SPCE 5025

20 Jan 2026

- Note: Class Time Next Week
  - ***MONDAY***, **26 January, 7:00 pm Mountain Time**

- Note: Possible shift in class time
  - ***MONDAY***, **2 February, 7:00 pm Mountain Time**
    - **Launch-dependent**

Introductory Information

- Instructor: Ed Brown
- Background
  - Space Shuttle Navigation
    - Mission Control console support, payload support, post-flight attitude and trajectory analysis
  - GPS Orbit Analysis/Mission Planning
    - Real-time launch, anomaly, disposal, normal ops support; Software development
  - P91/Argos Orbit Analysis/Mission Planning
    - Real-time mission support, Orbit analysis, mission concept development
  - Astrodynamics Analysis and Space Domain Awareness
  - Rendezvous/Proximity Operations analysis

Course Aims

- Provide familiarity with standard concepts in astronautics
  - Orbital mechanics
  - Coordinate transformations and pointing
  - Observing satellites
  - Common computations
- Focus is on Earth-orbiting satellites
- Learn by doing
  - Homework will stress practical applications
  - Computer-oriented: most useful computations too involved to be done by hand
- Exams
  - Similar approach, all will be comprehensive from beginning of class

Texts

- Syllabus refers to Vallado sections/chapters (5th Edition)
  - Vallado often leaves a lot of the details to Bate, Mueller, and White and others
  - https://astrobooks.com/vallado5hb.aspx

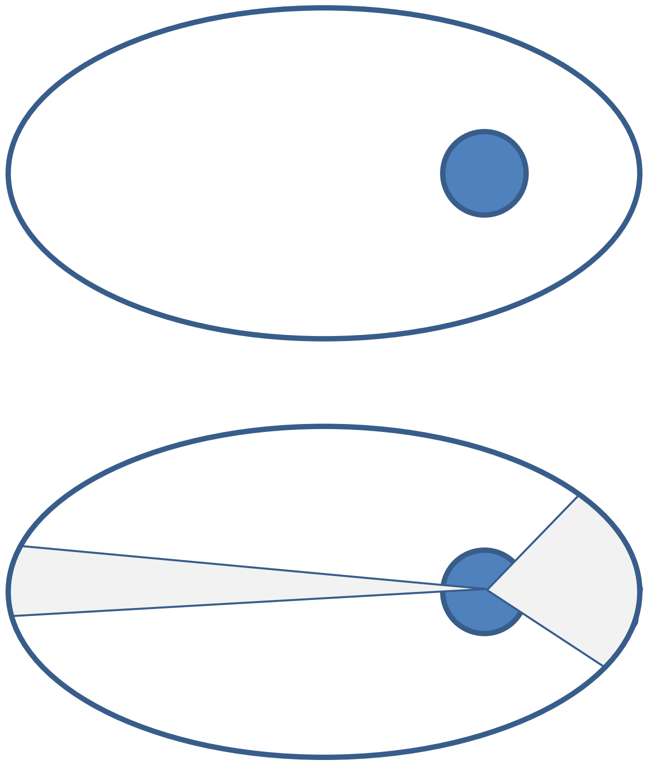
- A lot of excellent free stuff on-line, if you know what you're looking for
  - Recommended: Goddard Trajectory Determination System Mathematical Theory (1989)
    - Seminal work: basis for numerous other ground systems
    - Excellent reference for definitions and advanced concepts
    - Available in Course Modules folder

Lecture Schedule

- Normal Class Schedule
  - Tuesdays – Lecture
  - Thursdays – Office Hours
- Class time – 7:00 pm Mountain Time

Background – Kepler’s Laws

- Orbit of a satellite is an ellipse with the central body at one focus
- The line joining the central body and satellite sweeps out equal areas in equal time
- The square of the satellite’s orbit period is proportional to the cube of the mean distance from the central body



Background – Newton's Laws

- **Inertia**: Object in motion moves in straight line unless acted on by an outside force
- $\boldsymbol{F} = \boldsymbol{ma}$
  - Actually, $F = \frac{d}{dt}(mv)$
- Equal and opposite reactions

