Copula Entropy

Theory and Applications

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- Lecture 7: Generalizations
- **Lecture 8: Real applications and softwares**

Lecture 8: Real applications and softwares

1. Real applications
   - Astronomy
   - Physics
   - Chemistry
   - Geology
   - Biology
   - Medicine
   - Social sciences
   - Engineering
2. Softwares
   - Official
   - Third-party

天体物理学

- 测光红移
  - 测光红移准确度提升[1]

[1] Jian Ma. "Photometric Redshifts with Copula Entropy". In: *arXiv preprint arXiv:2310.16633* (2023).

理论物理学

- 相关粒子系统
  - 平衡态相关粒子系统中熵的推导和计算[2]

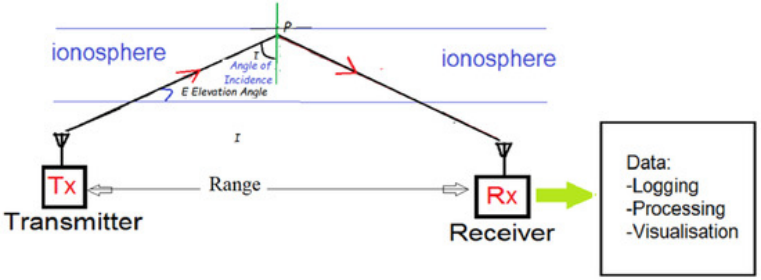
[2] Jian Ma. "On Thermodynamic Interpretation of Copula Entropy". In: *arXiv preprint arXiv:2111.14042* (2021). arXiv: 2111.14042 [cs.IT].

空间科学

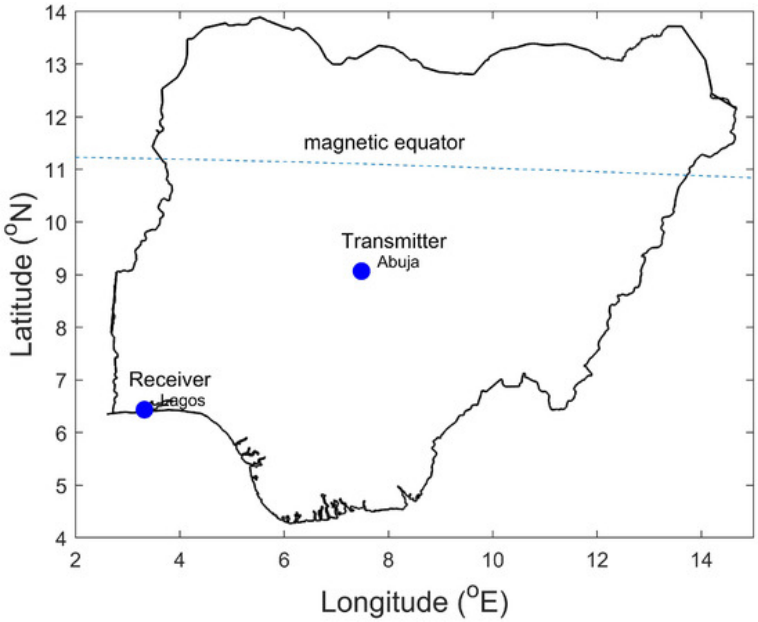
- 赤道电离层特性随季节变化分析
  - 基于 TE 分析赤道地区高频微波信号传输中 TEC 和 DFS 之间的非线性关系[3]

[3] Aderonke Akerele et al. "Complexity and Nonlinear Dependence of Ionospheric Electron Content and Doppler Frequency Shifts in Propagating HF Radio Signals within Equatorial Regions". In: *Atmosphere* 15.6 (2024), p. 654. DOI: 10.3390/atmos15060654. URL: https://www.mdpi.com/2073-4433/15/6/654.

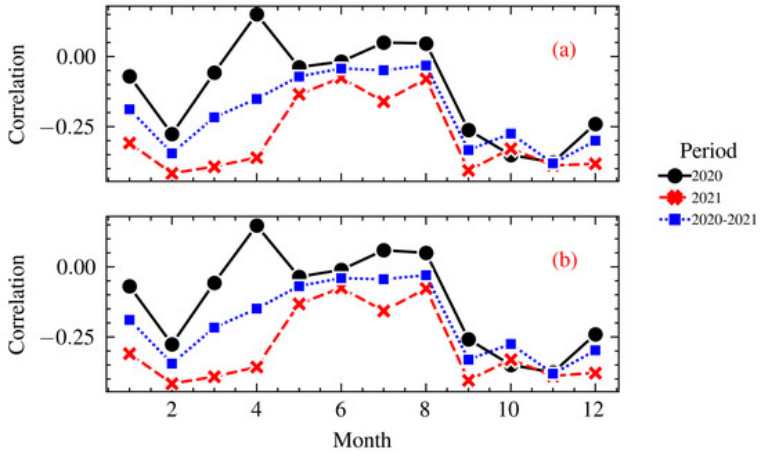
CASE 1: Ionosphere in equatorial regions by ICTP, etc.



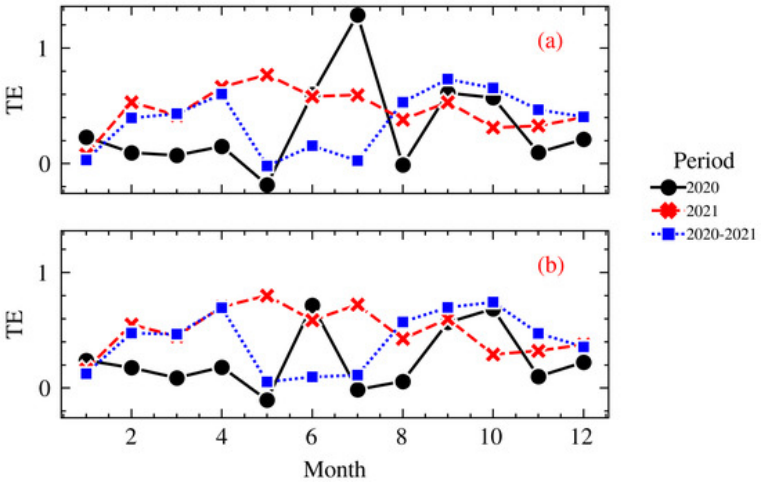
Experimental setup



Locations in Nigeria



Monthly correlation between VTEC and DFS



Monthly transfer entropy between VTEC and DFS

量子物理学

- 量子计算新概念
  - 提出了一种量子 CE 的理论框架[4]

[4] Carmen Maria Constantin and Andreas Doering. "Contextual Entropy and Reconstruction of Quantum States – an Unified Framework based on Copula Entropy". In: *arXiv preprint arXiv:1208.2046* (2021). Unpublished version at August 28, 2021.

CASE 2: Quantum copula entropy by University of Oxford

Definition (Quantum Copula Entropy)

$$C_S(\rho_{AB}) = I(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \tag{1}$$

Definition (Contextual Quantum Copula Entropy)

$$\mathcal{CH}(\rho_{AB}) = \{H_c^{C_A,C_B}(\rho_{AB})\}_{C_A,C_B} \tag{2}$$

Algorithm (Reconstructing Quantum States)

$$\min_{\rho\in\mathcal{D}} f(\rho) = \sum_{t=1}^{M} w_t \left[H_{C_t}(\rho) - H_{C_t}^{exp}\right]^2 \tag{3}$$

流体力学

- 离心泵性能预测
  - 利用 SMOGN 过采样技术和 GAKHO 超参数调节技术相结合的方法来进行离心泵性能预测[5]

[5] Yuhao Chen et al. "Research on performance prediction of a small sample centrifugal pump based on a pre-processing approach for imbalanced regression and key hyperparameters optimization". In: *Physics of Fluids* 37.6 (2025), p. 067148. DOI: 10.1063/5.0274888. eprint: https://pubs.aip.org/aip/pof/article-pdf/doi/10.1063/5.0274888/20565541/067148_1_5.0274888.pdf. URL: https://doi.org/10.1063/5.0274888.

热学

- 微热管设计和制造
  - 提出了一种基于 CE 和神经网络模型相结合的微热管设计和制造参数预测方法[6]
- 循环流化床锅炉工艺参数优化
  - 结合 CE、加权融合树模型和多目标树结构估计器的循环流化床锅炉工艺参数优化方法[7]

[6] 李勇 et al. "基于一维卷积残差网络的微热管几何结构及制造工艺参数预测". In: 中国科学: 技术科学 55.2 (2025), pp. 281–294, 张靖昊. "基于机器学习的微热管结构及工艺参数设计系统开发". 硕士学位论文. 华南理工大学, 2024.

[7] Yunpeng Ma et al. "Improve thermal efficiency and reduce NOx emission of circulating fluidized bed boiler based on multi-objective optimization framework". In: *Thermal Science and Engineering Progress* (2025), p. 103753. ISSN: 2451-9049. DOI: https://doi.org/10.1016/j.tsep.2025.103753. URL: https://www.sciencedirect.com/science/article/pii/S2451904925005438.

理论化学

- 变构效应研究
  - 变构效应配位点和激活点热力学耦合模型[8]

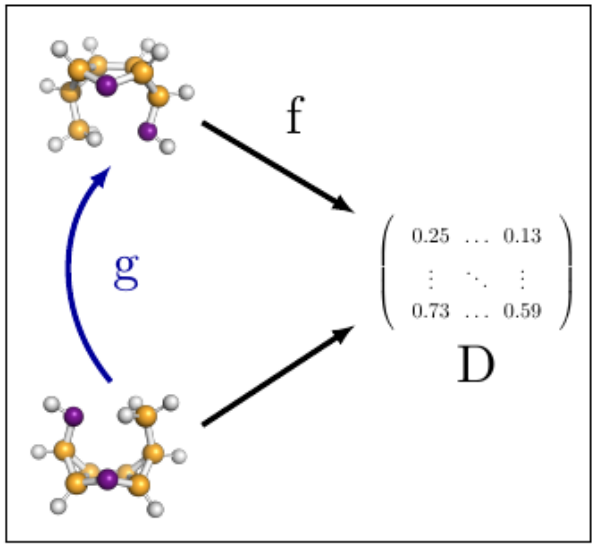
[8] Michel A. Cuendet, Harel Weinstein, and Michael V. LeVine. "The Allostery Landscape: Quantifying Thermodynamic Couplings in Biomolecular Systems". In: *Journal of Chemical Theory and Computation* 12.12 (2016), pp. 5758–5767. ISSN: 1549-9618. DOI: 10.1021/acs.jctc.6b00841. URL: https://doi.org/10.1021/acs.jctc.6b00841.

