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Higgs Boson in Superconductors

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Superfluid helium, describable by a two-component order parameter, exhibits only the Bogolubov mode with energy $\rightarrow$ 0 at long wavelengths, while a Lorentz-invariant theory with a two-component order parameter exhibits a finite energy mode at long wavelengths (the Higgs boson), besides the above mass-less mode. The mass-less mode moves to high energies if it couples to electromagnetic fields (the Anderson-Higgs mechanism). Superconductors, on the other hand have been theoretically and experimentally shown to exhibit both modes. This occurs because the excitations in superconductors have an (approximate) particle-hole symmetry and therefore show a similarity to Lorentz-invariant theories.

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1. INTRODUCTION

The order parameter for superfluidity or superconductivity may be written as

$$\psi(\mathbf{k}) = |\psi(\mathbf{k})| \exp(i\phi). \tag{1}$$

The free energy is invariant to the value of ϕ. Then in a neutral superfluid, the deviation of the phase $\delta\phi(\mathbf{k},\omega)$ from a chosen value $\phi = \phi_0$ follows a wave equation with the dispersion $\omega \propto k$ at long wavelengths. These facts were realized by Bogolubov[1], Anderson[2], Nambu[3] and others and are in accord with the more general Goldstone theorem for collective fluctuations in models with continuous broken symmetry.

It was also realised by Anderson that in a charged superconductor the phase mode moves to the plasma frequency of the metal, since the current $\mathbf{j}$, proportional to $\nabla\phi$ is related to the charge density fluctuations through the continuity equation. This is also the Higgs mechanism for mass generation in unified electro-weak field theories. Since in condensed matter physics,

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experiments are abundant, the underlying physics is often understood more vividly. Indeed the shift of the phase mode to the plasma frequency would appear inescapable; otherwise the color of a metal would change below the superconducting transition.

In $U(1)$ field theories and generalization to higher groups Higgs also predicted the existence of another massive mode—the so called Higgs boson. The experimental search for the Higgs boson in particle physics has assumed much importance. The equivalent of the Higgs boson is not found in superfluid ^{4}He but about twenty years ago, spurred by some anomalous experimental results[4], the theory of the equivalent of the Higgs boson in superconductors and its experimental identification was provided[5,6]. In this note I discuss first why such a mode is not present in superfluid ^{4}He, will not be found in the recently discovered weakly interacting Bose gases but is found in superconductors.

2. NO HIGGS IN SUPERFLUID HELIUM

For weakly interacting bosons (and as far as symmetry-related properties are concerned, ^{4}He as well) the minimal Hamiltonian for the complex scalar field ψ is

$$H = \int d\mathbf{r} \left(|\nabla\psi|^2 + r|\psi|^2 + u|\psi|^4 \right). \tag{2}$$

For $r < 0$, a superfluid condensation leads to a finite thermal expectation value

$$\langle \psi \rangle = [-r/2u]^{1/2} \equiv \rho_0^{1/2} \tag{3}$$

where ρ_0 is the superfluid density. The potential has the form of an inverted Mexican hat as shown in Fig. 1. in the space of the real and imaginary part of ψ. One then expands ψ about the condensed value

$$\psi(\mathbf{r}, t) = [\rho_0 + \delta\rho(\mathbf{r}, t)]^{1/2} \exp(i\phi(\mathbf{r}, t)) \tag{4}$$

and uses the non-relativistic Schrödinger equation

$$-i\partial\psi/\partial t = H\psi. \tag{5}$$

to find after linearising that

$$\partial\phi/\partial t = (\nabla^2\delta\rho + V\delta\rho)/2\rho_0 \tag{6}$$

$$(1/2\rho_0)\, \partial\delta\rho/\partial t = \nabla^2\phi \tag{7}$$

The latter is just the continuity equation. Solving Eqs. (6,7) to linear order leads to a pair of degenerate equations for ϕ and $\delta\rho$.

$$\frac{\partial^2(\phi, \delta\rho/\rho_0)}{\partial t^2} = \nabla^2(\phi, \delta\rho/\rho_0) + \nabla^4(\phi, \rho/\rho_0), \tag{8}$$

