APPENDIX
I. DERIVATION OF PSEUDO-BACKGROUND NOISE DIFFUSION

Given a data point x_0 sampled from the real data distribution $q(x)$, $(x_0 \sim q(x))$, the forward diffusion process can be defined as adding noise $\epsilon_0, \dots, \epsilon_{t-2}, \epsilon_{t-1} \sim \mathcal{N}(\mu, \sigma^2)$ stepwise to x_0. This diffusion process can be formulated as follows:

$$\begin{gathered} q(x_t \mid x_{t-1}) = \mathcal{N}(x_t; \sqrt{1-\beta_t} x_{t-1} + \mu\sqrt{\beta_t}, \sigma^2 \beta_t \mathbf{I}), \\ \text{s.t., } \beta_t = \frac{\lambda t}{T}, \end{gathered} \tag{1}$$

where x_t denotes the data after t-step diffusion, β_t denotes the noise intensity, λ is a hyperparameter to limit the magnitude of β_t, and $\mathbf{I}$ is the identity matrix. Note that $q(x_t \mid x_{t-1})$ still obeys the Gaussian distribution. Thus, the diffusion process from step 1 to T can be mathematically defined as:

$$q(x_{1:T} \mid x_0) = \prod_{t=1}^{T} q(x_t \mid x_{t-1}), \tag{2}$$

this is also the posterior probability. Similar to DDPMs, in order to make the above formulas computable, we apply the reparameterization trick to design a treatable closed-form sampling at any timestep. First, we define $\alpha_t = 1 - \beta_t$, $\overline{\alpha}_t = \prod_{i=1}^{t} \alpha_i$. In this way, x_t can be calculated as:

$$x_t = \sqrt{\alpha_t} x_{t-1} + \sqrt{\beta_t} \epsilon_{t-1}. \tag{3}$$

By substituting x_{t-1} into Equation 3, we have:

$$x_t = \sqrt{\alpha_t \alpha_{t-1}} x_{t-2} + \sqrt{\alpha_{t-1} \beta_t} \epsilon_{t-2} + \sqrt{\beta_t} \epsilon_{t-1}. \tag{4}$$

Since all time steps have the same Gaussian noise, we only use ϵ to denote the noise in the subsequent derivations:

$$x_t = \sqrt{\alpha_t \alpha_{t-1}} x_{t-2} + (\sqrt{\alpha_{t-1} \beta_t} + \sqrt{\beta_t}) \epsilon. \tag{5}$$

By deriving Equation 5 to $t = 1$, we have:

$$\begin{gathered} x_t = \sqrt{\overline{\alpha}_t} x_0 + \gamma_t \epsilon, \\ \text{s.t., } \gamma_t \epsilon \sim \mathcal{N}(\mu \gamma_t, (1 - \overline{\alpha}_t) \sigma^2), \end{gathered} \tag{6}$$

where γ_t is defined as:

$$\gamma_t = \sum_{k=0}^{t-1} \sqrt{\frac{\overline{\alpha}_t \beta_{t-k}}{\overline{\alpha}_{t-k}}}. \tag{7}$$

Thus, Equation 2 can be simplified to the distribution of x_t:

$$x_t \sim q(x_t \mid x_0) = \mathcal{N}(x_t; \sqrt{\overline{\alpha}_t} x_0 + \mu \gamma_t, (1 - \sqrt{\overline{\alpha}_t}) \sigma^2 \mathbf{I}). \tag{8}$$

Finally, the symbol $\oplus_t$ in Equation **??** can be denoted mathematically as:

$$\boldsymbol{H}^t = \boldsymbol{H} \oplus_t \boldsymbol{N} = \sqrt{\overline{\alpha}_t} \boldsymbol{H} + \gamma_t \boldsymbol{N}. \tag{9}$$

APPENDIX
II. DETAILED RESULTS OF THE GENERALIZABILITY STUDY

TABLE I
$\mathrm{AUC}_{(\mathrm{D,F})}$, $\mathrm{AUC}_{(\mathrm{F},\tau)}$ AND $\mathrm{AUC}_{\mathrm{BS}}$ FOR DIFFERENT COMPARISON METHODS ON THE HAD-100 DATASET.

