HW1-MajidDareini

November 25, 2023

1 Image Processing

1.1 Libraries

First of all, we include all libraries needed for this exercise.

[1]:
```
import numpy as np
import cv2 as cv
import math
import pywt
from matplotlib import pyplot as plt

Lena_img_path = "Lena.bmp"
```

```
/usr/lib/python3/dist-packages/scipy/__init__.py:146: UserWarning: A NumPy
version >=1.17.3 and <1.25.0 is required for this version of SciPy (detected
version 1.26.2
  warnings.warn(f"A NumPy version >={np_minversion} and <{np_maxversion}"
/home/majiddrn/.local/lib/python3.10/site-
packages/matplotlib/projections/__init__.py:63: UserWarning: Unable to import
Axes3D. This may be due to multiple versions of Matplotlib being installed (e.g.
as a system package and as a pip package). As a result, the 3D projection is not
available.
  warnings.warn("Unable to import Axes3D. This may be due to multiple versions
of "
```

1.2 Exercise 1

The method discussed in the exercise, LoG, is mainly used as an approach which removes the effect of noise before laplacian application. By reversing the process, the result will be different. But to elaborate, We apply these two different approaches in several different ways. **All approaches are by (5, 5) filter sizes**

First, we apply these two different approaches on grayscale image of `Lena`. We measure SSIM, PSNR and subtraction of two approaches.

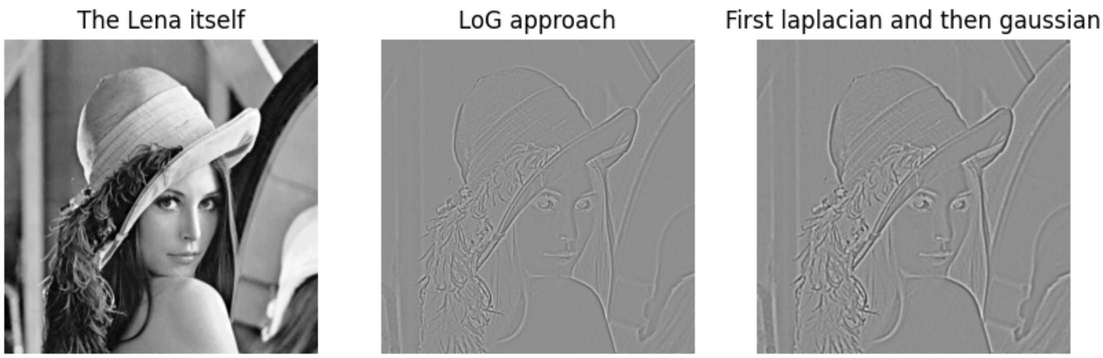
[2]:
```
lena_gray = cv.imread(Lena_img_path, cv.IMREAD_GRAYSCALE)
```

```python
# As stated, we want the laplacian and gaussian filters, both with kernel size␣
↪of (5, 5)
laplacian_kernel = np.array([[0, 1, 0, 1, 0],
                             [1, 1, 1, 1, 1],
                             [0, 1, -24, 1, 0],
                             [1, 1, 1, 1, 1],
                             [0, 1, 0, 1, 0]], dtype=np.float32)

# Firstly, applying LoG on the Lena image

blurred_lena_gray = cv.GaussianBlur(lena_gray, (5, 5), 0) # First applying␣
↪gaussian
log_lena_gray = cv.Laplacian(blurred_lena_gray, cv.CV_64F, laplacian_kernel)


# Secondly, applying laplacian first, instead of gaussian.

laplacian_lena_gray = cv.Laplacian(blurred_lena_gray, cv.CV_64F,␣
↪laplacian_kernel)
gol_lena_gray = cv.GaussianBlur(laplacian_lena_gray, (5, 5), 0)
```

Well, computation is over. Let's plot the result images.

[3]:
```python
fig = plt.figure(figsize=(10, 7))

fig.add_subplot(1, 3, 1)

plt.imshow(lena_gray, cmap='gray')
plt.axis("off")
plt.title("The Lena itself")

fig.add_subplot(1, 3, 2)

plt.imshow(log_lena_gray,  cmap='gray')
plt.axis('off')
plt.title('LoG approach')

fig.add_subplot(1, 3, 3)

plt.imshow(gol_lena_gray,  cmap='gray')
plt.axis('off')
plt.title('First laplacian and then gaussian')
```

[3]:
```
Text(0.5, 1.0, 'First laplacian and then gaussian')
```



Great, we saw some differences between two approaches in two stated different implementations. But to make sure about it, let's measure PSNR, SSIM and difference of LoG and GoL.

The implementation of PSNR in code and mathematics is as follow.

$$\mathrm{PSNR} = 10 \cdot \log_{10}\left(\frac{\mathrm{MAX}^2}{\mathrm{MSE}}\right)$$

Which MSE is mean square error.

$$\mathrm{SSIM}(x, y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)(\sigma_x^2 + \sigma_y^2 + C_3)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)(\sigma_x\sigma_y + C_3)}$$

Code implementation credit: CV Notes

[4]:
```
def calculate_psnr(img1, img2):
    # img1 and img2 have range [0, 255]
    img1 = img1.astype(np.float64)
    img2 = img2.astype(np.float64)
    mse = np.mean((img1 - img2)**2)
    if mse == 0:
        return float('inf')
    return 20 * math.log10(255.0 / math.sqrt(mse))
```

