Introduction to Modeling and Optimization of Sustainable Energy Systems:

Heat exchanger networks

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Energy and Process Systems Engineering

Since the second lecture, you are able to…

- ✓ explain the idea underlying **the heat integration problem**
- ✓ apply the **pinch rules** to heat integration problems.
- ✓ thermodynamically analyze **heat exchangers** with the **pinch method**.
- ✓ integrate **external utilities** by using the grand composite curve
- ✓ interpret **heat integration as optimization problem**

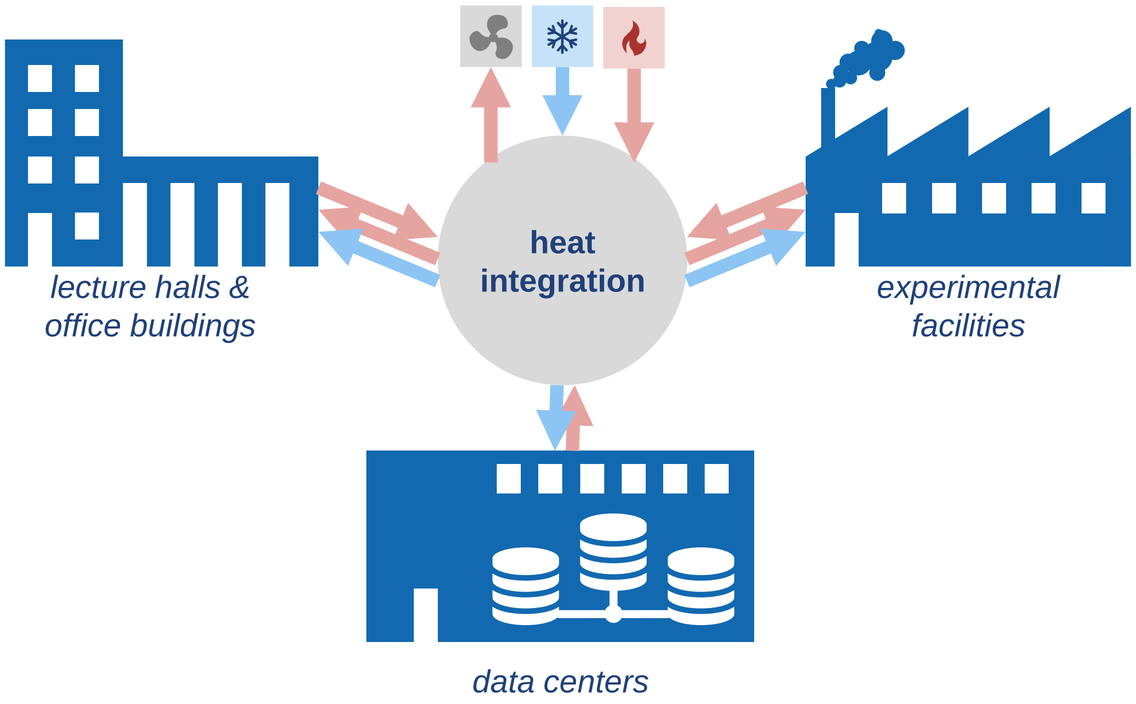
The heat integration problem

Given multiple hot streams that need cooling

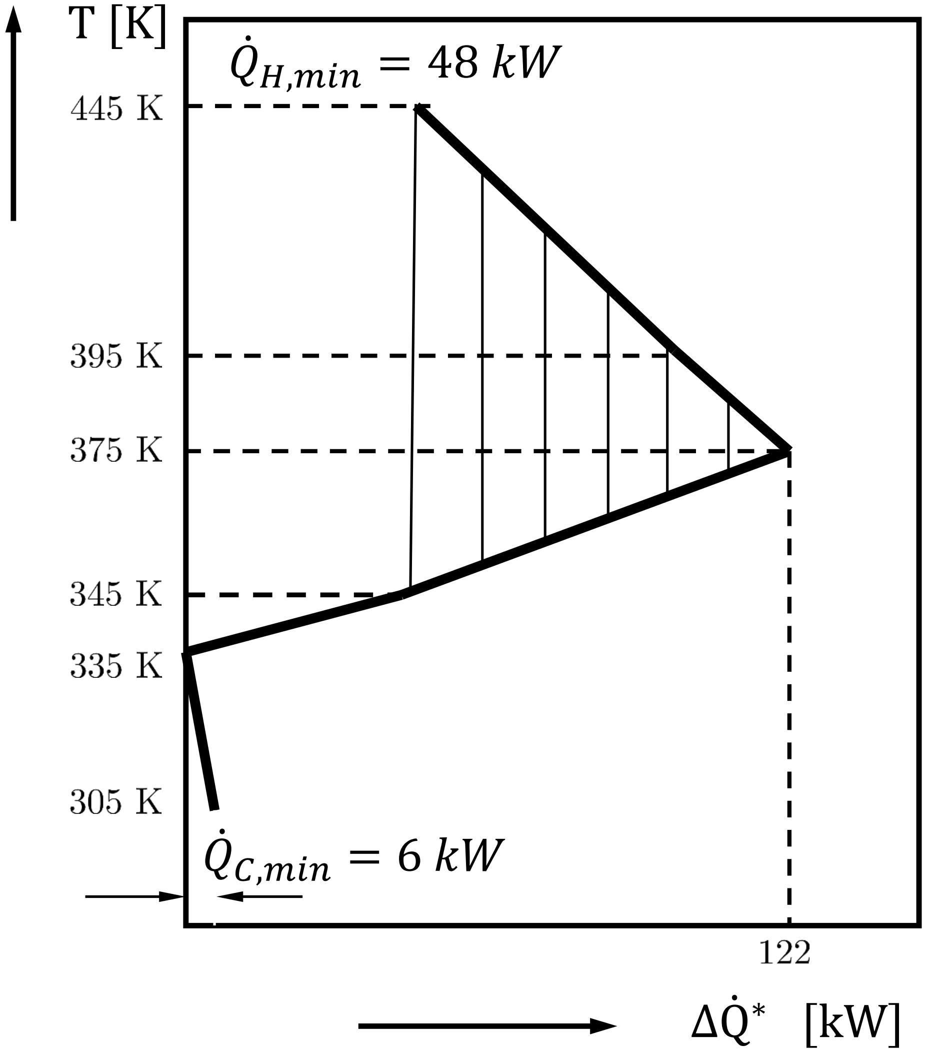
and multiple cold streams that need heating,

and considering the cost of external heating & cooling utilities,

how to optimally combine (integrate) heating & cooling demands to minimize cost?



