

LEO Coverage & Sun Synchronization

Comprehensive Study Guide - Chapters 5 & 6

How to Use This Guide

This study guide is designed using neuroscience-backed learning principles:

- **Spaced Repetition:** Review each section multiple times over days/weeks
- **Active Recall:** Test yourself with the questions before looking at answers
- **Chunking:** Information is broken into digestible sections
- **Dual Coding:** Visual descriptions complement mathematical concepts
- **Interleaving:** Related concepts are presented together for deeper understanding

CHAPTER 5: LEO COVERAGE

Core Concept: What is LEO Coverage?

Definition: The fraction of Earth's surface from where users can see, access, and communicate with a satellite.

Key Visualization

Coverage is the circular area on Earth from where the satellite can be seen:

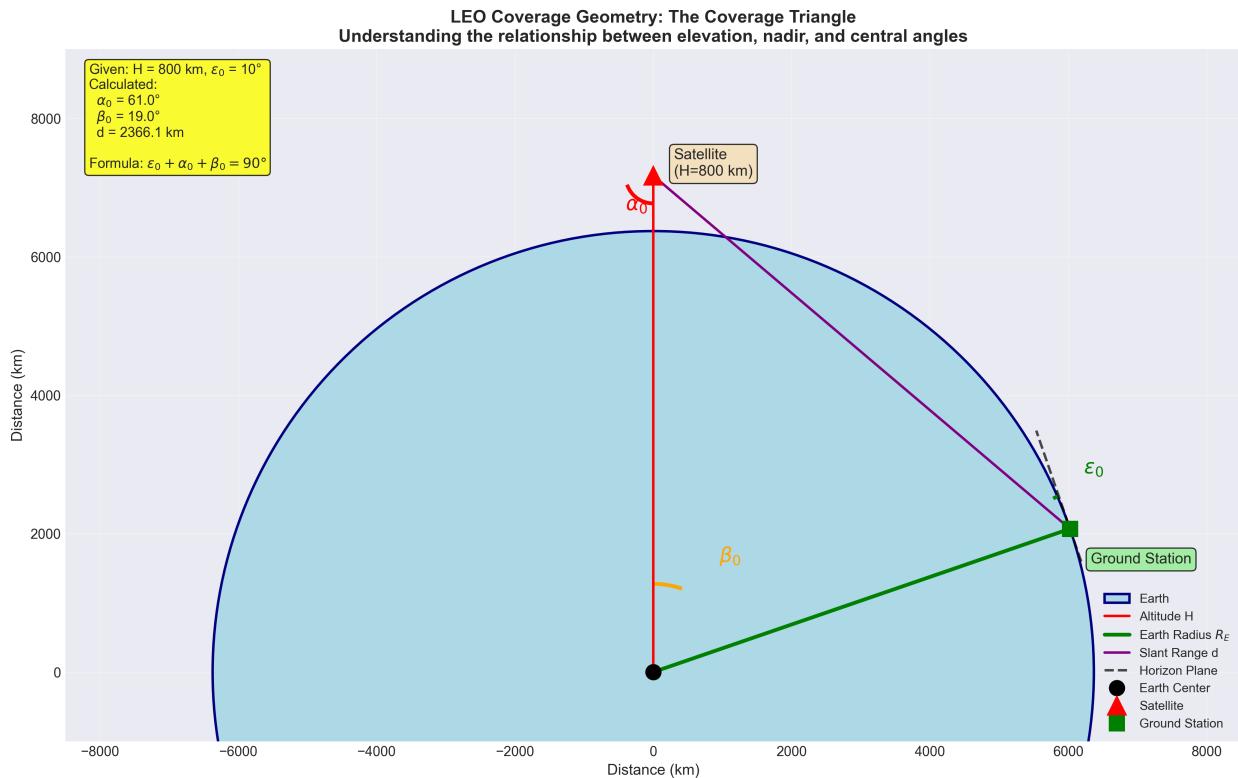


Figure 1: Coverage Geometry

This diagram shows the fundamental triangle relationship between the satellite, ground station, and Earth's center. Understanding this geometry is KEY to all coverage calculations!

5.1 Coverage Fundamentals

Essential Terms (Memorize These!)

Term	Definition	Why It Matters
Coverage Area	Circular area on Earth where satellite is visible	Determines who can communicate
Satellite Footprint	Same as coverage area - the "shadow" of coverage	Visual way to think about coverage
Elevation Angle (ε)	Angle from horizon to satellite	Minimum angle needed for clear signal
Ideal Horizon Plane	Virtual flat surface to Earth's radius vector	Theoretical maximum visibility
Designed Horizon Plane	Practical horizon with minimum elevation	Accounts for real-world obstacles

Key Insight

Each user within a satellite's coverage area has their own horizon plane - this is why different users see the satellite at different angles!

5.2 Coverage Geometry & Mathematics

The Coverage Triangle

The coverage triangle shows the geometric relationship between: - \mathbf{H} = Satellite altitude - \mathbf{RE} = Earth radius (6,371 km) - ε_0 = Elevation angle - α_0 = Nadir angle - β_0 = Central angle - \mathbf{d} = Slant range (distance to satellite)

See the detailed diagram above for the complete geometric visualization!

