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Reliability and Risk Engineering

Introduction to Modeling and Optimization of Sustainable Energy Systems:

Key Performance Indicators (KPIs) for security and multi-objective optimization

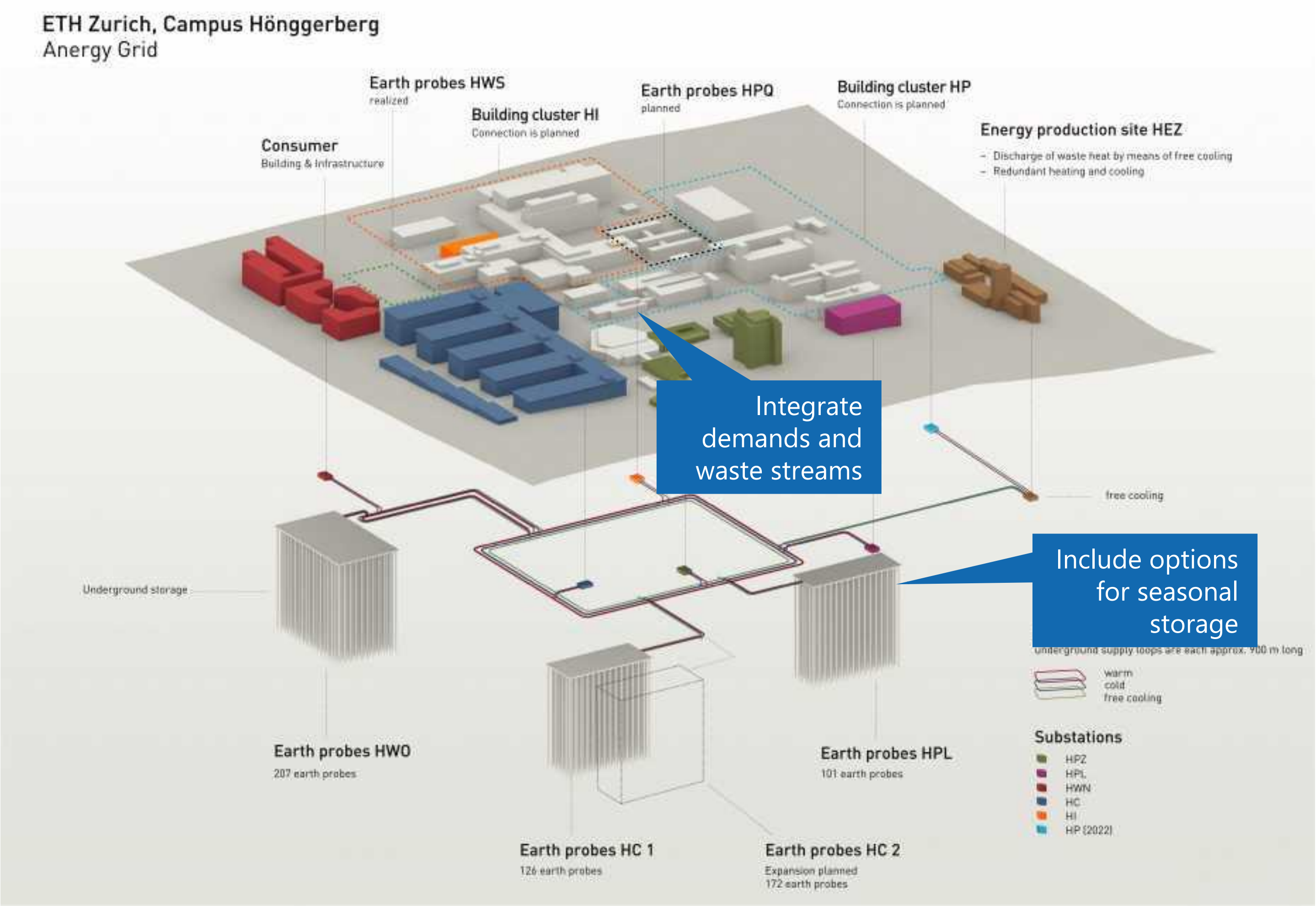
Prof. Dr. Giovanni Sansavini
Reliability and Risk Engineering

Field Trip to the Anergy Grid

- Dates:
  - Wednesday, 01.12.
  - Monday, 06.12.
  - Wednesday, 08.12.

- Meeting Point:
  - Campus Info Hönggerberg at 12:50 pm

- The tour will take approx. 2 hours
- Participants will be asked to provide a COVID certificate
- Sign up here until Nov. 28th



Since the last lecture, you are able to …

- ✓ Define and critically evaluate:
  - ✓ Environmental KPIs deriving from life cycles assessment (LCA)
  - ✓ Thermodynamic Key Performance Indicators (KPIs)
  - ✓ Equity KPIs such as net present value (NPV), total annual cost (TAC), …

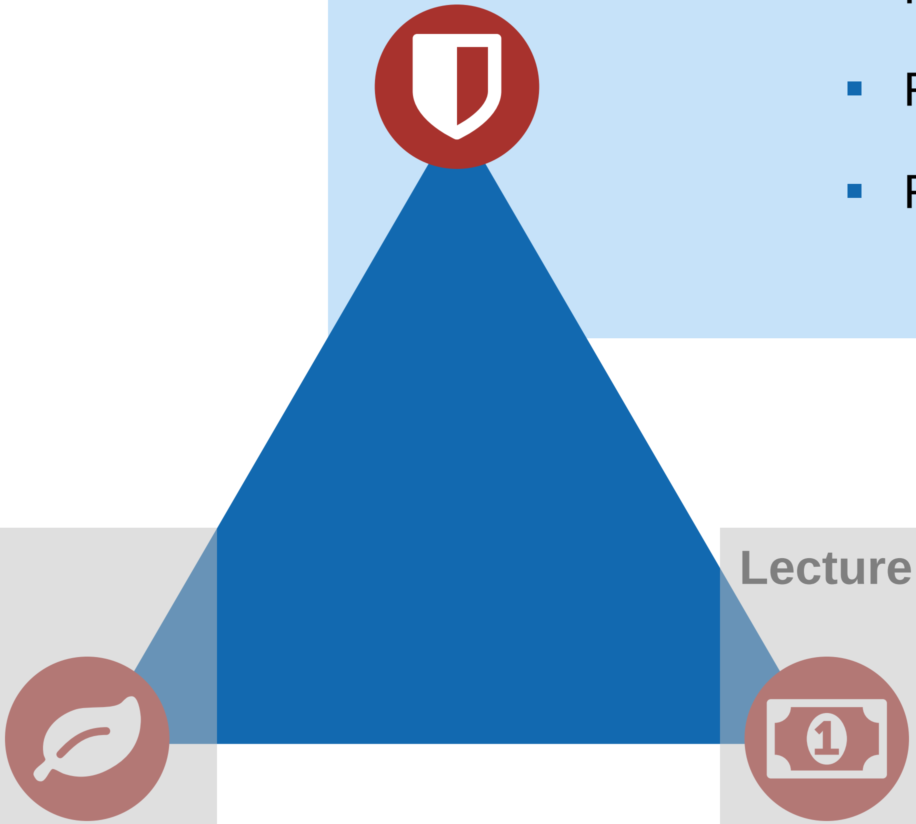
After this lecture, you are able to …

- Define and critically evaluate security KPIs for energy systems:
  - Reliability indicators
  - Risk indicators
  - Resilience indicators
- Use different KPIs for the formulation of multi-objective optimization problems

Energy trilemma: Different metrics for different requirements

availability of sources
ensuring supply of energy
Security

- Reliability
- Risk
- Resilience



Thermodynamic **Lecture 7**

- Cumulative energy demand (CED)
- Conversion and storage efficiency

Lecture 6

Sustainability
climate protection
life cycle

Lecture 7

Equity
price development
competition

- Net Present Value (NPV)
- Internal Rate of Return (IRR)
- Levelized Cost of Energy (LCOE)
- Total Annual Cost (TAC)

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Security KPIs:
Reliability, risk and resilience metrics

Why risk analysis is important?

- It allows to quantify the probability and the consequences of extreme events (e.g. blackouts)
- It allows to study complex dependencies and interdependencies (e.g. between different energy networks)
- It allows to limit the expected loss of performance of the energy system due to a credible set of disruptive events
- It allows to price the desired level of security of supply that we want our energy system to achieve

Reliability: Definition

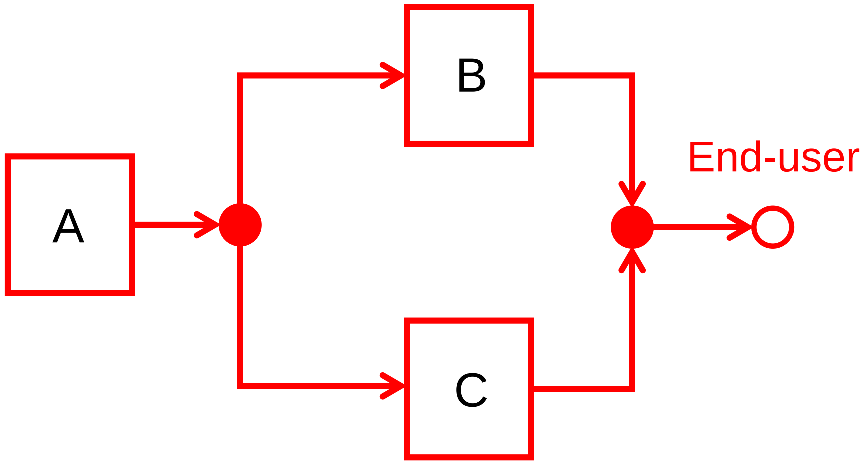
- *Reliability* is a characteristic of a system or sub-system, expressed by the **probability** that it performs its **required function** under given conditions **DURING a stated time interval**, i.e. (0, t]
- From a qualitative point of view, reliability is defined as the ability to **REMAIN** functional
- Quantitatively, reliability specifies the probability that **NO** operational **INTERRUPTIONS** will occur during a stated time interval
- To make sense, a numerical statement of reliability (e.g., R = 0.9) must be accompanied by the definition of the required function and the mission duration. We sometimes use $R(t)$ indicating the reliability in the time interval (0, t]:

$$R(t) = \text{Prob}\{\text{Failure occurs after } t\} = \cdots = e^{-\int_0^t \lambda(s)ds}$$

- $\lambda(t)$ is the ***failure rate*** function (or *hazard rate* function) measured in [# failures / time]
- In this form, reliability is also called survivor function

Structure reliability: Reliability block diagrams

- For a component of type k within the mission time T, we know $R_k(T) = e^{-\int_0^T \lambda_k(t)dt} = R_k = 1 - Q_k$
- We can compute the reliability of a parallel-series structure applying AND, OR logical operators:



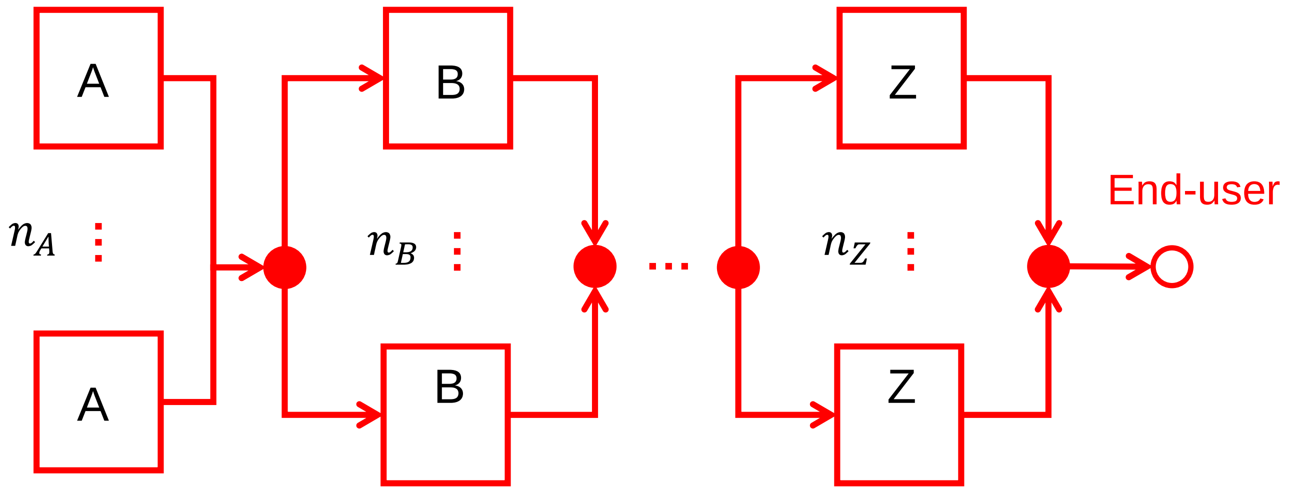
- In this example, the end-user will receive energy services during the mission time, if component A is operational AND component B OR component C is operational
- Alternatively, the client will not receive energy services during the mission time, if component A is failed OR component B AND component C is failed:

$$R = R_A \cdot R_{BC} = R_A \cdot (1 - Q_B \cdot Q_C) = R_A \cdot \big(1 - (1 - R_B) \cdot (1 - R_C)\big) = (1 - Q_A) \cdot (1 - Q_B \cdot Q_C)$$