[5]:
```
def ssim(img1, img2):
    C1 = (0.01 * 255)**2
    C2 = (0.03 * 255)**2

    img1 = img1.astype(np.float64)
    img2 = img2.astype(np.float64)
    kernel = cv.getGaussianKernel(11, 1.5)
    window = np.outer(kernel, kernel.transpose())

    mu1 = cv.filter2D(img1, -1, window)[5:-5, 5:-5]  # valid
    mu2 = cv.filter2D(img2, -1, window)[5:-5, 5:-5]
    mu1_sq = mu1**2
```

```
    mu2_sq = mu2**2
    mu1_mu2 = mu1 * mu2
    sigma1_sq = cv.filter2D(img1**2, -1, window)[5:-5, 5:-5] - mu1_sq
    sigma2_sq = cv.filter2D(img2**2, -1, window)[5:-5, 5:-5] - mu2_sq
    sigma12 = cv.filter2D(img1 * img2, -1, window)[5:-5, 5:-5] - mu1_mu2

    ssim_map = ((2 * mu1_mu2 + C1) * (2 * sigma12 + C2)) / ((mu1_sq + mu2_sq +
 ↪C1) *
                                                            (sigma1_sq +
 ↪sigma2_sq + C2))
    return ssim_map.mean()

def calculate_ssim(img1, img2):
    '''calculate SSIM
    the same outputs as MATLAB's
    img1, img2: [0, 255]
    '''
    if not img1.shape == img2.shape:
        raise ValueError('Input images must have the same dimensions.')
    if img1.ndim == 2:
        return ssim(img1, img2)
    elif img1.ndim == 3:
        if img1.shape[2] == 3:
            ssims = []
            for i in range(3):
                ssims.append(ssim(img1, img2))
            return np.array(ssims).mean()
        elif img1.shape[2] == 1:
            return ssim(np.squeeze(img1), np.squeeze(img2))
    else:
        raise ValueError('Wrong input image dimensions.')
```

Well, let's calculate the PSNR and SSIM factors. First to compare the LoG approach with the Lena image and then for GoL approach and Lena image itself comparsion.

[6]:
```
psnr_log = calculate_psnr(log_lena_gray, lena_gray)
ssim_log = calculate_ssim(log_lena_gray, lena_gray)

psnr_gol = calculate_psnr(gol_lena_gray, lena_gray)
ssim_gol = calculate_ssim(gol_lena_gray, lena_gray)

print(f"For Lena and its LoG approach comparsion:\nPSNR:{psnr_log} and SSIM:
 ↪{ssim_log}")
print("\n-----\n")
print(f"For Lena and its GoL approach comparsion:\nPSNR:{psnr_gol} and SSIM:
 ↪{ssim_gol}")
```

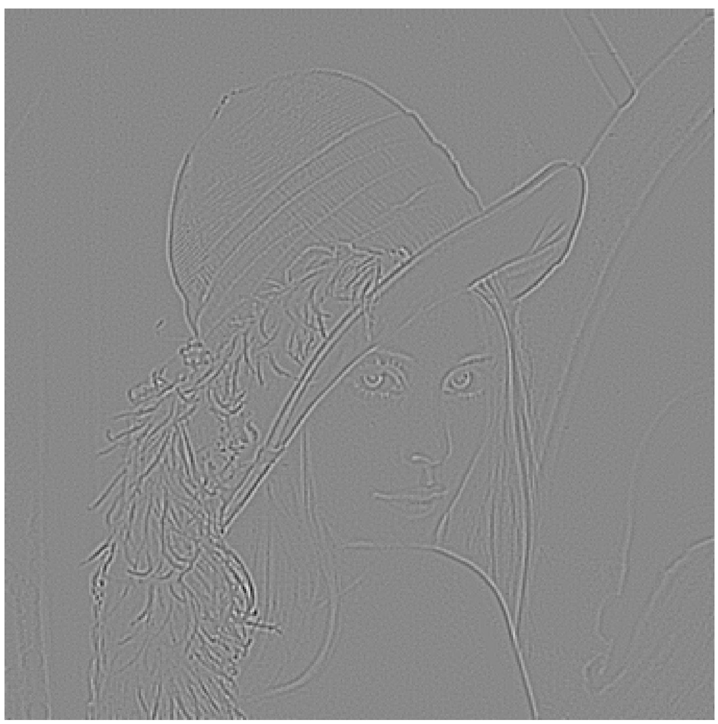
```
For Lena and its LoG approach comparsion:
PSNR:5.814888863609525 and SSIM:-0.00022580376884029653

-----

For Lena and its GoL approach comparsion:
PSNR:5.833012211438813 and SSIM:0.0008247144778262229
```

While there is not much difference in PSNR factor, we can see a noteable difference in their SSIM factor which obviously shows us the difference between these two approaches.

Let's simply compute the subtraction of two approach results.

[7]:
```
log_gol_lena_diff = log_lena_gray - gol_lena_gray

plt.imshow(log_gol_lena_diff, cmap='gray')
plt.axis('off')
```

[7]: (-0.5, 511.5, 511.5, -0.5)



The above difference, simply shows the result of LoG and GoL(Gaussian of Laplacian) are not the same.

Anyway, let's examine the computation on color image of Lena as well.

[8]:
```python
lena_color = cv.imread(Lena_img_path)

# LoG approach for colored Lena

lena_color_gaussian = cv.GaussianBlur(lena_color, (5, 5), 0)
log_lena_color = cv.Laplacian(lena_color_gaussian, cv.CV_64F, laplacian_kernel)␣
↪# We have implemented the kernel above

# GoL approach for colored Lena

lena_color_laplacian = cv.Laplacian(lena_color, cv.CV_64F, laplacian_kernel)
gol_lena_color = cv.GaussianBlur(lena_color_laplacian, (5, 5), 0)
```

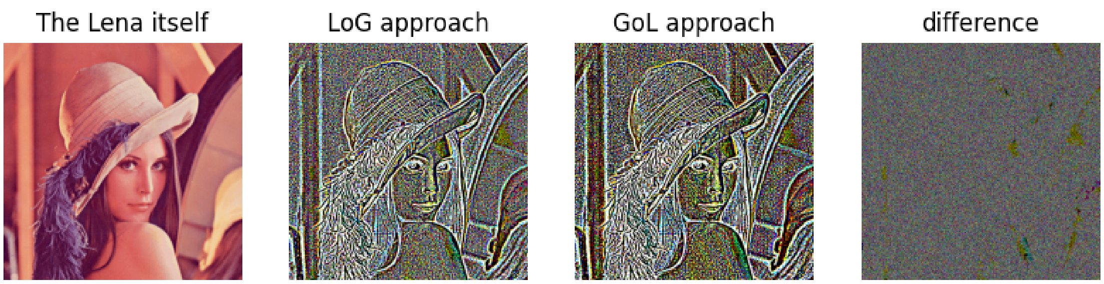
Computation is over now, let's plot the results.

