A study on the timeseries forecasting method: ARIMA

In [ ]:
```python
# initial imports
import pandas as pd

from TSErrors import FindErrors

import plotly.express as px
import plotly.graph_objects as go

import plotly.io as pio
pio.renderers.default='notebook'

import holidays

import warnings
warnings.filterwarnings("ignore")

from statsmodels.tsa.stattools import adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf

import pmdarima as pm
from statsmodels.tsa.statespace.sarimax import SARIMAX
```

Data Preparation

The daily data (for confirmed cases in each) has already been preprocessed in update_data.py and is stored the data directory

Now we write a function that takes this cumulative time series and returns a new time series with number of new daily cases for a singule country, for:

1. Easier trend, seasonality identification, and to
2. Reduce chances for the cumulative number to go down

In [ ]:
```python
confirmed_global = pd.read_csv(r"./data/country_confirmed.csv")
```

In [ ]:
```python
def get_data(country, confirmed=confirmed_global):
    confirmed = confirmed.groupby("country").sum().T
    confirmed.index = pd.to_datetime(confirmed.index, infer_datetime_format=True)
    data = pd.DataFrame(
            index=confirmed.index, data=confirmed[country].values, columns=["Total"]
        )
    # we remove the beginninhg 0s as they do not contribute
    data = data[(data != 0).all(1)]

    data_diff = data.diff()

    # removing the first value from data_diff
    # it had no previous value and is a NaN after taking the difference
    data_diff = data_diff[1:]

    return data, data_diff
```

For the sake of this study we will be using the time series for India to build our model, but in the process we will also write generic functions and a workflow that can be used for any of the countries in the main dataset, with near optimal parameters.

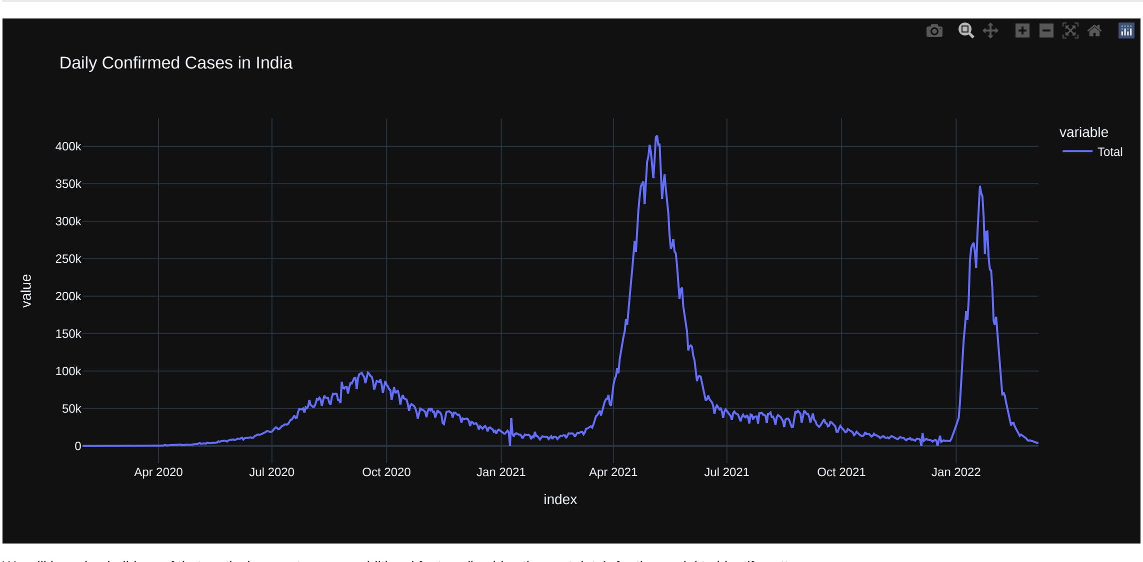
In [ ]:
```python
country = "India"
confirmed_dfs = get_data(country)
confirmed_daily = confirmed_dfs[1] # taking the daily data, not the cumulative
```

In [ ]:
```python
confirmed_daily.tail(10)
```

Out[ ]:

	Total
2022-02-27	8013.0
2022-02-28	6915.0
2022-03-01	7554.0
2022-03-02	6561.0
2022-03-03	6396.0
2022-03-04	5921.0
2022-03-05	5476.0
2022-03-06	4362.0
2022-03-07	3993.0
2022-03-08	4575.0

In [ ]:
```python
# plotting our daily cases
px.line(confirmed_daily, title=f"Daily Confirmed Cases in {country}", template="plotly_dark")
```



We will be using holidays of that particular country as an additional feature (besides the past data), for the model to identify patterns.