Grand composite curve of the example process

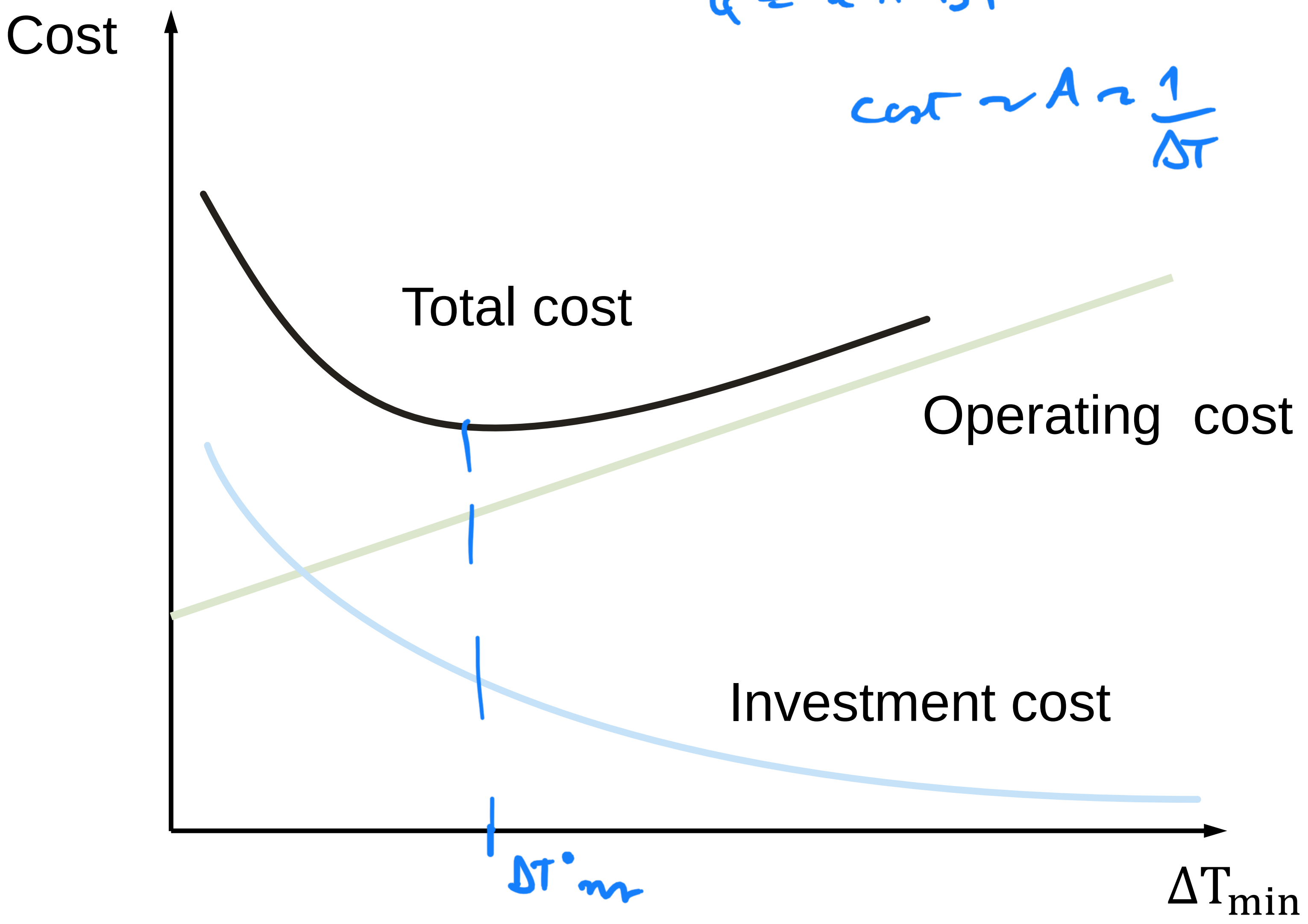


Total Cost vs. ΔT_{min}

total costs = investment costs + operation costs

$\dot{Q} = \alpha A \Delta T$

$\text{cost} \sim A \sim \frac{1}{\Delta T}$



Literature Recommendations

Additional reading material in MOODLE

Heat integration

- R. Smith, 2016, Chemical Process Design and Integration, 2nd Edition, Wiley.
- S. Papoulias, I.E. Grossmann, 1983, A structural optimization approach in process synthesis – II Heat Recovery Networks, *Computers & Chemical Engineering*, 7(6), 707–721.

Optimization

- I.E. Grossmann, 2021, Advanced Optimization for Process Systems Engineering, Cambridge: Cambridge University Press.
- R. Sioshansi, A.J. Conejo, 2017, Optimization in Engineering, Springer International Publishing.
- J. Kallrath, 2013, Gemischt-ganzzahlige Optimierung: Modellierung in der Praxis, 2nd Edition, Springer Vieweg.

Life-Cycle Analysis

- J.B. Guinee, 2002, Handbook on Life Cycle Assessment, Springer, Dordrecht.
- N. von der Assen, P. Voll, M. Peters, A. Bardow, 2014, Life cycle assessment of CO2 capture and utilization: a tutorial review, *Chem. Soc. Rev.*, 43, 7982-7994.
- J. Kleinekorte, L. Fleitmann, M. Bachmann, A. Kätelhön, A. Barbosa-Póvoa, N. von der Assen, A. Bardow, 2020, Life Cycle Assessment for the Design of Chemical Processes, Products, and Supply Chains, *Annual Review of Chemical and Biomolecular Engineering*, 11(1), 203-233.