Master Formula Set

Triangle Relationship:

$$(\varepsilon_0 + 90^\circ) + \alpha_0 + \beta_0 = 180^\circ$$

(Equation 5.1)

Simplified:

$$\varepsilon_0 + \alpha_0 + \beta_0 = 90^\circ$$

(Equation 5.2)

Sine Law Application:

$$\frac{\sin \alpha_0}{R_E} = \frac{\sin(90 + \varepsilon_0)}{R_E + H}$$

(Equation 5.3)

Nadir Angle Formula:

$$\sin \alpha_0 = \frac{R_E}{R_E + H} \cos \varepsilon_0$$

(Equation 5.4)

Maximum Nadir Angle (at $\theta = 0^\circ$):

$$\alpha_{0,max} = \sin^{-1} \left(\frac{R_E}{R_E + H} \right)$$

(Equation 5.5)

THE COVERAGE FORMULA

Coverage as percentage of Earth's surface:

$$C[\%] = \frac{1}{2}(1 - \cos \beta_0)$$

(Equation 5.7)

Where: - $SAT_{COVERAGE} = 2\pi R_E^2 (1 - \cos \beta_0)$ - $S_{EARTH} = 4\pi R_E^2$

Active Recall Question #1

Before looking at the answer below, try to answer:

Q: Why is LEO coverage only a few percent of Earth's surface?

Click for Answer

A: Because: 1. LEO satellites are relatively close to Earth (600-1200 km vs Earth's radius 6,371 km) 2. The coverage circle is determined by the central angle 3. Even at 0° elevation (maximum coverage), the nadir angle is limited by geometry 4. Result: Only 1.69% to 7.95% of Earth covered at typical LEO altitudes

Mnemonic: “Low and Local” - LEOs fly low, so they see locally, not globally

5.3 Coverage at Low Elevation - Critical Data

Why Elevation Matters

- **0° elevation:** Maximum coverage BUT obstacles (buildings, mountains) block signal
- **Practical minimum:** 2° to 10° elevation for reliable communication
- **Trade-off:** Higher elevation = Better signal quality, Smaller coverage area

Coverage Table (Memorize Key Values)

Altitude	Elevation	Coverage
600 km	0°	4.30%
600 km	10°	1.69%
1200 km	0°	7.95%
1200 km	10°	4.20%

Pattern Recognition

Two key relationships (inverse proportional): 1. \uparrow Elevation angle $\rightarrow \downarrow$ Coverage (for same altitude) 2. \uparrow Altitude $\rightarrow \uparrow$ Coverage (for same elevation)

Left plot shows the trade-off between elevation and coverage. Right plot shows how altitude affects coverage.

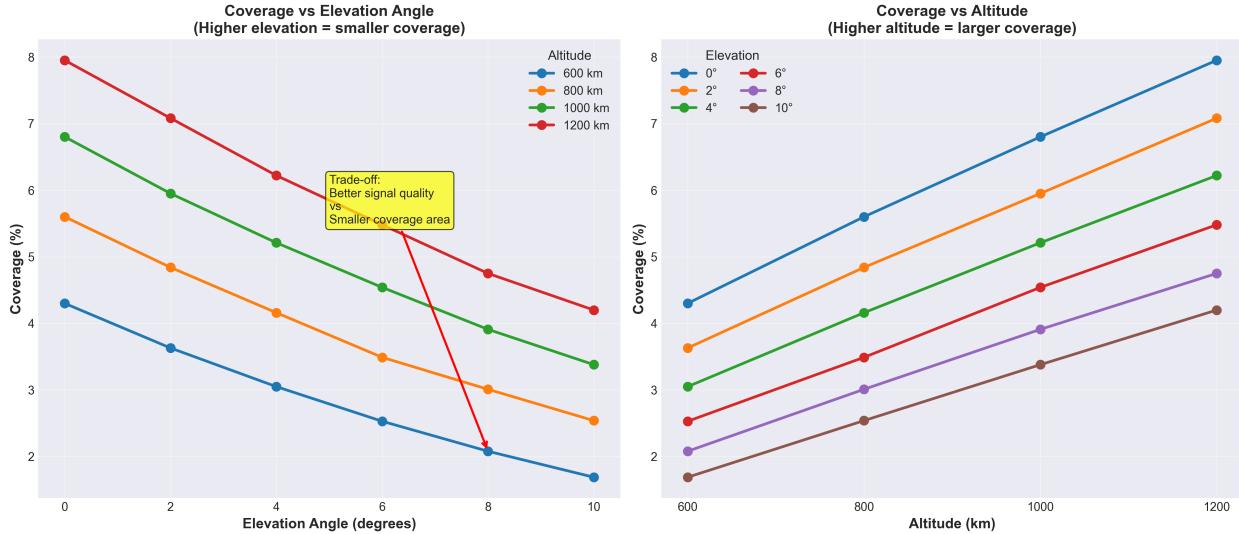


Figure 2: Coverage Analysis

5.4 Coverage Belt

Concept Visualization

This visualization shows how the satellite's coverage area sweeps across Earth's surface, creating a belt. The belt width determines the geographical area accessible during one orbital pass.

