ETH zürich

RRE
Reliability and Risk Engineering

Introduction to Modeling and Optimization of Sustainable Energy Systems:

Uncertainty in energy systems

Prof. Dr. Giovanni Sansavini

Reliability and Risk Engineering

Our path towards the exam

No.	Date	Content	
1-8	...	...	Methods/Metrics/Applications
9	24.11.	Multi-energy dimension: introduction	Methods & Applications
10	01.12.	Design dimensions: technology modeling	
11	08.12.	Space dimensions: energy networks	
12	15.12.	Uncertainty in energy systems	
13	22.12.	Open office hour with TAs	Zoom
	25.12. – 31.12.	Mockup exam is released on Moodle	Individual work
	20.01. 13:15 - 14:45	Recap lecture and discussion of mockup exam	Zoom
	27.01. 13:15 - 14:45	Open office hour with TAs	Zoom
	04.02. 09:00 - 11:00	Final Exam	tba

- Open office hours (Questions? Suggestions? → moses-edu@ethz.ch)
  - Breakout rooms, students ask questions about the exercises/lectures
  - Please, come early or let us know if you will be late, as we may close once questions are over

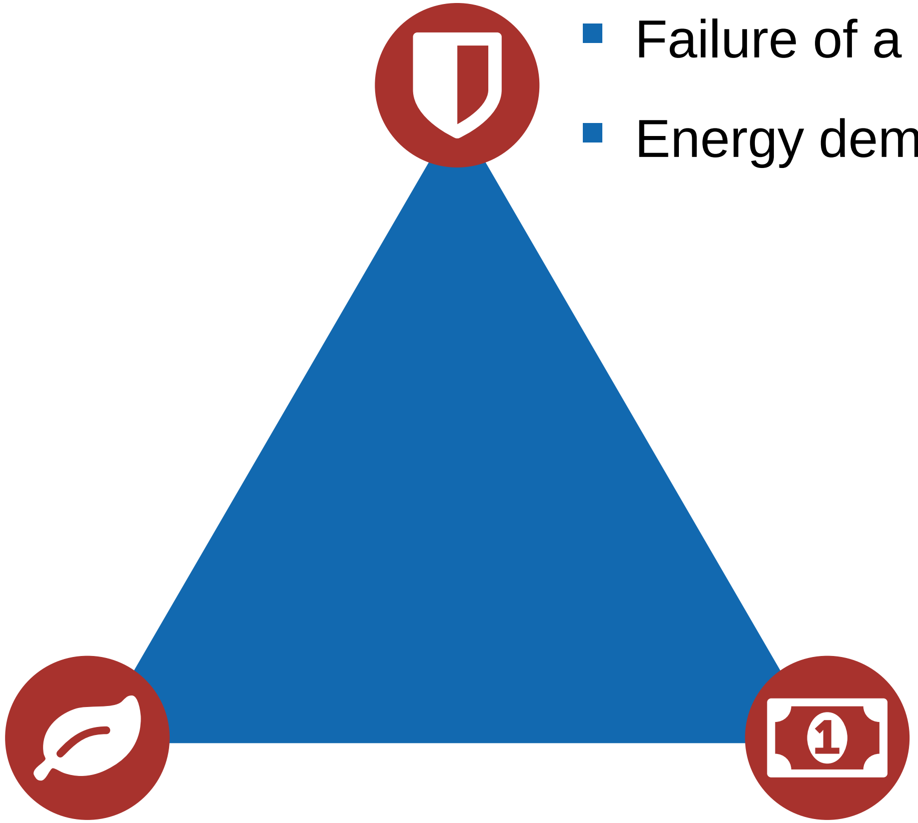
Since the last lecture, you are able to …

- ✓ Describe the energy network modeling process
- ✓ Define a generic energy network with
  - ✓ Graphs, node and link quantities
  - ✓ Decision variables and constraints
- ✓ Solve network equations with backward-forward sweep
- ✓ Describe the process of linearizing physical laws
- ✓ Define network optimization models for gas, electricity and thermal networks
- ✓ Model non-unique flow directions and topology changes

After this lecture, you are able to …

- Understand how uncertainty affects energy systems, their modeling and optimization
- Describe the uncertainty occurring within modeling and optimization of energy systems:
  - Aleatory and epistemic uncertainty
  - Probability distributions and uncertainty ranges
- Apply methods for analyzing uncertainty within modeling and optimization of energy systems:
  - Sensitivity analysis: local and global approaches
  - Uncertainty analysis: Monte Carlo simulation
- Interpret the results and making decisions under uncertainty:
  - Monte Carlo simulation
  - Stochastic optimization
  - Robust optimization

Uncertainty affects the entire energy trilemma



Security

- Failure of a component/connection
- Energy demand not supplied

Sustainability

- Uncertain carbon footprints
- Uncertain renewable generation

Equity

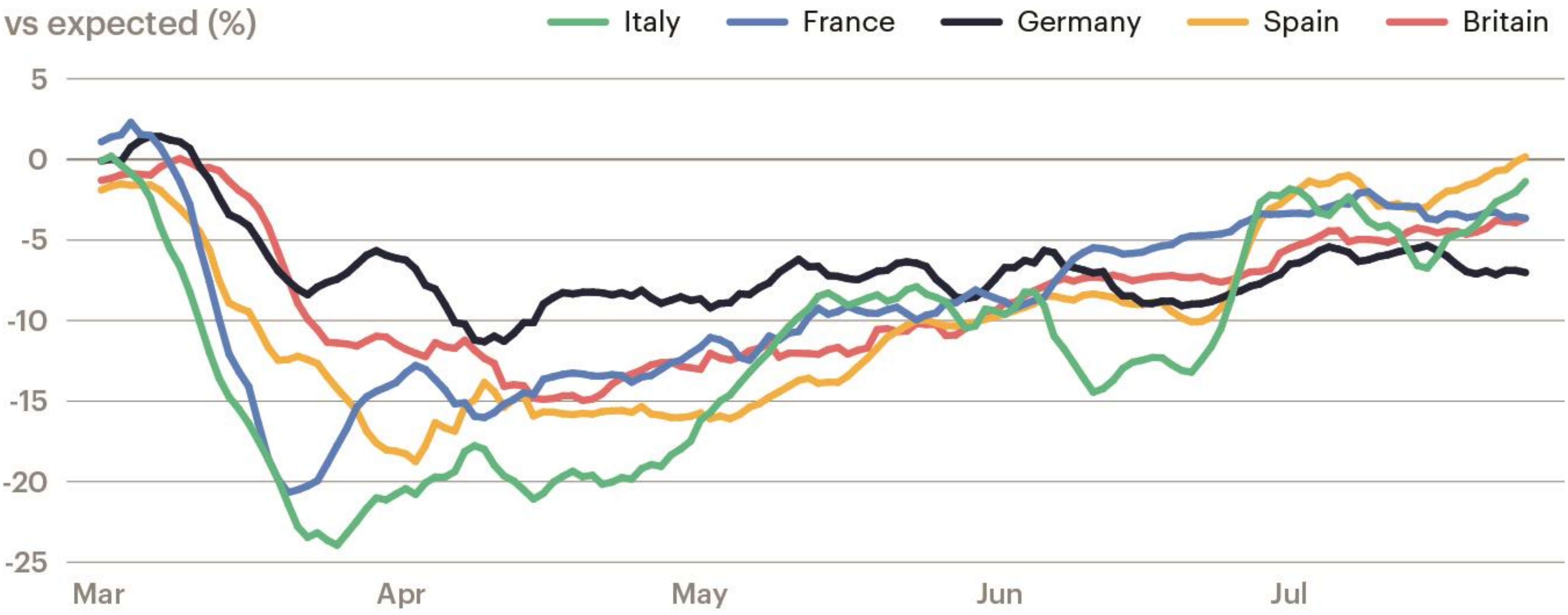
- Increased financial risk
- Uncertain energy prices
- Uncertain energy demands

Uncertainty affects energy systems

Identifying the uncertainty

Real-world example: unexpected variations of electricity demand

Impact of COVID-19 virus on European electricity demand



Source: Independent Commodity Intelligent System (ICIS)
https://www.icis.com/explore/resources/news/2020/03/19/10482507/topic-page-coronavirus-impact-on-energy-markets

Real-world example: unexpected interruption of energy supply


GRIDRAD

Home ▸ News und Blog

Current events in the European electr

Click to enlarge

From time to time major disturbances of national impo unnoticed by regular consumers. Such disturbances a disturbances, net time deviations to various other fau what we capture with our Wide Area Monitoring Syst monitoring system is independently run, so no trans is to fully capture power flows and frequency events, phase a to make them visible to non-experts.


Tages-Anzeiger

Front Zürich Schweiz International **Wirtschaft** Börse Sport Kultur Reisen Wissen Auto Blogs Panorama Mehr

Unternehmen Konjunktur Geld Karriere Recht & Konsum Rechtsfragen Vorbörse Weiterbildung World Economic Forum Bildstrecken Paradise Papers

Switzerland came this close to a blackout

Lights would've gone dark in Switzerland on Monday if it hadn't been for an emergency procedure. This could happen more often in the future.

Dreieinhalb Stunden musste Swissgrid bangen, ehe das Problem gelöst war. Foto: Keystone

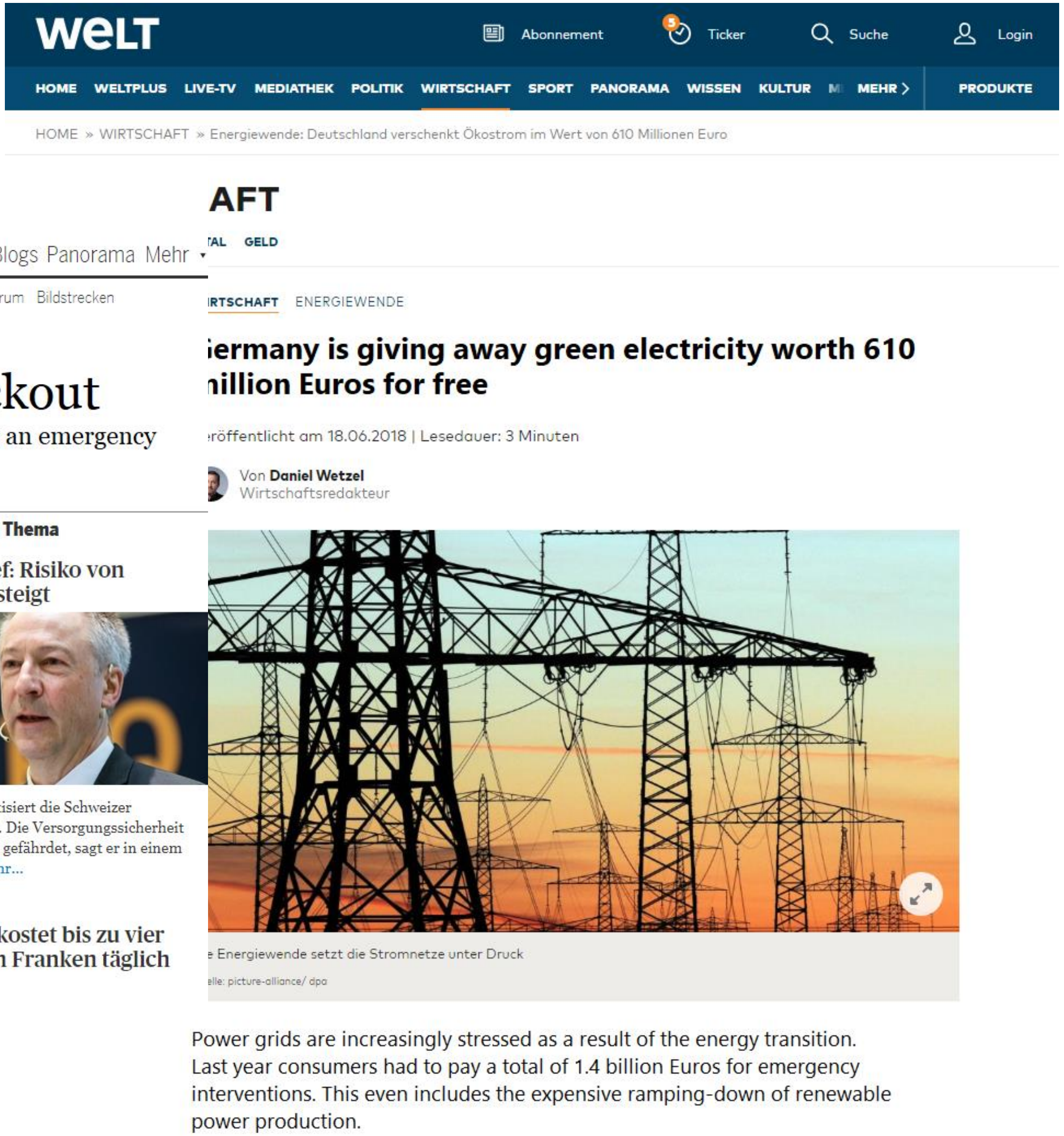
Artikel zum Thema

Alpiq-Chef: Risiko von Blackout steigt

Jens Alder kritisiert die Schweizer Energiepolitik. Die Versorgungssicherheit des Landes sei gefährdet, sagt er in einem Interview. Mehr...