In [ ]:
```python
holiday = [1 if i==True else 0 for i in [day in holidays.India() for day in confirmed_daily.index]]
confirmed_daily["holiday"] = holiday
```

In [ ]:
```python
df = confirmed_daily.copy()
df = df[["holiday", "Total"]]
df[df["holiday"] == 1].tail() # making sure there are holidays in the data that we are using
```

Out[ ]:

	holiday	Total
2021-10-02	1	22842.0
2021-11-04	1	12729.0
2021-12-25	1	6987.0
2022-01-14	1	268833.0
2022-01-26	1	286384.0

Model Identification

Depending on the type of our time series, there are 2 to 7 parameters that we will need to identify:

1. **Stationary time series:**

A staionary time series is a time series whose value(and covariance) does not depend on time at which the series is observed rather just the lag ***'k'*** wrt to some other point in the series, i.e one that has a constant mean and variance (and thus by default cannot have a non zero trend). An example of a perfectly stationary time series is a sine wave. But as time series are rarely perfectly stationary, we will set a threshold(p-value of 0.05 in adfuller) and use a test (Augmented Dicky-Fuller or adfuller) to identify if the time series is stationary or not.

Forecast of a stationary time series can be found using an ARMA model. It consists of two parts Autoreggressive(AR) and Moving Average(MA). Moving average is a technique that forecasts the future value of a time series data using the average (or of needed weighted average) of the past *n* values. The AR part is a regression on the time series itself measures/observed at different points with respect to a specified lag *k*. For example if we were use a AR model with lag 1 or AR(1), the model's equation would be given as follows:

Y(t+1) = μ + βY(t) + ε(t+1)

where μ is the mean of the time series, β is the coefficient of the previous value of the time series, and ε(t+1) is the extra residual term

In this model, we need to identify only 2 parameters/orders: **p** and **q**, the lags for the AR and MA processes respectively. The final model will be a ARMA(p,q) or ARIMA(p,0,q)

1. **Non stationary and non seasonal:**

If our time series does not satisfy the conditions for the stationary time series, we will use the method of differencing to convert our time series to a stationary time series. This is what the extra ***'I'*** in ARIMA stands for: Integration, i.e, order of differencing.

In this model, the 3 parameters/orders we will need to identify are: **p**, **q** (same as that from ARMA) and **d**, the order of differencing. The final model will be ARIMA(p,d,q).

1. **Seasonal time series:**

If our time series has an additional seasonality component, we will need to use SARIMA/SARIMAX model to forecast (Seasonal ARIMA) for the forecast, as seasonality doesnt work well with the standard ARIMA model. For this, in addition to the 3 parameters we found in ARIMA, we will need to identify:

1. ***P*** and ***Q***: the lags of the **seasonal component** of the time series
2. ***D***: the order of differencing for the seasonal component
3. ***s or m***: the number of seasonal periods in the time series. (i.e, if m is 4, each period will be 1/4 of a year, so quarterly. If 12, then monthly etc.)

Step 1

Checking if our time series is stationary or not, and if not, identify the order of differencing.

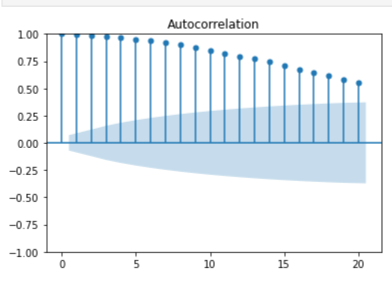
In [ ]:
```python
result=adfuller(df['Total'].dropna())
print(f'p-value: {result[1]}')
```

```
p-value: 0.08124784990657441
```

As our p-value is > 0.05, the time series is not stationary.

Another show of this, is that the decrease/decline in the auto correlations is very slow (shown below)

In [ ]:
```python
acf = plot_acf(df['Total'].dropna(), lags=20)
```

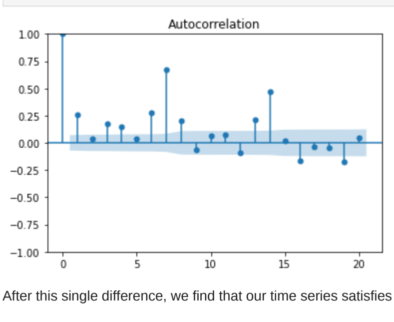


We will now try the same with the same series after a single difference (d = 1)

In [ ]:
```python
result=adfuller(df['Total'].diff().dropna())
print(f'p-value: {result[1]}')
```

```
p-value: 6.778172436582615e-09
```

In [ ]:
```python
acf = plot_acf(df['Total'].diff().dropna(), lags=20)
```



After this single difference, we find that our time series satisfies both conditions for being stationary.