Belt Width Formula

Maximum radius of coverage:

$$d_{(\varepsilon_0=0)} = d_{max} = R_E \left[\sqrt{\left(\frac{H + R_E}{R_E} \right)^2 - 1} \right]$$

(Equation 5.8)

Belt Width:

$$D_{BELT} = 2d_{max}$$

(Equation 5.9)

Belt Width Data

Altitude	Elevation	Belt Width
600 km	0°	5,633 km
600 km	10°	3,857 km
1200 km	0°	8,178 km
1200 km	10°	6,274 km

Key Takeaway Box

Coverage Belt Width depends on:

1. Altitude (H)

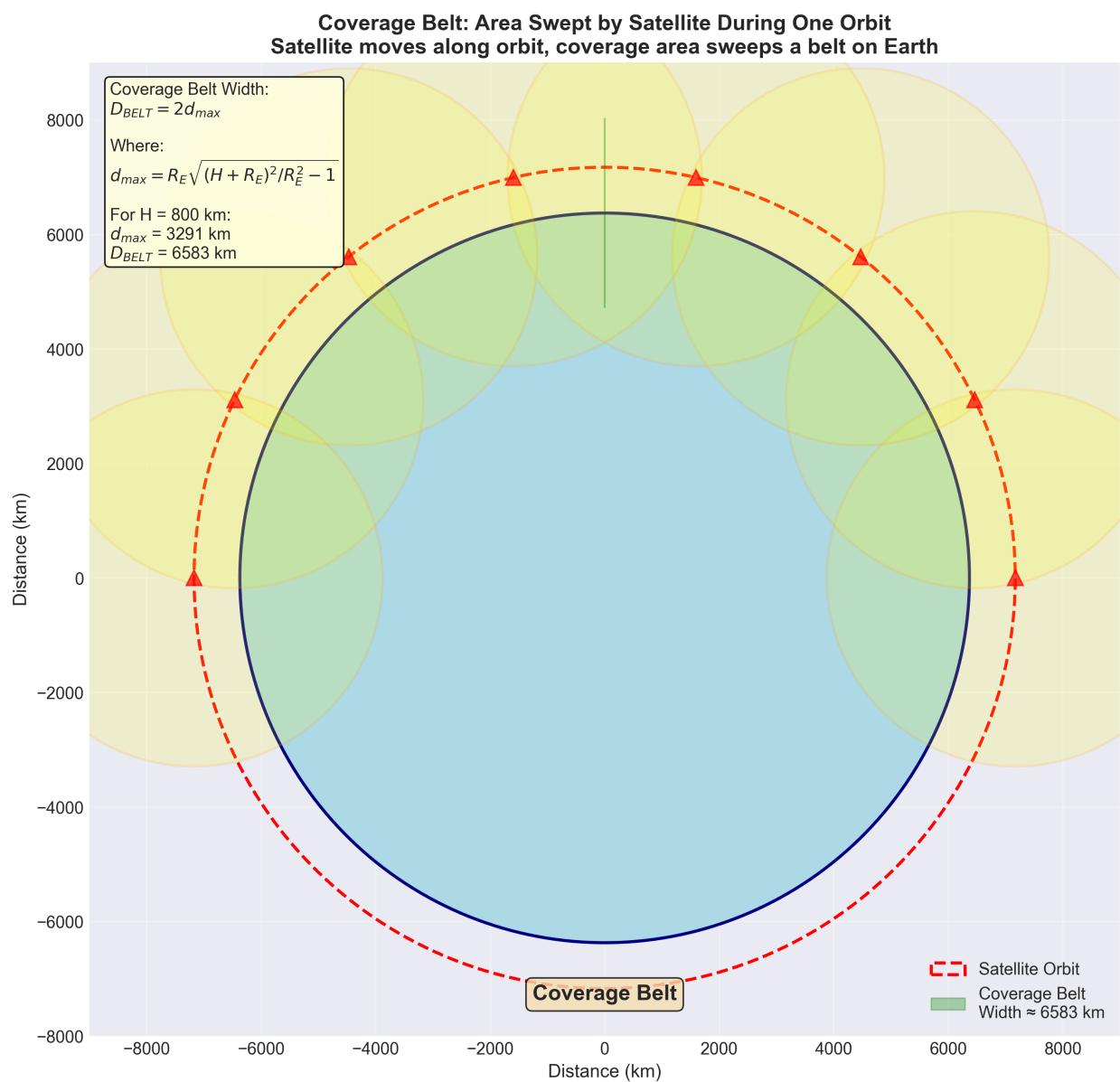


Figure 3: Coverage Belt

2. Elevation angle ()

Higher altitude OR lower elevation
→ WIDER belt

5.5 Global Coverage & Constellations

Individual vs. Global Coverage

Individual Coverage: Single satellite covers small area (few %)

Global Coverage: Multiple satellites in constellation provide: - Continuous service - Real-time communication - Worldwide accessibility

The Coverage Problem

Single Satellite:

[Coverage Area moving] → [Leaves region] → [Communication lost]

Constellation:

[Sat 1] → [Handover] → [Sat 2] → [Handover] → [Sat 3]
↓ ↓
[Continuous Service] [No gaps]

Critical Concepts

1. **Interoperability:** Satellites communicate with each other
 2. **Handover:** Transfer of user from one satellite to another
 3. **Overlapping Coverage:** Ensures smooth transitions (typically few degrees overlap)
 4. **Intersatellite Links (ISL):** Enable satellite-to-satellite communication
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5.6 Starlink Case Study

Three-Shell Architecture

Shell	Satellites	Altitude
First	1,440	550 km
Second	2,825	1,110 km
Third	7,500	340 km

Total planned: ~12,000 satellites

Coverage Comparison

550 km altitude: - 0° elevation: 4.003% - 40° elevation: 0.206%

1,110 km altitude: - 0° elevation: 7.461% - 40° elevation: 0.657%

Why So Many Satellites?

At 550 km altitude, 40° elevation:
 - Coverage per satellite: 0.206% of Earth
 - Earth surface: 510 million km²
 - Coverage area: ~1.05 million km² per satellite
 - Radius: ~580 km

To cover entire Earth with overlap → Thousands of satellites needed!