- Therefore, we can select the appropriate types of component to comply with a minimum reliability requirement $\underline{R}$ during the mission time, i.e. $R \geq \underline{R}$

Structure reliability: Redundancy allocation

- In general, we can select the cost-optimum number and types of component in a series-parallel structure via a structure reliability constraint (obtained by induction from the previous equation):



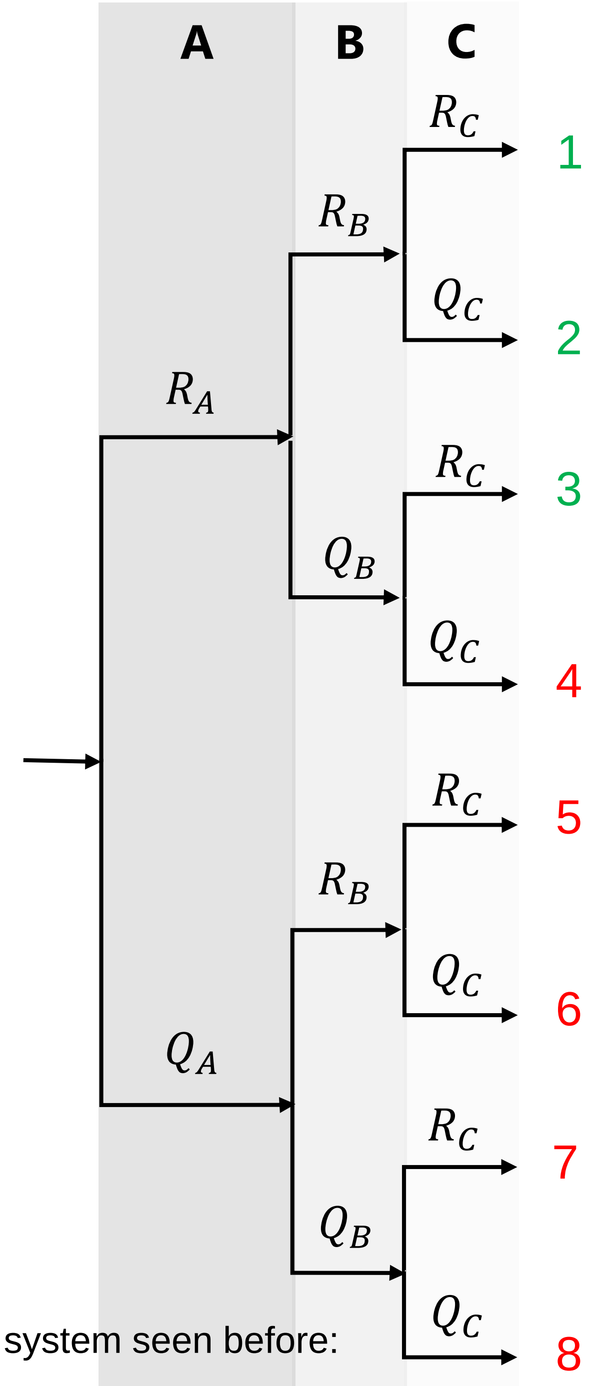
$$R = (1 - \underbrace{Q_A \cdot Q_A \ldots \cdot Q_A}_{Q_A^{n_A}}) \cdot (1 - \underbrace{Q_B \cdot Q_B \ldots \cdot Q_B}_{Q_B^{n_B}}) \cdot \cdots \cdot (1 - \underbrace{Q_Z \cdot Q_Z \ldots \cdot Q_Z}_{Q_Z^{n_Z}}) = \prod_{k=A}^{Z} \left(1 - Q_k^{n_k}\right)$$

Q_k is a function of installation and scheduling of component k

Event trees

- Visualization of all the events which can occur in a system, deduced as a sequence of events involving success or failure of the system components
- The system outcome caused by the occurrence of each path can only be deduced from the knowledge of the operating requirements of the system
- The probability π_i of occurrence of each path i can be evaluated from the product of the appropriate event probabilities since they must all occur for the path to occur
- Since all the paths are mutually exclusive, the probability of a particular system outcome (success or failure) can be evaluated by summating the path probabilities leading to that outcome (success or failure)

For the 3 component system seen before:



$$\pi_1 = R_A \cdot R_B \cdot R_C$$

$$\pi_2 = R_A \cdot R_B \cdot Q_C$$

$$\pi_3 = R_A \cdot Q_B \cdot R_C$$

$$R = \pi_1 + \pi_2 + \pi_3$$

Failure probability
$Q = 1 - R$

Risk: Definition

RISK = POTENTIAL DAMAGE ⊗ UNCERTAINTY

Dictionary: RISK = possibility of damage or injury to people or things

1. **What undesired conditions may occur?** ➡ **Accident Scenario, S**
2. **With what probability do they occur?** ➡ **Probability, π**
3. **What damage do they cause?** ➡ **Consequence, c**

⬇

We define risk as this TRIPLET

$$\text{RISK} = \{S, \pi, c\}$$

Total risk and risk reduction measures

$$\text{Total RISK} = \sum_i \pi_i c_i$$

WARNING: the same risk stems from different combinations of π_i and c_i

RISK (A) = RISK (B) but A = (π_A, c_A) ≠ ≠ ≠ B = (π_B, c_B)

LARGE small

RISK MEASURES: depend whether you act on π_i or c_i

Case A (reduce π_A): Prevention

Case B (reduce π_B): Protection, Mitigation

Expected value and standard deviation of cash flows

- Let us go back to the calculation of net present value (NPV) seen in Lecture 7 and consider n possible scenarios, i.e. possible values, of cash flow C_t during year t, each characterized by their probability, π.
- The expected value of the cash flow during year t, μ_t, can be calculated as

$$\mu_t = \sum_{i=1}^{n} \pi_{t,i} C_{t,i}$$

- The standard deviation of the cash flow during year t, σ_t, can be calculated as

$$\sigma_t = \sqrt{\sum_{i=1}^{n} \pi_{t,i} \left(C_{t,i} - \mu_t\right)^2}$$

Expected value and standard deviation of net present value (NPV)

- Assume the yearly cash flows are independent of each other (so-called *Hillier Model*)
- The expected value of the NPV can be calculated as

$$\mu_{\mathrm{NPV}} = -I_0 + \sum_{t=1}^{T} \left(\frac{1}{1+r}\right)^t \mu_t$$

- The standard deviation of the NPV can be calculated as

$$\sigma_{\mathrm{NPV}} = \sqrt{\sum_{t=1}^{T} \left(\frac{1}{1+r}\right)^{2t} \sigma_t^2}$$

- Different correlations are suggested when the yearly cash flows are correlated

Reliability and risk metrics for energy systems

- Expected demand not supplied EDNS – average power not supplied due to possible outages $i \in \{1,\ldots,n\}$ occurring with probability π_i and causing the loss of power supply L_i [MW]:

$$\text{EDNS} = \sum_{i=1}^{n} \pi_i L_i \qquad \text{[MW]}$$

- Expected energy not supplied EENS – average energy not supplied due to possible outages $i \in \{1,\ldots,n\}$ occurring with probability π_i, for D_i [hours] and causing the loss of power supply L_i [MW]:

$$\text{EENS} = \sum_{i=1}^{n} \pi_i L_i\, D_i \qquad \text{[MWh]}$$

System average interruption frequency index (λ_j and N_j, outage rate and end-users at load point j):

$$\text{SAIFI} = \frac{\sum_j \lambda_j N_j}{\sum_j N_j} \qquad \left[\frac{\text{Interruptions}}{\text{Customer Year}}\right]$$

System average interruption duration index (U_j and N_j, annual outage time and end-users at load point j):

$$\text{SAIDI} = \frac{\sum_j U_j N_j}{\sum_j N_j} \qquad \text{[h]}$$

After this lecture, you are able to …

- Define and critically evaluate security KPIs for energy systems:
  - ✓ Reliability indicators
  - ✓ Risk indicators
  - Resilience indicators

- Use different KPIs for the formulation of multi-objective optimization problems

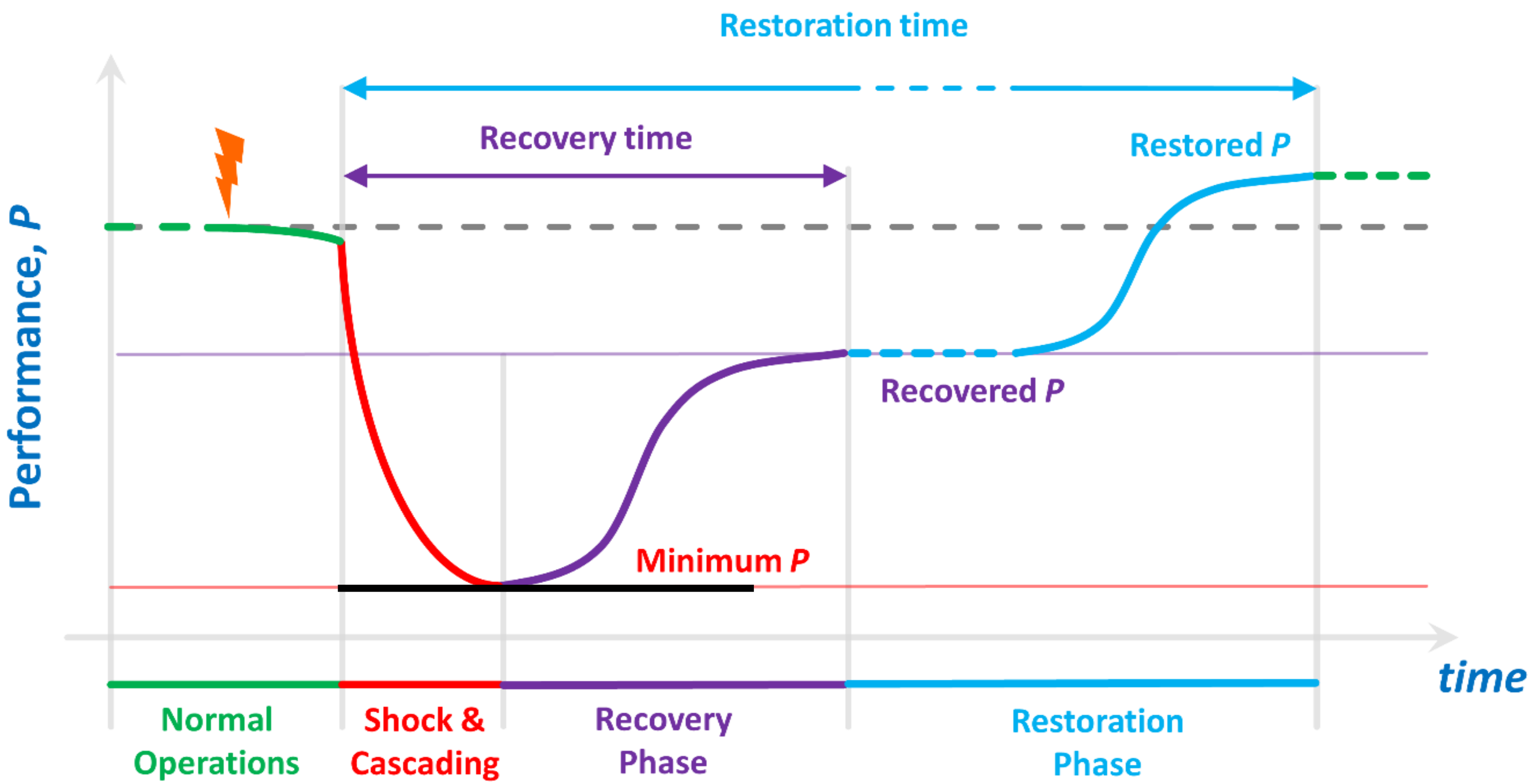
Resilience: An elusive definition

- Resilience can be defined as the ability of a system to **resist** a disruptive event and the ability to **reduce** deviation of the system performance

- US National Academy of Sciences defines it as “the ability to prepare and plan for, absorb, recover from, or more successfully adapt to actual or potential adverse events.”