[9]:
```python
fig = plt.figure(figsize=(10, 7))

fig.add_subplot(1, 4, 1)

lena_color_rgb = cv.cvtColor(lena_color, cv.COLOR_BGR2RGB) # To have the␣
↪lena_color in RGB space
plt.imshow(lena_color_rgb)
plt.axis("off")
plt.title("The Lena itself")

fig.add_subplot(1, 4, 2)

plt.imshow(log_lena_color)
plt.axis('off')
plt.title('LoG approach')

fig.add_subplot(1, 4, 3)

plt.imshow(gol_lena_color)
plt.axis('off')
plt.title('GoL approach')

fig.add_subplot(1, 4, 4)

plt.imshow(log_lena_color - gol_lena_color)
plt.axis('off')
plt.title('difference')
```

```
Clipping input data to the valid range for imshow with RGB data ([0..1] for
floats or [0..255] for integers).
Clipping input data to the valid range for imshow with RGB data ([0..1] for
floats or [0..255] for integers).
Clipping input data to the valid range for imshow with RGB data ([0..1] for
```

```
floats or [0..255] for integers).
```

[9]: Text(0.5, 1.0, 'difference')



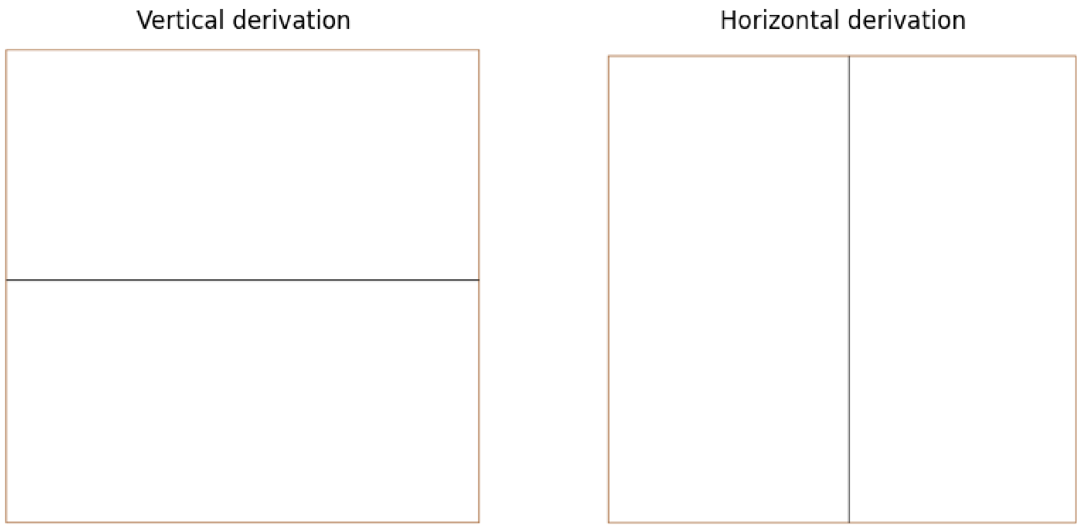
Well, we saw the two approaches difference in color images as well.

1.3 Exercise 2

Clearly, applying the first derivation on the image gives us the edges. Horizontal derivation gives the vertical edges and vertical derivation gives us the horizontal edges. As stated, I myself will choose the first derivation vertically when I need the horizontal edges and I will use the first derivation horizontally when I need the vertical edges.

[10]:
```
vertical_derivation_result = cv.imread('horizontal.png')
horizontal_derivation_result = cv.imread('vertical.png')

fig = plt.figure(figsize=(10, 7))

fig.add_subplot(1, 2, 1)

plt.imshow(vertical_derivation_result, cmap='gray')
plt.axis("off")
plt.title("Vertical derivation")

fig.add_subplot(1, 2, 2)

plt.imshow(horizontal_derivation_result, cmap='gray')
plt.axis("off")
plt.title("Horizontal derivation")
```

[10]: Text(0.5, 1.0, 'Horizontal derivation')



To elaborate, let's examine in code.

First of all we should implement the kernel needed for first derivative application and a function for convolution.

[11]:
```python
def convolution(img, kernel):
    width, height = img.shape
    width_k, height_k = kernel.shape

    output_height = height - height_k + 1
    output_width = width - width_k + 1

    output = np.zeros((output_height, output_width))

    for i in range(output_height):
        for j in range(output_width):
            summation = np.sum(img[i:i + height_k, j:j + width_k] * kernel)
            output[i, j] = summation

    return output
```

Well, let's apply the horizontal derivation and vertical derivation on the image.

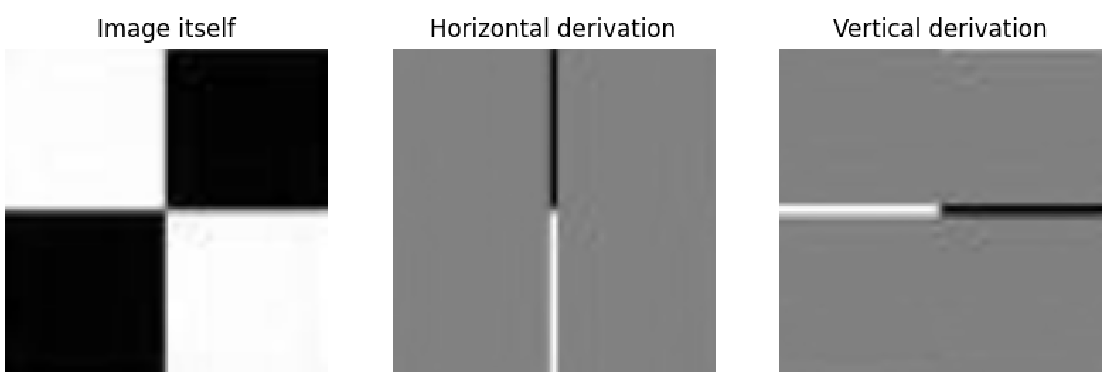
[12]:
```python
first_derivative_kernel_vertical = np.array([[1, 2, 1],
                                              [0, 0, 0],
                                              [-1, -2, -1]])
first_derivative_kernel_horizontal = np.array([[-1, 0, 1],
                                              [-2, 0, 2],
                                              [-1, 0, 1]])
square_gray = cv.imread('squre.png', cv.IMREAD_GRAYSCALE)
```

```
first_derivative_horizontal_output = convolution(img=square_gray,␣
↪kernel=first_derivative_kernel_horizontal)
first_derivative_vertical_output = convolution(img=square_gray,␣
↪kernel=first_derivative_kernel_vertical)
```

Now, let's simply plot the results.

