

# Copula 熵的多学科应用

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# Copula 熵

- Jian Ma and Zengqi Sun. Mutual information is copula entropy. *Tsinghua Science & Technology*, 16(1):51–54, 2011. See also arXiv preprint arXiv:0808.0845 (2008).

$$H_c(\mathbf{x}) = - \int_{\mathbf{u}} c(\mathbf{u}) \log c(\mathbf{u}) d\mathbf{u}.$$

# 综述论文

- 马健 . Copula 熵：理论和应用 . ChinaXiv:202105.00070, 2021.

统计独立性是统计学和机器学习领域的基础性概念，如何表示和度量统计独立性是该领域的基本问题。Copula 理论提供了统计相关关系表示的理论工具，而 Copula 熵理论则给出了度量统计独立性的概念工具。本文综述了 Copula 熵的理论和应用，概述了其基本概念定义、定理和性质，以及估计方法。介绍了 Copula 熵研究的最新进展，包括其在统计学的六个基本问题（结构学习、关联发现、变量选择、因果发现、域自适应和正态性检验等）上的理论应用。讨论了前四个理论应用之间的关系，以及其对应的深层次的相关性和因果性概念之间的联系，并将 Copula 熵的（条件）独立性度量框架与基于核函数和距离相关的同类框架进行了对比。简述了 Copula 熵在理论物理学、理论化学、化学信息学、水文学、气候学、气象学、环境学、生态学、动物形态学、农学、认知神经学、运动神经学、计算神经学、心理学、系统生物学、生物信息学、临床诊断学、老年医学、精神病学、公共卫生学、经济学、管理学、社会学、教育学、计算语言学、新闻传播学、法学、政治学、军事学，以及能源工程、食品工程、土木建筑、交通工程、制造工程、可靠性工程、化学工程、航空航天、车辆工程、电子工程、通信工程、高性能计算、测绘工程和金融工程等领域的实际应用。

# 理论物理学

- 相关粒子系统
  - 平衡态相关粒子系统中熵的推导和计算 [Ma2021b]

# 理论化学

- 变构效应研究
  - 变构效应配位点和激活点热力学耦合模型 [Cudndet2016]
    - 丙氨酸二肽的 C 端和 N 端

# 化学信息学

- 分子设计
  - 设计具有特定属性的分子结构 [Wieser2020]
  - 有机分子属性 QM9 数据库

# 水文学（1）

- 洪水预报
  - 金沙江流域洪水预报 [Chen2013, Chen2014]
- 河流相关性
  - 长江上游河段（金沙江、岷江、沱江、嘉陵江）相关性 [Chen2019]
- 水沙关系分析
  - 黄河西柳沟河流域径流量和输沙量数据分析 [Qian2022]
- 干旱研究
  - 黄河流域（河南和甘肃）干旱分析和预测 [温 2019, Huang2019, 黄 2021b]
  - 印度达布蒂（Tapti）河流域干旱指数设计 [Kanthavel2022]
  - 伊朗城市（扎黑丹、恩泽利和马什哈德）干旱数据分析 [Mohammadi2021]
- 水文事件风险建模
  - 黄河流域干旱事件识别 [Ni2020]
- 中长期径流预报
  - 长江上游水文站月径流预测 [Li2022]
  - 南水北调工程丹江口水库入库径流预报 [黄 2021]
  - 洪泽湖和骆马湖长期径流预报 [Mo2023]

## 水文学（2）

- 水文观测网络选址和优化
  - 上海雨量观测网 [Xu2017]
  - 伊洛河流域水文观测网 [王 2019]
  - 汾河流域观测网 [Li2020]
  - 北京市区水文观测网 [Li2020]
  - 太湖盆地流域雨量观测网 [Li2020]
  - 淮河流域雨量观测网 [徐 2022]
  - 美国查克托哈奇（Choctawhatchee）河流域水文观测网 [杨 2019]
- 流域分区
  - 鄱阳湖流域 [刘 2022]
- 多站点径流生成
  - 巴西雅瓜拉比 - 大都市水库系统 [Porto2021]