Inverse Square Law

- At planetary scales, gravity obeys inverse square law

$$\mathrm{f} = \mathrm{ma} \propto \frac{1}{\mathrm{r}^2}$$

- Gravitational force is exerted by mass m_2 on m_1 and vice versa

$$f_{grav,1} = \mathrm{m_1 a} = \mathrm{m_1}\left(\frac{\mathrm{Gm_2}}{\mathrm{r}^2}\right) = \frac{\mathrm{Gm_1 m_2}}{\mathrm{r}^2}$$

- In vector form:

$$\vec{\mathrm{f}}_{\mathrm{grav,1}} \rightarrow \left(\frac{\mathrm{Gm_1 m_2}}{\mathrm{r}^2}\right) \cdot \left(\frac{\vec{\mathrm{r}}}{\mathrm{r}}\right) = \frac{\mathrm{Gm_1 m_2}}{\mathrm{r}^3}\vec{\mathrm{r}}$$ (which direction?)

Accelerations

- Gravitational acceleration due to a mass M separated from m by $\vec{r}$

$$a_{grav} = \frac{\mathrm{GM}}{\mathrm{r}^2}$$

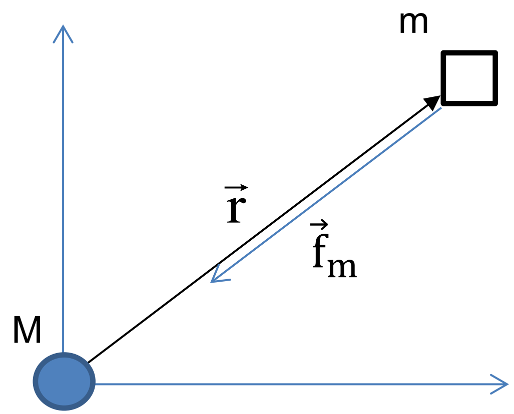
- Scalar force acting on a body of mass m due to mass M

$$\mathrm{f_m} = \mathrm{m}a_{grav} = \mathrm{m}\frac{\mathrm{GM}}{\mathrm{r}^2} = \frac{\mathrm{GMm}}{\mathrm{r}^2}$$

- Impose directionality to create vector quantity

$$\vec{\mathrm{f}}_\mathrm{m} = -\frac{\mathrm{GMm}}{\mathrm{r}^2}\left(\frac{\vec{\mathrm{r}}}{\mathrm{r}}\right) = -\frac{\mathrm{GMm}}{\mathrm{r}^3}\vec{\mathrm{r}}$$

  - Directionality depends on definitions



Acceleration acting on mass m due to gravitational attraction from mass M

Goal of Derivation

- Find a way to describe the motion through space of a satellite subject to an inverse square gravitational force

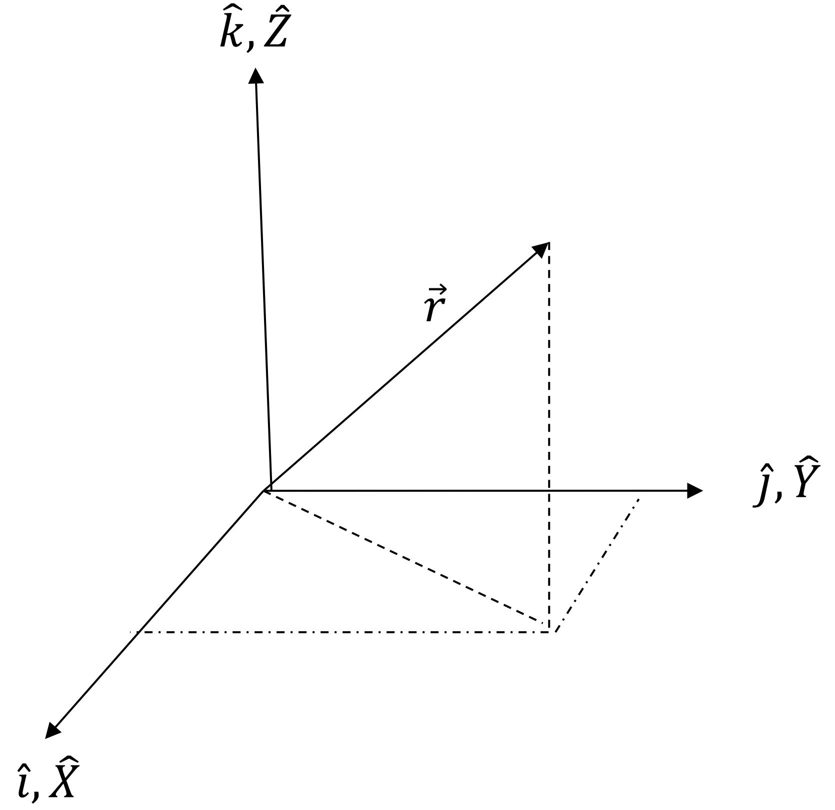
- Note: requires no assumption that mass m is “in orbit” about M
  - Idealized problem involving the motion of infinitesimal point masses in space
  - In real world the bodies have finite extent, especially the central body
    - “In orbit” $\rightarrow$ moving without hitting the ground