化学信息学

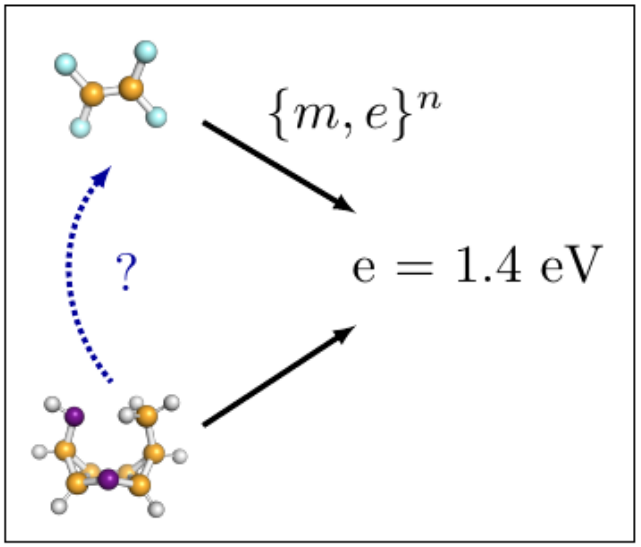
- 分子设计
  - 设计具有特定属性的分子结构[9]

[9] Mario Wieser. "Learning Invariant Representations for Deep Latent Variable Models". PhD thesis. University of Basel, 2020, Mario Wieser et al. "Inverse Learning of Symmetries". In: *Advances in Neural Information Processing Systems*. Vol. 33. 2020, pp. 18004–18015.

CASE 3: Symmetry of molecules by Harvard and Basel



(a) Rotational symmetry transformation.



(b) Unknown symmetry transformation.

Figure 1: **Left:** a molecule is rotated by g admitting an analytical form. The distance matrix D between atoms is calculated by a known function f and remains unchanged for all rotations. **Right:** n samples $\{m, e\}^n$ where m is the molecule and e the bandgap energy. These samples approximate the function f whereas the class of functions g leading to the same bandgap energy is unknown.

材料学

- 耐热型含能材料分子设计
  - 含能材料热分解温度预测[10]
- 近 β 钛合金加工
  - 研究了近 β 钛合金在 β 区形变机制和材质演化之间的关系[11]
- 难熔多元合金分子设计
  - 提出将 EMTO 第一性原理计算与基于 CE 的特征选择方法相结合，构建 RMPEAs 物理属性预测模型[12]

[10] 田杰. "基于机器学习的耐热型含能材料设计方法研究". 硕士学位论文. 西南科技大学, 2023.

[11] Song Zhang et al. "Correlation between plastic deformation mechanism and texture evolution of a near β-Ti alloy deformed in β region". In: *Intermetallics* 170 (2024), p. 108333. ISSN: 0966-9795. DOI: https://doi.org/10.1016/j.intermet.2024.108333. URL: https://www.sciencedirect.com/science/article/pii/S0966979524001523.

[12] Chengchen Jin et al. "High-Throughput Calculation and Machine Learning-Assisted Prediction of the Mechanical Properties of Refractory Multi-Principal Element Alloys". In: *Advanced Theory and Simulations* (2025), e00784.

环境学

- 大气污染传播路径分析
  - 兰州市大气污染传播路径预测[13]
- 火电厂排放污染物管控
  - 火电厂氮氧化物排放浓度预测[14]
- 氨气污染
  - 长三角地区氨气浓度长期变化及其驱动因素[15]
- SO_2 排放浓度预测
  - 提出了一种 SO_2 提前预测的软测量技术[16]

[13] 吴京鹏. "基于图嵌入表示的节点无特征网络链路预测研究". 硕士学位论文. 西北师范大学, 2022.

[14] 金秀章, 乔鹏, and 史德金. "基于 VMD-Bayes-Lasso 算法带误差补偿的火电厂 NOx 浓度软测量". In: 华北电力大学学报 (自然科学版) 52.235 (2023), pp. 117–124+142.

[15] Jingkai Xue et al. "Long-term spatiotemporal variations of ammonia in the Yangtze River Delta region of China and its driving factors". In: *Journal of Environmental Sciences* 150 (2025), pp. 202–217. ISSN: 1001-0742. DOI: https://doi.org/10.1016/j.jes.2024.02.021. URL: https://www.sciencedirect.com/science/article/pii/S1001074224001153.

[16] 乔鹏. "基于 CatBoost-Bayes 带误差补偿的脱硫出口SO_2 浓度软测量研究". 硕士学位论文. 华北电力大学, 2024.

地质学

- 岩相分类
  - 利用 CE 构建岩相分类模型[17]

[17] Jian Ma. "Facies Classification with Copula Entropy". In: *arXiv preprint arXiv:2501.14351* (2025).

地球物理学

- 土地干旱度分类
  - 考虑蒸散过程的土地干旱度分类[18]
- 北欧山地陆气耦合分析
  - 利用 CE 分析了芬瑟两个观测站的气象观测变量之间 CE 和解耦度量之间的关系[19]

[18] Yufeng Shan, Jiangfeng Wei, and Beilei Zan. "Improving Estimates of Land–Atmosphere Coupling Through a Novel Framework of Land Aridity Classification". In: *Geophysical Research Letters* 51.2 (2024), e2023GL106598. DOI: 10.1029/2023GL106598.

[19] Laura Mack, Marvin Kähnert, and Norbert Pirk. "Probabilistic modelling of atmosphere-surface coupling with a copula Bayesian network". In: *Environmental Modelling & Software* (2026). arXiv: 2509.11975 [physics.ao-ph]. URL: https://arxiv.org/abs/2509.11975.

CASE 4: Land-atmosphere coupling by University of Oslo

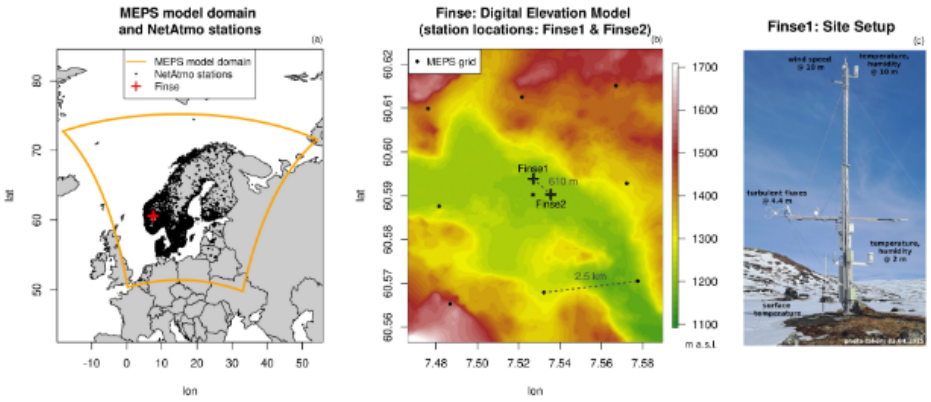


Figure 1: Used data sources: (a) Model domain of the NWP model MEPS and locations of the NetAtmo stations. (b) Surrounding of Finse and station locations. (c) Setup of the main site Finse1.

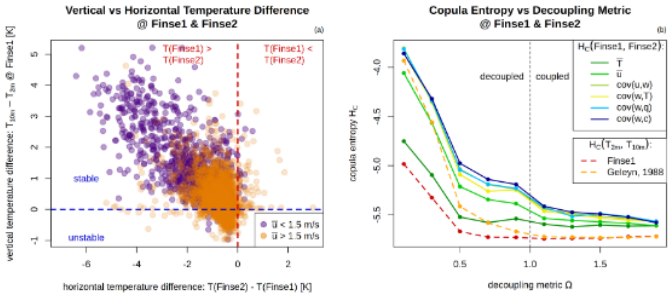


Figure 2: Relation of vertical and horizontal coupling: (a) Horizontal temperature difference between Finse1 and Finse2 versus vertical temperature difference at Finse1 color-coded based on a wind speed threshold (30 minutes temporal average), (b) decoupling metric at Finse1 versus copula entropy between Finse1 and Finse2 of temperature, wind speed, and the covariances corresponding to the fluxes of momentum cov(u,w) sensible heat cov(w,T), latent heat cov(q,w) and CO_2 cov(c,w) (30 minutes temporal average). Entropy calculation is based on time series of the same length to avoid a sampling size bias.

水文学 1

- 洪水预报
  - 金沙江流域洪水预报[20]
  - 天山北麓奎屯河洪水分析[21]
- 河流相关性
  - 长江上游河段（金沙江、岷江、沱江、嘉陵江）相关性[22]
  - 长江上游河段复合洪水事件分析[23]
- 水沙关系分析
  - 黄河西柳沟河流域径流量和输沙量数据分析[24]

[20] Lu Chen, Vijay P. Singh, and Shenglian Guo. "Measure of Correlation between River Flows Using the Copula-Entropy Method". In: *Journal of Hydrologic Engineering* 18.12 (2013), pp. 1591–1606, Lu Chen et al. "Copula entropy coupled with artificial neural network for rainfall–runoff simulation". In: *Stochastic Environmental Research and Risk Assessment* 28.7 (2014), pp. 1755–1767, 陈佳雷 et al. "一种基于多要素注意力时空图卷积网络的径流预报方法". Pat. CN117151285A. CN117151285A. 2023.

[21] Yingying Han et al. "Non-Stationary Flood Characteristics and Joint Risk Analysis in Inland China with Uncertainty Considerations". In: *Atmosphere* 17.3 (2026), p. 281.

[22] Lu Chen and Shenglian Guo. *Copulas and its Application in Hydrology and Water Resources*. Springer Singapore, 2018.