	$\mathrm{AUC}_{(\mathrm{D,F})}$			$\mathrm{AUC}_{(\mathrm{F},\tau)}$			$\mathrm{AUC}_{\mathrm{BS}}$		
Index	**DFAE**	**AETNet**	**BSDM**	**DFAE**	**AETNet**	**BSDM**	**DFAE**	**AETNet**	**BSDM**
1	0.9911	0.9892	0.9938	0.0000	0.0107	0.0078	0.9911	0.9785	0.9819
2	0.9983	0.9962	0.9996	0.0003	0.0296	0.0177	0.9980	0.9667	0.9844
3	0.9341	0.9800	0.9874	0.0000	0.0163	0.0030	0.9341	0.9637	0.9920
4	0.9998	0.9981	0.9998	0.0000	0.0142	0.0077	0.9998	0.9839	0.9541
5	0.9986	0.9920	0.9706	0.0002	0.0653	0.0165	0.9984	0.9268	0.9860
6	0.9973	0.9950	0.9955	0.0000	0.0199	0.0095	0.9973	0.9751	0.9835
7	0.9993	0.9988	0.9943	0.0001	0.0234	0.0108	0.9992	0.9754	0.9907
8	0.9998	0.9997	1.0000	0.0001	0.0359	0.0093	0.9997	0.9638	0.9888
9	0.5485	0.9984	0.9998	0.0000	0.0430	0.0110	0.5485	0.9554	0.9841
10	0.9988	0.9992	0.9952	0.0003	0.0464	0.0111	0.9985	0.9527	0.9904
11	0.9990	0.9988	0.9982	0.0000	0.0141	0.0077	0.9990	0.9847	0.9901
12	0.9977	0.9978	0.9988	0.0000	0.0126	0.0087	0.9977	0.9852	0.9737
13	0.9281	0.9738	0.9811	0.0000	0.0177	0.0075	0.9281	0.9560	0.9910
14	1.0000	0.9990	1.0000	0.0000	0.0177	0.0090	1.0000	0.9814	0.9813
15	1.0000	0.9998	0.9988	0.0000	0.0272	0.0175	1.0000	0.9726	0.9933
16	0.9989	0.9974	0.9997	0.0000	0.0125	0.0063	0.9989	0.9849	0.9850
17	0.9910	0.9993	0.9997	0.0005	0.0672	0.0147	0.9905	0.9322	0.9670
18	0.9833	0.9979	0.9944	0.0002	0.0934	0.0273	0.9831	0.9044	0.9908
19	0.9553	0.9956	0.9995	0.0000	0.0110	0.0087	0.9553	0.9846	0.9796
20	0.9977	0.9952	0.9890	0.0000	0.0171	0.0094	0.9977	0.9781	0.9899
21	1.0000	0.9982	0.9998	0.0000	0.0175	0.0099	1.0000	0.9807	0.9887
22	0.9985	0.9986	0.9991	0.0001	0.0237	0.0104	0.9984	0.9749	0.9847
23	0.9973	0.9944	0.9980	0.0000	0.0206	0.0133	0.9973	0.9737	0.9930
24	0.9988	0.9984	0.9996	0.0000	0.0185	0.0066	0.9988	0.9799	0.9956
25	0.9969	0.9948	0.9996	0.0000	0.0124	0.0041	0.9969	0.9825	0.9838
26	0.9992	0.9995	0.9996	0.0000	0.0226	0.0158	0.9992	0.9768	0.9911
27	0.9992	0.9976	0.9997	0.0000	0.0091	0.0086	0.9992	0.9885	0.9746
28	0.9978	0.9990	0.9973	0.0000	0.0428	0.0227	0.9978	0.9562	0.9788
29	0.9924	0.9972	0.9914	0.0001	0.0286	0.0126	0.9923	0.9687	0.9889
