ETH zürich

RRE
Reliability and Risk Engineering

Introduction to Modeling and Optimization of Sustainable Energy Systems:

Spatial dimension and energy networks

Prof. Dr. Giovanni Sansavini

Reliability and Risk Engineering

Administration

- Exam Office hours
  - Wed. Dec 22. 16:15 – 14:45 via Zoom
  - Thu. Jan 27. 13:15 – 14:45 via Zoom

- Recap Lecture
  - Thu. Jan 20. 13:15 – 14:45 via Zoom

- Questions: write to moses-edu@ethz.ch

Since the last lecture, you are able to …

- ✓ Formulate multi-objective optimization problem for MES optimal design
- ✓ Model energy conversion technologies within MES optimization
- ✓ Model energy storage technologies within MES optimization
- ✓ Understand the different degrees of complexity when optimizing MES
- ✓ Model energy conversion dynamics (***optional material, no exam***)

After this lecture, you are able to …

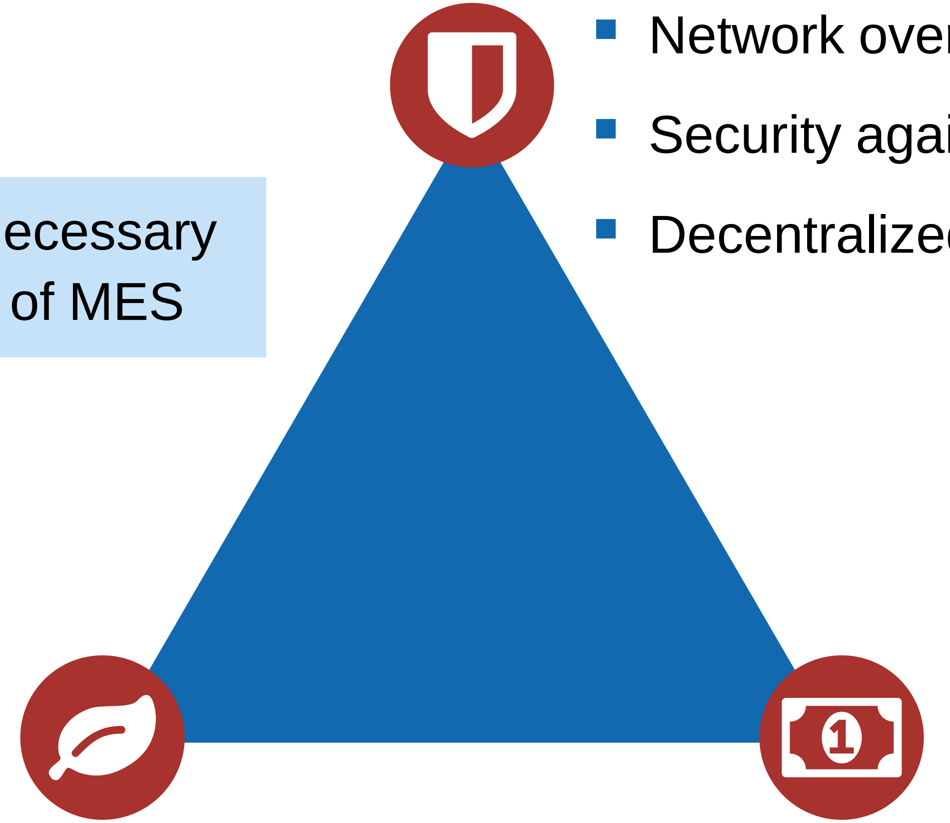
- Describe the energy network modeling process
- Define a generic energy network with
  - Graphs, node and link quantities
  - Decision variables and constraints
- Solve network equations with backward-forward sweep
- Describe the process of linearizing physical laws
- Define network optimization models for gas, electricity and thermal networks
- Model non-unique flow directions and topology changes

Importance of energy networks

The spatial dimension is necessary to model different aspects of MES

Security

- Network overloads
- Security against network failures
- Decentralized vs. centralized designs



Sustainability

- Spatial balancing of renewable energy
- Transportation losses

Equity

- Ensuring access to energy
- Network investments

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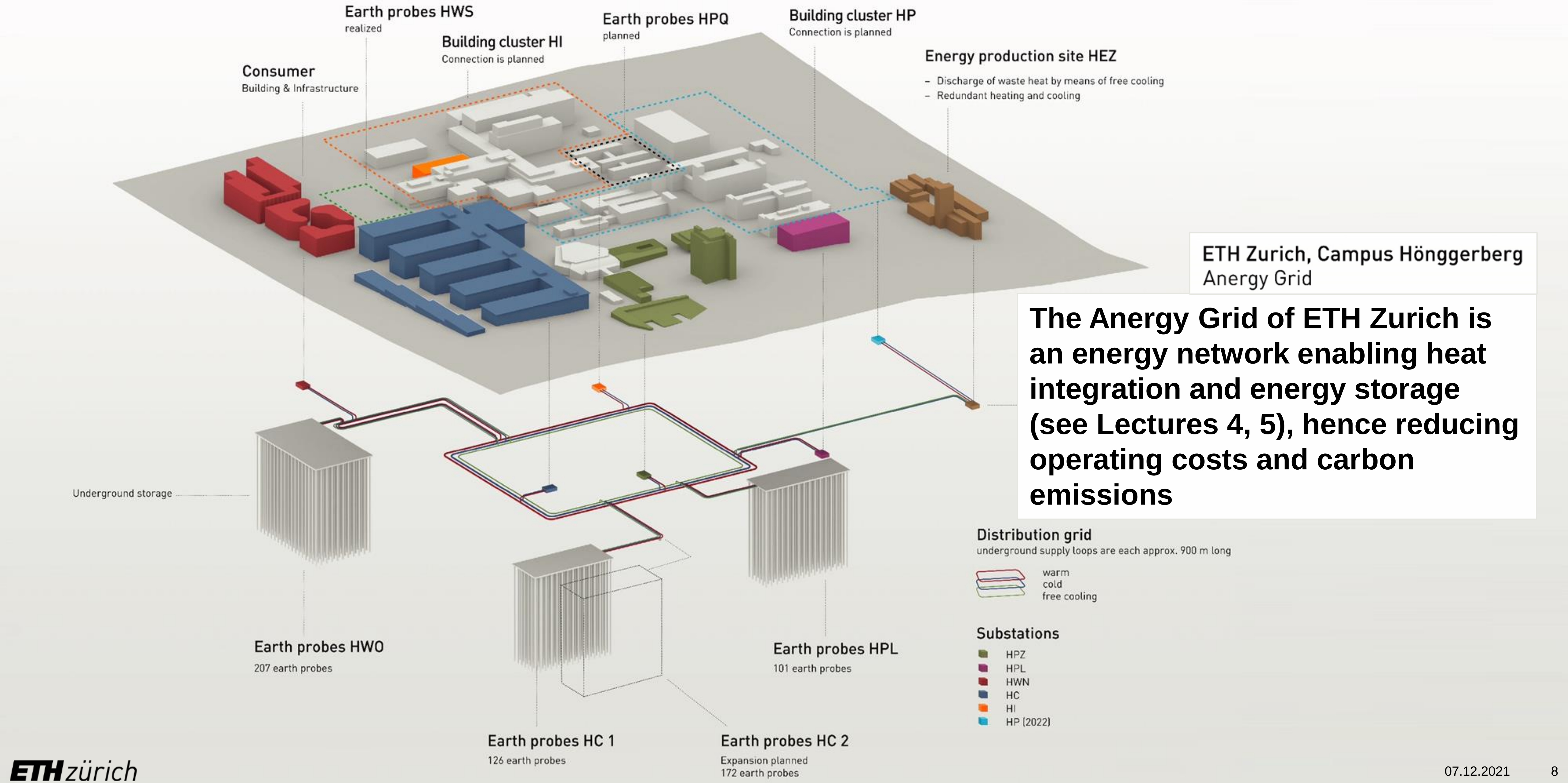
Modeling MES with spatial dimension: Energy Networks

