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SPCE_5400 - Homework #2

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Given a satellite in circular, polar LEO at an altitude of 1000 km, with the following parameters:
Satellite Tx Signal:
• Tx power: 10 W (10 dBW)
• Frequency: 2.0 GHz
• Satellite antenna Gain: 3 dBi
• Atmospheric Conditions: No rain, no adverse weather
Ground Station:
• Location: Pikes Peak, CO [no visible obstructions]
• Lock-on Design: AOS [Az, 30°]

Find the following:
1. Estimate Satellite speed in orbit (km/s) [10 pts]
2. Estimate the Satellite orbit period (min/sec) [10 pts]
3. Estimate visible time duration from acquisition of Signal AOS [Az, El] = AOS [0, 0] to loss of signal LOS [180,0]. (The satellite passes over a ground station (directly overhead, North to South, with near 90 Max El). [20 pts]
4. Estimate the percent of overhead (percent) the signal is actually "locked-on", based upon the design lock-on AOS[Az, 30]. [20 pts]
5. Calculate and plot the power received (dBW) [Sat Tx EIRP – Free Space Loss] at the ground station over throughout the pass (in 10 deg increments). [20]
6. At what degree elevation would the signal carry the maximum and minimum Doppler; what value (kHz) of maximum Doppler would be expected? [20]

1.) Estimate Satellite speed in orbit (km/s)

From chap. 4.1 eq 4.6: $V=\sqrt{\frac{\mu}{r}}$

- V = Satellite velocity in circular orbit (km/s)
- r = orbital radius = radius of earth, $R_E + H$

  where H = altitude

  $\Rightarrow 6371 + 1000 = 7371$ km

- μ = gravitational constant $= 3.986\times10^5\ km^3/s^2$

$$V=\sqrt{\frac{3.986\times10^5\ km^{\cancel{3}}/s^2}{7371\ \cancel{km}}} = \boxed{7.354\ km/s = V}$$

2.) Satellite orbital period (min/sec)

From Kepler's laws, Eq. 4.7: $T = 2\pi\sqrt{\frac{r^3}{\mu}}$

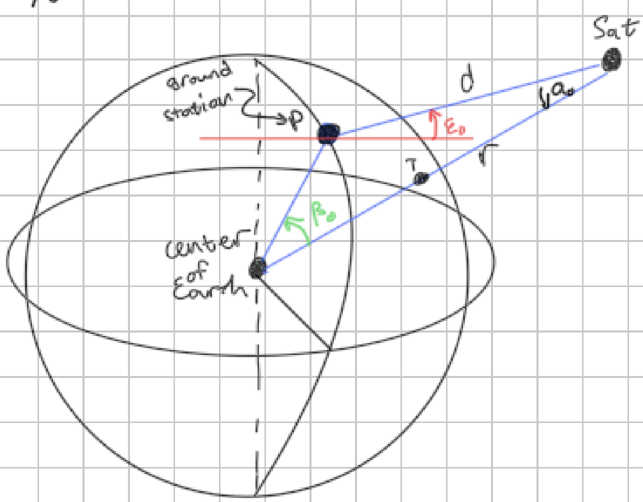
- $T = \text{period} = 2\pi\sqrt{\frac{7371^3\ km}{3.986\times10^5\ km^3/s^2}} = 6295.5\ s$

