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IEOR 174 Semester Project Final Report: Save the Stray Cats!

Youtube Presentation link: https://youtu.be/YIjjYylj8BY
Google Drive Presentation link: https://tinyurl.com/174Cats

Problem and Motivation

The feral cat population has increased over the years with a current population ranging to 60 million to 160 million cats. This huge feral cat population brings many environmental and economic consequences. Their hunting damages our wildlife ecosystem and costs the United States billions of dollars each year. Additionally, as the population grows, deadly diseases spread easily among the feral community, harming more cats and other stray animals. While there are currently non-profit animal shelters and organizations that aim to rescue feral cats, we wonder if there is an optimal economic policy to manage this growing population.

Objectives

We aim to answer the following questions by comparing feral cat population growth and the economic costs associated under current and hypothetical animal management policies. The desired result of the project is to identify an optimal policy (amount of funding and operation to fund, if any) that minimizes the national economic costs due to the feral cat population growth.

1. What is the predicted population of feral cats for 2022-2026 under current animal management policies?
2. What are the national economic costs due to this population growth? How does this cost change as the feral cat population changes under current animal management policies?
3. How will the feral cat population change if funding for animal management increases?
4. What is the optimal policy (amount of funding and operation to fund) for animal care such that it minimizes expected national costs?

Analysis of the Problem & Analytic Methods Used

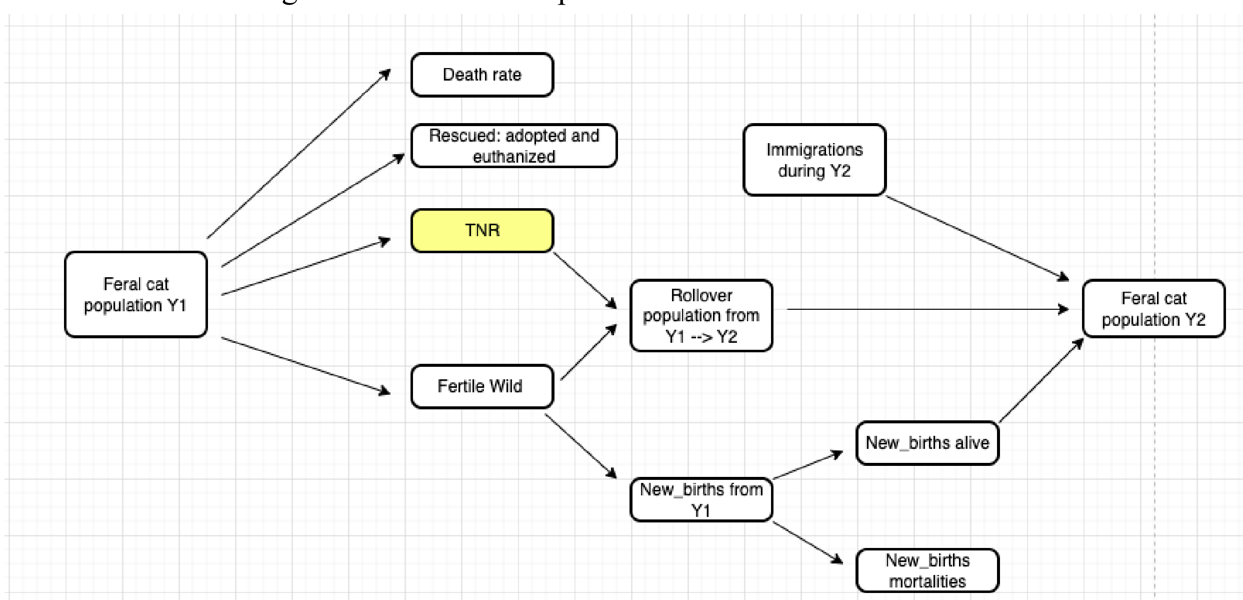
To simulate the growth of the feral cat population, we considered many factors impacting the arrivals and departures. For arrivals, we considered the annual feral cat reproduction rate (number of newborns) and the annual cat immigration rate (how many domestic cats are abandoned into the wild). For departures, we considered the annual death rate in the wilderness, annual euthanization and annual adoption rates of feral cats. For later policies, we add in the trap-neuter-rescue (TNR) policy aimed to curtail the arrival or birth rate. Under this policy, some feral cats are inoculated, unable to reproduce in the future, and released back into the wild.

Three policies are simulated with the objective of identifying the most effective policy in curtailing feral cat population:

1. **Current** (no TNR) policy: This policy accounts for births, deaths, immigrations, and shelter adoptions and euthanizations of the feral cat population.
2. **TNR-facility** policy: In addition to the processes accounted for in the current policy, TNR-facility policy simulates new buildings built for TNR-only operations. This is different from shelter operations as TNR-facilities do not house rescued animals for adoption or euthanization, which may lead to a difference in operations cost and higher annual TNR rates.
3. **TNR-shelter** policy: This policy is similar to the current policy, except that a proportion of funding allocated for shelter euthanizations are converted to fund TNR operations within existing shelters. This policy may be ideal since it does not require new buildings to be built, but may impact the annual adoption/euthanization rescue rates at a shelter.

The following flow chart illustrates the population growth dynamic for all 3 policies. The TNR component will not factor into the current (no TNR) policy. Here, ‘Y1’ represents ‘Year 1’ and ‘Y2’ represents ‘Year 2’ to illustrate the population change from one year to the next.

Diagram 1 - Feral Cat Population Growth Model Flow Chart



In our model, we assume a logistic growth curve as the basis to simulate a population carrying capacity due to finite resources and environmental limits of supporting feral cats. As a result, the feral cat population growth rate decreases as it approaches the carrying capacity.

Differential Equations

We use differential equations to model the change in feral cat population. We consider the following factors that may affect population growth.

Arrivals into the feral cat population are from:
- Births
- Immigrations into the feral system (ie. runaways from domestic cat population)

Departures from the feral cat population are from:
- Deaths in wilderness
- Animal Rescue: adoptions and euthanizations
- TNR-facility operations
- TNR-shelter operations

A complete definition, description, and assumed values for all variables used in our model are found in Table A.3 found under the Appendix. The rationale for building our model (differential equations) follows.

The carrying capacity is modeled by $K(t) = K_B(1 + \frac{E(t)}{100})$, in which we assume the carrying capacity of the feral cat population for a given year $K(t)$ is the base carrying capacity K_B linearly scaled by the GDP growth of year *t.* Feral cats rely mainly on food waste and cat food offered by people as their food source. We assume these components are dependent on the success of the economy; if the economy is doing well, consumers are likely to have more spare cash to buy cat food for stray cats and spend more on eating (and thus, produce greater quantities of food waste).

We assume the fertile population in 2021 $P_F(t = 0)$ is 70 million cats, so the 2021 feral cat population P(t = 0) = $P_F(t = 0)$ = 70M. We also assume that we begin to account for immigrations and TNR operations curtailment beginning in the year 2022 (*t = 1*).

The contributors to population growth are only the fertile feral cats. For simplicity, we assume that all annual TNR operations happen before reproductions for the current year. To quantitatively model this, we subtract the number of cats that have undergone TNR in a given year from the fertile population of the previous year. This modified fertile population value aims to reflect the number of contributors to population growth for the current year. The term $P_F(t - 1) - h(y(t))$ reflects this interpretation.

$$\frac{dP_F}{dt}(t) = f(P_F(t - 1) - h(y(t)), K(t) - P_N(t)) - g(E(t))$$

where

$$P(0) = P_F(0) = 70 \text{ million}$$

$$f(P_F(t-1) - h(y(t)),\ K(t) - P_N(t)) = r_0\left(1 - \frac{P_F(t-1) - h(y(t))}{K(t) - P_N(t)}\right)[P_F(t-1) - h(y(t))]$$

Additionally, $g(E(t))$ represents the annual rescue rate, which is influenced by the annual base funding to shelters c_F , the average cost of rescuing 1 animal c_R , and the annual GDP growth, *E(t)*.

$$g(E(t)) = \frac{c_F}{c_R}\left(1 + \frac{E(t)}{100}\right)$$

Similarly, the annual TNR rate is the base TNR funding *y(t)* linearly scaled by the annual GDP growth *E(t)* and divided by the cost of performing TNR for 1 stray cat, c_{TNR}.

$$h(y(t)) = \frac{y(t) * \left(1 + \frac{E(t)}{100}\right)}{c_{TNR}}$$

Depending on the policy, $y(t)$ may equal to the funding given to TNR operations within an existing shelter $y_s(t)$ or the funding given to TNR operations at TNR-only facilities $y_F(t)$, where

$$y_s(t) = p_{TNR,s} * c_F, \quad p_{TNR,s} \text{ and } c_F \text{ are simulated}$$

$$y_F(t) \text{ is simulated}$$

The funding given to TNR operations within an existing shelter $y_s(t)$ is scaled by the proportion of regular shelter operations converted to TNR operations. With the differential equation to model the change in fertile population, we obtain the fertile population of a given year using

$$P_F(t) = P_F(t-1) - h(y(t)) + \int_{t-1}^{t} \frac{dP_F}{dx} dx$$

This is estimated using

$$P_F(t) = P_F(t-1) - h(y(t)) + \frac{dP_F}{dt}(t)$$

showing that the fertile population of year *t* is impacted by the fertile population of the previous year minus the TNR rate of year *t* and adding the population growth of year *t*. This estimation gives < 1% error in comparison to the equation using integration. A sample computation can be found in the Appendix under section 4.