Lecture plan

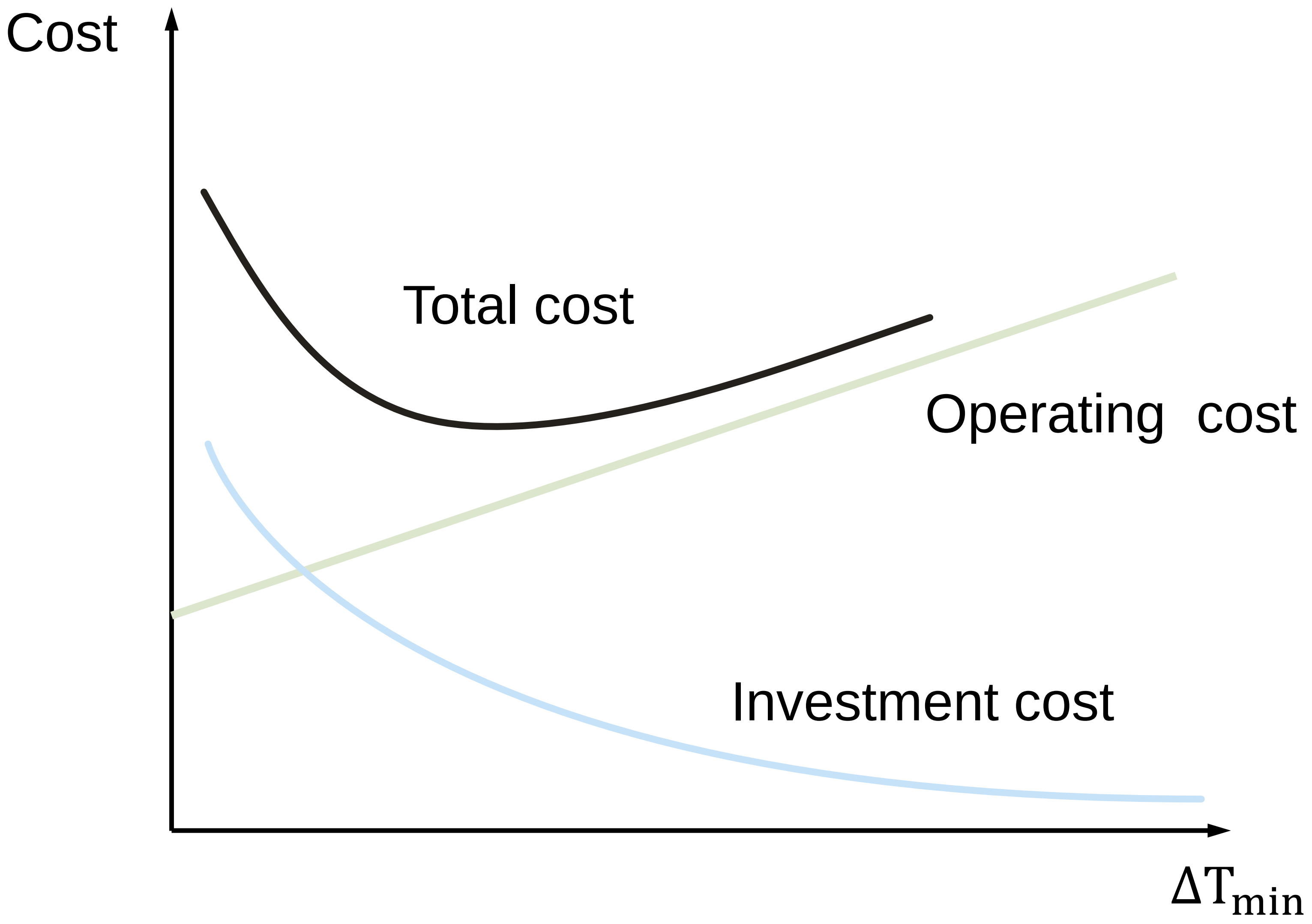
No.	Date	Content	
1	29.09.	Introduction & Models	
2	06.10.	Heat integration	Applications
3	13.10.	Continuous Optimization	Methods
4	20.10.	Heat exchanger networks	Applications
5	27.10.	Discrete Optimization	Methods
6	03.11.	Life Cycle Assessment (LCA)	Metrics
7	10.11.	Thermoeconomics	Metrics
8	17.11.	Risk Key Performance Indicators for Security	Metrics
9	24.11.	Multi-energy dimension: introduction	Methods & Applications
10	01.12.	Design dimensions: technology modelling	
11	08.12.	Space dimensions: energy networks	
12	15.12.	Uncertainty in energy systems	
13	22.12.	Recap (online)	

