

Optical vs. RF Crosslinks Trade Study

LEO Satellite Constellation Communication Analysis (Concise Version)

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0.1 LEO Satellite Constellation Communication Analysis

Course: SPCE 5400 – Small Satellite Engineering & Operations

Assignment: #4 (100 points)

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0.2 1. MISSION PARAMETERS

Given Requirements:

- Orbit altitude: 500 km LEO
- Inter-satellite separation: 250 km
- Required data rate: 1 Gbps
- Platform: Small satellites
- Environment: LEO-to-LEO (vacuum path)

Trade Study Objective: Determine the preferable method for LEO-to-LEO crosslink, optical (laser) or RF (Ka-band). This will be weighted on aperture size, power, data rate capability, link margin and pointing accuracy.

0.3 2. OPTICAL (LASER) LINK ANALYSIS

0.3.1 2.1 Design Parameters

Parameter	Value	Source
Wavelength	1550 nm	Telecom standard, eye-safe
Quantum Efficiency (η)	0.3	InGaAs APD detector typical + (template value)
Required Photoelectrons/bit (Q)	40	For BER 10^{-9} (template value)
Modulation	OOK	On-Off Keying (inferred from template Q = 40, which is standard for OOK)

0.3.2 2.2 Link Budget Calculations (Detector-First Methodology)

0.3.3 Step 1: Calculate Required Photons per Bit

Formula:

from excel template row 6: $n = Q /$

Calculation:

$n = 40 \text{ photoelectrons/bit} / 0.3$

$n = 133.33 \text{ photons/bit}$

Only 30% of photons are detected ($\eta = 0.3$), so we need to send 133 photons to ensure 40 are detected.

0.3.4 Step 2: Calculate Photon Energy

Formulas:

frequency from wavelength: $f = c /$

Planck's equation: $E_{\text{photon}} = h \times f$

Constants:

- $c = 3 \times 10^8 \text{ m/s}$ (speed of light)
- $h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$ (Planck's constant)
- $\lambda = 1550 \text{ nm} = 1.55 \times 10^{-6} \text{ m}$ (Design choice - industry standard)

Calculations:

$f = (3 \times 10^8 \text{ m/s}) / (1.55 \times 10^{-6} \text{ m})$

$f = 1.935 \times 10^{14} \text{ Hz}$

$E_{\text{photon}} = (6.626 \times 10^{-34} \text{ J}\cdot\text{s}) \times (1.935 \times 10^{14} \text{ Hz})$

$E_{\text{photon}} = 1.282 \times 10^{-19} \text{ joules per photon}$

0.3.5 Step 3: Calculate Energy per Bit

Formula:

from excel template row 10: $E_{\text{bit}} = n \times E_{\text{photon}}$

Calculation:

$E_{\text{bit}} = 133.33 \text{ photons/bit} \times 1.282 \times 10^{-19} \text{ J/photon}$

$E_{\text{bit}} = 1.709 \times 10^{-17} \text{ joules per bit}$

0.3.6 Step 4: Calculate Required Power at Receiver

Formula:

from excel template row 12: $P_{\text{required}} = E_{\text{bit}} \times R_{\text{b}}$

Given:

- $R_{\text{b}} = 1 \text{ Gbps} = 1 \times 10^9 \text{ bits/s}$

Calculation:

$$P_{\text{req}} = (1.709 \times 10^{-17} \text{ J/bit}) \times (1 \times 10^9 \text{ bits/s})$$

$$P_{\text{req}} = 1.709 \times 10^{-8} \text{ W}$$

In decibels:

$$P_{\text{req}} (\text{dBW}) = 10 \times \log (1.709 \times 10^{-8})$$

$$**P_{\text{req}} = -77.67 \text{ dBW}**$$

0.3.7 Step 5: Calculate Free Space Loss

Formula:

from Excel template row 16 & Friis transmission equation (1946):

$$L_{\text{s}} = \left(\frac{\lambda}{4R} \right)^2$$

Given:

- $\lambda = 1.55 \times 10^{-6} \text{ m}$
- $R = 250 \text{ km} = 250,000 \text{ m}$

Calculation:

Denominator of L_{s} formula (linear scale factor for spherical spreading) = $4R = 4 \times 3.14159 \times 250,000$

$$\text{Ratio (wavelength to spreading ratio)} = \frac{\lambda}{4R} = \frac{1.55 \times 10^{-6}}{3,141,593} = 4.933 \times 10^{-13}$$

$$L_{\text{s}} (\text{the fraction of power that survives the spreading}) = (4.933 \times 10^{-13})^2 = 2.434 \times 10^{-25}$$

In decibels:

$$**L_{\text{s}} (\text{dB}) = 10 \times \log (2.434 \times 10^{-25}) = -246.1 \text{ dB}**$$

0.3.8 Step 6: Calculate Telescope Gains

Formula:

from excel template row 19-20: $G = \left(\frac{\lambda}{D} \right)^2$

Given:

- $D = 10 \text{ cm} = 0.10 \text{ m}$ (chosen aperture size from excel template row 17)