5.7 Handover-Takeover Process

Geometric Understanding

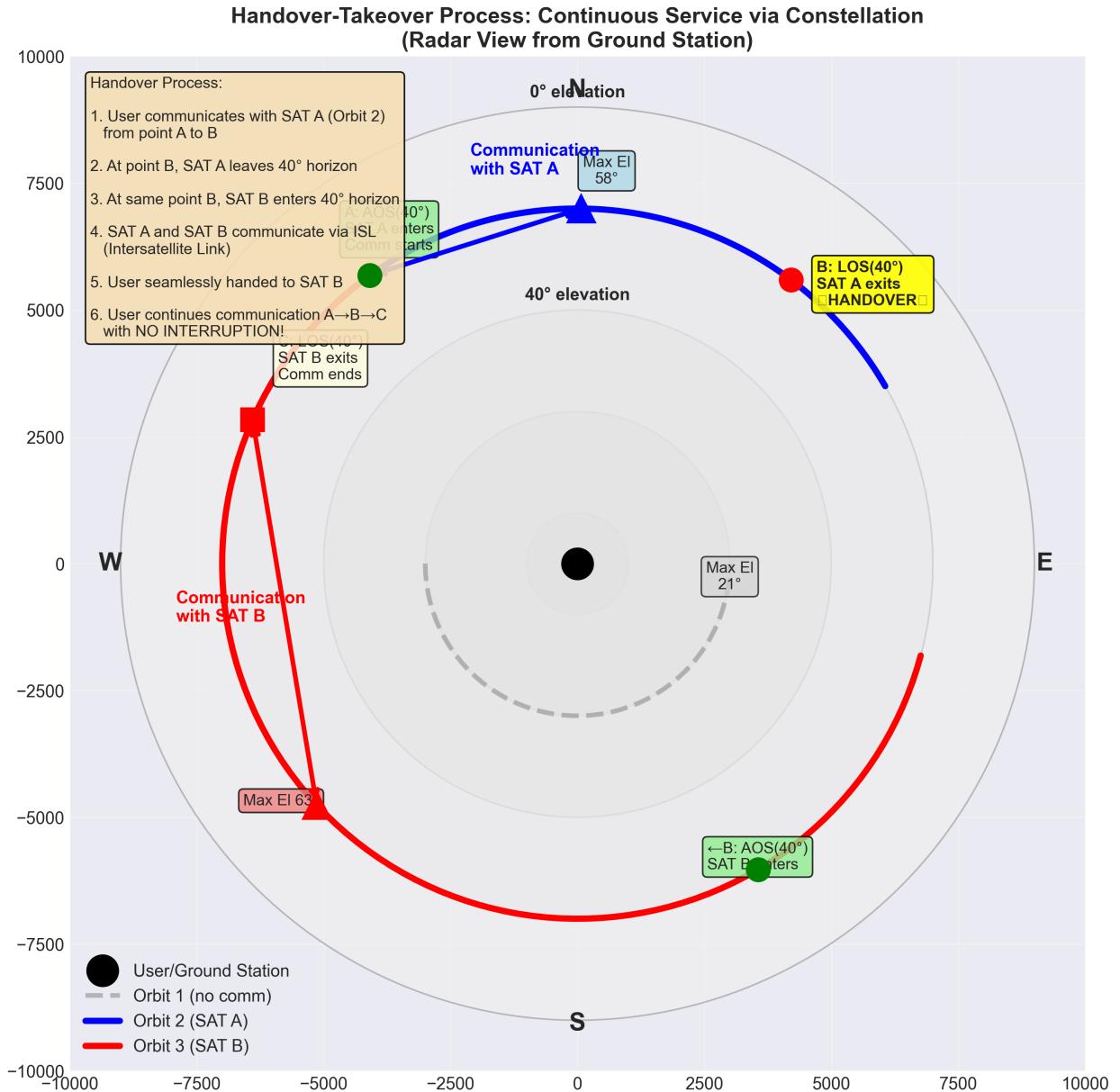


Figure 4: Handover Process

This radar view shows the handover zone where two satellites overlap in coverage. The blue satellite (Sat

1) is leaving while the red satellite (Sat 2) is entering the user's designed horizon. The green shaded area represents the critical overlap zone where seamless communication transfer occurs.

Key Events (Memorize!)

Event	Meaning	Significance
AOS(0)	Acquisition of Signal at 0°	Satellite enters ideal horizon
AOS(40)	Acquisition at 40°	Satellite enters designed horizon - Communication starts
Max-El	Maximum Elevation	Closest approach - strongest signal
LOS(40)	Loss of Signal at 40°	Satellite leaves designed horizon - Communication ends
LOS(0)	Loss at 0°	Satellite leaves ideal horizon

Handover Requirements

For successful handover at point B (overlap zone): 1. Sat 1 must reach LOS(40) 2. Sat 2 must reach AOS(40) 3. **Both satellites must be able to communicate with each other** 4. Distance between satellites: ~40 km (manageable for ISL)

Active Recall Question #2

Q: Calculate the coverage percentage for a satellite at 800 km altitude with 5° elevation. Use the formulas!