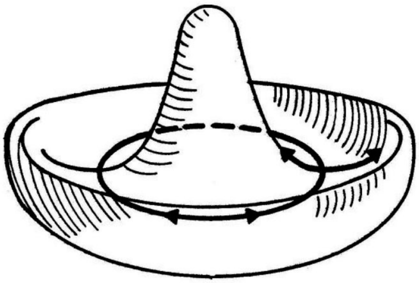

Fig. 1. Plot of the potential in Eqs. (2) and (10) as a function of real and imaginary parts of ψ.

which leads to the Bogolubov dispersion

$$\omega^2 = k^2 + k^4 \tag{9}$$

There is no other collective mode; the mode for amplitude ρ and phase ϕ have identical dispersion. It is not sufficient to have the Mexican hat potential of Fig. 1 to get two collective modes, one for the real and the other for the imaginary part of the order parameter.

3. COLLECTIVE MODES IN LORENTZ-INVARIANT THEORY

Contrast the above with the case that the problem is Lorentz invariant, so that the space and time derivatives have to be of the same order. More typically, one works with a Lorentz invariant Lagrangian[7]

$$\mathcal{L} = |\partial_\mu \psi|^2 + r|\psi|^2 + u|\psi|^4, \qquad \mu = (\mathbf{r}, t). \tag{10}$$

This has, of course, the same potential, of the form of a Mexican hat for $r < 0$ as in Fig. 1. But in contrast to the Lorentz non-invariant theory, besides the Goldstone mode which is massless at long wavelengths (just as the Bogolubov mode in the non-relativistic theory), there also exists a massive mode for the deviation of the amplitude of $|\psi|$ about the stationary value $(-r/2u)^{1/2}$ whose energy at long-wavelength is

$$\omega_{\text{Higgs}} = \sqrt{-4r}, \tag{11}$$

i.e. given by the curvature of the Mexican hat potential of Fig. 1. If ψ couples to the electromagnetic field, the phase mode moves up to high energy, alleviating a major headache of particle-theorists in the 1960's. In $SU(2)$ or higher group gauge symmetric theories, the basic physics is the same; depending on the structure of the theory some "phase" modes may remain massless while there may be a multitude of amplitude or Higgs modes. The amplitude or Higgs mode is yet to be discovered in elementary particle physics.

We see that with the same effective potential, the Schrödinger equation and the Lorentz-invariant theory give quite different answers. Why should there then be an amplitude mode in superconductors as in the Lorentz-invariant theory? We will come back to this question after reviewing briefly the microscopic derivation of the Higgs in superconductors.

4. MICROSCOPIC THEORY OF THE AMPLITUDE MODE IN SUPERCONDUCTORS

The theory of the phase mode in superconductors was developed soon after the BCS theory in order to show the gauge invariance of the theory. No attention was paid to the possibility of the amplitude mode, presumably because the experiments did not call for one. Then in 1980 Raman scattering experiments in the compound $NbSe_2$ appeared. $NbSe_2$ has a charge density wave (CDW) transition at 40 K and a superconducting transition at 7.2 K. Because of the doubling of the unit cell due to the change in periodicity, a new set of vibrational modes appear at long wavelengths and finite energies below the CDW transition, see Fig. 2. Some of these modes do not carry any dipole moment and are therefore seen in Raman scattering experiments but not in optical absorption experiments. It was found that below the superconducting transition the spectral weight of such modes is partialy transferred to a sharp mode mode at an energy of about twice the superconducting gap $2\Delta(T)$. This is shown in Fig. 3. Not only is the sum of the spectral weight in the two modes conserved, the ratio of the weight in the two peaks could be altered by applying a magnetic field which suppresses the superconducting gap.

The microscopic theory of this phenomena[5] was developed along the

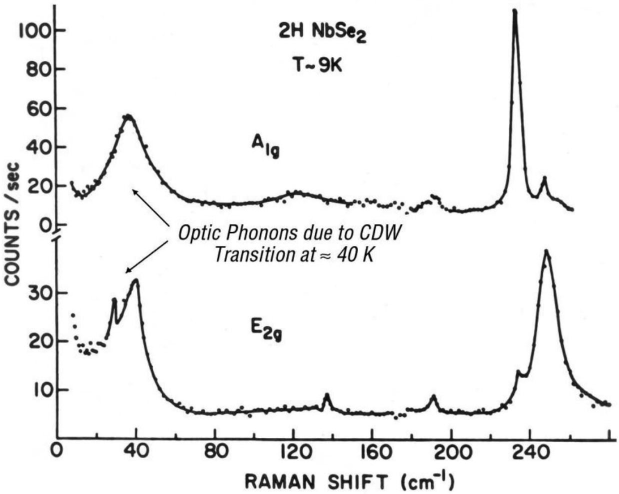

Fig. 2. Raman scattering intensity in $NbSe_2$ at $T = 9$ K, below the CDW transition at 40 K and above the superconducting transition at 7.2 K in two different symmetries. The peaks below 100 cm^{-1} arise only below the CDW transition. Data from Ref. 4.

