Chapter 4: Horizon Plane and Communication Duration

Summarized Notes

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Contents

1 4.1 LEO Satellite Tracking Principles

1.1 Executive Summary

Building on orbital mechanics from Chapter 1, this subsection introduces Kepler's laws (three principles describing planetary motion: orbits are ellipses, equal areas in equal times, and $T^2 \propto a^3$) and Newton's law of universal gravitation (force $F = G\frac{m_1 m_2}{r^2}$, where G is the gravitational constant) to explain Low Earth Orbit (LEO; orbits 200-2000 km altitude) satellite motion, defining key parameters for tracking and communication, essential for ground station design to predict and follow satellite paths accurately.

1.2 Key Takeaways (80/20 Insights)

- **Motion Equation and Trajectory**: Satellite movement follows the differential equation $\frac{d^2\mathbf{r}}{dt^2} + \frac{\mu}{r^3}\mathbf{r} = 0$, where $\mathbf{r}$ is the position vector from Earth's center, $r = |\mathbf{r}|$, and $\mu = GM$ (gravitational parameter, M Earth mass) yielding elliptical orbits with Earth at one focus; for LEO, circular orbits (eccentricity $e = 0$) simplify to constant velocity $v = \sqrt{\mu/r}$, linking to noise models in Chapter 3.

- **Orbital Parameters**: Semi-major axis a (half the major axis of the ellipse, equals radius for circular orbits) determines period $T = 2\pi\sqrt{a^3/\mu}$ and daily passes $n = T_{\text{sideral}}/T$ (sidereal day $T_{\text{sideral}} \approx 86164$ s, Earth's rotation relative to stars); eccentricity $e = (r_a - r_p)/(r_a + r_p)$ where r_a apogee (farthest point), r_p perigee (closest point); inclination i (tilt of orbital plane to Earth's equator); Right Ascension of Ascending Node (RAAN, Ω) orients the ascending node (equator crossing northward) to vernal equinox (Sun's position at spring equinox).

- **Argument of Perigee ω and True Anomaly θ**: ω positions perigee in the orbital plane; θ (or ν) locates satellite from perigee; these enable prediction of azimuth (Az; horizontal angle from north) and elevation (El; vertical angle above horizon) for antenna pointing, interleaving with horizon concepts in 4.2.

- **Tracking Software Functions**: Includes telemetry (satellite health/data transmission), command/-control (sending instructions), database (storing parameters), analysis (processing logs); uses Keplerian elements (six parameters: $a, e, i, \Omega, \omega, \theta$) to predict Acquisition of Signal (AOS; start of contact) and Loss of Signal (LOS; end of contact), control rotors (motors for antenna rotation); radar maps (2D projections of satellite ground track) show path for real-time operations.

- **Practical Implications**: For Multispectral Optical Science Telescope (MOST; example LEO satellite for astronomy) at H 830 km, $v \approx 7.43$ km/s, $T \approx 101$ min, $n \approx 14.2$ passes/day; professionals use software for acquisition a few minutes early to ensure lock (stable signal connection), critical for data transfer efficiency.

1.3 Important Equations

1. **Motion Equation**: $\frac{d^2\mathbf{r}}{dt^2} + \frac{\mu}{r^3}\mathbf{r} = 0$, where $\mathbf{r}$ is position vector, $\mu = GM = 3.986 \times 10^{14}$ $\mathbf{m^3/s^2}$; describes trajectory; example: for circular, balances centripetal force.

2. **Velocity in Circular Orbit**: $v = \sqrt{\mu/r}$, $r = H + R_E$ ($R_E = 6378$ km Earth radius); represents speed; example: H=830 km, $v = 7.43$ km/s.

3. **Orbital Period**: $T = 2\pi\sqrt{a^3/\mu}$, $a = r$ for circular; time for one orbit; example: $a = 7208$ km, $T = 6085$ s (101 min).

4. **Daily Passes**: $n = T_{\text{sideral}}/T$, $T_{\text{sideral}} = 86164$ s; number of orbits per sidereal day; example: $T = 6085$ s, $n = 14.16$.

1.4 Supporting Details

- Examples: MOST tracking at Vienna station uses Graphical User Interface (GUI; interactive software screen) for operations, radar maps for visualization; real-world trades: higher i (polar orbits near 90°) increases global coverage but complicates tracking vs. equatorial ($i = 0°$). - Applications: Parameters predict visibility for remote sensing (Earth observation); cross-references constellation design in Chapter 5 for handover (switching satellites). - Trades: Circular orbits simplify calculations but limit flexibility vs. elliptical (varying altitude); software must handle perturbations (small orbit deviations due to drag/gravity) for accuracy.