After this lecture, you are able to…

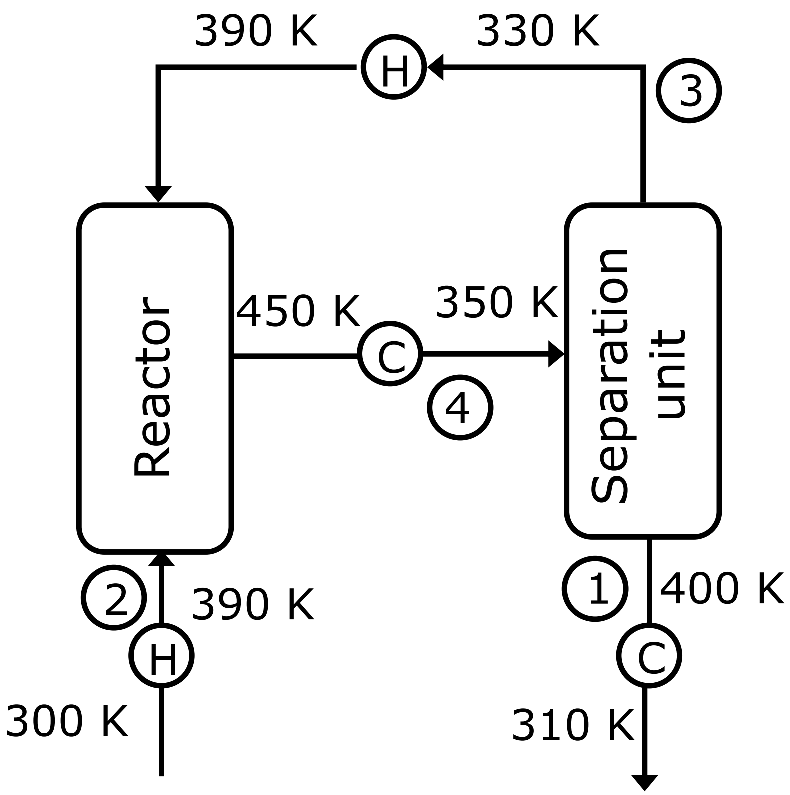
- formulate **heat integration as optimization problem**
- **design networks** for heat integration of hot and cold streams
  using pinch technology.
- find a **feasible solution** for the heat exchanger design problem using
  heuristics.
- formulate the **modelling equations**
  for designing **heat exchanger networks.**

Total Cost vs. ΔT_{min}

total costs = investment costs + operation costs



Hot and cold streams Process example



Stream table

Stream-No.	T_{in} [K]	T_{out} [K]	$\dot{C}_p$ [kW/K]
1(h)	400	310	2
2(c)	300	390	1.8
3(c)	330	370	4
4(h)	450	350	1

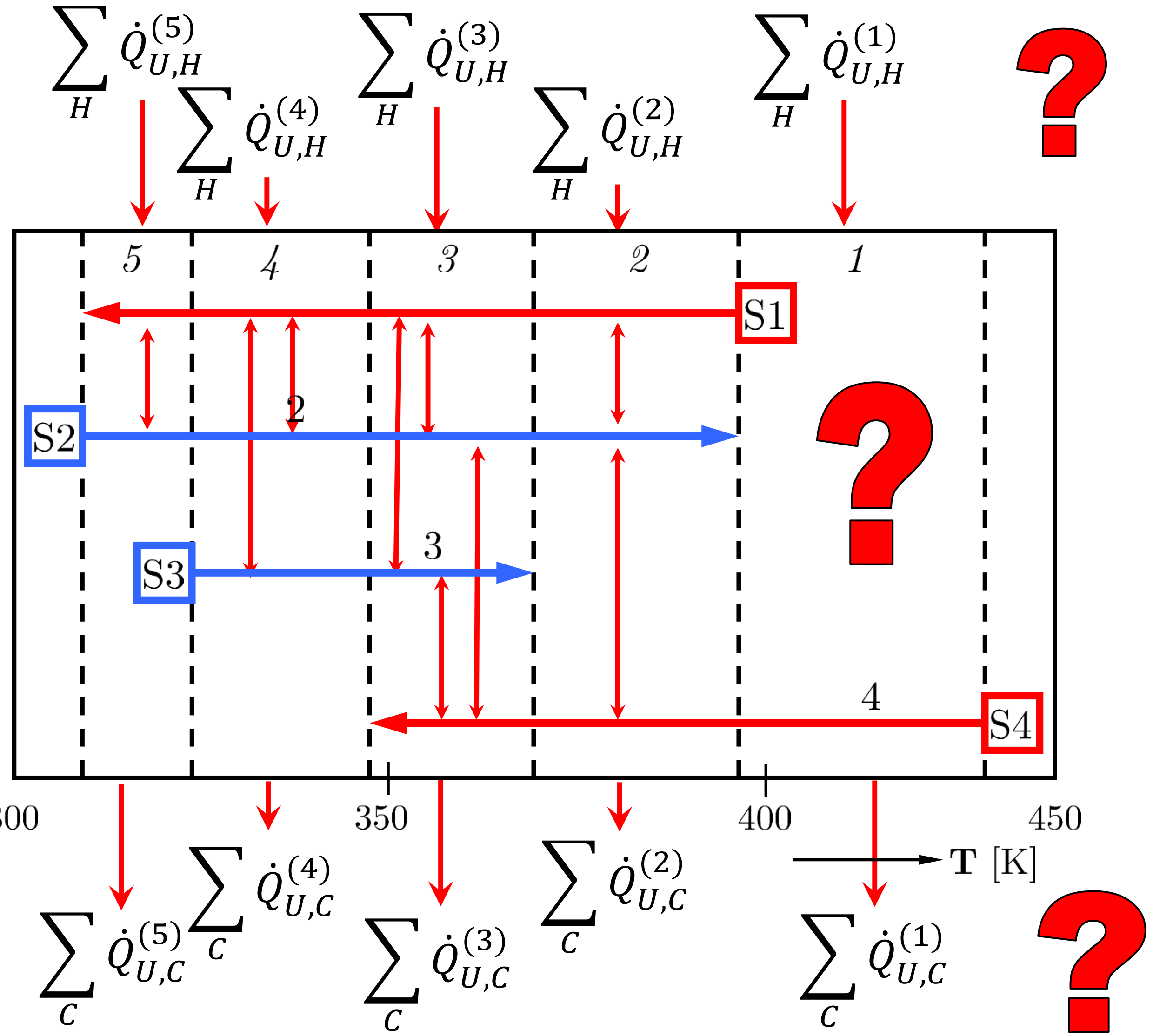
1. minimization of operating costs

H utilities for heating (prices: $c_{U,H}$)