This interpretation is reasonable since it takes the fertile population of the previous year, removes the number of cats who undergo TNR operations in the current year and factors in the growth rate according to this modified fertile population.

Next, we model the change in the non-fertile population, $P_N(t)$. Since we assume that TNR operations and immigrations are accounted for starting in the year 2022 *(t = 1)*, we obtain

$$P_N(0) = 0$$

The non-fertile population cannot reproduce, so we account only for TNR operations and immigrations as arrivals, and death rates as departures for this population. Each year, c_D% of the previous year non-fertile population will depart due to deaths and $(1 - c_D)$% of new arrivals from TNR operations in the current year $h(y(t))$ will enter along with the annual immigration from the current year, $i(E(t))$. We assume the immigration proportion r_i is impacted by GDP growth, since a better economy leads to less abandonment and runaways of domestic cats. We multiplied by the estimated number of domestic cats in the U.S to this rate to obtain the immigration rate. This is illustrated in the following differential equation for the neutered population.

$$\frac{dP_N}{dt} = - c_D P_N(t - 1) + (1 - c_D)h(y(t)) + i(E(t))$$

$$i(E(t)) = r_i (1 - \frac{E(t)}{100}) \cdot c_{C,H} \cdot H$$

We obtain the non-fertile population of a given year using

$$P_N(t) = P_N(t - 1) + \int_{t-1}^{t} \frac{dP_N}{dx} dx$$

and estimated using

$$P_N(t) = P_N(t - 1) + \frac{dP_N}{dt}(t)$$

showing that the non-fertile population of year *t* is impacted by the non-fertile population of the previous year and the growth rate of the non-fertile population in year *t.* The overall feral cat population for year *t* is estimated by summing the fertile and non-fertile population for year *t.*

$$P(t) = P_F(t) + P_N(t)$$

Primary Results and Corresponding Impact

The current policy is a no-TNR policy. Under this policy and assuming the parameter values in Table 1, we observe the following feral cat population estimates. A sample calculation to obtain the 2022 population estimates, P(*t = 1)* is found in section 5 of the Appendix.

Table 1: Feral Cat Population Estimates under Current Policy, 2021-2026

Year	Feral Cat Population (in millions)	Year-Over-Year Percent Change (%)
2021	70	-
2022	74.98	7.12
2023	79.18	5.60
2024	82.62	4.34
2025	85.57	3.57
2026	88.01	2.86

We observe that the population growth rate or 'Year-Over-Year Percent Change…" for each year is slowing. By 2026, our model is expecting a 88M feral cat population, a 26% increase compared to the 2021 population. We see this as reasonable as the results aligns with our expectations of high growth and other published estimations of feral cat population over the years.

Next, we obtain the population estimates for the TNR-facility policy. We choose to invest $100M as base funding into implementing the TNR-facility policy. This is equivalent to setting $y_F(t) = 100M$ and keeping all other parameters the same as used in Table 1. We obtain the results in Table 2.

Table 2: Feral Cat Population Estimates under TNR Facility Operations, 2021-2026

Year	Feral Cat Population (in millions)	Year-Over-Year Percent Change (%)
2021	70	-
2022	74.79	6.84
2023	78.68	5.20

2024	81.69	3.83
2025	84.19	3.07
2026	86.10	2.26

Similar to the Current policy, we see a slowing in the population growth rates and an overall smaller percent increase from 2021-2026. Under the TNR-facility policy, we expect a feral cat population of 86M by 2026, a 23% increase from 2021. It's reasonable to see that TNR-facility policy is more effective in curtailing feral cat population, since we added an additional $100M funding to implement the policy while keeping the $225M base funding for shelters to carry out their annual rescue rates.

Lastly, we obtain the following population estimates for TNR-shelter policy when we convert 56.8% of current shelter operations into TNR operations. Here, the funding is $225M base funding for shelters, which is split 56.8% for TNR operations and the rest for adoption/euthanization rescue efforts. We obtain the results in Table 3.

Table 3: Feral Cat Population Estimates under TNR Shelter Operations, 2021-2026

Year	Feral Cat Population (in millions)	Year-Over-Year Percent Change (%)
2021	70	-
2022	76.24	8.91
2023	81.27	6.61
2024	85.35	5.02
2025	88.62	3.83
2026	91.42	3.16

Similar to the other 2 policies, we see a slowing in the population growth rates. However, the population growth rates under TNR-shelter policy are greater than those of the other 2 policies. Under TNR-shelter policy, we expect to see a 91M feral cat population by 2026, which is greater than the 2026 population estimates under Current and TNR-Facility policy.

We compare the 5-year feral cat population values that result from our 3 policies. The percent reduction columns in Table 4 show the percentage decrease in population compared to the current policy for each TNR policy.

Table 4: Feral Cat Population Under Each Policy and Percent Reductions

Year	Current Population (in millions)	TNR-Facility Population (in millions)	TNR-Shelter Population (in millions)	Percent Reduction - TNR Facility (%)	Percent Reduction - TNR Shelter (%)
2021	70	70	70	0	0
2022	74.98	74.79	76.24	0.26	-1.67
2023	79.18	78.68	81.27	0.64	- 2.64
2024	82.62	81.69	85.35	1.12	- 3.31
2025	85.57	84.19	88.62	1.60	- 3.57
2026	88.01	86.10	91.42	2.17	- 3.87

We observe that implementing the TNR - shelter operations is not effective in curtailing the feral cat populations. In fact, it leads to a slightly higher feral cat population compared to the current (no-TNR) policy. This may be due to the fact that TNR - shelter operations take away partial funding from the overall rescue funding given to shelters to fund the TNR operations, which leaves less funding for animals rescued for adoption or euthanization. In turn, this means that under TNR - Shelter policy, less animals rescued actually leave the feral system completely because a majority of them undergo TNR and are returned to the wild.

Investing in additional TNR operations performed in TNR-focused facilities show a small, but positive reduction on the feral cat population. This is reasonable, since we also assume that TNR facilities operate independently from existing shelters; thus, under the TNR-facility policy, feral cats undergo TNR that help curtail the population while shelters continue to devote all of their funding and resources towards rescuing animals and removing them from the feral cat population. Here, we do not divide the resources for TNR operations within the shelter systems, which should lead to a greater reduction in feral cat population overall. However, additional funding for new TNR-focused facilities are required to implement the TNR-facility policy, which is assumed to be separate from the funding for shelter rescues. Looking at the curtailment from a base value of $100M invested annually (which converts to about 670,000 cats who undergo TNR operations annually), this policy may not be the cost-effective to implement in reality.

Sensitivity Analysis

We dive deeper into testing a range of values for the following parameters to gain insight on the impact of these factors on population values.

- Overall base shelter funding (c_F) under current policy
- TNR-facility base funding (y_F) under TNR-facility policy
- Base shelter funding (c_F) and TNR-shelter base funding (y_s) under TNR-shelter policy
- Proportion of shelter operations converted to TNR, $p_{TNR,\,s}$
- GDP growth, E(t)

For the following analysis, we may simulate values outside of the expected range in reality. For most variables like funding, we believe our simulation runs in a reasonable range of values and may be observed in reality. For other variables, we simulate a larger range of values for experimental purposes and analysis.

