

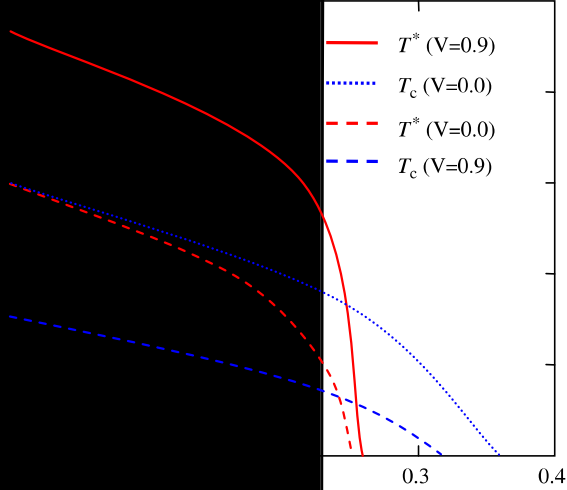


For comparison we discuss the pure superconducting transition in the absence of DDW order. In this case the gap equation for the DSC order parameter  $\Delta_B$  can be written as

$$\frac{1}{-V+P} = \frac{1}{N} \sum_{\mathbf{k}} \frac{d_{\mathbf{k}}^2}{2[(\epsilon_{\mathbf{k}}-\mu)^2 + \Delta_B^2(\mathbf{k})]^{1/2}} \tanh \frac{[(\epsilon_{\mathbf{k}}-\mu)^2 + \Delta_B^2(\mathbf{k})]^{1/2}}{2T}, \quad (15)$$

This is the well-known BCS equation for the DSC gap induced by spin fluctuations. In the limit of  $V=0$  and at half-filling ( $\mu=0$ ) two Eqs. (14) and (15) are the same with each other, yielding  $T^*=T_c$ . For a finite repulsion  $V$ ,  $T^*$  is higher than  $T_c$  at small dopings. Therefore, the introduction of the nearest-neighbor Coulomb repulsion  $V$  yields a possibility of the DDW state dominating in the small region of doping. In Fig. 2 we present the doping dependence of the temperatures  $T^*$  and  $T_c$  for  $V=0.0$  and  $0.9$  in the uncoupled case.  $T^*$  decreases rapidly with increase

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dopings.



Below  $T^*$  we calculate the DDW gap parameter  $\Delta_D$ , the spin fluctuation spectrum  $\text{Im } \chi_s(\mathbf{q}, \nu)$ , and the coupling parameter  $P$  by solving the Eqs. (14) and (12) self-consistently within the FLEX approximation. The resulting temperature dependence of  $\Delta_D$  is shown in Fig. 3 along with the one of the conventional weak-coupling theory where the coupling parameter is constant. We find that the DDW gap develops much more rapidly within the FLEX approximation (solid line) as compared with the weak-coupling theory (dashed line). This behavior is associated with the feedback effect on spin fluctuations of the DDW gap.

Fig. 4 shows a typical result for the spin fluctuation spectrum  $\text{Im } \chi_s(\mathbf{Q}, \nu)$  at the antiferromagnetic wave vector obtained within FLEX approximation. In the normal state the spectrum is comparatively structureless, as shown by the dashed line. The solid line shows results in the DDW state. The opening of the DDW gap leads to a suppression of spectral weight at low frequencies, but at higher frequencies a strong resonance-like peak appears in  $\text{Im } \chi_s(\mathbf{Q}, \nu)$ . The appearance of such a strong peak in the spin fluctuation spectrum yields an enhancement of the coupling parameter  $P$  in Eq. (15), as shown in Fig. 5. This in turn leads to an increase of the DDW gap, resulting in a positive feedback effect. This is the reason for the almost jump-like increase of the DDW gap in company with the pairing interaction below  $T^*$ . Such a strong enhancement of the spin fluctuation pairing interaction due to feedback of the DDW gap will be especially important in explaining high-temperature superconductivity induced by DDW as shown below.

### 3.2. Superconducting transition from the DDW state

We now study the superconducting transition from the DDW state within FLEX theory, neglecting the  $\pi$ -pairing order parameter. In the DDW state the equation for the superconducting transition temperature  $T_c$  is given from Eq. (10) as follows:

$$\frac{1}{-V+P} = \frac{1}{N} \sum_{\mathbf{k}} \frac{1}{\epsilon_{\mathbf{k}} - T_c}$$

where the coupling parameter  $P$  is calculated in the pure DDW state by solving Eqs. (14) and (12) self-consistently. Since the coupling parameter  $P$  is strongly enhanced due to the feedback effect on spin fluctuations of the DDW gap as shown above, the superconducting transition temperature  $T_c$  in the DDW state can be much larger than expected in the spin fluctuation theory without DDW order.

In Fig. 6 we show the result for the  $T_c$  calculated from Eq. (16) in the DDW state. Near half-filling  $T_c$  is zero. With increasing doping  $T_c$  starts to increase at a finite doping  $\delta_1$ , reaching a maximum at an optimal doping  $\delta_m$ , then decreases with further increase of doping as  $T_c = T^*$  as far as the QCP  $\delta_c$ . After passing  $\delta_c$ ,  $T_c$  becomes the pure superconducting transition temperature in the absence of DDW order.

respectively. After the growth of the DSC gap below  $T_c$ , the DDW gap is slightly suppressed and remains almost constant with lowering temperature. The DDW-induced-DSC gap develops at higher temperature, but its magnitude is small compared to the pure DSC gap. Below  $T_c$  the DDW and DSC gaps coexist.

In Fig. 8 we show the doping dependence of the gap parameters  $\Delta_D$  and  $\Delta_B$ , calculated at  $T/T^*=0.02$ . The uncoupled DDW gap (dotted line) first decreases monotonically with increasing  $\delta$ , then vanishing rapidly at the QCP  $\delta_c=0.26$ . The uncoupled DSC gap (dashed line) is smaller than the DDW gap at lower dopings  $\delta < \delta_c$  due to the nearest-neighbor Coulomb repulsion  $V$ , but it decreases so rather slowly to become nonzero up to large dopings  $\delta_c < \delta_2=0.32$ . The solid lines show the doping dependence of competing gap parameters  $\Delta_D$  and  $\Delta_B$ . Near half filling the DSC gap  $\Delta_B$  is completely suppressed due to the strong competition with the DDW gap  $\Delta_D$ . Upon further doping,  $\Delta_B$  first appears at a doping  $\delta_1=0.12$  and increases with increasing  $\delta$ , coexisting with the DDW gap up to  $\delta_c$ . The DDW gap is slightly suppressed by the appearance of the DSC gap in the doping region  $\delta_1 < \delta < \delta_c$ . After passing  $\delta_c$  the DDW gap is zero and only the pure DSC gap exists. Thus the QCP  $\delta_c$  separates a pure superconducting state at large dopings from a ground state containing both the DDW and superconducting gaps.

Finally we discuss the dependence of the phase diagram on the nearest-neighbor Coulomb repulsion  $V$ . Fig. 9 shows the phase diagrams calculated for  $V=0.0, 0.9$ , and  $1.3$ . The solid line shows the superconducting transition temperature  $T_c$ . The dashed-dotted line describes the DDW transition temperature  $T^*$ .

one crosses the solid line and enters the pure superconducting region. Increasing  $V$  the pure superconducting state is more and more suppressed whereas