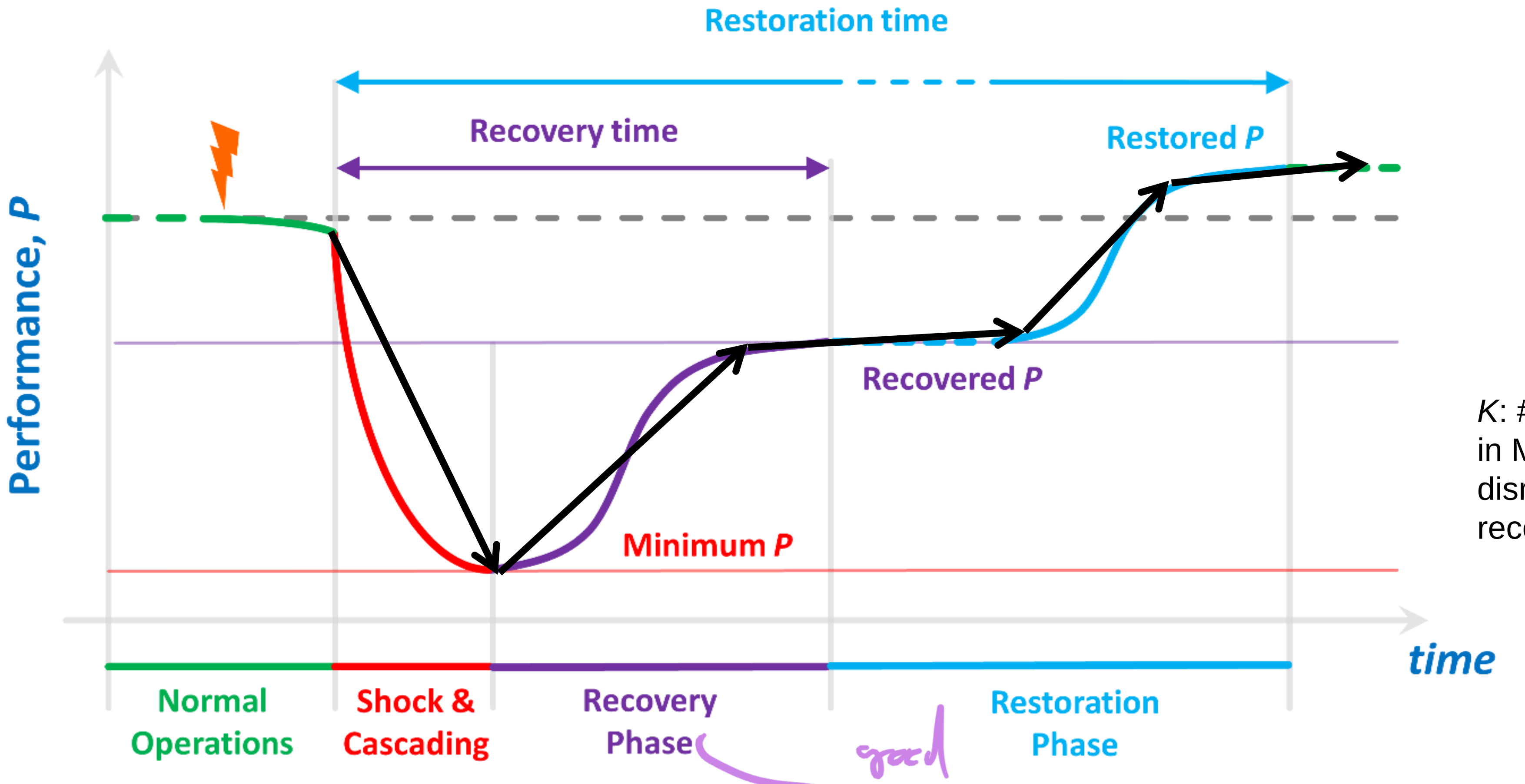
- However, there is no consensus definition to date

Quantifying resilience capabilities: Robustness (ROB)



Strength to resist initial impact $\mathrm{ROB} = \min\{P(t)\}$

Quantifying resilience capabilities: Rapidity (RAPI)



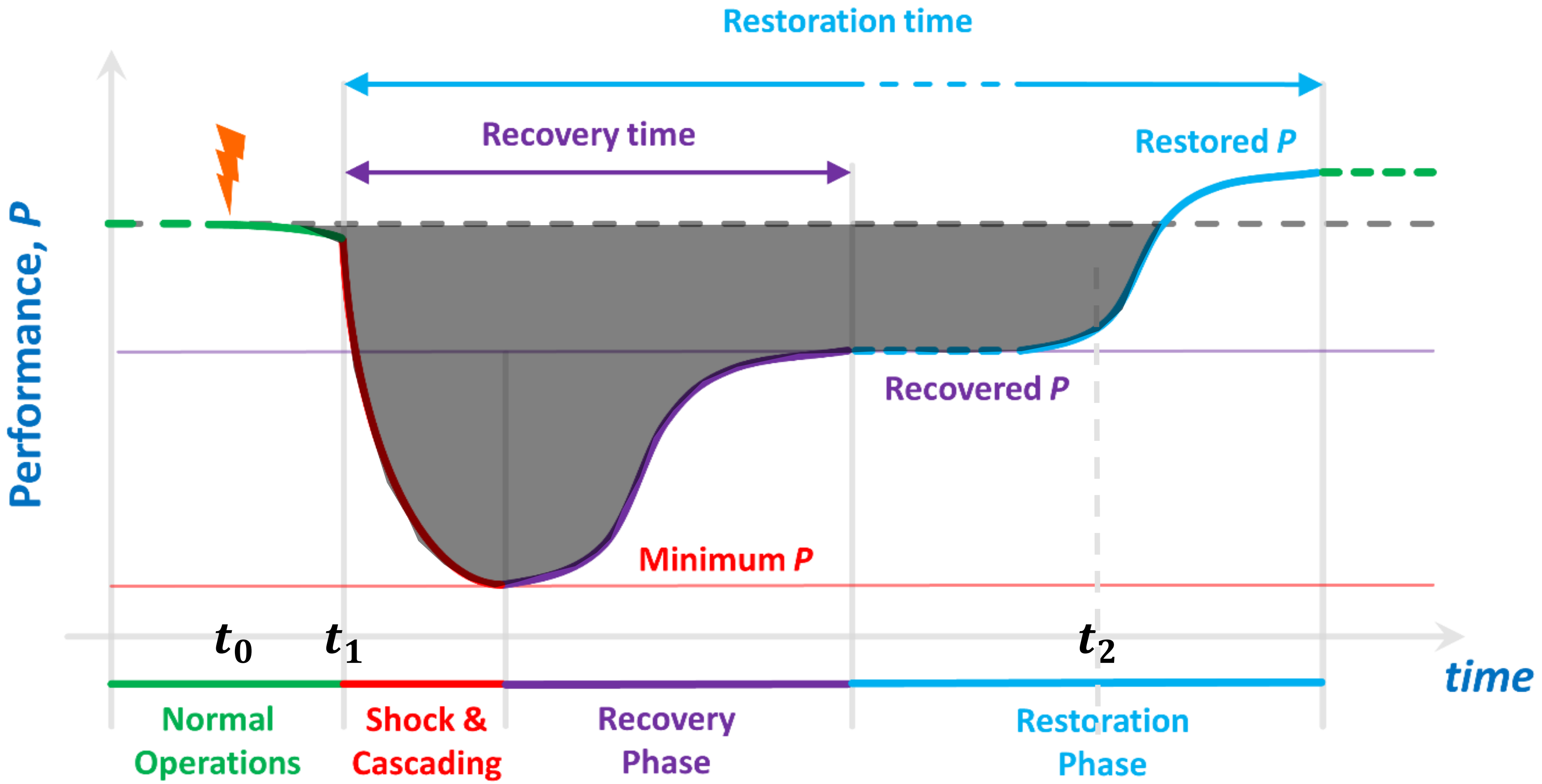
K: # of relevant changes in MOP slope during the disruption (DP) and the recovery phases (RP)

How fast performance degrades and recovers

$$\mathrm{RAPI}_{\mathrm{DP/RP}} = \frac{\left|\sum_{i=1}^{K} \frac{P(t_i) - P(t_i - \Delta t)}{\Delta t}\right|}{K}$$

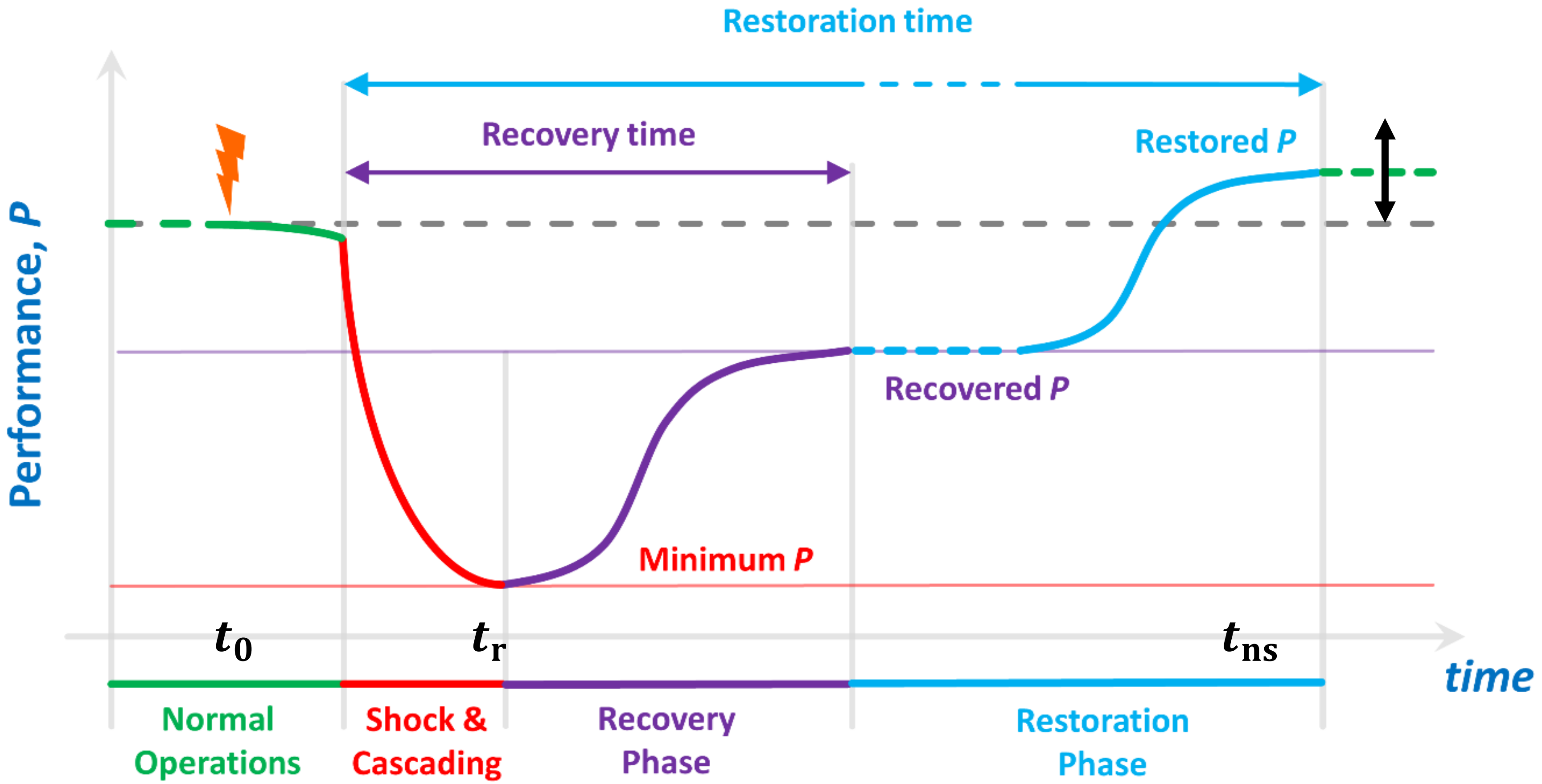
average

Quantifying resilience capabilities: Performance Loss (TAPL)