1.5 Simplified Analogy

Think of a satellite as a ball thrown in a circular path around a central post (Earth): the rope length (r) sets speed and lap time; tracking is like aiming a camera using predicted angles ($i, \Omega, \omega, \theta$). Verbalize aloud: "The ball circles faster closer to the post, just like LEO satellites at lower altitudes."

1.6 Visual Aid Suggestion

- Sketch: Elliptical orbit with perigee/apogee, nodes, i as tilt, Ω as rotation, ω as perigee angle, θ as position.
- Graph: Plot altitude H (600-1200 km) vs. period T (min); key points: H=600 km T 96 min, H=1200 km T 109 min.

Orbital Elements Diagram

Shows: orbital plane, perigee, apogee, inclination i, RAAN Ω, argument of perigee ω, true anomaly θ

Figure 1: Keplerian orbital elements for LEO satellite tracking. Reference: Textbook Figure 4.1

- Python Code Snippet:

```python
import matplotlib.pyplot as plt
import numpy as np
mu = 3.986e14  # m3/s2
Re = 6378000  # m
h = np.linspace(600000, 1200000, 7)
a = Re + h
t = 2 * np.pi * np.sqrt(a**3 / mu) / 60  # min
plt.plot(h/1000, t, marker='o')
plt.xlabel('Altitude (km)')
plt.ylabel('Period (min)')
plt.title('Orbital Period vs. Altitude')
plt.grid(True)
plt.show()
```

1.7 Retention Aid

Mnemonic Recap: "All Elephants In Orbit Really Twist" $(a, e, i, \Omega, r, \theta)$. Dual-code by drawing orbit while verbalizing: "Circle tight for fast laps." Interleave: Link velocity to Doppler shift (frequency change due to motion) in Chapter 8. Multisensory: Say equations aloud in 90-min session with breaks. Spaced repetition: Review in 24 hours, quizzes in 1 week.

Self-Quiz Questions: 1. Multiple-Choice: What parameter tilts the orbital plane? (a) Eccentricity, (b) Inclination, (c) True anomaly. Answer: (b) Inclination. 2. Fill-in: Orbital period $T = 2\pi\sqrt{\_\_\_/\mu}$. Answer: a^3. 3. Application: For H=800 km, calculate v (km/s) if $\mu = 3.986 \times 10^5$ $\mathrm{km}^{\mathbf{3}}/\mathrm{s}^{\mathbf{2}}$, r=7178 km. Answer: 7.46 ($v = \sqrt{\mu/r}$). 4. What: Explain why circular orbits are preferred for LEO. Answer: Constant velocity/altitude simplifies tracking/communication. 5. Calculate: Daily passes for T=100 min ($T_{\text{sideral}} \approx 1440$ min). Answer: 14.4.

2 4.2 Ideal Horizon Plane and Communication Duration with LEO Satellites

2.1 Executive Summary

Extending tracking from 4.1, this subsection defines the ideal horizon plane (theoretical visibility circle at 0° elevation with no barriers) for maximum visibility, calculating communication duration based on satellite path and Max-El (maximum elevation; peak angle during pass), crucial for optimizing data transfer in LEO systems.

2.2 Key Takeaways (80/20 Insights)

- **Ideal Horizon Plane**: Flat circle at minimum elevation $\epsilon_0 = 0°$, diameter 6000-8000 km; AOS/LOS at horizon edges determine visibility; links to practical adjustments in 4.4.
- **Communication Duration**: Ideal duration t_{ideal} = LOS time - AOS time, depends on Max-El; higher Max-El (e.g., 90°) gives 15 min, low (e.g., 5°) 8 min; interleaves with power needs in 4.7.
- **Path Projection**: Radar map shows curved ground track from AOS (Az) to LOS (Az); parameters like Max-El dictate duration; for MOST, records show variability.
- **Altitude Impact**: Higher H lengthens duration (e.g., H=1200 km ¿ H=600 km at same Max-El); professionals use for pass prediction.
- **Implications**: Ideal duration assumes no barriers; real is shorter, informing designed plane in 4.6 for reliability.

2.3 Important Equations

1. **Duration**: $t_{\text{ideal}} = t_{\text{LOS}} - t_{\text{AOS}}$; time from horizon to horizon; example: Max-El=84° $t = 15:21$ min.
2. **Slant Range d**: $d = R_E\sqrt{(H/R_E + 1)^2 - \cos^2\epsilon_0} - \sin\epsilon_0$; at $\epsilon_0 = 0°$, $d_{\max} = \sqrt{H(2R_E + H)}$; radius of plane.
3. **Elevation Function**: $\sin\epsilon_0 = [H(H + 2R_E) - d^2]/(2dR_E)$; inverts distance to elevation.
4. **Plane Width**: $D = 2d_{\max}$; diameter; example: H=800 km D 6579 km.