The decision-making context determines the optimization problem

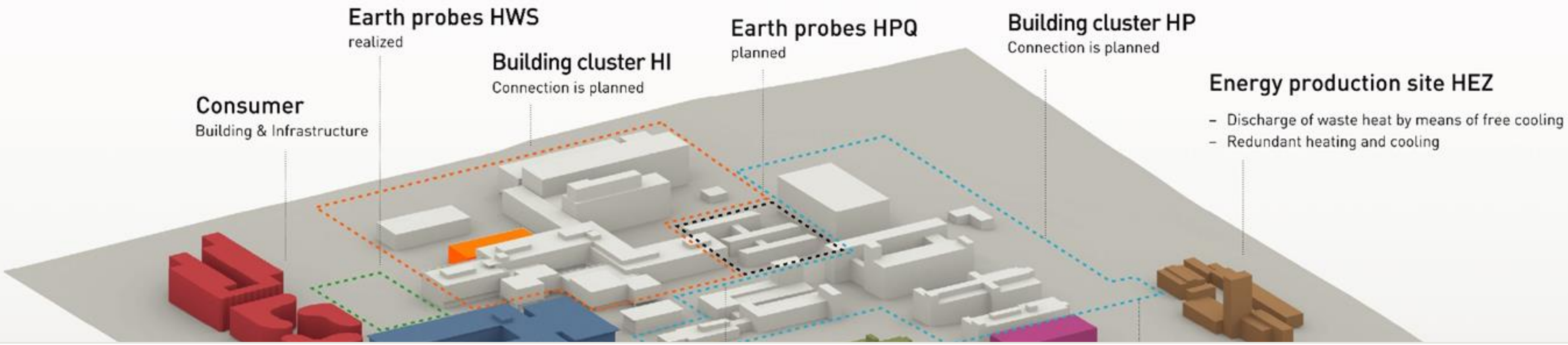
	easier	more difficult
Decision variables: Design vs Operation	Technology operation for given technology selection and size	Selection and size of technologies, coupled with their operation
Time dimension	Instantaneous	Time-dependent
Space dimension	Single node	Multi-node energy networks
Uncertainty	Deterministic optimization	Robust or stochastic optimization
Objective function	Single objective	Multiple objectives

Energy networks for heat integration: The Anergy Grid of ETH Zurich



The Anergy Grid of ETH Zurich is an energy network enabling heat integration and energy storage (see Lectures 4, 5), hence reducing operating costs and carbon emissions

Energy networks for heat integration: The Anergy Grid of ETH Zurich



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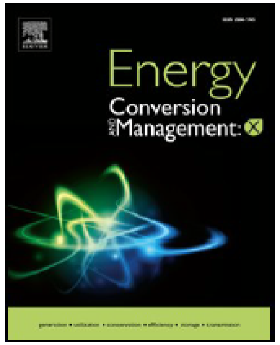


Optimization of low-carbon multi-energy systems with seasonal geothermal energy storage: The Anergy Grid of ETH Zurich

Check for updates

Paolo Gabrielli[a], Alberto Acquilino[a], Silvia Siri[b], Stefano Bracco[c], Giovanni Sansavini[a], Marco Mazzotti[d,*]

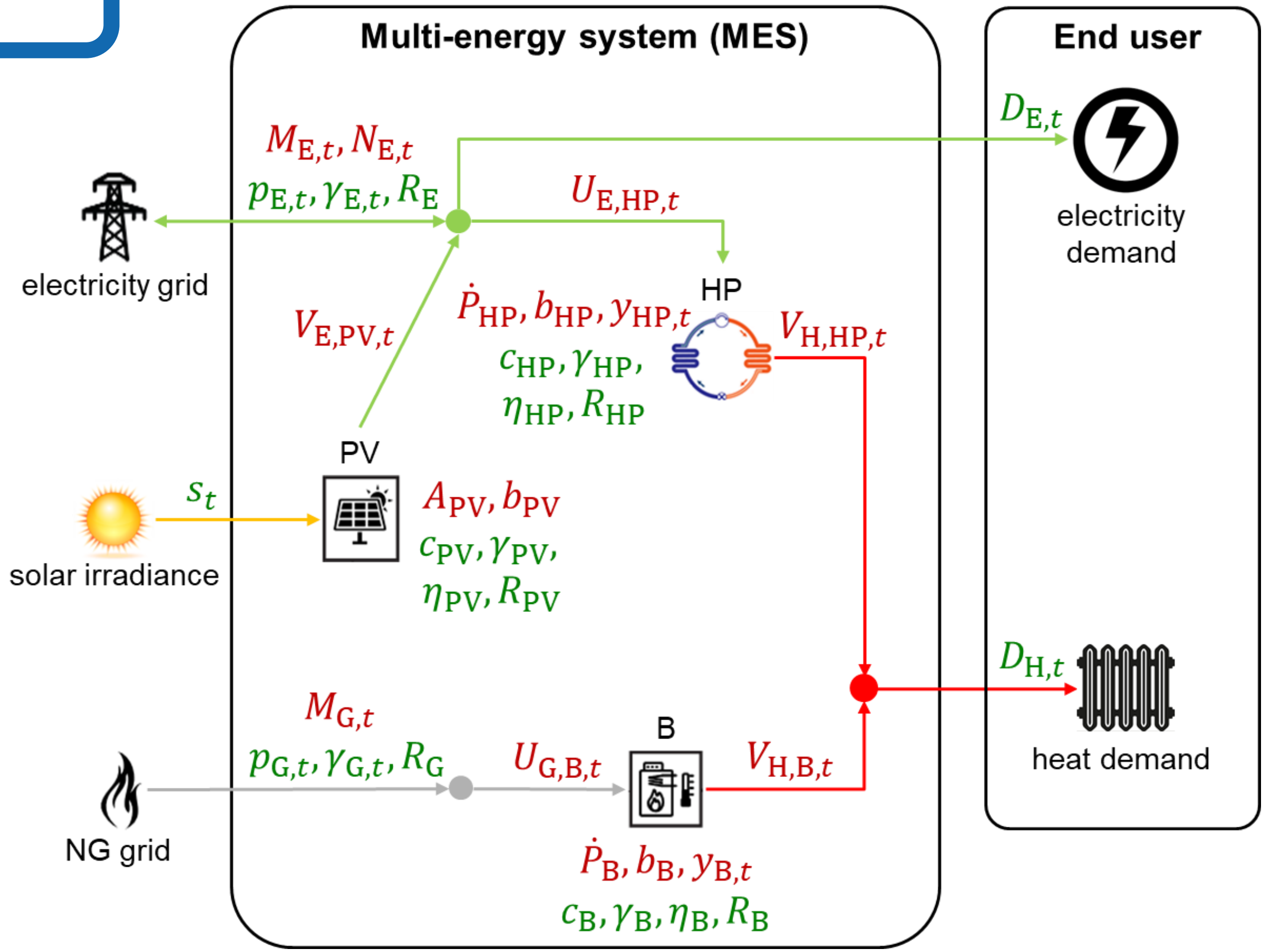
From single node formulation: Energy balances for all energy carriers…