Heat integration

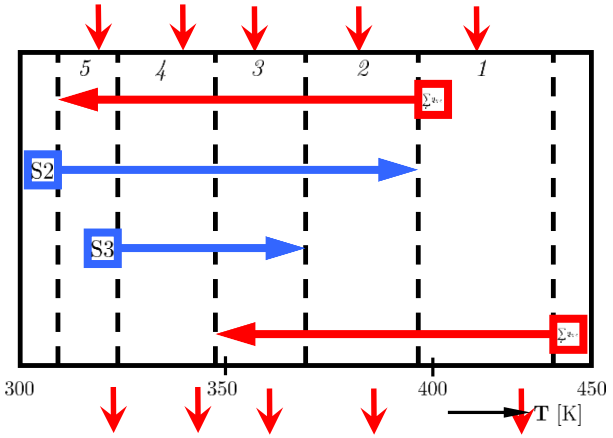
h hot streams (1, 4) and c cold streams (2, 3)

C utilities for cooling (prices: $c_{U,C}$)



Energy balances

$\sum_H \dot{Q}_{U,H}$



$\sum_C \dot{Q}_{U,C}$

Int. No.	Temperature	$\sum \dot{m}c_p$ $[kW/K]$	$\dot{Q}_z$ $[kW]$	$\Delta\dot{Q}_z$	$\Delta\dot{Q}_z^*$
	445 K			+0 [kW]	+48 [kW]
1		+1.0	+50	+50 [kW]	+98 [kW]
	395 K				
2		+1.2	+24	+74 [kW]	+122 [kW]
	375 K				
3		-2.8	-84	-10 [kW]	+38 [kW]
	345 K				

$Energy\ balances\ \forall\ intervals\ z$:

Residual heat flows: $\Delta\dot{Q}_{z-1}^*$ (in), $\Delta\dot{Q}_z^*$ (out)

hot/cold streams: $\sum_h (\dot{m}c_p)_h \Delta T^{(z)}$ → interval z → $\sum_c (\dot{m}c_p)_c \Delta T^{(z)}$

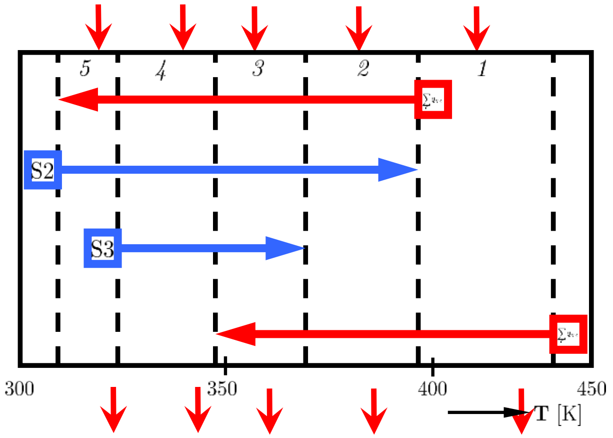
hot/cold utilities: $\sum_H \dot{Q}_{U,H}^{(z)}$ → interval z → $\sum_C \dot{Q}_{U,C}^{(z)}$

$$0 = \sum_h (\dot{m}c_p)_h \Delta T^{(z)} + \sum_H \dot{Q}_{U,H}^{(z)} - \sum_c (\dot{m}c_p)_c \Delta T^{(z)} - \sum_C \dot{Q}_{U,C}^{(z)} + \Delta\dot{Q}_{z-1}^* - \Delta\dot{Q}_z^*,$$

$$\Delta\dot{Q}_0^* = \Delta\dot{Q}_{z_{max}}^* = 0$$

Formulation of Optimization problem

$$\sum_H \dot{Q}_{U,H}$$



$$\sum_C \dot{Q}_{U,C}$$

costs

$$\min_{\dot{Q}_{U,H},\dot{Q}_{U,C}} \sum_H c_{U,H} \cdot \dot{Q}_{U,H} + \sum_C c_{U,C} \cdot \dot{Q}_{U,C}$$

$s.t.$

$$\forall H \quad \dot{Q}_{U,H} = \sum_{z=1}^{z_{max}=5} \dot{Q}_{U,H}^{(z)},$$

$$\forall C \quad \dot{Q}_{U,C} = \sum_{z=1}^{z_{max}=5} \dot{Q}_{U,C}^{(z)},$$

$$\sum_{intervals}$$

Papoulias, Grossmann. *A structural optimization approach in process synthesis II: Heat recovery networks.* Comput Chem Eng, *7*:707-721, 1983.

Operating costs minimization (LP)

$$\min_{\dot{Q}_{U,H},\dot{Q}_{U,C},\Delta\dot{Q}_z^*} \sum_H c_{U,H} \cdot \dot{Q}_{U,H} + \sum_C c_{U,C} \cdot \dot{Q}_{U,C}$$

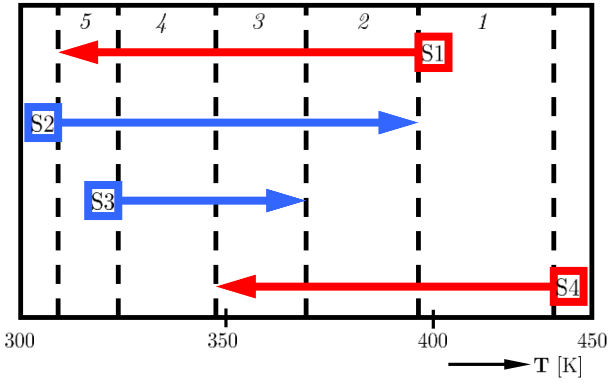
$s.t.$

$$\forall H \quad \dot{Q}_{U,H} = \sum_{z=1}^{z_{max}=5} \dot{Q}_{U,H}^{(z)},$$

$$\forall C \quad \dot{Q}_{U,C} = \sum_{z=1}^{z_{max}=5} \dot{Q}_{U,C}^{(z)},$$

$$(\dot{m}c_p)_h \Delta T^{(z)}$$

$$(\dot{m}c_p)_c \Delta T^{(z)}$$



$$\forall z \quad 0 = \sum_h (\dot{m}c_p)_h \Delta T^{(z)} + \sum_H \dot{Q}_{U,H}^{(z)} - \sum_c (\dot{m}c_p)_c \Delta T^{(z)} - \sum_C \dot{Q}_{U,C}^{(z)} + \Delta\dot{Q}_{z-1}^* - \Delta\dot{Q}_z^*,$$

$$\Delta\dot{Q}_0^* = 0, \Delta\dot{Q}_{z_{max}}^* = 0,$$

$$\Delta\dot{Q}_z^* \geq 0, \dot{Q}_{U,H}^{(z)} \geq 0, \dot{Q}_{U,C}^{(z)} \geq 0,$$

unknown

known

Operating costs minimization (LP)

$$\min_{\dot{Q}_{U,H}, \dot{Q}_{U,C}, \Delta\dot{Q}_z^*} \sum_H c_{U,H} \cdot \dot{Q}_{U,H} + \sum_C c_{U,C} \cdot \dot{Q}_{U,C}$$