2.4 Supporting Details

- Examples: MOST passes: orbit 2235 Max-El=84° t=15:21, orbit 2467 Max-El=6° t=9:38; trades: longer duration vs. higher path loss (signal weakening over distance) at low El. - Applications: Predict data volume; cross-references attenuation (signal loss) at low El in Chapter 2. - Trades: High Max-El maximizes t but rarer; low H shortens t but stronger signal.

2.5 Simplified Analogy

Satellite pass is like a plane flying over: horizon plane is runway edges (AOS/LOS); Max-El is peak altitude overhead for longest view. Verbalize aloud: "Plane highest overhead stays visible longer, like high Max-El extends talk time."

2.6 Visual Aid Suggestion

- Sketch: Radar map with curved path, labels AOS, Max-El, LOS. - Graph: Max-El (0-90°) vs. t (min) for H=800 km; key: 90° t 15 min, 5° t 8 min.

- Python Code Snippet:

Ideal Horizon Plane Geometry

Shows: circular horizon plane at $\epsilon_0 = 0°$, slant range d, satellite path from AOS to LOS through Max-El

Figure 2: Ideal horizon plane and communication duration. Reference: Textbook Figure 4.7

```python
import matplotlib.pyplot as plt
import numpy as np
max_el = np.arange(5, 91, 5)
t_min = [8 + 7 * (e/90) for e in max_el]  # Approximate
plt.plot(max_el, t_min, marker='o')
plt.xlabel('Max-El (  )')
plt.ylabel('Duration (min)')
plt.title('Duration vs. Max-El for H=800 km')
plt.grid(True)
plt.show()
```

2.7 Retention Aid

Mnemonic Recap: "AOS Low, LOS Gone, Max-El Long" for events. Dual-code: Map path while saying "High peak, long speak." Interleave: Connect to tracking in 4.1. Multisensory: Trace path aloud. Spaced repetition: Review tomorrow, quizzes weekly.

Self-Quiz Questions: 1. Multiple-Choice: What maximizes ideal duration? (a) Low H, (b) High Max-El, (c) High e. Answer: (b) High Max-El. 2. Fill-in: Ideal plane at $\epsilon_0 =$ ___. Answer: 0°. 3. Application: For H=800 km, $\epsilon_0 = 0°$ d 3294 km; calculate D. Answer: 6588 km ($2d$). 4. What: Why is t variable per pass? Answer: Differs with Max-El/path. 5. Calculate: Approx t for Max-El=45° if 90°=15 min (linear approximation). Answer: 7.5 min.

3 4.3 The Range and Horizon Plane Simulation for Ground Stations of LEO Satellites

3.1 Executive Summary

Following ideal plane in 4.2, this subsection simulates slant range (line-of-sight distance d) and plane dimensions for varying altitudes, showing how higher H expands horizon for longer communication, key for link budget (signal power accounting) planning.

3.2 Key Takeaways (80/20 Insights)

- **Slant Range Variation**: d shortest at 90° (equals H), longest at 0° ($\sqrt{H(2R_E + H)}$); increases with H; interleaves with loss in 4.8.
- **Plane Dimension**: Width D = $2\sqrt{H(2R_E + H)}$; 6000-8000 km for H=600-1200 km; simulation confirms expansion with H.
- **Elevation Impact**: Higher ϵ_0 shortens d; table shows d decreases 10-15% from 0° to 90°.
- **Simulation Method**: Step H by 100 km, ϵ_0 by 10°; professionals use for worst-case (0°) budget.
- **Implications**: Larger plane with higher H increases visibility but weakens signal; trade for constellations in Chapter 5.

3.3 Important Equations

1. **Slant Range**: $d = R_E[\sqrt{(H/R_E + 1)^2 - \cos^2 \epsilon_0} - \sin \epsilon_0]$; distance; example: H=800 km, $\epsilon_0 = 0°$ d=3289 km.
2. **Elevation from d**: $\sin \epsilon_0 = [H(H + 2R_E) - d^2]/(2dR_E)$; inverse.
3. **Plane Radius**: $d_{\max} = \sqrt{H(2R_E + H)}$; at 0°.
4. **Width**: $D = 2d_{\max}$; diameter; example: H=800 km D=6578 km.

3.4 Supporting Details

- Examples: Table 4.1: H=600 km 90° d=600 km, 0° d=2830 km; trades: High H wider plane but higher loss. - Applications: Predict communication windows; cross-references duration in 4.2. - Trades: Low H short range/strong signal vs. high H more coverage.