Simulating base shelter funding (c_F) under current policy

We simulate values for the base shelter funding c_F in the interval [\$225M, \$525M] incremented by \$20M to understand how shelter funding impacts the feral cat population in 2026 under the Current policy. Since there are no TNR operations under the Current policy, TNR funding is \$0. This means $p_{TNR,\,s} = 0$ and $y_F = 0$.

Table 5: 2026 Feral Cat Population as Shelter Funding Increases under Current Policy

	Base Shelter Funding (in millions \$)	Population 2026 (in millions)	Population Change (in millions)
0	225.0	88.014451	NaN
1	245.0	86.718166	-1.30
2	265.0	85.418084	-1.30
3	285.0	84.114190	-1.30
4	305.0	82.806470	-1.31
5	325.0	81.494909	-1.31
6	345.0	80.179494	-1.32
7	365.0	78.860209	-1.32
8	385.0	77.537042	-1.32
9	405.0	76.209976	-1.33
10	425.0	74.878998	-1.33
11	445.0	73.544094	-1.33
12	465.0	72.205248	-1.34
13	485.0	70.862446	-1.34
14	505.0	69.515673	-1.35
15	525.0	68.164915	-1.35

The column ‘Population Change (in millions)’ shows the change in 2026 population as funding increases by \$20M. We take the average of ‘Population Change (in millions)’ and obtain that the 2026 feral cat population is expected to **decrease by 1.32 million cats for every additional $20 million added to the annual base shelter funding c_F under the current (no-TNR) policy.** This is reasonable, as increasing the baseline shelter funding by $20M equates to a total increase of $100M over 5-years, which equates to rescuing about 1.33 million additional feral cats.

Simulating base funding under TNR-Facility policy

We simulate values for additional funding on top of the $225M funded to shelters annually for their shelter operations. This simulation went through values of y_F in the interval [$0M, $300M] incremented by $20M to understand how shelter funding impacts the feral cat population in 2026 under the TNR-Facility policy.

Table 6: 2026 Feral Cat Population as TNR-Facility Funding Increases under TNR-facility Policy

	TNR-Facility Funding (in millions $)	Population 2026 (in millions)	Population Change (in millions)
0	0.0	88.014451	NaN
1	20.0	87.633724	-0.38
2	40.0	87.251995	-0.38
3	60.0	86.869256	-0.38
4	80.0	86.485499	-0.38
5	100.0	86.100714	-0.38
6	120.0	85.714895	-0.39
7	140.0	85.328033	-0.39
8	160.0	84.940119	-0.39
9	180.0	84.551145	-0.39
10	200.0	84.161102	-0.39
11	220.0	83.769982	-0.39
12	240.0	83.377775	-0.39
13	260.0	82.984474	-0.39
14	280.0	82.590068	-0.39
15	300.0	82.194549	-0.40

The column ‘Population Change (in millions)’ shows the change in 2026 population as funding increases by $20M. We take the average of ‘Population Change (in millions)’ and obtain that the 2026 feral cat population is expected to **decrease by 390,000 cats for every additional**

$20 million added to the annual base fund for TNR-Facility policy.

This is reasonable, as while increasing the funding for TNR-facility operations by $20M does result in a direct decrease in the population, TNR only serves to limit the growth of the feral cat population and does not fully remove them from the wild. As such, the $100M funding over 5-years only results in decreasing the feral cat population by 390,000 additional feral cats.

Simulating base shelter and TNR-shelter funding under TNR-Shelter policy

We simulate values for the base shelter funding c_F in [$225M, $525M] incremented by $20M to understand how shelter funding impacts the feral cat population in 2026 under the TNR-shelter policy compared to the current (no-TNR) policy. This is similar to the sensitivity analysis performed for c_F under the current policy, now factoring in TNR-shelter performance. The proportion of shelter operations converted to TNR is fixed at $p_{TNR,s}$ = 0.568 for this simulation.

The column 'Base Shelter Funding…" is c_F , 'Current Policy: Population 2026…" is the 2026 population corresponding to c_F , 'Current Policy: Pop Change…" is the change in 2026 population as base shelter funding increases by $20M under current policy, 'TNR-Shelter Funding…" is $p_{TNR,s} * c_F$, 'TNR-Shelter Policy: Population 2026…' and 'TNR-Shelter Policy: Pop Change…" are the 2026 population and population changes as funding increases under the TNR-shelter policy.

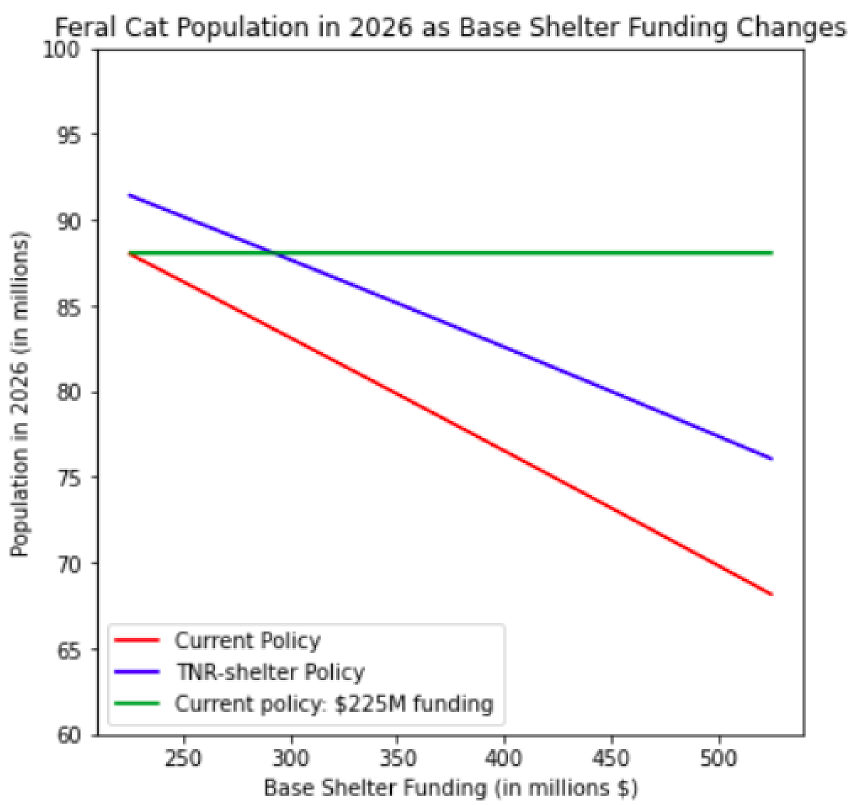
Table 7: 2026 Feral Cat Population as Shelter Funding Increases under Current and TNR-Shelter Policy

	Base Shelter Funding (in millions $)	Current Policy: Population 2026 (in millions)	Current Policy: Pop Change (in millions)	TNR-Shelter Funding (in $ millions)	TNR-Shelter Policy: Population 2026 (in millions)	TNR-Shelter Policy: Pop Change (in millions)
0	225.0	88.014451	NaN	127.80	91.424766	NaN
1	245.0	86.718166	-1.30	139.16	90.430982	-0.993784
2	265.0	85.418084	-1.30	150.52	89.433170	-0.997812
3	285.0	84.114190	-1.30	161.88	88.431287	-1.001883
4	305.0	82.806470	-1.31	173.24	87.425287	-1.006000
5	325.0	81.494909	-1.31	184.60	86.415124	-1.010163
6	345.0	80.179494	-1.32	195.96	85.400753	-1.014371
7	365.0	78.860209	-1.32	207.32	84.382126	-1.018627
8	385.0	77.537042	-1.32	218.68	83.359196	-1.022930
9	405.0	76.209976	-1.33	230.04	82.331914	-1.027282
10	425.0	74.878998	-1.33	241.40	81.300231	-1.031682
11	445.0	73.544094	-1.33	252.76	80.264098	-1.036133
12	465.0	72.205248	-1.34	264.12	79.223465	-1.040634
13	485.0	70.862446	-1.34	275.48	78.178279	-1.045186
14	505.0	69.515673	-1.35	286.84	77.128490	-1.049789
15	525.0	68.164915	-1.35	298.20	76.074044	-1.054446

Main insights from Table 7:

- Funding under current policy performs better in mitigating the population growth of feral cats than TNR-shelter policy. This is reasonable, since about 57% of the annual funding for shelters are allocated for TNR under the TNR-shelter policy, and this policy does not completely rescue the feral cat from the feral population. Cats who undergo TNR operation are released back into the wild and contribute themselves to the wilderness population. This policy aims to curtail the growth of the feral cat population.
- As a larger reduction in 2026 population is desired, more funding will be needed under the TNR-sheler policy than the current policy. This is illustrated in Graph 2.