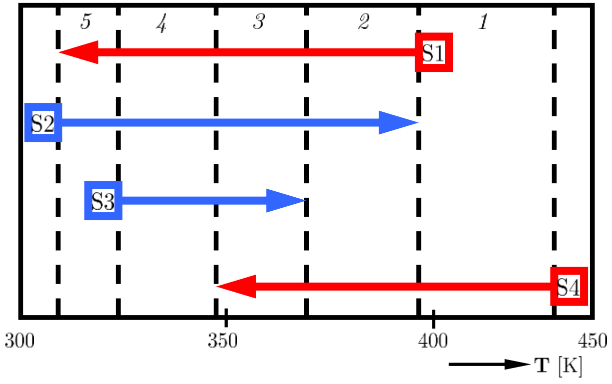
$s.t.$

$$\forall H \quad \dot{Q}_{U,H} = \sum_{z=1}^{z_{max}=5} \dot{Q}_{U,H}^{(z)},$$

$$\forall C \quad \dot{Q}_{U,C} = \sum_{z=1}^{z_{max}=5} \dot{Q}_{U,C}^{(z)},$$

$$\Delta\dot{H}_h^{(z)} = (\dot{m}c_p)_h \cdot \Delta T_h^{(z)}$$

$$\Delta\dot{H}_c^{(z)} = (\dot{m}c_p)_c \cdot \Delta T_c^{(z)}$$



$$\forall z \quad 0 = \sum_h \Delta\dot{H}_h^{(z)} + \sum_H \dot{Q}_{U,H}^{(z)} - \sum_c \Delta\dot{H}_c^{(z)} - \sum_C \dot{Q}_{U,C}^{(z)} + \Delta\dot{Q}_{z-1}^* - \Delta\dot{Q}_z^*,$$

$$\Delta\dot{Q}_0^* = 0, \Delta\dot{Q}_{z_{max}}^* = 0,$$

$$\Delta\dot{Q}_z^* \geq 0, \dot{Q}_{U,H}^{(z)} \geq 0, \dot{Q}_{U,C}^{(z)} \geq 0,$$

unknown

known

Heat integration as LP (complete)

$$\min_{\dot{Q}_{U,Steam},\dot{Q}_{U,Brine},\Delta\dot{Q}_Z^*} \quad c_{U,Steam} \cdot \dot{Q}_{U,Steam} + c_{U,Brine} \cdot \dot{Q}_{U,Brine}$$

$$s.t. \quad 0 = \underbrace{\sum_h \Delta\dot{H}_h^{(1)} - \sum_c \Delta\dot{H}_c^{(1)}}_{= (\dot{m}c_p)_4 \cdot \Delta T_1 - 0} + \dot{Q}_{U,Steam}^{(1)} - \dot{Q}_{U,Brine}^{(1)} + \Delta\dot{Q}_0^* - \Delta\dot{Q}_1^*,$$

$$\ldots$$

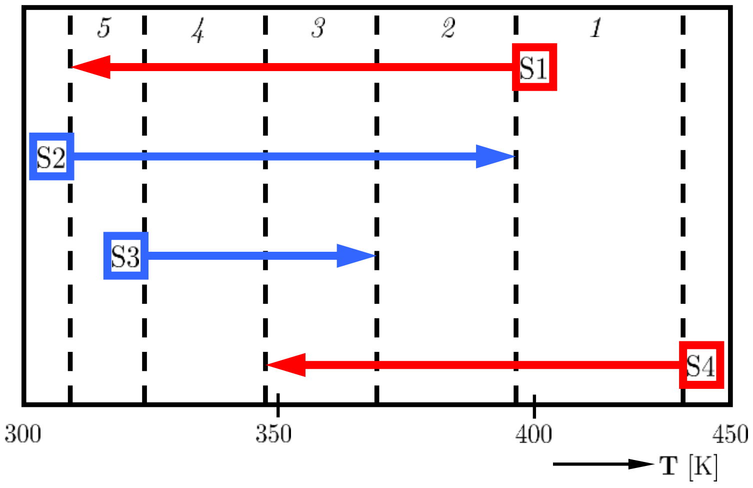
$$0 = \sum_h \Delta\dot{H}_h^{(5)} - \sum_c \Delta\dot{H}_c^{(5)} + \dot{Q}_{U,Steam}^{(5)} - \dot{Q}_{U,Brine}^{(5)} + \Delta\dot{Q}_4^* - \Delta\dot{Q}_5^*,$$

$$0 = \Delta\dot{Q}_0^*,$$

$$0 = \Delta\dot{Q}_5^*,$$

$$\dot{Q}_{U,Steam} = \dot{Q}_{U,Steam}^{(1)} + \dot{Q}_{U,Steam}^{(2)} + \dot{Q}_{U,Steam}^{(3)} + \dot{Q}_{U,Steam}^{(4)} + \dot{Q}_{U,Steam}^{(5)},$$

$$\dot{Q}_{U,Brine} = \dot{Q}_{U,Brine}^{(1)} + \dot{Q}_{U,Brine}^{(2)} + \dot{Q}_{U,Brine}^{(3)} + \dot{Q}_{U,Brine}^{(4)} + \dot{Q}_{U,Brine}^{(5)}$$