[13]:
```
fig = plt.figure(figsize=(10, 7))

fig.add_subplot(1, 3, 1)

plt.imshow(square_gray, cmap='gray')
plt.axis("off")
plt.title("Image itself")

fig.add_subplot(1, 3, 2)

plt.imshow(first_derivative_horizontal_output, cmap='gray')
plt.axis("off")
plt.title("Horizontal derivation")

fig.add_subplot(1, 3, 3)

plt.imshow(first_derivative_vertical_output, cmap='gray')
plt.axis("off")
plt.title("Vertical derivation")
```

[13]:
```
Text(0.5, 1.0, 'Vertical derivation')
```



Well, We saw the results are as expected.

1.4 Exercise 3

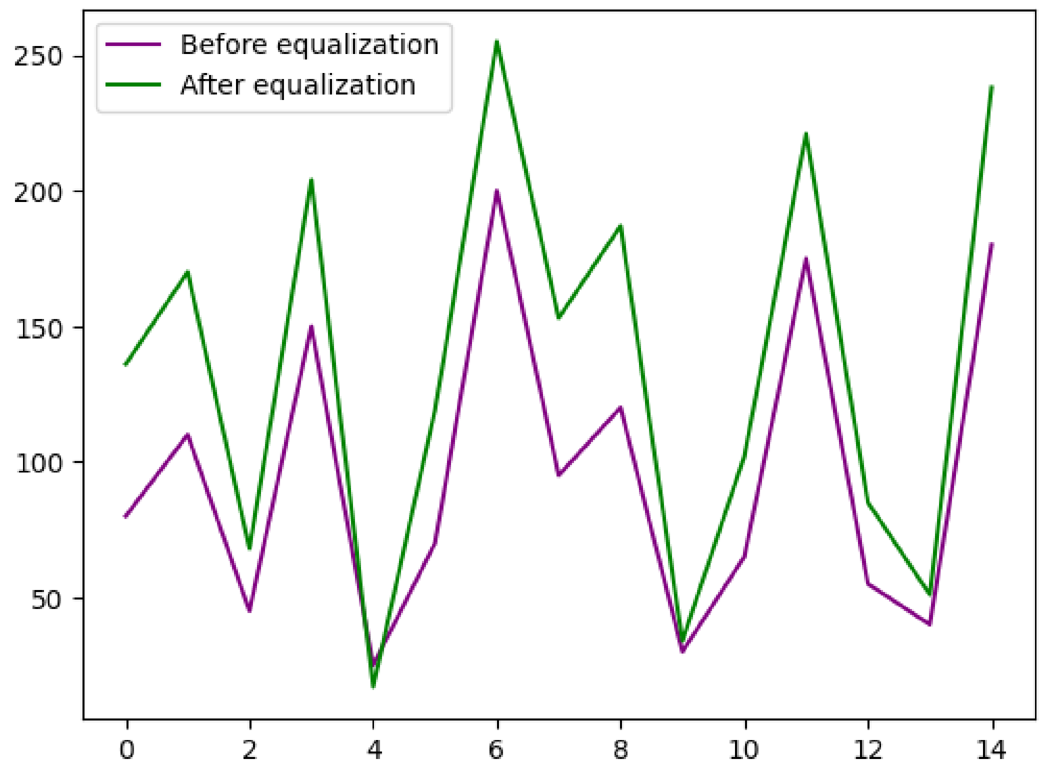
First of all we should compute the frequency of each gray scale in the image. After that, the probablity and PDF of each. Finally, by multiplying each PDF by 255(which is L-1), we can have new intensities.

Gray Level	Frequency	Probability	PDF	$PDF ×255 = $ new gray level
25	1	$\frac{1}{15}$	$\frac{1}{15}$	17
30	1	$\frac{1}{15}$	$\frac{2}{15}$	34
40	1	$\frac{1}{15}$	$\frac{3}{15}$	51
45	1	$\frac{1}{15}$	$\frac{4}{15}$	68
55	1	$\frac{1}{15}$	$\frac{5}{15}$	85
65	1	$\frac{1}{15}$	$\frac{6}{15}$	102
70	1	$\frac{1}{15}$	$\frac{7}{15}$	119
80	1	$\frac{1}{15}$	$\frac{8}{15}$	136
95	1	$\frac{1}{15}$	$\frac{9}{15}$	153
110	1	$\frac{1}{15}$	$\frac{10}{15}$	170
120	1	$\frac{1}{15}$	$\frac{11}{15}$	187
150	1	$\frac{1}{15}$	$\frac{12}{15}$	204
175	1	$\frac{1}{15}$	$\frac{13}{15}$	221
180	1	$\frac{1}{15}$	$\frac{14}{15}$	238
200	1	$\frac{1}{15}$	$\frac{15}{15} = 1$	255

The image before and after equalization is as follow.

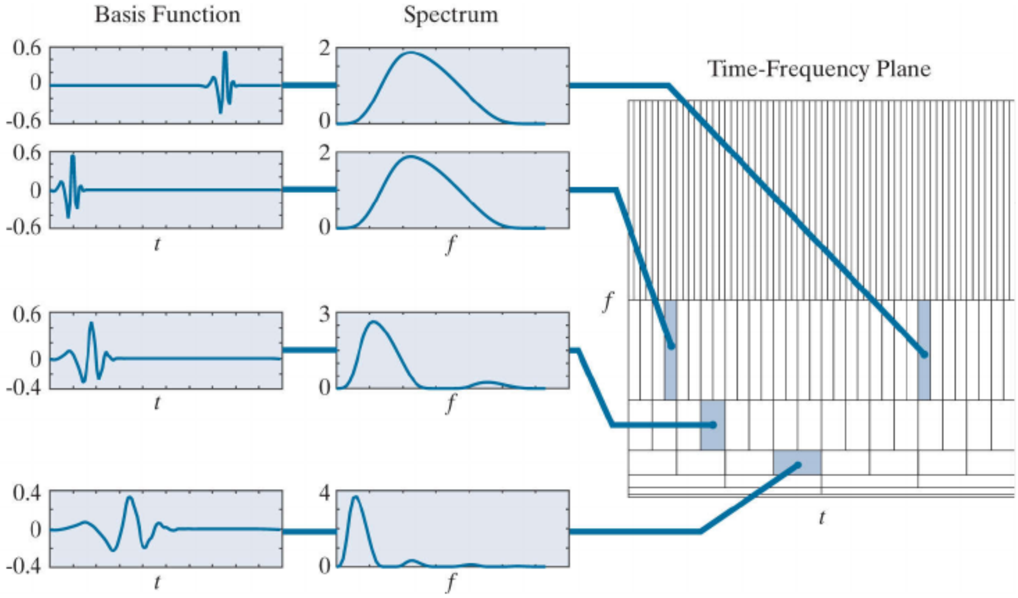
[14]:
```python
image_before_equal = np.array([80, 110, 45, 150, 25, 70, 200, 95, 120, 30, 65,␣
↪175, 55, 40, 180])
image_after_equal = np.array([136, 170, 68, 204, 17, 119, 255, 153, 187, 34,␣
↪102, 221, 85, 51, 238])

plt.plot(image_before_equal, color='purple', label='Before equalization')
plt.plot(image_after_equal, color='green', label='After equalization')
plt.legend(loc='upper left')
```