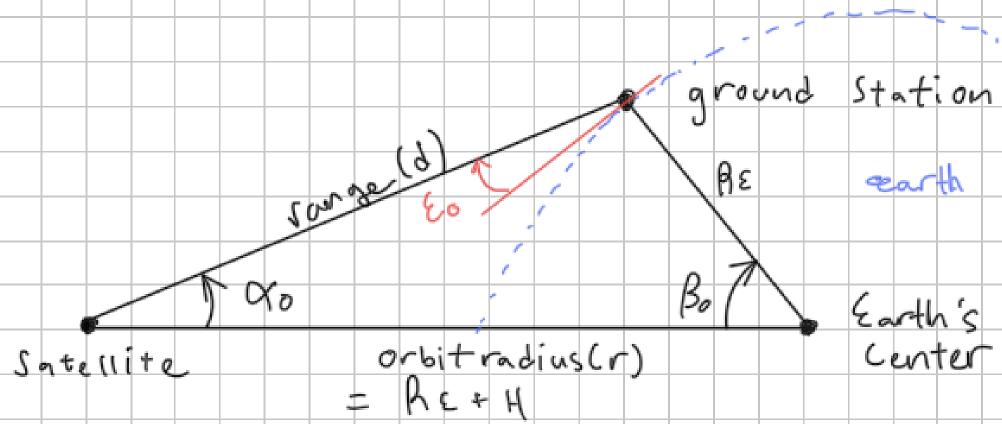
$= 104.9\ min = 104\ min\ 54\ se$

$\boxed{\text{The orbital period (T)} = 104\ min\ 54\ sec}$

3.) Estimate visible time duration from AOS [0,0] to LOS [180,0]

- the satellite passe directly overhead N/S with EL max ~ 90°





Step 1 : calc max angle under which the satellite sees the ground station (nadir angle[α])

$$\sin(\alpha_{.max}) = \frac{R_E}{R_E + H}$$

- From ch 5 Eq 5.5: Max nadir angle $\alpha_{0,max}$

$$\alpha_{0,max} = \sin^{-1}\left(\frac{R_E}{R_E + H}\right) \Rightarrow \sin^{-1}\left(\frac{6371\,km}{7371\,km}\right)$$

$$\alpha_{0,max} = \underline{59.75}$$

Step 2 : Calculate Central angle

ch5 eq 5.2: Fundamental triangle relationship

$$\cancel{\varepsilon_0}^{\,0} + \alpha_0 + \beta_0 = 90 \quad @ \text{ horizon } \varepsilon_0 = 0$$

$$\Rightarrow \alpha_0 + \beta_0 = 90 \Rightarrow \beta_0 = 90 - \alpha_0$$

- since the satellite travels from $-\beta_{0,max}$ to $+\beta_{0,max}$ in a symmetric overhead pass, the total central angle

$\therefore$ total visibility arc $(\gamma) = 2\beta_{0,max} \Rightarrow 2(90 - \alpha_{0,max})$

$$= 2(90 - 59.75) = \underline{60.5^\circ}$$

Step 3 : Calc visible duration

- angular v $(\omega) = \frac{360}{T}$

- visible time is time it takes to traverse the central angle $(60.5°)$

$$\therefore t_{visible} = \frac{\gamma}{\omega} = \frac{\gamma}{\frac{360}{T}} \Rightarrow t_{visible} = \frac{\gamma}{360} \times T$$

$$t_{visible} = \frac{60.5}{360}(6295.5\text{ sec}) = 1058\text{ sec}$$

$$= 17.6\text{ min} \approx 17\text{ min } 38\text{ sec}$$

$$\boxed{\text{Visible duration} = 17\text{ min } 38\text{ sec}}$$

4.) Estimate percent lock on time

Step 1: Calc nadir angle at 30° elevation

From ch 5.2 eq. 5.4 : $\sin \alpha_0 = \frac{R_E}{R_E + H} \cos \varepsilon_0$

$\sin \alpha_0 = \frac{6371}{7371} \cos 30 = .7485$

$\Rightarrow \alpha_0 = 48.45$

Step 2: Calc central angle @ el 30°

From ch 5.2 eq 5.2: $\varepsilon_0 + \alpha_0 + \beta_0 = 90$

$\beta_0 = 90 - \varepsilon_0 - \alpha_0$ where $\varepsilon_0 = 30$, $\alpha_0 = 48.45$

