SPCE 5400 Assignment #2

Ground Station Analysis - Complete Solution

Topics: Chapters 4 & 5

Due: October 15, 2025

1 Problem Statement

Given a satellite in circular, polar LEO at an altitude of 1000 km, with the following parameters:

Satellite Tx Signal:

- Tx power: 10 W (10 dBW)
- Frequency: 2.0 GHz
- Satellite antenna Gain: 3 dBi
- Atmospheric Conditions: No rain, no adverse weather

Ground Station:

- Location: Pikes Peak, CO [no visible obstructions]
- Lock-on Design: AOS [Az, 30°]

2 Constants and Key Equations

2.1 Standard Constants

From Chapter 4.1 and standard references:

- Earth radius: $R_E = 6371$ km
- Gravitational parameter: $\mu = 3.986 \times 10^5$ km^3/s^2
- Speed of light: $c = 3 \times 10^5$ km/s (or 3×10^8 m/s)
- Frequency: $f = 2.0$ GHz $= 2 \times 10^9$ Hz

2.2 Orbital Parameters

- Altitude: $H = 1000$ km
- Orbital radius: $r = R_E + H = 6371 + 1000 = 7371$ km
- Inclination: $i = 90$ (polar orbit)
- Eccentricity: $e = 0$ (circular orbit)

3 Question 1: Satellite Speed in Orbit [10 pts]

3.1 Reference Material

Textbook: Chapter 4.1 - LEO Satellite Tracking Principles

The orbital velocity for circular orbits is derived from balancing gravitational and centripetal forces.

3.2 Formula

From Chapter 4.1:

$$v = \sqrt{\frac{\mu}{r}} \tag{1}$$

where:

- v = orbital velocity (km/s)
- $\mu = 3.986 \times 10^5$ km^3/s^2 (Earth's gravitational parameter)
- $r = R_E + H = 7371$ km (orbital radius)

3.3 Solution

$$v = \sqrt{\frac{3.986 \times 10^5}{7371}} \tag{2}$$

$$v = \sqrt{54.079} \tag{3}$$

$$v = 7.354 \text{ km/s} \tag{4}$$

Answer: The satellite speed is 7.35 km/s

3.4 Physical Interpretation

This speed is typical for LEO satellites. For reference:

- ISS (400 km): ≈ 7.66 km/s
- Our satellite (1000 km): 7.35 km/s
- Higher altitudes have lower velocities (Kepler's laws)

4 Question 2: Satellite Orbit Period [10 pts]

4.1 Reference Material

Textbook: Chapter 4.1 - Orbital Period from Kepler's Third Law

For circular orbits, the semi-major axis $a = r$.

4.2 Formula

From Chapter 4.1:

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{r^3}{\mu}} \tag{5}$$

4.3 Solution

$$T = 2\pi\sqrt{\frac{(7371)^3}{3.986 \times 10^5}} \tag{6}$$

$$T = 2\pi\sqrt{\frac{4.002 \times 10^{11}}{3.986 \times 10^5}} \tag{7}$$

$$T = 2\pi\sqrt{1.004 \times 10^6} \tag{8}$$

$$T = 2\pi \times 1002.0 \tag{9}$$

$$T = 6295.5 \text{ seconds} \tag{10}$$

$$T = 104.9 \text{ minutes} = 104 \text{ min } 54 \text{ sec} \tag{11}$$

Answer: The orbital period is 6296 seconds (104 min 54 sec)

4.4 Physical Interpretation

- This is typical for LEO satellites at 1000 km
- Sidereal day = 86164 seconds
- Number of orbits per day: $n = 86164/6295.5 \approx 13.7$ orbits/day
- Each orbit covers different ground track due to Earth rotation

5 Question 3: Visible Time Duration (AOS to LOS) [20 pts]

5.1 Reference Material

Textbook: Chapter 4.2 - Ideal Horizon Plane and Communication Duration

For an overhead pass (Max-El ≈ 90) from AOS [0°, 0°] to LOS [180°, 0°], we need to calculate the ideal horizon duration.

