

Copula熵的多学科应用

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Copula熵

- Jian Ma and Zengqi Sun. Mutual information is copula entropy. *Tsinghua Science & Technology*, 16(1):51–54, 2011. See also arXiv preprint arXiv:0808.0845 (2008).

$$H_c(\mathbf{x}) = - \int_{\mathbf{u}} c(\mathbf{u}) \log c(\mathbf{u}) d\mathbf{u}.$$

综述论文

- 马健. Copula熵：理论和应用. ChinaXiv:202105.00070, 2021.

统计独立性是统计学和机器学习领域的基础性概念，如何表示和度量统计独立性是该领域的基本问题。Copula理论提供了统计相关关系表示的理论工具，而Copula熵理论则给出了度量统计独立性的概念工具。本文综述了Copula熵的理论和应用，概述了其基本概念定义、定理和性质，以及估计方法。介绍了Copula熵研究的最新进展，包括其在统计学的六个基本问题（结构学习、关联发现、变量选择、因果发现、域自适应和正态性检验等）上的理论应用。讨论了前四个理论应用之间的关系，以及其对应的深层次的相关性和因果性概念之间的联系，并将Copula熵的（条件）独立性度量框架与基于核函数和距离相关的同类框架进行了对比。简述了Copula熵在理论物理学、理论化学、化学信息学、水文学、环境气象学、生态学、动物形态学、农学、认知神经学、运动神经学、计算神经学、心理学、系统生物学、生物信息学、临床诊断学、老年医学、精神病学、公共卫生学、经济学、社会学、教育学、新闻传播学、法学、政治学，以及能源工程、土木工程、制造工程、可靠性工程、航空航天、电子工程、通信工程、测绘工程和金融工程等领域的实际应用。

理论物理学

- 相关粒子系统
 - 平衡态相关粒子系统中熵的推导和计算 [Ma2021b]

理论化学

- 变构效应研究
 - 变构效应配位点和激活点热力学耦合模型 [Cudndet2016]
 - 丙氨酸二肽的C端和N端

化学信息学

- 分子设计
 - 设计具有特定属性的分子结构 [Wieser2020]
 - 有机分子属性QM9数据库

水文学 (1)

- 洪水预报
 - 金沙江流域洪水预报 [Chen2013, Chen2014]
- 干旱研究
 - 黄河流域(河南和甘肃)干旱分析和预测 [温2019, Huang2019, 黄2021b]
 - 印度达布蒂 (Tapti) 河流域干旱指数设计 [Kanthavel2022]
- 水文事件风险建模
 - 黄河流域干旱事件识别 [Ni2020]
- 中长期径流预报
 - 长江上游水文站月径流预测 [Li2022]
 - 南水北调工程丹江口水库入库径流预报 [黄2021]

水文学（2）

- 水文观测网络选址和优化
 - 上海雨量观测网 [Xu2017]
 - 伊洛河流域水文观测网[王2019]
 - 汾河流域观测网 [Li2020]
 - 北京市区水文观测网 [Li2020]
 - 太湖盆地流域雨量观测网 [Li2020]
 - 淮河流域雨量观测网[徐2022]
 - 美国查克托哈奇（Choctawhatchee）河流域水文观测网 [杨2019]
- 流域分区
 - 鄱阳湖流域 [刘2022]
- 多站点径流生成
 - 巴西雅瓜拉比-大都市水库系统 [Porto2021]

环境气象学

- 大气污染气象成因分析
 - 北京地区气象因素对PM2.5浓度的因果关系分析[Ma2019a]
 - 北京地区PM2.5和气象观测数据
 - 上海和广州大气污染预测预警 [Wang2022]
 - 上海和广州PM2.5和气象观测数据

生态学

- 动物运动轨迹分析
 - Cylcop算法包 [Hodel2021]