Click for Answer

Given: - H = 800 km - $\alpha_0 = 5^\circ$ - RE = 6,371 km

Step 1: Calculate $\sin(\alpha_0)$

$$\sin \alpha_0 = \frac{6371}{6371 + 800} \cos(5^\circ) = \frac{6371}{7171} \times 0.996 = 0.884$$

Step 2: Calculate

$$\alpha_0 = \sin^{-1}(0.884) = 62.1^\circ$$

Step 3: Calculate

$$\beta_0 = 90^\circ - 5^\circ - 62.1^\circ = 22.9^\circ$$

Step 4: Calculate Coverage

$$C = \frac{1}{2}(1 - \cos(22.9^\circ)) = \frac{1}{2}(1 - 0.921) = 0.0395 = 3.95\%$$

Answer: 3.95% of Earth's surface

CHAPTER 6: LEO SUN SYNCHRONIZATION

6.1 Sun Synchronization Concept

The Core Problem

For **photo imagery missions**, we need: - Same lighting conditions for all images - Consistent illumination across different passes - Comparable data over time

The Solution: Sun-Synchronous Orbits (SSO)

Definition: An orbit that maintains a **constant angle** between: - The orbital plane normal vector, AND - The Sun direction vector

Throughout the entire year!

Seasonal Consistency

As Earth orbits the Sun throughout the year, the satellite's orbital plane precesses (via nodal regression) at exactly the right rate to keep a constant angle with the Sun direction.

This 3D visualization shows a sun-synchronous orbit (in blue) with Earth at the center. The yellow arrow points toward the Sun, and the orbital plane maintains a constant orientation relative to this Sun direction throughout the year. The equatorial plane is shown in light gray.

6.2 Earth's Shape & Nodal Regression

Why Earth's Shape Matters

Perfect sphere: Orbital plane would be fixed in space **Reality (Oblate ellipsoid):** Earth bulges at equator (~21 km)

This equatorial bulge creates an asymmetric gravitational field that causes orbital perturbations.

J Harmonic Coefficient

$$J_2 = 1.0827 \times 10^{-3}$$

This tiny number describes Earth's oblateness and causes **nodal regression!**

Nodal Regression Explained

What is Nodal Regression?

The **line of nodes** (intersection of orbital plane with equatorial plane) shifts over time due to Earth's bulge.

Visual Understanding

These plots show how nodal regression varies with inclination (left) and the critical sun-synchronous inclination window (right, shaded in green). The 600-1200 km altitude range creates a narrow 2.6° window for sun-synchronous orbits.

Direction Rules

Inclination	$\cos(i)$ sign	Regression Direction
$i < 90^\circ$ (Prograde)	Negative	Westward (opposite to Earth rotation)
$i = 90^\circ$ (Polar)	Zero	No regression
$i > 90^\circ$ (Retrograde)	Positive	Eastward (same as Earth rotation)

3D View: Sun-Synchronous Orbit
Orbital plane maintains constant angle with Sun direction