Performance loss during transient $\mathrm{TAPL} = \dfrac{\int_{t_1}^{t_2} \big(P(\mathrm{t}_0) - P(t)\big)dt}{t_2 - t_1}$

Quantifying resilience capabilities: Recovery Ability (RA)



New target performance level $$\mathrm{RA} = \left| \frac{P(t_\mathrm{ns}) - P(t_\mathrm{r})}{P(t_0) - P(t_\mathrm{r})} \right|$$

An integrated, general resilience metric (GR)

$$\mathrm{GR} = f(\mathrm{ROB}, \mathrm{RAPI_{DP}}, \mathrm{RAPI_{RP}}, \mathrm{TAPL}, \mathrm{RA})$$

$$= \mathrm{ROB}\ \frac{\mathrm{RAPI_{RP}}}{\mathrm{RAPI_{DP}}}\ \frac{1}{\mathrm{TAPL}}\ \mathrm{RA}$$

- Not system-specific and combines resilience capabilities
- Dimensionless and most useful in a comparative manner
- Consistent with resilience definition

Source: C. Nan, G. Sansavini. A quantitative method for assessing resilience of interdependent infrastructures. *Reliability Engineering & System Safety*, 2017, **157**, 35-53

Summary on security metrics

Indicator	Description	Formula	Decision criterion
Reliability	Ability of a system to remain functional during a time interval	$\text{Reliability} = \prod_{i=A}^{Z}\left(1 - Q_i^{n_i}\right)$	Choose/design system with maximum reliability
Total risk	Expected possibility of damage	$\text{Risk} = \sum_{i=1}^{n} \pi_i\, c_i^k, k \geq 1$	Design/operate the system to minimize total risk
Uncertainty variation	Standard deviation with respect to a quantity expected value, *μ*	$\sigma = \sqrt{\sum_{i=1}^{n} \pi_i (c_i - \mu)^2}$	For a positive quantity: minimize uncertainty of its expected value
EDNS	Expected demand not supplied following a power outage	$\text{EDNS} = \sum_{i=1}^{n} \pi_i L_i$	Choose/design system with minimum EDNS
SAIFI	System average interruption frequency index at a load point	$\text{SAIFI} = \frac{\sum_j \lambda_j \cdot N_i}{\sum_j N_j}$	Choose/design system with minimum SAIFI
TAPL	Loss of performance during transient system operation	$\text{TAPL} = \frac{\int_{t_d}^{t_{ns}} \left(\text{MOP}(\text{t}_\text{o}) - \text{MOP}(t)\right) dt}{t_{ns} - t_d}$	Design/operate the system to minimize TAPL

After this lecture, you are able to …

- ✓ Define and critically evaluate security KPIs for energy systems:
  - ✓ Reliability indicators
  - ✓ Risk indicators
  - ✓ Resilience indicators

- Use different KPIs for the formulation of multi-objective optimization problems

Security metrics as objectives and constraints of optimization problem

Remember Lecture 3/5

$$\boxed{\min_{\boldsymbol{x}} \quad z = f(\boldsymbol{x})}$$

$$\boxed{\begin{aligned} \text{s.t.} \quad & g_j(x) \leq 0, && j = 1, \ldots, n \\ & h_i(x) = 0, && i = 1, \ldots, o \end{aligned}}$$

objective function		z	$\mathbb{R}^m \to \mathbb{R}$	quantity to be minimized (performance loss, energy demand not supplied, …)
decision variables		$\boldsymbol{x}$	$\in \mathbb{R}^m$	optimal decisions (technology installed, amount of energy used, …)
constraints	equality	$h_i(\boldsymbol{x}) \ = \ 0$	$i = 1, \ldots, o$	balance equations, component models
	(in)equality	$g_j(\boldsymbol{x}) \leq 0$	$j = 1, \ldots, n$	limitations (SAIDI, SAIFI, performance loss, energy demand not supplied, …)

Critical infrastructure: Maximizing resilience

Optimum post-disruption restoration under uncertainty for enhancing critical infrastructure resilience

Yi-Ping Fang[a], Giovanni Sansavini[b,*]

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A R T I C L E I N F O

Keywords:
Critical infrastructures
System resilience
Restoration planning
Uncertainty
Stochastic programming

A B S T R A C T

The planning of post-disruption restoration in critical infrastructure systems often relies on deterministic assumptions such as complete information on resources and known duration of the repair tasks. In fact, the uncertainties faced by restoration activities, e.g. stemming from subjective estimates of resources and costs, are rarely considered. Thus, the solutions obtained by a deterministic approach may be suboptimal or even infeasible under specific realizations of the uncertainties. To bridge this gap, this paper investigates the effects of uncertain repair time and resources on the post-disruption restoration of critical infrastructure. Two-stage stochastic optimization provides insights for prioritizing the intensity and time allocation of the repair activities with the objective of maximizing system resilience. The inherent stochasticity is represented via a set of scenarios capturing specific realizations of repair activity durations and available resources, and their probabilities. A multi-mode restoration model is proposed that offers more flexibility than its single-mode counterpart. The restoration framework is applied to the reduced British electric power system and the results demonstrate the added value of using the stochastic model as opposed to the deterministic model. Particularly, the benefits of the proposed stochastic method increase as the uncertainty associated with the restoration process grows. Finally, decision-making under uncertainty largely impacts the optimum repair modes and schedule.

Source: Yiping Fang, Giovanni Sansavini. Optimum Post-Disruption Restoration under Uncertainty for Enhancing Critical Infrastructure Resilience. *Reliability Engineering and System Safety*, 2019, **185**, 1-11

Critical infrastructure: Maximizing resilience

Optimum post-disruption restoration under uncertainty for enhancing critical infrastructure resilience

Check for updates

Yi-Ping Fang[a], Giovanni Sansavini[b,*]

Expected system resilience loss

objective function

$$\min \mathbb{E}_{\xi(s)}\ \mathrm{TAPL} \quad (8)$$

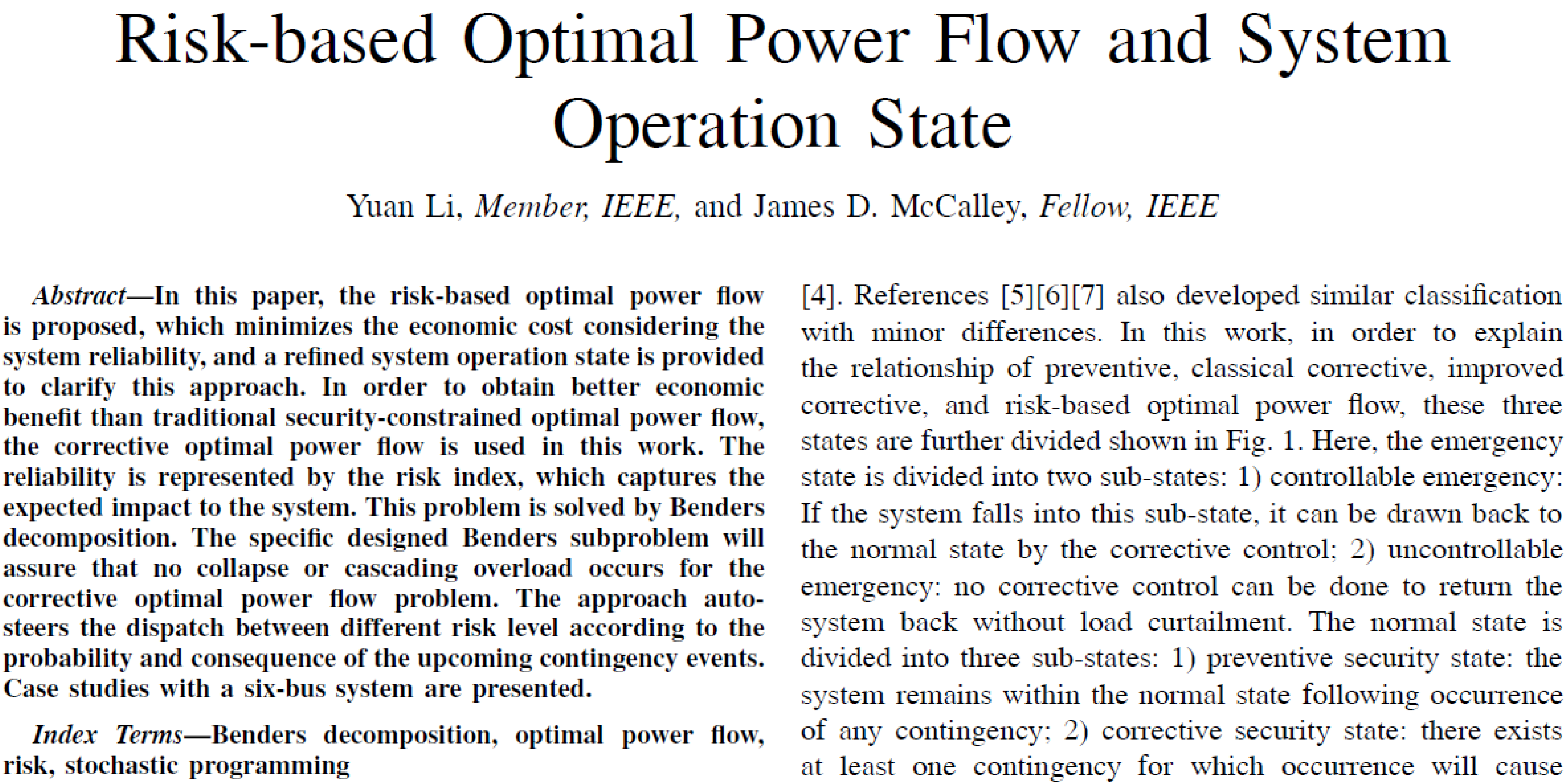
$$\mathrm{TAPL} = \frac{\int_{t_1}^{t_2} \left(\mathrm{P}(\mathrm{t}_0) - \mathrm{P}(t)\right) dt}{t_2 - t_1} \quad (2)$$

$$\mathrm{P}(t) = \frac{\sum_{n \in V_D} (\bar{p}_{nt} - \Delta p_{nt})}{\sum_{n \in V_D} \bar{p}_{nt}} \quad (1)$$

Problem typically formulated as an **MILP** (linear constraints)

Source: Yiping Fang, Giovanni Sansavini. Optimum Post-Disruption Restoration under Uncertainty for Enhancing Critical Infrastructure Resilience. *Reliability Engineering and System Safety*, 2019, **185**, 1-11

Power systems: Minimizing cost while considering system reliability


Risk-based Optimal Power Flow and System Operation State

Yuan Li, *Member, IEEE,* and James D. McCalley, *Fellow, IEEE*

Abstract—In this paper, the risk-based optimal power flow is proposed, which minimizes the economic cost considering the system reliability, and a refined system operation state is provided to clarify this approach. In order to obtain better economic benefit than traditional security-constrained optimal power flow, the corrective optimal power flow is used in this work. The reliability is represented by the risk index, which captures the expected impact to the system. This problem is solved by Benders decomposition. The specific designed Benders subproblem will assure that no collapse or cascading overload occurs for the corrective optimal power flow problem. The approach auto-steers the dispatch between different risk level according to the probability and consequence of the upcoming contingency events. Case studies with a six-bus system are presented.

Index Terms—Benders decomposition, optimal power flow, risk, stochastic programming

[4]. References [5][6][7] also developed similar classification with minor differences. In this work, in order to explain the relationship of preventive, classical corrective, improved corrective, and risk-based optimal power flow, these three states are further divided shown in Fig. 1. Here, the emergency state is divided into two sub-states: 1) controllable emergency: If the system falls into this sub-state, it can be drawn back to the normal state by the corrective control; 2) uncontrollable emergency: no corrective control can be done to return the system back without load curtailment. The normal state is divided into three sub-states: 1) preventive security state: the system remains within the normal state following occurrence of any contingency; 2) corrective security state: there exists at least one contingency for which occurrence will cause

Source: Y. Li and J. D. McCalley, "Risk-based optimal power flow and system operation state," *2009 IEEE Power & Energy Society General Meeting*, Calgary, AB, 2009, pp. 1-6, doi: 10.1109/PES.2009.5275724.