LP

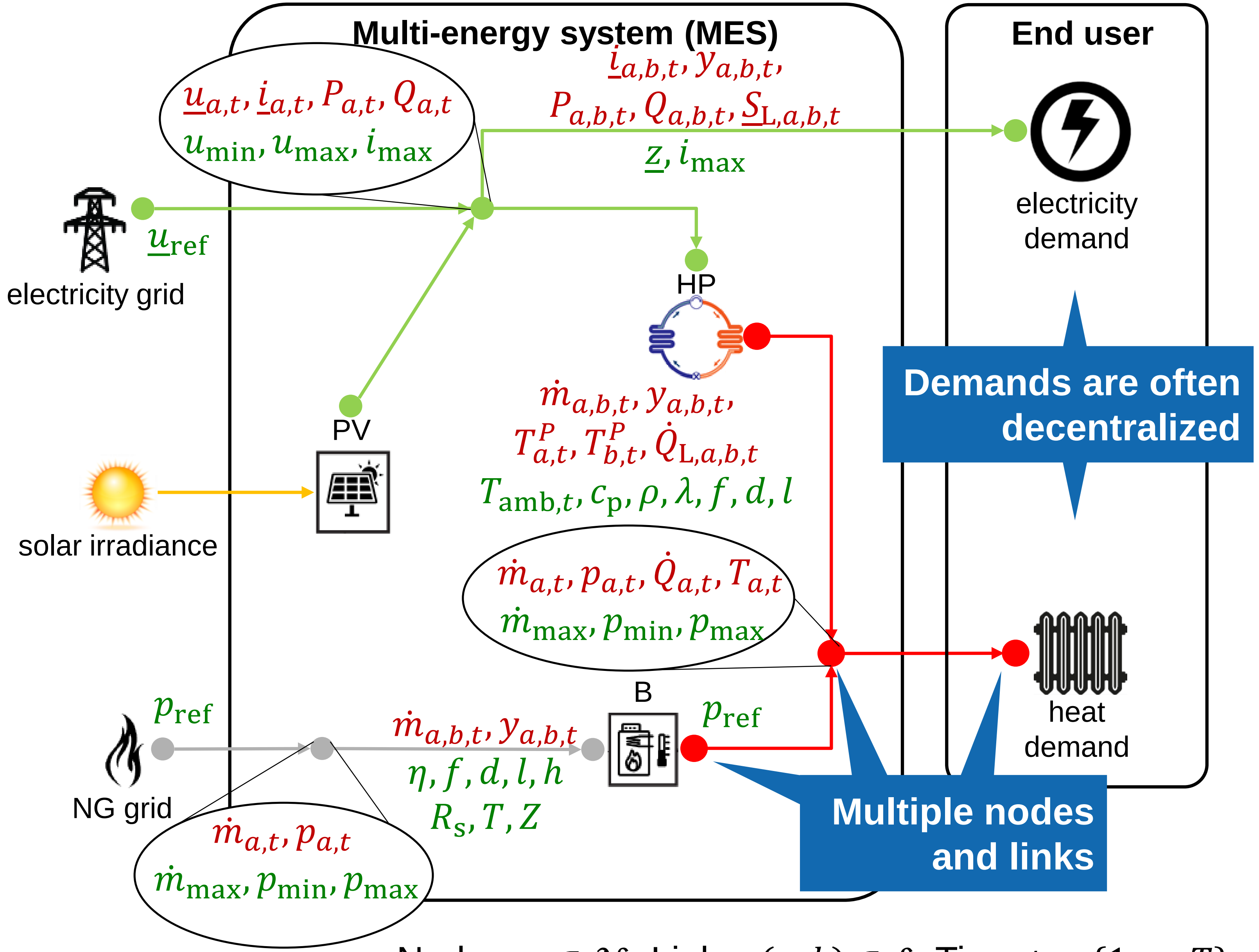
Generic energy carrier, *j*

$$\sum_{k\in\mathcal{K}} \left(V_{j,k,t} - U_{j,k,t}\right) + M_{j,t} - N_{j,t} = D_{j,t} \ ,$$

$$\forall j \in \mathcal{J}, \forall t \in \{1, \dots, T\}$$



…to introducing networks



Nodes: $a \in \mathcal{N}$. Links: $(a, b) \in \mathcal{L}$. Time $t = \{1, \dots, T\}$

Additional variables	
Mass flow	$\dot{m}$
Pressure	p
Temperature	T
Thermal power	$\dot{Q}$
Complex voltage	$\underline{u}$
Complex current	$\underline{i}$
Active electrical power	P
Reactive electrical power	Q
Apparent electrical power	$\underline{S}$
Link status (ON/OFF)	y
Losses	$\blacksquare_{\mathrm{L}}$

Additional Input data	
Lower limits	$\blacksquare_{\min}$
Upper limits	$\blacksquare_{\max}$
Reference potential	$\blacksquare_{\mathrm{ref}}$
Gas pipe properties	η, f, d, l, h
Gas properties	R_{s}, T, Z
Thermal pipe properties	λ, f, d, l
Fluid properties	c_{p}, ρ
Ambient temperature	T_{amb}
Electrical impedance	$\underline{z}$

Network optimization process: General procedure

1. Definition and mathematical modeling of quantities of interest
   a. Identifying decision variables and parameters
   b. Identifying degrees of freedom (see lecture 2)

2. Translating the model into optimization
   a. Classifying constraints (equality/inequality, linear/nonlinear)
   b. Simplifications and linearizations to achieve a suitable problem type (NLP, MILP, LP)

3. Solving

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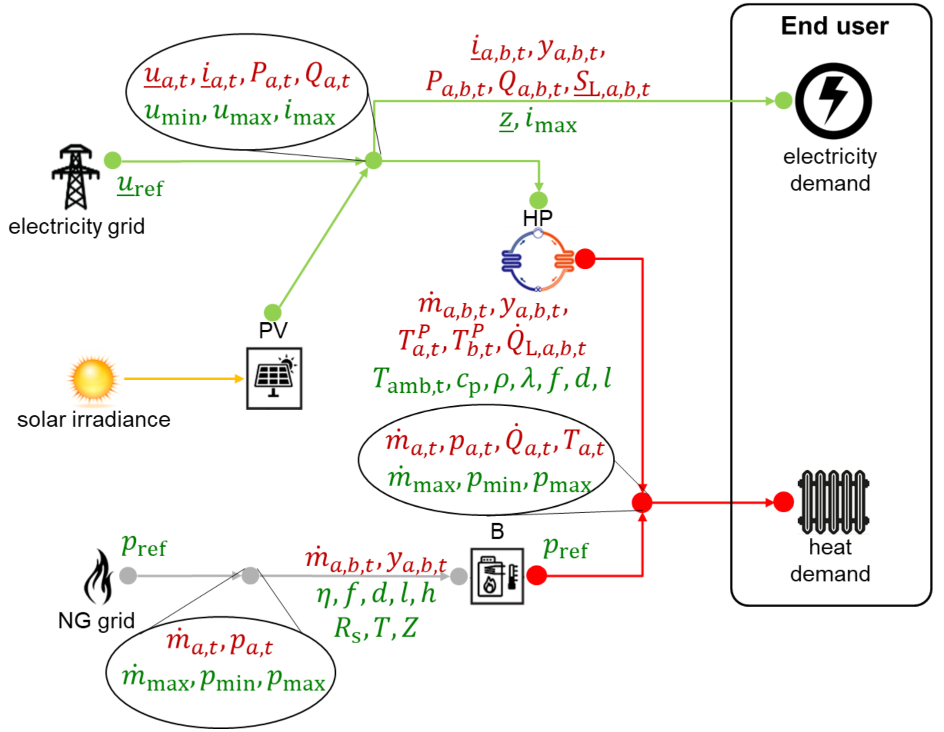
Defining and modeling energy networks

Energy network models

- Modeled as graphs with N nodes connected by L **directed** links
- Modeled quantities:

	Variables	Equations
Nodes	Potential (pressure, voltage)	Potential limits
	Energy injection and extraction	Injection and extraction limits (Capacity)
	Temperature	Conservation of mass, charge and energy
		Reference Potential at one node
Links	Energy Flows	
	Energy Losses	Flow limits (Capacity)
	Temperature	Physical laws linking potential, flows and losses



$$\boldsymbol{x} = [\underbrace{\underline{u}_{a,t}, p_{a,t}}_{\text{Potential}}, \underbrace{\underline{i}_{a,t}, \dot{m}_{a,t}, P_{a,t}, Q_{a,t}, \dot{Q}_{a,t}}_{\text{Injection}}, \underbrace{T_{a,t}, T^P_{a,t}}_{\text{Temperature}}, \underbrace{\underline{i}_{a,b,t}, \dot{m}_{a,b,t}, P_{a,b,t}, Q_{a,b,t}}_{\text{Flows}}, \underbrace{\dot{Q}_{L,a,b,t}, \underline{S}_{L,a,b,t}}_{\text{Losses}}, \underbrace{y_{a,b,t}}_{\text{On/Off}}]$$

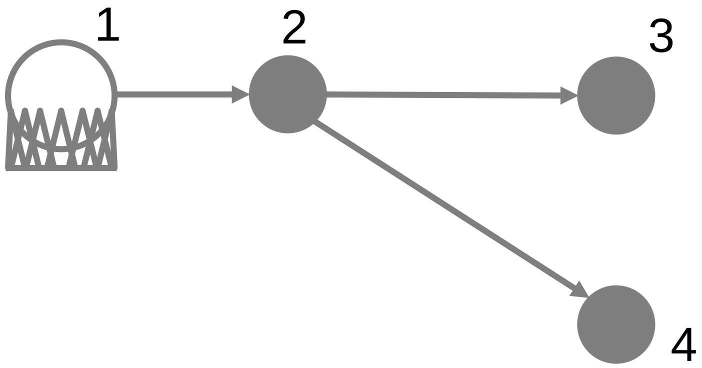
Exists in all 3 networks

Highlighted quantities exist in both gas and thermal networks

$$\boldsymbol{x} \in [\mathbb{R} \ or \ \{0,1\}]^{m(T,N,L)}, \qquad m(T,N,L) = T(12N + 14L)$$