[23] Xu Wang and Yong-Ming Shen. "A Framework of Dependence Modeling and Evaluation System for Compound Flood Events". In: *Water Resources Research* 59.8 (2023), e2023WR034718. DOI: https://doi.org/10.1029/2023WR034718.

[24] Longxia Qian et al. "A New Estimation Method for Copula Parameters for Multivariate Hydrological Frequency Analysis With Small Sample Sizes". In: *Water Resources Management* 36.4 (2022), pp. 1141–1157. ISSN: 1573-1650. URL: https://doi.org/10.1007/s11269-021-03016-w.

水文学 2

- 干旱研究
  - 黄河流域干旱分析和预测[25]
  - 黄河流域干旱事件识别[26]
  - 印度达布蒂（Tapti）河流域干旱指数设计[27]
  - 伊朗城市（扎黑丹、恩泽利和马什哈德）干旱数据分析[28]
  - 美国科罗拉多河流域干旱传播分析[29]
  - 黑龙江讷谟尔河流域干旱形成和演化分析[30]
  - 巴基斯坦旁遮普省干旱分析[31]

[25] 温云亮 et al. "基于 Copula 熵理论的干旱驱动因子选择". In: 华北水利水电大学学报 (自然科学版) 40.4 (2019), pp. 51–56, C.Y. Huang and Y.P. Zhang. "Prediction based on Copula Entropy and General Regression Neural Network". In: *Applied Ecology and Environmental Research* 17.6 (2019), pp. 14415–14424, 黄春艳. "黄河流域的干旱驱动及评估预测研究". 博士学位论文. 西安理工大学, 2021, 牛犇. "黄河流域气象-农业-水文干旱的时空传递特征". 硕士学位论文. 西北农林科技大学, 2023.

[26] Lingling Ni et al. "Vine copula selection using mutual information for hydrological dependence modeling". In: *Environmental Research* 186 (2020), p. 109604.

[27] P. Kanthavel, C.K. Saxena, and R.K. Singh. "Integrated Drought Index based on Vine Copula Modelling". In: *International Journal of Climatology* 42.16 (2022), pp. 9510–9529. DOI: https://doi.org/10.1002/joc.7840. eprint: https://rmets.onlinelibrary.wiley.com/doi/pdf/10.1002/joc.7840. URL: https://rmets.onlinelibrary.wiley.com/doi/abs/10.1002/joc.7840.

[28] Morteza Mohammadi, Mahdi Emadi, and Mohammad Amini. "Bivariate Dependency Analysis using Jeffrey and Hellinger Divergence Measures based on Copula Density Estimation by Improved Probit Transformation". In: *Journal of Statistical Sciences* 15.1 (2021), pp. 233–254. DOI: 10.29252/jss.15.1.233.

[29] 徐袁. "基于 GAMLSS 框架下非平稳气象干旱和水文干旱指数的构建及干旱传播分析". 硕士学位论文. 东北农业大学, 2025.

[30] 刘明阳. "讷谟尔河流域干旱形成及演化机制研究". 硕士学位论文. 东北农业大学, 2024.

[31] Muhammad Owais Khan et al. "Assessing drought dynamics across climatic zones using parametric and entropy copula models". In: *Stochastic Environmental Research and Risk Assessment* 40.8 (July 2026), p. 191. ISSN: 1436-3259. URL: https://doi.org/10.1007/s00477-026-03313-z.

CASE 5: Drought in Pakistan by Quaid-i-Azam, PKU, etc.

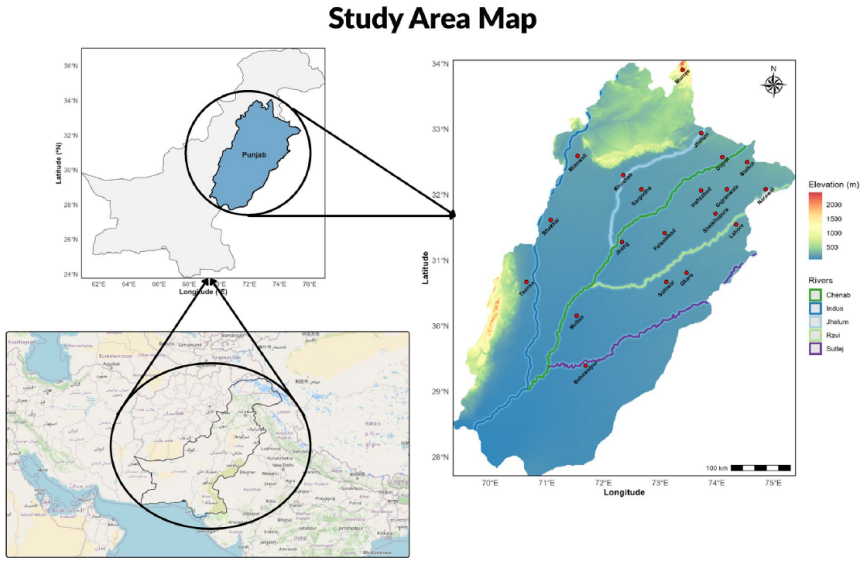


Fig. 1 Study area of 20 meteorological stations Punjab: Pakistan

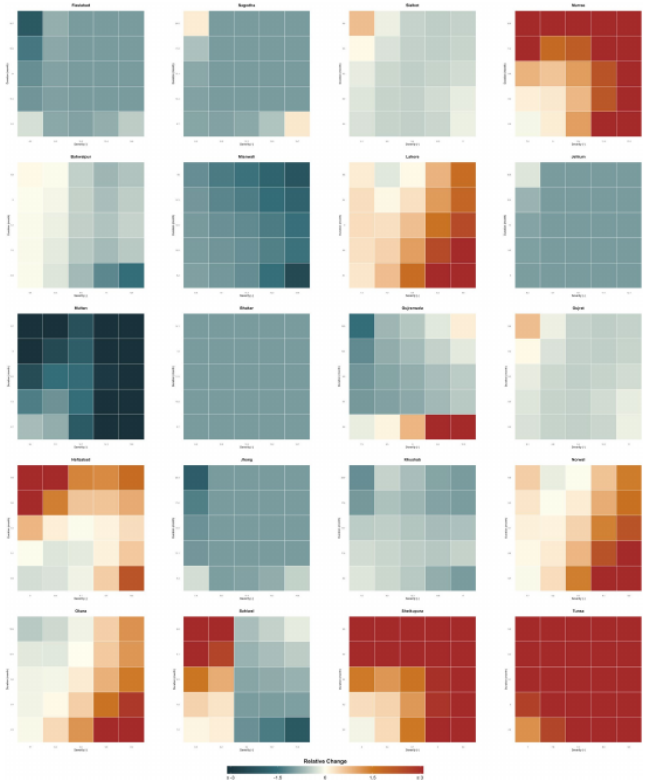


Fig. 6 Tile plot heatmaps of relative change in joint return period (ΔT_r) between the Reference (1981–2000) and Current (2001–2020) periods from the entropy-based copula. Brown indicates positive change (more frequent droughts), and teal indicates negative change (less frequent droughts)

水文学 3

- 中长期径流预报
  - 南水北调工程丹江口水库入库径流预报[32]
  - 洪泽湖和骆马湖长期径流预报[33]
  - 长江上游水文站月径流预测[34]
- 流域分区[35]
  - 鄱阳湖流域
- 多站点径流生成
  - 巴西雅瓜拉比-大都市水库系统[36]
- 河冰管理
  - 黑龙江[37]

[32] 黄朝君 et al. "基于 Copula 熵-随机森林的中长期径流预报研究". In: 人民长江 52.11 (2021), pp. 81–85.

[33] Ran Mo et al. "Long-term probabilistic streamflow forecast model with "inputs–structure–parameters" hierarchical optimization framework". In: *Journal of Hydrology* 622 (2023), p. 129736. DOI: https://doi.org/10.1016/j.jhydrol.2023.129736. URL: https://www.sciencedirect.com/science/article/pii/S0022169423006789.

[34] Xiao Li et al. "Predicting Monthly Runoff of the Upper Yangtze River Based on Multiple Machine Learning Models". In: *Sustainability* 14.18 (2022), p. 11149. ISSN: 2071-1050. DOI: 10.3390/su141811149. URL: https://www.mdpi.com/2071-1050/14/18/11149.

[35] 刘磊 et al. "基于非线性相关性和复杂网络的径流相似性分区". In: 水科学进展 33.3 (2022), pp. 442–451.

[36] Victor Costa Porto et al. "A GLM copula approach for multisite annual streamflow generation". In: *Journal of Hydrology* 598 (2021), p. 126226.

[37] Ruofei Xing et al. "Temporal variation in river ice phenology of the Heilongjiang River in response to climate change". In: *Journal of Hydrology: Regional Studies* 54 (2024), p. 101868. ISSN: 2214-5818. DOI: https://doi.org/10.1016/j.ejrh.2024.101868. URL: https://www.sciencedirect.com/science/article/pii/S2214581824002167.