25.03.2019

Blackout kostet bis zu vier Milliarden Franken täglich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HOME » WIRTSCHAFT » Energiewende: Deutschland verschenkt Ökostrom im Wert von 610 Millionen Euro

AFT

TAL GELD

IRTSCHAFT ENERGIEWENDE

iermany is giving away green electricity worth 610 nillion Euros for free

eröffentlicht am 18.06.2018 | Lesedauer: 3 Minuten

Von **Daniel Wetzel**
Wirtschaftsredakteur

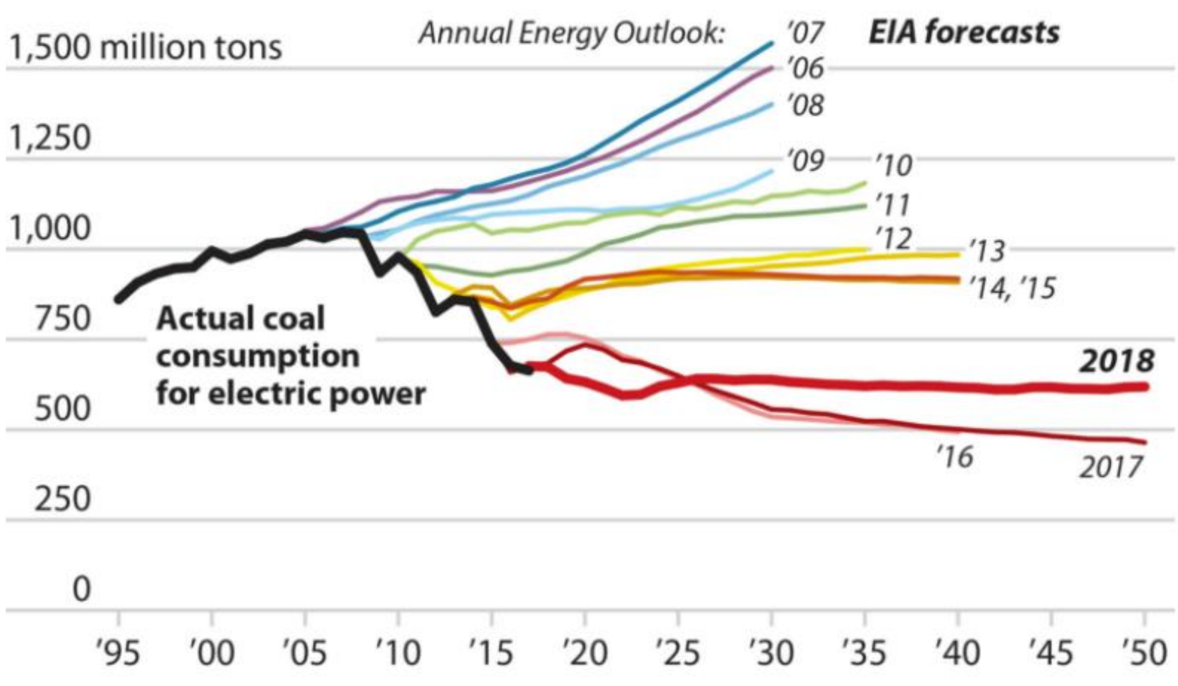
e Energiewende setzt die Stromnetze unter Druck

elle: picture-alliance/ dpa

Power grids are increasingly stressed as a result of the energy transition. Last year consumers had to pay a total of 1.4 billion Euros for emergency interventions. This even includes the expensive ramping-down of renewable power production.

Energy forecasts: Often associated with significant uncertainty

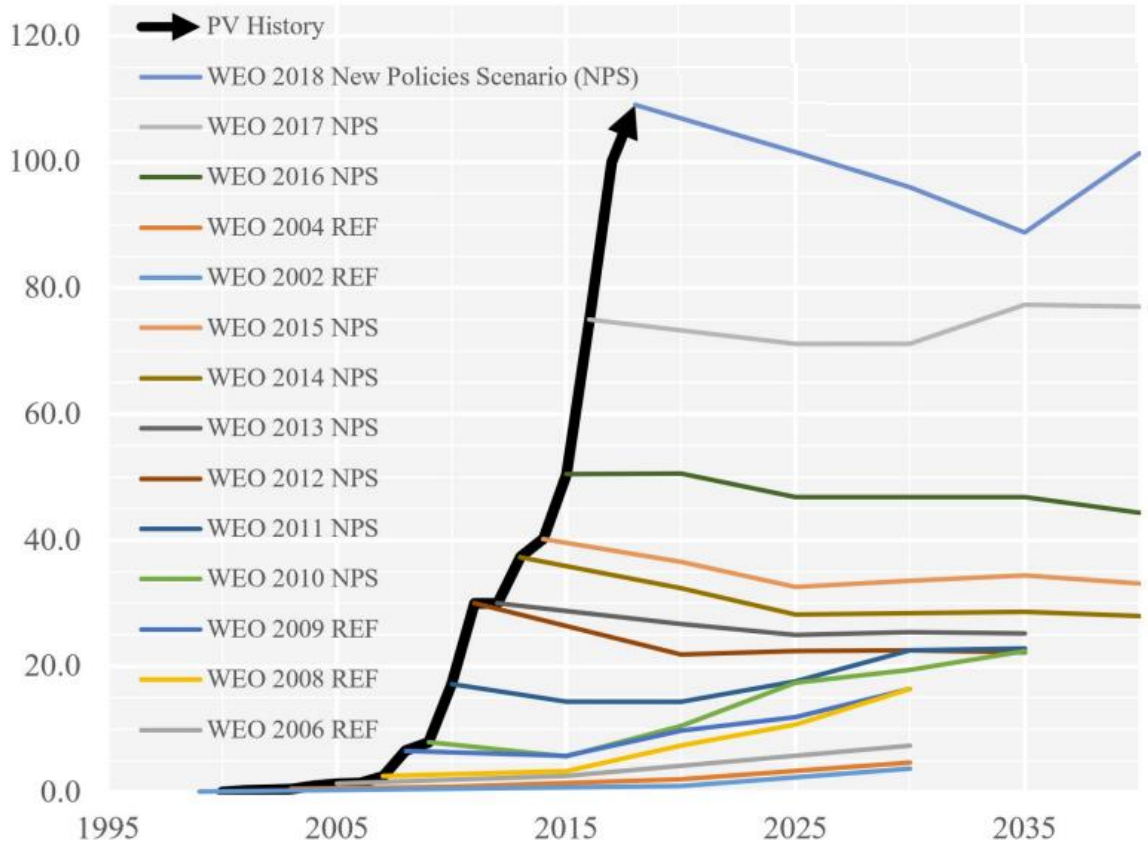
EIA coal consumption forecasts 2006-2018



Source: Energy Information Administration (EIA), 2019

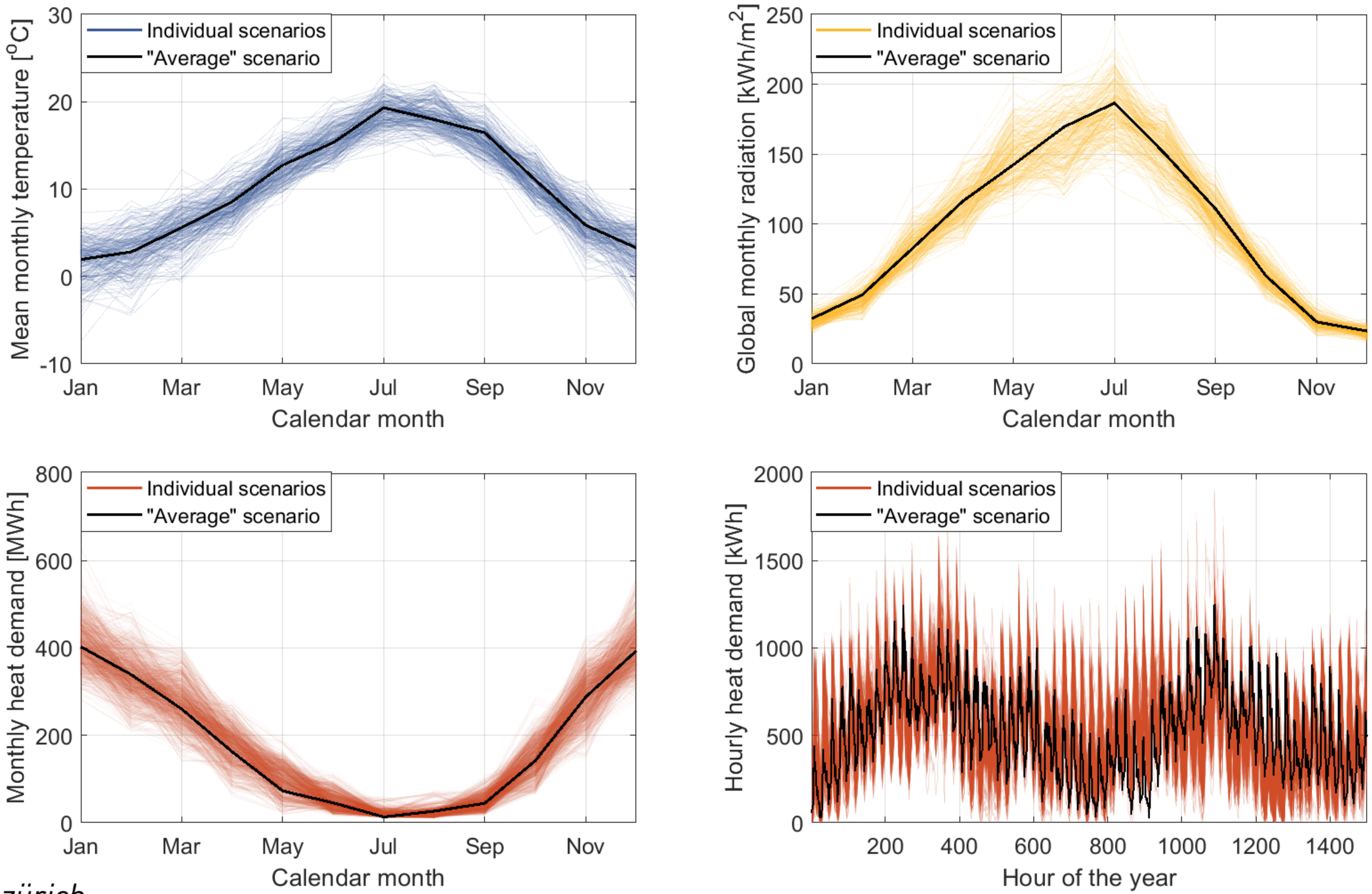
Each year, international institutions such as EIA and IEA are releasing long-term forecasts of several energy quantities. However, such forecasts are often quite inaccurate even in the short term.

IEA annual PV addition forecasts 2006-2018



Source: International Energy Agency (IEA), 2019

Input data for energy system modeling: “Natural” variability



Weather conditions (monthly average) and energy demands for a MES in Zurich.