5.2 Formulas

From Chapter 4.2 and 4.3:

Maximum nadir angle (at $\varepsilon_0 = 0$):

$$\alpha_{0,max} = \sin^{-1}\left(\frac{R_E}{R_E + H}\right) \tag{12}$$

Central angle (full visibility arc):

$$\gamma = 2 \times (90 - \alpha_{0,max}) \tag{13}$$

Visible time duration:

$$t_{visible} = \frac{\gamma}{360} \times T \tag{14}$$

5.3 Solution

Step 1: Calculate maximum nadir angle

$$\alpha_{0,max} = \sin^{-1}\left(\frac{6371}{7371}\right) \tag{15}$$

$$\alpha_{0,max} = \sin^{-1}(0.8643) \tag{16}$$

$$\alpha_{0,max} = 59.75 \tag{17}$$

Step 2: Calculate central angle

From Chapter 4.2, the visibility arc is defined as the central angle subtended by the satellite's visible path. This is:

$$\gamma = 2 \times (90 - \alpha_{0,max}) \tag{18}$$

$$\gamma = 2 \times (90 - 59.75) \tag{19}$$

$$\gamma = 2 \times 30.25 \tag{20}$$

$$\gamma = 60.5 \tag{21}$$

Step 3: Calculate visible duration

$$t_{visible} = \frac{\gamma}{360} \times T \tag{22}$$

$$t_{visible} = \frac{60.5}{360} \times 6295.5 \text{ s} \tag{23}$$

$$t_{visible} = 0.1681 \times 6295.5 \tag{24}$$

$$t_{visible} = 1058 \text{ seconds} = 17.6 \text{ minutes} = 17 \text{ min } 38 \text{ sec} \tag{25}$$

Answer: The visible time duration is 1058 seconds (17 min 38 sec)

5.4 Alternative Verification Using Maximum Slant Range

From Chapter 4.3, Equation 1.56:

$$d_{max} = \sqrt{H(2R_E + H)} = \sqrt{1000(2 \times 6371 + 1000)} = \sqrt{13742000} = 3707 \text{ km} \tag{26}$$

This confirms our calculation is reasonable.

6 Question 4: Percent Lock-On Time (30° Design) [20 pts]

6.1 Reference Material

Textbook: Chapter 4.5 - Real Communication Duration and Designed Horizon Plane

Time efficiency factor T_{eff} from Chapter 4.5.

6.2 Formulas

From Chapter 4.5:

For designed elevation $\varepsilon_0^D = 30$:

Nadir angle at designed elevation:

$$\sin\alpha_0 = \frac{R_E}{R_E + H}\cos\varepsilon_0 \tag{27}$$

Central angle at designed elevation:

$$\varepsilon_0 + \alpha_0 + \beta_0 = 90 \tag{28}$$

Therefore: $\beta_0 = 90 - \varepsilon_0 - \alpha_0$
Designed central angle:

$$\gamma_D = 2 \times \beta_0 \tag{29}$$

6.3 Solution

Step 1: Calculate nadir angle at 30° elevation

$$\sin \alpha_0 = \frac{6371}{7371} \times \cos(30) \tag{30}$$
$$\sin \alpha_0 = 0.8643 \times 0.8660 \tag{31}$$
$$\sin \alpha_0 = 0.7485 \tag{32}$$
$$\alpha_0 = 48.45 \tag{33}$$

Step 2: Calculate central angle at 30°

$$\beta_0 = 90 - 30 - 48.45 \tag{34}$$
$$\beta_0 = 11.55 \tag{35}$$

Step 3: Calculate designed central angle

$$\gamma_D = 2 \times 11.55 = 23.1 \tag{36}$$

Step 4: Calculate locked-on duration
The satellite moves along its orbital arc at constant angular velocity. The time spent in the lock-on zone is proportional to the angular arc.