Derivations to Follow

- Angular Momentum
  - Inclination, Right Ascension of Ascending Node
- Orbit Energy
- Trajectory Equation
- Energy and Semi-major Axis
- Orbit Period

Defining "Space"

- Derivations will be based on vector quantities
- Vectors measured relative to a Cartesian coordinate system
  - Three mutually-orthogonal characteristic directions
- Characteristic directions are fixed in space
  - "Inertial reference frame"
  - Greatly simplifies math to deal with non-moving coordinate axes



Total Acceleration = Sum of Forces

- Define vectors with respect to arbitrary origin

$$\vec{r}_m = \vec{r}_M + \vec{r}$$

$$\vec{r} = \vec{r}_m - \vec{r}_M$$

$$\ddot{\vec{r}} = \ddot{\vec{r}}_m - \ddot{\vec{r}}_M$$

$$\ddot{\vec{r}}_m = -\frac{GM}{r^3}\vec{r}$$

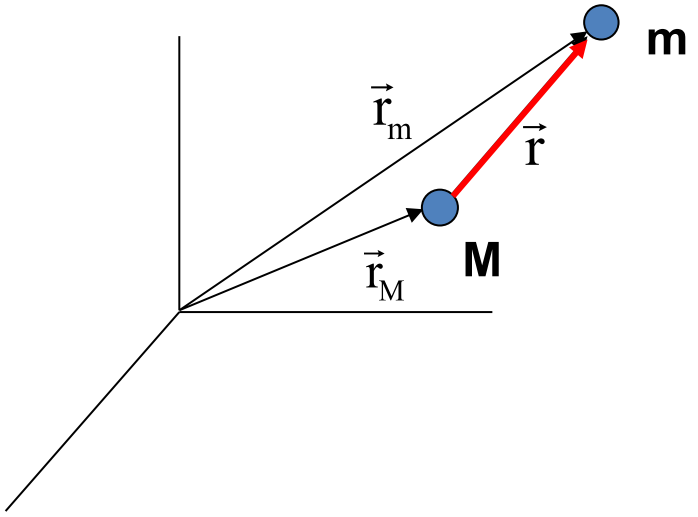
$$\ddot{\vec{r}}_M = \frac{Gm}{r^3}\vec{r}$$

$$\ddot{\vec{r}} = -\frac{GM}{r^3}\vec{r} - \frac{Gm}{r^3}\vec{r}$$

$$\ddot{\vec{r}} = -\frac{G(M+m)}{r^3}\vec{r}$$

if $M >> m$,

$$\boxed{\ddot{\vec{r}} = -\frac{GM}{r^3}\vec{r}}$$



Significant assumptions:
- **M >> m**
- **Gravitational field is smooth, spherically symmetric**
- **Gravitational force emanates from point at center of M**
- **No other forces acting on the bodies**

Some definitions

- G
  - Gravitational Constant = $6.673 \times 10^{-11}\, {m^3}/{kg \cdot s^2}$
  - Poorly defined – hard to measure
- M
  - Mass of central body
  - Also poorly defined – where do you put the scales?
- $\mu = GM$
  - Gravitational parameter
  - $\mu_{Earth} = \mu_{\oplus} = 3.986004418 \times 10^{14}\ m^3/s^2$ (WGS84 value)
  - Well-defined – can be derived from orbital motion