$\beta_0 = 90 - 30 - 48.45 = 11.55 = \beta_0$

Step 3: Calc designed visibility arc γ_0

$\gamma_D = 2\beta_0$ (derived in question 3)

$\gamma_D = 2(11.55) = 23.1°$

Step 4: Calc locked on duration

- Lock on Zone is when satellite is above 30° in this zone satellite travels the visibility arc $\gamma_0 = 23.1^\circ$

we need time based on visible pass duration

$$\frac{\gamma}{\gamma_0} = \frac{t_{vis}}{t_{locked}} \Rightarrow t_{locked} = \frac{\gamma_0}{\gamma} t_{vis}$$

$$t_{locked} = \frac{\text{designed vis arc}(\gamma_0)}{\text{total vis arc }(\gamma)}\left(\text{visible duration}(t_{vis})\right)$$

$$\Rightarrow \frac{23.1^\circ}{60.5^\circ}(1058\text{ sec}) = 404\text{ sec} = t_{locked}$$

Step 5: Calc the $\% = \frac{t_{locked}}{t_{vis}}(100)$

$$\Rightarrow \frac{404}{1058}(100) = \boxed{38.2\% = \text{Percent loked}}$$

5.) Power received vs elevation

Step1: we need the distance from ground Station to satellite for signal loss

From Ch 4.3 eq 1.56:

$$d(\varepsilon_0) = R_E\left[\sqrt{\left(\frac{H}{R_E}+1\right)^2 \cos^2\varepsilon_0} - \sin\varepsilon_0\right]$$

$$\Rightarrow 6371\left[\sqrt{\frac{1000}{6371}+1 - \cos^2 30} - \sin 30\right]$$

$d(30) = 1702$ km @ $30°$ elevation, the sat is 1702 km away from ground station

Step2: Free Space loss (L_s)

- radio waves spread as the travel

$$L_s = 20\log_{10}\left(\frac{4\pi d f}{c}\right) dB \Rightarrow$$

$d = 1702$ km $= 1.702 \times 10^6$ m

$f = 2$ GHz $= 2\times10^9$ Hz

$c = 3\times10^8$ m/s

$$L_s = 20\log_{10}\left(\frac{4\pi(1.702\times10^6)(2\times10^9)}{3\times10^8}\right)$$

$\Rightarrow \underline{L_s = 163.1\ dB}$ → the signal looses 163.1 dB of power traveling 1702 km through free space.

Step 3: Calc received power @ 30° el.