# 气候学

- 气候变化对水文气象变量相关性的影响
  - 美国德州达拉斯地区降水和气温相关性关系分析 [Hao2015]
    - 达拉斯市沃斯堡（ Fort Worth ）地区降水和气温观测数据
- 气候评估
  - 欧洲城市气候分类 [Condino2009]
    - 欧洲 25 座城市气温和降水数据

# 气象学

- 大气污染气象成因分析
  - 北京地区气象因素对 PM2.5 浓度的因果关系分析 [Ma2019a]
    - 北京地区 PM2.5 和气象观测数据
  - 上海和广州大气污染预测预警 [Wang2022]
    - 上海和广州 PM2.5 和气象观测数据
- 气象灾害预测
  - 广西地区台风灾情预测 [陈 2019]
    - 广西地区台风灾害数据

# 环境学

- 大气污染传播路径分析
  - 兰州市大气污染传播路径预测 [吴 2022]
  - 兰州市环境气象检测网络 2017 年 PM2.5 观测数据
- 火电厂排放污染物管控
  - 火电厂氮氧化物排放浓度预测 [金 2023]
  - 宁夏某燃煤电厂运行数据

# 生态学

- 动物运动轨迹分析
  - Cylcop 算法包 [Hodel2021]

# 动物学

- 动物形态学
  - 鱼类形态相似度研究 [Escolano2017]
    - GatorBait 海洋鱼类外形数据库
  - 鲍鱼生长过程的形态学研究 [Purkayastha2022]
    - UCI 鲍鱼数据集

# 农学

- 作物产量预测
  - 气候变化对我国南方两季稻产量的影响及对策 [Zhang2023]
    - 南方（江南和华南） 54 个地点未来气候变化数据和作物数据

# 神经科学

- 认知神经学
  - 分析大脑认知活动的多模态数据 [Kayser2015, Ince2016, Ince2017, Combrisson2022]
    - 人脸检测任务 EEG 数据
    - 听觉语音刺激任务 MEG 数据
    - 认知行为映射任务 MEG 数据
    - 奖惩学习任务前脑岛 (anterior Insula) S EEG 数据
  - 语音信息的编码和解析 [DeClercq2022]
    - 故事讲述语音及相应的 EEG 数据
  - 因果关系脑连接网络分析 [Redondo2023, 汪 2023]
    - 注意缺陷多动障碍患者 EEG 数据
    - 葡萄牙老年人静息态 fMRI 数据
- 运动神经学
  - 分析运动的肌肉组合协同策略 [吴 2021, Wu2022a, Reilly2022, Zhu2022, 金 2022]
    - 伸手运动时肌肉 sEMG 数据
    - 自主运动肌肉疲劳状态 sEMG 数据
- 计算神经学
  - 神经元可塑性建模 [Leugering2018]
  - 神经网络信息传输关系分析 [Parman2021]

# 心理学

- 生物心理学
  - 情绪刺激下心跳诱发脑电位的时间交互现象 [Ravijts2019]
  - 用于情绪分析的生理信号 DEAP 数据集



# 生物学

- 系统生物学
  - 生物信号调控和传导 [Charzynska2015]
    - 癌症分子机制数据
  - 生物现象动态网络结构和功能 [Farhangmehr2013]
    - 酵母细胞周期数据
- 生物信息学
  - 分析基因数据，研究生命和疾病机理 [Wieczorek2016]
    - 肝炎病毒感染治疗基因表达谱数据
  - 筛选与癌症有关的变异基因 [Wu2022b]
    - cBioPortal 癌症基因组数据
    - 亚利桑那州立大学癌症基因组数据