Equations of Motion

- Goal: find a solution to this Differential Equation that describes motion of satellite in space

$$\ddot{\vec{r}} = -\frac{GM}{r^3}\vec{r} = -\frac{\mu}{r^3}\vec{r}$$

  – Need 6 "constants of the motion" to fully describe motion
  – E.g., Cartesian coordinates
    - 3 position components
    - 3 velocity components
  – Other sets of elements also available
- We will derive the Keplerian (Classical) elements
  – This week and next week

Angular Momentum

- Definition

$$\vec{h} = \vec{r} \times \dot{\vec{r}}$$

$$\frac{d}{dt}(\vec{r} \times \dot{\vec{r}}) = \cancelto{0}{\dot{\vec{r}} \times \dot{\vec{r}}} + \vec{r} \times \ddot{\vec{r}}$$

Perform cross product of $\vec{r}$ with respect to EOM

$$\vec{r} \times \ddot{\vec{r}} = -\frac{\mu}{r^3}\cancelto{0}{\vec{r} \times \vec{r}} = 0$$

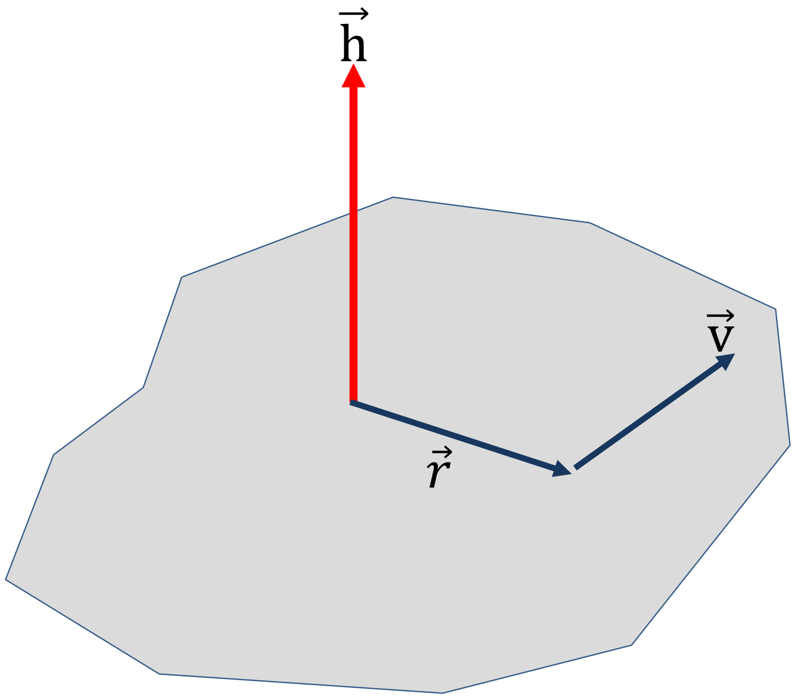
$$\frac{d}{dt}(\vec{r} \times \dot{\vec{r}}) = \vec{r} \times \ddot{\vec{r}} = 0$$