30	0.8024	0.9959	0.9998	0.0000	0.0199	0.0109	0.8024	0.9760	0.9840
31	0.9973	0.9930	0.9903	0.0000	0.0165	0.0063	0.9973	0.9765	0.9906
32	0.9585	0.9943	0.9992	0.0000	0.0127	0.0085	0.9585	0.9817	0.9713
33	0.8143	0.9852	0.9800	0.0000	0.0139	0.0087	0.8143	0.9713	0.9746
34	0.7394	0.9920	0.9979	0.0000	0.0418	0.0233	0.7394	0.9502	0.9492
35	0.9789	0.9764	0.9602	0.0000	0.0139	0.0110	0.9789	0.9625	0.9841
36	0.9999	0.9989	0.9992	0.0000	0.0160	0.0152	0.9999	0.9829	0.9906
37	0.9977	0.9998	0.9999	0.0003	0.0173	0.0093	0.9974	0.9825	0.9922
38	0.9840	0.9936	0.9995	0.0000	0.0126	0.0073	0.9840	0.9810	0.9817
39	0.9440	0.9943	0.9868	0.0000	0.0286	0.0051	0.9440	0.9657	0.9842
40	0.7446	0.9910	0.9972	0.0000	0.0289	0.0130	0.7446	0.9621	0.9746
41	0.9961	0.9977	0.9948	0.0001	0.0427	0.0202	0.9960	0.9550	0.9370
42	0.9972	0.9811	0.9584	0.0001	0.0715	0.0214	0.9971	0.9096	0.9900
43	0.9998	0.9997	0.9998	0.0000	0.0279	0.0098	0.9998	0.9718	0.9299
44	0.9889	0.9811	0.9432	0.0000	0.0352	0.0132	0.9889	0.9459	0.9802
45	0.9970	0.9934	0.9940	0.0000	0.0220	0.0139	0.9970	0.9714	0.9912
46	0.9630	0.9976	0.9999	0.0000	0.0176	0.0087	0.9630	0.9800	0.8543
47	0.9781	0.9099	0.8902	0.0000	0.1516	0.0360	0.9781	0.7584	0.9809
48	0.9908	0.9878	0.9954	0.0006	0.0454	0.0146	0.9902	0.9424	0.9787
49	0.9962	0.9962	0.9997	0.0001	0.0350	0.0210	0.9961	0.9612	0.9671
50	0.8755	0.9724	0.9896	0.0009	0.1143	0.0225	0.8746	0.8581	0.9901
51	0.9957	0.9934	0.9973	0.0000	0.0197	0.0072	0.9957	0.9738	0.9866
52	0.9998	0.9999	1.0000	0.0000	0.0299	0.0134	0.9998	0.9700	0.9899
53	0.9954	0.9861	0.9993	0.0011	0.0536	0.0094	0.9943	0.9326	0.9832
54	0.6569	0.9925	0.9985	0.0000	0.0372	0.0153	0.6569	0.9553	0.9478
55	0.9341	0.9748	0.9932	0.0000	0.2041	0.0455	0.9341	0.7706	0.9542
56	0.7490	0.9728	0.9899	0.0000	0.0919	0.0357	0.7490	0.8809	0.9899
57	0.8921	0.9826	0.9963	0.0000	0.0267	0.0064	0.8921	0.9559	0.9894
58	0.8545	0.9859	0.9955	0.0000	0.0203	0.0061	0.8545	0.9657	0.9800
59	0.7706	0.9997	1.0000	0.0000	0.0265	0.0200	0.7706	0.9732	0.9718
60	0.9368	0.9986	0.9859	0.0000	0.0285	0.0142	0.9368	0.9701	0.9767
61	0.7928	0.9956	0.9967	0.0000	0.0637	0.0200	0.7928	0.9320	0.9612
62	0.8491	0.9593	0.9668	0.0000	0.0108	0.0056	0.8491	0.9485	0.9752
