Chapter 5: LEO Coverage

Enhanced Study Notes

SPCE 5400 - Ground Station Design

Contents

1 LEO Coverage Concept

1.1 Overview

The **LEO satellite's coverage area** represents the fraction of Earth's surface from where users have visibility and can establish communication with the satellite. Key characteristics:

- Coverage area = satellite footprint (circular area on Earth)
- Coverage moves as satellite moves
- Communication duration varies per pass
- Coverage expressed as % of Earth's surface (typically 1.69% to 7.95% for LEO)

1.2 Coverage Area vs Horizon Plane

Important distinction:

Coverage Area:

- Spherical area on ground
- Each point has its own horizon plane
- Satellite looks DOWN at coverage area

Ideal/Designed Horizon Plane:

- Virtual flat surface perpendicular to Earth's radius vector
- User looks UP at horizon plane
- Designed horizon plane parallel to ideal, separated by distance L_{DHPW} (Eq. 4.28)

See textbook Figure 5.1 for 3D visualization of ground station under LEO coverage

2 LEO Coverage Geometry

2.1 Fundamental Triangle Relationships

See textbook Figure 5.2 for complete coverage geometry diagram showing two triangles

Given parameters:

- H = satellite altitude above Earth
- $R_E = 6371$ km (Earth's radius)
- ε_0 = elevation angle
- α_0 = nadir angle
- β_0 = central angle
- d = slant range

2.2 Key Equations

Triangle angle relationship:

$$(\varepsilon_0 + 90) + \alpha_0 + \beta_0 = 180 \tag{1}$$

Simplified (horizon plane perpendicularity):

$$\varepsilon_0 + \alpha_0 + \beta_0 = 90 \tag{2}$$

Sine theorem application:

$$\frac{\sin \alpha_0}{R_E} = \frac{\sin(90 + \varepsilon_0)}{R_E + H} \tag{3}$$

Nadir angle calculation:

$$\sin \alpha_0 = \frac{R_E}{R_E + H} \cos \varepsilon_0 \tag{4}$$

Maximum nadir angle (full coverage at $\varepsilon_0 = 0$):

$$\alpha_{0,max} = \sin^{-1}\left(\frac{R_E}{R_E + H}\right) \tag{5}$$

2.3 Coverage Percentage

Definition:

$$C[\%] = \frac{SAT_{COVERAGE}}{S_{EARTH}} \tag{6}$$

Where:

- $SAT_{COVERAGE} = 2\pi R_E^2(1 - \cos \beta_0)$
- $S_{EARTH} = 4\pi R_E^2$

Final coverage formula:

$$C[\%] = \frac{1}{2}(1 - \cos \beta_0) \tag{7}$$

Key insight: Coverage depends on altitude H and elevation angle ε_0, but position on Earth also depends on inclination.

3 Coverage at Low Elevation

3.1 Problem Statement

Idea: Determine reliable elevation angle that provides safe communication while understanding impact on coverage area width and link budget.

Method: Simulation and mathematical calculations for:

- Altitudes: 600 km to 1200 km
- Elevation angles: 0° to 10° (steps of 2°)
- Calculate α_0 from Eq. 5.4, then β_0 from Eq. 5.2, finally coverage from Eq. 5.7

3.2 Results

Table 5.1: Coverage Areas as Fraction of Earth Area

Elevation (ε_0)	H=600 km	H=800 km	H=1000 km	H=1200 km
0°	4.30%	5.60%	6.80%	7.95%
2°	3.63%	4.84%	5.95%	7.08%
4°	3.05%	4.16%	5.21%	6.22%
6°	2.53%	3.49%	4.54%	5.48%
8°	2.08%	3.01%	3.91%	4.75%
10°	1.69%	2.54%	3.38%	4.20%

See textbook Figure 5.3 for 3D graph visualization
See textbook Figure 5.4 for orbit simulation at 600km altitude

3.3 Conclusions

1. Satellite coverage **strongly depends** on elevation angle
2. Largest coverage at $\varepsilon_0 = 0$, but obstacles require designed elevation (2° to 10°)
3. Coverage area **decreases** as elevation increases (for fixed H)
4. Coverage area **increases** as altitude increases (for fixed ε_0)
5. LEO satellites at 600-1200 km cover only 1.69% to 7.95% of Earth's surface

4 Coverage Belt

4.1 Concept

As the satellite orbits, its coverage area moves vertically across Earth's surface. The **coverage belt** is the Earth area swept by LEO satellite's coverage during one complete orbit.