CASE 6: South-to-North Water Diversion Project

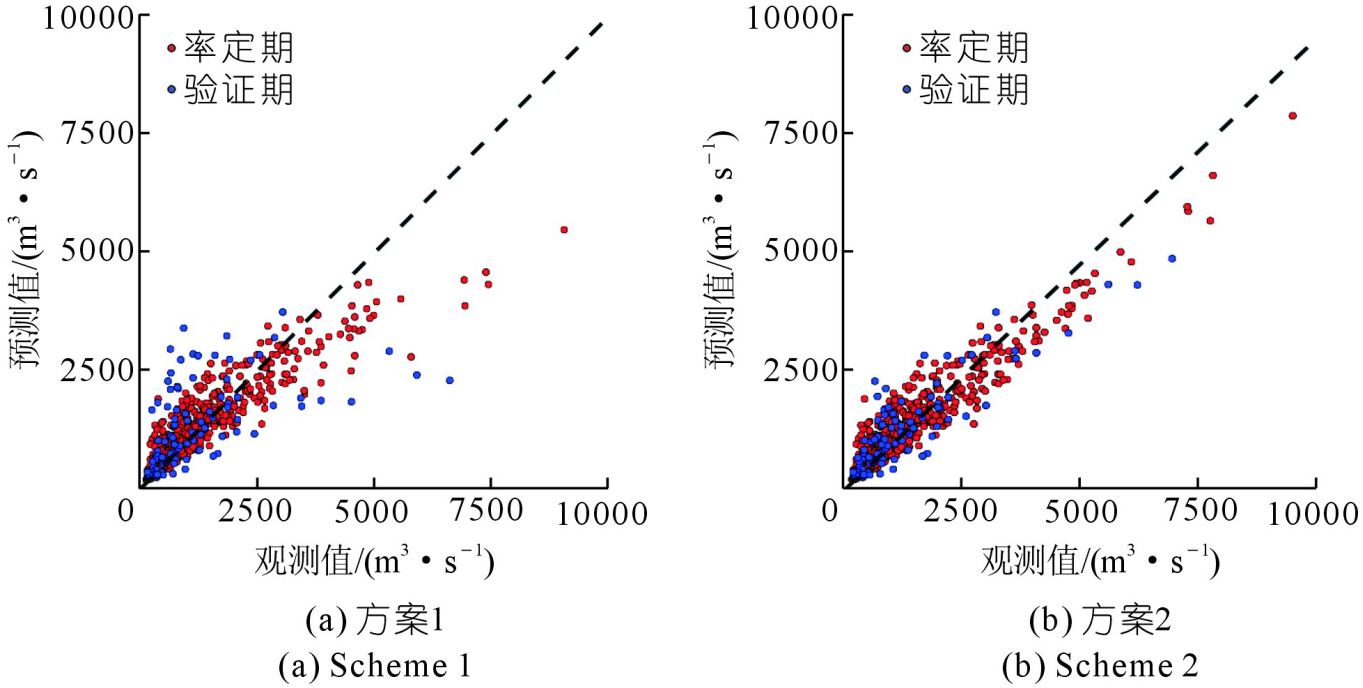


Figure: Comparison between observations and predictions by correlation coefficients and copula entropy on monthly inflow in Danjiangkou reservoir.

水文学 4

- 水文观测网络选址和优化
  - 上海雨量观测网[38]
  - 伊洛河流域水文观测网[39]
  - 北京市区水文观测网；汾河流域观测网；太湖盆地流域雨量观测网[40]
  - 淮河流域雨量观测网[41]
  - 美国查克托哈奇（Choctawhatchee）河流域水文观测网[42]
- 气候变化和人类活动的水文系统响应[43]

[38] Pengcheng Xu et al. "A two-phase copula entropy-based multiobjective optimization approach to hydrometeorological gauge network design". In: *Journal of Hydrology* 555 (2017), pp. 228–241.

[39] 王栋 et al. "一种基于 Copula 熵的水文站网优化模型的优化方法". Pat. CN106897530B. CN106897530B. 2019.

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[41] 徐鹏程 et al. "基于 C-Vine Copula 熵多目标优化模型的水文气象站网优化研究". In: 中国农村水利水电 2 (2022), pp. 16–21. DOI: 10.12396/znsd.220702.

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气候学

- 气候变化对水文气象变量相关性的影响
  - 美国德州达拉斯地区降水和气温相关性关系分析[44]
- 气候评估
  - 欧洲城市气候分类[45]

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气象学

- 大气污染气象成因分析
  - 北京地区气象因素对 PM2.5 浓度的因果关系分析[46]
  - PM2.5 浓度预测[47]
  - 上海和广州大气污染预测预警[48]
- 气象灾害预测
  - 广西地区台风灾情预测类[49]

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生态学

- 动物运动轨迹分析
  - Cylcop 算法包[50]
- 土壤生态学
  - 利用微生物-酶分解模型研究了土壤湿度变化对土壤碳氮动态性的长期影响[51]
- 社会生态学
  - 利用基于 CE 的 TE 和网络分析等工具研究西南喀斯特地区社会生态系统脆弱性演化[52]

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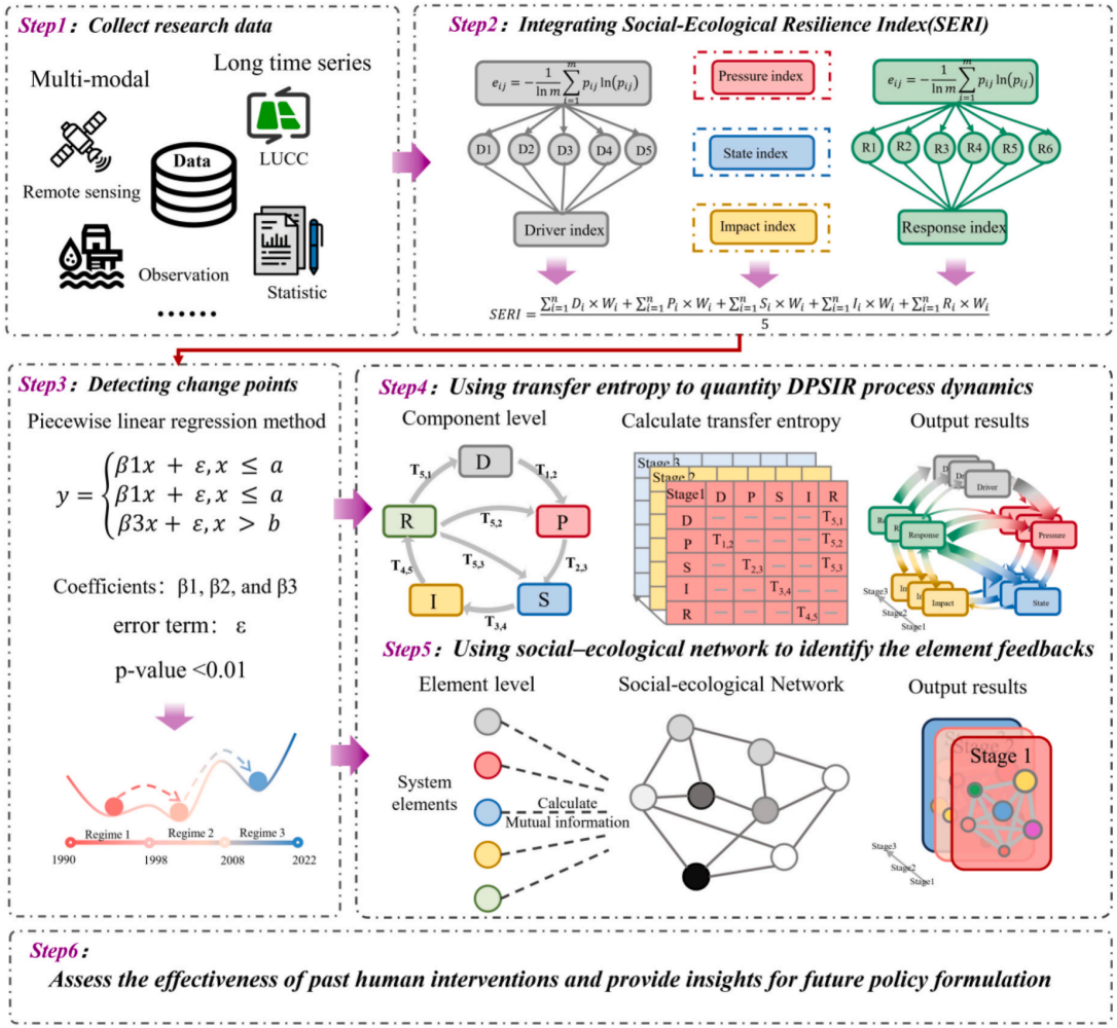
CASE 7: Social-ecological resilience in Karst region by CAS



Research methodology with copula entropy based transfer entropy

动物学

- 动物形态学
  - 鱼类形态相似度研究[53]
  - 鲍鱼生长过程的形态学研究[54]

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植物学

- 树干液流预测
  - 提出了一种基于历史环境因子预测树干液流的方法[55]

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农学

- 气候变化对我国南方两季稻产量的影响及对策[56]
- 南方水稻产量预测[57]
- 南极磷虾分布预测[58]

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免疫学

- 淋巴细胞间通信机制研究
  - 提出利用 TE 度量来分析小鼠 B 细胞活体钙离子成像[59]

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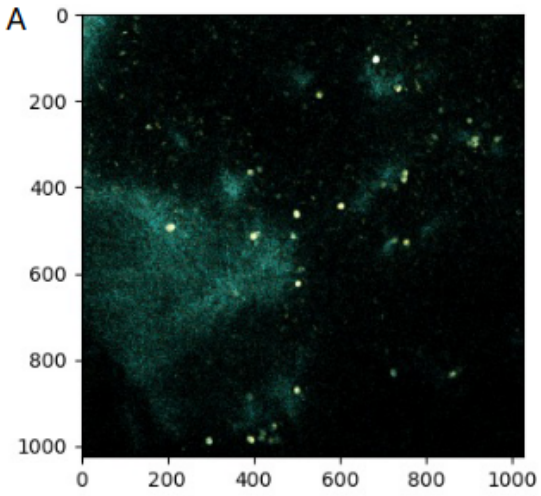
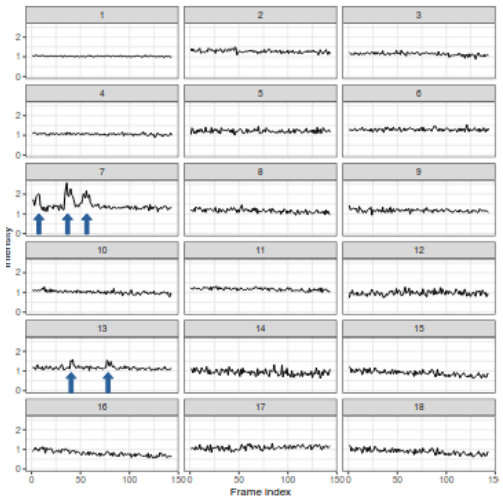
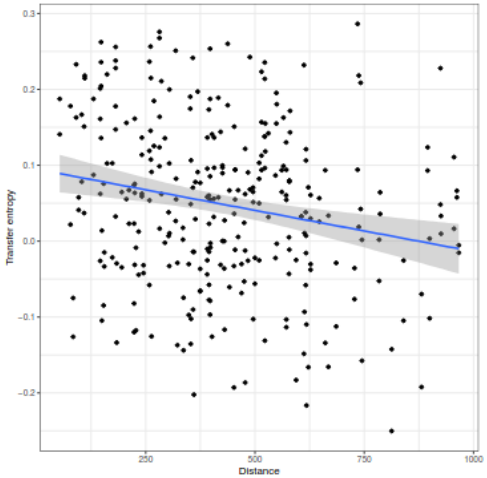
CASE 8: Intracellular calcium signals by Science Tokyo