We have successfully identified the order of differencing to be 1. (i.e, **d = 1**)

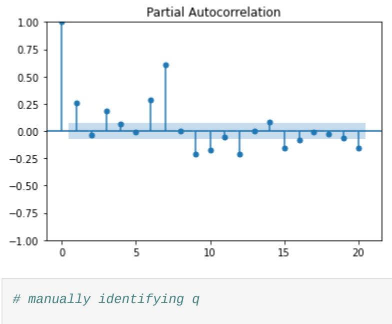
Step 2:

To identify the lags **p** and **q** manually, we can follow the steps:

1. The partial autocorrelation is significant only for the first p-values/lags and cuts off to zero.
2. The autocorrelation values is significant only for the first q-values/lags and cuts off to zero.

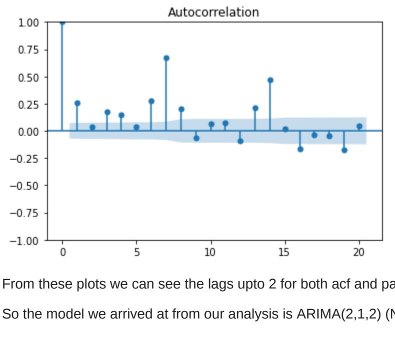
In [ ]:
```python
# manually identifying p

pacf = plot_pacf(df['Total'].diff().dropna(), lags=20)
```



In [ ]:
```python
# manually identifying q

acf = plot_acf(df['Total'].diff().dropna(), lags=20)
```



From these plots we can see the lags upto 2 for both acf and pacf are significant, therfore our manual arrived values of p and q are 2 and 2 respectively.

So the model we arrived at from our analysis is ARIMA(2,1,2) (NOTE: we do not need to and so haven't yet checked or accounted for the seasonal component)

Model Evaluation

In [ ]:
```python
df.index.freq = "D"
arima = SARIMAX(df['Total'], order=(2,1,2), exog=df['holiday'])
model = arima.fit(method="powell")
```

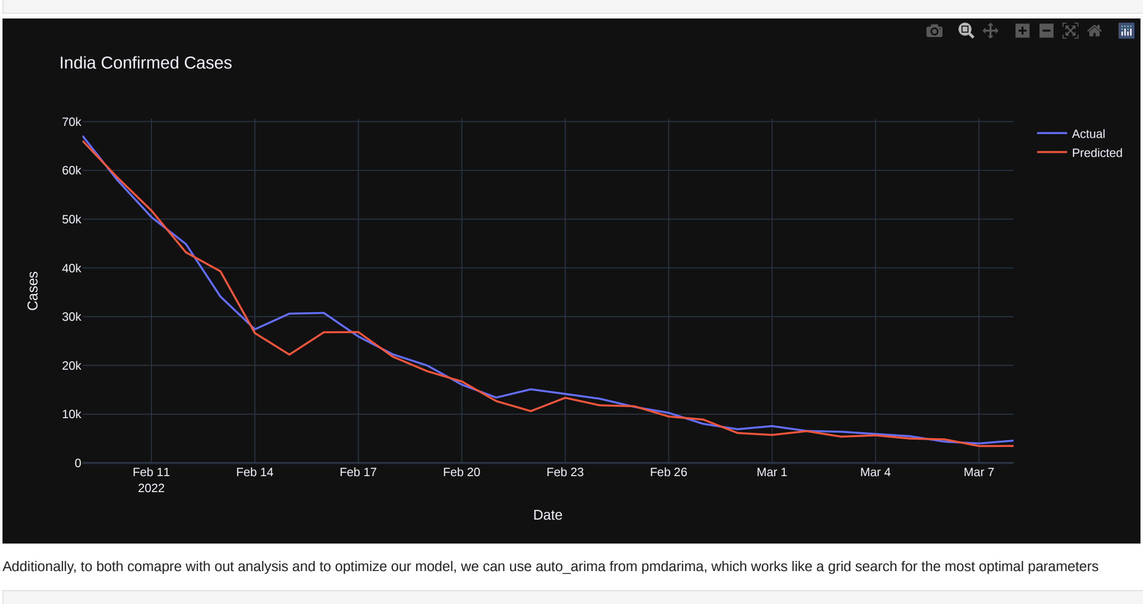
```
Optimization terminated successfully.
         Current function value: 10.392263
         Iterations: 2
         Function evaluations: 145
```