Table 1: Slant Range d (km) vs. Altitude and Elevation

Altitude (km)	**0°**	**10°**	**30°**	**60°**	**90°**	**IHPW (km)**
600	2830	1350	1070	730	600	5660
700	3050	1470	1160	800	700	6070
800	3290	1590	1250	870	800	6580
900	3470	1700	1330	940	900	6940
1000	3660	1810	1420	1010	1000	7320
1100	3840	1910	1500	1080	1100	7680
1200	4020	2010	1580	1150	1200	8040

3.5 Simplified Analogy

Horizon plane is like view from hilltop: higher hill (H) widens horizon circle; range is hike distance. Verbalize aloud: "Taller hill sees farther, like higher orbit expands comm window."

3.6 Visual Aid Suggestion

- Sketch: Triangle with R_E, H, d, ϵ_0. - Graph: H (600-1200 km) vs. D (km) at 0°; key: 600 km D=5660 km, 1200 km D=8176 km. - Python Code Snippet:

```
import matplotlib.pyplot as plt
import numpy as np
Re = 6371
h = np.arange(600, 1300, 100)
d_max = np.sqrt(h * (2*Re + h))
D = 2 * d_max
plt.plot(h, D, marker='o')
plt.xlabel('Altitude (km)')
plt.ylabel('Plane Width (km)')
plt.title('Ideal Horizon Width vs. Altitude')
plt.grid(True)
plt.show()
```

3.7 Retention Aid

Mnemonic Recap: "Double Root High Twice Radius" for $d_{\max}$. Dual-code: Draw triangle saying "Long d low El." Interleave: Link to 4.2 duration. Multisensory: Calculate aloud. Spaced repetition: Review equations daily.

Self-Quiz Questions: 1. Multiple-Choice: d min at? (a) 0°, (b) 90°. Answer: (b) 90°. 2. Fill-in: D = 2 ___. Answer: $d_{\max}$. 3. Application: H=1000 km $d_{\max}$? ($R_E = 6371$). Answer: 3708 km ($\sqrt{1000 \times (12742 + 1000)}$). 4. What: Why higher H larger D? Answer: Wider tangent circle. 5. Calculate: For H=600 km, $\epsilon_0 = 30°$ approx d (from table pattern). Answer: 1070 km.

4 4.4 Practical Horizon Plane for LEO Ground Stations

4.1 Executive Summary

Contrasting ideal in 4.2, this subsection accounts for barriers (obstructions like trees/buildings) raising effective horizon to 1-4°, forming broken plane (irregular visibility circle) and shortening duration, vital for urban stations to adjust tracking.

4.2 Key Takeaways (80/20 Insights)

- **Practical vs. Ideal**: Barriers raise lock (signal acquisition) and unlock (signal loss) El to 1-4°, creating broken circle; reduces visibility vs. ideal flat 0° plane; links to designed in 4.5.
- **Events Impact**: AOS/LOS at higher El shortens t; records show 2-4° average for Vienna.
- **Azimuth Variation**: Barriers differ by direction, distorting plane; simulation shows real t 80-95% of ideal.
- **Urban Challenges**: Low-cost stations hindered at low El; professionals record passes to map practical horizon.
- **Implications**: Real duration depends on lock El, not just Max-El; informs power savings in 4.7.

4.3 Important Equations

1. **Real Duration**: $t_{\text{real}} = t_{\text{unlock}} - t_{\text{lock}}$; shorter than ideal.
2. **Lock El**: Average 1-4° from records; example: orbit 2467 $t_{\text{real}} = 5$ min vs. ideal 9:38.

4.4 Supporting Details

- Examples: MOST orbit 2467: lock 4° t$_loss = 4 : 38 min; trades : Urban higher practical El vs. open sites. - Applications : Adjust for Synthetic Aperture Radar (SAR; high-resolution imaging); cross-references simulation in 4.3. - Trades : Higher practical El reliable but shorter t.$

4.5 Simplified Analogy

Practical plane is fenced yard with trees: view blocked low, effective horizon raised. Verbalize aloud: "Trees hide low-flying plane, like barriers shorten satellite talk."

4.6 Visual Aid Suggestion

- Sketch: Broken circle plane with varying El points. - Graph: Az (0-360°) vs. practical El (1-4°); random points.