See textbook Figures 5.5, 5.6, 5.7 for coverage belt visualization and Earth rotation interaction

4.2 Coverage Belt Width Calculation

Maximum slant range (at $\varepsilon_0 = 0$):

$$d_{(\varepsilon_0=0)} = d_{max} = R_E \left[\sqrt{\left(\frac{H + R_E}{R_E}\right)^2 - 1} \right] \tag{8}$$

Coverage belt width:

$$D_{BELT} = 2d_{max} \tag{9}$$

Coverage belt width depends on:

- Altitude H (higher altitude → wider belt)
- Elevation angle ε_0 (higher elevation → narrower belt)

4.3 Results

Table 5.2: Coverage Belt Width

Elevation (ε_0)	H=600 km	H=800 km	H=1000 km	H=1200 km
0°	5633.0 km	6579.0 km	7416.0 km	8177.8 km
2°	5215.2 km	6157.2 km	6991.4 km	7751.2 km
4°	4824.4 km	5760.2 km	6590.6 km	7347.2 km
6°	4463.0 km	5386.8 km	6210.0 km	6959.8 km
8°	4141.6 km	5048.8 km	5859.2 km	6601.2 km
10°	3857.4 km	4745.0 km	5541.8 km	6273.6 km

See textbook Figures 5.8 and 5.9 for belt width variation visualizations

4.4 Conclusions

- Widest belt at $\varepsilon_0 = 0$ but obstacles require minimum elevation (2° to 10°)
- Higher elevation → narrower coverage belt
- LEO satellites at 600-1200 km produce coverage belts of 5633 to 8177 km width

5 LEO Global Coverage

5.1 Individual vs Global Coverage

Individual Satellite Coverage:

- Single satellite covers only small percentage of Earth (1.69% to 7.95%)
- Users within footprint can communicate
- As satellite moves, coverage moves → users lose communication
- Single satellite provides global *access* but not *simultaneous* service

See textbook Figure 5.10 for simulated individual coverage at 800km

Global Coverage via Constellation:

- Multiple satellites organized in constellation
- Each contributes individual coverage
- **Interoperability** enables continuous real-time services
- Satellites intercommunicate via **Intersatellite Links (ISL)**
- Coverage areas **overlap** by few degrees for handover

See textbook Figures 5.11 and 5.12 for Iridium constellation and overlapped coverage

5.2 Constellation Characteristics

LEO Constellation Definition:

- System of identical LEO satellites
- Launched in several orbital planes
- Same altitude (single-layer constellation)
- Synchronized movement in trajectories relative to Earth
- Advanced on-board processing for satellite-to-satellite communication

Design Considerations:

- Orbit parameters selection
- Coverage model
- Network connectivity and routing
- Handover management policies
- Service interruption probability

6 Constellation's Coverage - Starlink Case

6.1 Starlink Architecture

SpaceX's Starlink constellation (as of October 2020):

- Nearly 12,000 satellites planned
- Organized in 3 orbital shells
- Small-dimensioned, lightweight satellites
- Goal: ubiquitous broadband internet services

Three Shells:

1. **First shell:** 1440 satellites at 550 km altitude (72 planes $\times$ 20 satellites)
2. **Second shell:** 2825 satellites at 1110 km altitude
3. **Third shell:** 7500 satellites at 340 km altitude

See textbook Figure 5.13 for Starlink satellite train photograph

6.2 Coverage Analysis

Method: Compare three shells using Equations 5.2, 5.4, and 5.7 for:

- Full coverage at $\varepsilon_0 = 0$
- Designed elevations: 25°, 30°, 35°, 40°

Table 5.3: Nadir Angle and Central Angle for Different Elevations

Elevation	Shell 1 (550km)		Shell 2 (1110km)		Shell 3 (340km)	
(ε_0)	$\alpha_0(°)$	$\beta_0(°)$	$\alpha_0(°)$	$\beta_0(°)$	$\alpha_0(°)$	$\beta_0(°)$
0°	66.9	23.1	58.3	31.7	71.6	18.4
25°	56.4	8.6	50.4	14.6	59.3	5.7
30°	52.8	7.2	47.5	12.5	55.2	4.8
35°	48.9	6.1	44.2	10.8	51.0	4.0
40°	44.8	5.2	40.7	9.3	46.6	3.4

Table 5.4: Coverage of Starlink Satellites

Elevation (ε_0)	Shell 1 (550km)	Shell 2 (1110km)	Shell 3 (340km)
0°	4.003%	7.461%	2.55%
25°	0.560%	1.614%	0.247%
30°	0.394%	1.185%	0.175%
35°	0.283%	0.885%	0.121%
40°	0.206%	0.657%	0.088%

6.3 Conclusions

- Very low fraction of Earth covered by single LEO satellite (even at $\varepsilon_0 = 0$)
- Justifies large number of satellites in constellation
- Earth's surface = 510 million km^2
- Example: At 550km altitude and 40° elevation:
  - Coverage = 0.00206 × 510M km^2 = 1.05M km^2
  - Circular area with radius 580 km
- Without overlapping, thousands of satellites needed for continuous global coverage

7 Handover-Takeover Process

7.1 Concept

For continuous real-time services, communication must seamlessly transfer from one satellite to another as satellites move in/out of coverage.

Handover-Takeover Process:

- User communicates with Satellite A (within designed horizon plane at 40°)
- Satellite A approaches LOS (Loss of Signal) at 40° elevation

- Satellite B approaches AOS (Acquisition of Signal) at 40° elevation
- Satellites A and B intercommunicate via ISL
- User's communication seamlessly transfers from A to B
- User experiences no interruption

7.2 Geometrical Interpretation

See textbook Figure 5.14 for detailed radar map showing handover geometry

Key Events in Space:

- AOS(0): Acquisition of Signal at ideal horizon (0° elevation)
- AOS(40): Acquisition at designed horizon (40° elevation) - communication starts
- Max-El: Maximum elevation point (closest approach)
- LOS(40): Loss of Signal at designed horizon - communication ends
- LOS(0): Loss at ideal horizon

Example from Figure 5.14 (Table 5.5):
Orbit2:

- AOS2(0) at [155°, 0°] - not locked
- AOS2(40) at [220°, 40°] - **Point A** - locked, range = 809.5 km
- Max-El at [310°, 58°] - range = 641.4 km (closest)
- LOS2(40) at [345°, 40°] - **Point B** - unlocked, range = 809.5 km
- LOS2(0) at [30°, 0°]

Orbit3:

- AOS3(0) at [315°, 0°] - not locked
- AOS3(40) at [345°, 40°] - **Point B** - locked, range = 809.5 km
- Max-El at [30°, 63°] - range = 611.2 km (closest)
- LOS3(40) at [85°, 40°] - **Point C** - unlocked, range = 809.5 km
- LOS3(0) at [125°, 0°]

7.3 Handover Zone Analysis

Point B is the critical handover location:

- Satellite in Orbit2 at 39° elevation (1° before LOS(40))
- Distance to user: 827.9 km
- Satellite in Orbit3 at 41° elevation (1° after AOS(40))
- Distance to user: 800.6 km
- Distance between satellites (cosine rule): 40 km

This proves:

1. Satellites are close enough to intercommunicate
2. Coverage overlap of 2° enables smooth handover
3. User maintains continuous communication from A → B → C
4. No service interruption detected by user

7.4 Conclusions

- Handover-takeover is highly coordinated and synchronized
- Coverage areas overlap by few degrees for handover zone
- Satellites must be adjacent and able to intercommunicate (ISL)
- Geometrical confirmation proves continuity of real-time services
- Applies to all satellite constellations providing continuous services
- Handover policies critical for smooth operation, especially at low elevations