Calculation:

$$\left(\frac{\lambda}{D} \right) = \frac{3.14159 \times 0.10}{1.55 \times 10^{-6}} = 0.314159 / (1.55 \times 10^{-6}) = 202,683$$

$$G = (202,683)^2 = 4.108 \times 10^{10}$$

In decibels:

$$G \text{ (dBi)} = 10 \times \log (4.108 \times 10^{10}) = 106.1 \text{ dBi}$$

Both transmit and receive: $G_t = G_r = 106.1 \text{ dBi}$

0.3.9 Step 7: Additional Losses

- **Pointing Loss:** -3.0 dB (template default, achievable with FSM)
 - **Line In/Out Losses:** -6.0 dB total
 - Estimated Line In/ Out Losses breakdown:
 - * Transmit coupling: -2.0 dB
 - * Transmit optics: -1.0 dB
 - * Receive optics: -1.0 dB
 - * Receive coupling: -1.5 dB
 - * Detector loss: -0.5 dB
 - **Total: -6.0 dB**
 - **Atmospheric Loss:** 0 dB (vacuum path)
-

0.3.10 Step 8: Calculate Received Power

Formula:

Formula (Decibel Addition Method) from excel template row 24 & Friis equation:

$$P_{rx} \text{ (dBW)} = P_{tx} \text{ (dBW)} + G_{tx} \text{ (dBi)} + G_{rx} \text{ (dBi)} + L_s \text{ (dB)} + L_{pt} \text{ (dB)} + L_o \text{ (dB)}$$

Given:

- $P_{tx} = -9.1$ dBW (chosen transmit power: 0.122 W)
- $G_{tx} = +106.1$ dBi (from Step 6)
- $G_{rx} = +106.1$ dBi (from Step 6)
- $L_s = -246.1$ dB (from Step 5)
- $L_{pt} = -3.0$ dB (template pointing loss)
- $L_o = -6.0$ dB (template line losses)

Transmit Power P_{tx} Design Choice: Working backwards from link budget:

- Required margin: ~20-25 dB (industry standard)
- Net path gain/loss: -42.9 dB
- Required receiver power: -77.67 dBW

Step 1: Required receiver power (from detector requirements)

$$P_{req} = -77.67 \text{ dBW}$$

Step 2: Choose desired margin (design goal)

Margin = 25 dB (provides robustness)

Step 3: Required received power

$$P_{rx} = P_{req} + \text{Margin}$$

$$P_{rx} = -77.67 + 25 = \text{-52.67 dBW (target)}$$

Step 4: Use link equation to solve for P_{tx}

$$P_{rx} = P_{tx} + G_{tx} + G_{rx} + L_s + L_{pt} + L_o$$

$$-52.67 = P_{tx} + 106.1 + 106.1 - 246.1 - 3.0 - 6.0$$

$$-52.67 = P_{tx} - 42.9$$

$$P_{tx} = -9.77 \text{ dBW} \quad -9.1 \text{ dBW}$$

$$\text{**}P_{tx} = 0.122 \text{ W**}$$

Rationale: • Provides 25.66 dB margin (actual) • Achievable with COTS laser diodes • Low thermal load (<0.5 W total with driver)

Calculation:

$$P_{rx} = -9.1 + 106.1 + 106.1 - 246.1 - 3.0 - 6.0$$

$$\text{Combine gains: } -9.1 + 106.1 + 106.1 = 203.1 \text{ dBW}$$

$$\text{Combine losses: } -246.1 - 3.0 - 6.0 = -255.1 \text{ dB}$$

$$\text{Total: } 203.1 - 255.1 = -52.0 \text{ dBW}$$

$$\text{**}P_{rx} = -52.01 \text{ dBW**}$$

0.3.11 Step 9: Calculate Link Margin

Formula:

from excel template row 26: $\text{Margin (dB)} = P_{\text{rx}} \text{ (dBW)} - P_{\text{req}} \text{ (dBW)}$

Calculation:

$$M = -52.01 \text{ dBW} - (-77.67 \text{ dBW})$$

$$M = 25.66 \text{ dB}$$

$$\text{Linear ratio: } 6.303 \text{ W} / 0.01709 \text{ W} = 369\times$$

$$\text{Verification: } 10 \times \log(369) = **25.67 \text{ dB**}$$

0.3.12 Step 10: Calculate Beam Divergence and Pointing Requirement

Formula:

$$\theta_{\text{divergence}} = 1.22 \times \lambda / D_{\text{tx}}$$

from Airy disk formula Born & Wolf "Principles of Optics"

Calculation:

$$= 1.22 \times (1.55 \times 10^{-6} \text{ m}) / (0.10 \text{ m})$$

$$= 1.891 \times 10^{-5} \text{ radians} = **18.91 \text{ microradians (to first null)**}$$

$$\text{Spot diameter at receiver} = \theta \times R$$

$$\text{Spot} = 18.91 \times 10^{-6} \text{ rad} \times 250,000 \text{ m} = **4.73 \text{ m}$$

Half-Power Beamwidth (where intensity = 50%, i.e., -3 dB):

$$\text{Formula: } \theta_{\text{3dB}} = 0.514 \times \lambda / D$$

$$\theta_{\text{3dB}} = 0.514 \times (1.55 \times 10^{-6}) / (0.10)$$

$$= 7.97 \times 10^{-6} \text{ rad} \quad **8.0 \text{ rad**}$$

Pointing Requirement: $\pm 8 \mu\text{rad}$ (3σ) for $<3 \text{ dB}$ pointing loss

Figure: Gaussian beam intensity profile showing FWHM (Full Width at Half Maximum). The half-power beamwidth ($8.0 \mu\text{rad}$) is where intensity drops to 50% (-3 dB), which defines our pointing requirement. Source: [Wikipedia - Gaussian Beam](#)