Graph 2



As seen in Graph 2, the gap between the blue and red line increases as base shelter funding increases, which shows that as funding increases, current policy becomes more and more efficient in reducing feral cat population compared to TNR-shelter policy. This is also equivalent to demonstrating that an increasing additional amount of funding is needed to supplement TNR-shelter policy in order to achieve the same performance as the current policy as 2026 population reduction improves. For example, from Table 7, it's noted that about $70M additional spending is needed under TNR-shelter policy to achieve the same 2026 population (around 88M cats) as setting an annual base funding of $225M under the current policy. This is also seen in the intersection between the green and blue line in Graph 2. However, a difference much greater than

$70M will be needed if organizations are aiming to reduce the 2026 population to 78M cats (around $100M).

Taking the difference of the averages of columns 'Current Policy: Pop Change…' and TNR-shelter Policy: Pop Change…", we observe that current policy reduces about 300,000 more feral cats for the 2026 population compared to TNR-shelter policy given the same funding.

Comparing 3 policies at Equal Funding

After performing sensitivity analysis on funding under each policy, we compare the 3 policies at equal funding to see if there's an optimal, cost-effective policy. We compare all 3 policies are $325M funding.

- Under Current policy, c_F = $325M
- Under TNR-Facility policy, c_F = $225M and y_S = $100M
- Under TNR-Shelter policy, the overall funding is $325M. The funding for TNR at shelters is a proportion $p_{TNR,s}$ of $325, where $p_{TNR,s}$ = 0.568. The rest goes towards funding regular shelter rescue efforts.

Under current policy, a $325M funding gives a 2026 population of 81.5M feral cats. Under TNR-Facility policy, an annual base shelter funding of $225M and additional $100M for TNR-facility operations (total $325M) gives a 2026 population of 86.1M feral cats. Under TNR-Shelter policy, an annual base shelter funding of $325M split at 56.8% for TNR operations in shelters give a 2026 population of 86.4M feral cats. Considering only cost, Current policy is most efficient, followed up TNR-Facility policy and lastly TNR-Shelter policy.

Simulating proportion of shelter operations converted to TNR under TNR-shelter policy

We simulate values for $p_{TNR,s}$ in the interval [0, 0.568] in increments of 0.01 while keeping the base shelter funding c_F fixed at $225M. Table 8 shows a preview of our results.

Table 8: 2026 Feral Cat Population as Proportion of TNR Operations at Shelter Increases

	TNR Operation Proportion at Shelter	Population 2026 (in millions)	Pop Change (in millions)
0	0.00	88.014451	NaN
1	0.01	88.074518	0.060067
2	0.02	88.134584	0.060066
3	0.03	88.194649	0.060065
4	0.04	88.254714	0.060064
5	0.05	88.314777	0.060063
55	0.55	91.316740	0.060016
56	0.56	91.376755	0.060015
57	0.57	91.436769	0.060014

Main insights from Table 8:

- When $p_{TNR,\,s} = 0$, we achieve the same 2026 population as under the current policy, which is expected.
- As $p_{TNR,\,s}$ increases, 2026 population increases. This is reasonable, since as $p_{TNR,\,s}$ increases, a higher proportion of the annual funding shelters receive are allocated for TNR operations within the shelter. Again, TNR operations do not completely rescue a cat from the feral population since cats are reintroduced back into the wild after undergoing TNR. More fund allocations for TNR operations equates to less funding allocated for rescuing cats who are adopted or euthanized. This also means that less cats leave the wild completely through shelter rescue efforts, hence a higher population in 2026 under TNR-shelter policy.
- Taking the average of the ‘Pop Change…” column in Table 8, it’s observed that the feral **cat population in 2026 increases about 60,041 cats per 1% increase in TNR operations at shelters.**

Simulating GDP growth percentage

We obtained the 5-year population estimates in Table 1-3 under the simulated yearly GDP growth percentages $E(t)$ in Table A.3 where t = $\{0, 1, 2, 3, 4, 5\}$. In Table 9, we simulate a range of values for annual GDP growth in the interval [-10, 10] incremented by 1 to represent an increase or decrease of up to 10% in annual GDP. As we assume the base year to be 2021, the GDP growth is 0%, hence a constant 70M feral cat population in Table 9.

Here, we simulate under current policy, setting c_F fixed at \$225M and no funding for TNR ($y_F = 0$ and $y_S = 0$). Table 9 shows our results.

Table 9: Yearly Feral Cat Population (in millions) as Annual GDP Growth Changes

Year	2021	2022	2023	2024	2025	2026
GDP Growth Percent (%)						
-10	70.0	75.15	79.27	82.62	85.36	87.63
-9	70.0	75.14	79.27	82.63	85.40	87.71
-8	70.0	75.13	79.27	82.65	85.44	87.78
-7	70.0	75.12	79.26	82.66	85.47	87.84
-6	70.0	75.11	79.26	82.66	85.50	87.90
-5	70.0	75.10	79.25	82.67	85.52	87.95
-4	70.0	75.09	79.24	82.67	85.54	87.99
-3	70.0	75.07	79.22	82.66	85.56	88.03
-2	70.0	75.06	79.20	82.65	85.57	88.07
-1	70.0	75.04	79.19	82.64	85.57	88.09
0	70.0	75.03	79.17	82.63	85.57	88.12
1	70.0	75.01	79.14	82.61	85.57	88.13
2	70.0	74.99	79.12	82.59	85.56	88.14
3	70.0	74.97	79.09	82.57	85.55	88.15
4	70.0	74.95	79.06	82.54	85.53	88.15
5	70.0	74.93	79.03	82.51	85.51	88.15
6	70.0	74.91	79.00	82.48	85.49	88.14
7	70.0	74.88	78.97	82.45	85.46	88.13
8	70.0	74.86	78.93	82.41	85.43	88.11
9	70.0	74.83	78.90	82.37	85.40	88.09
10	70.0	74.81	78.86	82.33	85.36	88.06

This is graphically visualized in Graph 3.

Graph 3

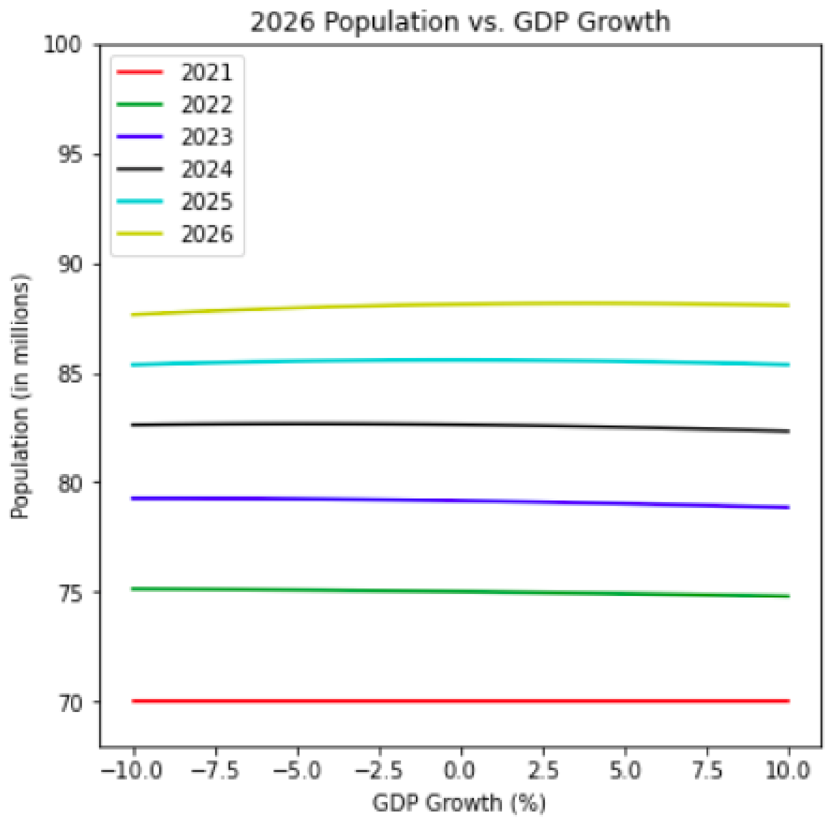


Table 10 provides the slope of each line.