[14]: <matplotlib.legend.Legend at 0x7fe4bd32f3a0>



1.5 Exercise 4

In wavelet transform, we try to have both frequency and spatial information of the image with the least cost. As a result, there would be a kernel with a variable size, moving across the signal(here, the signal is our image). The kernel computes the frequency of each area it consists in every iteration. One important factor in the iteration is that the size of wavelet window is dependent on the resolution and frequency of that area. If the area has high frequencies, the window size will be wider. Otherwise, if the area has lower frequencies, the spatial information to be remained, window size will be narrower. Well, It was simply the decomposition phase.



After decomposition, we have a set of wavelet coefficients. Quantization is the process of mapping these coefficients to a smaller set of values, which can be represented with fewer bits.

1.6 Exercise 5

To downsample the image, we simply drop rows/columns decussately. To acheive this goal, we implement a function for the rest.

[15]:
```
def downsample2(img):
    rows_keep = img.shape[0] // 2
    cols_keep = img.shape[1] // 2

    downsampled_img = img[:rows_keep * 2, :cols_keep * 2]
    downsampled_img = downsampled_img.reshape((rows_keep, 2, cols_keep, 2))
    downsampled_img = downsampled_img.mean(axis=(1, 3))

    return downsampled_img
```

Now, let's impelement a function for upsampling the image with the replication or bilinear interpolation method.

[16]:
```
def upsample2(img, method):
    upsampled_img = np.zeros((img.shape[0] * 2, img.shape[1] * 2), dtype=img.
 ↪dtype)
    img_ = img.astype(np.float64)

    if method == 'replication':
        for i in range(img_.shape[0]):
```

```
            for j in range(img_.shape[1]):
                upsampled_img[i * 2, j * 2] = img_[i, j]
                upsampled_img[i * 2 + 1, j * 2] = img_[i, j]
                upsampled_img[i * 2, j * 2 + 1] = img_[i, j]
                upsampled_img[i * 2 + 1, j * 2 + 1] = img_[i, j]
    elif method == 'bilinear':
        for i in range(img_.shape[0]):
            for j in range(img_.shape[1]):
                upsampled_img[i * 2, j * 2] = img_[i, j]

                # Handle the last column and last row separately
                if j < img_.shape[1] - 1:
                    upsampled_img[i * 2, j * 2 + 1] = (img_[i, j] + img_[i, j +␣
↪1]) / 2
                else:
                    upsampled_img[i * 2, j * 2 + 1] = img_[i, j]

                if i < img_.shape[0] - 1:
                    upsampled_img[i * 2 + 1, j * 2] = (img_[i, j] + img_[i + 1,␣
↪j]) / 2
                else:
                    upsampled_img[i * 2 + 1, j * 2] = img_[i, j]

                if i < img_.shape[0] - 1 and j < img_.shape[1] - 1:
                    upsampled_img[i * 2 + 1, j * 2 + 1] = (img_[i, j] + img_[i␣
↪+ 1, j] + img_[i, j + 1] + img_[i + 1, j + 1]) / 4
                elif i < img_.shape[0] - 1:
                    upsampled_img[i * 2 + 1, j * 2 + 1] = (img_[i, j] + img_[i␣
↪+ 1, j]) / 2
                elif j < img_.shape[1] - 1:
                    upsampled_img[i * 2 + 1, j * 2 + 1] = (img_[i, j] + img_[i,␣
↪j + 1]) / 2
                else:
                    upsampled_img[i * 2 + 1, j * 2 + 1] = img_[i, j]

    return upsampled_img
```

Well, let's pass the Lena image to the functions above.

[17]:
```
lena_downsampled2 = downsample2(lena_gray)

lena_upsampled2_replication = upsample2(lena_gray, 'replication') # First with␣
↪the replication method
lena_upsampled2_bilinear = upsample2(lena_gray, 'bilinear') # Then with the␣
↪bilinear interpolation method
```

Now, let's plot the results.

[18]:

```
fig = plt.figure(figsize=(10, 7))

fig.add_subplot(1, 3, 1)

plt.imshow(lena_gray, cmap='gray')
plt.axis("off")
plt.title("The Lena itself")

fig.add_subplot(1, 3, 2)

plt.imshow(lena_upsampled2_replication, cmap='gray')
plt.axis('off')
plt.title('Replication method')

fig.add_subplot(1, 3, 3)

plt.imshow(lena_upsampled2_bilinear, cmap='gray')
plt.axis('off')
plt.title('Bilinear interpolation method')
```

[18]:

```
Text(0.5, 1.0, 'Bilinear interpolation method')
```

Let's simply compute the difference of two method results.

[19]:

```
plt.imshow(lena_upsampled2_replication - lena_upsampled2_bilinear, cmap='gray')
plt.axis("off")
```

[19]:

```
(-0.5, 1023.5, 1023.5, -0.5)
```

The difference, obviously shows that the two methods have noteable differences.

1.7 Exercise 6

In this exercise, we simply use the previous convolution function implemented for the spatial domain. And for the fourier computations, we simply implement the following function to calculate the fourier transform of any filter.