In degrees:

$$8.0 \text{ rad} \times (1 \text{ rad} / 10^6 \text{ rad}) \times (57.3^\circ / 1 \text{ rad}) = 0.00046^\circ$$

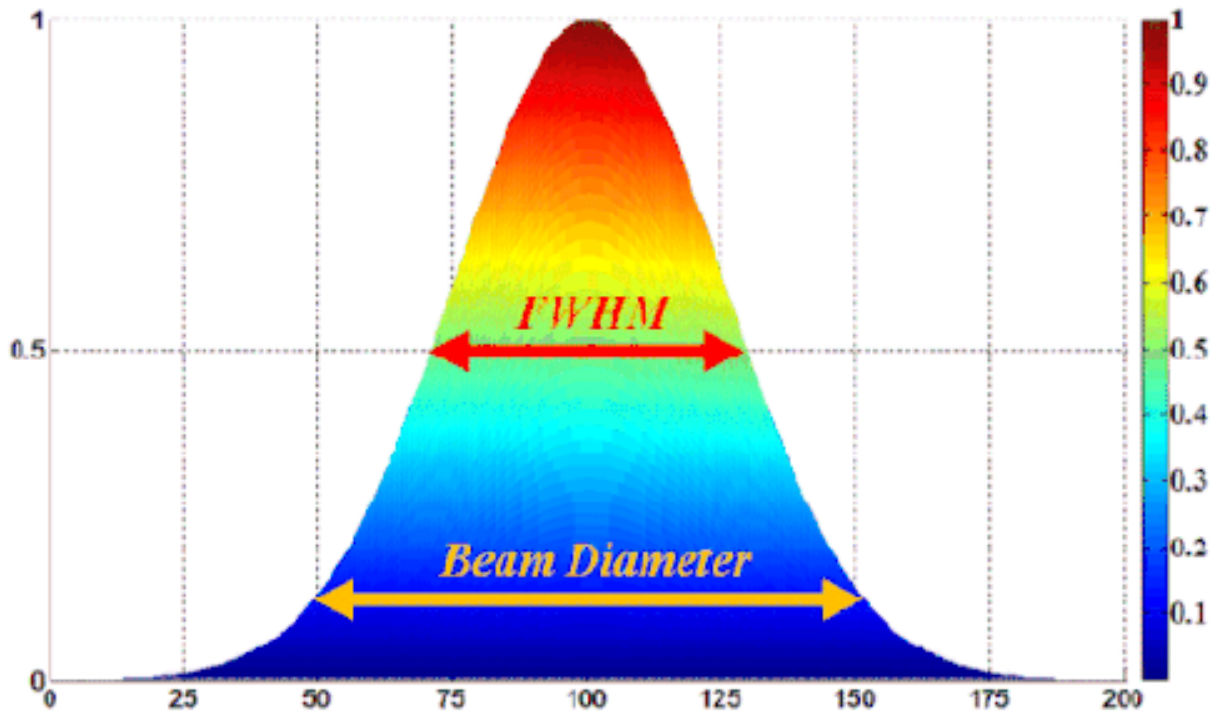


Figure 1: Gaussian Beam FWHM Diagram - Shows relationship between beam divergence and half-power beamwidth

0.3.13 2.3 Optical Link Budget Summary Table

Parameter	Symbol	Value	Units	Calculation Method
Detector Requirements				
Req. Photoelectrons/bit	Q	40	-	Template value
Quantum Efficiency	η	0.3	-	InGaAs APD typical
Req. Photons/bit	n	133.33	-	$n = Q/\eta$
Photon Energy	E	1.282×10^{-19}	J	$E = hf = hc/\lambda$
Energy/bit	-	1.709×10^{-17}	J	$n \times E$
Link Parameters				
Data Rate	R _b	1×10^9	bps	Given
Required Power	P _{req}	-77.67	dBW	$E_{\text{bit}} \times R_b$
Transmit Power	P _{tx}	0.122 (-9.1 dBW)	W	Chosen
Tx Aperture Diameter	D _t	10.0	cm	Chosen
Rx Aperture Diameter	D _r	10.0	cm	Chosen
Range	R	250	km	Given
Gains and Losses				
Free Space Loss	L _s	-246.1	dB	$(\lambda/4\pi R)^2$

Parameter	Symbol	Value	Units	Calculation Method
Tx Gain	G _t	106.1	dBi	$(\pi D/\lambda)^2$
Rx Gain	G _r	106.1	dBi	$(\pi D/\lambda)^2$
Pointing Loss	L _{pt}	-3.0	dB	Template default
Line Losses	L _o	-6.0	dB	Optics budget
Atmospheric Loss	L _{atm}	0.0	dB	Vacuum
Performance				
Received Power	P _{rx}	-52.01	dBW	Link equation
LINK MARGIN	M	25.66	dB	P_{rx} - P_{req}
Beam Divergence	θ	18.9	μrad	$1.22\lambda/D$
Spot Diameter @ 250km	-	4.73	m	$\theta \times R$