Intravital Ca^{2+} imaging of lymphocytes



Time series of intracellular Ca^{2+} signals



Scattering plot of estimated transfer entropy

神经科学 1

- 神经病学
  - 注意缺陷多动障碍[60]
  - 癫痫症[61]
  - 脑卒中[62]

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CASE 9: Information transfer within brain by KAUST

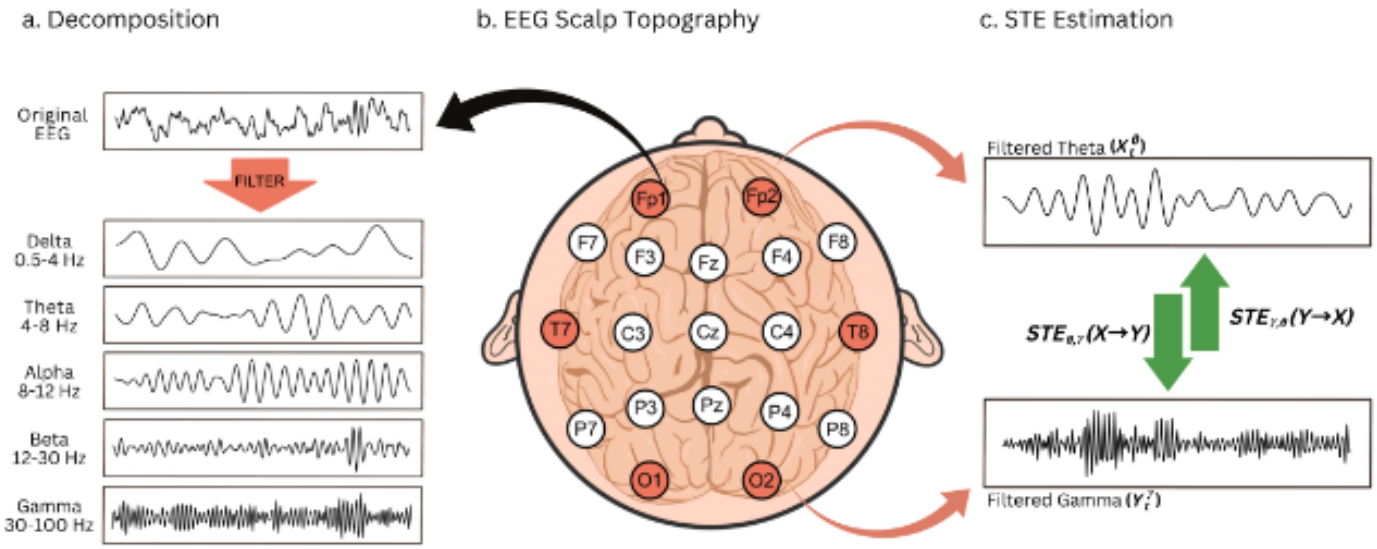

FIG 9. *Building blocks of STE. (**a**) Application of bandpass filter on original EEG series to obtain frequency band-specific filtered series. (**b**) Standard 10–20 EEG scalp topography where the highlighted channels are included in the analysis. (**c**) Illustration of STE estimation between two filtered series.*

神经科学 2

- 认知神经学
  - 分析大脑认知活动的多模态数据[63]
  - 语音信息的编码和解析[64]
  - 神经元特异化识别[65]

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神经科学 3

- 运动神经学
  - 分析运动的肌肉组合协同策略[66]
- 计算神经学
  - 神经元可塑性建模[67]
  - 神经网络信息传输关系分析[68]

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心理学

- 生物心理学
  - 情绪刺激下心跳诱发脑电位的时间交互现象[69]

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生物学

- 系统生物学
  - 生物信号调控和传导[70]
  - 生物现象动态网络结构和功能[71]
- 生物信息学
  - 分析基因数据，研究生命和疾病机理[72]
  - 筛选与癌症有关的变异基因[73]
  - 单细胞测序基因调控网络构建[74]

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医学 1

- 临床医学 1
  - 心脏病诊断[75]
  - 糖尿病病情管理[76]
  - 癌症预后[77]

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医学 2

- 临床医学 2
  - 白内障术后角膜水肿风险预测[78]
  - 主动脉瓣置换手术射血分数分析[79]
  - 脑肿瘤医学影像组学诊断模型构建[80]
  - 基于脉搏波的高血压和糖尿病健康状态分类[81]

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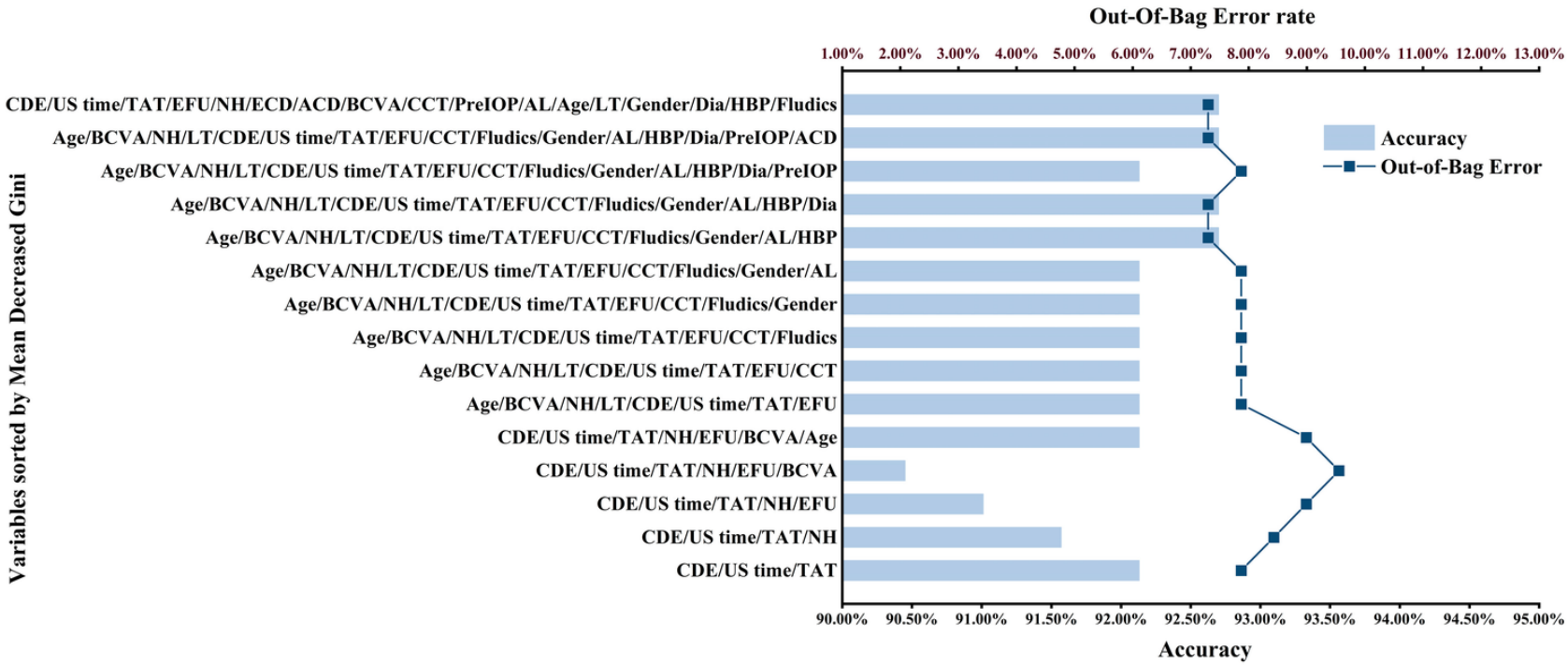
CASE 10: Corneal edema prediction by PLA 301 Hospital



The predictive accuracy and out-of-bag error rate of each random forest model generated after selecting variables on the basis of copula entropy

医学 3

- 认知医学
  - 认知能力评估 / 痴呆症筛查[82]
- 运动医学
  - 运动能力评估 / 跌倒风险预测[83]
  - 重复经颅磁刺激对帕金森病改善神经机制分析[84]
- 精神病学
  - 抑郁症患者识别[85]

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医学 4

- 法医学
  - 基于 DNA 分析推断人身高[86]

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药学

- 药物-靶点相互作用预测
  - 提出了一种交互式多特征融合的 DTIs 预测算法[87]

[87] 高浩田 et al. "基于交互式多特征融合算法的药物靶标预测". In: 太原理工大学学报 55.4 (2024), pp. 751–758.

公共卫生学

- 新冠肺炎流行病 (COVID19)
  - 发热症状疑似病人筛查诊断[88]
- 高血压
  - 高血压关联基因研究[89]

[88] Radko Mesiar and Ayyub Sheikhi. "Nonlinear Random Forest Classification, a Copula-Based Approach". In: *Applied Sciences* 11.15 (2021), p. 15. ISSN: 2076-3417. DOI: 10.3390/app11157140. URL: https://www.mdpi.com/2076-3417/11/15/7140.