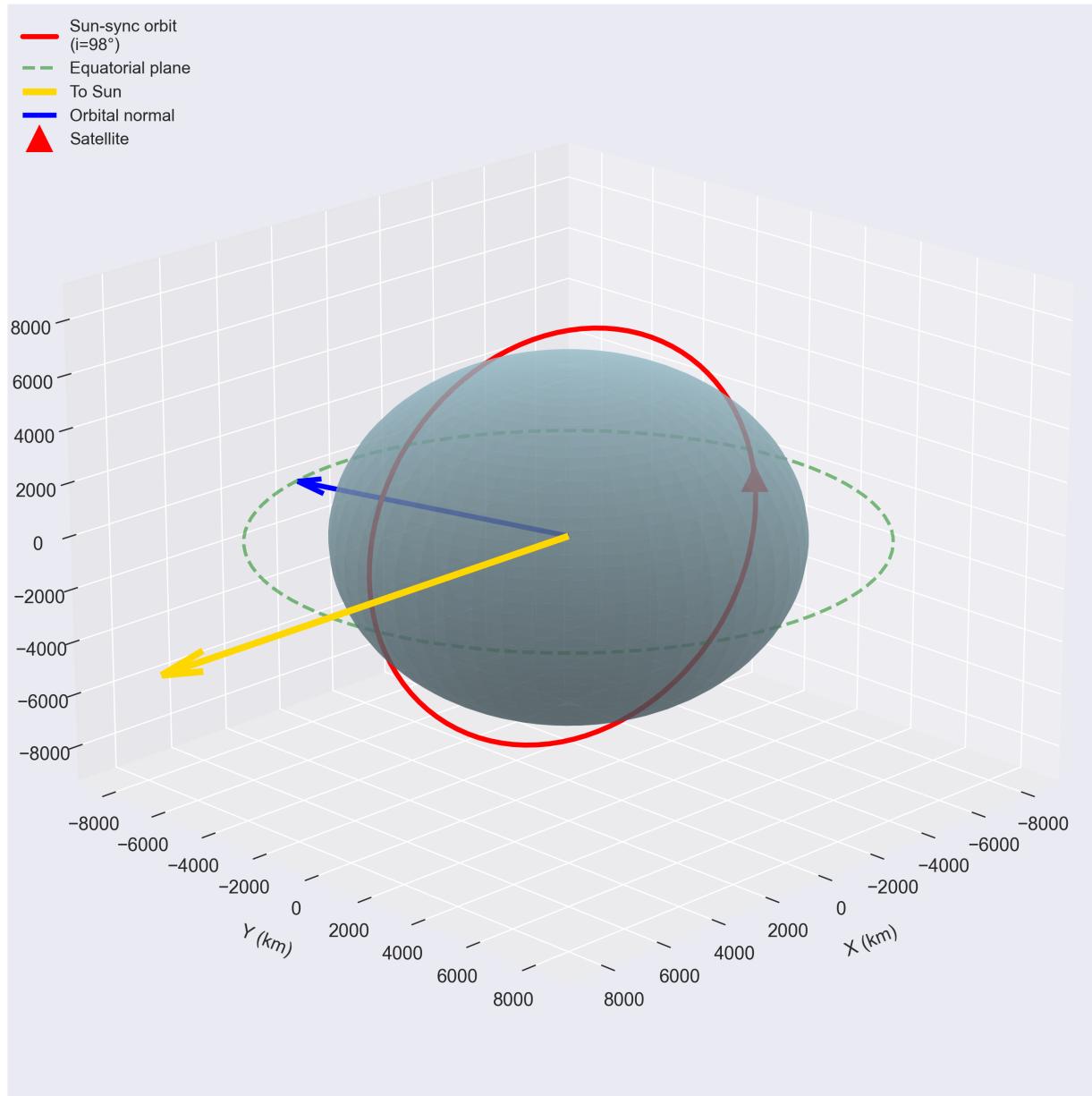


Figure 5: 3D Sun-Synchronous Orbit

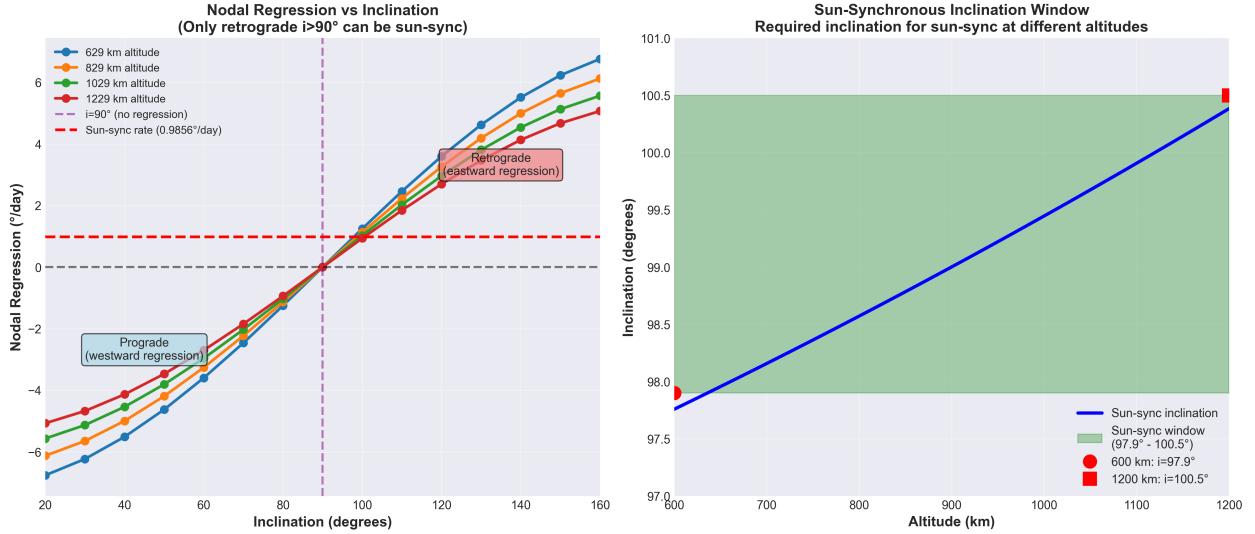


Figure 6: Nodal Regression Analysis

Master Formula for Nodal Regression

$$\frac{d\Omega}{dt} = - \left(\frac{3}{2} \right) n_0 A J_2 \cos i$$

(Equation 6.2)

Where: - $n_0 = \frac{2\pi}{T}$ (mean satellite movement) - $A = \frac{R_E^2}{r^2}$ (for circular orbits) - T = orbital period

Simplified for Circular LEO

$$\Delta\Omega = -2.06474 \times 10^{14} \cdot \frac{\cos i}{r^{7/2}} \quad [\text{°}/\text{day}]$$

(Equation 6.7)

6.3 Nodal Regression Data

Critical Values to Remember

Altitude	Inclination	Nodal Regression
600 km	90°	0°/day
600 km	100°	+1.24°/day
600 km	20°	-6.74°/day
1200 km	100°	+0.94°/day

Pattern Box

Nodal Regression Patterns:

1. Higher altitude → LESS regression
2. $i=90^\circ$ → ZERO regression
3. $i>90^\circ$ → POSITIVE (eastward)

4. $i < 90^\circ \rightarrow$ NEGATIVE (westward)

Key: Only RETROGRADE orbits ($i > 90^\circ$)
can be Sun-synchronized!

6.3 Achieving Sun Synchronization

The Magic Number

Earth revolves around Sun once per year (365.25 days)

Angular rotation rate:

$$\frac{360^\circ}{365.25 \text{ days}} = 0.986^\circ/\text{day}$$

The Synchronization Condition

$$\frac{d\Omega}{dt} = 0.9856^\circ/\text{day}$$

(Equation 6.8)

Solving for Inclination

From Equation 6.9:

$$-2.06474 \times 10^{14} \cdot \frac{\cos i}{r^{7/2}} = 0.9856$$

For 600 km altitude ($r = 7000$ km):

$$i_1 = 97.9^\circ$$

For 1200 km altitude ($r = 7600$ km):

$$i_2 = 100.5^\circ$$

6.3 The Inclination Window

Sun-Synchronous Range

For LEO altitudes 600-1200 km:

$$97.9^\circ \leq i \leq 100.5^\circ$$

Window width: 2.6°

Why This Matters

Below 97.9° :	Within Window:	Above 100.5° :
Not enough regression	Perfect Sun-sync!	Too much regression

Active Recall Question #3

Q: Why can't MEO or GEO satellites be Sun-synchronized?