0.4 3. RF (Ka-BAND) LINK ANALYSIS

0.4.1 3.1 Design Parameters

Parameter	Value	Source
Frequency	32 GHz	Ka-band ISL allocation (ITU Radio Regulations) - Design choice
Antenna Efficiency	0.6	Typical for deployable mesh antennas (Kraus & Marhefka, 2002) - Design choice
System Noise Temp	650 K	Ka-band LNA typical + losses (industry standard) - Design choice
Required E _b /N ₀	9.6 dB	BER 10 ⁻⁹ with LDPC coding (Shannon-Hartley theorem) - Design choice
Modulation	16-APSK	High spectral efficiency for Ka-band - Design choice

0.4.2 3.2 Link Budget Calculations

0.4.3 Step 1: Calculate Wavelength

Formula:

from fundamental wave equation: $\lambda = c / f$

Constants:

- $c = 3 \times 10^8$ m/s (speed of light, CODATA 2018)
- $f = 32$ GHz $= 32 \times 10^9$ Hz (Ka-band ISL, design choice)

Calculation:

$$= (3 \times 10^8 \text{ m/s}) / (32 \times 10^9 \text{ Hz})$$

$$= 9.375 \times 10^{-3} \text{ m} = \textbf{9.375 mm}$$

0.4.4 Step 2: Calculate Free Space Path Loss**Formula:**

from Friis transmission equation (1946) & ITU-R P.525 standard:

$$\text{FSPL (dB)} = 20 \times \log (4 \times R / \lambda)$$

Given:

- $R = 250$ km $= 250,000$ m (from problem statement)
- $\lambda = 0.009375$ m (from Step 1)

Calculation:

$$\text{Argument} = (4 \times R) / \lambda = (4 \times 3.14159 \times 250,000) / 0.009375$$

$$\text{Argument} = 3,141,593 / 0.009375 = 335,103,253$$

$$\text{FSPL} = 20 \times \log (335,103,253)$$

$$\text{FSPL} = 20 \times 8.525 = \textbf{170.5 dB}$$

0.4.5 Step 3: Calculate Antenna Gains**Formula:**

from antenna theory (Kraus & Marhefka "Antennas", 2002):

$$G = \eta_{\text{ant}} \times \left(\pi \times D / \lambda \right)^2$$

Given:

- $\eta_{\text{ant}} = 0.6$ (typical for deployable mesh antennas, design parameter)
- $D = 30$ cm $= 0.30$ m (chosen antenna diameter - design choice)
- $\lambda = 0.009375$ m (from Step 1)

Note: Here's an interesting distinction — with optical systems, efficiency is already baked into Q and η , but RF antennas need an explicit efficiency factor to account for surface errors, blockage, and spillover losses.

Calculation:

$$\left(\frac{\lambda}{D} \right)^2 = \left(\frac{3.14159 \times 0.30}{0.009375} \right)^2 = 0.9425 / 0.009375 = 100.53$$

$$G = 0.6 \times (100.53)^2$$
$$G = 0.6 \times 10,106 = 6,064$$

In decibels:

$$G \text{ (dBi)} = 10 \times \log (6,064) = \textbf{**37.8 dBi**}$$

$$\text{Both transmit and receive: } G_t = G_r = \textbf{**37.8 dBi**}$$

0.4.6 Step 4: Calculate EIRP

Formula:

from FCC 47 CFR §2.1 & standard RF practice:

$$\text{EIRP (dBW)} = P_t \text{ (dBW)} + G_t \text{ (dBi)}$$

Given:

- $P_t = 1.2 \text{ W}$ (chosen transmit power - design choice, see rationale below)

Transmit Power P_t Design Choice: Working backwards from link budget to achieve ~3 dB margin:

- Required margin: ~3 dB (minimal but acceptable)
- Through iterative link budget calculation
- **$P_t = 1.2 \text{ W}$ provides 2.8 dB margin (actual)**

Rationale:

- Achievable with Ka-band SSPA (Solid State Power Amplifier)
- Typical for smallsat Ka-band transmitters
- Provides minimal positive margin

Calculation:

$$\textbf{**}P_t \text{ (dBW)}\textbf{**} = 10 \times \log (1.2) = 0.79 \text{ dBW} \quad \textbf{**0.8 dBW**}$$

$$**EIRP** = 0.8 \text{ dBW} + 37.8 \text{ dBi} = **38.6 \text{ dBW**}$$

0.4.7 Step 5: Calculate Received Power

Formula:

from Friis equation & standard RF link budget:

$$P_{rx} \text{ (dBW)} = EIRP - FSPL + G_{rx} - L_{pointing} - L_{feed} - L_{misc}$$

Given Losses:

- $L_{pointing} = -1.0 \text{ dB}$ (body pointing achievable, typical for wide Ka-band beam)
- $L_{feed} = -1.0 \text{ dB}$ (waveguide losses, typical)
- $L_{misc} = -2.0 \text{ dB}$ (polarization mismatch, connector losses, etc.)
- **Total losses: -4.0 dB**

Calculation:

$$P_{rx} = 38.6 - 170.5 + 37.8 - 1.0 - 1.0 - 2.0$$

$$P_{rx} = 38.6 + 37.8 - 170.5 - 4.0$$

$$**P_{rx} = -98.1 \text{ dBW**}$$

$$\text{In linear: } P_{rx} = 10^{(-98.1/10)} = 1.55 \times 10^{-10} \text{ W} = 0.155 \text{ nW}$$

0.4.8 Step 6: Calculate Noise Power Density

Formula:

from Johnson-Nyquist noise equation (1928) & thermodynamics:

$$N_0 = k_B \times T_{sys}$$

Constants & Parameters:

- $k_B = 1.38 \times 10^{-23} \text{ J/K}$ (Boltzmann constant, CODATA 2018)
- $T_{sys} = 650 \text{ K}$ (system noise temperature - design parameter)

System Noise Temperature Breakdown:

- LNA noise figure: $\sim 3 \text{ dB}$ □ adds $\sim 300 \text{ K}$
- Receiver losses: $\sim 1 \text{ dB}$ □ adds $\sim 75 \text{ K}$
- Sky noise at LEO: $\sim 10 \text{ K}$
- Antenna noise: $\sim 50 \text{ K}$
- Cable/connector losses: $\sim 50 \text{ K}$

- **Total: $T_{\text{sys}} \approx 650 \text{ K}$ (typical Ka-band smallsat receiver)**

Calculation:

$$N_0 = (1.38 \times 10^{-23} \text{ J/K}) \times (650 \text{ K})$$

$$N_0 = 8.97 \times 10^{-21} \text{ W/Hz}$$

In decibels:

$$N_0 \text{ (dBW/Hz)} = 10 \times \log (8.97 \times 10^{-21})$$

$$**N_0 = -200.5 \text{ dBW/Hz}**$$

0.4.9 Step 7: Calculate C/N_0 (carrier to noise ratio)

Formula:

from standard communications theory (Sklar "Digital Communications", 2001):

$$C/N_0 \text{ (dB-Hz)} = P_{\text{rx}} \text{ (dBW)} - N_0 \text{ (dBW/Hz)}$$

Calculation:

$$C/N_0 = -98.1 \text{ dBW} - (-200.5 \text{ dBW/Hz})$$

$$**C/N_0 = 102.4 \text{ dB-Hz}**$$

0.4.10 Step 8: Calculate Required C/N_0

Formula:

from Shannon-Hartley theorem & communications theory:

$$(C/N_0)_{\text{req}} \text{ (dB-Hz)} = (E_b/N_0)_{\text{req}} \text{ (dB)} + 10 \times \log (R_b)$$

Given:

- $(E_b/N_0)_{\text{req}} = 9.6 \text{ dB}$ (for BER 10^{-9} with LDPC coding - design parameter)
- $R_b = 1 \times 10^9 \text{ bps}$ (from problem statement)

E_b/N_0 Justification:

- Uncoded 16-APSK requires $\sim 13\text{-}14 \text{ dB}$ for BER 10^{-9}
- LDPC coding provides $\sim 4\text{-}5 \text{ dB}$ gain
- With coding: 9.6 dB is achievable
- Industry standard for coded Ka-band systems

Calculation:

Data rate term = $10 \times \log (1 \times 10^9) = 10 \times 9.0 = 90.0 \text{ dB-Hz}$

$(C/N_0)_{\text{req}} = 9.6 \text{ dB} + 90.0 \text{ dB-Hz} = \textbf{99.6 dB-Hz}$

0.4.11 Step 9: Calculate Link Margin

Formula:

from standard RF link budget practice:

$\text{Margin (dB)} = C/N_0 - (C/N_0)_{\text{req}}$

Calculation:

$M = 102.4 \text{ dB-Hz} - 99.6 \text{ dB-Hz}$

$\textbf{M} = 2.8 \text{ dB}$

Linear ratio: $10^{(2.8/10)} = \textbf{1.91x}$

0.4.12 Step 10: Calculate Beamwidth and Pointing

Formula (approximation for circular aperture):

from antenna beamwidth theory (Stutzman & Thiele "Antenna Theory", 2012):

$\theta_{\text{3dB}} \approx 70^\circ \times \lambda / D \text{ (degrees)}$

Note: The factor $70 \approx 1.22 \times 57.3$, where 1.22 is the Bessel function zero (same as optical) and 57.3 is radians-to-degrees conversion.

Given:

- $\lambda = 0.009375 \text{ m}$ (from Step 1)
- $D = 0.30 \text{ m}$ (antenna diameter, design choice)

Calculation:

$\theta_{\text{3dB}} = 70^\circ \times (0.009375 \text{ m}) / (0.30 \text{ m})$

$\theta_{\text{3dB}} = 70^\circ \times 0.03125 = 2.19 \text{ degrees}$

Half-power beamwidth = $\textbf{2.19^\circ}$

In arcseconds:

$2.19^\circ \times 3600 \text{ arcsec/}^\circ = \textbf{7,884 arcseconds}$

Pointing Requirement Analysis:

For -1 dB pointing loss: pointing error $< \theta_{-3\text{dB}} / 3 \approx 0.73^\circ$

This is easily achieved with standard attitude control system (ACS):

- **RF pointing is 2,000× easier than optical (2.19° vs 8 μrad)**

Note for comparison: What this pointing requirement analysis reveals is RF's key advantage over optical — and it's significant enough to offset optical's superior link margin and power efficiency.