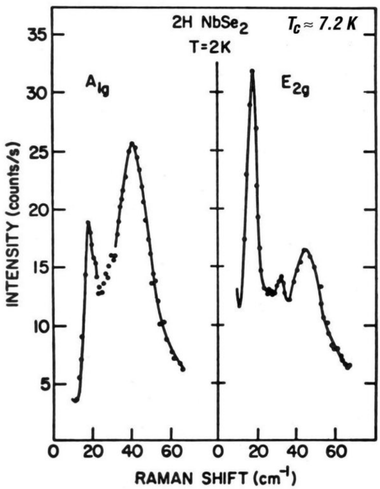

Fig. 3. Same as Fig. 2 at $T = 2$ K. The intensity of the new peaks grows below the superconducting transition and at low temperatures their energy is at approximately twice the superconducting gap. Spectral weight can be transferred back to the mother peaks by applying a magnetic field.

following lines: in terms of the two component vectors

$$\Psi_{\mathbf{k}} = \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c^{\dagger}_{-\mathbf{k}\downarrow} \end{pmatrix} \qquad \Psi^{\dagger}_{\mathbf{k}} = \begin{pmatrix} c^{\dagger}_{\mathbf{k}\uparrow} & c_{-\mathbf{k}\downarrow} \end{pmatrix} \tag{12}$$

the Hamiltonian of a system of fermions interacting with a non-retarded potential is

$$\mathcal{H} = \sum_{\mathbf{k}} \Psi^{\dagger}(\mathbf{k})\tau_3\Psi_{\mathbf{k}} \tag{13}$$

$$+ \sum_{\mathbf{k},\mathbf{k}'.\mathbf{q}} V(\mathbf{k},\mathbf{k}',\mathbf{q})\boldsymbol{\Psi}^{\dagger}_{\mathbf{k}+\mathbf{q}}\boldsymbol{\tau_3}\boldsymbol{\Psi}^{\dagger}_{\mathbf{k}}\boldsymbol{\Psi}^{\dagger}_{\mathbf{k}'-\mathbf{q}}\boldsymbol{\tau_3}\boldsymbol{\Psi}^{\dagger}_{\mathbf{k}'}, \tag{14}$$

where the τ's are Pauli matrices in the space of (12). We may write

$$\mathcal{H} = \mathcal{H}_{BCS} + \mathcal{H}_1, \tag{15}$$

$$\mathcal{H}_{BCS} = \sum_{\mathbf{k}} \Psi^{\dagger}_{\mathbf{k}} \left(\epsilon_{\mathbf{k}}\tau_3 + \Delta_{\mathbf{k}}\tau_1\right) \Psi_{\mathbf{k}}. \tag{16}$$

In (16), the amplitude of the gap parameter has been arbitrarily chosen in the τ_1 direction and phase in the τ_2 direction. Identical physical results should be obtained for any choice of phase in the $\tau_1 - \tau_2$ plane. Long wavelength variations about this condition

$$\Psi \to \exp(i\alpha(\mathbf{r},t)\tau_3)\psi \tag{17}$$

should satisfy the continuity equation for slowly varying $\alpha(\mathbf{r},t)$:

$$\frac{\partial}{\partial t}(\psi^{\dagger}\tau_3\psi) + \nabla \cdot \Psi^{\dagger}\frac{\mathbf{P}}{m}\Psi = 0. \tag{18}$$

This guarantees the existence of the Anderson-Bogolubov mode. But since the longitudinal part of the electromagnetic field couples to the charge density $\Psi^{\dagger}\tau_3\Psi$, this mode is pushed to the plasma frequency.