Power systems: Minimizing cost while considering system reliability

Risk-based Optimal Power Flow and System Operation State

Yuan Li, *Member, IEEE*, and James D. McCalley, *Fellow, IEEE*

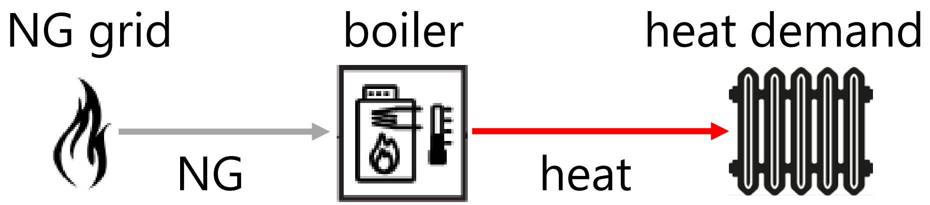
		Economic cost / System reliability, represented by the risk index, *R*		
objective function	Min	$f_0(x_0, u_0) + w * R(x_0, u_0)$		(2a)
constraints: equality	$s.t.$	$g_k(x_k, u_k) = 0$	$k = 0, \cdots, c$	(2b)
		$g_k^0(x_k^0, u_0) = 0$	$k = 1, \cdots, c$	(2c)
constraints: (in)equality		$h_k(x_k, u_k) \leq h^{max}$	$k = 0, \cdots, c$	(2d)
		$h_k^0(x_k^0, u_0) \leq p_k h^{max}$	$k = 1, \cdots, c$	(2e)
		$\lvert u_k - u_0 \rvert \leq \Delta_k^{max}$	$k = 1, \cdots, c$	(2f)
		$g_k^1(x_k^1, u_0 + s_k) = 0$	$k = 1, \cdots, c$	(2g)
		$h_k^1(x_k^1, u_0 + s_k) \leq h^{max}$	$k = 1, \cdots, c$	(2h)

Original **MINLP**, often linearized as solved as **MILP**

Source: Y. Li and J. D. McCalley, "Risk-based optimal power flow and system operation state," *2009 IEEE Power & Energy Society General Meeting*, Calgary, AB, 2009, pp. 1-6, doi: 10.1109/PES.2009.5275724.

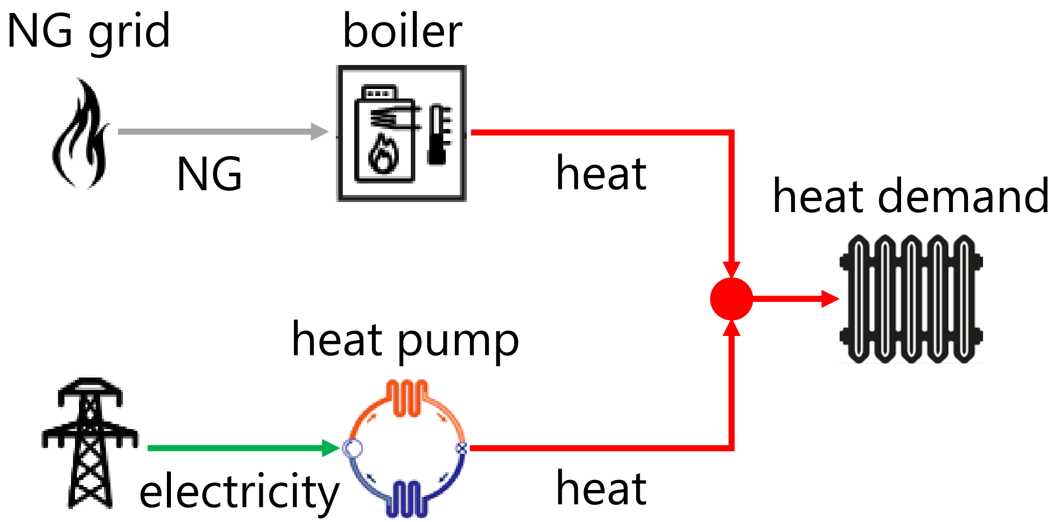
Optimal configurations of energy systems with security of supply

System A: Cost-optimal, not considering risk



- Optimal system configuration in terms of minimum total annual cost (TAC)
- Less resilient to failures of the energy supply (only one technology and one type of supply)

System B: Cost-optimal, considering risk



- System configuration leading to higher values of total annual cost (TAC)
- More resilient to failures of the energy supply (less likely that more technologies fail at once)

N-1 criterion: Secure dispatch of energy systems

$$\min_{\boldsymbol{x}} \quad z = f(\boldsymbol{x})$$

$$\text{s.t.} \quad \left.\begin{array}{ll} g_j(x) \leq 0, & j = 1, \dots, nN \\ h_i(x) = 0, & i = 1, \dots, oN \end{array}\right\}$$

System constraints are formulated *N* times (i.e. number of contingencies), to ensure the system operates properly without any of the *N* components considered (one at a time – deterministic approach)

objective function		z	$\mathbb{R}^m \to \mathbb{R}$	Equity and/or security metrics
decision variables		$\boldsymbol{x}$	$\in \mathbb{R}^m$	optimal quantities (technology investments, amount of energy used, …)
constraints	equality	$h_i(\boldsymbol{x}) = 0$	$i = 1, \dots, oN$	additional constraints ensuring system operation without any of *N* components
	(in)equality	$g_j(\boldsymbol{x}) \leq 0$	$j = 1, \dots, nN$	additional constraints ensuring system operation without any of *N* components

Decision-making according to multiple KPIs: Multi-objective optimization

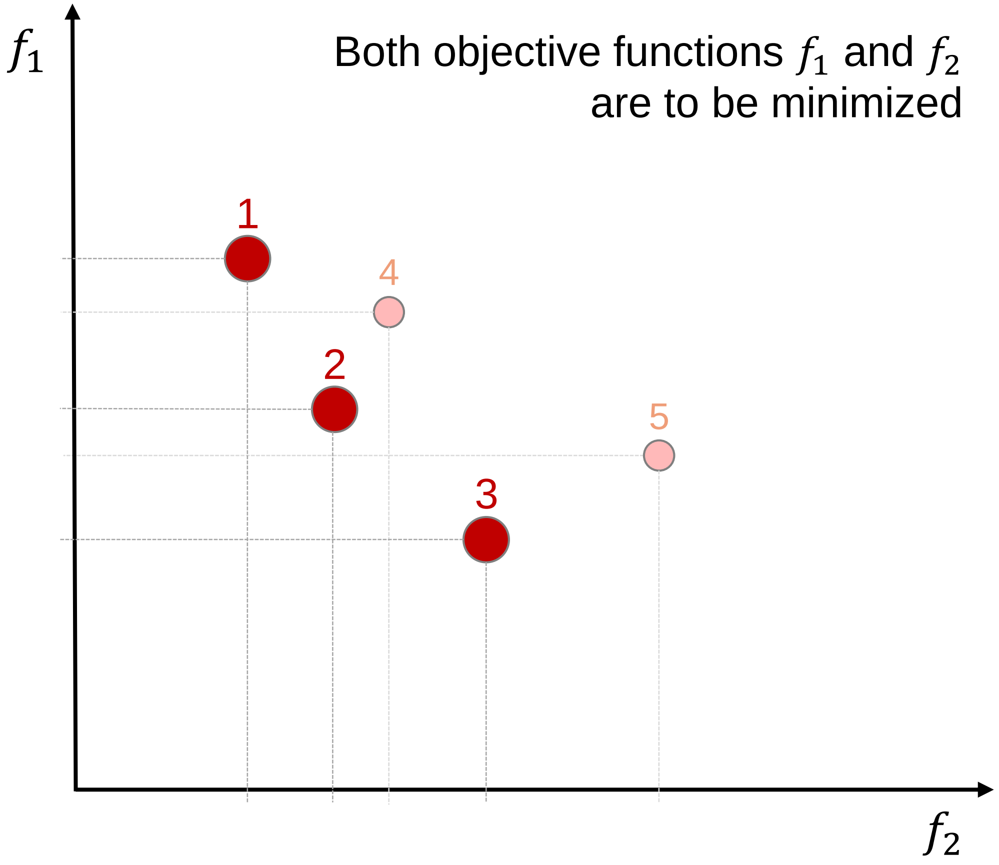
Formulation of the optimization problem

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & z_q = f_q(\boldsymbol{x}), && q = 1, \dots, Q \\ \text{s.t.} \quad & g_j(\boldsymbol{x}) \leq 0, && j = 1, \dots, n \\ & h_i(\boldsymbol{x}) = 0, && i = 1, \dots, o \end{aligned}$$

objective functions		z_q	$\mathbb{R}^m \rightarrow \mathbb{R}$	System total annual cost, environmental impact, reliability and resilience
decision variables		$\boldsymbol{x}$	$\in \mathbb{R}^m$	Technology installation and operation, energy exchanged with grids
constraints	equality	$h_i(\boldsymbol{x}) = 0$	$i = 1, \dots, o$	Energy conversion efficiency (typically nonlinear)
	(in)equality	$g_j(\boldsymbol{x}) \leq 0$	$j = 1, \dots, n$	Min/max-power constraints (typically nonlinear)

The concept of dominance

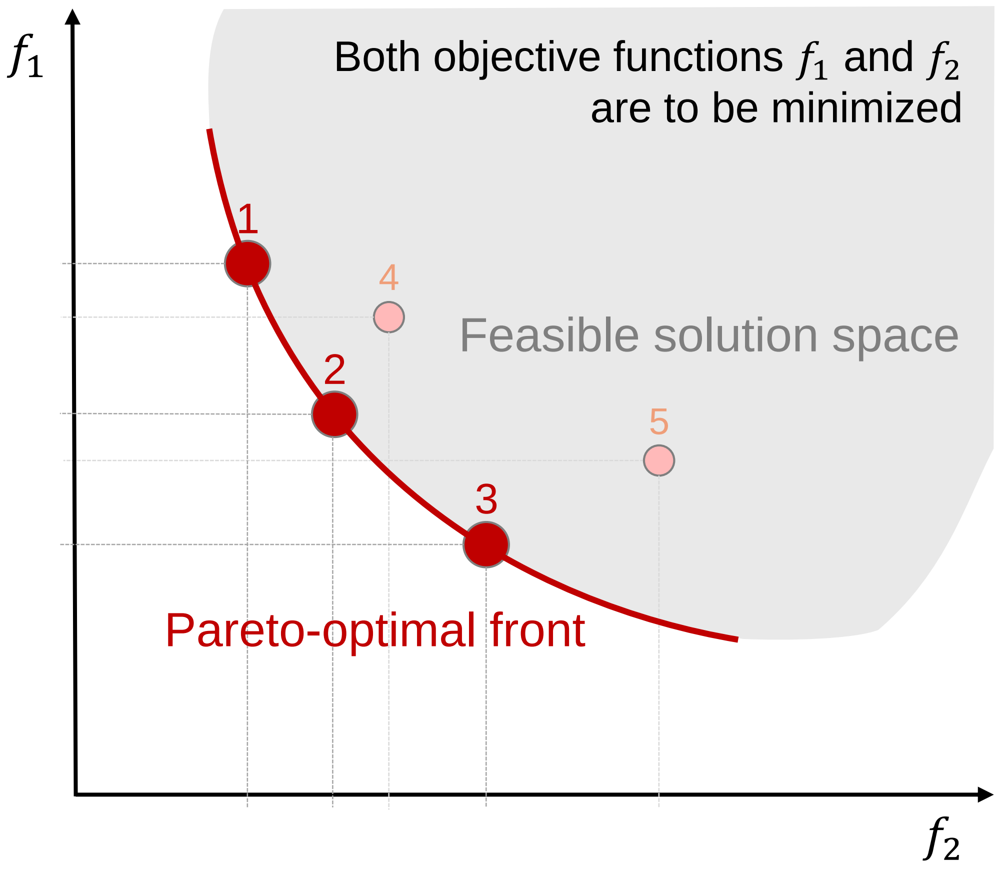
- Single-objective optimization: the superiority of a solution over others is determined by comparing their objective function values
- Multi-objective optimization: the superiority of a solution over others is determined by the dominance
- *Definition of dominance*. Given two solutions $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$, $\boldsymbol{x}_1$ dominates over $\boldsymbol{x}_2$ if:
  - $\boldsymbol{x}_1$ is no worse than $\boldsymbol{x}_2$ in all objectives
  - $\boldsymbol{x}_1$ is strictly better than $\boldsymbol{x}_2$ in at least one objective