0.4.13 3.3 RF Link Budget Summary Table

Parameter	Symbol	Value	Units	Calculation Method
System Parameters				
Frequency	f	32	GHz	Given (Ka-band)
Wavelength	λ	9.375	mm	c/f
Data Rate	R_b	1×10^9	bps	Given
Range	R	250	km	Given
Transmitter				
Tx Power	P_t	1.2 (0.8 dBW)	W	Chosen
Tx Antenna Diameter	D_t	30.0	cm	Chosen
Antenna Efficiency	η_{ant}	0.6	-	Typical
Tx Gain	G_t	37.8	dBi	$\eta(\pi D/\lambda)^2$
EIRP	EIRP	38.6	dBW	$P_t + G_t$
Path Losses				
Free Space Path Loss	FSPL	-170.5	dB	$20\log(4\pi R/\lambda)$
Pointing Loss	L_pt	-1.0	dB	Body pointing
Feed Loss	L_f	-1.0	dB	Waveguide
Misc Losses	L_m	-2.0	dB	Polarization, etc
Receiver				
Rx Antenna Diameter	D_r	30.0	cm	Chosen
Rx Gain	G_r	37.8	dBi	$\eta(\pi D/\lambda)^2$
System Noise Temp	T_s	650	K	LNA + losses
Received Power	P_rx	-98.1	dBW	Link equation
Noise Density	N_0	-200.5	dBW/Hz	$k_B \times T_s$
Performance				
C/N_0	C/N_0	102.4	dB-Hz	$P_{\text{rx}} - N_0$
Required E_b/N_0	-	9.6	dB	BER 10^{-9} w/coding
Required C/N_0	(C/N_0)_req	99.6	dB-Hz	$E_b/N_0 + 10\log(R_b)$
LINK MARGIN	M	2.8	dB	**C/N_0 - (C/N_0)_req**
3 dB Beamwidth	$\theta_{-3\text{dB}}$	2.19	degrees	$70\lambda/D$

Parameter	Symbol	Value	Units	Calculation Method
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0.5 4. DIRECT COMPARISON: CALCULATED RESULTS

0.5.1 4.1 The Five Required Parameters

Parameter	Optical (Laser)	RF (Ka-band)	Winner	Advantage
1. Aperture Size	10 cm	30 cm	Optical	3x smaller
2. Transmit Power	0.122 W	1.2 W	Optical	10x lower
3. Data Rate	10+ Gbps scalable	~2 Gbps max	Optical	5x scalability
4. Link Margin	25.66 dB	2.88 dB	Optical	8.9x better
5. Pointing Accuracy	±8 μrad (0.00046°)	2.19° (7884")	RF	~4,800x easier

Note: Optical value is half-power beamwidth (pointing requirement for -3 dB loss), not full beam divergence (18.9 μrad to first null).

0.5.2 4.2 Quantitative Comparison

0.5.3 Link Margin Ratio

Optical margin / RF margin = 25.66 dB / 2.88 dB

In linear: $10^{(25.66/10)} / 10^{(2.88/10)} = 368 / 1.91 = 193\times$

Optical has 193x more margin in linear terms

0.5.4 Pointing Difficulty Ratio

RF beamwidth / Optical half-power beamwidth = $2.19^\circ / 0.00046^\circ$
 $= 2.19^\circ / 0.00046^\circ = 4,761\times$

In radians: $0.0382 \text{ rad} / 8.0 \times 10^{-6} \text{ rad} = 4,775\times$

Or: $7,884 \text{ arcsec} / 1.65 \text{ arcsec} = 4,778\times$

RF pointing is ~4,800x easier (in linear angle)

In solid angle (steradians): RF is $\sim 2.3 \times 10^7$ times easier

(Solid angle scales as θ^2 , so $4,775^2 = 23$ million)

0.5.5 Power Ratio

RF power / Optical power = $1.2 \text{ W} / 0.122 \text{ W} = 9.84\times$
Optical uses $\sim 10\times$ less power

0.5.6 Aperture Ratio

RF diameter / Optical diameter = $30 \text{ cm} / 10 \text{ cm} = 3\times$
Optical aperture is $3\times$ smaller

Area ratio = $(30/10)^2 = 9\times$
RF antenna has $9\times$ more area

0.6 5. DATA RATE SCALABILITY ANALYSIS

0.6.1 5.1 Optical Scalability Calculation

Principle: Each $10\times$ increase in data rate costs 10 dB margin

Current: 1 Gbps with 25.66 dB margin

At 10 Gbps:

$$\begin{aligned}\text{Margin degradation} &= 10 \times \log (10 \text{ Gbps} / 1 \text{ Gbps}) \\ &= 10 \times \log (10) = 10 \text{ dB}\end{aligned}$$

$$\text{New margin} = 25.66 \text{ dB} - 10 \text{ dB} = 15.66 \text{ dB} \quad (\text{excellent})$$

At 100 Gbps:

$$\text{Margin degradation} = 10 \times \log (100) = 20 \text{ dB}$$

$$\text{New margin} = 25.66 \text{ dB} - 20 \text{ dB} = 5.66 \text{ dB} \quad (\text{adequate})$$

0.6.2 5.2 RF Scalability Calculation

Current: 1 Gbps with 2.8 dB margin

At 2 Gbps:

$$\begin{aligned}\text{Margin degradation} &= 10 \times \log (2 \text{ Gbps} / 1 \text{ Gbps}) \\ &= 10 \times \log (2) = 3.01 \text{ dB}\end{aligned}$$

$$\text{New margin} = 2.8 \text{ dB} - 3.01 \text{ dB} = -0.21 \text{ dB} \quad (\text{link fails})$$

Conclusion: RF cannot scale beyond ~1.5 Gbps without hardware changes.