Heat integration as LP

For example 1 CHF /kW

$$\min_{\dot{Q}_{U,Steam},\dot{Q}_{U,Brine},\Delta\dot{Q}_z^*} \; c_{U,Steam}\cdot\dot{Q}_{U,Steam} + c_{U,Brine}\cdot\dot{Q}_{U,Brine}$$

$$s.t.\quad 0 = \underbrace{\sum_h \Delta\dot{H}_h^{(1)} - \sum_c \Delta\dot{H}_c^{(1)}} + \dot{Q}_{U,Steam}^{(1)} - \dot{Q}_{U,Brine}^{(1)} + \Delta\dot{Q}_0^* - \Delta\dot{Q}_1^*,$$

$$= (\dot{m}c_p)_4 \cdot \Delta T_1 - 0$$

$$= 50\text{kW} = \dot{Q}_z$$

Int. No.	Temperature	$\sum\dot{m}c_p$ $[kW/K]$	$\dot{Q}_z$ $[kW]$	$\Delta\dot{Q}_z$	$\Delta\dot{Q}_z^*$
	445 K			+0 [kW]	+48 [kW]
1		+1.0	+50	+50 [kW]	+98 [kW]
	395 K				
2		+1.2	+24	+74 [kW]	+122 [kW]
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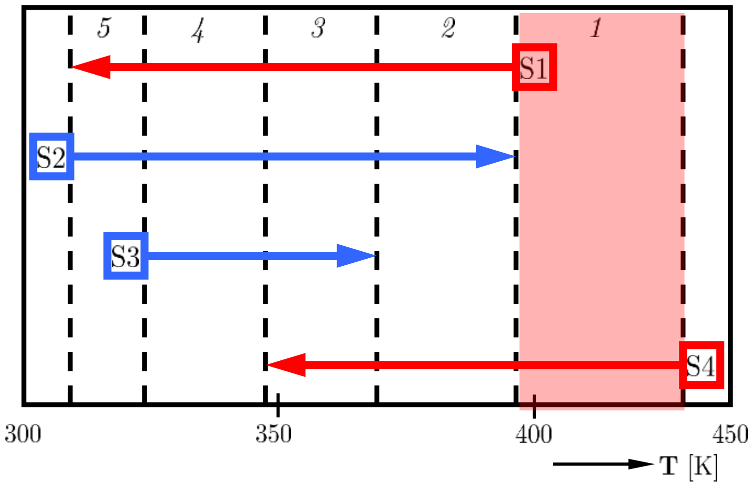
3		-2.8	-84	-10 [kW]	+38 [kW]
	345 K				



Online optimization tool

Tutorial & Code will be provided in MOODLE

*copy source code … website when you want to play around … he model

Assumption:
Only 1 utility each for heating (steam) and cooling (brine).

LP solution of the heat integration

$$\dot{Q}_{U,Steam} = \underbrace{\dot{Q}^{(1)}_{U,Steam}}_{=48\text{kW}} + \underbrace{\dot{Q}^{(2)}_{U,Steam}}_{=0\text{kW}} + \underbrace{\dot{Q}^{(3)}_{U,Steam}}_{=0\text{kW}} + \underbrace{\dot{Q}^{(4)}_{U,Steam}}_{=0\text{kW}} + \underbrace{\dot{Q}^{(5)}_{U,Steam}}_{=0\text{kW}} = 48\text{ kW}$$

$$\dot{Q}_{U,Brine} = \underbrace{\dot{Q}^{(1)}_{U,Brine}}_{=0\text{kW}} + \underbrace{\dot{Q}^{(2)}_{U,Brine}}_{=0\text{kW}} + \underbrace{\dot{Q}^{(3)}_{U,Brine}}_{=0\text{kW}} + \underbrace{\dot{Q}^{(4)}_{U,Brine}}_{=0\text{kW}} + \underbrace{\dot{Q}^{(5)}_{U,Brine}}_{=6\text{kW}} = 6\text{kW}$$

$\Delta\dot{Q}_1^* = 98\text{ kW}$

$\Delta\dot{Q}_2^* = 122\text{ kW}$

$\Delta\dot{Q}_3^* = 38\text{ kW}$

$\Delta\dot{Q}_4^* = 0\text{ kW}$ ⇔ Pinch

Int. No.	Temperature	$\sum\dot{m}c_p$ $[kW/K]$	$\dot{Q}_z$ $[kW]$	$\Delta\dot{Q}_z$	$\Delta\dot{Q}_z^*$
				+0 [kW]	+48 [kW]
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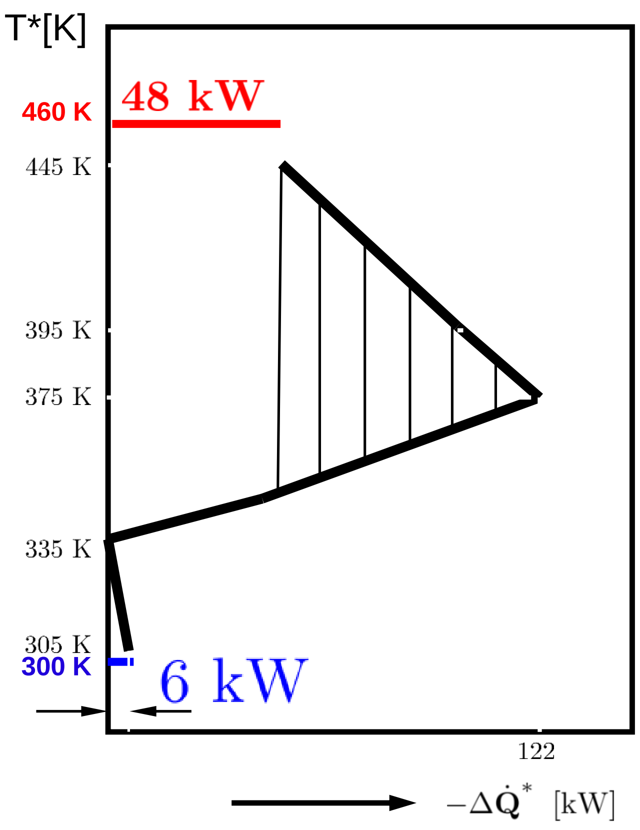
2		+1.2	+24	+74 [kW]	+122 [kW]
	375 K				
3		-2.8	-84	-10 [kW]	+38 [kW]
	345 K				
4		-3.8	-38	-48 [kW]	+0 [kW]
	335 K				
5		+0,2	+6	-42 [kW]	+6 [kW]
	305 K				