Click for Answer

A: Because nodal regression depends on $r^{-7/2}$:

1. **LEO** ($r \approx 7,000$ km): Regression $1^\circ/\text{day}$
2. **MEO** ($r \approx 20,000$ km): Regression $0.02^\circ/\text{day}$
3. **GEO** ($r \approx 42,000$ km): Regression $0.001^\circ/\text{day}$

The regression becomes **negligible** at higher altitudes - not enough to match Earth's $0.986^\circ/\text{day}$ rotation around the Sun!

Mnemonic: "High and Dry" - High orbits have dried up (negligible) nodal regression

6.4 Perigee Deviation

What is Argument of Perigee ()?

The angle from the line of nodes to the orbital perigee (closest approach point).

Why It Changes

Just like the line of nodes, Earth's oblateness causes the **major axis to rotate** within the orbital plane!

This diagram shows how the argument of perigee () changes over time due to Earth's oblateness. The major axis of the orbit rotates within the orbital plane, causing the perigee point to drift. For sun-synchronous orbits, this deviation is approximately 10-13 arcminutes per orbit.

General Formula

$$\Delta\omega = 0.29 \left[\frac{4 - 5 \sin^2 i}{(1 - e^2)^2} \right] \left[\left(\frac{D}{r_a + r_p} \right)^2 \right]$$

(Equation 6.12)

Where: - D = Earth's diameter - ra, rp = apogee, perigee radii - e = eccentricity

For Circular LEO ($e = 0$)

$$\Delta\omega = 0.29 \left(\frac{R_E}{a} \right) [4 - 5 \sin^2 i]$$

(Equation 6.14)

Special Case: No Deviation!

$$\sin^2(63.4^\circ) = 0.8 \rightarrow 4 - 5(0.8) = 0$$

At $i = 63.4^\circ$, there's NO perigee deviation! (Used for Molnya orbits)

Perigee Deviation for SSO

For Sun-Sync Window ($97.9^\circ - 100.5^\circ$)

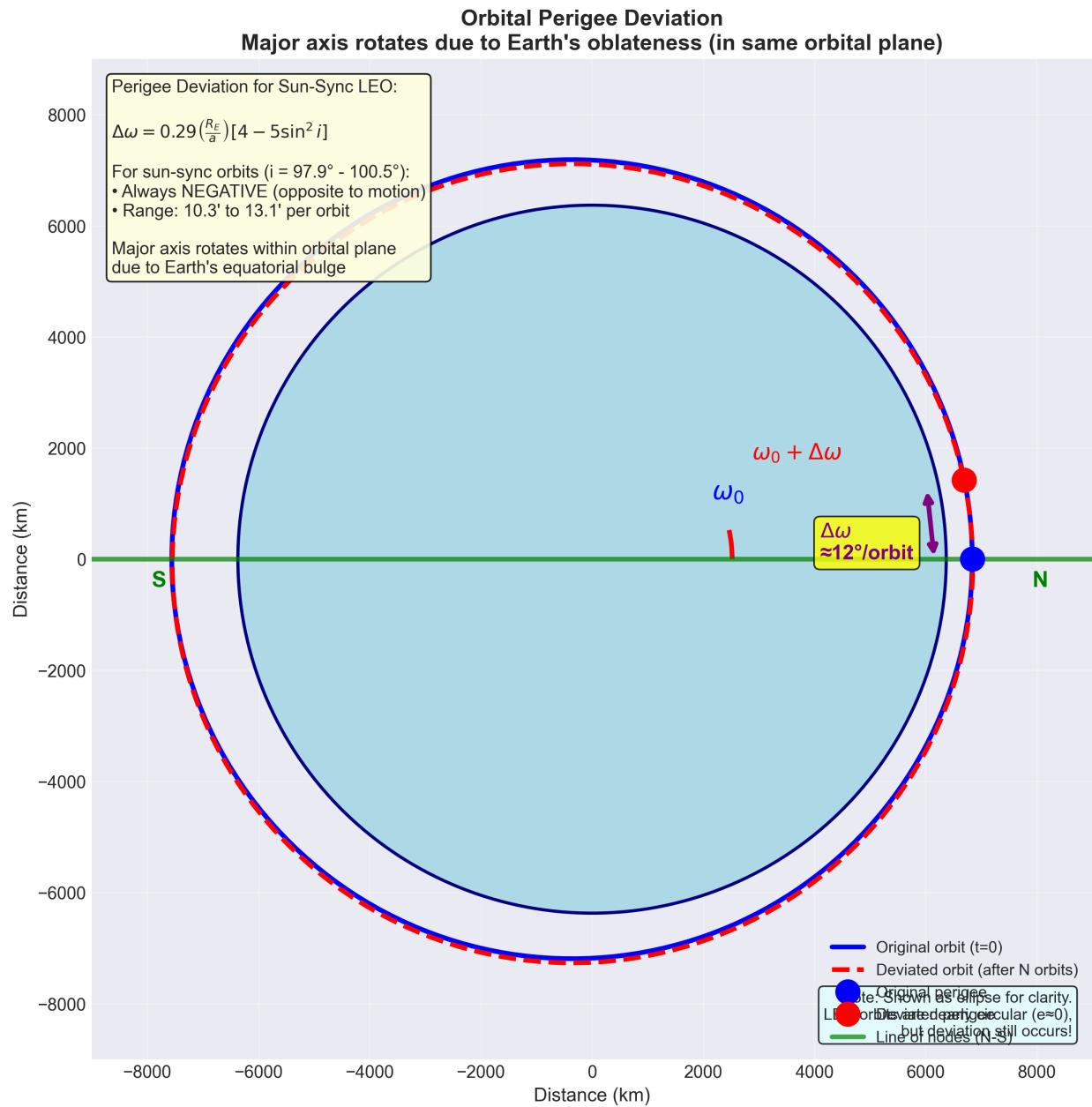


Figure 7: Perigee Deviation

Altitude	Inclination	Perigee Deviation
600 km	97.9°	-0.219°/orbit = -13.1' / orbit
600 km	100.5°	-0.202°/orbit = -12.1' / orbit
1200 km	97.9°	-0.186°/orbit = -11.2' / orbit
1200 km	100.5°	-0.171°/orbit = -10.3' / orbit