动物学

- 鱼类形态学
 - 鱼类形态相似度研究 [Escolano2017]
 - GatorBait海洋鱼类外形数据库

农学

- 葡萄酒质量评价
 - 葡萄酒质量与理化成分关系分析 [Lasserre2021,Lasserre2022]
 - 葡萄牙绿酒葡萄酒理化成分与质量评价数据

神经科学

- 认知神经学
 - 分析大脑认知活动的多模态数据 [Kayser2015, Ince2016, Ince2017, Combrisson2022]
 - 人脸检测任务EEG数据
 - 听觉语音刺激任务MEG数据
 - 认知行为映射任务 MEG数据
 - 奖惩学习任务前脑岛(anterior Insula)SEEG 数据
 - 语音信息的编码和解析 [DeClercq2022]
 - 故事讲述语音及相应的EEG数据
- 运动神经学
 - 分析运动的肌肉组合协同策略 [吴2021, Wu2022a, Reilly2022, Zhu2022]
 - 伸手运动时肌肉EMG数据
- 计算神经学
 - 神经元可塑性建模 [Leugering2018]
 - 神经网络信息传输关系分析 [Parman2021]

心理学

- 生物心理学
 - 情绪刺激下心跳诱发脑电位的时间交互现象 [Ravijts2019]
 - 用于情绪分析的生理信号DEAP数据集

生物学

- 系统生物学
 - 生物信号调控和传导 [Charzynska2015]
 - 癌症分子机制数据
 - 生物现象动态网络结构和功能 [Farhangmehr2013]
 - 酵母细胞周期数据
- 生物信息学
 - 分析基因数据，研究生命和疾病机理 [Wieczorek2016]
 - 肝炎病毒感染治疗基因表达谱数据
 - 筛选与癌症有关的变异基因 [Wu2022b]
 - cBioPortal癌症基因组数据
 - 亚利桑那州立大学癌症基因组数据

医学

- 临床医学
 - 心脏病诊断 [Ma2021a]
 - UCI心脏病数据
 - 糖尿病病情管理 [Mesiar2021]
 - 美国Health Facts糖尿病救治网络数据
 - 癌症预后 [Ma2022b]
 - UCI肺癌数据
- 认知医学
 - 认知能力评估 / 痴呆症筛查 [Ma2019b]
 - 北京和天津痴呆症老年人数据
- 运动医学
 - 运动能力评估 / 跌倒风险预测 [Ma2020a, Ma2020b, Ma2022]
 - 天津和成都跌倒人群老年人数据
- 精神病学
 - 抑郁症患者识别 [张2022]
 - 江苏常州抑郁症青少年患者EEG数据

公共卫生学

- 新冠肺炎流行病 (COVID19)
 - 发热症状疑似病人筛查诊断 [Mesiar2021]
 - 新冠临床数据

社会科学 (I)

- 经济学

- 扶贫政策效果评估，用于政策目标人口鉴别 [Shan2020,罗2022]
 - 2018年政府贫困家庭状况普查数据（四川省）
- 议价机制中的互惠行为和时间效应 [Bossemeyer2021]
 - eBay的Best Offer平台数据
- 产业链内部相关性分析 [韦2021]
 - 国内畜禽养殖产业链主要上市企业股票价格数据

- 社会学

- 分析教育、职业和收入上的性别不平等问题 [Ma2022a]
 - 美国国家成年人收入调查数据（1994年）

- 教育学

- 高中数学成绩与其他学科成绩相关性分析 [柳2018]
 - 某市2013级理科学生高一、高二期末成绩和高三两次模考成绩

- 新闻传播学

- 上海新冠疫情下的公众情绪变化 [Zhang2022]
 - 微博平台“上海疫情”主题数据

社会科学（II）

- 法学
 - 社区属性与社区犯罪关系分析 [Wieser2020]
 - 美国社区与犯罪数据集
- 政治学
 - 分析政权领导力因素和政权危机之间关系 [Card2011]
 - 雪城大学莫伊尼汉全球事务研究所国际政治领导力数据集