# 医学（1）

- 临床医学
  - 心脏病诊断 [Ma2021a]
    - UCI 心脏病数据
  - 糖尿病病情管理 [Mesiar2021]
    - 美国 Health Facts 糖尿病救治网络数据
  - 癌症预后 [Ma2022b]
    - UCI 肺癌数据
  - 白内障术后角膜水肿风险预测 [Luo2023]
    - 临床白内障超声乳化手术患者数据
  - 主动脉瓣置换手术射血分数分析 [Sunoj2023]
    - 临床主动脉瓣置换手术前后射血分数数据
- 认知医学
  - 认知能力评估 / 痴呆症筛查 [Ma2019b]
    - 北京和天津痴呆症老年人数据

## 医学（2）

- 运动医学
  - 运动能力评估 / 跌倒风险预测 [Ma2020a, Ma2020b, Ma2022]
    - 天津和成都跌倒人群老年人数据
  - 重复经颅磁刺激对帕金森病改善神经机制分析 [李 2023]
    - 帕金森患者经颅磁刺激前后 EEG 数据
- 精神病学
  - 抑郁症患者识别 [张 2022a]
    - 江苏常州抑郁症青少年患者 EEG 数据

# 公共卫生学

- 新冠肺炎流行病 (COVID19)
  - 发热症状疑似病人筛查诊断 [Mesiar2021]
  - 新冠临床数据
- 高血压
  - 高血压关联基因研究 [Purkayastha2022]
  - ELEMENT 数据集

# 社会科学（1）

- 经济学

- 扶贫政策效果评估，用于政策目标人口鉴别 [Shan2020, 罗 2022]
  - 2018 年政府贫困家庭状况普查数据（四川省）
- 议价机制中的互惠行为和时间效应 [Bossemeyer2021]
  - eBay 的 Best Offer 平台数据
- 产业链内部相关性分析 [韦 2021]
  - 国内畜禽养殖产业链主要上市企业股票价格数据
- 投资者情绪分析 [Han2022]
  - 中国新能源汽车上市公司的百度搜索数据

- 管理学

- 商品期货价格预测 [An2023]
  - 国家统计局猪肉价格数据和大连商品交易所大豆期货价格数据
- 单周期库存管理 [Tian2023]
  - 大众朗逸汽车销售数据

## 社会科学（2）

- 社会学
  - 分析教育、职业和收入上的性别不平等问题 [Ma2022a]
  - 美国国家成年人收入调查数据（1994 年）
- 教育学
  - 高中数学成绩与其他学科成绩相关性分析 [柳 2018]
  - 某市 2013 级理科学生高一、高二期末成绩和高三两次模考成绩
- 计算语言学
  - 城市热线派单系统知识图谱构建 [陈 2023]
  - 济南市民热线数据
- 新闻传播学
  - 上海新冠疫情下的公众情绪变化 [Zhang2022]
  - 微博平台“上海疫情”主题数据

## 社会科学（3）

- 法学
  - 社区属性与社区犯罪关系分析 [Wieser2020]
  - 美国社区与犯罪数据集
- 政治学
  - 分析政权领导力因素和政权危机之间关系 [Card2011]
  - 雪城大学莫伊尼汉全球事务研究所国际政治领导力数据集
- 军事学
  - 目标意图识别 [张 2022b]
  - 空中飞行目标示例

# 工程 (1)

- 能源工程
  - 能源网络管理，研究天气因素与能源网络的耦合 [Fu2017]
    - 北方某地区能源系统运行数据
  - 光伏发电功率预测 [朱 2022a]
    - 澳大利亚 Yulara 地区光伏电站数据
  - 风电机组工况划分 [崔 2022]
    - 广东某海上风电场 SCADA 数据
  - 电力负荷预测 [Ma2023]
    - 摩洛哥缔头万城电力消费数据
  - 风光储协同规划 [董 2022]
    - 某工业园区风光火储联合发电系统
  - 电网频率稳定性预测 [Liu2022]
    - 贵州电网数据
  - 电力系统宽频振荡影响因素和传播路径分析 [冯 2022]