- 1 vs 4: neither solution dominates
- 1 vs 2: neither solution dominates
- 2 vs 4: 2 dominates 4 (4 is dominated by 2)
- 3 vs 5: 3 dominates 5 (5 is dominated by 3)

Pareto-optimal set and Pareto-optimal front

- *Pareto-optimal set*. Set of feasible and non-dominated solutions
- *Pareto-optimal front*. Boundary of the feasible solution space, defined by the mapping of the Pareto-optimal set
- The scope of multi-objective optimization is to **determine the Pareto-optimal front**, often called *Pareto front*
- Several methods are available for multi-objective optimization. In general, the problem is first transformed into a single-objective optimization problem and then solved



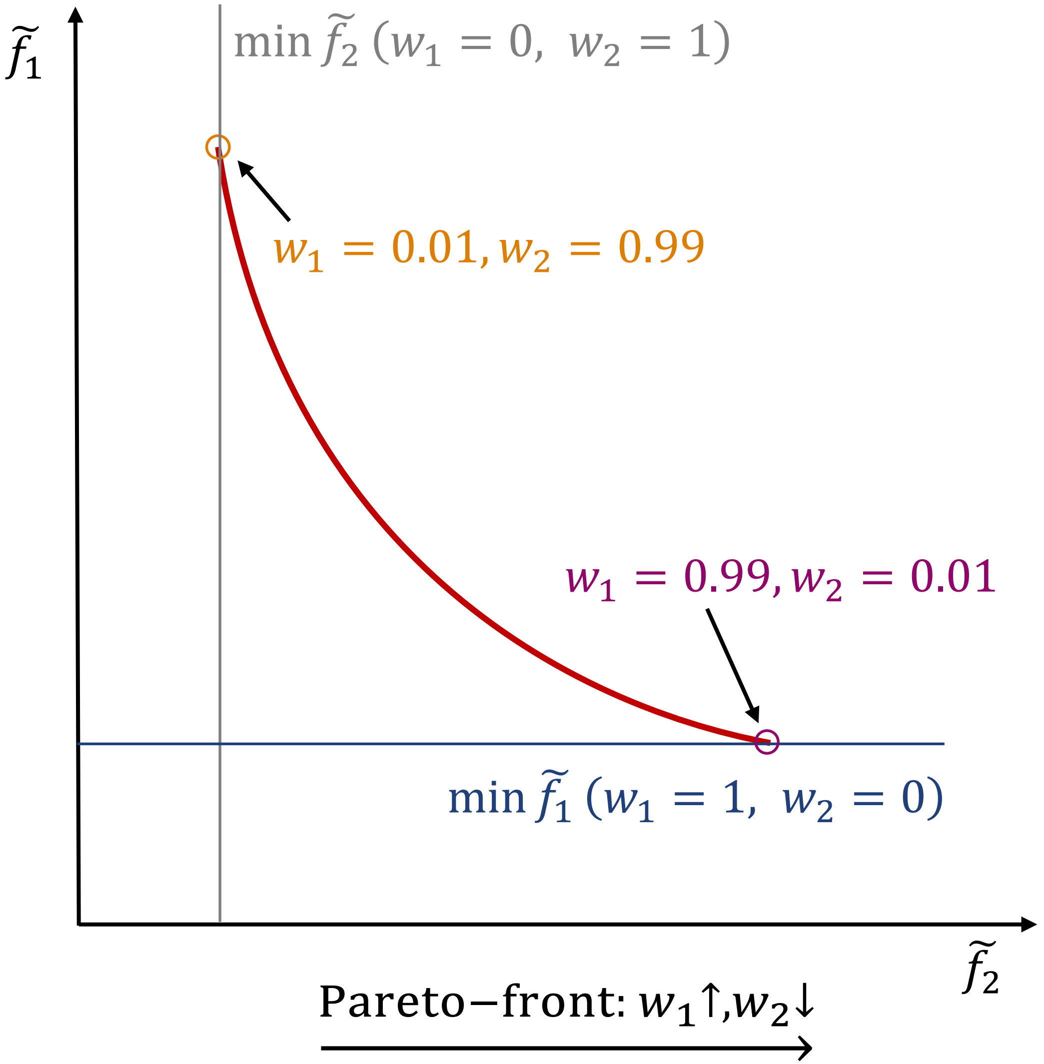
- 1 vs 4: neither solution dominates
- 1 vs 2: neither solution dominates
- 2 vs 4: 2 dominates 4 (4 is dominated by 2)
- 3 vs 5: 3 dominates 5 (5 is dominated by 3)

Multi-objective optimization methods: Weighted sum

- A single objective function is built as a linear combination of the original objective functions, multiplied by a set of weights, w_q

$$
\begin{aligned}
&\min_{\boldsymbol{x}} && z = \sum_{q=1}^{Q} w_q\, f_q(\boldsymbol{x}) && \\
&\text{s.t.} && g_j(\boldsymbol{x}) \le 0, && j = 1, \dots, n \\
& && h_i(\boldsymbol{x}) = 0, && i = 1, \dots, o
\end{aligned}
$$

- Given two objectives, f_1 and f_2, the Pareto front is built via different combinations of w_1 and w_2
- Challenge: The objective functions need to be normalized to 1, i.e. $\widetilde{f}_q$, for setting values of weights significant relative to each other and relative to the objective functions values

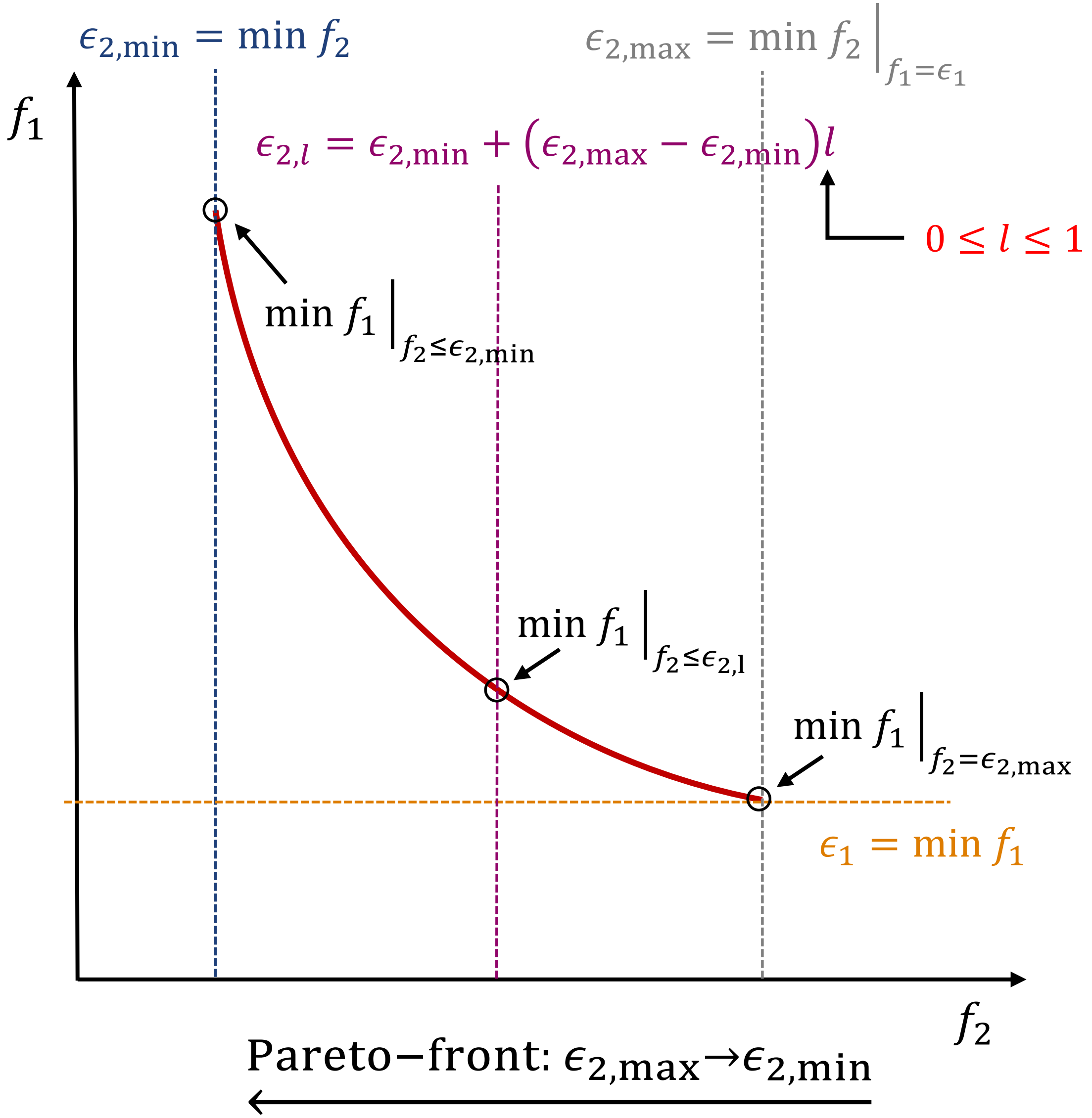


Multi-objective optimization methods: ϵ-constraint

- One objective function is considered only, while the values of the others are constrained

$$
\begin{aligned}
\min_{\boldsymbol{x}} \quad & z = f_p(\boldsymbol{x}) \\
\text{s.t.} \quad & f_q(\boldsymbol{x}) \leq \epsilon_q, && q = 1, \dots, Q, q \neq p \\
& g_j(\boldsymbol{x}) \leq 0, && j = 1, \dots, n \\
& h_i(\boldsymbol{x}) = 0, && i = 1, \dots, o
\end{aligned}
$$

- Given two objectives, f_1 and f_2, Pareto front is built by:
    - considering the minimum value of both objectives
    - varying the values of the ϵ-constraint (e.g. on f_2) from its maximum to its minimum value



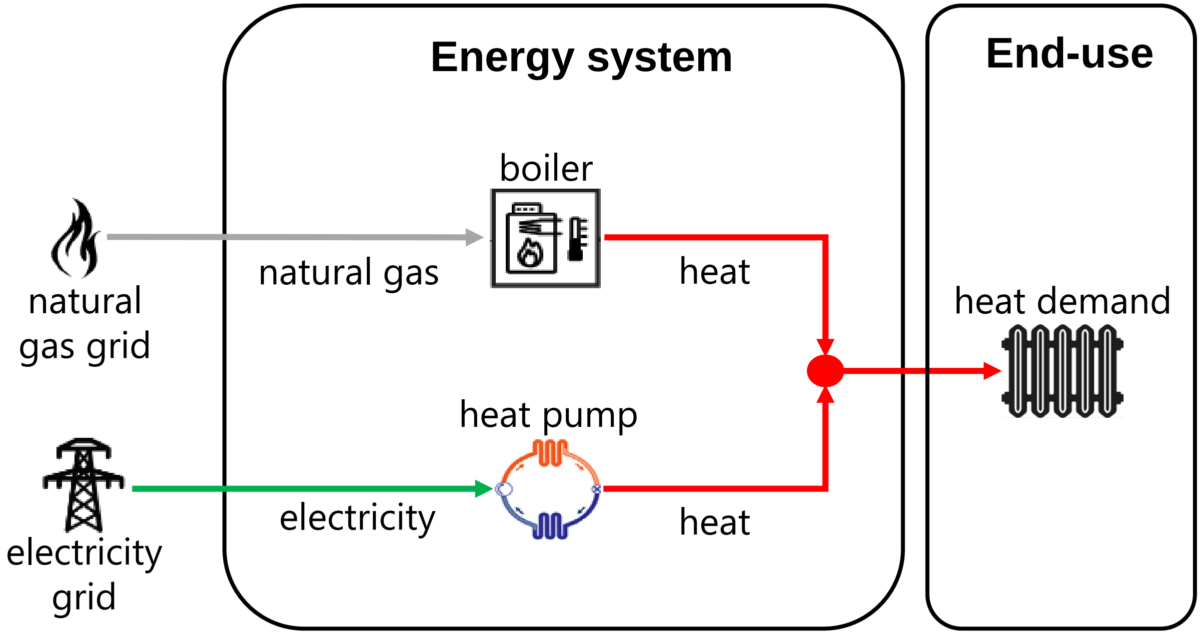
Multi-objective optimization methods: Pros and Cons