Table 10: Yearly Population Change for 1% GDP Growth

	Year	Pop Change (in M cats) per 1% GDP inc
0	2021	0.00
1	2022	-16824.48
2	2023	-20743.32
3	2024	-14341.46
4	2025	299.65
5	2026	21500.49

Based on Table 10, we observe that for every 1% increase in GDP Growth:

- The 2022 population decreases an average of 16,824 cats
- The 2023 population decreases an average of 20,743 cats
- The 2024 population decreases an average of 14,341 cats
- The 2025 population increases an average of 300 cats
- The 2026 population increases an average of 21,500 cats

As annual GDP growth increases, populations for years 2022-2024 decreases while populations for year 2026 increases. Recall, a higher GDP is assumed to translate to more food waste and people having more money to spend on cat food for stray cats, thus leading to population increase. However, a higher GDP also provides an increase in funding for shelter rescuing and thus, an increase in the number of rescued animals per year. For years 2022-2024, it's feasible that the increased animal rescuing efforts by shelters out-weigh the population growth as a result of GDP growth. For years 2025-2026, we see an increasing presence of population growth outweighing rescue efforts, hence an increase in population.

Cost Analysis

We aim to see if investing money in any of these policies will yield a net savings in the 5-year period we're observing. Since feral cats do damage to the natural and urban environments, we give monetary estimates to their damage and compare it with our simulated funding values to see if there is a positive difference (hence, worthwhile to invest money in curtailment policies).

We initially simulated a no-funding policy. We then compared how the population changed when funding was added incrementally for each policy and calculated the population reduction per fund value for each year. We multiply this population reduction by the annual damage per cat (which we estimated it to be $51.50) to yield the savings per year per fund value.

We subtract annual funding value by the annual savings to acquire the annual net savings. Lastly, we cumulative the 5-year spendings and 5-year savings and find the difference to obtain the 5-year net savings. First, we implement our above analysis to the current (no TNR) policy. Below are the results.

Table 11: 5-year Spending, Savings, and Net Savings as Funding Increases under the Current Policy

Base Shelter Spending (in M $)	5-year Spending (in millions $)	5-year Saving (in millions $)	5-Year Net Saving (in millions $)
0.0	0.0	0.000	0.000
25.0	125.0	123.230	-1.770
50.0	250.0	246.460	-3.540
75.0	375.0	370.720	-4.280
100.0	500.0	494.980	-5.020
125.0	625.0	619.755	-5.245
150.0	750.0	745.045	-4.955
175.0	875.0	871.365	-3.635
200.0	1000.0	998.715	-1.285
225.0	1125.0	1125.035	0.035
250.0	1250.0	1252.900	2.900
275.0	1375.0	1380.250	5.250
300.0	1500.0	1509.145	9.145
325.0	1625.0	1638.555	13.555
350.0	1750.0	1767.450	17.450
375.0	1875.0	1897.890	22.890
400.0	2000.0	2029.360	29.360
425.0	2125.0	2160.315	35.315
450.0	2250.0	2291.785	41.785
475.0	2375.0	2424.800	49.800
500.0	2500.0	2557.300	57.300
525.0	2625.0	2690.830	65.830

As we can see from this table, the current policy doesn't yield a positive net saving until the base shelter funding exceeds $225 million dollars. These results are reasonable, since more funding goes into this policy, we see a greater removal of feral cats from the system. Given that this policy completely removes the feral cat from the system, as opposed to reintroducing it into the wild, they can no longer contribute damages to the system. However, it is important to note that in reality, shelters have a limited capacity. Up to a certain capacity, any more funding won't result in a higher rescue rate. Additionally, for our system, we only simulated the population change over the course of 5 years, so we could witness varying results if we increase our observation interval. Overall, from these results, we can state that this policy is worthwhile to invest in if annual base shelter funding is at least $225 million dollars.

Next, we perform the same analysis under the TNR-Facility policy. The 'TNR-Facility Spending…" column is the additional funding y_F we're adding on top of the annual base shelter funding c_F = $225M. Below are the results.

Table 12: Cumulative Spending, Savings, and Net Savings as Funding Increases under the TNR-Facility Policy

TNR-Facility Spending (in M $)	5-year Spending (in millions $)	5-year Saving (in millions $)	5-Year Net Saving (in millions $)
0.0	0.0	0.000	0.000
20.0	100.0	50.985	-49.015
40.0	200.0	100.425	-99.575
60.0	300.0	151.410	-148.590
80.0	400.0	201.365	-198.635
100.0	500.0	252.865	-247.135
120.0	600.0	303.850	-296.150
140.0	700.0	354.835	-345.165
160.0	800.0	405.305	-394.695
180.0	900.0	456.805	-443.195
200.0	1000.0	507.790	-492.210
220.0	1100.0	559.290	-540.710
240.0	1200.0	610.790	-589.210
260.0	1300.0	662.805	-637.195
280.0	1400.0	713.790	-686.210
300.0	1500.0	765.805	-734.195

Interestingly, we see that for the TNR-Facility funding, regardless of the amount of annual funding this policy receives, we will not witness any positive-net savings. This can be due to how the cost of a TNR operation under this policy is $150 dollars, which is far greater than the $51.5 dollars in damages per cat. Additionally, when compared to the TNR-Shelter split operations policy and the current policy (no TNR), the implementation of the TNR-Facility policy costs twice as much. This policy does not completely remove the feral cat from the system. As such, we see that the costs outweigh the savings regardless of the amount of funding put into the policy.

Now, looking into the TNR-Shelter policy, we apply the same analysis. The column 'TNR-Shelter Spending…" is c_F and $p_{TNR,s} = 0.568$, so the TNR-Shelter Spending is $p_{TNR,s} \cdot c_F$. Below are the results for each simulated value for c_F.

Table 13: Cumulative Spending, Savings, and Net Savings as Funding Increases under the TNR-Shelter Policy

TNR-Shelter Spending (in M $)	5-year Spending (in millions $)	5-year Saving (in millions $)	5-Year Net Saving (in millions $)
0.0	0.0	0.000	0.000
25.0	125.0	51.645	-73.355
50.0	250.0	102.775	-147.225
75.0	375.0	154.935	-220.065
100.0	500.0	207.095	-292.905
125.0	625.0	261.315	-363.685
150.0	750.0	314.505	-435.495
175.0	875.0	368.725	-506.275
200.0	1000.0	423.460	-576.540
225.0	1125.0	479.225	-645.775
250.0	1250.0	534.990	-715.010
275.0	1375.0	591.785	-783.215
300.0	1500.0	648.580	-851.420
325.0	1625.0	705.375	-919.625
350.0	1750.0	763.715	-986.285
375.0	1875.0	823.085	-1051.915
400.0	2000.0	882.455	-1117.545
425.0	2125.0	942.340	-1182.660
450.0	2250.0	1003.255	-1246.745
475.0	2375.0	1064.170	-1310.830
500.0	2500.0	1126.115	-1373.885
525.0	2625.0	1189.090	-1435.910

Similar to the TNR-Facility policy, the TNR-Shelter policy would not give net-positive savings, regardless of the amount of funding it receives. However, this is to be expected as it aligns with the previous sensitivity analysis for TNR-shelter policy. The primary reason for this is due to the funds being split, resulting in funds for shelter rescue operations being lower compared to Current policy. This policy is inherently less cost efficient, and as such, we see the costs outweighing the savings regardless of the amount of funding put into the policy.

In conclusion of our cost analysis, we see that any additional investments should be made under Current policy and used to fund regular shelter rescue efforts instead of TNR.