Example: Gas distribution network

- Gas flows from source (node 1) to consumers with fixed gas demands
- Assumptions: Steady state, no compressors, isothermal flow, no energy losses, fixed flow directions
- Mathematical model with **variables** and **parameters**



Nodes $\mathcal{N} = \{1, 2, \ldots N\}$

Links $\mathcal{L} = \{(1,2), \ldots L\}$

Variables

Pressure: $p_a \; \forall \, a \in \mathcal{N}$
Injected mass: $\dot{m}_a \; \forall \, a \in \mathcal{N}$

It becomes a parameter in load nodes

Link flow $\dot{m}_{a,b} \; \forall \, (a,b) \in \mathcal{L}$
No energy losses

Fundamental Equations, Engineering Limits

Injection limits: $-\dot{m}_{a,\max} \leq \dot{m}_a \leq \dot{m}_{a,\max} \forall \, a \in \mathcal{N}$
Pressure limits: $p_{a,\min} \leq p_a \leq p_{a,\max} \forall \, a \in \mathcal{N}$
Reference pressure: $p_1 = p_{\text{ref}}$
Mass conservation: $\dot{m}_a + \sum_b \dot{m}_{b,a} - \sum_b \dot{m}_{a,b} = 0 \; \forall \, a \in \mathcal{N}$

Links are directed: Either outgoing ($\dot{m}_{a,b} \neq 0$) or incoming ($\dot{m}_{b,a} \neq 0$)

Flow limits: $0 \leq \dot{m}_{a,b} \leq \dot{m}_{a,b,\max} \; \forall \, (a,b) \in \mathcal{L}$
Physical law linking pressures and flows:

$$\dot{m}_{a,b} = \frac{\pi \eta \sqrt{d^5}}{8\sqrt{f R_s l T Z}} \sqrt{(p_a^2 - p_b^2) - \frac{(p_a + p_b)^2 g h_{a,b}}{2 Z R_s T}} \; \forall \, (a,b) \in \mathcal{L}$$

Friction factor f depends on $\dot{m}_{a,b}$

Units and Parameters:

p_a in bar and $\dot{m}$ in kg/h

$R_s = \frac{\mathrm{R}}{\mathrm{M}}$ specific gas constant, R is the universal gas constant, M is the molar mass

Z = Compressibility factor

μ = viscosity

f = dimensionless friction factor

l, d = pipe length (l) in m and diameter (d) in mm

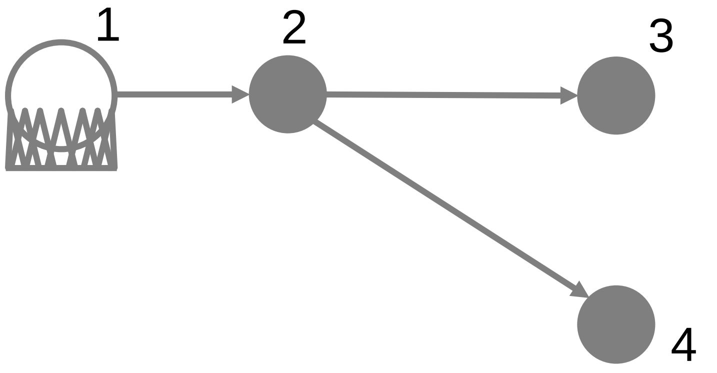
g = gravitational acceleration

η = Efficiency factor (0.8 – 1) from degradation and non-ideal surfaces

Osiadacz, A. *Simulation and Analysis of Gas Networks*. London: E. & F.N. Spon, 1987.

Gas network simulation: Degree of freedom analysis

- Recap: Degree of freedom d of a model is the difference of number of variables n_x minus number of independent equations n_e, $d = n_x - n_e$

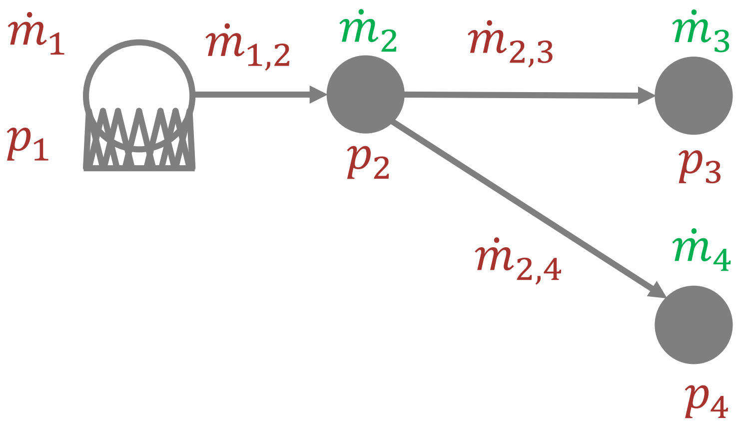


	Variables	Fundamental Equations, Engineering Limits
Nodes $\mathcal{N}$	Pressure: $p = [p_1 \quad p_2 \quad p_3 \quad p_4]^T$	Injection limits: $-\dot{m}_{a,\max} \leq \dot{m}_a \leq \dot{m}_{a,\max} \forall\, a \in \mathcal{N}$
	Injected mass: $\dot{m} = [\dot{m}_1 \quad \dot{m}_2 \quad \dot{m}_3 \quad \dot{m}_4]^T$	Pressure limits: $p_{a,\min} \leq p_a \leq p_{a,\max} \forall\, a \in \mathcal{N}$
		Mass conservation: $\dot{m}_a + \sum_b \dot{m}_{b,a} - \sum_b \dot{m}_{a,b} = 0 \;\forall\, a \in \mathcal{N}$
		Reference pressure: $p_1 = p_{ref}$
Links $\mathcal{L}$	Link flow: $\dot{m} = [\dot{m}_{1,2} \quad \dot{m}_{2,3} \quad \dot{m}_{2,4}]^T$	Flow limits: $0 \leq \dot{m}_{a,b} \leq \dot{m}_{a,b,\max} \forall\, (a,b) \in \mathcal{L}$
		Physical law linking pressures and flows: $\dot{m}_{a,b} = \frac{\pi\eta\sqrt{d^5}}{8\sqrt{f R_s l T Z}} \sqrt{(p_a^2 - p_b^2) - \frac{(p_a+p_b)^2 g h_{a,b}}{2 Z R_s T}} \;\forall\, (a,b) \in \mathcal{L}$

- $n_x = 4 \text{ pressures } + \ 1 \text{ mass injection } + \ 3 \text{ link flows} = 8$
- $n_e = 4 \text{ mass conservation } + 3 \text{ flow laws} + 1 \text{ reference pressure} = 8$

$d = 0$: There is a unique solution

Gas network simulation: Backward forward sweep calculation



- Compute pressures p_a and mass flows $\dot{m}_{a,b}$ from gas demands $\dot{m}_a$
- Backward Sweep: Starting at the sink nodes, calculate mass flows using mass conservation moving towards the reference (source) node

Stage 1 (Sink nodes):

$$\dot{m}_3 + \dot{m}_{2,3} = 0$$
$$\dot{m}_4 + \dot{m}_{2,4} = 0$$

Stage 2:

$$\dot{m}_2 - \dot{m}_{2,3} - \dot{m}_{2,4} + \dot{m}_{1,2} = 0$$

Stage 3:

$$\dot{m}_1 - \dot{m}_{1,2} = 0$$

- Forward sweep: Starting at reference node with given pressure ($p_1 = p_{\mathrm{ref}}$), calculate pressures using the equation linking pressures and mass flows

Stage 1 (reference node):

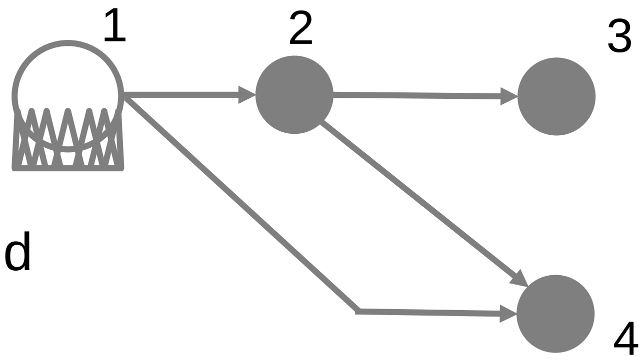
Calculate p_2 using $\dot{m}_{1,2} = \frac{\pi\eta\sqrt{d^5}}{8\sqrt{fR_s lTZ}}\sqrt{(p_1^2 - p_2^2) - \frac{(p_1+p_2)^2 g h_{1,2}}{2ZR_sT}}$

Stage 2:

Calculate p_3 from p_2 and $\dot{m}_{2,3}$

Calculate p_4 from p_2 and $\dot{m}_{2,4}$

Gas network simulation: Loops



- What if there are loops in the network?
  - Cannot use backward forward sweep, as equations for pressures and flows in the loop need to be solved together.