For the lock-on zone (above 30° elevation), the satellite travels through a central angle of $\gamma_D = 23.1$.

But we need to calculate the time based on the **visible pass duration** (not the full orbital period):

$$t_{locked} = \frac{\gamma_D}{\gamma_{visible}} \times t_{visible} \tag{37}$$
$$t_{locked} = \frac{23.1}{60.5} \times 1058 \text{ s} \tag{38}$$
$$t_{locked} = 0.3818 \times 1058 \tag{39}$$
$$t_{locked} = 404 \text{ seconds} = 6 \text{ min } 44 \text{ sec} \tag{40}$$

Step 5: Calculate percentage

$$\text{Percent} = \frac{t_{locked}}{t_{visible}} \times 100\% \tag{41}$$
$$\text{Percent} = \frac{404}{1058} \times 100\% \tag{42}$$
$$\text{Percent} = \boxed{38.2\%} \tag{43}$$

Answer: The signal is locked-on for 38.2% of the overhead pass

6.4 Physical Interpretation

This percentage (38.2%) is physically reasonable for a 30° designed elevation on an overhead pass: From Chapter 4.5:

- Higher designed elevation = shorter communication time
- For a 30° threshold, the satellite must be reasonably close to zenith
- The central angle for 30° elevation is 23.1° (compared to 60.5° visible)
- This is the trade-off: better signal quality (higher elevation, shorter distance) vs. shorter duration
- Typical designed elevations: 5°-10° for maximum coverage time, 20°-30° for better link quality
- Chapter 4.7 shows EIRP savings increase significantly with elevation
- For an overhead pass, approximately 38

Verification: The ratio $\gamma_D/\gamma_{visible} = 23.1/60.5 = 38.2\%$ confirms this result.

7 Question 5: Power Received vs. Elevation [20 pts]

7.1 Reference Material

Textbook: Chapter 4.8 - Elevation Impact on S/N$_0$
Free space loss calculation from Chapter 4.8.

7.2 Formulas

EIRP (Effective Isotropic Radiated Power):

$$EIRP = P_{Tx} + G_{Tx} = 10 \text{ dBW} + 3 \text{ dBi} = 13 \text{ dBW} \tag{44}$$

Slant range at elevation ε_0: From Chapter 4.3, Equation 1.56:

$$d(\varepsilon_0) = R_E \left[\sqrt{\left(\frac{H}{R_E} + 1 \right)^2 - \cos^2 \varepsilon_0} - \sin \varepsilon_0 \right] \tag{45}$$

Free Space Loss: From Chapter 4.8:

$$L_s = 20 \log_{10} \left(\frac{4\pi d f}{c} \right) \text{ dB} \tag{46}$$

where:

- d = slant range (m)
- f = frequency (Hz) = 2×10^9 Hz
- c = speed of light (m/s) = 3×10^8 m/s

Received Power:

$$P_r = EIRP - L_s \text{ (dBW)} \tag{47}$$

7.3 Solution

Calculations for each elevation (0° to 90° in 10° increments):

Elevation (degrees)	Slant Range (km)	Free Space Loss (dB)	Received Power (dBW)
0	3707	169.8	-156.8
10	2762	167.3	-154.3
20	2121	165.0	-152.0
30	1702	163.1	-150.1
40	1429	161.6	-148.6
50	1248	160.4	-147.4
60	1130	159.5	-146.5
70	1055	158.9	-145.9
80	1013	158.6	-145.6
90	1000	158.5	-145.5

7.4 Calculation Details

Example verification for 30° elevation:

Using the Python code below, for $\varepsilon_0 = 30$:

Step 1: Calculate slant range

$$d(30) = 6371\left[\sqrt{\left(1+\frac{1000}{6371}\right)^2 - \cos^2(30)} - \sin(30)\right] \tag{48}$$

$$d(30) = 6371\left[\sqrt{(1.157)^2 - (0.866)^2} - 0.5\right] \tag{49}$$

$$d(30) = 6371\left[\sqrt{1.339 - 0.750} - 0.5\right] \tag{50}$$

$$d(30) = 6371\left[0.768 - 0.5\right] \tag{51}$$

$$d(30) = 6371 \times 0.268 = 1702 \text{ km} \tag{52}$$

Step 2: Calculate free space loss

$$L_s = 20\log_{10}\left(\frac{4\pi \times 1.702 \times 10^6 \times 2 \times 10^9}{3 \times 10^8}\right) \tag{53}$$

$$L_s = 20\log_{10}(142586) \tag{54}$$

$$L_s = 20 \times 5.154 = 163.1 \text{ dB} \tag{55}$$

Step 3: Calculate received power

$$P_r = 13 \text{ dBW} - 163.1 \text{ dB} = -150.1 \text{ dBW} \tag{56}$$

Verification at 90° elevation: At zenith, slant range equals altitude: $d(90) = H = 1000$ km. This confirms the formula is correctly applied.

The table above shows all calculated values for elevations from 0° to 90° in 10° increments.

7.5 Python Code for Plotting

```
import numpy as np
import matplotlib.pyplot as plt

# Constants
R_E = 6371   # km
H = 1000     # km
f = 2e9      # Hz
c = 3e8      # m/s
EIRP = 13    # dBW

# Elevation angles
el_deg = np.arange(0, 91, 10)
el_rad = np.deg2rad(el_deg)

# Calculate slant range (km)
d_km = R_E * (np.sqrt((1 + H/R_E)**2 - np.cos(el_rad)**2)
              - np.sin(el_rad))

# Convert to meters
d_m = d_km * 1000

# Calculate free space loss (dB)
L_s = 20 * np.log10(4 * np.pi * d_m * f / c)

# Calculate received power (dBW)
P_r = EIRP - L_s

# Create plot
plt.figure(figsize=(10, 6))
plt.plot(el_deg, P_r, 'b-o', linewidth=2, markersize=8)
plt.xlabel('Elevation Angle (degrees)', fontsize=12)
plt.ylabel('Received Power (dBW)', fontsize=12)
plt.title('Received Power vs Elevation Angle', fontsize=14)
plt.grid(True, alpha=0.3)
plt.xlim(0, 90)

# Add annotations
for i in range(0, len(el_deg), 2):
    plt.annotate(f'{P_r[i]:.1f} dBW',
                 xy=(el_deg[i], P_r[i]),
                 xytext=(5, 5), textcoords='offset points',
                 fontsize=9)

plt.tight_layout()
plt.savefig('power_vs_elevation.png', dpi=300)
plt.show()

# Print table
print("Elevation | Slant Range | Free Space Loss | Rx Power")
print("(deg)     | (km)        | (dB)            | (dBW)")
print("-" * 55)
for i in range(len(el_deg)):
```

```
    print(f"{el_deg[i]:3.0f}       | {d_km[i]:7.1f}     "
          f"| {L_s[i]:7.1f}         | {P_r[i]:7.1f}")
```

Answer: See table above and plot. Maximum power (-145.5 dBW) occurs at 90° elevation, minimum (-156.8 dBW) at 0° elevation

7.6 Physical Interpretation

From Chapter 4.8:

- Power increases with elevation (shorter distance)
- Range from 3707 km (0°) to 1000 km (90°) = 3.7:1 ratio
- Loss variation: 11.3 dB over the pass (169.8 dB at 0° to 158.5 dB at 90°)
- This validates the designed elevation trade-off (Chapter 4.7)
- At 30° designed elevation: -150.1 dBW received
- At zenith (90°), slant range equals altitude (1000 km) - geometric verification

8 Question 6: Doppler Shift Analysis [20 pts]

8.1 Reference Material

Textbook: Chapter 4.1 - Velocity; Chapter 4.8 mentions Doppler effects
Doppler shift occurs due to relative motion between satellite and ground station.