63	0.9860	0.9998	0.9888	0.0002	0.0183	0.0137	0.9858	0.9815	0.9865
64	0.9963	0.9977	0.9996	0.0001	0.0229	0.0131	0.9962	0.9748	0.9887
65	0.9965	0.9981	0.9998	0.0001	0.0147	0.0110	0.9964	0.9834	0.9906
66	0.9938	0.9969	0.9986	0.0001	0.0188	0.0079	0.9937	0.9781	0.9691
67	0.9646	0.9960	0.9767	0.0004	0.0259	0.0077	0.9642	0.9701	0.9564
68	0.9932	0.9922	0.9689	0.0000	0.0156	0.0124	0.9932	0.9766	0.9813
69	0.9671	0.9922	0.9902	0.0015	0.0150	0.0090	0.9656	0.9772	0.9618
70	0.9555	0.9952	0.9949	0.0001	0.0688	0.0331	0.9554	0.9264	0.9873
71	0.9863	0.9936	0.9962	0.0000	0.0143	0.0088	0.9863	0.9793	0.9506
72	0.9913	0.9899	0.9632	0.0000	0.0186	0.0126	0.9913	0.9713	0.9830
73	0.7123	0.9394	0.9986	0.0002	0.0292	0.0156	0.7121	0.9102	0.9947
74	0.6347	0.9987	0.9999	0.0001	0.0455	0.0052	0.6346	0.9532	0.9553
75	0.9889	0.9991	0.9664	0.0002	0.0258	0.0112	0.9887	0.9733	0.9882
76	0.9947	1.0000	0.9939	0.0003	0.0263	0.0056	0.9944	0.9737	0.9456
77	0.9874	0.9970	0.9617	0.0004	0.0373	0.0161	0.9870	0.9597	0.9241
78	0.9155	0.9052	0.9561	0.0004	0.0735	0.0320	0.9151	0.8317	0.9728
79	0.9250	0.9954	0.9930	0.0004	0.0635	0.0203	0.9246	0.9319	0.9809
80	0.9994	0.9981	0.9986	0.0001	0.0377	0.0177	0.9993	0.9604	0.9791
81	0.9968	0.9965	0.9918	0.0001	0.0223	0.0127	0.9967	0.9742	0.8174
82	0.7250	0.8736	0.8617	0.0000	0.0375	0.0443	0.7250	0.8362	0.9905
83	0.8992	0.9803	0.9955	0.0000	0.0125	0.0049	0.8992	0.9677	0.9881
84	0.9966	0.9969	0.9994	0.0003	0.0297	0.0113	0.9963	0.9672	0.8688
85	0.9871	0.9604	0.9149	0.0002	0.0852	0.0461	0.9869	0.8752	0.9840
86	0.4797	0.9955	0.9974	0.0000	0.0340	0.0134	0.4797	0.9615	0.9932
87	0.9988	0.9958	0.9999	0.0000	0.0094	0.0067	0.9988	0.9864	0.9814
88	0.9966	0.9892	0.9879	0.0001	0.0200	0.0066	0.9965	0.9692	0.9930
89	0.9999	0.9975	1.0000	0.0000	0.0145	0.0070	0.9999	0.9830	0.9910
90	0.9627	0.9961	1.0000	0.0003	0.0219	0.0090	0.9624	0.9742	0.9806
91	0.9993	0.9837	0.9931	0.0000	0.0277	0.0124	0.9993	0.9559	0.9848
92	0.9637	0.9920	0.9951	0.0002	0.0217	0.0103	0.9635	0.9703	0.9896
93	0.9993	0.9995	0.9967	0.0000	0.0344	0.0071	0.9993	0.9651	0.9808
94	0.9814	0.8543	0.9907	0.0001	0.0566	0.0099	0.9813	0.7977	0.9808