Source: P. Gabrielli et al., Robust and optimal design of multi-energy systems with seasonal storage through uncertainty analysis, *Applied Energy*, 2019, **238**, 1192-1210

Causes and relevance of uncertainty

- The uncertainty associated with the modeling and optimization of sustainable energy systems stems from various sources, such as:
  - Natural variability and volatility of the quantities of interest (e.g. weather conditions, energy prices)
  - Random events (e.g. device failures and natural hazards resulting in damages/blackouts)
  - Human behavior, which affects energy demands and system management
  - Unknown future developments, e.g. future technology costs and deployment, interest rates
  - Model approximations due to impossibility of capturing and representing interdependencies among energy sectors and physical interactions among technical components

- Such uncertainty significantly affects the optimal design and operation of energy systems (and MES) and must be taken into account when formulating and solving the optimization problem

Uncertain quantities for modeling energy systems (partial)

Relevant input parameters		
Weather conditions		
Energy demands		
Energy prices		
Grid energy mix		
Technology carbon footprint		
Technology cost		
Technology performance		
Component failure rate		

After this lecture, you are able to …

✓ Understand how uncertainty affects energy systems, their modeling and optimization

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Describing the uncertainty

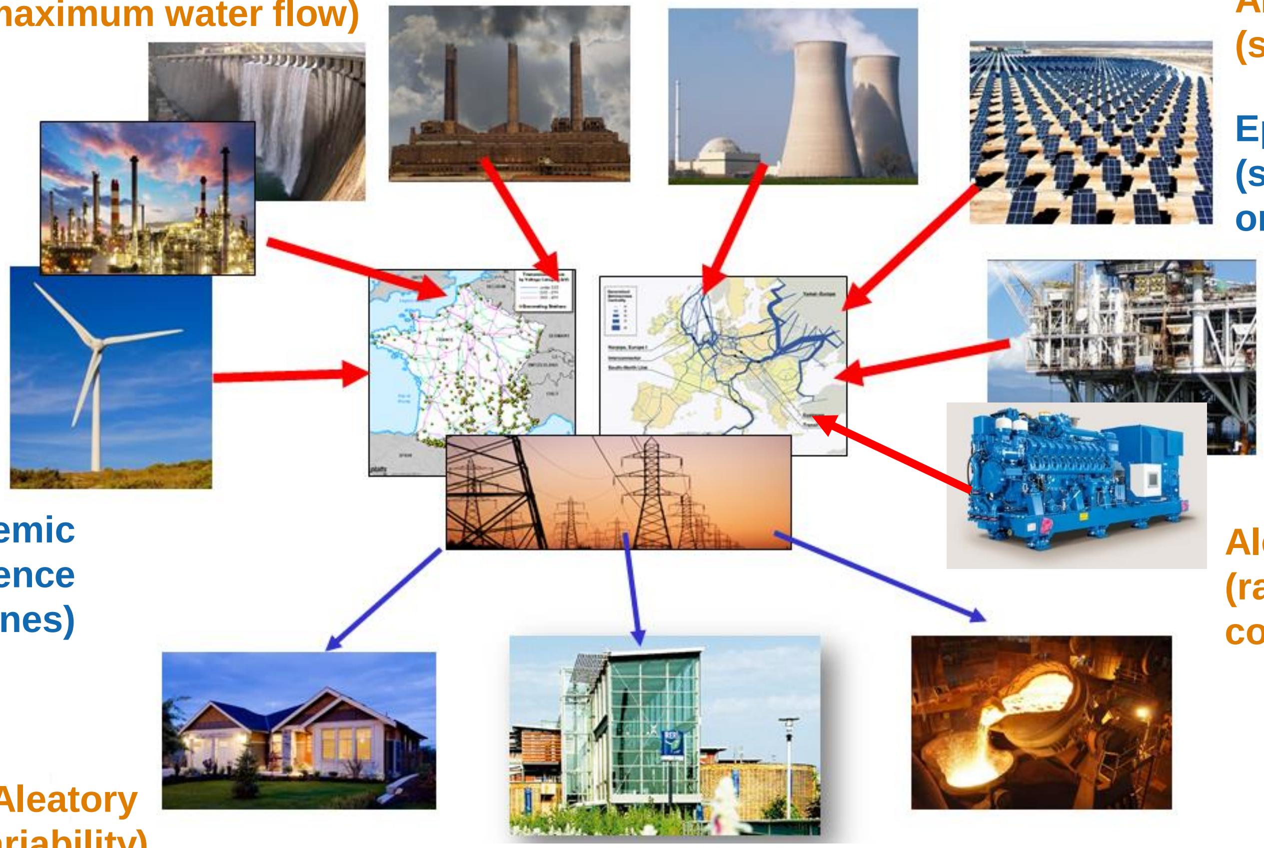
Types of uncertainty: Aleatory and epistemic

- **Aleatory variability** is the inherent randomness in the system behavior
  - For discrete parameters, randomness is parameterized by the probability of each possible value
  - For continuous parameters, randomness is parameterized by the probability density function
- **Epistemic uncertainty** is the scientific uncertainty in the model of the system, and it is due to lack of data and knowledge
  - For discrete parameters, epistemic uncertainty can be modelled by ranges or alternative probability distributions
  - For continuous parameters, epistemic uncertainty can be modelled by ranges or alternative probability density functions
  - **Model approximations** when actual behavior (e.g. nonlinear performance of a technology) cannot be modeled

In this class, we do not deal with model approximation, i.e. we are confident that our model describes well the reality

Types of uncertainty: Aleatory and epistemic in energy systems

ALEATORY (Randomness) Example: renewable generation, mechanical failure, …	**EPISTEMIC** (Imprecision / lack of knowledge) Example: failure rate, conversion efficiency, …



Uncertain quantities for modeling energy systems (partial)

Relevant input parameters	Type of uncertainty	
Weather conditions	Aleatory	
Energy demands	Aleatory	
Energy prices	Aleatory	
Grid energy mix	Aleatory/Epistemic	
Technology carbon footprint	Epistemic	
Technology cost	Epistemic	
Technology performance	Epistemic	
Component failure rate	Aleatory/Epistemic	

Types of uncertainty: Probability distributions and intervals

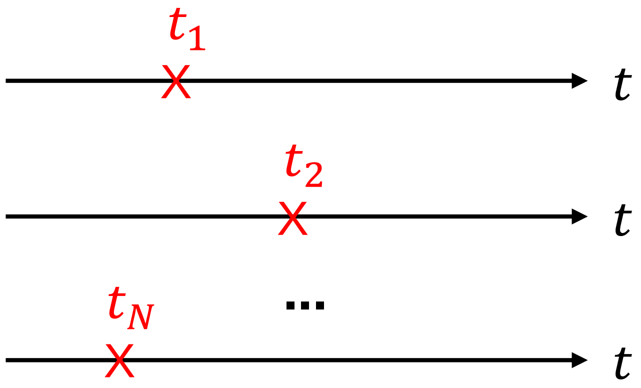
ALEATORY

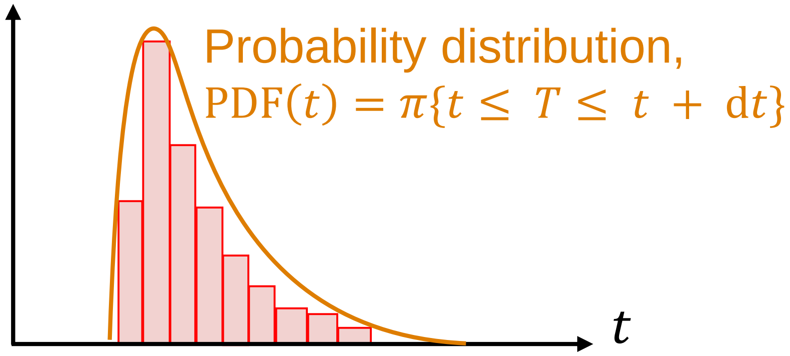
(Randomness)

Described via precise probability distributions

Random events
(e.g. mechanical failure) → Random parameter
(e.g. time to failure, t)

Well-known CHP technology





Types of uncertainty: Probability distributions and intervals

ALEATORY (Randomness) Described via precise probability distributions	**EPISTEMIC** (Imprecision / lack of knowledge) Described via ranges

Random events (e.g. mechanical failure) → Random parameter (e.g. time to failure, t)

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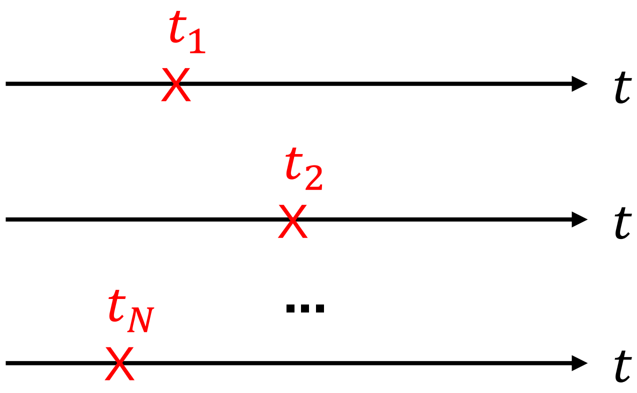
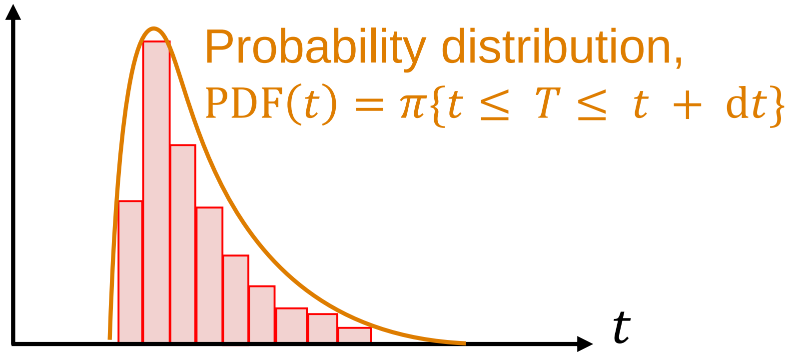
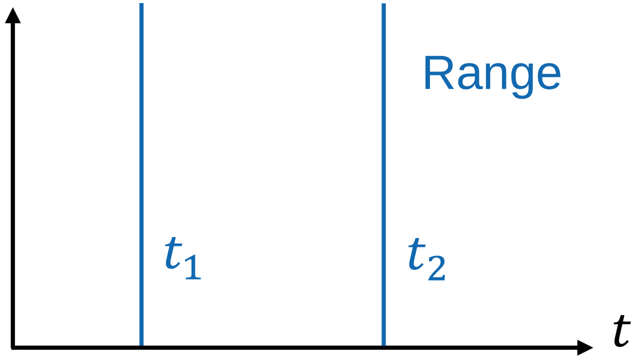




Random events (e.g. mechanical failure) → Random parameter (e.g. time to failure, t)

Poorly-known wind technology

- Scarce data availability
- Qualitative data, e.g. expert judgment



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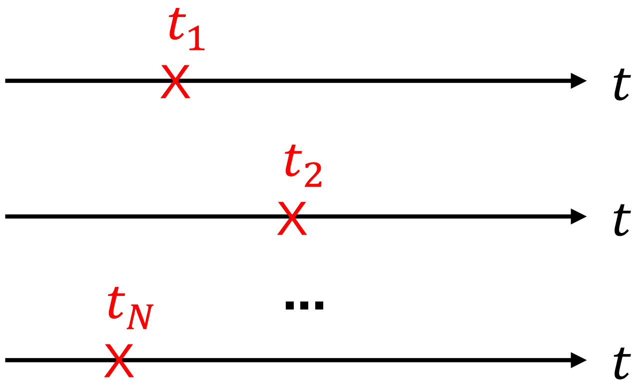

