



Relational Algebra

Chapter 4.1-4.2 Part II

Complete Chapter 5 worksheet before Monday's class





Division (not an operator in SQL)

- Useful for expressing queries like: Find sailors who have reserved all boats.
- Let A have 2 fields, x and y; B have only field y:
 - $A/B = \{\langle x \rangle \mid \exists \langle x, y \rangle \in A \ \forall \langle y \rangle \in B\}$
 - i.e., *A/B* contains all *x* tuples (sailors) such that for *every y* tuple (boat) in *B*, there is an *xy* tuple in *A*.
 - If the set of *y* values (boats) associated with an *x* value (sailor) in A contains all y values in B, the x value is in A/B.







sno	pno
s1	p1
s1	p2
s1	р3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

pno







sno	pno	
s1	p1	
s1	p2	
s1	р3	
s1	p4	
s2	p1	
s2	p2	
s3	p2	
s4	p2	
s4	p4	

pno

B1

sno s1 **s**2 s3 **s**4







sno	pno
s1	p1
s1	p2
s1	р3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

pno p2

B1

sno s1 s2 s3 s4

A/B1

pno p2 p4

B2







sno	pno	pno	pno
s1	p1	p2	p2
s1	p2	D 1	p4
s1	р3	<i>B1</i>	חס
s1	p4		<i>B2</i>
s2	p1	sno	
s2	p2	s1	
s3	p2	s2	sno
s4	p2	s3	s1
s4	p4	s4	s 4
	A	A/B1	A/B2







sno	pno
s1	p1
s1	p2
s1	р3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

pno p2	
<i>B1</i>	l
sno	
sno s1	
s1	





A

A/B1

A/B2





Examples of Division A/B

sno	pno
s1	p1
s1	p2
s1	р3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

$$A_1$$

pno
p2

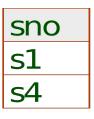
B1

sno
s1
s2
s3
s4

<u> A1÷B1</u>

pno
p2
р4

B2



<u>A÷B2</u>

pno
р1
p2
р4

B3

sno s1





Expressing A/B Using Basic Operators

- Division is not essential; it's just a useful shorthand.
 - (Also true of joins, but joins are so common that systems implement joins specially.)





Expressing A/B Using Basic Operators

- ❖ *Idea*: For *A/B*, compute all *x* values that are not "disqualified" by some *y* value in *B*.
 - x value is *disqualified* if by attaching y value from B, we obtain an xy tuple that is not in A.

Disqualified *x* values:
$$\pi_{\chi}((\pi_{\chi}(A) \times B) - A)$$

A/B:
$$\pi_{x}(A)$$
 – disqualified x values





Relational Algebra Examples

- Assume the following extended schema:
 - Sailors(sid: integer, sname: string, rating: integer, age: real)
 - Reserves(sid: integer, bid: integer, day: date)
 - Boat(bid: integer, bname: string, bcolor: string)





Relational Algebra Examples

- Objective: Write a relational algebra expression whose result instance satisfies the specified conditions
 - May not be unique
 - Some alternatives might be more efficient (in terms of time and/or space)



Names of sailors who reserved boat #103

Solution 1: $\pi_{\text{sname}}(\sigma_{\text{bid=103}}(\text{Reserves} \boxtimes \text{Sailors}))$

Solution 2: π_{sname} (($\sigma_{\text{bid=103}}$ Reserves) \bowtie Sailors

Solution 3:
$$\rho(T1, \sigma_{\text{bid=103}} \text{Reserves})$$

 $\rho(T2, T1 \bowtie \text{Sailors})$
 $\sigma_{\text{sname}} T2$

Sailors who reserved boat 103



Names of sailors who reserved a red boat

Information about boat color only available in Boats; so need an extra join:



Names of sailors who've reserved a red boat

- Information about boat color only available in Boats; so need an extra join:
 - π_{sname} (($\sigma_{\text{color=red}}$ Boats) \bowtie Reserves \bowtie Sailors)
- * A more efficient solution $\pi_{\text{sname}}(\pi_{\text{sid}}(\sigma_{\text{color=red}}) \otimes \text{Reserves}) \otimes \text{Sailors})$
- A query optimizer could find this from first.

Sailors who've reserved a red boat



Sailors who've reserved a red or a green boat

Can identify all red or green boats, then find sailors who've reserved one of these boats:



Sailors who've reserved a red or a green boat

Can identify all red or green boats, then find sailors who've reserved one of these boats:

$$ho$$
(T, $\sigma_{\text{color=red} \lor \text{color=green}}$ Boats)

 π_{sname} (T \bowtie Reserves \bowtie Sailors)



Sailors who've reserved a red and a green boat

Previous approach won't work! Must identify sailors who've reserved red boats, sailors who've reserved green boats, then find the intersection (note that sid is a key for Sailors):



Sailors who've reserved a red and a green boat

Previous approach won't work! Must identify sailors who've reserved red boats, sailors who've reserved green boats, then find the intersection (note that sid is a key for Sailors)

$$\rho(\text{T1}, \boldsymbol{\pi}_{\text{sid}}((\boldsymbol{\sigma}_{\text{color=red}}\text{Boats}) \bowtie \text{Reserves}))$$
 $\rho(\text{T2}, \boldsymbol{\pi}_{\text{sid}}((\boldsymbol{\sigma}_{\text{color=green}}\text{Boats}) \bowtie \text{Reserves}))$
 $\boldsymbol{\pi}_{\text{sname}}((\text{T1} \cap \text{T2}) \bowtie \text{Sailors})$

Sailors who reserved a red and green boat

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Summary

- The relational model has rigorously defined query languages that are simple and powerful.
- Relational algebra is more operational; useful as internal representation for query evaluation plans.
- Several ways of expressing a given query; a query optimizer should choose the most efficient version.