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社会科学 1

- 经济学
  - 扶贫政策效果评估，用于政策目标人口鉴别[90]
  - 议价机制中的互惠行为和时间效应[91]
  - 产业链内部相关性分析[92]
  - 投资者情绪分析[93]
  - 通胀与预期关系分析[94]
  - 马来西亚宏观经济分析[95]

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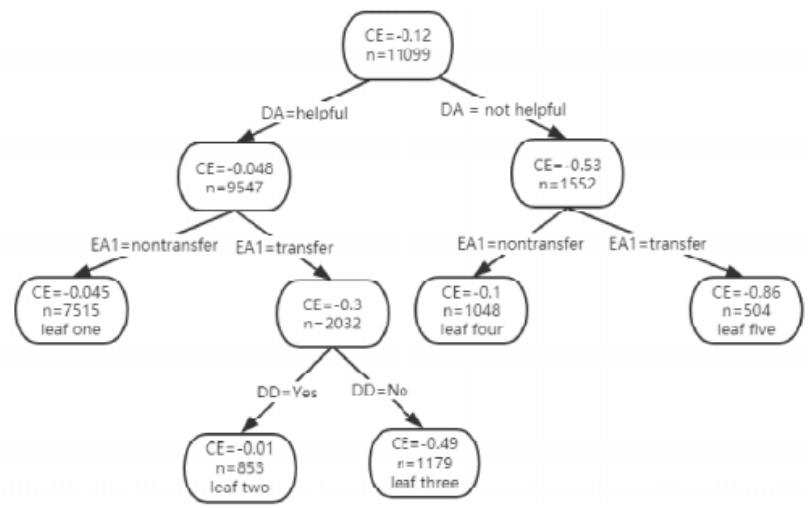
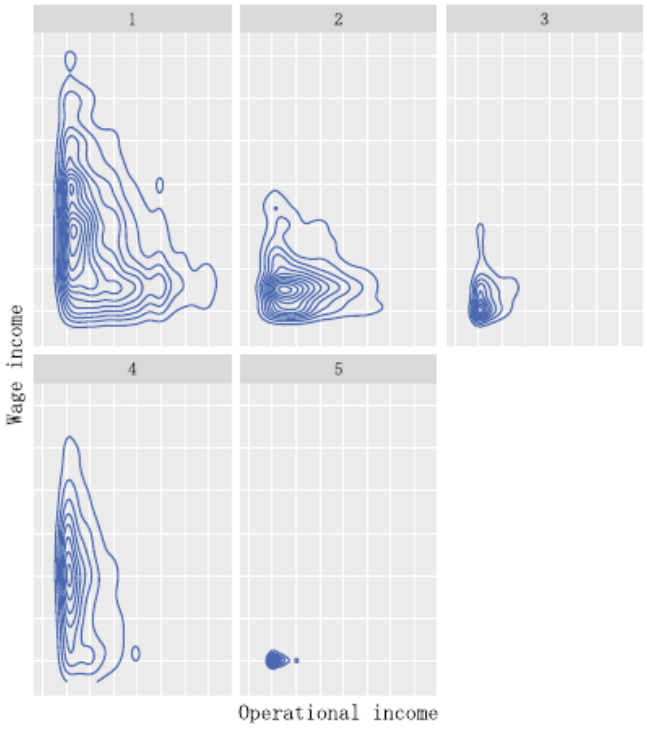
CASE 11: Poverty policy evaluation by JXUFE



Binary decision tree by copula entropy



Joint density of two incomes of the families in each leaf

社会科学 2

- 管理学
  - 商品期货价格预测[96]
  - 单周期库存管理[97]
  - 巴西糖、乙醇和汽油三种大宗商品价格分析[98]
  - 中国企业海外并购影响因素分析[99]
- 社会学
  - 分析教育、职业和收入上的性别不平等问题[100]

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CASE 12: Commodity market analysis by UNESP

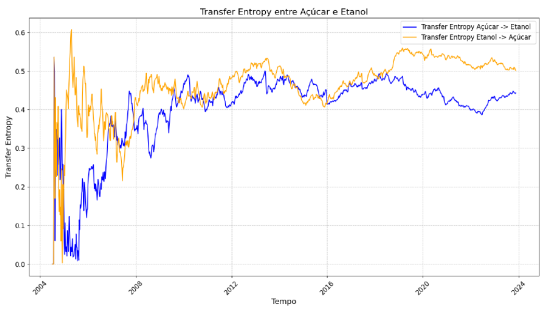

Figura 5.0.5: Transfer Entropy entre as séries da volatilidade de preços do açúcar e etanol.

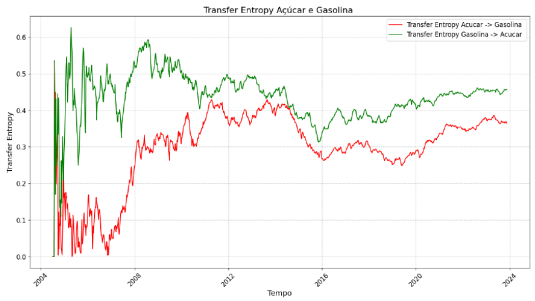

Figura 5.0.6: Transfer Entropy entre as séries da volatilidade de preços do açúcar e gasolina.

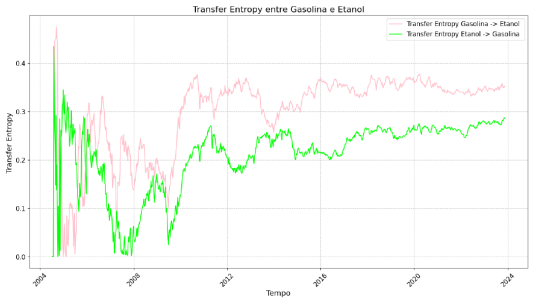

Figura 5.0.7: Transfer Entropy entre as séries da volatilidade de preços de etanol e gasolina.

社会科学 3

- 教育学
  - 高中数学成绩与其他学科成绩相关性分析[101]
- 计算语言学
  - 城市热线派单系统知识图谱构建[102]
  - 大模型词嵌入质量评估[103]
- 新闻传播学
  - 上海新冠疫情下的公众情绪变化[104]

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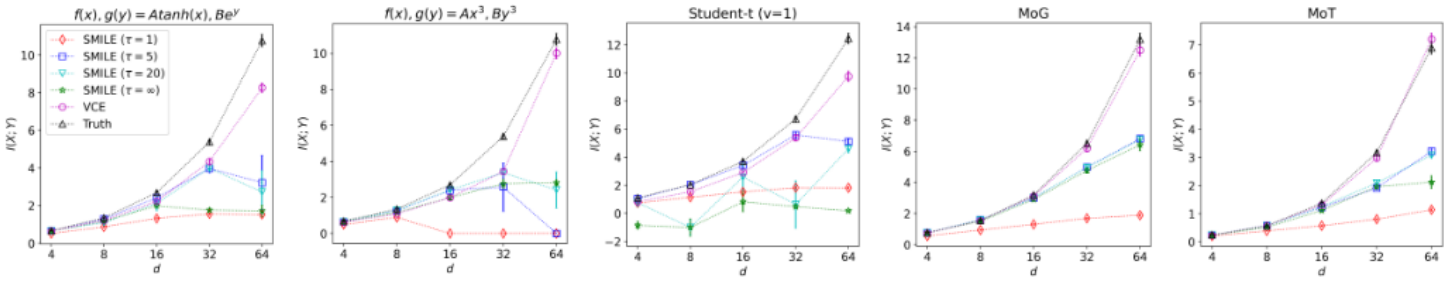
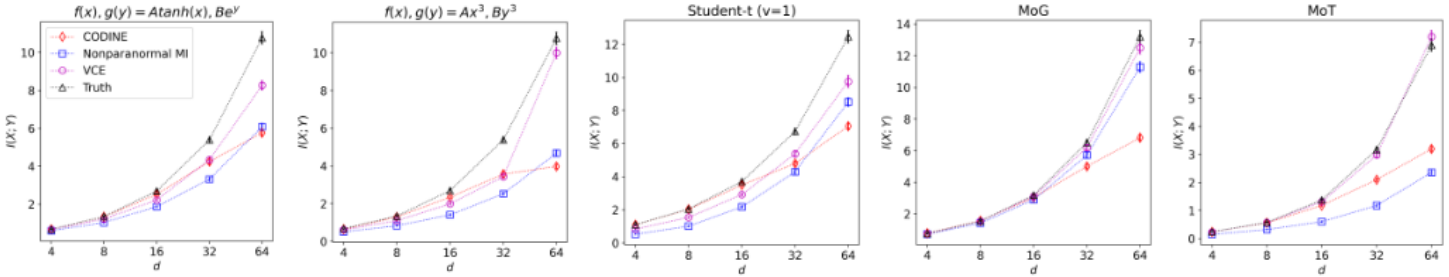
CASE 13: LLMs evaluation by University of Cambridge



(a) $\mathbf{A}\tanh(X), \mathbf{B}e^Y$ (b) $\mathbf{A}X^3, \mathbf{B}Y^3$ (c) Student-t (d) MoG (e) MoT

Figure 8: Comparison with the SMILE estimator under different clipping values τ.



(a) $\mathbf{A}\tanh(X), \mathbf{B}e^Y$ (b) $\mathbf{A}X^3, \mathbf{B}Y^3$ (c) Student-t (d) MoG (e) MoT

Figure 9: Comparing VCE with classic copula-based estimators e.g. nonparanormal MI (which uses a Gaussian copula to estimate MI) and CODINE (equivalent to classic copula transformation + MINE).