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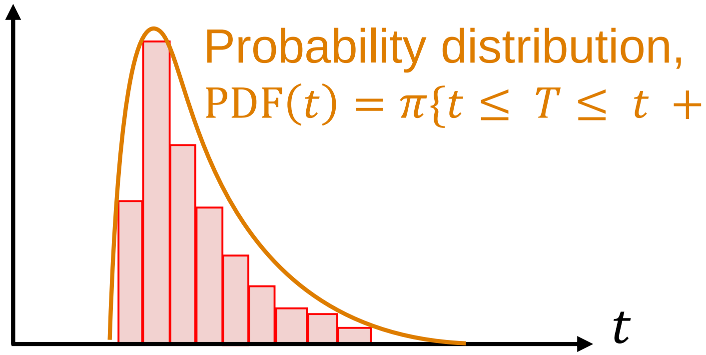
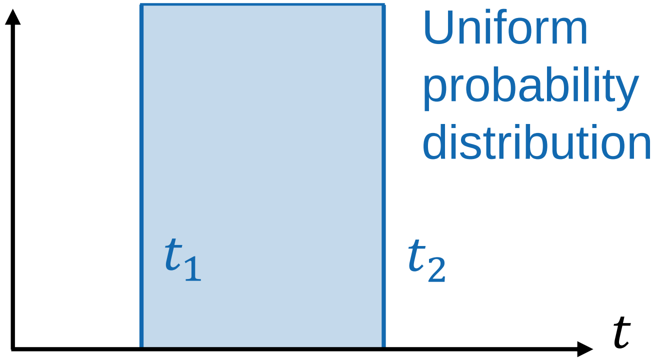


Principle of Insufficient reason (Laplace): if there is no reason to value one probability distribution over another, a uniform distribution must be used



S. M. Stigler, *The history of statistics: the measurement of uncertainty before 1900*, Harvard University Press (1986)

Types of uncertainty: Probability distributions and intervals

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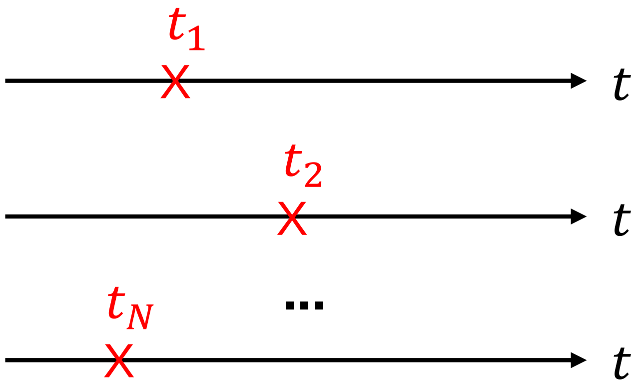

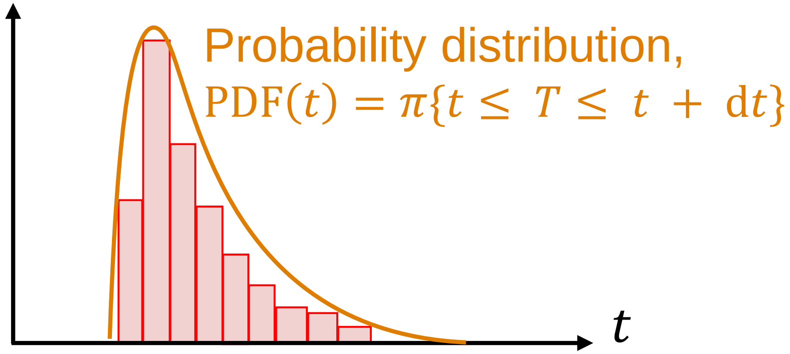

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Poorly-known wind technology

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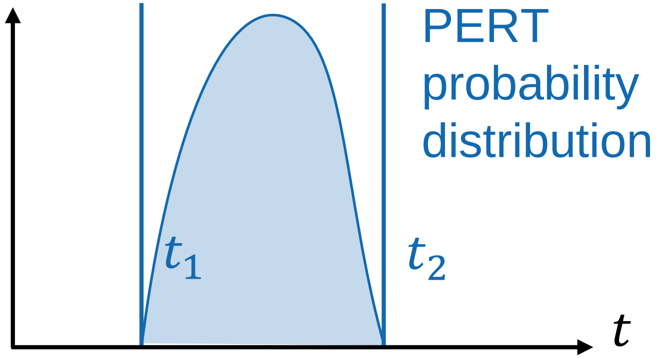
PERT distribution requires minimum, maximum and most likely values, and it is often recommended to model qualitative data



D. Vose, *Risk analysis: A quantitative guide*, John Wiley & Sons (2008)

Types of uncertainty: Probability distributions and intervals

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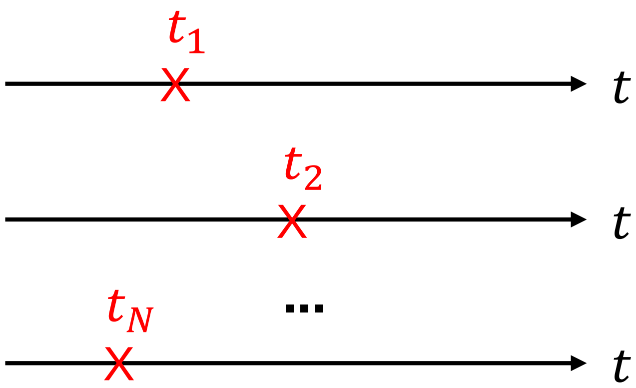
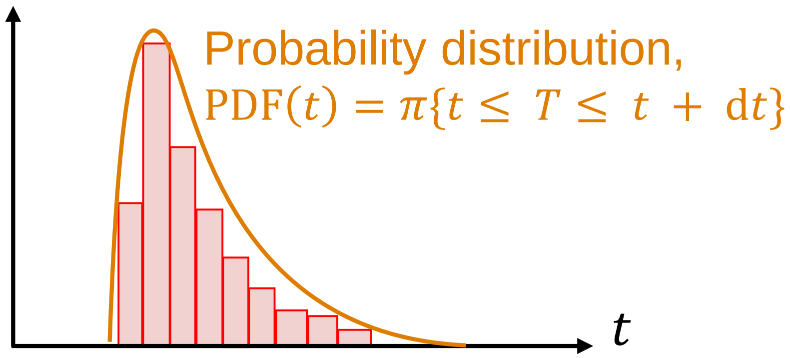


Poorly-known wind technology

- Scarce data availability
- Qualitative data, e.g. expert judgment

For both aleatory and epistemic uncertainty, the combination of several effects is often addressed via scenarios



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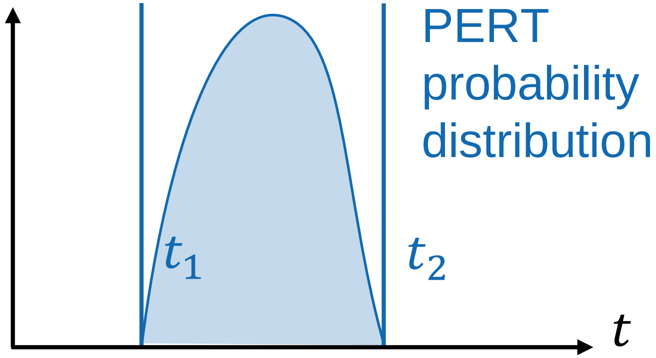

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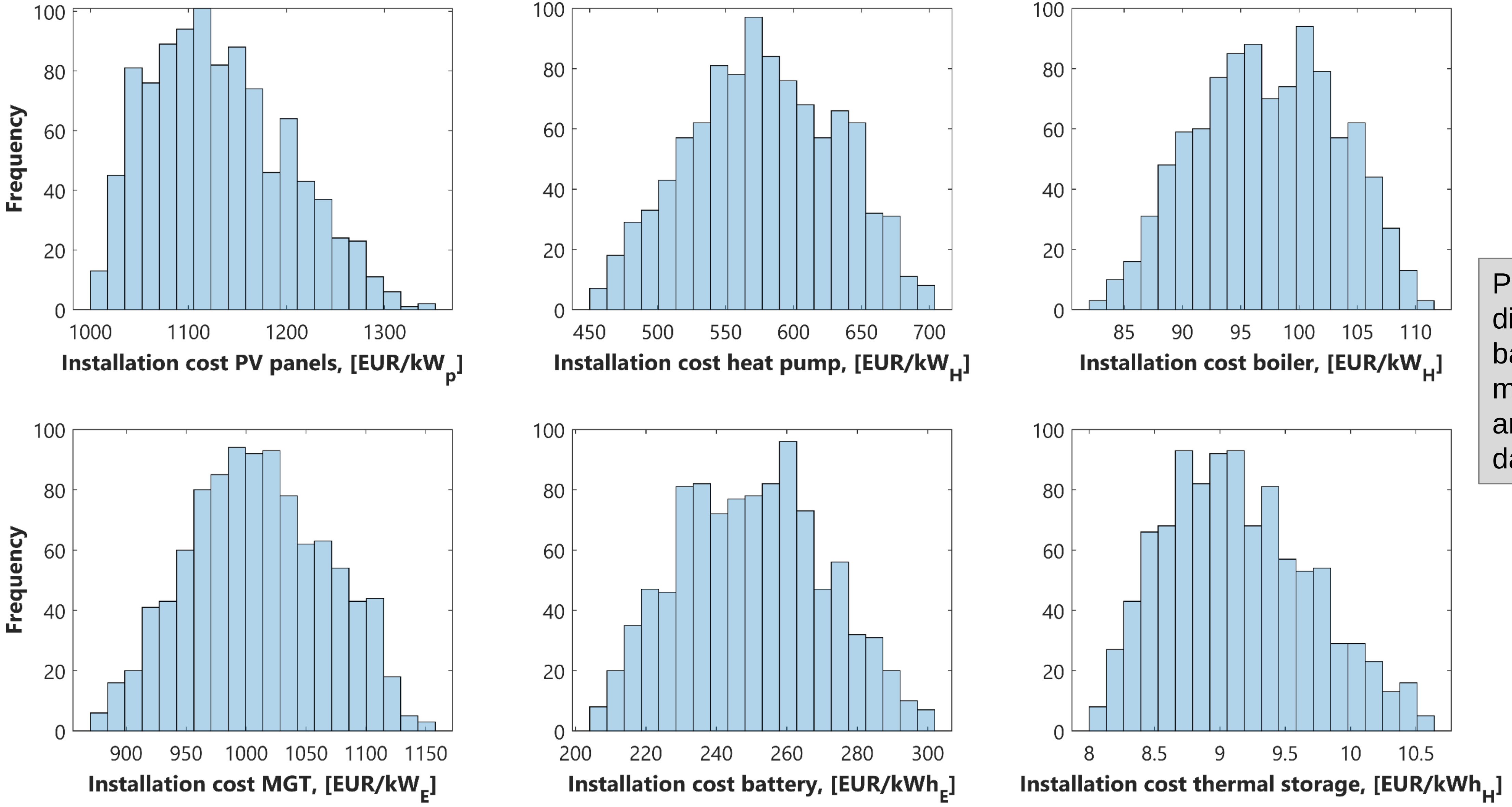
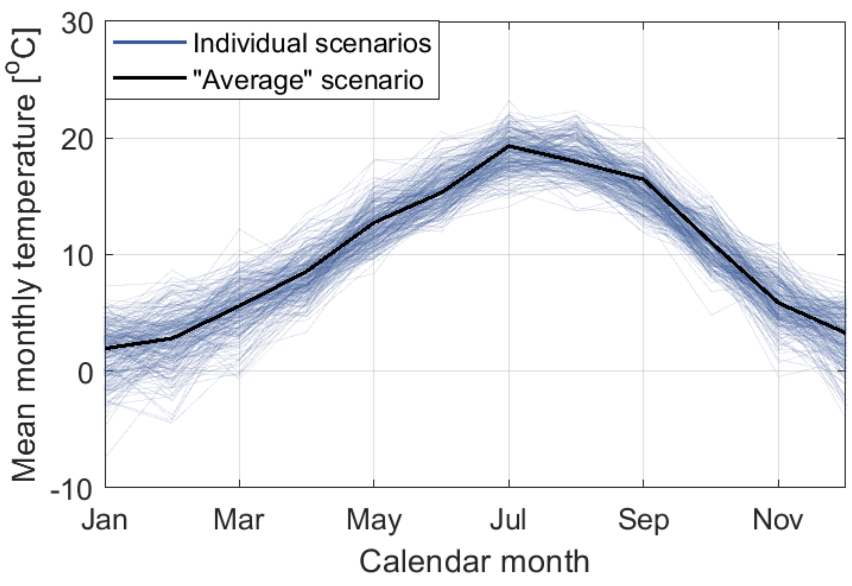
Uncertain quantities for modeling energy systems: Examples (1)