$P_r = EIRP - L_s$

$EIRP = P_{Tx} + G_{Tx}$

- P_{Tx} = Transmitter power = 10W = 10dBW
- G_{Tx} = Antenna gain = 3dBi

$EIRP = 10 + 3 = 13\,dBW$

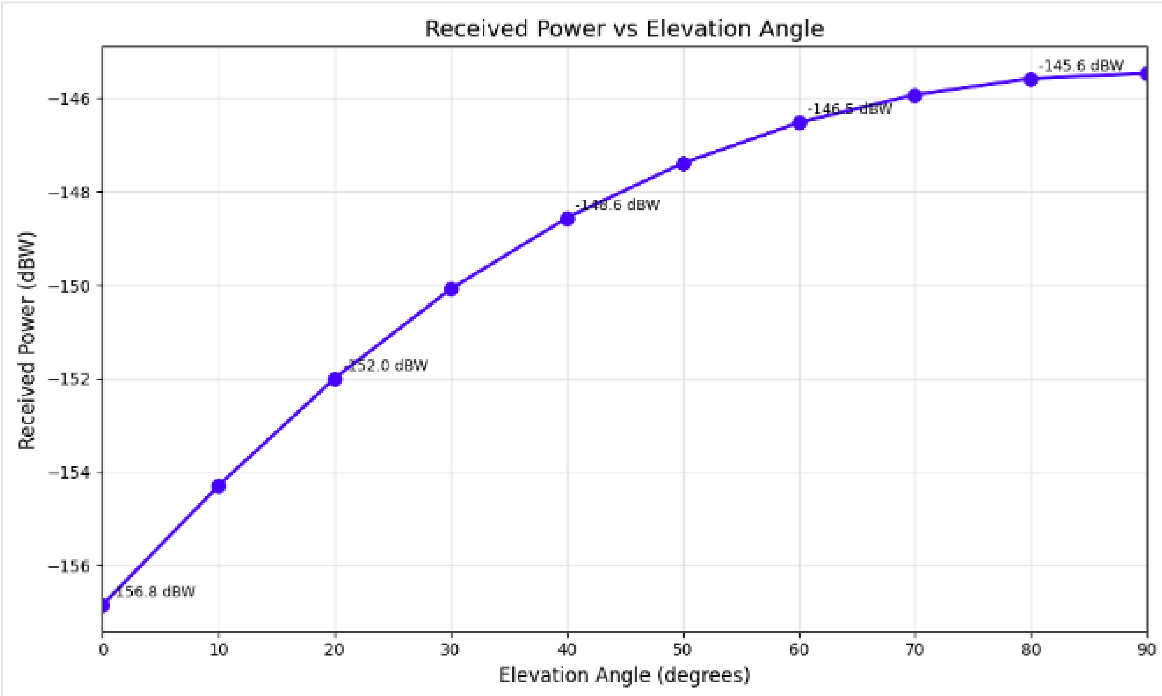
$P_r = 13 - 163.1 = \underline{-150.1\,dBW} = P_r$

- @ 30° el, the station receives −150.1dBW

Summary table for elevations 0-90 in increments of 10:

Elevation (deg)	Slant Range (km)	Free Space Loss (dB)	Received Power (dBW)
0	3707.0	169.8	−156.8
10	2762.3	167.3	−154.3
20	2121.0	165.0	−152.0
30	1702.2	163.1	−150.1
40	1428.6	161.6	−148.6
50	1248.2	160.4	−147.4
60	1129.7	159.5	−146.5
70	1054.8	158.9	−145.9
80	1013.3	158.6	−145.6
90	1000.0	158.5	−145.5

Plot of summary table:



Question 6: Doppler Shift Analysis

The doppler shift occurs because the satellite is moving relative to the ground station. Need to find out where this effect is maximum and minimum, then calculate the actual frequency shift. Overall strategy is to understand the geometry of satellite motion during pass, identify where radial velocity is max and min, then calculate the doppler sift at these extremes.

Step 1 - Geometry: Maximum doppler occurs at 0 degrees elevation (horizon) and minimum doppler occurs at 90 degrees elevation (zenith)

Step 2 - Calculate maximum doppler shift at horizon:

$$\Delta F = \frac{V_r}{c} f = \frac{V \cos \varepsilon_0}{c} f$$

at horizon $\Delta f_{max} = \frac{V \cos 0}{c} f = \frac{V}{c} f$

- $V = 7.354$ km/s (from Q1)
- $f = 2 \times 10^9$ Hz
- $c = 3 \times 10^5$ km/s

$$\Delta f_{max} = \frac{7.354 \text{ km/s}}{3 \times 10^5 \text{ km/s}} (2)(10^9)$$

$$\Delta f_{max} = 49{,}020 \text{ Hz} = 49 \text{ kHz}$$

Step 3: Calc min Doppler shift at zenith ($\varepsilon_0 = 0$)

$$\Delta f_{min} = \frac{V \cos 90}{c} f = \frac{V(0)}{c} f = 0 \text{ Hz}$$

This means that the satellite is directly overhead, there is no frequency shift. The received frequency equals the transmitted frequency.

Summary:

1. Maximum Doppler occurs at 0 degrees elevation (horizon, both AOS and LOS)
2. Minimum Doppler occurs at 90 degrees elevation (zenith)
3. Maximum Doppler value is 49kHz