2+B^2}}\right)\right]^{\frac{1}{2}} & \text{for} \quad A < 0, \end{cases}$$

Ekscentričnost

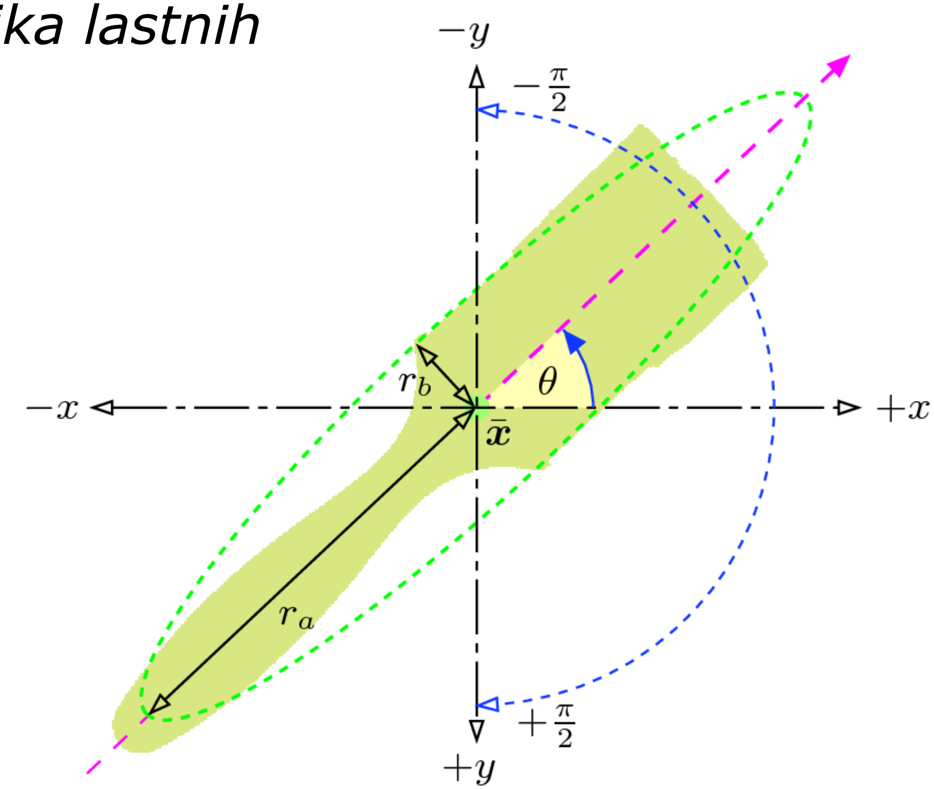
- Poldogovatost regije

$$\mathrm{Ecc}(\mathcal{R}) = \frac{a_1}{a_2} = \frac{\mu_{20} + \mu_{02} + \sqrt{(\mu_{20} - \mu_{02})^2 + 4 \cdot \mu_{11}^2}}{\mu_{20} + \mu_{02} - \sqrt{(\mu_{20} - \mu_{02})^2 + 4 \cdot \mu_{11}^2}}$$

- *$a_1 = 2\lambda_1$, $a_2 = 2\lambda_2$ sta večkratnika lastnih vrednosti matrike*

$$\boldsymbol{A} = \begin{pmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{pmatrix}$$

- Vrednosti med 1 in ∞
  - Ecc=1 => krog
  - Ecc>1 => podolgovata regija
- Invariantna na orientacijo in velikost



Ekscentričnost

- Vizualizacija elipse, ki ponazarja podolgovatost
- Dolžine osi elipse:

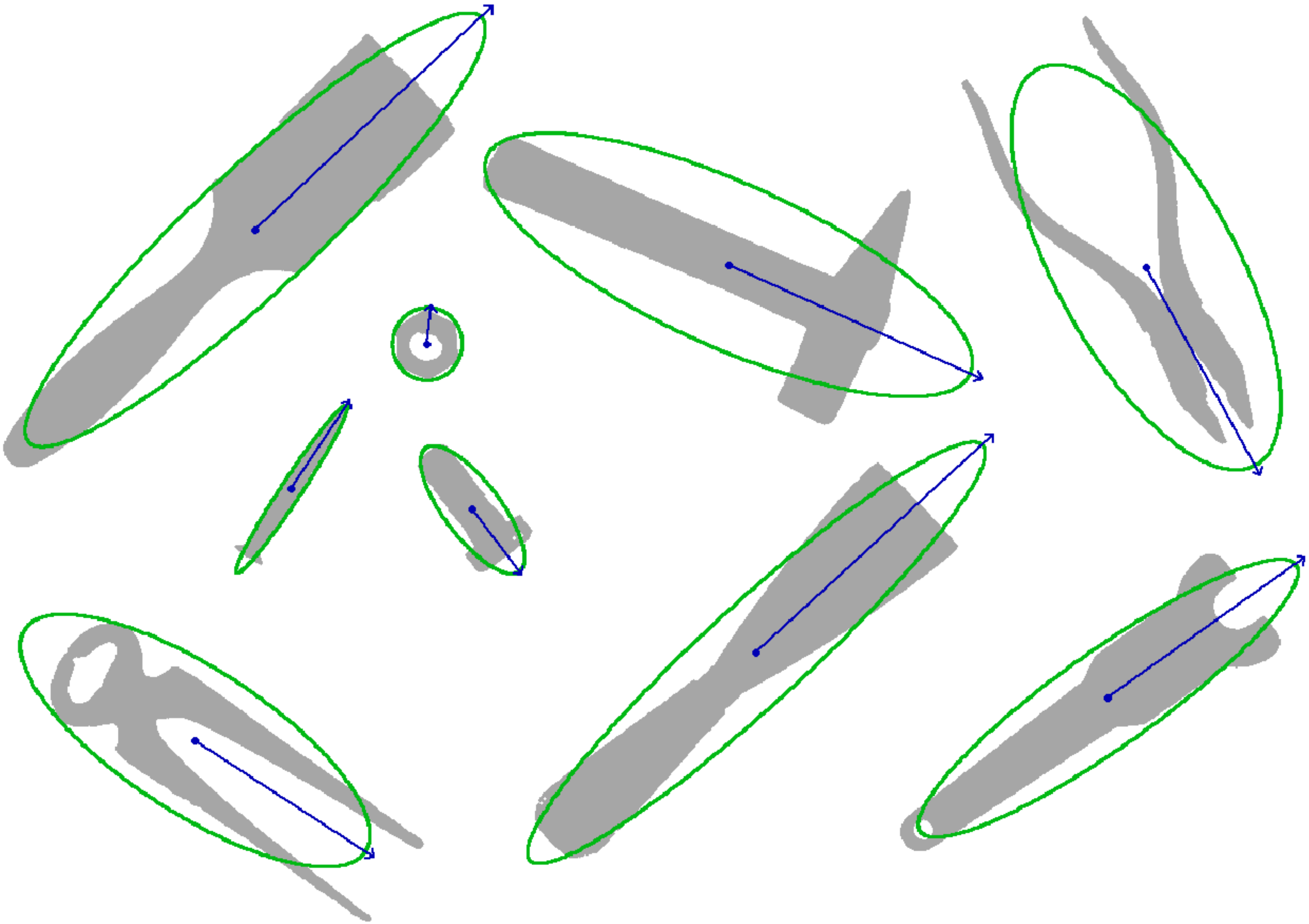
$$r_a = 2 \cdot \left(\frac{\lambda_1}{|\mathcal{R}|}\right)^{\frac{1}{2}} = \left(\frac{2\,a_1}{|\mathcal{R}|}\right)^{\frac{1}{2}}$$

$$r_b = 2 \cdot \left(\frac{\lambda_2}{|\mathcal{R}|}\right)^{\frac{1}{2}} = \left(\frac{2\,a_2}{|\mathcal{R}|}\right)^{\frac{1}{2}}$$