- Integrate

$$\boxed{\vec{r} \times \dot{\vec{r}} = \vec{h} = \text{Constant}}$$

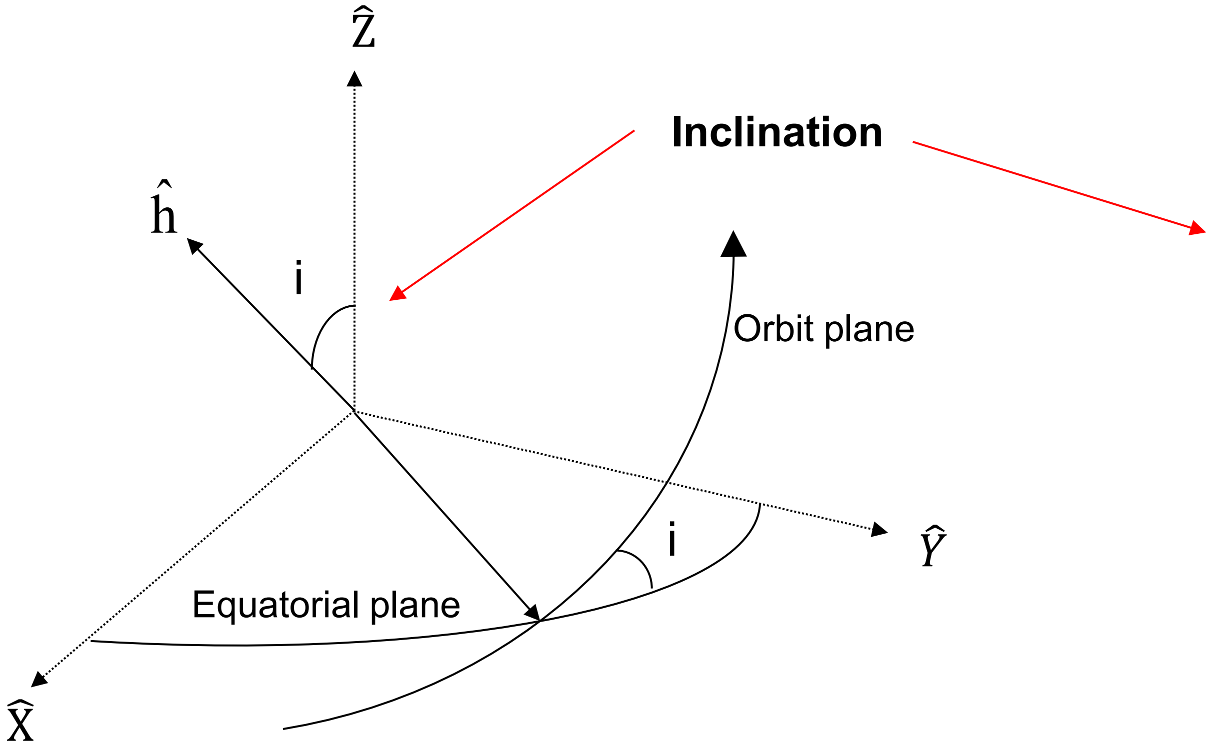
Angular Momentum Considerations

- Angular momentum vector is normal to plane containing position and velocity
- “Constant” angular momentum includes both direction and magnitude
- **Constant direction**: plane has fixed orientation in inertial space
- **Constant magnitude**: related to Kepler’s “equal areas in equal times”