工程 (1)

- 能源工程
 - 能源网络管理，研究天气因素与能源网络的耦合 [Fu2017]
 - 北方某地区能源系统运行数据
 - 风光储协同规划 [董2022]
 - 某工业园区风光火储联合发电系统
 - 电网频率稳定性预测 [Liu2022]
 - 贵州电网数据
 - 电力系统宽频振荡影响因素和传播路径分析 [冯2022]
- 土木工程
 - 建筑能源系统节能技术 [Li2022]
 - 大连某教学楼供热监测数据
- 制造工程
 - 制造质量管理，研究优化制造过程参数，预测产品质量 [Sun2021]
 - 富士康生产线制造过程数据
 - 装配质量控制 [王2015]
 - 江淮汽车某型汽油发动机关键零部件装配过程数据

工程 (2)

- 可靠性工程
 - 系统退化过程建模 [Sun2019]
 - 微波电子组件数据
 - 风电机组健康状态评估 [齐2019]
 - 内蒙古某风场的风机SCADA数据
- 航空航天
 - 飞行器总体参数分析和优化 [Krishnankutty2020]
 - 美国喷气战斗机总体设计参数数据
 - 卫星在轨健康状态监测 [Liu2022, Zeng2022]
 - 真实卫星遥测数据
 - NASA公开的 SMAP 和 MSL 数据集
 - 机场间航班延误因果关系分析 [吴2020]
 - 民航信息系统

工程 (3)

- 电子工程
 - 集成电路封装材料物理性能预测 [刘2022]
 - CuNi合金体系材料强度和稳定性计算数据
- 通信工程
 - 通讯网络加密技术研究 [Wang2016]
- 测绘工程
 - 高光谱遥感数据分析 [Zeng2009]
 - 美国印第安纳Indian Pine高光谱遥感数据

- 量化金融工具箱 MLFinLab
 - Hudson and Thames Quantitative Research [HudsonThames2021]
 - 非线性相关分析算法
- 投资组合优化
 - 股票资产相关性网络分析 [Wang2015]
 - 沪深A股指数、沪深300指数数据
- 金融问题建模
 - Copula函数模型选择 [Calsaverini2009]
 - 标普500指数数据
- 股票相关性建模
 - R-vine copula结构建模 [Alanazi2021]
 - 德国DAX指数数据
- 信用风险评价
 - 信用风险卡模型建立 [孔2021]
 - 信用卡客户数据