- Parametrična enačba elipse:

$$\begin{aligned}
\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} &= \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} + \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \cdot \begin{pmatrix} r_a \cdot \cos(t) \\ r_b \cdot \sin(t) \end{pmatrix} \\
&= \begin{pmatrix} \bar{x} + \cos(\theta) \cdot r_a \cdot \cos(t) - \sin(\theta) \cdot r_b \cdot \sin(t) \\ \bar{y} + \sin(\theta) \cdot r_a \cdot \cos(t) + \cos(\theta) \cdot r_b \cdot \sin(t) \end{pmatrix}
\end{aligned}$$

Primer



Invariantni momenti

- Hujevi momenti oz. Invariantni momenti
- So invariantni tudi na orientacijo

$$
\begin{aligned}
H_1 &= \bar{\mu}_{20} + \bar{\mu}_{02} \\
H_2 &= (\bar{\mu}_{20} - \bar{\mu}_{02})^2 + 4\,\bar{\mu}_{11}^2 \\
H_3 &= (\bar{\mu}_{30} - 3\,\bar{\mu}_{12})^2 + (3\,\bar{\mu}_{21} - \bar{\mu}_{03})^2 \\
H_4 &= (\bar{\mu}_{30} + \bar{\mu}_{12})^2 + (\bar{\mu}_{21} + \bar{\mu}_{03})^2 \\
H_5 &= (\bar{\mu}_{30} - 3\,\bar{\mu}_{12}) \cdot (\bar{\mu}_{30} + \bar{\mu}_{12}) \cdot \left[(\bar{\mu}_{30} + \bar{\mu}_{12})^2 - 3(\bar{\mu}_{21} + \bar{\mu}_{03})^2\right] \\
&\quad + (3\,\bar{\mu}_{21} - \bar{\mu}_{03}) \cdot (\bar{\mu}_{21} + \bar{\mu}_{03}) \cdot \left[3\,(\bar{\mu}_{30} + \bar{\mu}_{12})^2 - (\bar{\mu}_{21} + \bar{\mu}_{03})^2\right] \\
H_6 &= (\bar{\mu}_{20} - \bar{\mu}_{02}) \cdot \left[(\bar{\mu}_{30} + \bar{\mu}_{12})^2 - (\bar{\mu}_{21} + \bar{\mu}_{03})^2\right] \\
&\quad + 4\,\bar{\mu}_{11} \cdot (\bar{\mu}_{30} + \bar{\mu}_{12}) \cdot (\bar{\mu}_{21} + \bar{\mu}_{03}) \\
H_7 &= (3\,\bar{\mu}_{21} - \bar{\mu}_{03}) \cdot (\bar{\mu}_{30} + \bar{\mu}_{12}) \cdot \left[(\bar{\mu}_{30} + \bar{\mu}_{12})^2 - 3\,(\bar{\mu}_{21} + \bar{\mu}_{03})^2\right] \\
&\quad + (3\,\bar{\mu}_{12} - \bar{\mu}_{30}) \cdot (\bar{\mu}_{21} + \bar{\mu}_{03}) \cdot \left[3\,(\bar{\mu}_{30} + \bar{\mu}_{12})^2 - (\bar{\mu}_{21} + \bar{\mu}_{03})^2\right]
\end{aligned}
$$

- Ponavadi se uporablja logaritme vrednosti

Projekcije

- Enodimenzionalna predstavitev vsebine slike
- Slikovne elemente projiciramo na x in y koordinato:

$$P_{\text{hor}}(v_0) = \sum_{u=0}^{M-1} I(u, v_0) \qquad \text{for} \quad 0 < v_0 < N$$

$$P_{\text{ver}}(u_0) = \sum_{v=0}^{N-1} I(u_0, v) \qquad \text{for} \quad 0 < u_0 < M$$

- Včasih tako lahko ločimo dokument na smiselne celote

- Projiciramio lahko tudi na glavni osi regije

Topološke lastnosti

- Zajamejo strukturne lastnosti
- So invariantne tudi na zelo močne transformacije slike

- Konveksnost regije

- Število lukenj: $N_L(\mathcal{R})$

- Eulerjevo število: $N_E(\mathcal{R}) = N_R(\mathcal{R}) - N_L(\mathcal{R})$