In [ ]:
```python
# we now evaluate our model
pred = model.predict(start=len(df['Total'])-28, end=len(df['Total'])-1, exog=df["holiday"][-28:],dynamic=False)
```

In [ ]:
```python
errors = FindErrors(df[-28:]["Total"], pred)
print(f"MAPE: {errors.mape()}") # mean absolute pecentage error
print(f"RMSE: {errors.rmse()}") # root mean square error
```

```
MAPE: 9.505075716252787
RMSE: 2344.112358582104
```

In [ ]:
```python
fig = go.Figure()
fig.add_trace(go.Scatter(x=df[-28:].index, y=df[-28:]['Total'], name="Actual"))
fig.add_trace(go.Scatter(x=df[-28:].index, y=pred, name="Predicted"))
fig.update_layout(title_text=f"{country} Confirmed Cases", xaxis_title="Date", yaxis_title="Cases", template="plotly_dark", hovermode="x")
```



Additionally, to both compare with our analysis and to optimize our model, we can use auto_arima from pmdarima, which works like a grid search for the most optimal parameters

In [ ]:
```python
results=pm.auto_arima(df["Total"], start_p=1, d=1, start_q=1, max_p=3, max_q=3,
                      information_criterion='bic', trace=True, error_action='ignore',
                      exogenous=df[["holiday"]], stepwise=True, freq="D")
```

```
Performing stepwise search to minimize bic
[illegible]

Best model:  ARIMA(2,1,2)(0,0,0)[0]
Total fit time: [illegible] seconds
```

This agrees with our analysis!

NOTES:

1. Setting a higher max_p and max_q might result in auto_arima resulting in a different model, but by experimenting with those values, I observed that the model was getting overfitted, and hence stuck to these max values being 2 and 3 respectively.
2. We do not take into account seasonality here as ARIMA/SARIMAX is known to face many problems in high frequency (even weekly data (m=52) causes problems, and ours is daily data (m=365)) and in data where the seasonal cycles are too long, as mentioned in these links:

a. Poor performance on long cycles

b. SARIMAX too blunt for high frequency data(Answer written by one of the authors of statsmodels himself)

Forecasting

In [ ]:
```python
dateList = pd.date_range(df.index[-1], periods=8, freq="D")[1:]
```

In [ ]:
```python
is_holiday = [1 if i==True else 0 for i in [day in holidays.India() for day in dateList]]
```

In [ ]:
```python
forecast = model.get_forecast(steps=7, exog = is_holiday)
mean_forecast=forecast.predicted_mean
```

In [ ]:
```python
print(mean_forecast)
```

```
2022-03-09    [illegible]
2022-03-10    [illegible]
2022-03-11    [illegible]
2022-03-12    [illegible]
2022-03-13    [illegible]
2022-03-14    [illegible]
2022-03-15    [illegible]
Freq: D, Name: predicted_mean, dtype: float64
```

In [ ]:
```python
# converting to cumulative form for final plot and prediction formation
start = confirmed_dfs[0]["Total"][-1]
predictions_cumulative = []
for i in mean_forecast:
    start = start + i
    predictions_cumulative.append(start)
```

In [ ]:
```python
fig = go.Figure()
fig.add_trace(go.Scatter(x=confirmed_dfs[0][-30:].index, y=confirmed_dfs[0][-30:]["Total"],
                         mode="lines",
                         name="Up till now"))
fig.add_trace(go.Scatter(x=dateList, y=predictions_cumulative,
                         mode="lines",
                         name="Predictions"))
fig.update_layout(title_text=f"{country} Confirmed Cases", xaxis_title="Date", yaxis_title="Cases", template="plotly_dark", hovermode="x")
```

In [ ]:
```python
# function to format the number for easy readability
def format_number(number):
    s, *d = str(number).partition(".")
    r = ",".join([s[x-2:x] for x in range(-3, -len(s), -2)][::-1] + [s[-3:]])
    return "".join([r] + d)
```

In [ ]:
```python
predictions = pd.DataFrame()
predictions.index = dateList
predictions["Total"] = [format_number(int(i)) for i in predictions_cumulative]
```

In [ ]:
```python
predictions
```

Out[ ]:

	Total
2022-03-09	[illegible]
2022-03-10	[illegible]
2022-03-11	[illegible]
2022-03-12	[illegible]
2022-03-13	[illegible]
2022-03-14	[illegible]
2022-03-15	[illegible]