社会科学 4

- 法学
  - 社区属性与社区犯罪关系分析[105]
- 政治学
  - 分析政权领导力因素和政权危机之间关系[106]
- 军事学
  - 目标意图识别[107]
- 情报学
  - 颠覆性技术科学-技术-产业互动模式分析[108]
  - 高价值专利评估和发现[109]

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工学 1

- 能源工程 1
  - 能源网络管理，研究天气因素与能源网络的耦合[110]
  - 光伏发电功率预测[111]
  - 风电机组工况划分[112]
  - 电力负荷预测[113]
  - 风光储协同规划[114]

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工学 2

- 能源工程 2
  - 电网频率稳定性预测[115]
  - 用户线损贡献分析[116]
  - 配电网拓扑辨识[117]
  - 电价预测[118]
  - 电力系统宽频振荡影响因素和传播路径分析[119]
  - 电力系统信息物理融合[120]

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CASE 14: Cyber-physical integration by State Grid

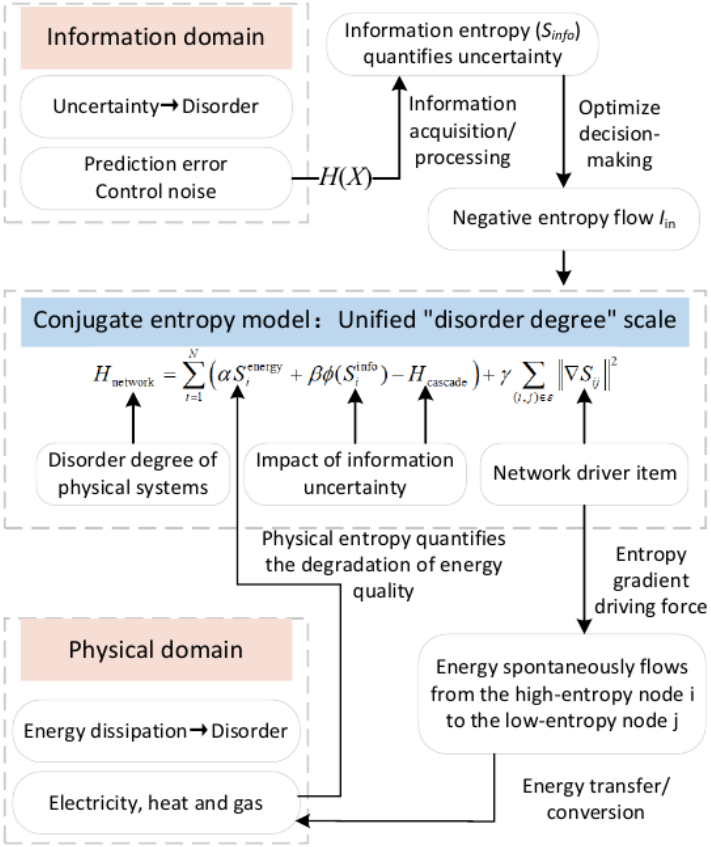

Fig. 1. The core concept diagram of the conjugate entropy theory.

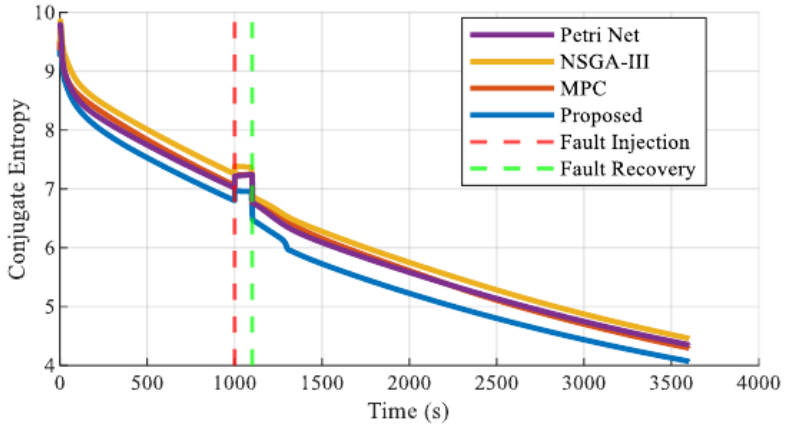

Fig. 7. Comparison of conjugate entropy.

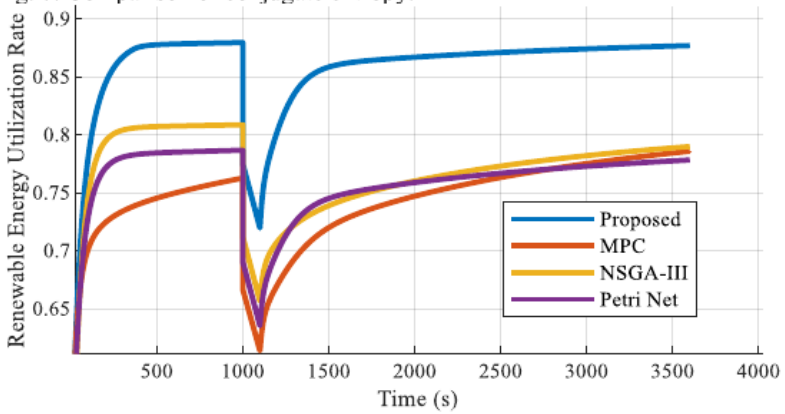

Fig. 8. Comparison of renewable energy utilization rate.

工学 3

- 能源工程 3
  - 锂电池容量估计[121]
  - 能效异常原因诊断[122]

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工学 4

- 纺织工程
  - 纱布生产过程质量控制[123]
  - 可穿戴织物传感器方案设计[124]
- 食品工程
  - 葡萄酒质量与理化成分关系分析[125]

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CASE 15: Wine quality by Sorbonne Université

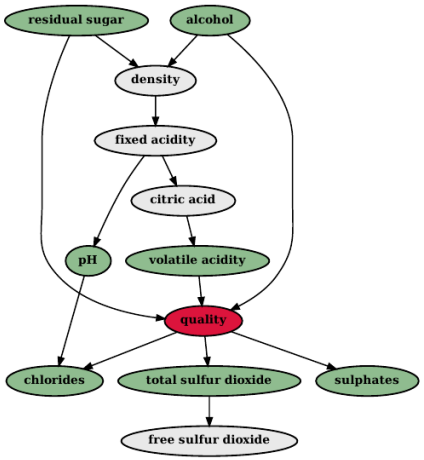

FIGURE 8.10 – Structure reconstruite avec l'algorithme B-CMIIC à partir de l'échantillon des vins rouges. Le paramètre α a été fixé à 0.04 en utilisant la méthode de *cross-validation*. La variable colorée en rouge est la variable cible tandis que l'ensemble des variables colorées en vert correspond à sa couverture de Markov.

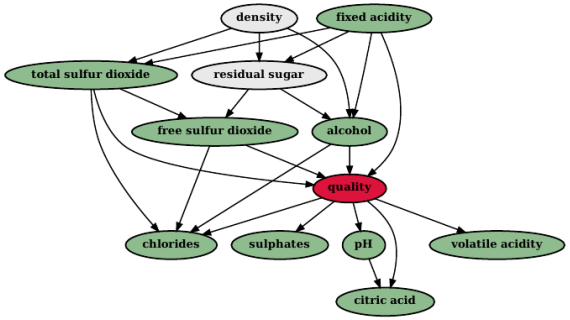

FIGURE 8.11 – Structure reconstruite avec l'algorithme B-CMIIC pour l'échantillon des vins blancs. Le paramètre α est fixé à 0.02. La variable colorée en rouge est la variable cible tandis que l'ensemble des variables colorées en vert correspond à sa couverture de Markov.

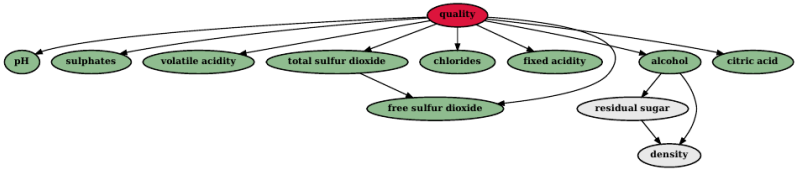

FIGURE 8.12 – Structure reconstruite avec l'algorithme B-CMIIC pour l'échantillon des vins blancs. Le paramètre α est fixé à 0.06. La variable colorée en rouge est la variable cible tandis que l'ensemble des variables colorées en vert correspond à sa couverture de Markov.

工学 5

- 土木建筑
  - 建筑能源系统节能技术[126]
  - 工程变形监测[127]
  - 钢筋混凝土梁地震性能等级预测[128]
  - 受剪承载力预测[129]
  - 医院建筑系统脆弱性评估[130]

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工学 6

- 交通运输
  - 大件货物运输方案制定[131]
  - 航空和高铁票价影响因素分析[132]
  - 城市轨道交通客流分析和预测[133]
  - 铁路客流量预测[134]

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工学 7

- 制造工程
  - 制造质量管理，研究优化制造过程参数，预测产品质量[135]
  - 装配质量控制[136]
  - 工业过程故障监测[137]
  - 钢铁工艺过程碳排放预测[138]
  - 液晶显示器质量预测[139]
  - 航空发动机涡轮盘制造工艺优化[140]

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工学 8

- 可靠性工程
  - 系统退化过程建模[141]
  - 风电机组健康状态评估[142]
  - 砂轮剩余寿命预测[143]
  - 滚动轴承剩余使用寿命预测[144]

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- 石油工程
  - 煤层气井产量预测[145]
  - 陆地原油生产碳排放分析[146]
- 矿业工程
  - 煤-岩识别[147]
  - 重介质选煤[148]

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工学 10

- 医学工程
  - 脑机接口[149]
- 冶金工程
  - 真空蒸馏法制备高纯金属工艺参数优化[150]

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工学 11

- 化学工程
  - 化学过程故障监测和诊断[151]
  - 化工过程因果网络构建[152]
  - 化工过程缺失数据补全[153]
  - 生物发酵过程软测量[154]

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工学 12

- 核工程
  - 核反应堆传感器故障与事故预防[155]

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工学 13

- 航空航天
  - 飞行器总体参数分析和优化[156]
  - 飞行器操纵品质评估[157]
  - 卫星在轨健康状态监测[158]
  - 涡扇发动机健康状态监测[159]
  - 机场间航班延误因果关系分析[160]
  - 航班到达时间预测[161]

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工学 14

- 兵器工程
  - 武器效能评估指标体系约简[162]
- 车辆工程
  - CAN 总线入侵检测[163]
  - 高速动车组关键部件轴承健康监测[164]
  - 自动驾驶感知模块优化[165]
- 控制工程
  - 航站楼室内温度控制[166]

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工学 15

- 电子工程
  - 集成电路封装材料物理性能预测[167]
- 通信工程
  - 通讯网络加密技术研究[168]
  - 6G 网络语义通信技术研究[169]
  - 游戏系统编解码技术[170]
- 高性能计算
  - 高性能计算能源效率优化[171]
- 信息安全
  - 深度神经网络对抗攻击防御算法[172]