	Weighted sum	ϵ-constraint
Pros	▪ Simple method ▪ Often more practical at the extremes of the Pareto front (i.e. minimize one objective function while accounting for another)	▪ Applicable to both convex and non-convex objective spaces ▪ Often more practical across the Pareto front since it can return non-convex shapes (can be the case with MILP optimization problems)
Cons	▪ Can be difficult to define weights ▪ It cannot find some sections of the Pareto front (and corresponding Pareto-optimal solutions) for non-convex objective space	▪ Complicated method when optimizing for several objective function since ϵ-constraints should be defined for all objective functions

Other multi-objective optimization methods are available

M. Ehrgott, *Multicriteria Optimization*, 2005, Springer-Verlag Berlin Heidelberg, pp. 323

Numerical Example: Multi-objective optimization



Input data			
Heat demand	D_{H}	1500	kWh
Electricity price	p_{E}	0.15	CHF/kWh
Natural gas price	p_{G}	0.03	CHF/kWh
Electricity grid carbon footprint	γ_{E}	100	gCO_2/kWh
Natural gas grid carbon footprint	γ_{G}	230	gCO_2/kWh
Technology sizes (all)	P	2000	kW
Heat pump efficiency	η_{HP}	4	-
Boiler efficiency	η_{B}	0.95	-

→ Minimize $f_1 = \text{total cost}$, and $\mathrm{f}_2 = \text{total emissions}$ and determine the Pareto optimal front

Numerical-example Multi-objective optimization

Determine the Pareto Optimal Front

1. Determining the minimum value of the objective functions:

   - $f_1 = \text{total cost} = p_\text{E} M_\text{E} + p_\text{G} M_\text{G}$
   - $f_2 = \text{total emissions} = \gamma_\text{E} M_\text{E} + \gamma_\text{G} M_\text{G}$
   - $\min\limits_{M_E, M_G} \; f_i(M_\text{E}, M_\text{G})$

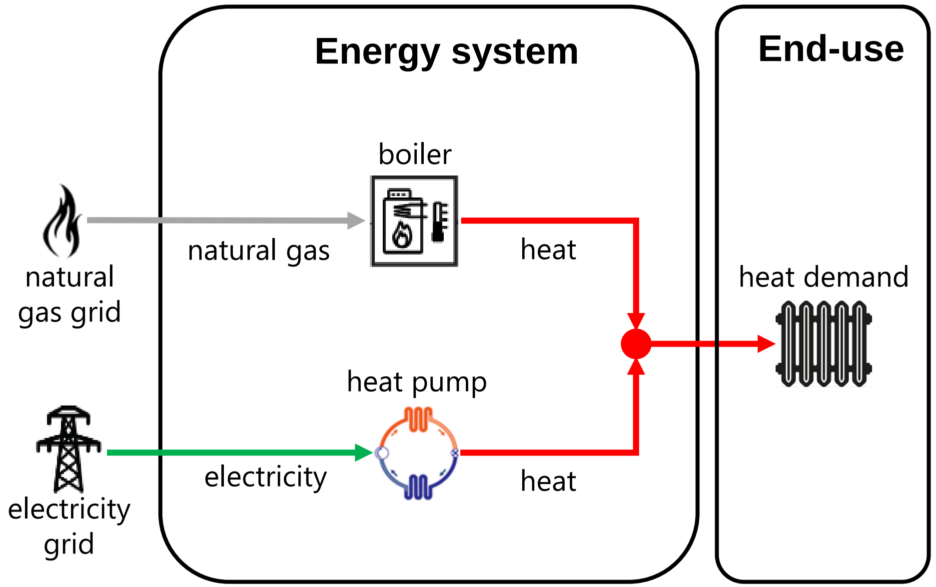
$$\text{s.t.} \quad V_\text{H,HP} + V_\text{H,B} = D_\text{H}$$

$$V_\text{H,HP} = M_\text{E} \eta_\text{HP}$$

$$V_\text{H,B} = M_\text{G} \eta_\text{G}$$

$$V_\text{H,HP}, V_\text{H,B} \le P$$

$$\text{M}_E, M_\text{G}, V_\text{H,HP}, V_\text{H,B} \ge 0$$



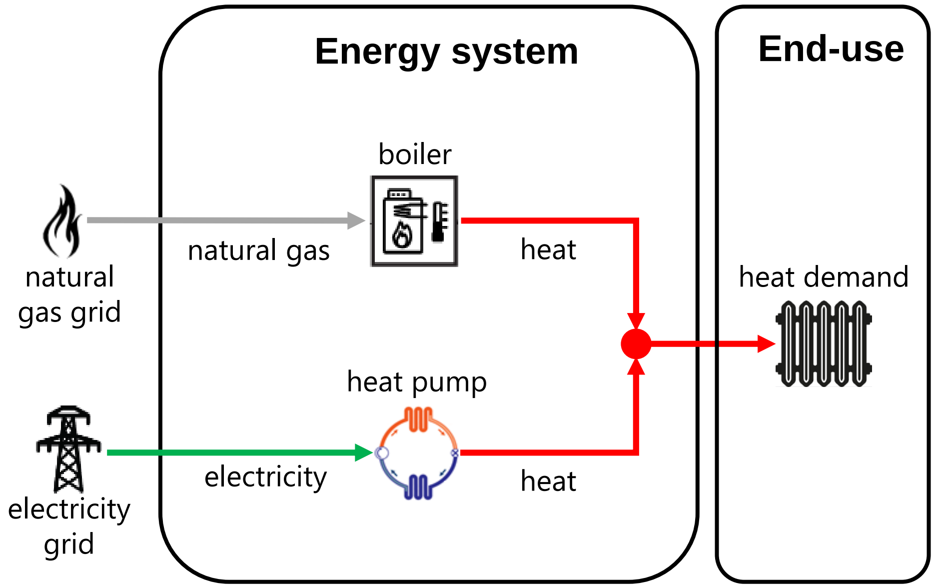
Decision variables (minimum-cost optimization)	
Heat output, heat pump	$V_\text{H,HP}$
Heat output, boiler	$V_\text{H,B}$
Imported electricity	M_E
Imported natural gas	M_G

Numerical-example Multi-objective optimization

Determine the Pareto Optimal Front

1. Determining the minimum value of the objective functions:
   - $f_1 = \text{total cost} = p_E M_E + p_G M_G$
   - $f_2 = \text{total emissions} = \gamma_E M_E + \gamma_G M_G$
   - Solve the optimization problem for both objective functions individually:

	total cost	total emissions
heat pump	CHF 56	$38\ kg_{CO_2}$
boiler	CHF 47	$363\ kg_{CO_2}$



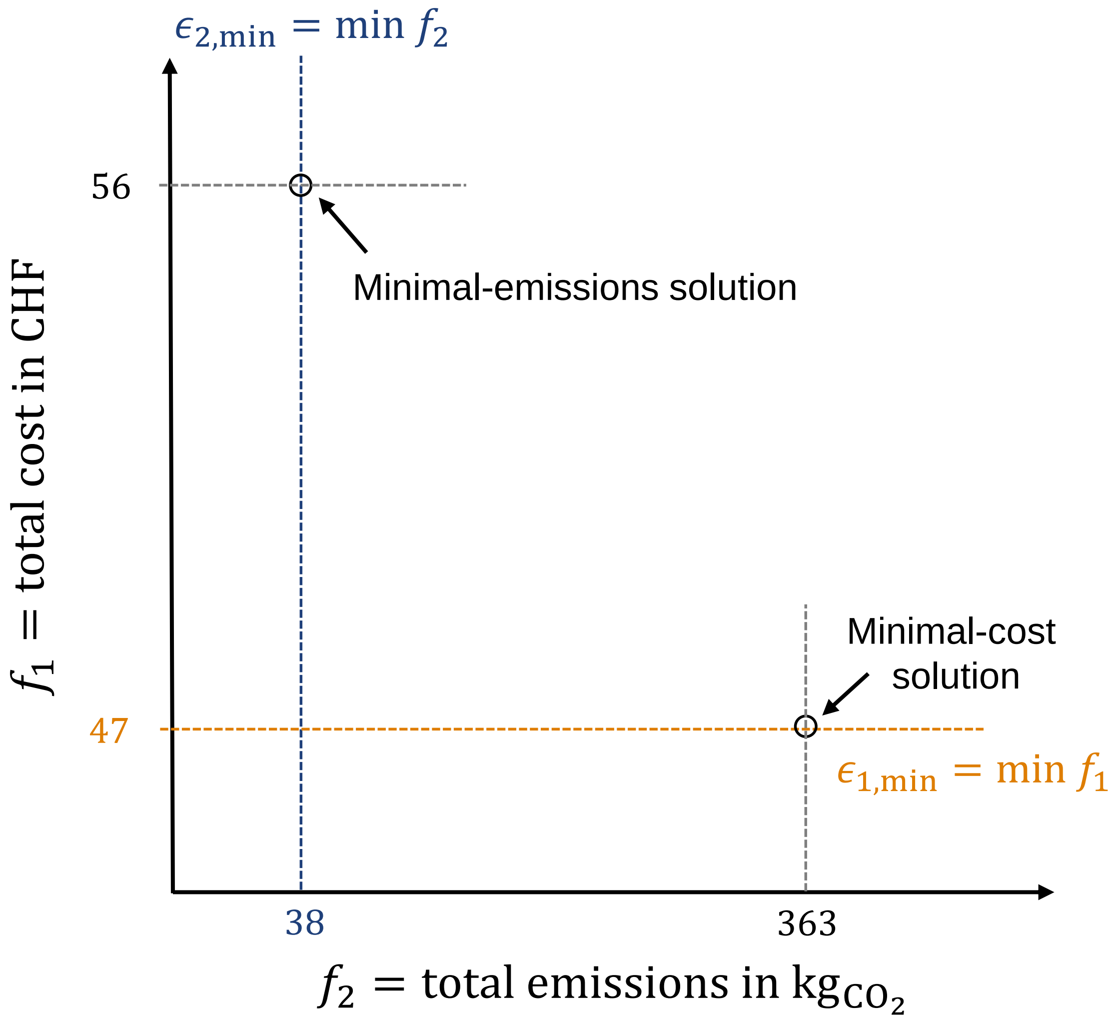
Decision variables (minimum-cost optimization)	
Heat output, heat pump	$V_{H,HP}$
Heat output, boiler	$V_{H,B}$
Imported electricity	M_E
Imported natural gas	M_G

Numerical-example Multi-objective optimization

Determine the Pareto Optimal Front

1. Determining the minimum value of the objective functions:
   - $f_1 = \text{total cost} = p_\mathrm{E} M_\mathrm{E} + \mathrm{p_G M_G}$
   - $f_2 = \text{total emissions} = \gamma_\mathrm{E} M_\mathrm{E} + \gamma_\mathrm{G} M_\mathrm{G}$
   - Solve the optimization problem for both objective functions individually:

	total cost	**total emissions**
heat pump	CHF 56	38 $\mathrm{kg_{CO_2}}$
boiler	CHF 47	363 $\mathrm{kg_{CO_2}}$