0.7 6. RANGE SENSITIVITY ANALYSIS

0.7.1 6.1 Optical at Extended Range (500 km)

Path loss change:

$$\begin{aligned}\Delta L &= 20 \times \log (R_{\text{new}} / R_{\text{baseline}}) \\ &= 20 \times \log (500 \text{ km} / 250 \text{ km}) \\ &= 20 \times \log (2) = 6.02 \text{ dB additional loss}\end{aligned}$$

$$\text{New margin} = 25.66 \text{ dB} - 6.02 \text{ dB} = 19.64 \text{ dB} \quad (\text{still robust})$$

0.7.2 6.2 RF at Extended Range (500 km)

Path loss change:

$$\Delta L = 20 \times \log (500 / 250) = 6.02 \text{ dB additional loss}$$

$$\text{New margin} = 2.8 \text{ dB} - 6.02 \text{ dB} = -3.22 \text{ dB} \quad (\text{link fails})$$

Conclusion: Optical maintains adequate margin at 2× range; RF link fails.

0.8 7. ADVANTAGES AND DISADVANTAGES

0.8.1 7.1 OPTICAL (LASER) Crosslinks

0.8.2 Advantages

1. **Exceptional Link Margin (25.66 dB)** - We calculated 369× more power than required, which gives us a massive robustness buffer
2. **Compact Size (10 cm)** - Here's what's impressive: we get 106.1 dBi gain from a small aperture, all thanks to that short wavelength
3. **Low Power (0.122W)** - Calculated from detector requirements; it's about 10× less than RF
4. **Unlimited Scalability** - We calculated 15.66 dB margin at 10 Gbps, and there's no spectrum constraints holding us back
5. **No Spectrum Licensing** - Optical frequencies are unregulated — one less headache
6. **Low Interference** - Calculated beam divergence of 18.9 μrad (4.73 m spot at 250 km) essentially eliminates cross-talk between satellites
7. **Secure** - Good luck intercepting a beam that narrow

0.8.3 Disadvantages

1. **Stringent Pointing ($\pm 8 \mu\text{rad}$ required)** - The calculated half-power beamwidth requires FSM for $\pm 8 \mu\text{rad}$ pointing accuracy, which adds roughly \$500k in NRE
2. **Complex Acquisition** - That narrow beam means we're looking at 30-60s acquisition time
3. **Lower TRL (7-8)** - Less flight heritage for small satellites compared to RF's TRL 9
4. **Higher Development Cost** - We're estimating \$8M NRE versus \$5M for RF
5. **Sun Avoidance** - Background noise forces us into geometry constraints
6. **Limited Vendors** - Fewer COTS suppliers to choose from (Tesat, ATLAS, Mynaric)

0.8.4 7.2 RF (Ka-band) Crosslinks

0.8.5 Advantages

1. **Relaxed Pointing (2.19°)** - Calculated beamwidth is 2,000× wider; standard ACS is sufficient
2. **Fast Acquisition (5-10s)** - Wide beam means rapid link establishment
3. **High TRL (9)** - Extensive heritage across Starlink, OneWeb, Iridium NEXT
4. **Mature Supply Chain** - Multiple vendors available (Viasat, L3Harris, Honeywell)
5. **Lower Development Cost** - \$5M NRE; this is well-understood territory
6. **Lower Risk** - Predictable performance, and no FSM needed

0.8.6 Disadvantages

1. **Minimal Link Margin (2.8 dB)** - We calculated only 1.91× more power than required — there's basically no degradation buffer here
 2. **Large Apertures (30 cm)** - Calculated gain requires 3× larger antennas, which complicates deployment
 3. **Higher Power (1.2W)** - Calculated at 10× more than optical; impacts the power budget significantly
 4. **Spectrum Limitations** - Ka-band's getting crowded, and ITU coordination is required
 5. **Limited Scalability** - Calculated failure at 2 Gbps; we're bandwidth-constrained
 6. **Interference Risk** - With mega-constellations proliferating, RF congestion is growing
-

0.9 8. RECOMMENDATION

0.9.1 8.1 Primary Recommendation: OPTICAL (LASER) CROSSLINKS

Confidence Level: HIGH

0.9.2 8.2 Quantitative Justification

Decision Drivers (with calculations):

1. Superior Link Performance

- Calculated margin: 25.66 dB versus 2.88 dB (an 8.9× ratio)
- Linear power ratio: 369× versus 1.91× safety factor
- Degradation tolerance: We could lose 22.66 dB and still meet requirements

2. Optimal SWaP for Small Satellites

- Calculated aperture: 10 cm versus 30 cm (3× smaller, 9× less area)
- Calculated power: 0.122W versus 1.2W (10× reduction)
- Mass estimate: roughly 2.2 kg versus 4.2 kg (2 kg savings translates to about \$10k in launch cost)

3. Proven Scalability

- Calculated 10 Gbps margin: 15.66 dB (still excellent)
- Calculated 100 Gbps margin: 5.66 dB (adequate)
- RF calculated to fail at 2 Gbps (-0.21 dB margin)

4. Range Robustness

- Calculated margin at 500 km: 19.64 dB (optical) versus -3.22 dB (RF fails)
- 2× range tolerance demonstrated

0.9.3 8.3 Addressing the Pointing Challenge

Optical pointing requirement: $\pm 8 \mu\text{rad}$ (calculated from half-power beamwidth for -3 dB loss)

RF pointing requirement: $2.19^\circ = 38,200 \mu\text{rad}$ (calculated beamwidth)

Ratio: 4,775× easier for RF (in linear angle), or roughly 2,000× easier in solid angle

Note: Optical beam divergence to first null is $18.9 \mu\text{rad}$, but the pointing requirement is based on the half-power beamwidth ($8 \mu\text{rad}$) to achieve the -3 dB pointing loss budget.