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CASE 16: Energy efficiency in HPC by TU Dresden

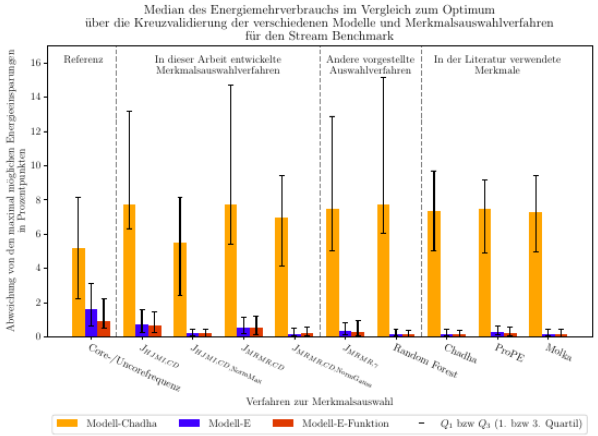

Abbildung 5.9: Median des Energiemehrverbrauchs bezogen auf die maximal möglichen Energieeinsparungen für Stream. Je geringer der Median, desto besser ist das neuronale Netz zusammen mit den Merkmalsauswahlverfahren. Der Median sowie die Quartile wurden über alle Kreuzvalidierungsversuche der jeweiligen Modelle gebildet. Bei dem Merkmalsauswahlverfahren Core-/Uncorefrequenz wurden zum Training des Modells neben der Core- und Uncorefrequenz keine weiteren Merkmale verwendet. Jedes Training mit Merkmalen sollte besser sein als diese Referenz. Die Verfahren Chadha, ProPE und Molka beziehen sich auf die in Arbeiten von Chadha und Gerndt (2019), an Mey et al. (2019) und Molka et al. (2017) ausgewählten Merkmale.

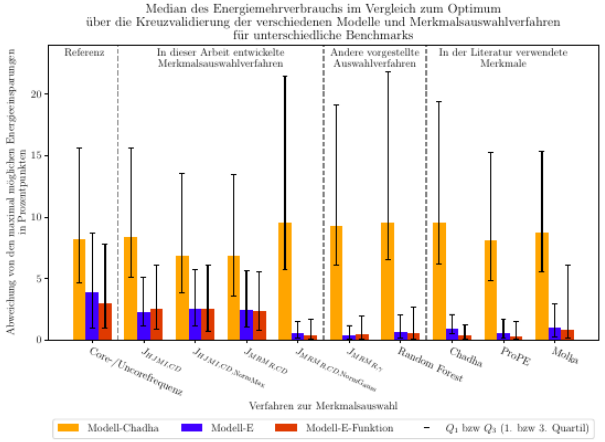

Abbildung 5.17: Median des Energiemehrverbrauchs bezogen auf die maximal möglichen Energieeinsparungen für verschiedene Benchmarks. Je geringer die Abweichung vom Optimum, also der Median, ist, desto besser ist das neuronale Netz zusammen mit den Merkmalsauswahlverfahren. Der Median sowie die Quartile wurden über alle Kreuzvalidierungsversuche der jeweiligen Modelle gebildet. Bei dem Merkmalsauswahlverfahren Core-/Uncorefrequenz wurden zum Training des Modells neben der Core- und Uncorefrequenz keine weiteren Merkmale verwendet. Jedes Training mit Merkmalen sollte besser sein als diese Referenz. Die Verfahren Chadha, ProPE und Molka verweisen auf die in Arbeiten von Chadha und Gerndt (2019), an Mey et al. (2019) und Molka et al. (2017) ausgewählten Merkmale.

工学 16

- 测绘遥感
  - 高光谱遥感数据分析[173]
  - 大型工程变形体内部监测[174]
- 海洋工程
  - 海洋底质信息探测[175]

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金融工程 1

- 投资组合优化
  - 股票资产相关性网络分析[176]
  - ST 股票分类[177]
- 金融问题建模
  - Copula 函数模型选择[178]
- 股票相关性建模
  - R-vine copula 结构建模[179]

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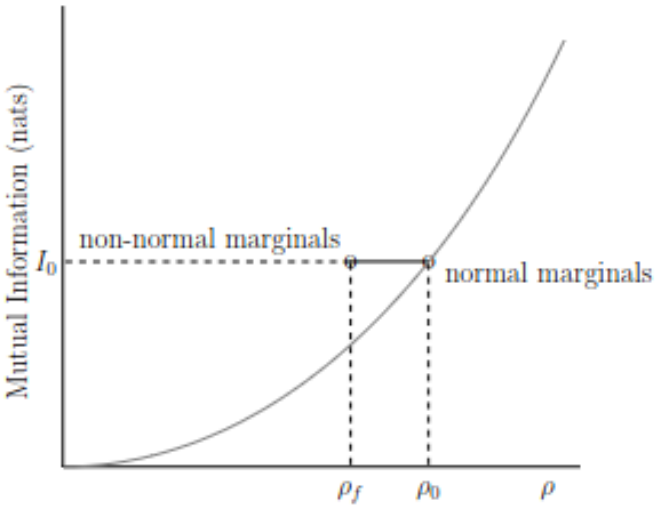
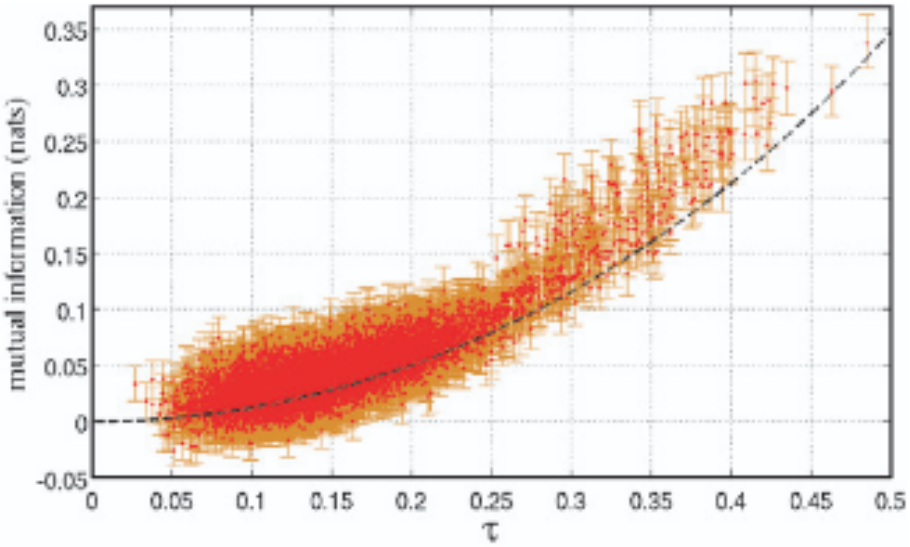
CASE 17: Copula modeling by Universidade de São Paulo





金融工程 2

- 量化金融工具箱 MLFinLab[180]
- 金融系统性风险
  - 行业风险溢出效应分析[181]
  - 金融脆弱性度量[182]
- 信用风险评价
  - 信用风险卡模型建立[183]
  - P2P 借贷个人信用风险模型[184]

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金融工程 3

- 金融产品价格预测
  - 基于因果关系的迁移学习价格预测模型构建[185]
- 疫情对金融市场的影响
  - 疫情对股票指数相关性的影响[186]
- 金融市场预测
  - 加密货币市场预测[187]

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Softwares

- **copent**[188]
  The **copent** package in R and Python for estimating copula entropy, transfer entropy, the statistic for multivaritate normality test, two-sample test, and change point detection are available on CRAN and PyPI respectively. The source codes are provided on GitHub.

  https://cran.r-project.org/package=copent

  https://pypi.org/project/copent/

  https://github.com/majianthu

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Softwares

Third-party

- R: `cylcop`[189] (by University of Zürich and University of Minnesota);
- Python: `MLFinLab`[190] and `ArbitrageLab`[191] (authored by Prof. Marcos López de Prado from Cornell University, Github 4500+ and 670+ stars);

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Softwares

Third-party

- Python: `gcmi`[192] (by Glasgow University, Radboud University Nijmegen, University of Amsterdam, Kyiv AI group, Github 100+ stars), `pytorch-mighty`[193] (by Kyiv AI group), `HOI`[194] and `Frites`[195] (by CNRS, INT, and Aix-Marseille Université), `Tensorpac`[196] (by Université de Montréal, INT, and Aix-Marseille Université), `driada`[197] (by Moscow State University);

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Softwares

Third-party

- Python: `CopulaGP`[198] (by University of Edinburgh), `Polars-ds`[199] (by New York University, 1.45+ million downloads, Github 650+ stars), `copul`[200] (by Albert-Ludwigs-Universität Freiburg), `rcopula`[201], and `pycopcor`[202] (by Technische Universität Dresden);

[198] Nina Kudryashova et al. "Parametric Copula-GP model for analyzing multidimensional neuronal and behavioral relationships". In: *PLoS computational biology* 18.1 (2022), e1009799, Nina Kudryashova. *Parametric Copula-GP framework*. GitHub. URL: https://github.com/NinelK/CopulaGP. 2022.

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[200] Marcus Rockel. *The copul package in Python*. Github. URL: https://github.com/Corrram/copul. 2026.

[201] Christopher Bell. *The rcopula package in Python*. GitHub. URL: https://github.com/Chrisebell24/python-copula. 2026.

[202] Andreas Gocht-Zech. *The pycopcor package in Python*. Github. URL: https://github.com/AndreasGocht/pycopcor. 2023.

Softwares

Third-party

- Julia: `CopEnt.jl`[203] (by University of Edinburgh), `CausalityTools.jl`[204] (by JuliaDynamics), and `Copulas.jl`[205] (by Aix-Marseille Université, Github 110+ stars);
- Matlab: `gcmi`[206] (by Glasgow University, Radboud University Nijmegen, University of Amsterdam, Kyiv AI group, Github 100+ stars), and `FieldTrip`[207] (by Radboud University Nijmegen);
- C++: `NPStat`[208] [209] (by Texas Tech University).

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Monograph

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Thanks!

Q&A