- Use numerical method (e.g. Newton) to solve the system of equations:

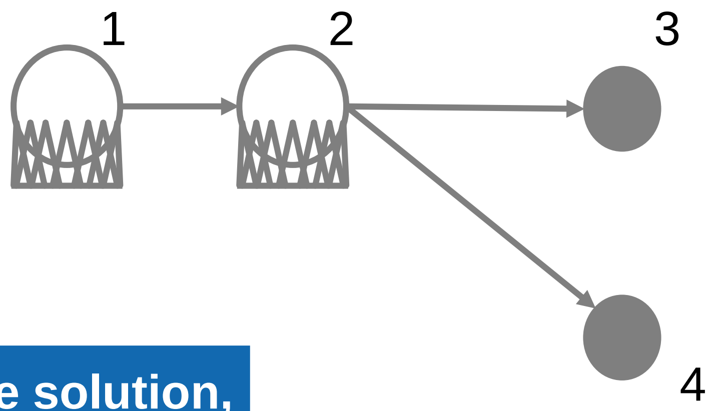
$$\begin{cases} \dot{m}_{a,b} - \dfrac{\pi\eta\sqrt{d^5}}{8\sqrt{f R_s l T Z}} \sqrt{(p_a^2 - p_b^2) - \dfrac{(p_a + p_b)^2 g h_{a,b}}{2 Z R_s T}} = 0 \; \forall \, (a,b) \in \mathcal{L} \\ \dot{m}_a + \sum_b \dot{m}_{b,a} - \sum_b \dot{m}_{a,b} = 0 \; \forall \, a \in \mathcal{N} \end{cases}$$

$d > 0$: No unique solution, optimization possible

- What if the flow can be controlled with valves?
  - Degree of freedom > 0, flows can be optimized to reduce pressure loss

Gas network optimization: Multiple sources and engineering limits

- What happens if there is more than one source?



$$n_x = 4 \text{ pressures } + 2 \text{ mass injections } + \ 3 \text{ link flows} = 9$$
$$n_e = 4 \text{ mass conservation } + 3 \text{ flow laws} + 1 \text{ reference pressure} = 8$$

$d > 0$: No unique solution, optimization possible

- What if the solution exceeds pressure or mass flow limits?
- Gas demand cannot be fully supplied
  a) Use optimization and compute gas demand not supplied (see constraints in Lectures 8, 10)
  b) In a MES, use optimization to possibly supply demand via other energy carriers and technologies

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Translating the mathematical model into optimization

Elements of continuous optimization (see Lecture 3)

objective function		z_q	$\mathbb{R}^m \rightarrow \mathbb{R}$	quantity to be minimized/maximized
decision variables		$\boldsymbol{x}$	$\in \mathbb{R}^m$	Values that decision makers need to determine
constraints	equality	$h_i(\boldsymbol{x}) = 0$	$i = 1, \ldots, o$	conservation equations (*mass, energy*), component models (*performance curves*)
	inequality	$g_j(\boldsymbol{x}) \leq 0$	$j = 1, \ldots, n$	limitations (*physical, technical, mathematical, legal*)

Adding networks to energy system optimization

- Networks introduce additional decision variables, parameters, equality and inequality constraints

objective function	z_q
decision variables	$\boldsymbol{x}$
constraints: equality	$h_i(\boldsymbol{x}) = 0$
constraints: inequality	$g_j(\boldsymbol{x}) \leq 0$

Nodes	Links
Potential (Pressure, Voltage)	Energy Flows
Energy injections	Energy losses
Temperatures	
Conservation of mass, charge and energy	Physical laws linking potential, flows and losses
Reference Potential at 1 node	
Injection and extraction limits	Flow limits
Potential limits	

Often nonlinear

- How to transform network optimization from NLP to (MI-)LP?

Linearizing physical laws

Gas flow law: Reorganization and simplification

- Assumptions
  - Mass flow and pressures are decision variables, everything else is a parameter
  - Horizontal pipe (height difference $h_{a,b} = 0$)
  - Pressure between 0.75 and 7 bar → Friction factor
  - Natural gas at T = 288 K

Horizontal pipe

$$\dot{m}_{a,b} = \frac{\pi\eta\sqrt{d^5}}{8\sqrt{fR_s lTZ}}\sqrt{(p_a^2 - p_b^2) - \cancel{\frac{(p_a+p_b)^2 g h_{a,b}}{2ZR_sT}}}$$

$$\sqrt{\frac{1}{f}} = 5.338(Re)^{0.076}\eta \text{ where } Re = \frac{4\dot{m}_{a,b}}{\mu d\pi}$$

- Linearization
  - Use squared pressure instead of pressure as decision variable (adjust pressure bounds and reference pressure)
  - Piecewise affine approximation of $\dot{m}_{a,b}^{1.848}$

$$p_a^2 - p_b^2 = k\,\dot{m}_{a,b}^{1.848} \text{ where } k = 56.94\frac{l}{\eta^2 d^{4.848}}$$

Typical linearization process

1. Identify decision variables and parameters in the fundamental equation
2. Simplify equations by lumping parameters
3. Make simplifying assumptions, such as:
   a. Approximations for numbers close to 0 or close to 1 (e.g. $\sin x \approx x$ for small x)
   b. Binary instead of continuous variables in products of variables (many components such as pumps just have on/off controllers, can be modelled with a binary variable)
4. Use linearization tricks for the remaining nonlinearities (Lecture 10)
   a. Piecewise Affine Approximations
   b. Products of binary and continuous variables
5. Test and validate different linearization options. Performance criteria:
   a. Solving speed
   b. Constraint violations when plugging linearized optimal solution into original equations
   c. Change in objective function between linearized and original model

Several iterations may be required

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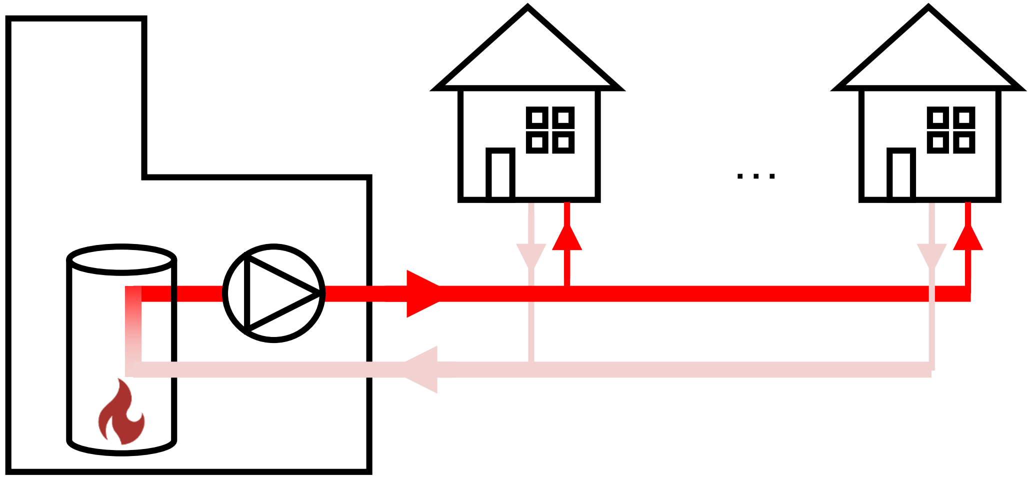
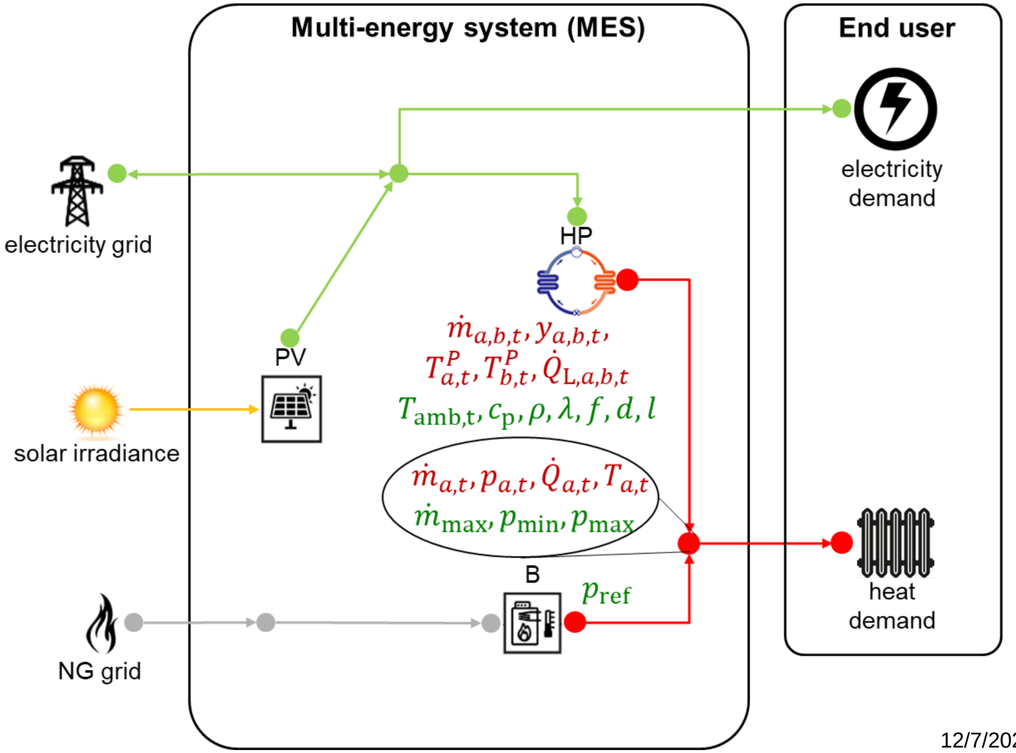
District heating networks