[20]:
```
def fourier__magnitude(img_in, shape=(0, 0)):
    if (shape == (0, 0)):
        fdft = np.fft.fft2(img_in)
    else:
        fdft = np.fft.fft2(img_in, shape)
    fdftshift = np.fft.fftshift(fdft)
    magnitude_spectrum_fdft = np.log(np.abs(fdftshift))
    return (fdftshift, magnitude_spectrum_fdft)

def fourier__pad_kernel(kernel, new_size):
    fdftshift__kernel,_ = fourier__magnitude(kernel, new_size)

    return fdftshift__kernel
```

Now, let's implement a function to compute the application of a the filters on the image in frequency domain.

[21]:
```
def apply_filter_freq(img, kernel):
    img__,_  = fourier__magnitude(img)

    result_freq = img__ * kernel

    return np.uint8(np.abs(np.fft.ifft2(np.fft.ifftshift(result_freq))))
```

Now, with the help of the functions we implemented before, let's apply the filters on the image.

[22]:
```
# We simply implement the kernels we need

kernel_A = np.array([[1/9, 1/9, 1/9],
                     [1/9, 1/9, 1/9],
                     [1/9, 1/9, 1/9]])

kernel_B = np.array([[-1, -1, -1],
                     [-1,  8, -1],
                     [-1, -1, -1]])

kernel_C = np.array([[0, -1,  0],
                     [-1, 5, -1],
                     [0, -1,  0]])


# Now, let's apply the kernel A on the Lena image with two spatial and fourier␣
↪approaches

kernel_A_fourier = fourier__pad_kernel(kernel_A, lena_gray.shape)

lena__A_spatial_builtin = cv.filter2D(lena_gray, -1, kernel_A)
lena__A_fourier = apply_filter_freq(lena_gray, kernel_A_fourier)

# Now, let's apply the kernel B on the Lena image with two spatial and fourier␣
↪approaches

kernel_B_fourier = fourier__pad_kernel(kernel_B, lena_gray.shape)

lena__B_spatial_builtin = cv.filter2D(lena_gray, -1, kernel_B)
lena__B_fourier = apply_filter_freq(lena_gray, kernel_B_fourier)


# Now, let's apply the kernel C on the Lena image with two spatial and fourier␣
↪approaches

kernel_C_fourier = fourier__pad_kernel(kernel_C, lena_gray.shape)

lena__C_spatial_builtin = cv.filter2D(lena_gray, -1, kernel_C)
```

```
lena__C_fourier = apply_filter_freq(lena_gray, kernel_C_fourier)
```

```
/tmp/ipykernel_11133/1749126714.py:7: RuntimeWarning: divide by zero encountered
in log
  magnitude_spectrum_fdft = np.log(np.abs(fdftshift))
```

Now, let's plot the result of applying kernel A on the image in both spatial and frequency domain.

[23]:
```
fig = plt.figure(figsize=(10, 7))

fig.add_subplot(1, 3, 1)

plt.imshow(lena_gray, cmap='gray')
plt.axis("off")
plt.title("The Lena itself")

fig.add_subplot(1, 3, 2)

plt.imshow(lena__A_spatial_builtin, cmap='gray')
plt.axis('off')
plt.title('Box filter builtin conv')

fig.add_subplot(1, 3, 3)

plt.imshow(lena__A_fourier, cmap='gray')
plt.axis('off')
plt.title('Box filter freq')
```

[23]:
```
Text(0.5, 1.0, 'Box filter freq')
```

Now, it's the kernel B turn.

[24]:
```
fig = plt.figure(figsize=(10, 7))
```

```
fig.add_subplot(1, 3, 1)

plt.imshow(lena_gray, cmap='gray')
plt.axis("off")
plt.title("The Lena itself")

fig.add_subplot(1, 3, 2)

plt.imshow(lena__B_spatial_builtin, cmap='gray')
plt.axis('off')
plt.title('Laplacian builtin conv')

fig.add_subplot(1, 3, 3)

plt.imshow(lena__B_fourier, cmap='gray')
plt.axis('off')
plt.title('Laplacian freq')
```

[24]: Text(0.5, 1.0, 'Laplacian freq')

And finally, it's the kernel C's turn which is a simple sharpenning kernel.

[25]:
```
fig = plt.figure(figsize=(10, 7))

fig.add_subplot(1, 3, 1)

plt.imshow(lena_gray, cmap='gray')
plt.axis("off")
plt.title("The Lena itself")

fig.add_subplot(1, 3, 2)

plt.imshow(lena__C_spatial_builtin, cmap='gray')
plt.axis('off')
```

```python
plt.title('Sharpenning builtin conv')

fig.add_subplot(1, 3, 3)

plt.imshow(lena__C_fourier, cmap='gray')
plt.axis('off')
plt.title('Sharpenning freq')
```

[25]: Text(0.5, 1.0, 'Sharpenning freq')

1.8 Exercise 7

We have the box filter as follow.

$$K = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

Let's make the pyramid as wanted.

[26]:
```python
kernel__average22 = np.array([[1/4, 1/4],
                              [1/4, 1/4]])

img__L1 = lena_gray
approx__L1 = downsample2(cv.filter2D(img__L1, -1, kernel__average22))
prediction__L1 = img__L1 - upsample2(approx__L1, 'replication')

img__L2 = approx__L1
approx__L2 = downsample2(cv.filter2D(img__L2, -1, kernel__average22))
prediction__L2 = img__L2 - upsample2(approx__L2, 'replication')

img__L3 = approx__L2
approx__L3 = downsample2(cv.filter2D(img__L3, -1, kernel__average22))
prediction__L3 = img__L3 - upsample2(approx__L3, 'replication')
```

Computation is over, let's plot the pyramid.