Conclusion & Policy Recommendation

Through the project and the corresponding simulations, we were able to better understand feral cat population dynamics under different policy conditions. Trap-Neuter-Return (TNR) is an interesting policy that helps curtail population, though the policy has its drawbacks. Upon projecting the population for the next 5 years, it is evident that TNR policies were not cost-effective despite its reducing the feral cat population. Beyond costs, there are also ethical components that need serious consideration, as TNR is more humane to feral cats at the expense of its prey (birds, etc) which are placed in risk of feral cat predation. The extent of TNR implementation affects the pendulum between feral cats and their prey, which requires serious consideration of cost and ethics.

Ultimately, under current funding, the current policy curtails the population the best. This is because TNR-Facility would require additional funding to initially be established in order to give better performance. With additional funding, the current policy would still perform better than others, followed by TNR-Facility and TNR-Shelter as shown under '*Comparing 3 policies at Equal Funding'* in the Sensitivity Analysis.

Funding plays a highly pivotal role in reducing the feral cat population, which makes it an area of interest for policy-makers. For instance, we project a decrease by 1.32 million for every additional $20 million added to the annual base shelter funding under Current policy. Each policy has its appeals and drawbacks depending on the objectives of those enacting the policy. The current policy is more cost-efficient in reducing the feral cat population, whereas TNR policies are more ethical for feral cats with a slower population curtailment rate. Within the TNR policies, the facility-oriented one is generally more performant than the shelter-oriented one with population impact.

What challenges did you encounter?

The most immediate challenge during the project was sourcing accurate and US specific feral cat data. There were limited data sources and often, the data was provided as generalizations with a lot of uncertainty. For example, when trying to quantify the size of the feral cat population in the US, we were unable to find a specific number, instead we were given a broad range of 60 to 160 million cats. Furthermore, another challenge was capturing the nuanced variables responsible for the cat population dynamics. Generalizing the variable relationships was challenging and we were even unable to capture all the variables. We often overlooked specific details such as the impact of weather, food resources or state-level geography. Lastly, our final challenge revolved around the simulation design as it was discrete in nature rather than having a continuous, meshed behavior. In our model, we assumed that certain processes like immigrations and TNR occurred in completion before other processes like reproduction, which is not representative of reality. Similarly, we modeled the adoptions such that they all happen together after new year reproduction. Hence, we found it difficult to model the more realistic continuous, interwoven process where births, adoptions and trapping happen simultaneously year-round.

What did you learn through implementing the project? Any major takeaways?

Overall, this project was a great learning experience as it enabled us to develop an in-depth understanding of the feral cat epidemic in the US. As we versed ourselves with some key feral cat characteristics, we were surprised to realize the scale and sheer gravitas of the issue. This motivated us further to help curb the population growth through the findings of this project.

The project also exposed us to some of the fundamentals of simulation. In doing so, we developed great insights on how to conduct research effectively and optimally source and compile data sets. Once we built the foundation, the project naturally tended to a system dynamics simulation. Given our limited background, we had a lot of learning in building such an intricate yet powerful model from scratch. Specifically, we developed a strong theoretical understanding of such processes by creating differential equations and flowcharts. In addition, given the heavy use of computer based simulation we honed out Python programming expertise.

What are the “next steps” in the project if you were to continue working on it?

Going forward, we would like to improve the sophistication of our model by incorporating more key variables and better representing their relationships. To achieve this we would take a three-pronged strategy. Firstly, we would like to collect more granular and specific data that is directly relevant to our simulation purposes. Specifically, we would like to source more accurate cost breakdowns, shelter space constraints etc. To do this, we would like to conduct in-person samplified primary research as well as access paid data sources to get specific information. Next, we would like to account for more pinpointed variables that impact the feral cat population. For example, how could we realistically factor in the amount of food resources the cats have access to? Once we form this foundation, we would like to better represent the relationship between the variables. For example, we assumed that the GDP and government funding have a linear relationship with the cat population, however, this is a rough estimate. Going forward, we want to develop more realistic relationships by layering in more complicated base models.

How do the software packages you used work? Are there other approaches that might work? What are the trade-offs between these approaches?

For this project, the code and all pertaining quantitative analysis was undergone in Python. We wrote numerous functions that incorporated the differential equations to project our desired outputs (i.e. population, percent change, TNR proportion, etc.). Afterwards, we built in-depth for loops to extract the yearly development of our population/cost metrics. With sensitivity analysis, we tweaked the desired variable (i.e funding) while keeping all other variables constant, and analyzed how the population/costs change. To get results from our code, simply run all the cells!

Simio is one alternative software package that we considered in this project; it could theoretically work in our project's context. Simio would model continuous, meshed processes easier than Python, as well as achieving the yearly population change in a more direct manner. However, Python is more succinct when considering functions and several distinct models with their respective experiments. This is because we dealt with different policies which meant different models. We also believed Python would be easier to implement our simulation in and simpler to analyze/visualize. With those tradeoffs, we were confident that Python would be the better choice.

Do these overall results seem reasonable? Are they physically realistic? Do they match your intuition?

The results seem reasonable; every experiment outcome came with a grain of intuition. For example, when we compared population change under different policies with fixed funding, we noticed that TNR-Shelter had a slightly higher population per year than under current population. This may not seem reasonable at first, since TNR should reduce population size. However, this may be due to the fact that TNR-Shelter operations take away partial funding from the overall rescue funding given to shelters to fund the TNR operations, which leaves less funding for animals rescued for adoption or euthanization. Additionally, TNR operations rescue the treated animal back into the wild, so the policy does not immediately lower the population. TNR-Facility curtails population the best in this context, which makes sense because it does not divide resources as with TNR-Shelter. Our intuition is matched.

The results also appear to be physically realistic. Our simulations are in the same ballpark as expected feral cat population growth. The effect of TNR on the population is also realistic - not too large or too little under current funding. When funding is largely increased, we view a population decrease, which is also realistic since money is a crucial aspect of resource allocation.

What did you learn by running the experiments?

We learned a great ton about the intricate details that go into population dynamics. It is not as straightforward as births less deaths of feral cats, as there are different dynamic contributors that go into births and deaths alike. Through the experiments, we ultimately learned which contributors affect the feral cat population more significantly than others. A takeaway from the aforementioned is that funding plays a very pivotal role in the feral cat population. For instance, we project a decrease by 1.32 million cats for every additional $20 million added to the annual base shelter funding funded under the current policy. Considering feral cats' environmental and economic consequences, it makes one contemplate the cost-benefit analysis towards shelter funding. Funding is significant!

We also learned that although alternative policies are effective, such as TNR policies, they require further funding, which means they may not be the most cost-effective policies to implement in reality.

How does your project fit into the larger context of the class and the IEOR field?

The project pays dividends towards the design, programming, and statistical analysis of simulation methods for large-scale systems, with feral cat population in this case. It also shows how population simulations change drastically over time depending on the system (policy), arrivals (birth, domestic cats becoming feral), and departures (death, rescue, TNR operations). With the IEOR field, the project demonstrates how one can use simulation and differential equations to analyze and optimize population dynamics with respect to policies and funding. With that, one can optimize their respective objectives (population decrease, cost-benefit ratio, etc.) based on their policy and funding interests.

The tools we built can be translated into any general population simulation under differing policies. Our project is one example with feral cats, but can serve as a template for many population-related projects that can flourish within the class and the IEOR field at large.

Further Works to Be Done

Though we undertook a comparative analysis across multiple policies, we performed that under one estimate of the possible TNR rates. One path we would like to study further if time permits would be sensitivity analysis of the TNR rates and their implications on the optimal policy and population numbers. Specifically, we aim to understand how the population changes over time with different TNR rates and what underlying factors contribute to it, while possibly visualizing its connection to the rate of growth.

Another area of interest would be a cost analysis regarding feral cat related policies. Rather than utilizing population changes as our measuring point, we could build on that and see how the costs and economic factors directly change as an effect of feral cat related policies, a variation of end-to-end learning. Particularly with respect to the policy in which shelter operations/funds would be split with TNR operations.

Appendix

1. Estimating death proportion, c_D

We created a death probability distribution dependent on a cat's age to find the converged proportion of the feral cat population that will pass away in the wild due to diseases, old-age, human-caused accidents, and more. This excludes birth mortalities.