- Python Code Snippet:

```
import matplotlib.pyplot as plt
az = range(0, 361, 30)
el = [np.random.uniform(1,4) for _ in az]
plt.plot(az, el, marker='o')
plt.xlabel('Azimuth (  )')
plt.ylabel('Practical El (  )')
plt.title('Practical Horizon Variation')
plt.grid(True)
plt.show()
```

Practical Horizon Plane - Broken Circle

Shows: irregular horizon with varying lock/unlock elevations by azimuth due to barriers (buildings, trees)

Figure 3: Practical horizon plane showing barrier effects. Reference: Textbook Figure 4.11

4.7 Retention Aid

Mnemonic Recap: "Barriers Break Ideal Circle." Dual-code: Draw broken plane saying "Raised fence cuts view." Interleave: With ideal in 4.2. Multisensory: Discuss urban issues aloud. Spaced repetition: Review records weekly.

Self-Quiz Questions: 1. Multiple-Choice: Practical plane shape? (a) Flat circle, (b) Broken circle. Answer: (b). 2. Fill-in: Average lock El __-__. Answer: 1-4°. 3. Application: Ideal t=15 min, practical lock 2° t$_loss?Answer: 1min(estimate).4.What: Whyurbanchallenge?Answer: BarriershinderlowElcomm.5.Calculate: Realtifideal10min, loss2min.Answer: 8min.$

5 4.5 Real Communication Duration and Designed Horizon Plane Determination

5.1 Executive Summary

Using measurements from 4.4, this subsection quantifies duration loss with time efficiency factor (Teff or η; ratio real/ideal, 80-95%), defining designed plane (intentional minimum El to avoid risks) at 10° breaking point to ensure unbroken link, optimizing reliability.

5.2 Key Takeaways (80/20 Insights)

- **Time Efficiency Teff**: $\eta = t_{\text{real}}/t_{\text{ideal}}$; 80-100% for Max-El¿10°, drops below; interleaves with practical in 4.4.
- **Duration Loss Δt**: $\Delta t = t_{\text{ideal}} - t_{\text{real}}$; higher at low Max-El due to barriers.
- **Breaking Point**: At 10° Max-El, Teff linear ¿80%; design plane above for safe comm.
- **Designed Plane**: Set at min El (5-30°) to ensure unbroken link; NOAA (National Oceanic and Atmospheric Administration) uses 5°.
- **Implications**: Higher Teff better system use; professionals analyze passes for design El.

5.3 Important Equations

1. **Teff**: $\eta = (t_{\text{lock}} - t_{\text{unlock}})/(t_{\text{AOS}} - t_{\text{LOS}})$; ratio.
2. **Real t**: $t_{\text{real}} = t_{\text{unlock}} - t_{\text{lock}}$; example: 14:40 vs. ideal 15:29 loss 0:49.
3. Δt: $\Delta t = t_{\text{ideal}} - t_{\text{real}}$; loss.
4. **Teff vs. Max-El**: Graph shows break at 10°; example: Max-El=86° Teff 95%.

5.4 Supporting Details

- Examples: 3000 MOST passes: low Max-El high loss; trades: High design El reliable but less t. - Applications: Optimize for science/SAR; cross-references power in 4.7. - Trades: Design El 5° max visibility vs. 30° min risk.

5.5 Simplified Analogy

Teff is fuel efficiency: high Max-El like smooth highway (95%), low like bumpy road (drop). Verbalize aloud: "Smooth ride saves time, like high El cuts loss."

5.6 Visual Aid Suggestion

- Sketch: Efficiency curve with break point. - Graph: Max-El vs. Teff (%); key: 10° break, 90° 100%.
- Python Code Snippet:

```
import matplotlib.pyplot as plt
max_el = [5,10,20,40,60,80]
teff = [60,80,85,90,95,100]
plt.plot(max_el, teff, marker='o')
plt.xlabel('Max-El (  )')
plt.ylabel('Teff (\%)')
plt.title('Teff vs. Max-El')
plt.grid(True)
plt.show()
```

Time Efficiency Factor (Teff) vs. Max-El

Shows: Teff curve with breaking point at ∼10° Max-El, linear relationship above, degradation below

Figure 4: Time efficiency factor showing 10° breaking point. Reference: Textbook Figure 4.12

5.7 Retention Aid

Mnemonic Recap: ”Teff High, Loss Low.” Dual-code: Plot curve saying ”Break at 10 saves win.” Interleave: With 4.4 records. Multisensory: Calculate Teff aloud. Spaced repetition: Review graph daily.

Self-Quiz Questions: 1. Multiple-Choice: Teff break at? (a) 5°, (b) 10°. Answer: (b). 2. Fill-in: Teff = $t_{\text{real}}/\_\_\_$. Answer: t_{ideal}. 3. Application: Ideal 15 min, Teff=90% t_{real}? Answer: 13.5 min. 4. What: Why design plane ¿ break? Answer: Avoid barrier uncertainties. 5. Calculate: Δt if $t_{\text{ideal}} = 10$ min, Teff=80%. Answer: 2 min.