参考文献

- [Chen2013] Lu Chen, Vijay P. Singh, and Shenglian Guo. Measure of correlation between river flows using the copula-entropy method. *Journal of Hydrologic Engineering*, 18(12):1591–1606, 2013.
- [Chen2014] Lu Chen, Vijay P. Singh, Shenglian Guo, Jianzhong Zhou, and Lei Ye. Copula entropy coupled with artificial neural network for rainfall–runoff simulation. *Stochastic Environmental Research and Risk Assessment*, 28(7):1755–1767, 2014.
- [Ni2020] Lingling Ni, Dong Wang, Jianfeng Wu, Yuankun Wang, Yuwei Tao, Jianyun Zhang, Jiufu Liu, and Fei Xie. Vine copula selection using mutual information for hydrological dependence modeling. *Environmental Research*, 186:109604, 2020.
- [Xu2017] Pengcheng Xu, Dong Wang, Vijay P. Singh, Yuankun Wang, Jichun Wu, Lachun Wang, Xinqing Zou, Yuanfang Chen, Xi Chen, Jiufu Liu, Ying Zou, and Ruimin He. A two-phase copula entropy-based multiobjective optimization approach to hydrometeorological gauge network design. *Journal of Hydrology*, 555:228–241, 2017.
- [Li2020] Heshu Li, Dong Wang, Vijay P. Singh, Yuankun Wang, Jianfeng Wu, Jichun Wu, Ruimin He, Ying Zou, Jiufu Liu, and Jianyun Zhang. Developing a dual entropy-transformation criterion for hydrometric network optimization based on information theory and copulas. *Environmental Research*, 180:108813, 2020.
- [Porto2021] Victor Costa Porto, Francisco de Assis de Souza Filho, Taís Maria Nunes Carvalho, Ticiana Marinho de Carvalho Studart, and Maria Manuela Portela. A GLM copula approach for multisite annual streamflow generation. *Journal of Hydrology*, 598:126226, 2021.
- [Ma2019a] Jian Ma. Estimating transfer entropy via copula entropy. arXiv preprint arXiv:1910.04375, 2019.
- [Hodel2021] Florian H. Hodel and John R. Fieberg. Cylcop: An R package for circularlinear copulae with angular symmetry. bioRxiv, page 2021.07.14.452253, 2021.
- [Wieser2020] Mario Wieser, Sonali Parbhoo, Aleksander Wieczorek, and Volker Roth. Inverse learning of symmetries. In *Advances in Neural Information Processing Systems*, volume 33, pages 18004–18015, 2020.
- [Ince2017] Robin A.A. Ince, Bruno L. Giordano, Christoph Kayser, Guillaume A. Rousselet, Joachim Gross, and Philippe G. Schyns. A statistical framework for neuroimaging data analysis based on mutual information estimated via a gaussian copula. *Human Brain Mapping*, 38(3):1541–1573, 2017.
- [Ince2016] Robin A. A. Ince, Katarzyna Jaworska, Joachim Gross, Stefano Panzeri, Nicola J. van Rijsbergen, Guillaume A. Rousselet, and Philippe G. Schyns. The deceptively simple N170 reflects network information processing mechanisms involving visual feature coding and transfer across hemispheres. *Cerebral Cortex*, 26(11):4123–4135, 2016.
- [Kayser2015] Stephanie J. Kayser, Robin A.A. Ince, Joachim Gross, and Christoph Kayser. Irregular speech rate dissociates auditory cortical entrainment, evoked responses, and frontal alpha. *The Journal of Neuroscience*, 35(44):14691–14701, 2015.
- [Leugering2018] Johannes Leugering and Gordon Pipa. A unifying framework of synaptic and intrinsic plasticity in neural populations. *Neural Computation*, 30(4):945–986, 2018.
- [Parman2021] Ari Pakman, Amin Nejatbakhsh, Dar Gilboa, Abdullah Makkeh, Luca Mazzucato, Michael Wibral, and Elad Schneidman. Estimating the unique information of continuous variables in recurrent networks. *Advances in Neural Information Processing Systems*, 2021.
- [Charzynska2015] Agata Charzyńska and Anna Gambin. Improvement of the k-NN entropy estimator with applications in systems biology. *Entropy*, 18(1):13, 2015.

参考文献 (cont)