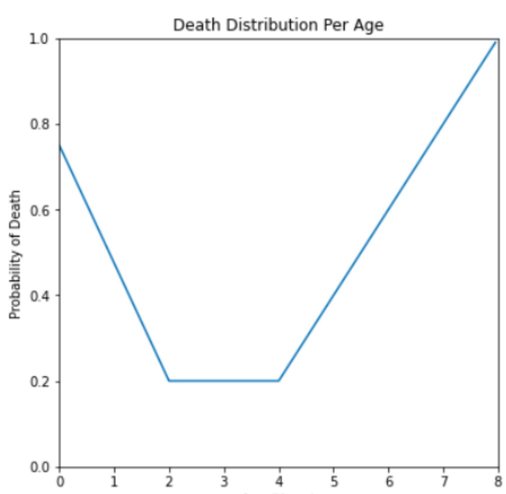
The probability of death for a feral cat given their age, p_D, follows the distribution:

$$p_D = -0.275(age) + 0.75,\ 0 \leq age \leq 2$$
$$p_D = 0.2,\ 2 \leq age \leq 4$$
$$p_D = 0.2(x - 8) + 1,\ 4 \leq age \leq 8$$

For simplicity, we assumed that

- cats range from age 0-8 years old
- cats are at their prime age when they're 2-4 years old, which is also when they have the lowest probability of death
- after age 4, cats leave their prime age and begin to enter old-age. From age 4-8, their probability of death increases linearly until their probability of death reaches 1 at age 8.
- all cats that survive through the birth mortality period have a probability of death ≥ 0.2; this accounts for the probability of death due to diseases, human-caused accidents, and related incidents
- newborn kittens have a 75% chance of death; we assume this probability decreases as they age

Graph A.1 - Probability Density Function Dependent on Age



Given the death probability distribution, we generate random ages for a population of cats following the age distribution ~ Normal(3, 1) so that about 68% of our population is within 2-4 years old and with a maximum age cutoff of 8 years old. Given the age of the cat, we find their corresponding probability of death and generate a Bernoulli random variable to decide if they live or pass away in our system. Running our simulation under our death probability and age distribution, we obtain the following results.

Table A.2 - Estimating Death Probability Regardless of Age

Initial Population	Number Cats Alive	Proportion of Cats Passed Away
10,000	7628	0.2472
100,000	76,184	0.2382
1,000,000	761,208	0.2388

We see from Table A.2 that the converged proportion of cats that leave the feral population due to non-birth mortalities is in the range [0.23, 0.25]. We round up and use 0.25 as our probability of death, c_D. This equates to 25% of the non-fertile population leaving the feral cat system annually.

2. Estimating r_0 as a parameter to the logistic growth function

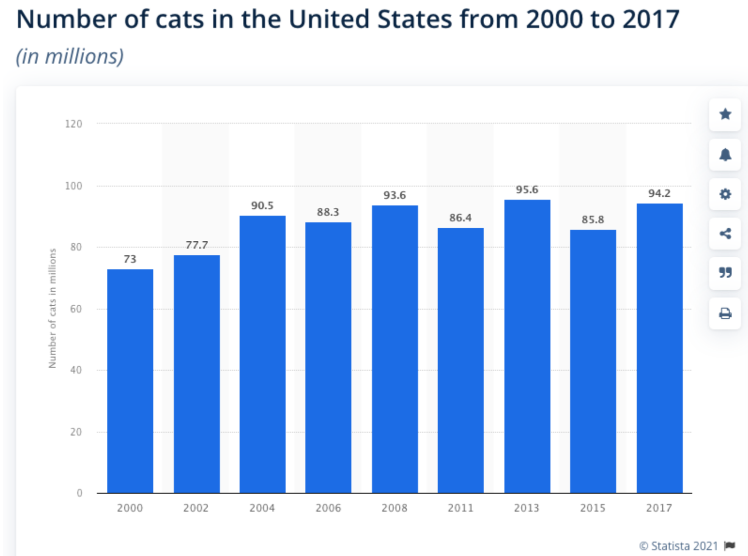
We estimate the logistic growth rate using Statista data on the feral cat population from 2000-2017 and fitting the data into a logistic growth model. We observe that the feral cat population is about 70M in 2000 and increased to about 90M in 4 years. Assuming a carrying capacity of 150M and an initial population of 70M, we plug these values into the general logistic growth model

$$P(t) = \frac{K}{1+Ae^{-r_0 t}}$$

$$A = \frac{K - P_0}{P_0}$$

where K = 150M is the carrying capacity, t = 4, and P_0 = 70M. The r_0 value that produces relatively close results to the above is around 0.13; thus, we proceeded our modeling and analysis setting r_0 = 0.13.

Diagram A.2 - Feral Cat Population Estimates, 2000-2017



Source: Statista

3. Model Variables Description and Values

Table A.3: Variables to Differential Equations

Variable	Description	Value	Rationale
t	Represents years since 2021, where t = 0.	t = {0, 1, 2, 3, 4, 5}	
r_0	Assumed growth rate from births and deaths in the feral cat population.	0.13	See section 2 of the appendix for rationale.
r_i	Base proportion of immigration from domestic cat population into feral population that survive	0.03	Arbitrary estimate.
$c_{C,H}$	Domestic cat to people ratio in the United States.	0.33	Studies show there are about 1 domestic cat to 3 people in the U.S.
H	Human population in the United States.	330 million	Assumed that any small changes to this value from

			2021 to 2026 are negligible to our analysis.
$E(t)$	GDP percent growth for year t.	E(0) = 0 E(1) = 2.36 E(2) = -3.41 E(3) = 1.21 E(4) = -8.50 E(5) = 3.77	Randomly generated.
c_R	Cost of rescuing a feral cat into the shelter.	$75	Arbitrary estimate.
c_F	Annual sum of funding for all U.S. shelters.	$225M	Statistics show that ~3M cats are rescued into shelters annually. Using c_R , this equates to an annual national base shelter budget of $225M. This value fluctuates based on GDP growth, E(t).
$c_{F,R}$	Annual sum of funding for all U.S. shelters for rescuing cats that end up adopted or euthanized; relevant when considering the TNR-shelter policy.	43.2% of $c_F$when considering in the context of TNR - shelter operations; else equivalent to c_F $(1 - p_{TNR,\,s}) * c_F$	Since $c_{TNR,s}$ and c_R are equivalent, the shelter funding allocated for adoptions/euthanizations ($c_{F,R}$) and TNR operations at shelters (y_s) are proportional to the fraction of rescued animals that undergo each process. A maximum of 56.8% of shelter operations may be converted into TNR operations to account for a historical 30% adoption rate and 13.2% euthanization rate at shelters.
c_{TNR}	Cost of performing a single TNR operation. This value is equivalent to $c_{TNR,s}$ in the context of shelters and $c_{TNR,F}$in the context of TNR	-	

	facilities.		
$c_{TNR,s}$	Cost of performing a single TNR operation at a shelter.	$75	Arbitrarily assume this is equivalent to c_R since the animal rescuing is done before the shelter knows whether the animal will be adopted, euthanized, or undergo TNR.
$c_{TNR,F}$	Cost of performing a single TNR operation at a TNR facility.	$150	Arbitrarily assume this is twice as expensive as to c_R to factor in costs of building new facilities for TNR-facility policy.
c_D	Probability of death in the feral cat population.	0.25	See section 1 of the appendix for rationale.
K_B	Base carrying capacity; expected to fluctuate based on $E(t)$	150M	Studies show that between 60M to 160M feral cats live in the U.S. This value of the base carrying capacity was arbitrarily chosen to be consistent with this range.
$K(E(t))$	Returns the carrying capacity of feral cats at time t.	-	Simulated across different values.
$y(t)$	Annual funding for TNR operations. performed by building new TNR-facilities. This value is equivalent to $y_s(t)$in the context of shelters and $y_F(t)$in the context of TNR facilities.	-	Simulated across a range of values.
$p_{TNR,\,s}$	Proportion of shelter operations converted to TNR	0.568	A maximum of 56.8% of shelter operations may be converted into TNR operations to account for a historical 30% adoption rate and 13.2% euthanization rate at shelters.