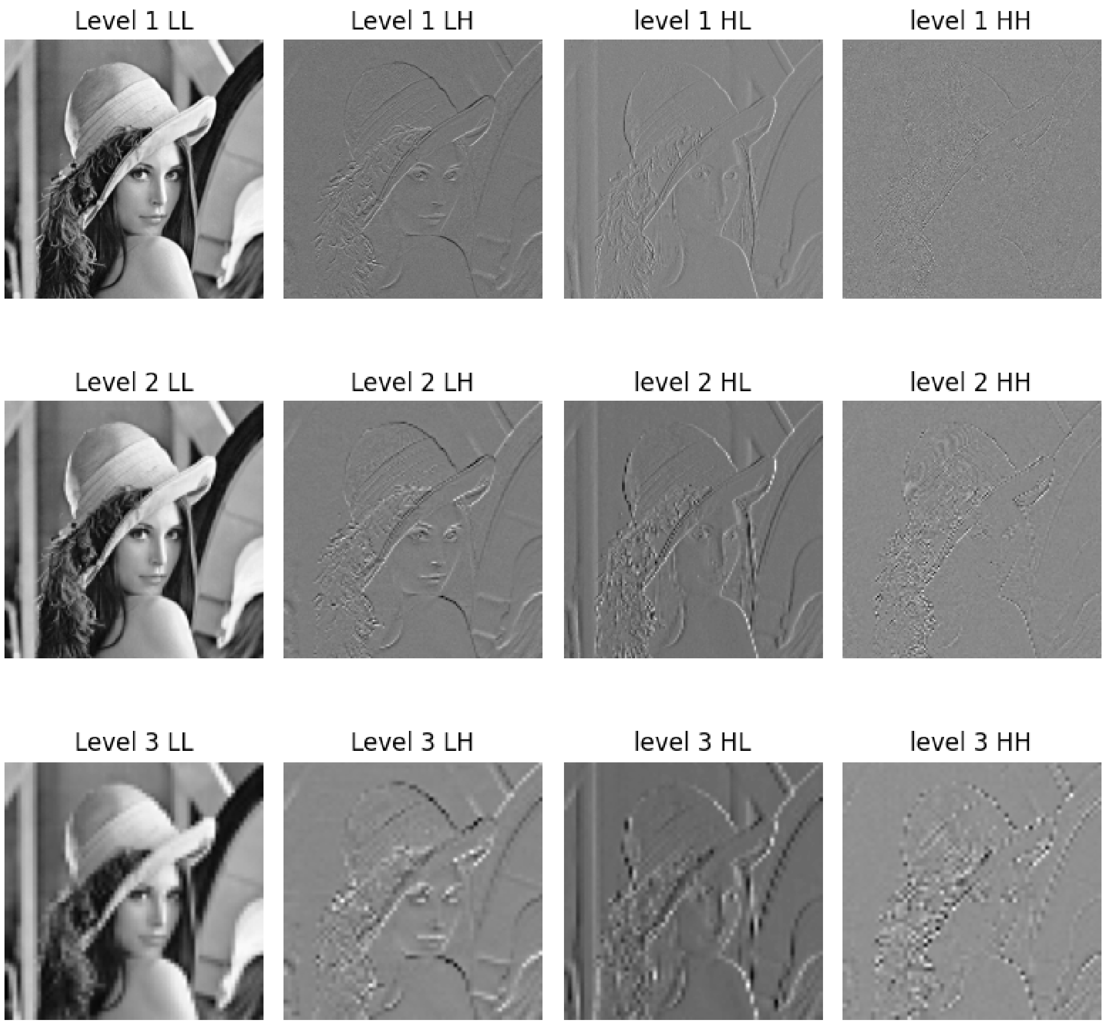
[27]:
```
fig, axes = plt.subplots(nrows=3, ncols=2, figsize=(8, 8))

approximation_pyramid = [approx__L1, approx__L2, approx__L3]
prediction_residual_pyramid = [prediction__L1, prediction__L2, prediction__L3]

for level in range(3):
    axes[level, 0].imshow(approximation_pyramid[level], cmap='gray')
    axes[level, 0].set_title(f'Level {level+1} Approximation')

    axes[level, 1].imshow(prediction_residual_pyramid[level], cmap='gray')
    axes[level, 1].set_title(f'Level {level+1} Prediction Residual')

for ax in axes.flat:
    ax.axis('off')

plt.tight_layout()
plt.show()
```


Level 1 Approximation
Level 1 Prediction Residual
Level 2 Approximation
Level 2 Prediction Residual
Level 3 Approximation
Level 3 Prediction Residual

1.9 Exercise 8

To acheive this goal, we simply install the library `pywt`. After the installation, usage is so easy. We store every level as follow.

[28]:
```
fig, axes = plt.subplots(nrows=3, ncols=4, figsize=(8, 8))

img__LL = lena_gray

for level in range(3):
    coeffs = pywt.dwt2(img__LL, 'haar')
    cA, (cH, cV, cD) = coeffs

    axes[level, 0].imshow(cA, cmap='gray')
    axes[level, 0].set_title(f'Level {level+1} LL')

    axes[level, 1].imshow(cH, cmap='gray')
    axes[level, 1].set_title(f'Level {level+1} LH')

    axes[level, 2].imshow(cV, cmap='gray')
    axes[level, 2].set_title(f'level {level+1} HL')

    axes[level, 3].imshow(cD, cmap='gray')
    axes[level, 3].set_title(f'level {level+1} HH')

    img__LL = cA

for ax in axes.flat:
    ax.axis('off')

plt.tight_layout()
plt.show()
```



Well, let's discuss on the differences.

- The wavelet transform, gives more infromation which contains vertical, horizontal and diagonal edges. While, the pyramid in the 7th question, doesn't give us horizontal or vertical edges seperately.
- The Haar wavelet transform provides a multi-resolution representation of the image, prepares both low-frequency and high-frequency details at different scales. This is useful for some image processing tasks, such as compression, denoising and feature extraction.
- Clearly, the Haar wavelet transform offers a more sophisticated analysis of the image content compared to a simple 2x2 filter.

1.10 Exercise 9

After some little changes in the `upsample2(img, method)` function implemented in the exercise 5, I added two other interpolation techniques, including bilinear and nearest neighbor interpolation.

There has been added another argument named `scale_factor` which upsamples the image in any scale needed.