- [Farhangmehr2013] Farzaneh Farhangmehr, Daniel M. Tartakovsky, Parastou Sadatmousavi, Mano R. Maurya, and Shankar Subramaniam. An information-theoretic algorithm to data-driven genetic pathway interaction network reconstruction of dynamic systems. In 2013 IEEE International Conference on Bioinformatics and Biomedicine, pages 214–217, 2013.
- [Wieczorek2016] Aleksander Wieczorek and Volker Roth. Causal compression. arXiv preprint arXiv:1611.00261, 2016.
- [Ma2021a] Jian Ma. Variable selection with copula entropy. Chinese Journal of Applied Probability and Statistics, 37(4):405–420, 2021. See also arXiv preprint arXiv:1910.12389 (2019).
- [Mesiar2021] Radko Mesiar and Ayyub Sheikhi. Nonlinear random forest classification, a copula-based approach. Applied Sciences, 11(15), 2021.
- [Ma2019b] Jian Ma. Predicting MMSE score from finger-tapping measurement. bioRxiv, page 817338, 2019.
- [Ma2020a] Jian Ma. Predicting TUG score from gait characteristics based on video analysis and machine learning. bioRxiv, page 963686, 2020.
- [Ma2020b] Jian Ma. Associations between finger tapping, gait and fall risk with application to fall risk assessment. arXiv preprint arXiv:2006.16648, 2020.
- [Shan2020] Qingsong Shan and Qianning Liu. Binary trees for dependence structure. IEEE Access, 8:150989–150998, 2020.
- [Card2011] Stuart William Card. Towards an information theoretic framework for evolutionary learning. Master’s thesis, Syracuse University, 2011.
- [Fu2017] Xueqian Fu, Hongbin Sun, Qinglai Guo, Zhaoguang Pan, Wen Xiong, and Li Wang. Uncertainty analysis of an integrated energy system based on information theory. Energy, 122(122):649–662, 2017.
- [Sun2021] Yan-Ning Sun, Yu Chen, Wu-Yin Wang, Hong-Wei Xu, and Wei Qin. Modelling and prediction of injection molding process using copula entropy and multi-output svr. In IEEE 17th International Conference on Automation Science and Engineering, 2021.
- [Sun2019] Fuqiang Sun, Wendi Zhang, Ning Wang, and Wei Zhang. A copula entropy approach to dependence measurement for multiple degradation processes. Entropy, 21(8):724, 2019.
- [Krishnankutty2020] Baby Alpettiyil Krishnankutty, Rajesh Ganapathy, and Paduthol Godan Sankaran. Non-parametric estimation of copula based mutual information. Communications in Statistics - Theory and Methods, 49(6):1513–1527, 2020.
- [Wang2016] Xu Wang, Liang Jin, Kaizhi Huang, Mingliang Li, and Yi Ming. Physical layer secret key capacity using correlated wireless channel samples. In 2016 IEEE Global Communications Conference (GLOBECOM), pages 1–6, 2016.
- [Zeng2009] Xuexing Zeng and T S Durrani. Band selection for hyperspectral images using copulas-based mutual information. In 2009 IEEE/SP 15th Workshop on Statistical Signal Processing, pages 341–344, 2009.
- [HudsonThames2021] Hudson and Thames. Machine learning financial laboratory (MLFinLab), 2021. URL: <https://github.com/hudson-and-thames/mlfinlab>.
- [Wang2015] Qiutong Wang. Social networks, asset allocation and portfolio diversification. Master’s thesis, University of Waterloo, 2015.
- [Calsaverini2009] Rafael Calsaverini and Renato Vicente. An information-theoretic approach to statistical dependence: Copula information. EPL (Europhysics Letters), 88(6):68003, 2009.

参考文献 (cont)