$y_s(t)$	Annual funding for TNR-shelter operations (ie. integration of TNR into shelter operations).	$p_{TNR,\, s} * c_F$	Same rationale as that for $c_{F,R}$.
$y_F(t)$	Annual funding for TNR operations performed in newly-built TNR-facilities.	-	Simulated across a range of values.
$P_F(t)$	Returns fertile population at time t.	-	Dependent variable.
$P_N(t)$	Returns the neutered population at time t. We assume cats that have undergone TNR and cats who immigrated into the system make up this population.	-	Dependent variable.
$f(P_F(t),\, K(t)$	Logistic growth differential equation that models the growth rate of the feral cat population given a year. This model attempts to factor in the curtailment effect of TNR operations.	-	Dependent variable.
$g(E(t))$	Annual rescue rate (in number of cats) for a given year t. This factors in how the economy impacts rescue rates.	-	Dependent variable.
$h(y(t))$	Annual TNR rate (in number of cats).	-	Dependent variable.
$i(t)$	Number of cats who immigrate into the feral	-	Simulated across a range of values.

	cat population at time t. (Ie. domestic cat runaways, abandonment, etc)		
$\frac{dP_F}{dt}$	Rate of change in fertile population at time t.	-	Dependent variable.
$\frac{dP_N}{dt}$	Rate of change in neutered population at time t.	-	Dependent variable.

4. Percent Difference for Integral Estimation

We use Equation A.2 to approximate Equation A.1 in our simulations and analysis for this project. We calculate a percent difference between the 2 equations to evaluate how accurate our estimation (results from Equation A.2) are to the actual results under our model (from Equation A.1).

$$P_F(t) = P_F(t-1) - h(y(t)) + \int_{t-1}^{t} \frac{dP_F}{dx} dx \qquad \text{Equation A.1}$$

$$P_F(t) = P_F(t-1) - h(y(t)) + \frac{dP_F}{dt}(t) \qquad \text{Equation A.2}$$

The difference between Equations A.1 and A.2 is in the last term, so we compare the integral $\int_{t-1}^{t} \frac{dP_F}{dx} dx$ with its approximation $\frac{dP_F}{dt}(t)$. We compute both values for year 2022 under the current policy as an example and insight to the percent difference between the 2.

$$\frac{dP_F}{dt}(t) = f(P_F(t-1) - h(y(t)),\ K(t) - P_N(t)) - g(E(t))$$

$$f(P_F(t-1) - h(y(t)),\ K(t) - P_N(t)) = r_0\left(1 - \frac{P_F(t-1) - h(y(t))}{K(t) - P_N(t)}\right)[P_F(t-1) - h(y(t))]$$

$$g(E(t)) = \frac{c_F}{c_R}\left(1 + \frac{E(t)}{100}\right)$$

In this example:

- t = 1 to represent 2022

- the fertile population at $P_F(t-1)$ equals $P_F(t=0)$ and is thus 70M cats
- $h(y(t))$ is 0 as we are simulating under current policy and thus, no TNR operations
- $K(t)$ follows its definition in the Differential Equations section, where K_B = 150M
- $P_N(t) = P_N(t=1)$ is the non-fertile population for year 2022, which includes only immigrations since TNR operations are nonexistent under current policy.
- All other variables are equal to its value in Table A.3.

Below is an intensive computation of $\int_{t-1}^{t} \frac{dP_F}{dx}dx$ when t = 1.

$$\int_0^1 \left(0.13\left(1-\frac{\frac{150\cdot10^6\cdot\left(1+\frac{2.36t}{100}\right)}{1+\left(\frac{\left(150\cdot10^6\cdot\left(1+\frac{2.36t}{100}\right)\right)-\left(70\cdot10^6\right)}{70\cdot10^6}\right)e^{-0.13t}}}{150\cdot10^6\cdot\left(1+\frac{2.36t}{100}\right)-\left(3.366\cdot10^6\cdot\left(1-\frac{2.36t}{100}\right)\right)}\right)\left(\frac{150\cdot10^6\cdot\left(1+\frac{2.36t}{100}\right)}{1+\left(\frac{\left(150\cdot10^6\cdot\left(1+\frac{2.36t}{100}\right)\right)-\left(70\cdot10^6\right)}{70\cdot10^6}\right)e^{-0.13t}}\right)-\left(\frac{225\cdot10^6\cdot\left(1+\frac{2.36t}{100}\right)}{75}\right)\right)dt$$

$$= 1785572.64686$$

Below is a computation of its approximation, $\frac{dP_F}{dt}(t=1)$.

$$0.13\left(1-\frac{70000000}{\left(150\cdot10^6\cdot\left(1+\frac{2.36t}{100}\right)\right)-\left(3.366\cdot10^6\cdot\left(1-\frac{2.36t}{100}\right)\right)}\right)(70000000)-\left(\frac{225\cdot10^6\cdot\left(1+\frac{2.36t}{100}\right)}{75}\right)$$

$$= 1789696.3309$$

We calculate the percent difference between the 2 equations with respect to the integral as the actual value.

$$\text{Percent difference} = \frac{|1{,}785{,}572.65 - 1{,}789{,}696.33|}{1{,}785{,}572.65} * 100 = 0.23\%$$

As seen, Equation A.2 is a very effective and close approximation to Equation A.1 with less than a 1% difference.

5. Sample Calculation for Population Estimate

We provide a sample calculation in obtaining the 2022 feral cat population estimate under Current Policy.

We recall that

$$P(t) = P_F(t) + P_N(t)$$

and

$$P_F(t) = P_F(t-1) - h(y(t)) + \frac{dP_F}{dt}(t)$$

$$P_N(t) = P_N(t-1) + \frac{dP_N}{dt}(t)$$

where

$$\frac{dP_F}{dt}(t) = f(P_F(t-1) - h(y(t)), K(t) - P_N(t)) - g(E(t))$$

$$f(P_F(t-1) - h(y(t)), K(t) - P_N(t)) = r_0(1 - \frac{P_F(t-1) - h(y(t))}{K(t) - P_N(t)})[P_F(t-1) - h(y(t))]$$

$$g(E(t)) = \frac{c_F}{c_R}(1 + \frac{E(t)}{100})$$

$$\frac{dP_N}{dt} = -c_D P_N(t-1) + (1 - c_D)h(y(t)) + i(E(t))$$

$$i(E(t)) = r_i(1 - \frac{E(t)}{100}) \cdot c_{C,H} \cdot H$$

We set the following variables to values consistent with Table A.3:

- t = 1
- r_0 = 0.13
- $P_F(t-1) = P_F(0)$ = 70M
- $h(y(t))$ = 0 since there is no TNR under Current Policy
- $E(t = 1)$ = 2.36
- $K(t) = K_B(1 + \frac{E(t)}{100})$ = 150M$(1 + \frac{2.36}{100})$ = 153.54M
- c_F = \$225M
- c_R = \$75
- c_D = 0.25
- r_i = 0.03
- $c_{C,H}$ = 0.33
- H = 330M

We compute

$$i(E(t)) = r_i(1 - \frac{E(t)}{100}) \cdot c_{C,H} \cdot H = 0.03 \cdot (1 - \frac{2.36}{100}) \cdot 0.33 \cdot 330M = 3{,}189{,}898.80$$

$$g(E(t)) = \frac{c_F}{c_R}(1 + \frac{E(t)}{100}) = \frac{\$225M}{\$75}(1 + \frac{2.36}{100}) = 3,070,800$$

$$P_N(t = 1) = i(E(t = 1)) = 3{,}189{,}898.80$$

$$f(P_F(0) - h(y(1)), K(1) - P_N(1)) = 0.13(1 - \frac{P_F(0) - h(y(1))}{K(1) - P_N(1)})[P_F(0) - h(y(1))]$$

$$f(P_F(0) - h(y(1)), K(1) - P_N(1)) = 0.13(1 - \frac{70M}{153.54M - 3.19M})[70M]$$

$$f(P_F(1) - h(y(1)), K(1) - P_N(1)) = 4{,}860{,}496.33$$

$$\frac{dP_F}{dt}(1) = f(P_F(0) - h(y(1)), K(1) - P_N(1)) - g(E(1)) = 4{,}863{,}222.01 - 3,070,800 = 1{,}789{,}696.33$$

$$P_F(1) = P_F(0) - h(y(1)) + \frac{dP_F}{dt}(1) = 70M + 0 + 1.8M = 71.8M$$

$$P(t) = P_F(t) + P_N(t) = 71.8M + 3.2M = 75M$$

This is consistent with the population estimates we see for 2022 under Current policy (see Table 1).

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