District heating: Typical elements

- Central heat source (e.g. waste incineration plant)
- Distribution with insulated water pipes
- Closed system: Supply and return network
- Circulation pumps

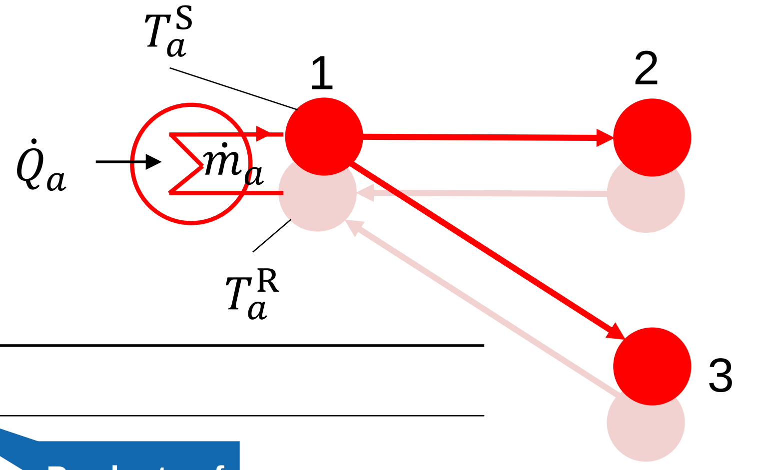
Difference to gas network where mass exits at loads





District heating: Modeling

- Assumptions: Steady state, identical supply (S) and return (R) networks



Decision Variables

Nodes $\mathcal{N}$	Temperature $T_a \forall a \in \mathcal{N}$
	Mass injection $\dot{m}_a \ \forall a \in \mathcal{N}$
	Pressure $p_a \ \forall a \in \mathcal{N}$
	Power injection $\dot{Q}_a \ \forall a \in \mathcal{N}$
Links $\mathcal{L}$	Mass flow $\dot{m}_{a,b} \forall (a,b) \in \mathcal{L}$
	Pipe end temperatures $T_a^P, T_b^P \forall (a,b) \in \mathcal{L}$
	Energy Losses $\dot{Q}_{L,a,b} \forall (a,b) \in \mathcal{L}$

It becomes a parameter in load nodes

Constraints

Node power injection $\dot{Q}_a = c_p \dot{m}_a (T_a^S - T_a^R) \forall a \in \mathcal{N}$
Injection limits: $-\dot{m}_{a,\max} \leq \dot{m}_a \leq \dot{m}_{a,\max} \forall a \in \mathcal{N}$
Pressure limit: $p_{a,\min} \leq p_a \leq p_{a,\max} \forall a \in \mathcal{N}$
Reference node: Fixed pressure $p_1 = p_{\mathrm{ref}}$
Mass conservation: $\dot{m}_a + \sum_b \dot{m}_{b,a} - \sum_b \dot{m}_{a,b} = 0 \ \forall a \in \mathcal{N}$
Mixing Energy Balance: $\left(\dot{m}_a^I T_a^I + \sum_b \dot{m}_{b,a} T_a^P\right) = \left(\dot{m}_a^E + \sum_b \dot{m}_{a,b}\right) T_a^{S/R} \ \forall a \in \mathcal{N}$

Products of variables: Nonlinear

Flow Limit: $0 \leq \dot{m}_{a,b} \leq \dot{m}_{a,b,\max} \forall (a,b) \in \mathcal{L}$
Pipe heat loss (energy): $\dot{Q}_{L,a,b} = c_p \dot{m}_{a,b} (T_a^P - T_b^P) \ \forall (a,b) \in \mathcal{L}$
Pipe heat loss (temperature drop): $T_b^P = (T_a^P - T_{\mathrm{amb}}) e^{-\frac{\lambda l}{c_p \dot{m}_{a,b}}} + T_{\mathrm{amb}} \ \forall (a,b) \in \mathcal{L}$
Fluid flow: $p_a - p_b = 0.01 \frac{8 l f}{\pi^2 \rho d^5} \dot{m}_{a,b}^2 \ \forall (a,b) \in \mathcal{L}$

Nonlinear

NLP

Pressure in bar, mass flows in kg/s

f Friction factor (depends on $\dot{m}_{a,b}$)

Parameters

c_p, λ, ρ Fluid heat capacity, thermal conductivity and density

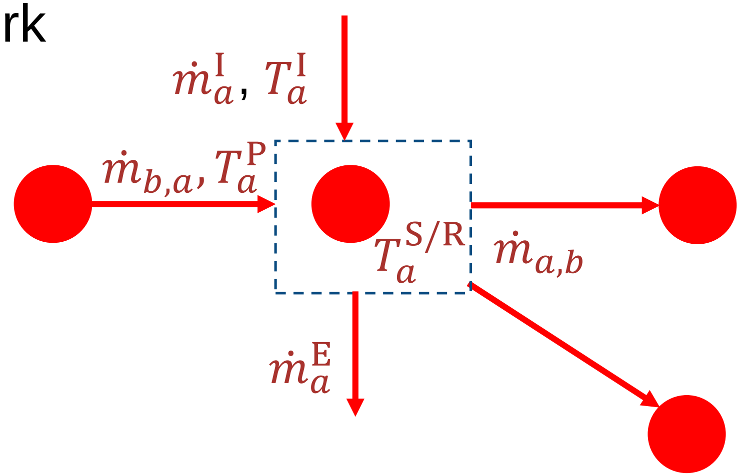
l, d pipe length (l) and diameter (d) in m

T_{amb} ambient temperature

Liu, Xuezhi. "Combined Analysis of Electricity and Heat Networks." Cardiff University, 2013.