## 工程 (2)

- 能源工程
  - 用户线损贡献分析 [Hu2022]
    - 辽宁电网数据
  - 电价预测 [Xiong2022]
    - 2017 年美国 PJM 电力市场电价数据
  - 锂电池容量估计 [Hu2023a]
    - NASA 锂电池退化数据
- 食品工程
  - 葡萄酒质量与理化成分关系分析 [Lasserre2021,Lasserre2022]
    - 葡萄牙绿酒葡萄酒理化成分与质量评价数据
- 土木建筑
  - 建筑能源系统节能技术 [Li2022]
    - 大连某教学楼供热监测数据
  - 工程变形监测 [Cao2023]
    - 某隧道工程施工段围堰监测数据

## 工程 (3)

- 交通工程
  - 大件货物运输方案制定 [黄 2021c]
    - 大件货物运输案例数据
  - 航空和高铁票价影响因素分析 [许 2023]
    - 京沪高铁和航空票价数据
- 制造工程
  - 制造质量管理, 研究优化制造过程参数, 预测产品质量 [Sun2021]
    - 富士康生产线制造过程数据
  - 装配质量控制 [王 2015]
    - 江淮汽车某型汽油发动机关键零部件装配过程数据
  - 工业过程故障监测 [Dong2023]
    - 鞍钢热轧带钢工艺过程数据
  - 钢铁工艺过程碳排放预测 [Hu2023b]
    - 某钢铁厂烧结过程数据

## 工程 (4)

- 可靠性工程
  - 系统退化过程建模 [Sun2019]
    - 微波电子组件数据
  - 风电机组健康状态评估 [齐 2019]
    - 内蒙古某风场的风机 SCADA 数据
- 化学工程
  - 化学过程故障监测和诊断 [Yin2022, Wei2022]
    - Tennessee Eastman 过程数据
  - 化工过程因果网络构建 [Bi2023]
    - 连续搅拌槽式反应器数据和 Tennessee Eastman 过程数据
- 电子工程
  - 集成电路封装材料物理性能预测 [刘 2022]
    - CuNi 合金体系材料强度和稳定性计算数据

## 工程 (5)

- 航空航天
  - 飞行器总体参数分析和优化 [Krishnankutty2020]
    - 美国喷气战斗机总体设计参数数据
  - 卫星在轨健康状态监测 [Liu2022, Zeng2022]
    - 真实卫星遥测数据
    - NASA 公开的 SMAP 和 MSL 数据集
  - 机场间航班延误因果关系分析 [吴 2020]
    - 民航信息系统
- 车辆工程
  - CAN 总线入侵检测 [Gao2023]
    - 现代汽车 YF 索纳塔 CAN 数据

## 工程 (6)

- 通信工程
  - 通讯网络加密技术研究 [Wang2016]
  - 6G 网络语义通信技术研究 [傅 2023]
    - ImageNet-1k 数据集和 VOC2012 数据集
- 高性能计算
  - 高性能计算能源效率优化 [Gocht-Zech2022]
- 测绘工程
  - 高光谱遥感数据分析 [Zeng2009]
    - 美国印第安纳 Indian Pine 高光谱遥感数据

# 金融工程

- 量化金融工具箱 MLFinLab
  - Hudson and Thames Quantitative Research [Hudson2021]
    - 非线性相关分析算法
- 投资组合优化
  - 股票资产相关性网络分析 [Wang2015]
    - 沪深 A 股指数、沪深 300 指数数据
  - ST 股票分类 [朱 2022b]
    - A 股市场 ST 股票数据
- 金融问题建模
  - Copula 函数模型选择 [Calsaverini2009]
    - 标普 500 指数数据
- 股票相关性建模
  - R-vine copula 结构建模 [Alanazi2021]
    - 德国 DAX 指数数据
- 信用风险评价
  - 信用风险卡模型建立 [孔 2021]
    - 信用卡客户数据



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# 谢谢

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