The Hamiltonian $\mathcal{H}$ obeys another invariance relation besides Eq. (17)[3,5]. It is invariant to the non-unitary transformation:

$$\Psi \to \exp(\alpha(\mathbf{r},t)\tau_1)\Psi \tag{19}$$

$$\nabla \to \nabla + \alpha(\mathbf{r},t)\tau_1, \tag{20}$$

leading to the pseudo-continuity equation

$$i\Psi^{\dagger}\tau_1\left(\frac{\overleftarrow{\partial}}{\partial t} - \frac{\vec{\partial}}{\partial t}\right)\Psi + \nabla\cdot\Psi^{\dagger}\tau_2\left(\frac{\overleftarrow{P}}{m} + \frac{\vec{P}}{m}\right)\Psi = 0. \tag{21}$$

A treatment of $\mathcal{H} - \mathcal{H}_{BCS}$ obeying this continuity equation reveals the existence of the amplitude mode (so-called because its eigenvector is proportional to τ_1), i.e. the Higgs boson. At $q = 0$, its energy ν is given by the solution of

$$1 + V \sum_{\mathbf{k}} \frac{\epsilon_k^2}{E_k(\nu^2/4 - E_k^2)} = 0, \tag{22}$$

where V is the appropriate partial wave component of the potential.

For s-wave superconductors with Δ independent of k, the solution is $\nu = 2\Delta$. For $q \ll k_F$, the mode is found at

$$\nu_q^2 \approx 4\Delta^2 + v_F^2 q + (\pi^2/12)\, i\, \Delta\, v_F q. \tag{23}$$

For d-wave superconductors, or generally when a continuum of single-particle excitations exist at low energies, the mode is below twice the maximum superconducting gap and is heavily overdamped. One might wish to worry about the damping of the Higgs boson in elementary particle physics.

Since the amplitude mode is a fluctuation of the Cooper pair density (it exists in the τ_1 channel), there is no coupling to electromagnetic waves. The lattice vibrational modes of the CDW oscillate the density of states at the Fermi-energy and thereby couple to the superconducting gap and the amplitude mode. This coupling also pushes the amplitude mode below 2Δ and provide it spectral weight. Given the change of the CDW and the superconducting transition with pressure, the coupling of the CDW mode and the amplitude mode can be found and the observed phenomena has been explained quantitatively without any free parameters[5].

I now return to the earlier question of why such a Higgs mode, a property of a Lorentz-invariant theory, is found in non-relativistic condensed matter physics.

5. APPROXIMATE PARTICLE-HOLE SYMMETRY IN SUPERCONDUCTORS

The mathematical reason for the difference in the results for the non-relativistic and the Lorentz-invariant theory is of-course that in the former the equation of motion is first order in time and in the second, it is second-order in time. The reason the equation of motion for superconductivity is effectively second-order in time is the particle-hole symmetry of the theory. This is obvious from the BCS Hamiltonian, which is formally identical to the Dirac Hamiltonian. The elaborations on the BCS theory discussed above preserve this invariance. Even in the absence of Lorentz-invariance, the approximate particle-hole symmetry of the theory requires the equation of motion to be second-order. (Relaxing particle-hole symmetry in the microscopic theory sketched above produces a corresponding damping in the dispersion of the amplitude mode.) In fact phenemenological Lagrangians for superconductors at low tempertures with a second-derivative in time have been derived[8]. Near T_c, the damping of the modes is usually handled with a phenemenological first order time derivative in a Landau-Ginzburg theory, making the theory of superconductors and superfluids look alike. This is somewhat misleading.

Just like in the Dirac equation, the theory of superconductivity deals with conserved charge as well as unconserved particle number. Correspondingly, there is an important physical difference of weakly interacting bosons or superfluid ^{4}He from superconductors. In the former, the physical variables, density and current are constrained by the continuity equation and no other degree of freedom exists (as long as we work at energies small compared to the dissociation of the boson into constituent fermions). In superconductors, there is an additional degree of freedom, the Cooper pair density and therefore the possibility of an additional collective mode.

ACKNOWLEDGMENTS

I am pleased to be contributing this little note to the joyous occasion celebrating Peter Wölfle's sixtieth birthday and to his many contributions to Science, including work related to that described here: collective modes in superfluid ^{3}He [9]. Over the years I have had numerous profitable discussions with him in very many problems ranging from the collective mdoes discussed here to transport in heavy fermions to the theory of high temperature superconductivity. I have found him a valuable colleague and a scientist upholding the highest standards of science.

Discussions with Chetan Nayak contributed to my decision to write this note.

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