District heating: Mixing energy balance

- 2 equations per node, one on the supply (S) and return (R) network
- Control volume: Supply (or return) node
- Incoming flows
  - Mass flows from other nodes $\dot{m}_{b,a}$ at pipe end temperature T_a^{P}
  - Mass flow injected at node a $\dot{m}_a^{\mathrm{I}}$ at injection temperature T_a^{I} (load nodes inject into the return network, generation nodes inject into the supply network)
- Outgoing flows all at the same temperature $T_a^{\mathrm{S/R}}$
  - Mass flows to other nodes $\dot{m}_{a,b}$
  - Mass flow extracted at node a $\dot{m}_a^{\mathrm{E}}$ (load nodes extract from the supply network, generation nodes extract from the return network)



$$\left(\dot{m}_a^{\mathrm{I}} T_a^{\mathrm{I}} + \sum_b \dot{m}_{b,a}\, T_a^{\mathrm{P}}\right) = \left(\dot{m}_a^{\mathrm{E}} + \sum_b \dot{m}_{a,b}\right) T_a^{\mathrm{S/R}}$$

District heating: Linearization

- Problem would be linear, if mass flows were parameters:
  - Fix mass flows, assuming circulation pump that is always switched on → **LP**
  - Discretize mass flow, e.g. $\dot{m} = ay$ where $y \in \{0,1\}$, use linearization of a product of a binary and a continuous variable (see Lecture 10) → **MILP**

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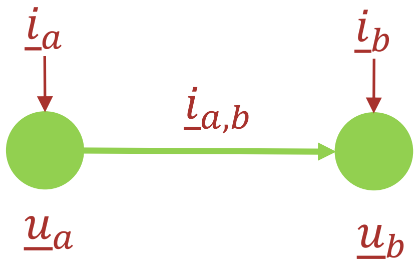
Alternating current electricity networks

AC electricity networks: Modeling

- Assumptions: Steady state, symmetrical 3-phase network, sinusoidal voltages & currents (complex numbers such as $\underline{u}$ to express magnitude u and phase angle θ). Complex power $\underline{S}$: $\mathrm{P} = \mathrm{real}\{\underline{S}\}, \mathrm{Q} = \mathrm{imag}\{\underline{S}\}$



Nodes $\mathcal{N}$

	Decision Variables
1	Current Injection $\underline{i}_a \forall\, a \in \mathcal{N}$
2	Voltages $\underline{u}_a \forall\, a \in \mathcal{N}$
3	Power Injection $\underline{S}_a \forall\, a \in \mathcal{N}$

Parameter in load nodes

Constraints	
Injection limits: $0 \leq S_a \leq S_{a,\max} \forall\, a \in \mathcal{N}$	
Voltage magnitude limit: $u_{a,\min} \leq u_a \leq u_{a,\max} \forall\, a \in \mathcal{N}$	a
Kirchhoff's Current Law: $\underline{i}_a + \sum_b \underline{i}_{b,a} - \sum_b \underline{i}_{a,b} = 0 \;\forall\, a \in \mathcal{N}$	b
Power Injection: $\underline{S}_a = \underline{u}_a \underline{i}_b^* \forall\, a \in \mathcal{N}$	c
Reference node: Fixed voltage $\underline{u}_1 = \underline{u}_{\mathrm{ref}}$	d

NLP

nonlinear product of variables. * := complex conjugate

Bidirectional flow possible

Links $\mathcal{L}$

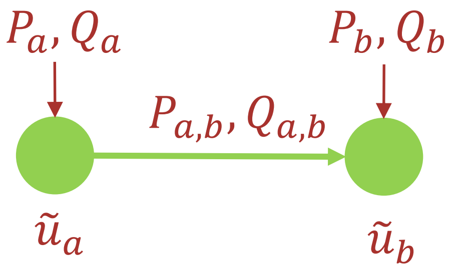
	Decision Variables
4	Currents $i_{a,b} \forall\, (a,b) \in \mathcal{L}$
	Losses $\underline{S}_{L,a,b} \forall\, (a,b) \in \mathcal{L}$

Constraints	
Flow Limit: $-i_{a,b,\max} \leq i_{a,b} \leq i_{a,b,\max} \forall\, (a,b) \in \mathcal{L}$	e
Ohm's Law: $\underline{u}_a - \underline{u}_b = \underline{z}\underline{i}_{a,b} \forall\, (a,b) \in \mathcal{L}$ where $\underline{z}$ is the impedance	f
Energy loss equation: $\underline{S}_{L,a,b} = \underline{z} i_{a,b}^2 \forall\, (a,b) \in \mathcal{L}$	g

Nonlinear and often not necessary in optimization: can be calculated after solving

AC linearization: Linear DistFlow

LP

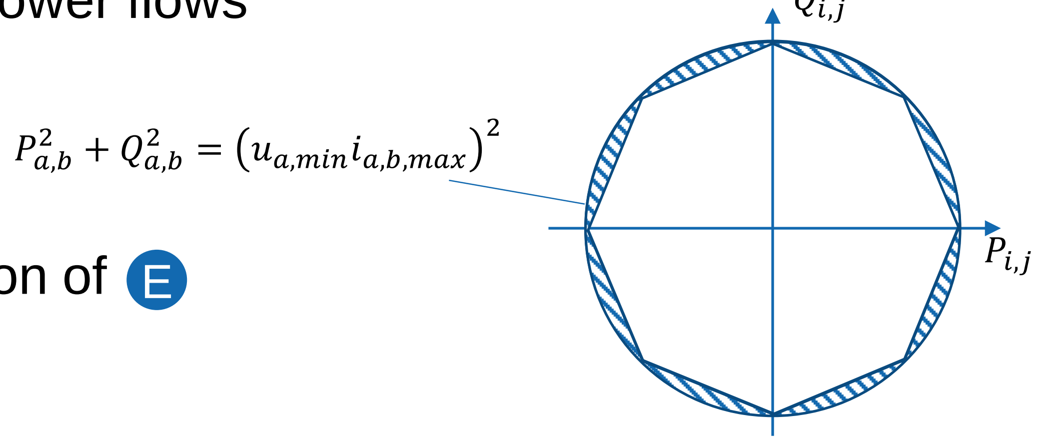


- Applicable in distribution systems
- Assumptions: No losses, radial topology (no loops)
- Remove: Current Injections 1, Power Injection law c, losses g
- Replace:
  - Voltages 2 by squared voltage magnitudes $\tilde{u}_a$
  - Apparent Power 3 by active P_a and reactive Q_a power injections
  - Line currents 4 by active $P_{a,b}$ and reactive $Q_{a,b}$ power flows on lines
  - Kirchhoff's law b by power balances B
  - Line flow limits e by polygonal inner approximation of E
  - Ohm's law f by F

B
$$P_a + \sum_b P_{b,a} - \sum_b P_{a,b} = 0 \; \forall \; a \in \mathcal{N}$$
$$Q_a + \sum_b Q_{b,a} - \sum_b Q_{a,b} = 0 \; \forall \; a \in \mathcal{N}$$

E
$$P_{a,b}^2 + Q_{a,b}^2 \leq \left(u_{a,\min} i_{a,b,\max}\right)^2 \; \forall \; (a,b) \in \mathcal{L}$$

F
$$\tilde{u}_a - \tilde{u}_b = 2\left(r_{a,b} P_{a,b} + x_{a,b} Q_{a,b}\right) \; \forall \; (a,b) \in \mathcal{L}$$



Nick, M. et al. *IEEE Transactions on Power Systems*, **33(2)**, 1577-1590, 2017

After this lecture, you are able to …

- ✓ Describe the energy network modeling process
- ✓ Define a generic energy network with
  - ✓ Graphs, node and link quantities
  - ✓ Decision variables and constraints
- ✓ Solve network equations with backward-forward sweep
- ✓ Describe the process of linearizing physical laws
- ✓ Define network optimization models for gas, electricity and thermal networks
- Model non-unique flow directions and topology changes

Network model extensions

Network topology changes

MILP

- Idea
  - Introduce binary variables $y_{a,b}$ indicating that link (a, b) is active
  - Force flows to be zero if $y_{a,b} = 0$
  - Relax flow laws with binary variable to avoid enforcing equal potential at the two ends of inactive links ($p_a^2 - p_b^2 = k0^{1.848}$)
- Applications
  - Non-unique flow directions
  - Switching links on/off for loss reduction
  - Network expansion
  - Modelling failures and repair in the network

$$0 \leq \dot{m}_{a,b} \leq \dot{m}_{a,b,\max}$$

$$\Downarrow$$

$$0 \leq \dot{m}_{a,b} \leq y_{a,b}\dot{m}_{a,b,\max}$$

$$p_a^2 - p_b^2 = k\,\dot{m}_{a,b}^{1.848}$$

$$\Downarrow$$

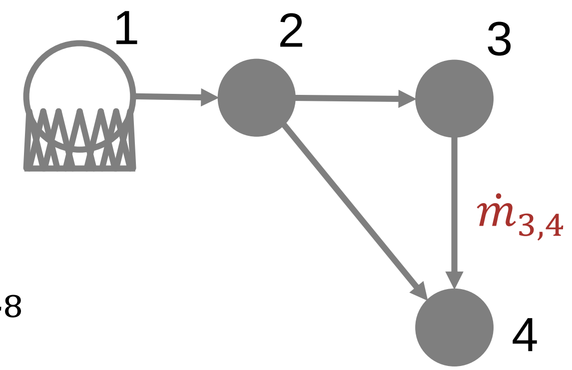
$$p_a^2 - p_b^2 \leq k\,\dot{m}_{a,b}^{1.848} + 10k\dot{m}_{a,b,\max}^{1.848}(1 - y_{a,b})$$

$$p_a^2 - p_b^2 \geq k\,\dot{m}_{a,b}^{1.848} - \underbrace{10k\dot{m}_{a,b,\max}^{1.848}}_{\text{“Sufficiently big” number}}(1 - y_{a,b})$$

“Sufficiently big” number: Large enough to be non-limiting, small enough for solver numerics

Non-unique flow directions

MILP



$$p_a^2 - p_b^2 = k\,\dot{m}_{a,b}^{1.848}$$

- What happens in **gas** and **thermal** networks if the flow direction is not defined a priori?