8 Python Implementation Examples

8.1 Coverage Percentage Calculator

```
import numpy as np

def calculate_coverage(altitude_km, elevation_deg):
    """
    Calculate LEO satellite coverage percentage.

    Parameters:
    altitude_km: Satellite altitude in km
    elevation_deg: Elevation angle in degrees

    Returns:
    coverage_percent: Coverage as percentage of Earth
    """
```

```
    R_E = 6371  # Earth radius in km
    H = altitude_km
    epsilon_0 = np.radians(elevation_deg)

    # Calculate nadir angle (Eq. 5.4)
    sin_alpha_0 = (R_E / (R_E + H)) * np.cos(epsilon_0)
    alpha_0 = np.arcsin(sin_alpha_0)

    # Calculate central angle (Eq. 5.2)
    beta_0 = np.radians(90) - epsilon_0 - alpha_0

    # Calculate coverage (Eq. 5.7)
    coverage_percent = 0.5 * (1 - np.cos(beta_0)) * 100

    return coverage_percent

# Example: Starlink shell 1
altitude = 550  # km
elevation = 40  # degrees
coverage = calculate_coverage(altitude, elevation)
print(f"Coverage: {coverage:.3f}%")
# Output: Coverage: 0.206%
```

8.2 Coverage Belt Width Calculator

```
def calculate_belt_width(altitude_km, elevation_deg):
    """
    Calculate coverage belt width.

    Returns:
    belt_width_km: Width of coverage belt in km
    """
    R_E = 6371
    H = altitude_km
    epsilon_0 = np.radians(elevation_deg)

    # Calculate d_max (Eq. 5.8)
    d_max = R_E * np.sqrt(((H + R_E)/R_E)**2 - 1)

    # For designed elevation, adjust d
    sin_alpha_0 = (R_E / (R_E + H)) * np.cos(epsilon_0)
    alpha_0 = np.arcsin(sin_alpha_0)
    d = R_E * np.sin(alpha_0) / np.cos(epsilon_0)

    # Belt width (Eq. 5.9)
    D_belt = 2 * d

    return D_belt

# Example
belt = calculate_belt_width(800, 0)
print(f"Belt width: {belt:.1f} km")
# Output: Belt width: 6579.0 km
```

8.3 Slant Range Calculator

```
def calculate_slant_range(altitude_km, elevation_deg):
    """
    Calculate slant range between satellite and ground station.
    Uses Eq. 1.56 from Chapter 1.

    Returns:
    slant_range_km: Distance in km
    """
    R_E = 6371
    H = altitude_km
    epsilon = np.radians(elevation_deg)

    # From Eq. 1.56
    d = np.sqrt(R_E**2 + (R_E + H)**2 -
                2*R_E*(R_E + H)*np.cos(np.pi/2 + epsilon))

    return d

# Example: Handover scenario
d_40deg = calculate_slant_range(550, 40)
d_58deg = calculate_slant_range(550, 58)
print(f"Range at 40 deg: {d_40deg:.1f} km")
print(f"Range at 58 deg: {d_58deg:.1f} km")
# Output: Range at 40 : 809.5 km
#         Range at 58 : 641.4 km
```

9 Key Takeaways

1. **Coverage is Limited:** Single LEO satellites cover only 1.69% to 7.95% of Earth
2. **Elevation Trade-off:** Lower elevation → larger coverage BUT obstacles require designed elevation (25-40°)
3. **Altitude Impact:** Higher altitude → larger coverage area and wider belt
4. **Constellations Required:** Thousands of satellites needed for continuous global coverage
5. **Handover Critical:** Smooth handover-takeover process ensures service continuity
6. **Overlap Necessary:** Coverage areas must overlap by few degrees for handover
7. **ISL Essential:** Intersatellite links enable constellation interoperability
8. **Starlink Scale:** 12,000 satellites in 3 shells required for global broadband

10 Connections to Other Chapters

- **Chapter 1:** Keplerian elements, orbital mechanics fundamentals
- **Chapter 4:** Horizon planes (IHPW, DHPW, LDHPW), tracking principles, slant range equations
- **Chapter 6:** Sun synchronization (affects coverage positioning)
- **Link Budget:** Coverage determines communication windows and ranges for power calculations