Numerical-example Multi-objective optimization

Determine the Pareto Optimal Front

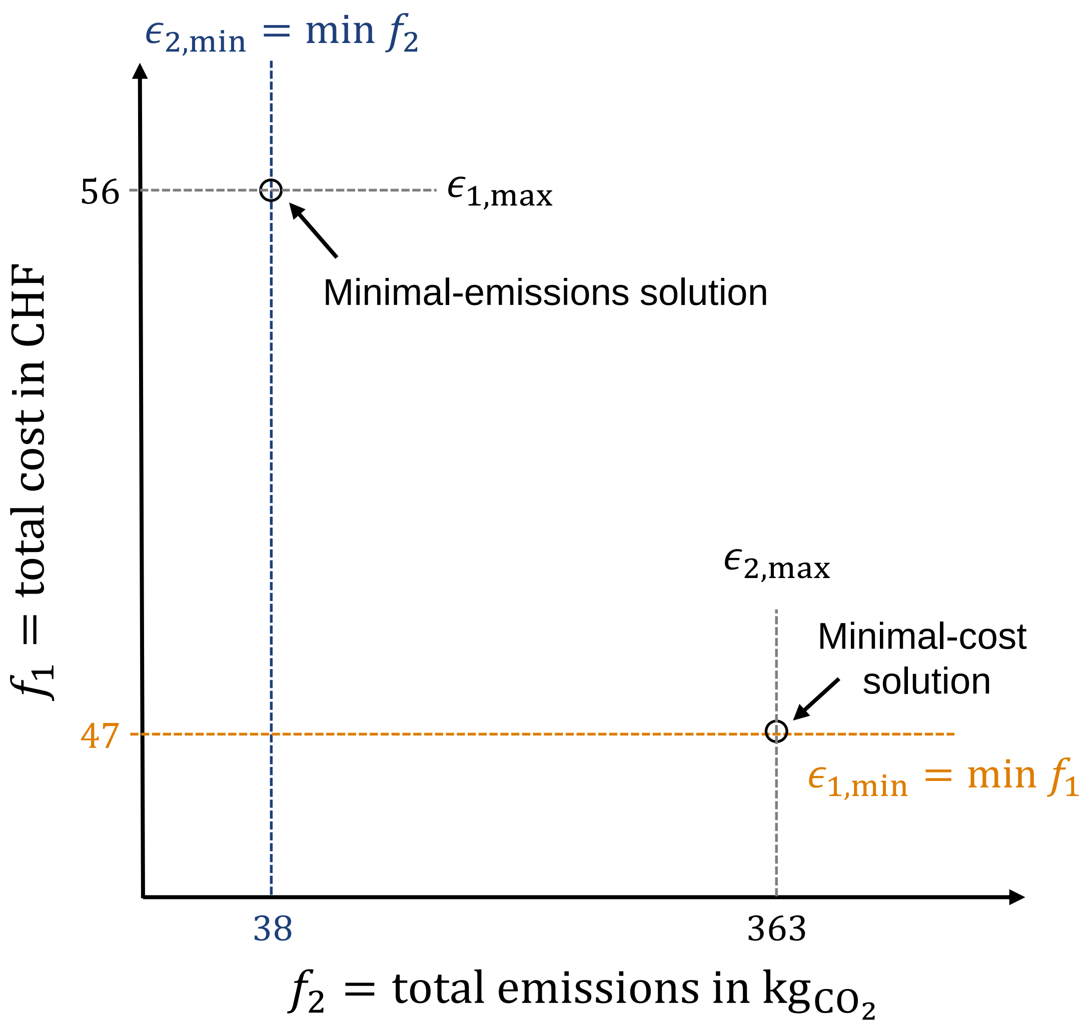
2. Varying the values of the ϵ-constraint for $f_2 =$ total emissions from its maximum ($\epsilon_{2,\max}$) to its minimum value ($\epsilon_{2,\min}$)
   - $\epsilon_{2,\max} = 363\ \mathrm{kg}_{\mathrm{CO_2}}$ and $\epsilon_{2,\min} = 38\ \mathrm{kg}_{\mathrm{CO_2}}$
   - $\epsilon_{2,l} = \epsilon_{2,\min} + \left(\epsilon_{2,\max} - \epsilon_{2,\min}\right) l$ and $0 \leq l \leq 1$
   - Formulate and solve the optimization problem:

$$\min_{M_\mathrm{E}, M_\mathrm{G}} \quad p_\mathrm{E} \cdot M_\mathrm{E} + p_\mathrm{G} \cdot M_\mathrm{G}$$

$$\text{s.t.} \quad \gamma_\mathrm{E} \cdot M_\mathrm{E} + \gamma_\mathrm{G} \cdot M_\mathrm{G} \leq \epsilon_{2,l}$$

$$\vdots$$

$$\mathrm{M_E}, M_\mathrm{G}, V_\mathrm{H,HP}, V_\mathrm{H,B} \geq 0$$

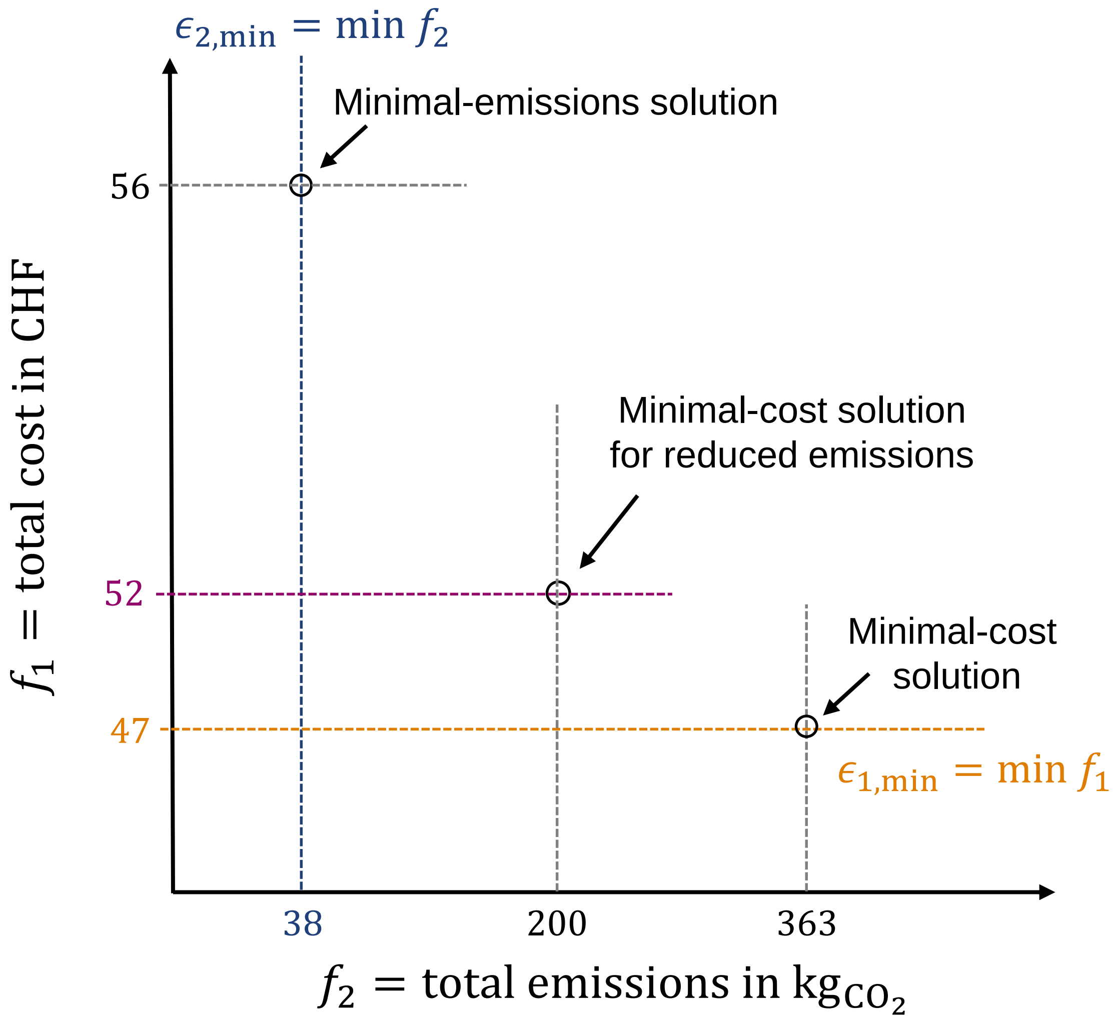


Numerical-example Multi-objective optimization

Determine the Pareto Optimal Front

2. varying the values of the ϵ-constraint for $f_2 =$ total emissions from its maximum ($\epsilon_{2,\max}$) to its minimum value ($\epsilon_{2,\min}$)
   - $\epsilon_{2,\max} = 363\ \mathrm{kg}_{\mathrm{CO}_2}$ and $\epsilon_{2,\min} = 38\ \mathrm{kg}_{\mathrm{CO}_2}$
   - $\epsilon_{2,l} = \epsilon_{2,\min} + \left(\epsilon_{2,\max} - \epsilon_{2,\min}\right) l$ and $0 \leq l \leq 1$
   - Formulate and solve the optimization problem:

$$\min f_1|_{f_2 \leq \epsilon_{2,l}} = \text{CHF } 52$$

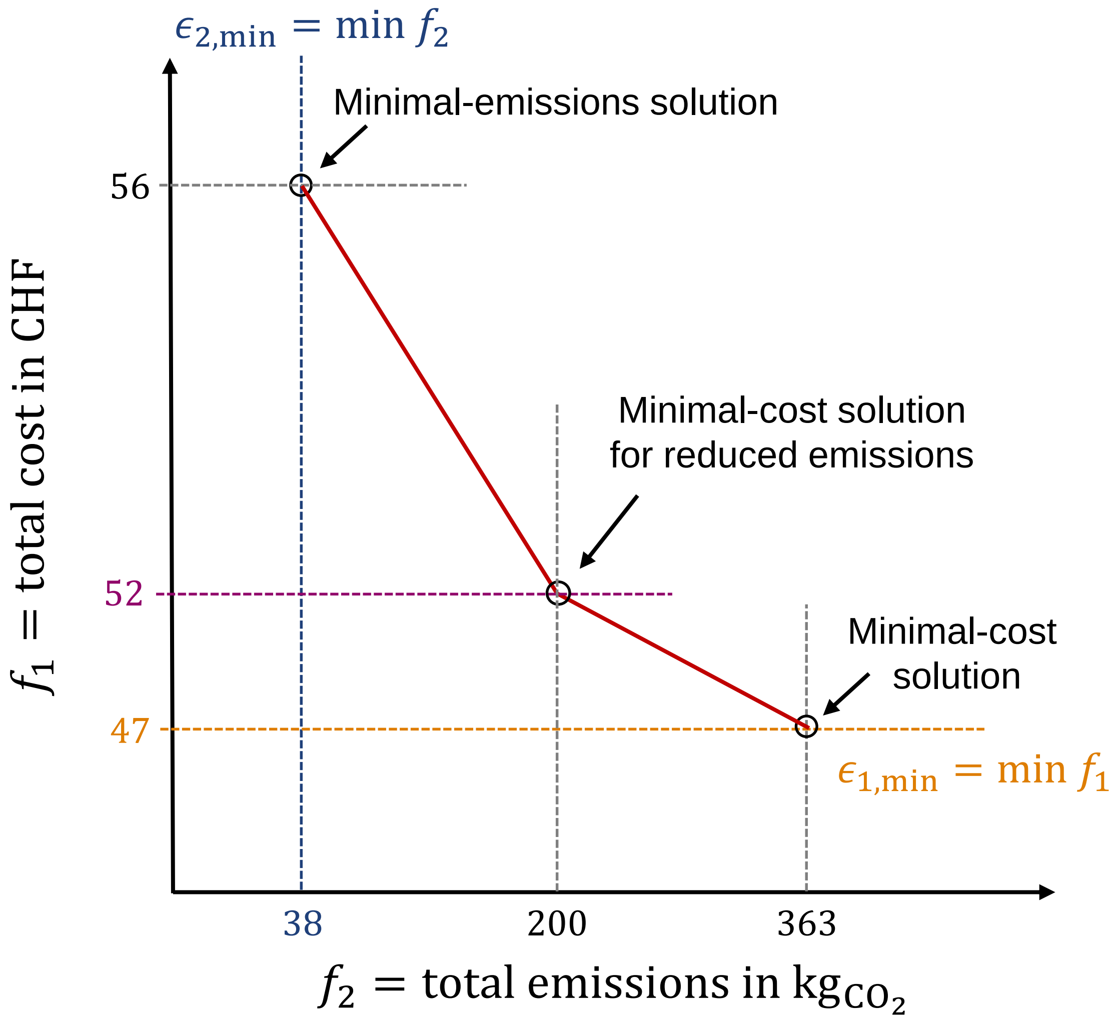


Numerical-example Multi-objective optimization: Summary

Determine the Pareto Optimal Front

2. varying the values of the ϵ-constraint for $f_2 =$ total emissions from its maximum ($\epsilon_{2,\max}$) to its minimum value ($\epsilon_{2,\min}$)
   - $\epsilon_{2,\max} = 363\ \mathrm{kg}_{CO_2}$ and $\epsilon_{2,\min} = 38\ \mathrm{kg}_{CO_2}$
   - $\epsilon_{2,l} = \epsilon_{2,\min} + \left(\epsilon_{2,\max} - \epsilon_{2,\min}\right) l$ and $0 \leq l \leq 1$
   - Formulate and solve the optimization problem:

$$\min f_1|_{f_2 \leq \epsilon_{2,l}} = \mathrm{CHF}\ 52$$



After this lecture, you are able to …

- ✓ Define and critically evaluate security KPIs for energy systems:
  - ✓ Reliability indicators
  - ✓ Risk indicators
  - ✓ Resilience indicators
- ✓ Use different KPIs for the formulation of multi-objective optimization problems