- [温2019] 温云亮, 李艳玲, 黄春艳, and 张泽中. 基于 copula 熵理论的干旱驱动因子选择. 华北水利水电大学学报 (自然科学版), 40(4):51–56, 2019.
- [黄2021b] 黄春艳. 黄河流域的干旱驱动及评估预测研究. 博士学位论文, 西安理工大学, 2021.
- [吴2021] 吴亚婷, 余青山, 高云园, 谭同才, and 范影乐. 多尺度肌间耦合网络分析. 生物医学工程学杂志, 38(4):742–752, 2021.
- [Wu2022a] Yating Wu, Qingshan She, Hongan Wang, Yuliang Ma, Mingxu Sun, and Tao Shen. R-vine copula mutual information for intermuscular coupling analysis. In Proceedings of the 11th International Conference on Computer Engineering and Networks, pages 526–534, 2022.
- [Reilly2022] David ó Reilly and Ioannis Delis. A network information theoretic framework to characterise muscle synergies in space and time. Journal of Neural Engineering, 19(1):016031, feb 2022.
- [Ma2021b] Jian Ma. On thermodynamic interpretation of copula entropy. arXiv preprint arXiv:2111.14042, 2021.
- [Ma2022a] Jian Ma. Causal domain adaptation with copula entropy based conditional independence test. arXiv preprint arXiv:2202.13482, 2022.
- [刘2022] 刘磊, 高超, 王志刚, 王晓艳, 章四龙, and 陈娜. 基于非线性相关性和复杂网络的径流相似性分区. 水科学进展, 2022.
- [Wu2022b] Qiang Wu and Dongxi Li. CRIA: An interactive gene selection algorithm for cancers prediction based on copy number variations. Frontiers in Plant Science, 13, 2022.
- [Liu2022] Hao Liu, Dechang Pi, Shuyuan Qiu, Xixuan Wang, and Chang Guo. Data driven identification model for associated fault propagation path. Measurement, 188:110628, 2022.
- [Zeng2022] Zefan Zeng, Guang Jin, Chi Xu, Siya Chen, Zhelong Zeng, and Lu Zhang. Satellite telemetry data anomaly detection using causal network and feature-attention-based lstm. IEEE Transactions on Instrumentation and Measurement, 71:1–21, 2022.
- [Huang2019] C.Y. Huang and Y.P. Zhang. Prediction based on copula entropy and general regression neural network. Applied Ecology and Environmental Research, 17(6):14415–14424, 2019.
- [Wang2022] Jujie Wang, Wenjie Xu, Yue Zhang, and Jian Dong. A novel air quality prediction and early warning system based on combined model of optimal feature extraction and intelligent optimization. Chaos, Solitons & Fractals, 158:112098, 2022.
- [Combrisson2022] Etienne Combrisson, Michele Allegra, Ruggero Basanisi, Robin A. A. Ince, Bruno Giordano, Julien Bastin, and Andrea Brovelli. Group-level inference of information-based measures for the analyses of cognitive brain networks from neurophysiological data. bioRxiv, 2022.
- [Li2022] Zhiwei Li, Peng Wang, Jili Zhang, and Hua Guan. A model-free method for identifying time-delay characteristics of HVAC system based on multivariate transfer entropy. Building and Environment, 217:109072, 2022.
- [黄2021a] 黄朝君, 贾建伟, 秦赫, 王栋. 基于Copula熵 - 随机森林的中长期径流预报研究. 人民长江, 2021, 52(11): 81-85.
- [罗2022] 罗良清, 平卫英, 单青松, and 王佳. 中国贫困治理经验总结 : 扶贫政策能够实现有效增收吗? . 管理世界, 38(2):70–83, 2022.

参考文献 (cont)