Range for Sun-Sync Orbit

$$10.3' \leq \Delta\omega \leq 13.1' \quad [{}'/orbit]$$

Direction: Always opposite to satellite motion (negative)

Memory Palace Technique

Create a mental journey through these concepts:

Room 1 - Coverage (Living Room): - Picture a satellite lamp casting a circular light (coverage area) on your floor - The higher you raise it (altitude), the wider the circle - Tilting your head (elevation angle) makes it harder to see the lamp

Room 2 - Belt (Hallway): - Imagine the satellite lamp moving down the hallway - It sweeps a belt of light as it moves - Width depends on lamp height

Room 3 - Constellation (Kitchen): - Multiple lamps (satellites) covering the whole kitchen - As one dims (LOS), another brightens (AOS) - Smooth handover like passing a baton

Room 4 - Sun Sync (Bedroom): - Window always faces the same direction to the Sun - Earth's lumpiness (pillow under sheets) makes the bed tilt - Tilt rate of 0.986°/day keeps Sun angle constant - Only retrograde beds (tilted backwards >90°) can maintain this!

Room 5 - Deviations (Bathroom): - Mirror rotates (nodal regression) due to Earth's bulge - Toothbrush rotates in holder (perigee deviation) - Both caused by Earth not being a perfect sphere

Summary Cheat Sheet

Quick Reference Formulas

Concept	Formula	Equation #
Coverage %	$C = \frac{1}{2}(1 - \cos \beta_0)$	5.7
Belt Width	$D_{BELT} = 2d_{max}$	5.9
Nodal Regression	$\Delta\Omega = -2.06474 \times 10^{14} \frac{\cos i}{r^{7/2}}$	6.7
Sun-Sync Condition	$\frac{d\Omega}{dt} = 0.9856^\circ/\text{day}$	6.8
Perigee Deviation	$\Delta\omega = 0.29 \left(\frac{R_E}{a} \right) [4 - 5 \sin^2 i]$	6.14

Key Constants

- Earth radius: RE = 6,371 km (use 6,400 for quick calcs)
- $J = 1.0827 \times 10^{-3}$
- Earth rotation around Sun: 0.9856°/day
- Typical LEO: 600-1200 km altitude

Critical Insights

1. **LEO coverage is TINY:** 1.69% - 7.95% per satellite
 2. **Constellations are NECESSARY:** Single satellite can't provide global real-time service
 3. **Only LEOs can be Sun-sync:** MEO/GEO have negligible nodal regression
 4. **SSOs are RETROGRADE:** $i = 97.9^\circ - 100.5^\circ$ for 600-1200 km
 5. **Overlapping coverage is ESSENTIAL:** Enables smooth handover
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Self-Test Questions

Level 1: Basic Understanding

1. What is the coverage area of a satellite?
2. Why do we use designed horizon planes instead of ideal horizon planes?
3. What is nodal regression?
4. What does "Sun-synchronous" mean?

Level 2: Conceptual Connections

5. Why does higher altitude increase coverage percentage?
6. Explain why only retrograde orbits can be Sun-synchronized
7. How does Earth's oblateness cause nodal regression?
8. Why is handover necessary for global coverage?

Level 3: Calculations

9. Calculate coverage % for $H=700$ km, $\epsilon=8^\circ$
10. Find the inclination for Sun-sync orbit at 900 km altitude
11. Determine belt width for $H=800$ km, $\epsilon=0^\circ$
12. Calculate nodal regression rate for $i=98^\circ$, $r=7200$ km

Level 4: Application

13. Design a constellation for global coverage with 95% uptime
 14. Explain why Starlink needs thousands of satellites
 15. What are the trade-offs between low and high elevation angles?
 16. How would you optimize a Sun-sync orbit for polar regions?
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Further Study Connections

Related Topics to Explore

- **Link Budget:** Uses coverage geometry for signal calculations
- **Doppler Shift:** Related to satellite velocity and user geometry
- **Ground Track:** How SSOs create repeating ground tracks
- **Molnya Orbits:** Use the 63.4° zero-perigee-deviation angle
- **Kepler Elements:** The 6 parameters that fully describe an orbit

Real-World Applications

- Weather satellites (NOAA series)
- Earth observation (Landsat, Sentinel)
- Communication constellations (Starlink, OneWeb)
- Military reconnaissance

- Climate monitoring
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Spaced Repetition Schedule

Day 1: Read entire guide **Day 3:** Review all summary boxes and formulas **Day 7:** Do all Level 1 & 2 self-test questions **Day 14:** Do all Level 3 & 4 questions **Day 30:** Full review of formulas and key concepts **Day 60:** Final comprehensive review

Created using neuroscience-backed learning principles including chunking, dual coding, active recall, and spaced repetition. For optimal retention, review regularly and test yourself without looking at answers first!