- Simplified gas network equation:
  - If $p_3 > p_4$, then $(p_3^2 - p_4^2) > 0$ and $\dot{m}_{3,4} > 0$
  - If $p_3 < p_4$, then $(p_3^2 - p_4^2) < 0$ and there is no real-valued solution

- Solution when solving manually: Use $\dot{m}_{4,3}$ instead of $\dot{m}_{3,4}$

- Solution when running an optimization: Relaxation of the flow law using binary variables $y_{3,4}$ and $y_{4,3}$

$$y_{3,4} + y_{4,3} = 1$$

$$0 \leq \dot{m}_{4,3} \leq y_{4,3}\dot{m}_{3,4,\text{max}}$$

$$0 \leq \dot{m}_{4,3} \leq y_{4,3}\dot{m}_{3,4,\text{max}}$$

$$p_3^2 - p_4^2 \leq k\,\dot{m}_{3,4}^{1.848} + 10k\dot{m}_{3,4,\text{max}}^{1.848}\left(1 - y_{3,4}\right)$$

$$p_3^2 - p_4^2 \geq k\,\dot{m}_{3,4}^{1.848} - 10k\dot{m}_{3,4,max}^{1.848}(1 - y_{3,4})$$

$$p_4^2 - p_3^2 \leq k\,\dot{m}_{4,3}^{1.848} + 10k\dot{m}_{3,4,\text{max}}^{1.848}\left(1 - y_{4,3}\right)$$

$$p_4^2 - p_3^2 \geq k\,\dot{m}_{4,3}^{1.848} - 10k\dot{m}_{3,4,max}^{1.848}(1 - y_{4,3})$$

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Combining MES technology and network optimization

Combined MES technology and network optimization

Objective function

$$\min_{\boldsymbol{x}} \; z_{\text{cost}}$$

$$\min_{\boldsymbol{x}} \; z_{\text{CO2}}$$

$$\min_{\boldsymbol{x}} \; z_{\text{risk}}$$

s.t.

RE (non-dispatchable) technologies

Lectures 9, 10

Conventional (dispatchable) technologies

Lectures 9, 10

Storage technologies

Lecture 10

⋮

MES structure

$\mathcal{J}$ = Set of energy carriers, {E, G, H}

$\mathcal{N}$ = Set of nodes

$\mathcal{L}$ = Set of links

$\mathcal{K}_{\text{a}}$ = Set of technologies, installed at node a

T = Length of the time horizon

Combined MES technology and network optimization

⋮

Energy balances

$$\sum_{k\in\mathcal{K}_a}\left(V_{G,k,t}-U_{G,k,t}\right)-D_{G,a,t}=\mathrm{LHV}\dot{m}_{a,t},$$

Lower heating value to convert between mass flow and power

$$\sum_{k\in\mathcal{K}_a}\left(V_{H,k,t}-U_{H,k,t}\right)-D_{H,a,t}=\dot{Q}_{a,t},$$

$$\sum_{k\in\mathcal{K}_a}\left(V_{E,k,t}-U_{E,k,t}\right)-D_{E,a,t}=P_{a,t},$$

$$D_{E,a,t}f_Q=Q_{a,t},$$

Reactive power at load nodes, proportional to active power demand

$$M_{Q,t}-N_{Q,t}=Q_{1,t}$$

$$M_{E,t}-N_{E,t}=P_{1,t}$$

$$M_{G,t}=\mathrm{LHV}\dot{m}_{1,t},$$

Node 1: Connection to the distribution grids

$$0\le M_{j,t}\le M_{j,\max},\quad 0\le N_{j,t}\le N_{j,\max}\qquad \forall a\in\mathcal{N}\backslash\{1\},t\in\{1,\dots,T\}$$

⋮

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Combined MES technology and network optimization

⋮

Gas network

$$-\dot{m}_{a,\max} \leq \dot{m}_{a,t} \leq \dot{m}_{a,\max}$$

$$p_{a,\min} \leq p_{a,t} \leq p_{a,\max}$$

$$p_{1,t} = p_{\mathrm{ref}}$$

$$\dot{m}_{a,t} + \sum_b \dot{m}_{b,a,t} - \sum_b \dot{m}_{a,b,t} = 0$$

$$p_{a,t}^2 - p_{b,t}^2 = k\, \dot{m}_{a,b,t}^{1.848} \qquad \forall a \in \mathcal{N}, (a,b) \in \mathcal{L}, t \in \{1, \dots, T\}$$

⋮

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⋮

Thermal network

$$\dot{Q}_{a,t} = c_p \dot{m}_{a,t} (T^{\mathrm{S}}_{a,t} - T^{\mathrm{R}}_{a,t})$$

$$-\dot{m}_{a,\max} \leq \dot{m}_{a,t} \leq \dot{m}_{a,\max}$$

$$p_{a,\min} \leq p_{a,t} \leq p_{a,\max}$$

$$p_{1,t} = p_{\mathrm{ref}}$$

$$\dot{m}_{a,t} + \textstyle\sum_b \dot{m}_{b,a,t} - \sum_b \dot{m}_{a,b,t} = 0$$

$$\left(\dot{m}^{\mathrm{E}}_{a,t} + \textstyle\sum_b \dot{m}_{a,b,t}\right) T^{\mathrm{S/R}}_{a,t} = \left(\dot{m}^{\mathrm{I}}_{a,t} T^{I}_{a,t} + \sum_b \dot{m}_{b,a,t}\, T^{\mathrm{P}}_{a,t}\right)$$

$$0 \leq \dot{m}_{a,b,t} \leq \dot{m}_{a,b,\max}$$

$$\dot{Q}_{\mathrm{L},a,b,t} = c_{\mathrm{p}} \dot{m}_{a,b,t} (T^{\mathrm{P}}_{a,t} - T^{\mathrm{P}}_{b,t})$$

$$T^{\mathrm{P}}_{b,t} = \left(T^{\mathrm{P}}_{a,t} - T_{\mathrm{amb},t}\right) e^{-\frac{\lambda l}{c_{\mathrm{p}} \dot{m}_{a,b,t}}} + T_{\mathrm{amb},t}$$

$$p_{a,t} - p_{b,t} = 0.01 \frac{8 l f}{\pi^2 \rho d^5} \dot{m}^2_{a,b,t}$$

$$\forall a \in \mathcal{N}, (a,b) \in \mathcal{L}, t \in \{1, \dots, T\}$$

⋮

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Electricity network (DistFlow)

$$-P_{a,\max} \leq P_{a,t} \leq P_{a,\max}$$

$$-Q_{a,\max} \leq Q_{a,t} \leq Q_{a,\max}$$

$$u_{a,\min}^2 \leq \tilde{u}_{a,t} \leq u_{a,\max}^2$$

$$\tilde{u}_{1,t} = u_{\mathrm{ref}}^2$$

$$P_{a,t} + \sum_b P_{b,a,t} - \sum_b P_{a,b,t} = 0$$

$$Q_{a,t} + \sum_b Q_{b,a,t} - \sum_b Q_{a,b,t} = 0$$

$$P_{a,b,t}^2 + Q_{a,b,t}^2 \leq \left(u_{a,\min} i_{a,b,\max}\right)^2$$

$$\tilde{u}_{a,t} - \tilde{u}_{b,t} = 2\left(r_{a,b} P_{a,b,t} + x_{a,b} Q_{a,b,t}\right)$$

$$\forall a \in \mathcal{N}, (a,b) \in \mathcal{L}, t \in \{1, \dots, T\}$$

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MES optimization: The full picture