- [Lasserre2021] Marvin Lasserre, Régis Lebrun, and Pierre-Henri Willemin. Learning continuous high-dimensional models using mutual information and copula bayesian networks. In Thirty-Fifth AAAI Conference on Artificial Intelligence, AAAI 2021, pages 12139–12146. AAAI Press, 2021.
- [Lasserre2022] Marvin Lasserre. Apprentissages dans les réseaux bayésiens à base de copules non-paramétriques. PhD thesis, Sorbonne Université, 2022.
- [王2015] 王小巧. 复杂机械产品装配过程质量自适应控制方法及支持系统研究. 博士学位论文, 合肥工业大学, 2015.
- [齐2019] 齐咏生, 景彤梅, 李永亭, 刘利强, and 刘月文. 一种基于多维度 SCADA 数据评估风电机组健康状态评估方法, 2019. CN110442833A.
- [吴2020] 吴格, 陈旭, 傅之凤, 李忠虎, and 杨程屹. 一种因果关系分析方法及装置, 2020. CN110766314A.
- [孔2021] 孔祥永, 王浩, 袁伟, and 蔡明. 一种自动化特征工程信用风险评价系统及方法, 2021. CN114049198A.
- [Cudndet2016] Michel A. Cuendet, Harel Weinstein, and Michael V. LeVine. The allosteric landscape: Quantifying thermodynamic couplings in biomolecular systems. *Journal of Chemical Theory and Computation*, 12(12):5758–5767, December 2016.
- [Escolano2017] Francisco Escolano, Edwin R. Hancock, Miguel A. Lozano, and Manuel Curado. The mutual information between graphs. *Pattern Recognition Letters*, 87:12–19, 2017.
- [董2022] 董海艳, 赵炳文, 王运韬, 田宇, 傅彦博, 孟德群, and 张铁. 一种含源荷时序相似度约束的源储协同规划配置方法, 2022. CN110766314A.
- [徐2022] 徐鹏程, 仇建春, 李帆, 刘赛艳, and 蒋新跃. 基于高维Copula熵和克里金的站网优化方法. 2022. CN114595556A.
- [王2019] 王栋, 徐鹏程, 王远坤, and 吴吉春. 一种基于Copula熵的水文站网优化模型的优化方法. 2019. CN106897530B.
- [张2022] 张婷婷, 王楠, 周天彤, 王苏弘, and 邹凌. 基于Couple熵的抑郁症相干性反馈指标提取. *电子测量技术*, 45(9): 160-167, 2022.
- [Alanazi2021] Alanazi, F. A. Truncating Regular Vine Copula Based on Mutual Information: An Efficient Parsimonious Model for High-Dimensional Data. *Mathematical Problems in Engineering*, 2021, 4347957.
- [柳2018] 柳琼. 基于Copula和MI理论的相关性度量及其应用研究. 硕士学位论文, 三峡大学, 2018.
- [Ravijts2019] Liesa Ravijts. Revealing temporal interactions around the heartbeat-evoked potential modulated by emotional perception. Master's thesis, Ghent Univeristy, 2019.
- [Bossemeyer2021] Leonie Bossemeyer. Machine Learning for Causal Discovery with Applications in Economics. Master's thesis, Ludwig-Maximilians-Universität München, 2021.
- [韦2021] 韦颖璐. 基于pair--copula熵的相关性度量. 硕士学位论文, 苏州大学, 2021.
- [Wieser2020] Mario Wieser. Learning Invariant Representations for Deep Latent Variable Models. PhD thesis, University of Basel, 2020.
- [Li2022] Xiao Li, Liping Zhang, Sidong Zeng, Zhenyu Tang, Lina Liu, Qin Zhang, Zhengyang Tang, and Xiaojun Hua. Predicting monthly runoff of the upper Yangtze river based on multiple machine learning models. *Sustainability*, 14(18):11149, 2022.
- [Kanthavel2022] P. Kanthavel, C.K. Saxena, and R.K. Singh. Integrated drought index based on vine copula modelling. *International Journal of Climatology*, 2022.
- [杨2019] 杨惜岁. 多目标准则下流域水文站网的优化与评价. 硕士学位论文, 武汉理工大学, 2019.

参考文献 (cont)

- [DeClercq2022] Pieter De Clercq, Jonas Vanthornhout, Maaike Vandermosten, and Tom Francart. Beyond linear neural envelope tracking: A mutual information approach. bioRxiv, 2022.
- [Ma2022b] Jian Ma. Copula entropy based variable selection for survival analysis. arXiv preprint arXiv:2209.01561, 2022.
- [Zhang2022] Bowen Zhang, Jinping Lin, Man Luo, Changxian Zeng, Jiajia Feng, Meiqi Zhou, and Fuying Deng. Changes in public sentiment under the background of major emergencies – taking the Shanghai epidemic as an example. International Journal of Environmental Research and Public Health, 19(19):12594, 2022.
- [刘2022] 刘勃. 基于机器学习的封装材料加速预测. 硕士学位论文, 哈尔滨理工大学, 2022.
- [Zhu2022] Shaojun Zhu, Jinhui Zhao, Yating Wu, and Qingshan She. Intermuscular coupling network analysis of upper limbs based on R-vine copula transfer entropy. Mathematical Biosciences and Engineering, 19(9):9437–9456, 2022.

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