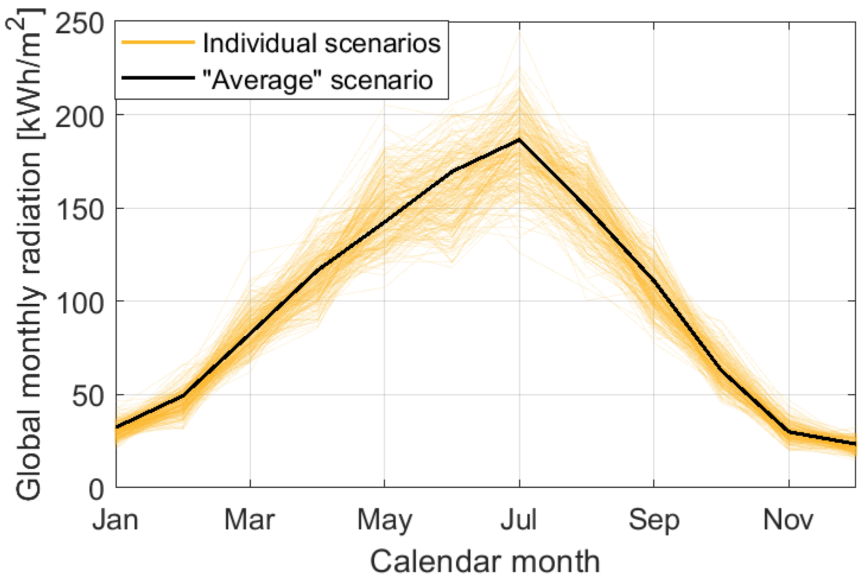


PERT distributions based on manufacturer and literature data

E. Petkov, P. Gabrielli, *Applied Energy* (2020), **274**, 115197

Uncertain quantities for modeling energy systems: Examples (2)



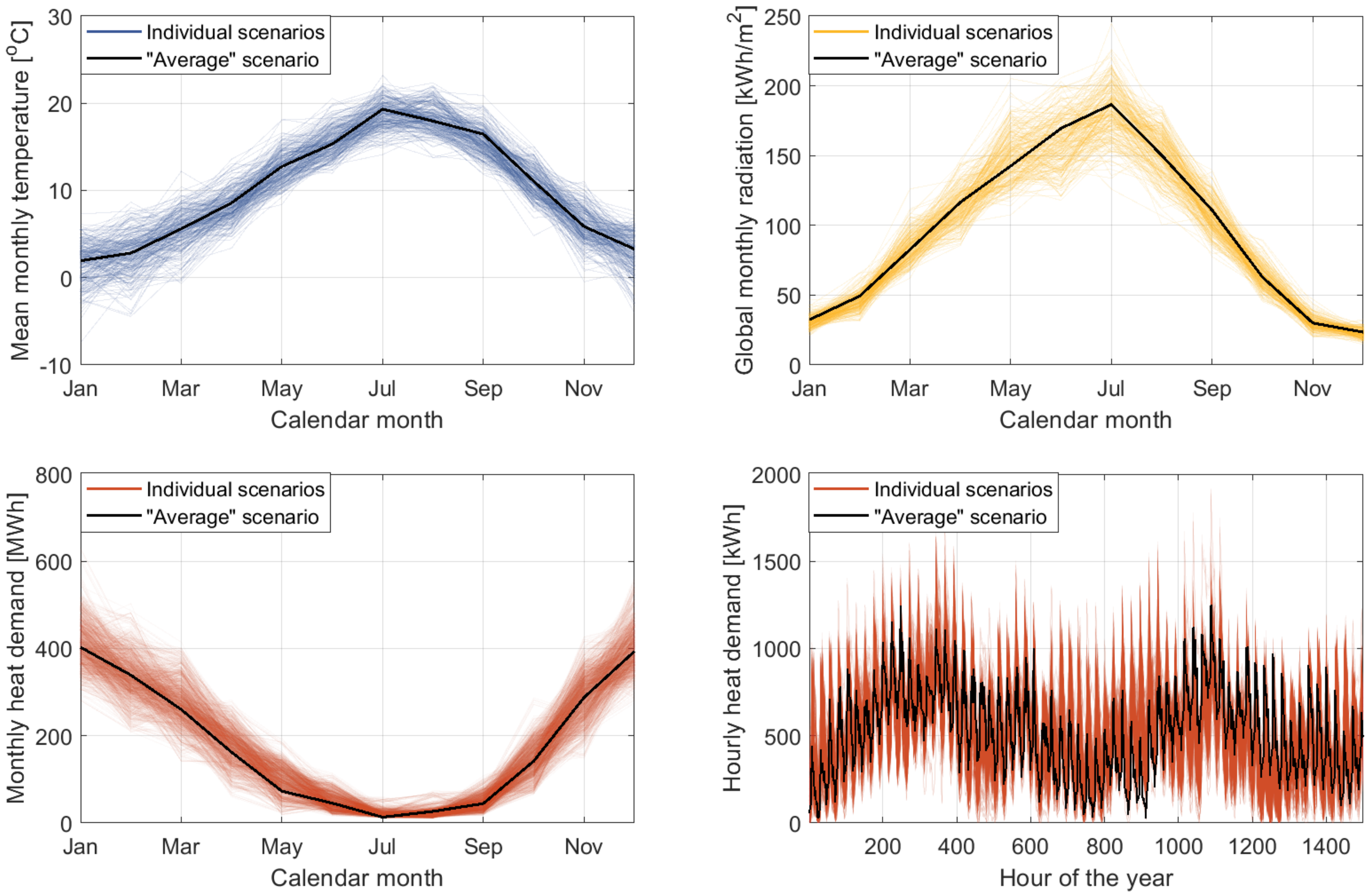


Scenarios of CO_2 concentration in atmosphere (human-caused), adding epistemic model uncertainty

Probability distributions embedded in climate models (e.g. normal or Rayleigh) to model aleatory weather uncertainty

Future weather scenarios

Uncertain quantities for modeling energy systems: Examples (3)



Epistemic uncertainty connected to building properties (e.g. materials, infiltration)

Probability distributions embedded in building models (e.g. normal or triangular distributions) to model aleatory occupancy and demand uncertainty

Future demand scenarios

Uncertain quantities for modeling energy systems (complete)

<table>
<tr><th>Relevant input parameters</th><th>Type of uncertainty</th><th>Quantitative modeling</th></tr>
<tr><td>Weather conditions</td><td>Aleatory</td><td rowspan="8">Definition of the probability distribution and its support are dependent on the specific system and modelling approach, e.g.
• Wind speed ~ Rayleigh distribution
• Failure rate ~ log-logistic distribution
• Energy demand ~ Gaussian or triangular distributions</td></tr>
<tr><td>Energy demands</td><td>Aleatory</td></tr>
<tr><td>Energy prices</td><td>Aleatory</td></tr>
<tr><td>Grid energy mix</td><td>Aleatory/Epistemic</td></tr>
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</table>

After this lecture, you are able to …

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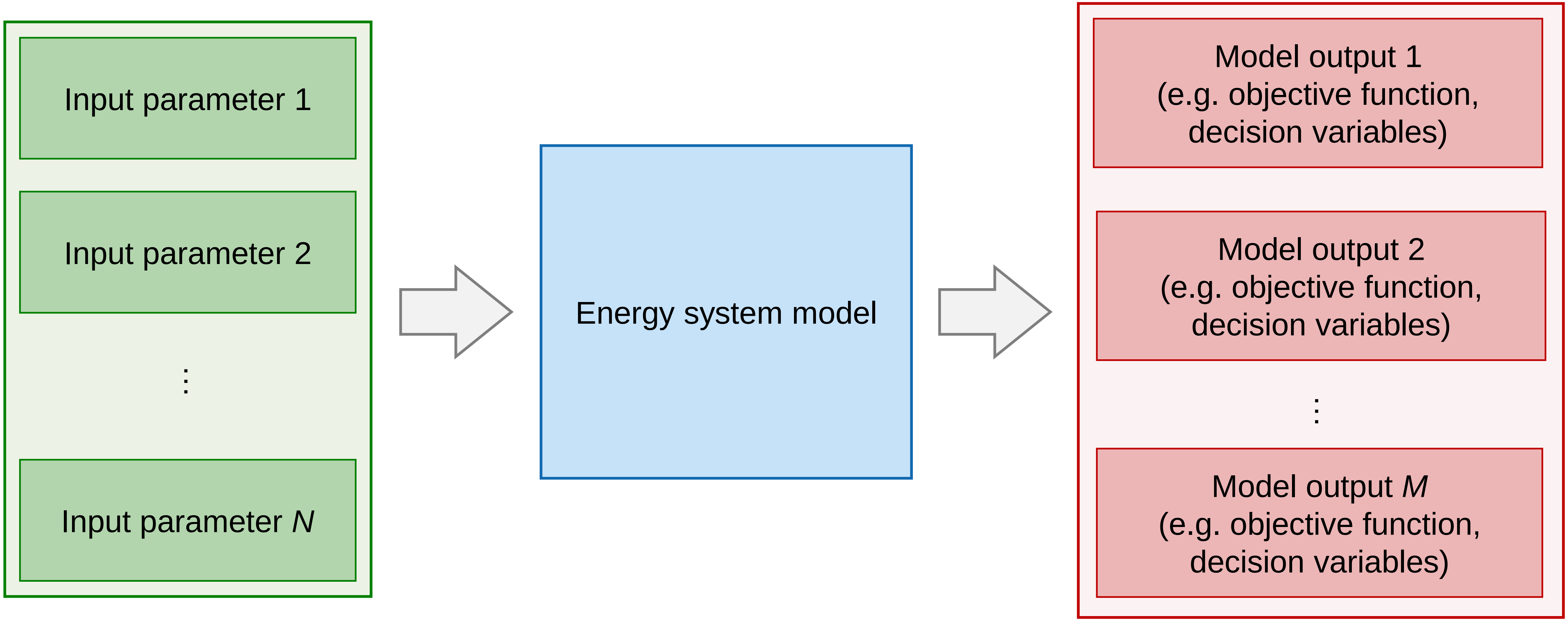
Quantifying uncertainty via sensitivity and uncertainty analysis

Sensitivity and uncertainty analysis: Basic idea

Scope: Study how the uncertainty in model output can be attributed to and explained by different sources of uncertainty in the model input (input parameters)



Sensitivity and uncertainty analysis: Basic questions

1. What factors contribute most / least to the output uncertainty (e.g. variation of system costs)

   ⇨ *Factor prioritization / Factor fixing*

2. What factors lead to a certain model output (e.g. installation of a technology of interest)

   ⇨ *Factor mapping*

3. How to reduce the uncertainty (the variance) of the model output (e.g. limit system costs at a certain probabilistic threshold)?