6 4.6 Ideal and Designed Horizon Plane Relation in Space

6.1 Executive Summary

Building on 4.5, this subsection mathematically correlates ideal and designed planes, deriving widths (IHPW/DHPW; ideal/designed horizon plane width) and separation (LDHPW; layer distance) for varying El, enabling precise geometry for Starlink-like systems.

6.2 Key Takeaways (80/20 Insights)

- **Plane Widths**: IHPW $= 2d(0°)$, DHPW $= 2d(X°)$; DHPW ¡ IHPW; for H=550 km 40° DHPW=1240 km vs. IHPW=5406 km.
- **Separation LDHPW**: LDHPW $= d(X°)\sin(90° - X°)$; increases with X°; interleaves with power savings in 4.7.
- **Geometry**: Designed plane parallel above ideal, up by LDHPW; cone base DHPW from user apex.
- **Starlink Example**: For shells (orbit groups at different H) 340-1110 km, X=25-40°; lower H shorter widths.
- **Implications**: New parameters for LEO geometry; professionals calculate for coverage trades in Chapter 5.

6.3 Important Equations

1. **IHPW**: IHPW $= 2\sqrt{H(H + 2R_E)}$; ideal width; example: H=550 km=5406 km.
2. **DHPW**: DHPW $= 2d(X°)\cos(90° - X°)$; designed; example: X=40° H=550 km=1240 km.
3. **LDHPW**: LDHPW $= d(X°)\sin(90° - X°)$; separation; example: 520 km for above.
4. **Min d**: $d_{\min} = H$ at 90°; unchanged.

6.4 Supporting Details

- Examples: Table 4.4: H=340 km 30° LDHPW=316 km; trades: High X short DHPW but safe. - Applications: Starlink user sites; cross-references simulation in 4.3. - Trades: Low X max visibility vs. high reliability.

Table 2: Horizon Plane Widths and Separation for Starlink Shells

Altitude (km)	Design El (°)	IHPW (km)	DHPW (km)	LDHPW (km)
340	25	4260	1620	680
340	30	4260	1360	316
340	40	4260	990	410
550	25	5406	2045	860
550	30	5406	1718	396
550	40	5406	1240	520
1110	25	7680	2906	1220
1110	30	7680	2440	1123
1110	40	7680	1765	730

6.5 Simplified Analogy

Planes are stacked pancakes: ideal bottom large, designed top smaller, separated by syrup layer (LDHPW). Verbalize aloud: "Thinner top pancake safer from edges, like high El avoids barriers."

6.6 Visual Aid Suggestion

- Sketch: Parallel planes with IHPW, DHPW, LDHPW. - Graph: X (25-40°) vs. DHPW (km) for H=550 km; key: 25°=2045, 40°=1240.

3D Geometry: Ideal and Designed Horizon Planes

Shows: two parallel planes, ground station at center, IHPW (large circle at 0°), DHPW (smaller circle at X°), vertical separation LDHPW, cone from user to designed plane

Figure 5: Spatial relationship between ideal and designed horizon planes. Reference: Textbook Figure 4.14

- Python Code Snippet:

```python
import matplotlib.pyplot as plt
x = [25,30,35,40]
dhpw = [2045,1718,1465,1240]
plt.plot(x, dhpw, marker='o')
plt.xlabel('Designed El (  )')
plt.ylabel('DHPW (km)')
plt.title('DHPW vs. X for H=550 km')
plt.grid(True)
plt.show()
```

6.7 Retention Aid

Mnemonic Recap: "Ideal Wide, Designed Narrow, Layer Between." Dual-code: Stack planes saying "Top short safe." Interleave: With 4.5 Teff. Multisensory: Measure aloud. Spaced repetition: Review table weekly.

Self-Quiz Questions: 1. Multiple-Choice: DHPW ¡ ___? (a) IHPW, (b) LDHPW. Answer: (a). 2. Fill-in: LDHPW $= d(X)\_\_\_(90 - X)$. Answer: sin. 3. Application: H=550 km X=30° DHPW? Answer: 1718 km (table). 4. What: Why parallel planes? Answer: User fixed, designed shifts up. 5. Calculate: IHPW for H=800 km ($R_E = 6371$). Answer: 6578 km ($2\sqrt{H(H + 2R_E)}$).

7 4.7 Savings on Transmit Power through Designed Horizon Plane at LEO Satellite Ground Stations

7.1 Executive Summary

From 4.6 geometry, this subsection calculates EIRP (Effective Isotropic Radiated Power; transmit power × antenna gain) savings (1-8 dB) by using designed plane to shorten max range, maintaining constant margin (extra signal strength), key for power-limited LEO satellites.