Inclination

- First element of the orientation of the orbit plane in space



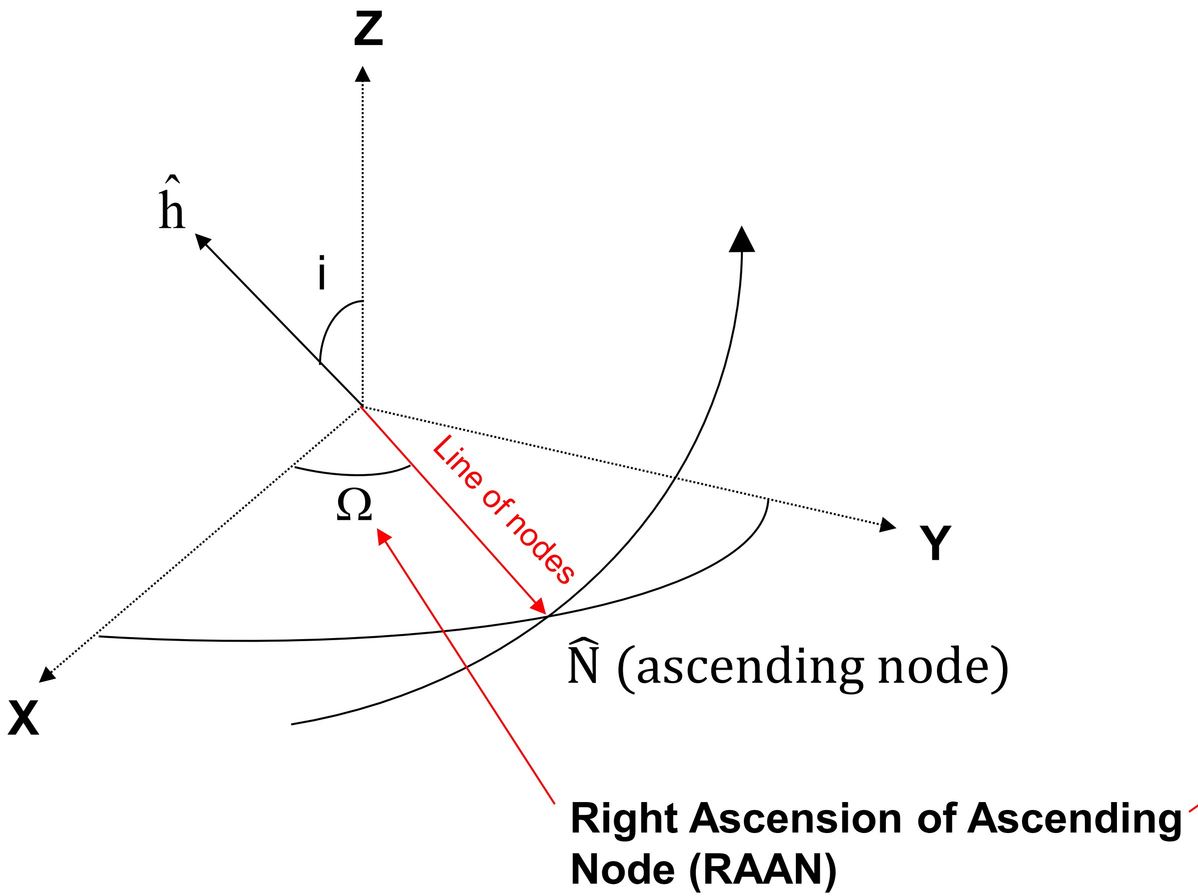
$$\hat{Z} \cdot \hat{h} = \cos i$$

$$i = \cos^{-1}(\hat{Z} \cdot \hat{h})$$

$$i \sim [0, 180°]$$

Right Ascension of Ascending Node

- Second element of the orientation of the orbit plane in space



$$\hat{N} = \frac{\hat{Z} \times \hat{h}}{|\hat{Z} \times \hat{h}|}$$

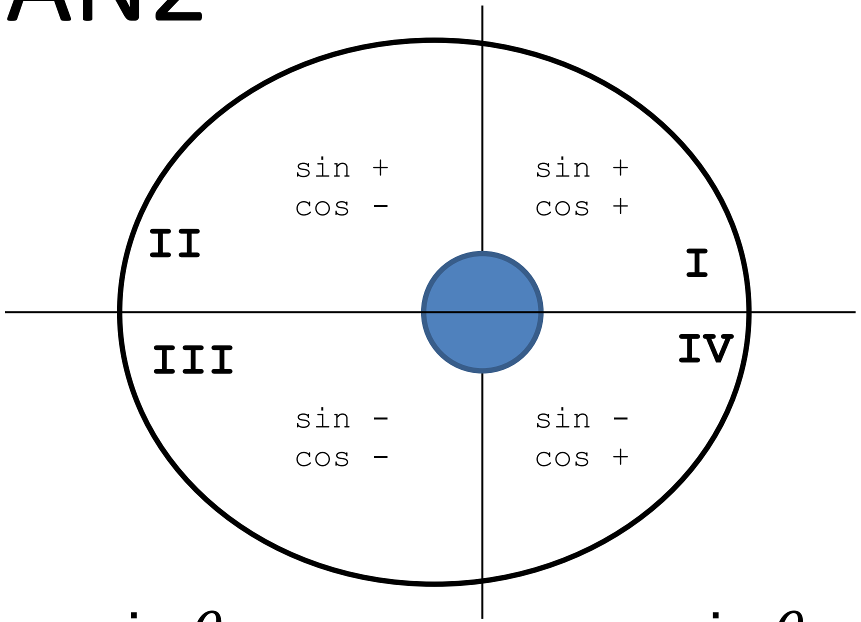
$$\hat{N} = (N_x, N_y, 0)$$

$$\Omega = \operatorname{atan2}(N_y, N_x)$$

$$\Omega \sim [0, 360°]$$

Unit Circle and ATAN2

- We deal with tangents of angles that span 2π radians within the unit circle



$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \rightarrow \quad \theta = \tan^{-1}\frac{\sin\theta}{\cos\theta}$$

- Software ATAN function returns values in range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
  - Does not return proper angle for values in Quadrants II or III

- ATAN2 → angle in range $[-\pi, \pi]$
  - Accounts for signs in different quadrants
  - If angle < 0, add 2π to get range $[0, 2\pi]$