   ⇨ *Variance cutting*

Sensitivity and uncertainty analysis: Basic steps

Scope: Study how the uncertainty in model output can be attributed to and explained by different sources of uncertainty in the model input (input parameters)

In general, any sensitivity analysis consists of four basic steps:

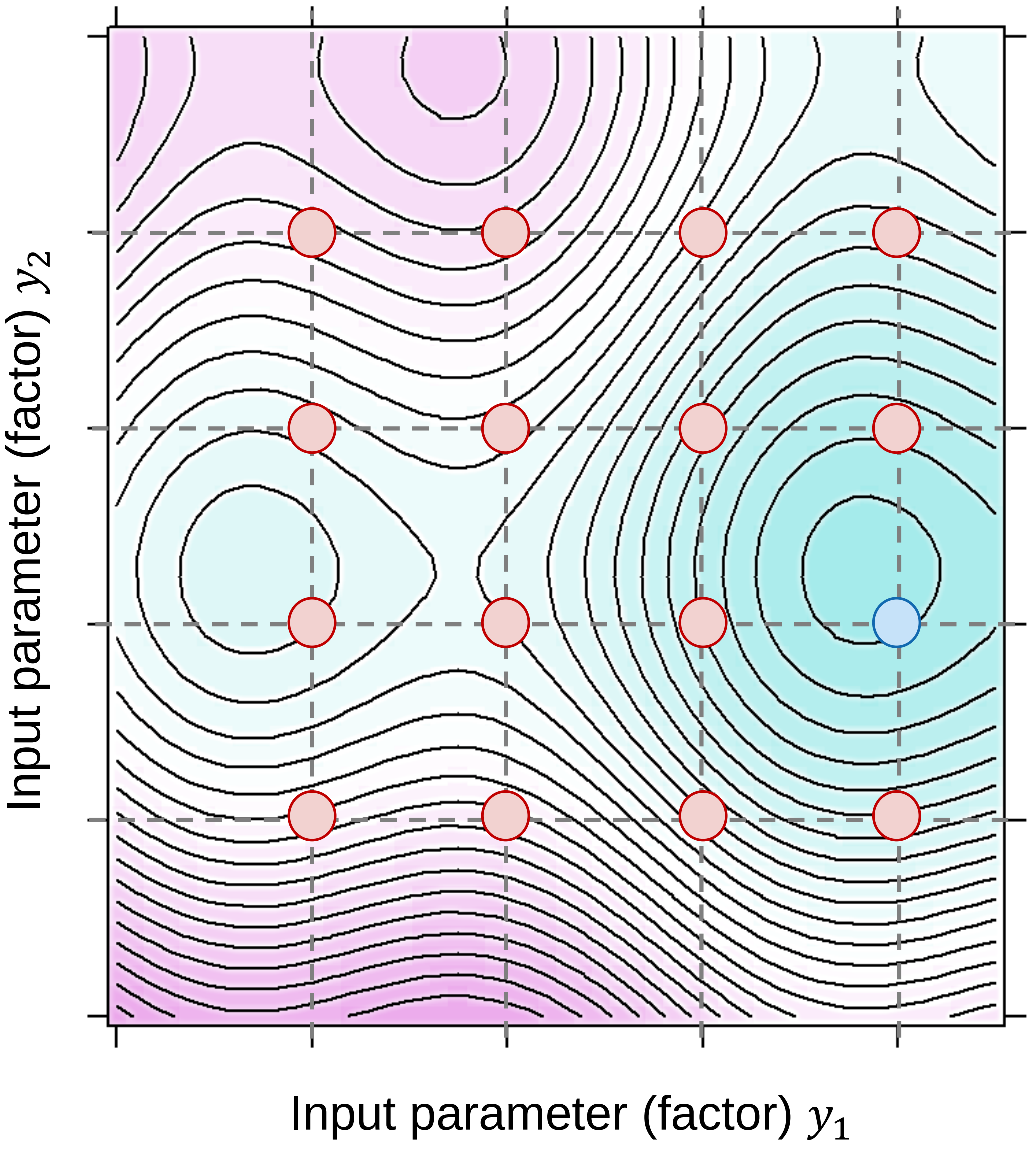
1. Sampling the space of the parameters (so-called *factor space*)
2. Defining and calculating an output-based metric (e.g. variation in objective functions vs. variation in decision variables)
3. Defining and adopting a sensitivity approach (local or global sensitivity analyses vs. uncertainty analysis)
4. Visualizing and interpreting the results

Sensitivity analysis: Local vs. global approaches

- Local sensitivity analysis analyzes sensitivity around some (often optimal) point in the factor space

- Global sensitivity analysis analyzes variability across the full factor space

Contour plot of objective function, z



Local sensitivity analysis

- Local sensitivity analysis analyzes sensitivity around a reference point in the factor space, $\boldsymbol{y^0} = (y_1^0, y_2^0, \dots, y_N^0)$
- All parameters are varied one-at-a-time, with small variations around $\boldsymbol{y^0}$, and with all other parameters at their reference value

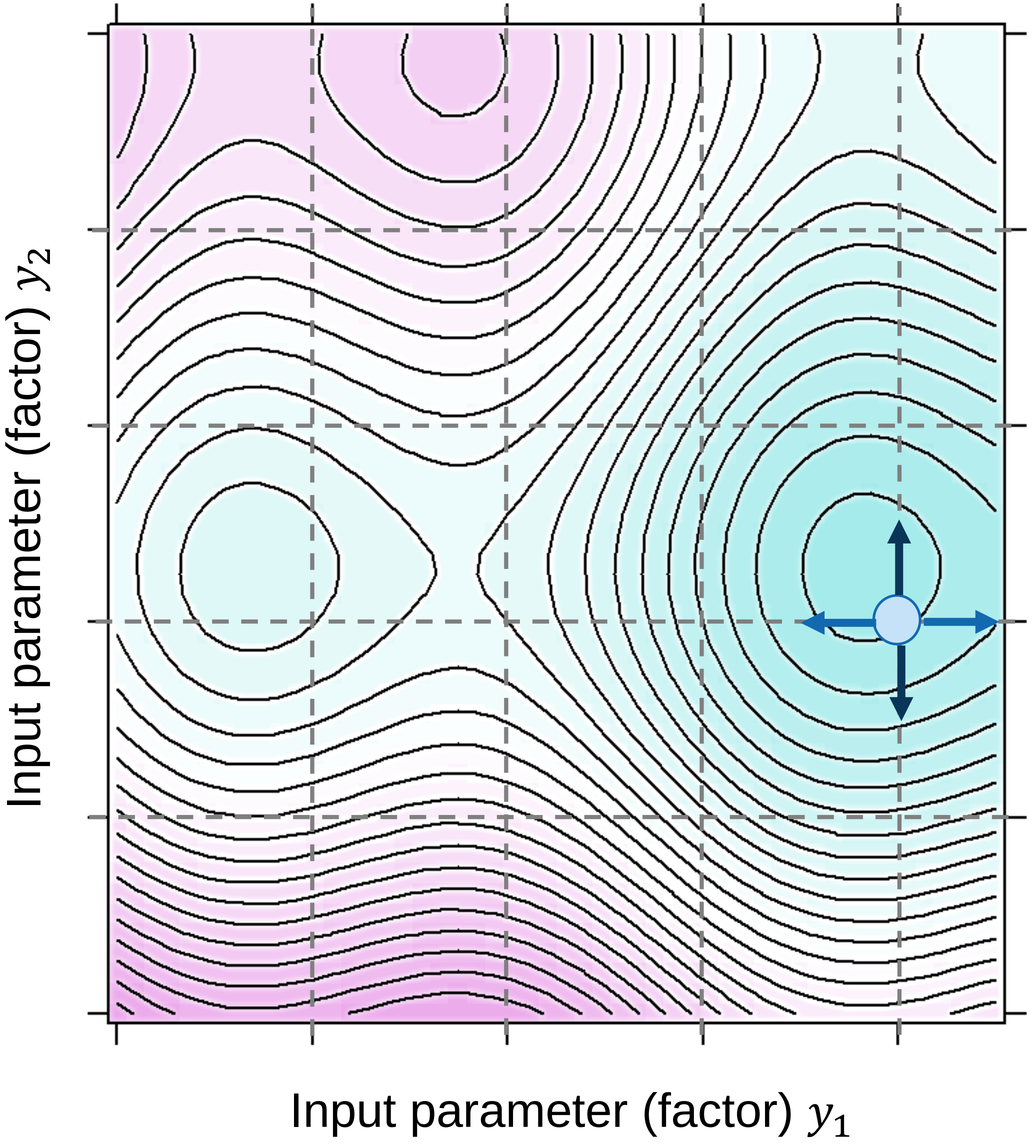
$$\left(\frac{\partial z}{\partial y_i}\right)_{y_{j\neq i}^0}, \forall i,j \in \{1, \dots, N\}$$

- Example (figure):