[29]:
```
def upsample(img, method, scale_factor):
    upsampled_img = np.zeros((int(img.shape[0] * scale_factor), int(img.shape[1] * scale_factor)), dtype=img.dtype)
    img_ = img.astype(np.float64)

    for i in range(img_.shape[0]):
        for j in range(img_.shape[1]):
            for p in range(int(scale_factor)):
                for q in range(int(scale_factor)):
                    if method == 'replication':
                        upsampled_img[i * int(scale_factor) + p, j * int(scale_factor) + q] = img_[i, j]
                    elif method == 'bilinear':
                        upsampled_img[i * int(scale_factor) + p, j * int(scale_factor) + q] = bilinear_interpolation(img_, i, j, p / scale_factor, q / scale_factor)
                    elif method == 'nearest':
                        upsampled_img[i * int(scale_factor) + p, j * int(scale_factor) + q] = nearest_neighbor_interpolation(img_, i + p / scale_factor, j + q / scale_factor)

    return upsampled_img

def bilinear_interpolation(img, i, j, p, q):
    i_floor, j_floor = int(np.floor(i)), int(np.floor(j))
    i_ceil, j_ceil = min(i_floor + 1, img.shape[0] - 1), min(j_floor + 1, img.shape[1] - 1)

    top_left = img[i_floor, j_floor]
    top_right = img[i_floor, j_ceil]
    bottom_left = img[i_ceil, j_floor]
    bottom_right = img[i_ceil, j_ceil]

    interpolated_value = (1 - p) * (1 - q) * top_left + (1 - p) * q * top_right + p * (1 - q) * bottom_left + p * q * bottom_right
    return interpolated_value

def nearest_neighbor_interpolation(img, i, j):
    i_round, j_round = int(round(i)), int(round(j))
    i_round = max(0, min(img.shape[0] - 1, i_round))
    j_round = max(0, min(img.shape[1] - 1, j_round))
    return img[i_round, j_round]
```

Implementation is over. Let's first downscale the image and then upscale the image by factor 5. First by *bilinear interpolation* and then by *nearest neighbor* interpolation.

[30]:
```
lena__up5_bilinear = upsample(lena_gray, method='bilinear', scale_factor=5)
lena__up5_nearest = upsample(lena_gray, method='nearest', scale_factor=5)
```

Application is over. Let's plot the result.

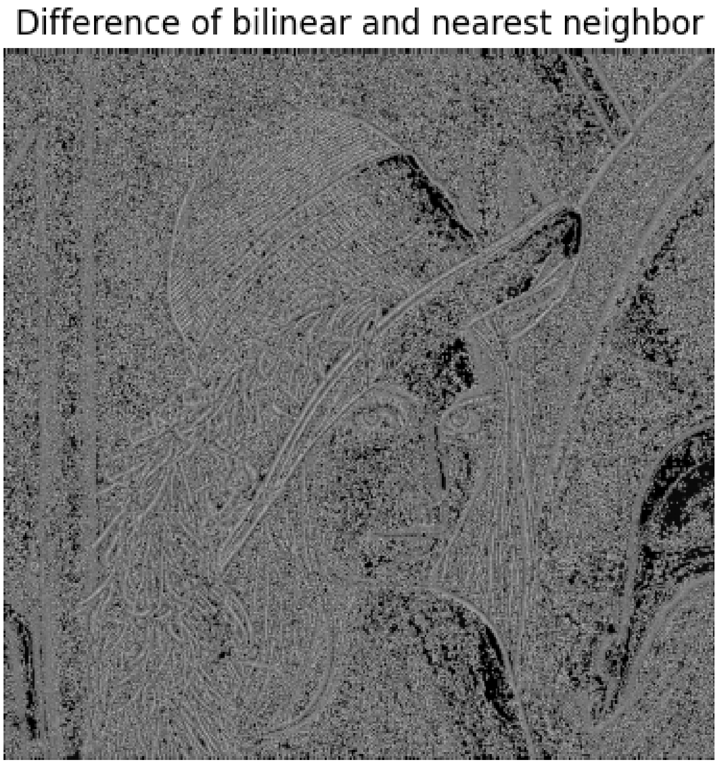
[31]:
```
fig = plt.figure(figsize=(10, 7))

fig.add_subplot(1, 2, 1)

plt.imshow(lena__up5_bilinear, cmap='gray')
plt.axis("off")
plt.title("Upscaled 3 times- bilinear interpolation")

fig.add_subplot(1, 2, 2)

plt.imshow(lena__up5_nearest, cmap='gray')
plt.axis('off')
plt.title('Upscaled 3 times- nearest neighbor interpolation')
```

[31]:
```
Text(0.5, 1.0, 'Upscaled 3 times- nearest neighbor interpolation')
```

Well, to compare the results, let's subtract one from the other.

[32]:
```
plt.imshow(lena__up5_bilinear - lena__up5_nearest, cmap='gray')
plt.axis("off")
plt.title("Difference of bilinear and nearest neighbor")
```

[32]:
```
Text(0.5, 1.0, 'Difference of bilinear and nearest neighbor')
```



Well, the difference is clear, let's examine again by builtin OpenCV functions.

[33]:
```python
# Nearest-Neighbor Interpolation
lena__up5_nearest__builtin = cv.resize(lena_gray, None, fx=5, fy=5,␣
↪interpolation=cv.INTER_NEAREST)

# Bilinear Interpolation
lena__up5_bilinear__builtin = cv.resize(lena_gray, None, fx=5, fy=5,␣
↪interpolation=cv.INTER_LINEAR)
```

Let's plot

[34]:
```python
fig = plt.figure(figsize=(10, 7))

fig.add_subplot(1, 2, 1)

plt.imshow(lena__up5_nearest__builtin, cmap='gray')
plt.axis("off")
plt.title("-builtin function- bilinear interpolation")

fig.add_subplot(1, 2, 2)

plt.imshow(lena__up5_bilinear__builtin, cmap='gray')
```

```
plt.axis('off')
plt.title('-builtin function- nearest neighbor interpolation')
```

[34]:
```
Text(0.5, 1.0, '-builtin function- nearest neighbor interpolation')
```

And now, let's subtract one from the other.

[35]:
```
plt.imshow(lena__up5_bilinear__builtin - lena__up5_nearest__builtin, cmap='gray')
plt.axis("off")
plt.title("Difference of bilinear and nearest neighbor -builtin-")
```

[35]:
```
Text(0.5, 1.0, 'Difference of bilinear and nearest neighbor -builtin-')
```

Generally, if speed is most important, nearest neighbor interpolation can be a good choice. However, if image quality is paramount, bilinear interpolation will generally yield better results.