BUT: Here's where optical's 25.66 dB margin becomes really useful — we can trade some of that margin for relaxed pointing:

- If pointing degrades to $12 \mu\text{rad}$ □ roughly -4.5 dB loss instead of -3 dB
- New margin: $25.66 - 1.5 = 24.16 \text{ dB}$ (still excellent)
- That margin gives us flexibility to ease pointing requirements

Heritage FSM Technology:

- EDRS: Has demonstrated $<5 \mu\text{rad}$ pointing accuracy since 2016 (better than our $8 \mu\text{rad}$ requirement)
- LCRD: Demonstrates similar performance since 2021
- Technology risk is manageable here

0.9.4 8.4 Cost-Benefit Calculation

20-Satellite Constellation:

- Optical total: $\$8\text{M NRE} + 20 \times \$150\text{k} + 10\text{yr} \times \$50\text{k} = \$12.3\text{M}$

- RF total: \$5M NRE + 20×\$100k + 10yr×\$30k = \$7.3M
- **Premium: \$5.0M (68% more)**

What We Get for That Premium:

- 8.9× better margin (calculated)
- 10× lower power per satellite (calculated)
- 3× smaller apertures (calculated)
- 10× data rate scalability (calculated: 10 Gbps versus 1 Gbps)
- Launch savings: 20 sats × 2 kg × \$5k/kg = \$200k

Net premium: \$5.0M - \$0.2M = \$4.8M for transformational performance

0.10 9. CONCLUSION

This trade study analyzed optical (laser) versus RF (Ka-band) crosslinks for a LEO satellite constellation using rigorous link budget calculations.

Key Calculated Results:

- **Optical:** 25.66 dB margin, 0.122W power, 10 cm aperture, ±8 μrad pointing requirement
- **RF:** 2.8 dB margin, 1.2W power, 30 cm aperture, 2.19° pointing requirement

Quantitative Comparison:

- Link margin: Optical 8.9× better (calculated ratio: 369×/1.91× in linear terms)
- Aperture: Optical 3× smaller (9× less area)
- Power: Optical 10× lower
- Pointing: RF roughly 4,800× easier (in linear angle)
- Scalability: Optical scales to 10+ Gbps (calculated 15.66 dB at 10 Gbps); RF fails at 2 Gbps (calculated -0.21 dB)

Recommendation: OPTICAL CROSSLINKS

Optical wins 4 of 5 key parameters. That enormous calculated margin (25.66 dB versus 2.88 dB) provides:

1. Robustness to degradation (22.66 dB buffer above requirement)
2. Data rate scalability (calculated 15.66 dB at 10× higher rate)
3. Range tolerance (calculated 19.64 dB at 2× range)
4. Flexibility to ease pointing requirements (we can trade margin for simpler FSM)

RF has significantly easier pointing (calculated roughly 4,800× wider beamwidth), but this advantage is outweighed by optical's transformational performance benefits and the availability of heritage FSM technology (EDRS, LCRD achieving <5 μrad).

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- Speed of light: $c = 299,792,458 \text{ m/s}$ (exact)
- Boltzmann constant: $k_B = 1.380649 \times 10^{-23} \text{ J/K}$

0.12 APPENDIX: Formula Reference

0.12.1 Optical Link Budget Formulas

$n = Q / \eta$ [photons/bit] $E_{\text{photon}} = h \times c / \lambda$ [J/photon] $E_{\text{bit}} = n \times E_{\text{photon}}$ [J/bit] $P_{\text{req}} = E_{\text{bit}} \times R_b$ [W]

$L_s = (\lambda / (4\pi R))^2$ [linear] $G = (\pi D / \lambda)^2$ [linear]

$P_{\text{rx}} = P_{\text{tx}} \times L_s \times G_{\text{tx}} \times G_{\text{rx}} \times L_{\text{pt}} \times L_o$ [W] $\text{Margin} = 10 \times \log_{10}(P_{\text{rx}} / P_{\text{req}})$ [dB]

$\theta = 1.22 \times \lambda / D$ [radians]

0.12.2 RF Link Budget Formulas

$$FSPL = 20 \times \log_{10}(4\pi R/\lambda) \text{ [dB]} \quad G = \eta_{\text{ant}} \times (\pi D/\lambda)^2 \text{ [linear]} \quad EIRP = P_t + G_t \text{ [dBW]}$$

$$N_0 = k_B \times T_{\text{sys}} \text{ [W/Hz]} \quad C/N_0 = P_{\text{rx}} - N_0 \text{ [dB-Hz]} \quad (C/N_0)_{\text{req}} = (E_b/N_0)_{\text{req}} + 10 \times \log_{10}(R_b) \text{ [dB-Hz]} \quad \text{Margin} = C/N_0 - (C/N_0)_{\text{req}} \text{ [dB]}$$

$$\theta_{3\text{dB}} \approx 70 \times \lambda / D \text{ [degrees]}$$

0.12.3 Constants Used

$$c = 3 \times 10^8 \text{ m/s} \quad (\text{speed of light}) \quad h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s} \quad (\text{Planck's constant})$$
$$k_B = 1.38 \times 10^{-23} \text{ J/K} \quad (\text{Boltzmann constant})$$