$$\left(\frac{\partial z}{\partial y_1}\right)_{y_2^0}$$

$$\left(\frac{\partial z}{\partial y_2}\right)_{y_1^0}$$

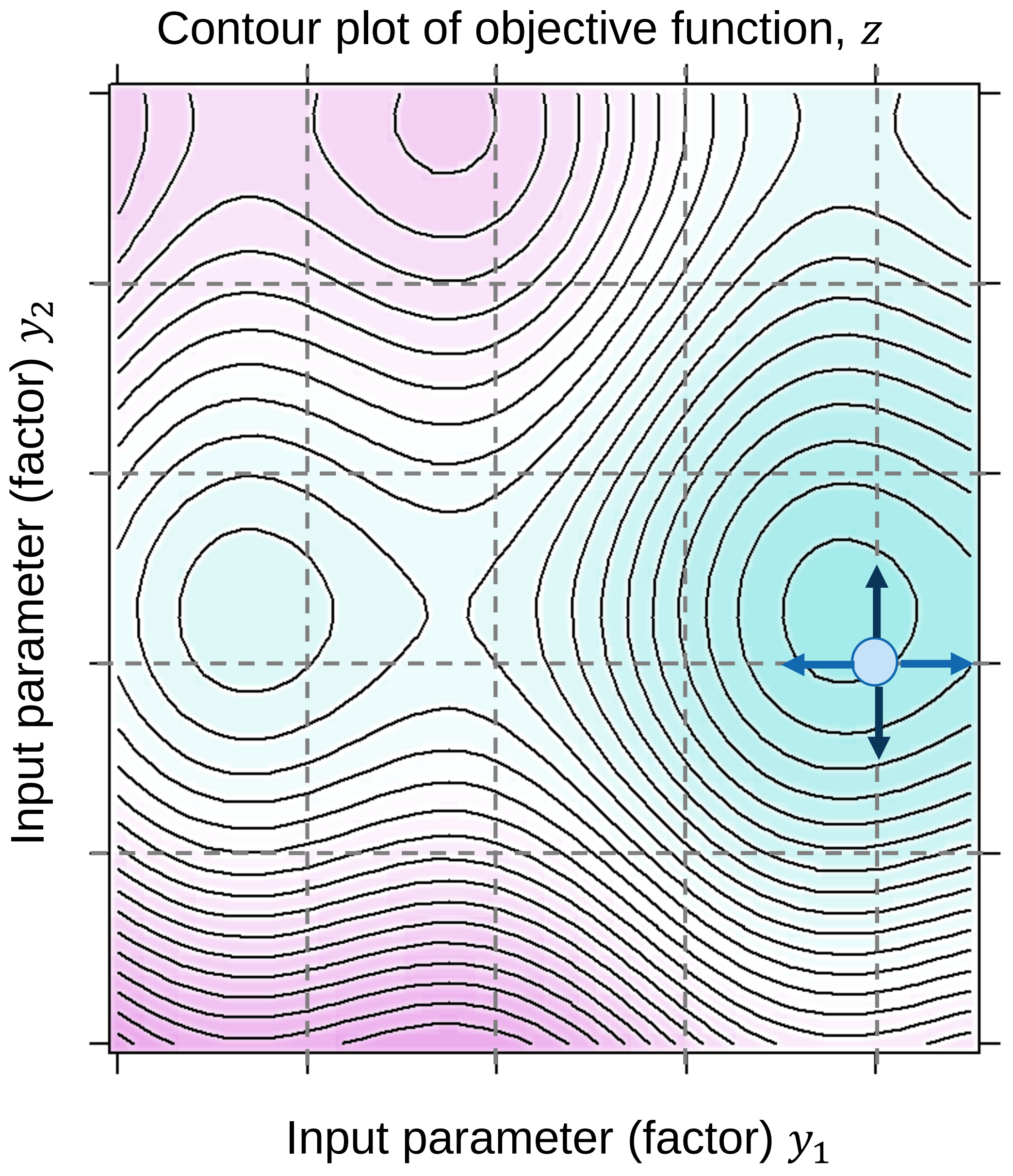
Contour plot of objective function, z



Local sensitivity analysis: One-at-a-time sensitivity analysis

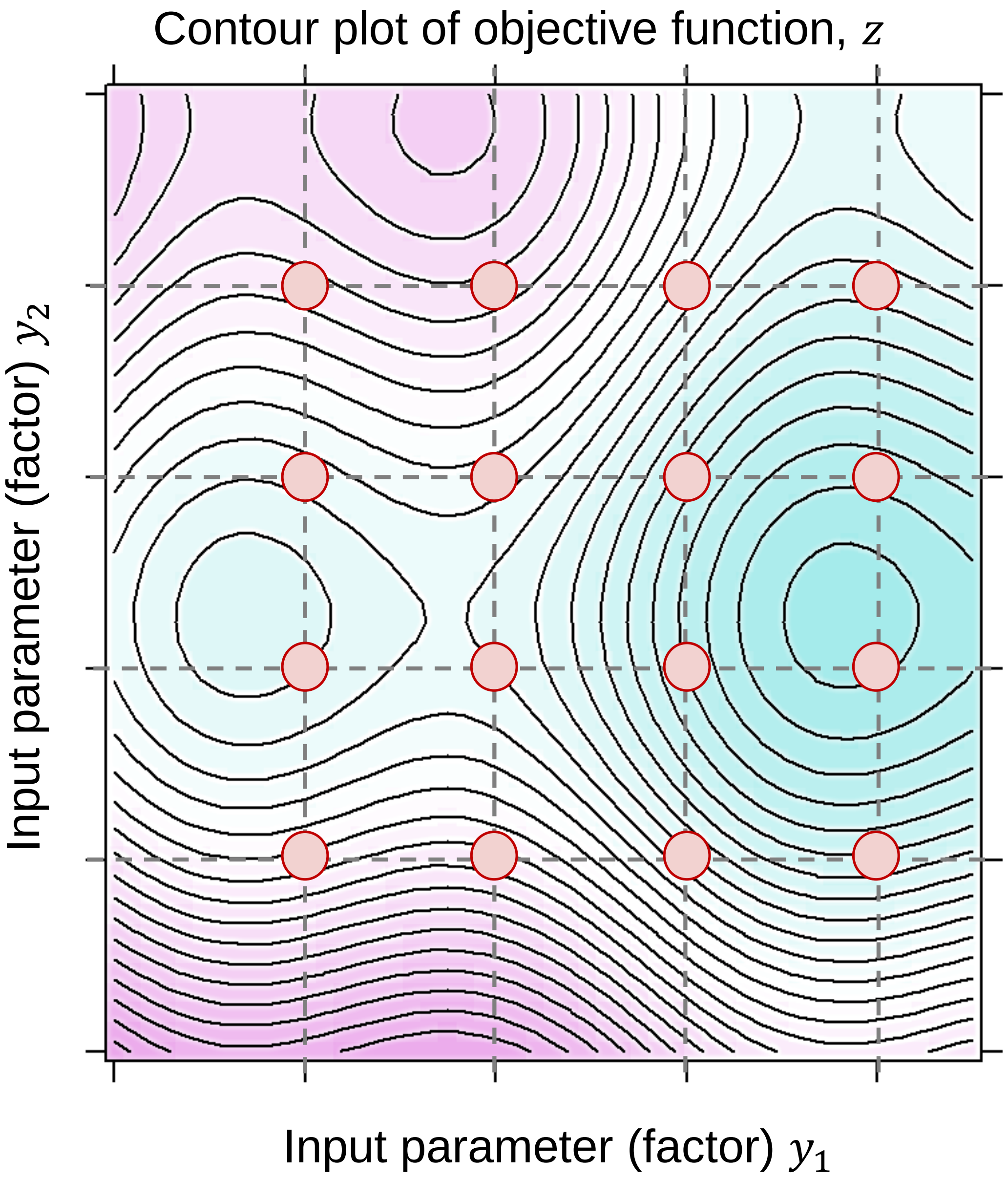
- Widely used method because,
  - reference values often a “safe” starting point where the model properties are well known
  - the effect observed on the output is due solely to the one changing factor
  - non-zero effect implies influence, hence it never detects uninfluential factors as relevant
  - computationally effective

- However,
  - local sensitivity analysis does not effectively cover the input parameter space
  - parameter interactions cannot be studied



Global sensitivity analysis

- Global sensitivity analysis varies all uncertain parameters simultaneously, hence
  - it offers a better coverage of the uncertain parameter space
  - it allows to study parameter interactions
- Global sensitivity analysis offers comprehensive results, but
  - it translates into more complex methods
  - it results in a higher computational burden
- Often used to screen the parameter space (several methods are available)



Global sensitivity analysis: Screening methods

- Screening methods
  - identify the most relevant parameters, so that these can be investigated more thoroughly
  - rank parameters in order of their importance
  - maintain a low computational expense
- However, they do not give any quantitative information about the output variance. They are useful to screen out parameters whose variability is uninfluential, when the number of parameters is too high to perform a quantitative analysis
- Idea: An “upgrade” of the local one-at-a-time sensitivity analysis would be such that
  - after moving one step in one direction, e.g. along y_1, one would straightway move along y_2, and, so on, until all factors up to y_n have been varied by one step each, and
  - the entire parameter space is covered
- This idea is the basis for **elementary effects methods**, such as the Morris screening method

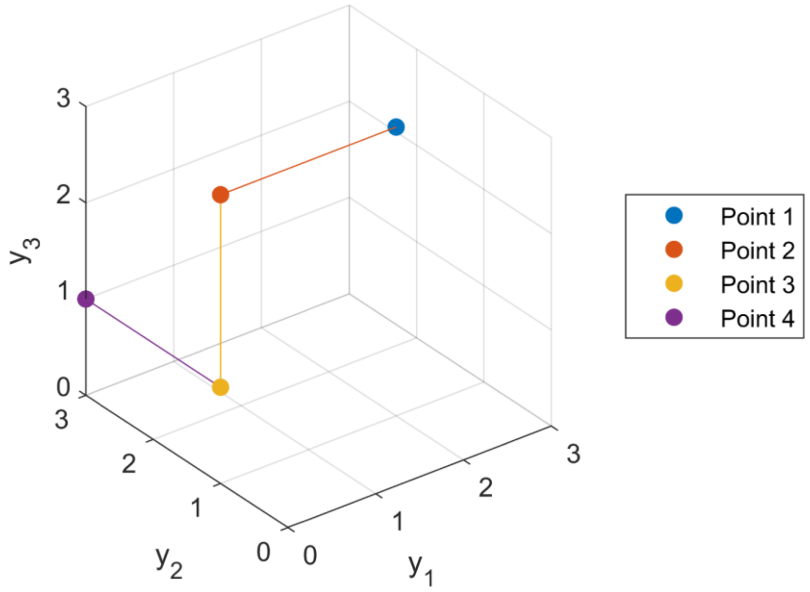
Global sensitivity analysis: Morris screening method (1)

- The Morris method quantifies global sensitivity using a set of local derivatives (approximated by finite differences), or elementary effects (EE)
- Each factor y_i, $i \in \{1, \dots, N\}$, is perturbed along a grid of size Δy_i to create a trajectory, k, through the factor space
- Each trajectory yields one estimate of the elementary effect for each factor, i.e. the ratio of the change in model output to the change in that parameter
- The elementary effect of factor i, (EE_i) is expressed as

$$\mathrm{EE}_i^k = \frac{z\left(y_1^k, y_2^k, \dots, \overbrace{y_i^k + \Delta y_i}^{\text{Perturbation of factor } i \text{ in trajectory } k}, \dots, y_N^k\right) - \overbrace{z\left(y_1^k, y_2^k, \dots, y_i^k, \dots, y_N^k\right)}^{\text{Point in trajectory } k \text{ before changing factor } i}}{\Delta y_i}$$

Global sensitivity analysis: Morris screening method (2)

- Process
  1. Start at a random point in the input space and evaluate the model $z(P_1)$
  2. Randomly select a factor y_i which has not yet been perturbed
  3. Add a perturbation Δy_i to y_i
     a) With a random sign, if y_i is not at its upper or lower limit
     b) Otherwise: Move it away from the limit
  4. Evaluate the model at this new point
  5. Repeat steps 2-4 until all factors have been perturbed once



Global sensitivity analysis: Morris screening method (3)

- The procedure above must be repeated for K trajectories in the factor space to avoid the dependence on a specific reference point
- The mean elementary effect of factor i, denoted as *first order effect*, μ_i, is expressed as

$$\mu_i = \frac{1}{K}\sum_{k=1}^{K} \mathrm{EE}_i^k$$

- Its standard deviation, denoted as interaction, σ_i, is expressed as

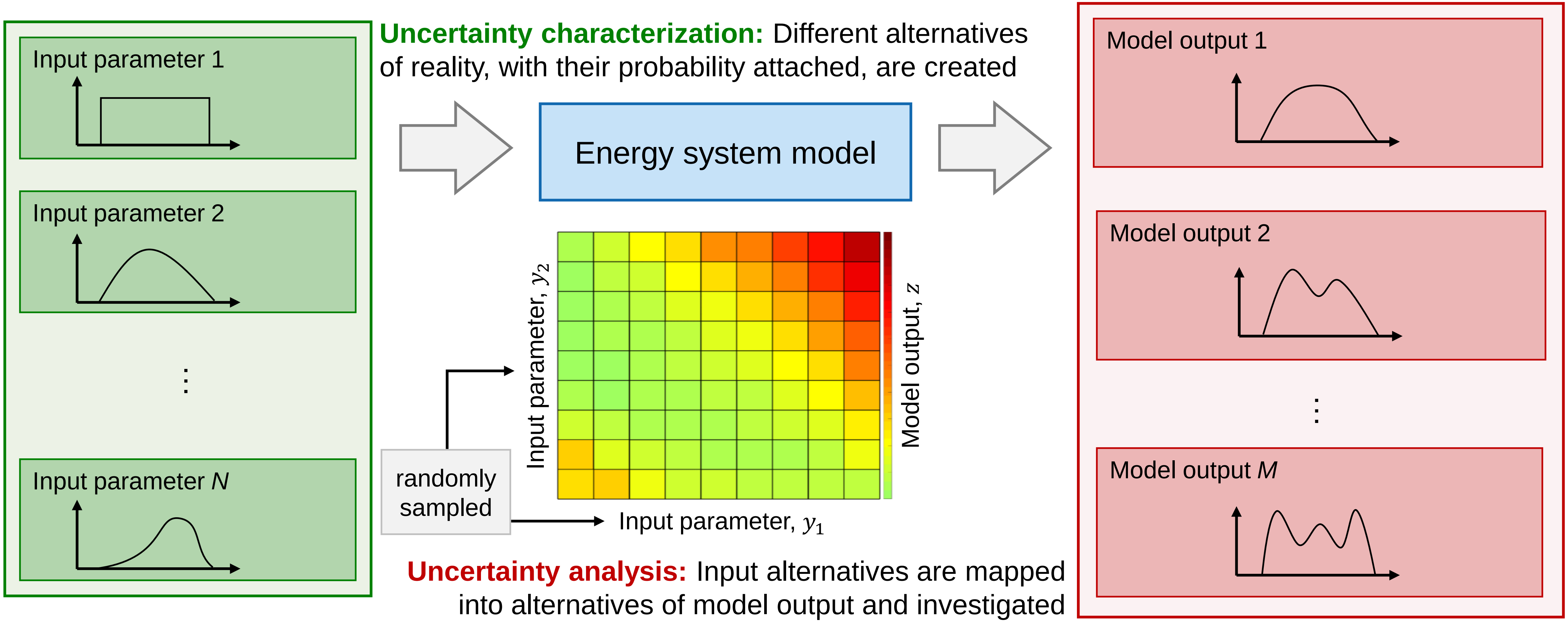
$$\sigma_i = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\left(\mathrm{EE}_i^k - \frac{1}{K}\sum_{k=1}^{K}\mathrm{EE}_i^k\right)^2}$$