Energy Concepts

- Kinetic Energy

$$KE = \frac{1}{2} m\, v^2 = \frac{1}{2} m\, \dot{\vec{r}} \cdot \dot{\vec{r}}$$

- Rearrange EOM

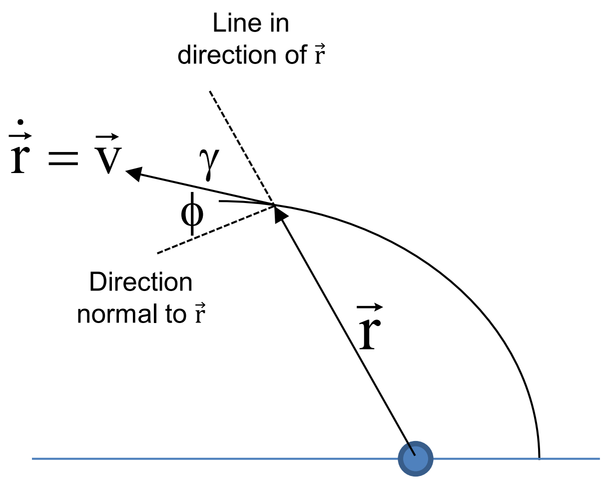
$$\ddot{\vec{r}} + \frac{\mu}{r^3} \vec{r} = 0$$

- Dot multiply by $\dot{\vec{r}}$

$$\dot{\vec{r}} \cdot \ddot{\vec{r}} + \frac{\mu}{r^3} \dot{\vec{r}} \cdot \vec{r} = 0$$

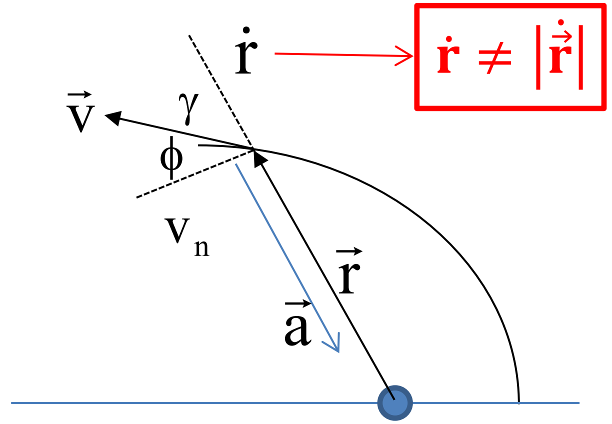
- Consider as scalar

$$\vec{r} \cdot \dot{\vec{r}} = \vec{r} \cdot \vec{v} = r\, v \cos\gamma$$



Energy Concepts (cont'd)

- Components of $\vec{v}$



$$v_n = v \sin\gamma$$
$$v_r = \dot{r} = v \cos\gamma$$

- So that

$$\boxed{\vec{r} \cdot \vec{v} = r\, v \cos\gamma = r\,(v \cos\gamma) = r\,\dot{r}}$$

- Similarly

$$\boxed{\dot{\vec{r}} \cdot \ddot{\vec{r}} = \vec{v} \cdot \dot{\vec{v}} = a\, v \cos\gamma = v\,\dot{v}}$$

- **Note:** $\dot{r}$ and $\dot{v}$ are rate of change of ***magnitude*** in radial and velocity directions, not rates of change of position and velocity vectors!

Energy Concepts (cont'd)

- Completing:

$$\dot{\vec{r}} \cdot \ddot{\vec{r}} + \frac{\mu}{r^3}\dot{\vec{r}} \cdot \vec{r} = v\,\dot{v} + \frac{\mu}{r^3} r\,\dot{r} = 0$$

$$v\,\dot{v} + \frac{\mu}{r^2}\,\dot{r} = 0$$

$$\frac{d}{dt}\left(\frac{v^2}{2}\right) = \frac{2v\,\dot{v}}{2} = v\,\dot{v}$$

$$\frac{d}{dt}\left(-\frac{\mu}{r}\right) = \frac{\mu}{r^2}\dot{r}$$

$$\frac{d}{dt}\left(\frac{v^2}{2} - \frac{\mu}{r}\right) = 0$$

$$\boxed{\underbrace{\frac{v^2}{2}}_{\text{KE}} - \underbrace{\frac{\mu}{r}}_{\text{PE}} = \xi = \text{Constant}}$$