7.2 Key Takeaways (80/20 Insights)

- **EIRP Savings ΔEIRP**: $\Delta\text{EIRP} = L_s(0°) - L_s(X°) = 20\log(d(0°)/d(X°))$; 1-8 dB for X=5-30°; interleaves with S/N0 in 4.8.
- **Constant Margin**: Vary EIRP to compensate range; designed shortens max d, saves power.
- **Altitude Effect**: Savings decrease with H (e.g., H=600 km 8.44 dB at 30°, H=1200 km 6.22 dB).
- **Agile Transmitter**: Satellite adjusts power per El; half power at 10° for H=800 km.
- **Implications**: Extends battery life; professionals "play" with X for trades.

7.3 Important Equations

1. **Downlink Margin DM**: $\text{DM} = (S/N)_{\text{received}} - (S/N)_{\text{required}}$; positive for link.
2. **S/N0**: $S/N_0 = \text{EIRP} - L_s - L_0 + G/T_s + 228.6$; constant requires EIRP vary (G/T_s figure of merit, antenna gain/noise temp).
3. **ΔEIRP**: $\Delta\text{EIRP} = 20\log(d_0/d_X)$; savings; example: X=30° H=800 km=7.47 dB.
4. **Range d**: $d(\epsilon_0) = R_E[\sqrt{(H/R_E+1)^2 - \cos^2\epsilon_0} - \sin\epsilon_0]$; basis.

7.4 Supporting Details

- Examples: Table 4.5: 5° 1.44 dB, 30° 7.47 dB for H=800 km; trades: High X more savings but less t. - Applications: Nano sats (small satellites); cross-references agile payloads (adaptive transmitters). - Trades: Power save vs. coverage reduction.

Table 3: EIRP Savings (ΔEIRP in dB) for Various Designed Elevations

Altitude (km)	**5°**	**10°**	**15°**	**20°**	**25°**	**30°**
600	1.57	3.10	4.52	5.85	7.03	8.44
700	1.52	2.99	4.37	5.66	6.79	7.73
800	1.44	2.85	4.16	5.40	6.46	7.47
900	1.40	2.77	4.05	5.23	6.28	7.19
1000	1.36	2.69	3.93	5.09	6.10	6.97
1100	1.32	2.61	3.82	4.95	5.93	6.78
1200	1.29	2.55	3.73	4.83	5.79	6.22

7.5 Simplified Analogy

Designed plane is shorter ladder: less effort (power) to climb same height. Verbalize aloud: "Shorter path saves battery, like designed El cuts EIRP."

EIRP Savings vs. Designed Elevation

Shows: bar chart or line graph of power savings (1-8 dB) as designed elevation increases from 5° to 30°, demonstrating link budget improvement

Figure 6: Power savings through designed horizon plane. Reference: Textbook Figure 4.16

7.6 Visual Aid Suggestion

- Sketch: Bricks stacking savings per X. - Graph: X (5-30°) vs. ΔEIRP (dB) for H=800 km; key: 30°=7.47. - Python Code Snippet:

```
import matplotlib.pyplot as plt
x = [5,10,15,20,25,30]
deirp = [1.44,2.85,4.16,5.4,6.46,7.47]
plt.bar(x, deirp)
plt.xlabel('Designed El (  )')
plt.ylabel('   EIRP (dB)')
plt.title('Power Savings for H=800 km')
plt.grid(True)
plt.show()
```

7.7 Retention Aid

Mnemonic Recap: "Delta EIRP Logs Range Ratio." Dual-code: Stack bricks saying "Higher X more save." Interleave: With 4.6 d. Multisensory: Compute savings aloud. Spaced repetition: Review table tomorrow.

Self-Quiz Questions: 1. Multiple-Choice: ΔEIRP from? (a) $L_s diff, (b) G/T diff. Answer: (a). 2. Fill-in$: ΔEIRP = 20 log(___ / d_X). Answer: d_0. 3. Application: H=800 km X=20° ΔEIRP? Answer: 5.4 dB (table). 4. What: Why agile transmitter? Answer: Vary EIRP for constant margin. 5. Calculate: 3 dB savings halves power? Answer: Yes (3 dB = factor of 2).

8 4.8 Elevation Impact on Signal-to-Noise Density Ratio for LEO Satellite Ground Stations

8.1 Executive Summary

Concluding chapter from 4.7, this subsection quantifies elevation's effect on S/N0 (signal-to-noise density ratio; signal power / noise density in dB-Hz), with higher El improving ratio due to shorter range, guiding adaptive modulation (changing data rate) for LEO links.