- The procedure requires $K(N+1)$ model evaluations and is suitable for computationally expensive models. In many applications, 4 to 10 trajectories are sufficient for an estimate of μ_i

Several other screening methods available. However, they only indicate whether an input parameter is important, but do not provide quantitative information about the output variance

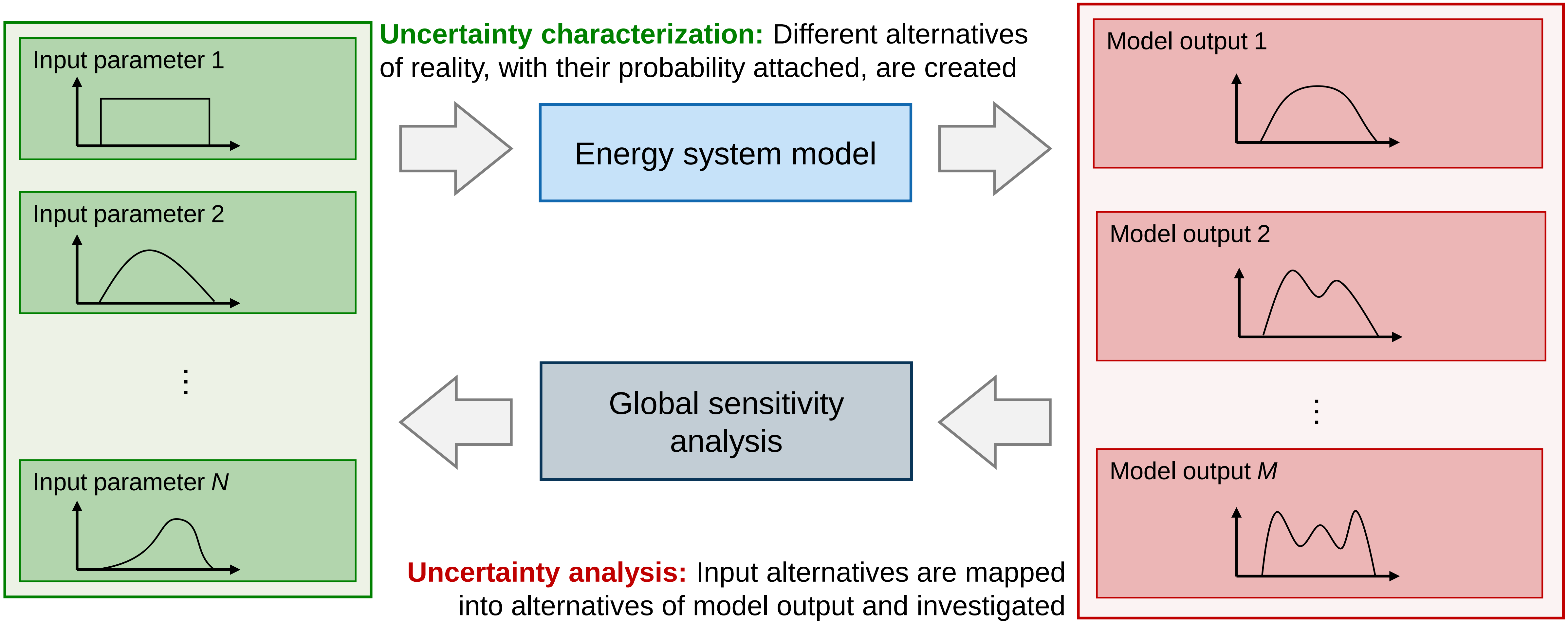
Uncertainty analysis

- Uncertain input parameters are sampled according to their probability distributions
- Model output is evaluated repeatedly for the different input samples (**Monte Carlo simulation**)



Uncertainty analysis for global sensitivity

- Basis for global sensitivity analysis via variance-based methods, which provide quantitative information about the impact of input parameters by decomposing the variance of model outputs



Uncertainty analysis for global sensitivity: Variance-based methods

- Variance-based methods entail:
  - characterization of uncertainty associated with input parameters
  - model evaluation (via Monte Carlo simulation) and uncertainty analysis of the model output
  - calculation of sensitivity indices quantifying the fraction of variance explained by each parameter
- Sensitivity indices describe:
  - The *main effect* of a parameter, which quantifies the portion of the variance of model output explained by that parameter when allowing all other parameters to vary at the same time
  - The *total effect* of a parameter, which quantifies the variance (i.e. the uncertainty) in the model output that is left after fixing all other parameters to their 'true', albeit unknown, value
- Several methods available, e.g. FAST (Fourier Amplitude Sensitivity Test) or Sobol's indices
- Variance-based methods provide a quantitative and thorough analysis of the model uncertainty but are computationally intensive

A. Saltelli et al., *Global sensitivity analysis. Gauging the worth of scientific models*, John Wiley & Sons (2007)

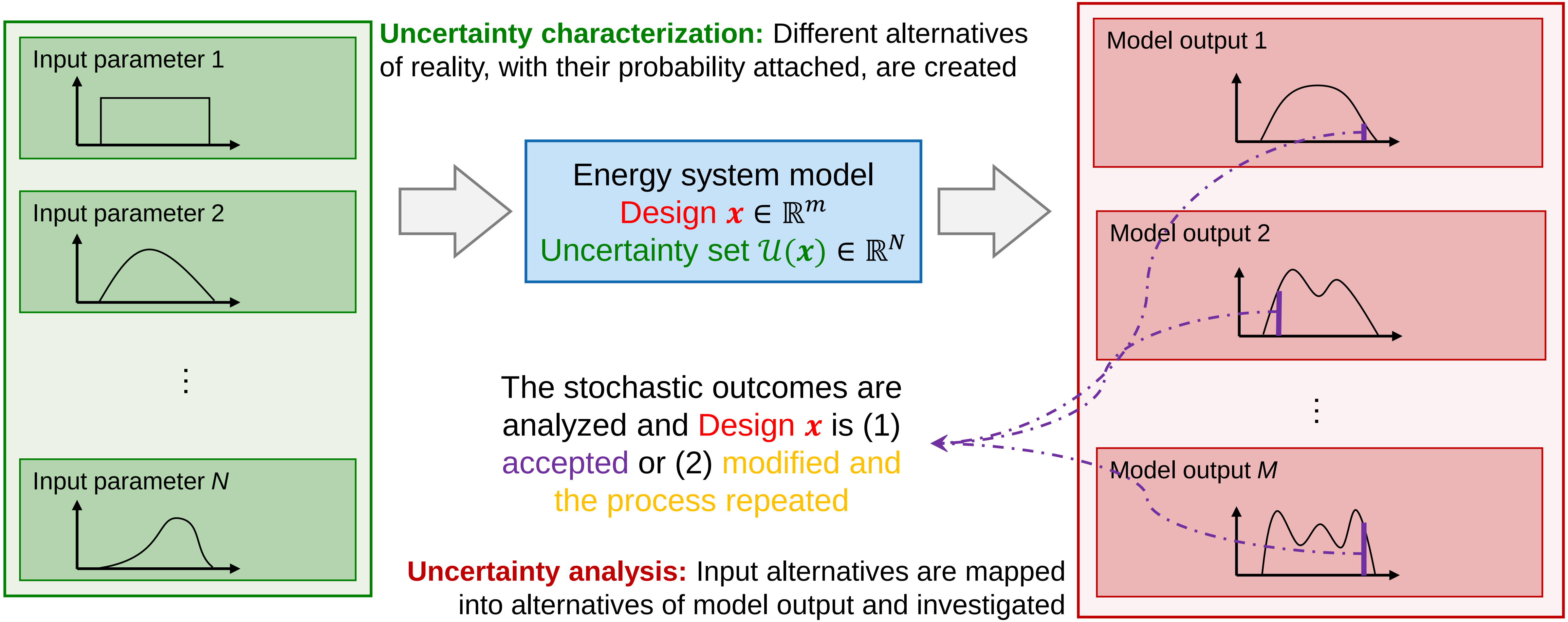
After this lecture, you are able to …

- ✓ Understand how uncertainty affects energy systems, their modeling and optimization
- ✓ Describe the uncertainty occurring within modeling and optimization of energy systems:
  - ✓ Aleatory and epistemic uncertainty
  - ✓ Probability distributions and uncertainty ranges
- ✓ Apply methods for analyzing uncertainty within modeling and optimization of energy systems:
  - ✓ Sensitivity analysis: local and global approaches
  - ✓ Uncertainty analysis: Monte Carlo simulation
- Interpret the results and making decisions under uncertainty:
  - Monte Carlo simulation
  - Stochastic optimization
  - Robust optimization

Decision-making under uncertainty

Monte Carlo simulation

- Uncertain input parameters are sampled according to their probability distributions
- Model output is evaluated repeatedly for the different input samples (**Monte Carlo simulation**)



Stochastic optimization

- Uncertainty characterizes objective and constraint functions
- $\mathcal{U}(\boldsymbol{x}) \in \mathbb{R}^N$ is modelled via random variables with known distributions
- Optimization is done for the expected values $\mathbb{E}_{\mathcal{U}(\boldsymbol{x})}$

$$
\begin{aligned}
\min_{\boldsymbol{x}} \quad & z = \mathbb{E}_{\mathcal{U}(\boldsymbol{x})} f(\boldsymbol{x}, \mathcal{U}(\boldsymbol{x})) \\
\text{s.t.} \quad & g_j(\boldsymbol{x}, \mathcal{U}(\boldsymbol{x})) \leq 0, \ \ j = 1, \dots, n \\
& h_i(\boldsymbol{x}, \mathcal{U}(\boldsymbol{x})) = 0, \ \ i = 1, \dots, o
\end{aligned}
$$

- To solve the stochastic optimization numerically, the random vector $\mathcal{U}(\boldsymbol{x})$ is discretized into a **finite** number of possible realizations, called **scenarios** $u_1 \dots u_K$ with probabilities $\pi_1 \dots \pi_K$ and $z = \sum_{k=1}^{K} \pi_k f(\boldsymbol{x}, u_k)$
- Two-stage formulation is widely used in stochastic programming: first stage decides before the realization of the uncertainties; second stage optimizes after the uncertainty is revealed
- Random (or chance) constraints also exist: $\pi(g_j(\boldsymbol{x}, \mathcal{U}(\boldsymbol{x})) \leq 0) \geq \eta$

Robust optimization

- Uncertainty characterizes objective and constraint functions
- The constraints must be satisfied for all possible values (including **the worst**) of parameters in $\mathcal{U}(\boldsymbol{x}) \in \mathbb{R}^N$
- The problem is cast as worst case analysis using the Wald's maximin (or **minimax**) **model**

$$
\begin{aligned}
\min_{\boldsymbol{x}} \quad & z = \min_{\boldsymbol{x}} \max_{\mathcal{U}(\boldsymbol{x})} f(\boldsymbol{x}, \mathcal{U}(\boldsymbol{x})) \\
\text{s.t.} \quad & g_j(\boldsymbol{x}, \mathcal{U}(\boldsymbol{x})) \leq 0, \; j = 1, \dots, n \\
& h_i(\boldsymbol{x}, \mathcal{U}(\boldsymbol{x})) = 0, \; i = 1, \dots, o
\end{aligned}
$$

- The min represents the decision-maker and the max represents the uncertainty, e.g.
  - we want to **minimize** the worst possible (**maximum**) system cost, or DNS, in the face of uncertainties
  - we want to **maximize** the worst possible (**minimum**) operation revenues in the face of uncertainties
- If the random vector $\mathcal{U}(\boldsymbol{x})$ is discretized into a **finite** number of possible scenarios $u_1 \dots u_K$ with probabilities $\pi_1 \dots \pi_K$ and the functions are linear, the problem becomes a linear programming $z = \min_{\boldsymbol{x}} \max_{\mathcal{U}(\boldsymbol{x})} \{f_1(\boldsymbol{x}), \dots, f_k(\boldsymbol{x})\}$
- The robust solution may be **excessively conservative** and come with a “high price”

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