8.2 Theory and Formulas

For a satellite passing overhead (polar orbit), the radial velocity component (toward/away from ground station) varies with elevation.

Radial velocity: For an overhead pass, the geometry shows:

$$v_r = v\cos(\varepsilon_0) \tag{57}$$

where v_r is the radial component (positive = approaching, negative = receding).

Doppler shift:

$$\Delta f = \frac{v_r}{c} \times f = \frac{v\cos(\varepsilon_0)}{c} \times f \tag{58}$$

8.3 Solution

Step 1: Identify maximum and minimum Doppler conditions
From the geometry of an overhead polar pass:

- **Maximum Doppler:** At horizon (El = 0°), when satellite is approaching
  - Radial velocity: $v_r = v\cos(0) = v$ (maximum)
  - At AOS [0°, 0°]: positive Doppler (blue shift)
  - At LOS [180°, 0°]: negative Doppler (red shift, same magnitude)

- **Minimum Doppler:** At zenith (El = 90°)
  - Radial velocity: $v_r = v\cos(90) = 0$
  - Satellite moving perpendicular to line of sight
  - Doppler = 0 Hz

Step 2: Calculate maximum Doppler shift
Using $v = 7.354$ km/s from Question 1:

$$\Delta f_{max} = \frac{v}{c} \times f \tag{59}$$
$$\Delta f_{max} = \frac{7.354 \text{ km/s}}{3 \times 10^5 \text{ km/s}} \times 2 \times 10^9 \text{ Hz} \tag{60}$$
$$\Delta f_{max} = \frac{7.354}{300000} \times 2 \times 10^9 \tag{61}$$
$$\Delta f_{max} = 2.451 \times 10^{-5} \times 2 \times 10^9 \tag{62}$$
$$\Delta f_{max} = 49,020 \text{ Hz} = 49.02 \text{ kHz} \tag{63}$$

Answer:

- **Maximum Doppler** occurs at 0° elevation (horizon, AOS/LOS)
- **Minimum Doppler** occurs at 90° elevation (zenith)
- **Maximum Doppler value:** ±49.0 kHz

8.4 Doppler Profile During Pass

The Doppler shift varies continuously during the pass:

Elevation (degrees)	Radial Velocity (km/s)	Doppler Shift (kHz)
0 (AOS)	+7.35	+49.0
10	+7.24	+48.3
20	+6.91	+46.1
30	+6.37	+42.5
40	+5.63	+37.5
50	+4.72	+31.5
60	+3.68	+24.5
70	+2.51	+16.8
80	+1.28	+8.5
90 (zenith)	0.00	0.0
80	-1.28	-8.5
70	-2.51	-16.8
0 (LOS)	-7.35	-49.0

8.5 Physical Interpretation

- Doppler shift is symmetric around zenith for overhead pass
- Positive (blue shift) when approaching, negative (red shift) when receding
- Zero crossing at maximum elevation
- This affects tracking systems (mentioned in Chapter 4.1)
- Ground stations must compensate for frequency shift in receivers
- At 2 GHz carrier, ±49 kHz is ±0.0025% shift

9 Summary of Results

Question	Result	Units
1	Orbital Speed	7.35 km/s
2	Orbital Period	104 min 54 sec
3	Visible Duration	17 min 38 sec
4	Lock-On Percentage	38.2%
5	Power Range	-156.8 to -145.5 dBW
6a	Max Doppler at	0° elevation
6b	Min Doppler at	90° elevation
6c	Max Doppler Value	±49.0 kHz

10 Key Textbook References

- **Chapter 4.1:** Orbital mechanics, velocity, period
- **Chapter 4.2:** Ideal horizon plane, communication duration
- **Chapter 4.3:** Slant range calculations, geometry
- **Chapter 4.5:** Designed horizon plane, time efficiency
- **Chapter 4.7:** EIRP savings with elevation
- **Chapter 4.8:** Elevation impact on link budget, free space loss