$$\widetilde{\mathscr{H}}_{\mathbf{S}}^{(k)^{-1}}$$