8.2 Key Takeaways (80/20 Insights)

- **S/N0 Variation**: Peaks at 90° (short d), drops 12 dB at 0°; decreases with H.
- **Free Space Loss L$_s$** : $L_s = 32.45 + 20\log(df)$**; main factor, 154-170 dB for LEO S-band (2 GHz range).**
- **Range Equation: S/N0 = EIRP - L$_s$** $- L_0 + G/T_s + 228.6; elevation drives L_s (L_0$ **other losses like atmosphere).**
- **Max ΔS/N0: 13.5 dB at H=600 km to 11.2 dB at 1200 km; compensate with EIRP.**
- **Implications: Higher El better performance; for constellations, higher orbits smoother S/N0.**

8.3 Important Equations

1. **L$_s$** : $L_s(\mathbf{dB}) = 20\log\left(\frac{4\pi df}{c}\right)$**; example: d=3289 km f=2 GHz 168.8 dB.**

2. S/N0: $S/N_0(\textbf{dB-Hz}) = \textbf{EIRP} - L_s - L_0 + G/T_s + 228.6$**; with EIRP=30 dBW G/T$_s$** $= 15\ 70 at 90$.

2. ΔS/N0: $\Delta S/N_0 = L_s(0°) - L_s(90°)$**; max diff; example: H=800 km 12.3 dB.**

3. G/T$_s$: $G/T_s = G - 10\log(T_{\textbf{ant}} + T_{\textbf{comp}})$**; figure of merit; example: G=35 T=100 K =15 dB/K.**

8.4 Supporting Details

- Examples: Table 4.6: 90° L$_s$ $= 156.5 dB H = 800 km, 0 168.8 dB; trades: Low El high loss vs. high El strong signal. - Applications: S-band LEO; cross-references rain in Chapter 2 adding L_0. - Trades: High H less \Delta$ but weaker overall.

Table 4: Free Space Loss L_s (dB) at S-band (2 GHz) vs. Elevation

Altitude (km)	**0°**	**10°**	**30°**	**60°**	**90°**	**ΔS/N$_0$ (dB)**
600	166.4	159.4	157.1	153.8	152.0	14.4
700	167.2	160.0	157.6	154.4	153.3	13.9
800	168.8	161.3	158.8	155.5	154.5	14.3
900	169.3	161.9	159.2	156.0	155.1	14.2
1000	169.8	162.6	159.8	156.5	155.5	14.3
1100	170.2	163.1	160.3	157.0	156.0	14.2
1200	170.6	163.6	160.7	157.4	156.4	14.2

8.5 Simplified Analogy

S/N0 is signal strength in noise storm: closer (high El) clearer call. Verbalize aloud: "Overhead close, loud and clear; horizon far, faint whisper."

8.6 Visual Aid Suggestion

- Sketch: Path with d varying, $L_s increasingdown. - Graph : El(0-90) vs. S/N0(dB-Hz) H = 800km; key :$ 0 58, 90 70.

S/N_0 vs. Elevation Angle

Shows: curve of signal-to-noise density ratio improving from 0° (lowest) to 90° (peak), demonstrating 12-14 dB variation

Figure 7: Elevation impact on S/N_0 for LEO ground stations. Reference: Textbook Figure 4.17

Link Budget Components

Shows: breakdown of EIRP, free space loss L_s, other losses L_0, G/T_s, resulting in S/N_0; visual flow diagram

Figure 8: Link budget analysis for LEO satellite downlinks. Reference: Textbook Figure 4.18

- Python Code Snippet:

```python
import matplotlib.pyplot as plt
el = range(0, 91, 10)
sn0 = [58 + 12 * (e/90) for e in el]  # Approx
plt.plot(el, sn0, marker='o')
plt.xlabel('El (  )')
plt.ylabel('S/N0 (dB-Hz)')
plt.title('S/N0 vs. El for H=800 km')
plt.grid(True)
plt.show()
```

8.7 Retention Aid

Mnemonic Recap: "Signal High El Low Loss." Dual-code: Graph while saying "Up El up signal." Interleave: With 4.7 EIRP. Multisensory: Explain graph aloud. Spaced repetition: Review diff table weekly.

Self-Quiz Questions: 1. Multiple-Choice: S/N0 max at? (a) 0°, (b) 90°. Answer: (b). 2. Fill-in: L_s 20 log(___). Answer: d f. 3. Application: H=1000 km ΔS/N0? Answer: 11.3 dB (table). 4. What: Why compensate? Answer: Keep constant margin. 5. Calculate: L_s at d=2000 km f=2 GHz (approx). Answer: 162 dB.