LP solution of the heat integration

ΔT_{min} = 10 K



Online optimization tool*

Economic Utilities Targeting without warm water
Economic Utilities Targeting with warm water

Utilities:
- Steam 465 K 0.033 CHF/kWh
- Brine 295 K 0.023 CHF/kWh

Steady-state operation during 8760 h/a.

Currently (no heat integration):
→ 131‘000 CHF/a

Heat integrated solution:

$\dot{Q}_{U,Steam} = 48\text{kW}$ $\dot{Q}_{U,Brine} = 6\text{kW}$

→ 15‘100 CHF/a (-88 %)

Additional heating utility:
- Warm water 345 K 0.005 CHF/kWh

Optimal Solution:

$\dot{Q}_{U,Steam} = 10\text{kW}$ $\dot{Q}_{U,WW} = 38\text{kW}$

$\dot{Q}_{U,Brine} = 6\text{kW}$

→5‘760 CHF/a (-62 %)

*Tutorial & Code will be provided in MOODLE

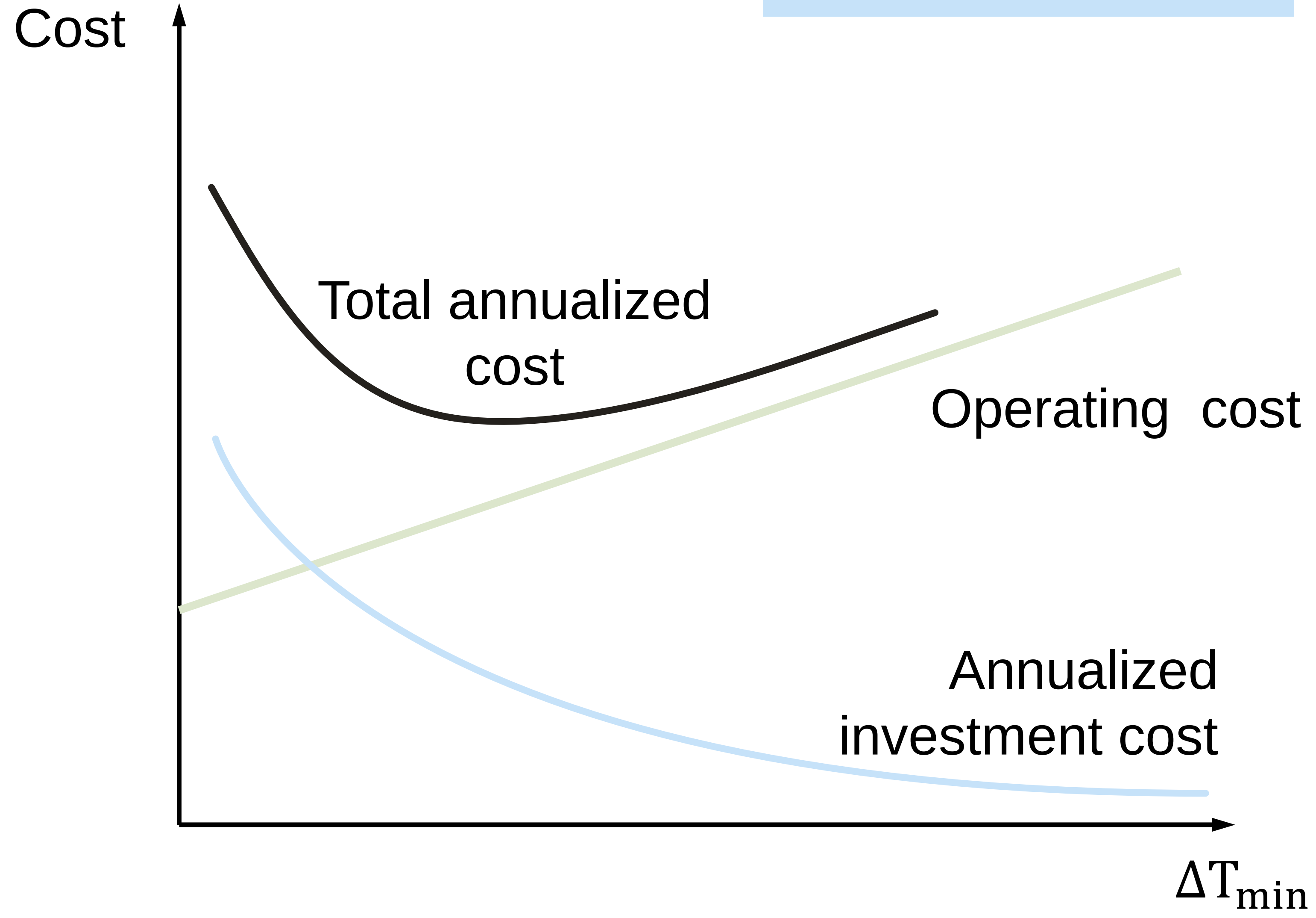
After this lecture, you are able to…

- ✓ formulate **heat integration as optimization problem**
- ▪ **design networks** for heat integration of hot and cold streams
  using pinch technology.
- ▪ find a **feasible solution** for the heat exchanger design problem using
  heuristics.
- ▪ formulate the **modelling equations**
  for designing **heat exchanger networks.**

Total Cost vs. ΔT_{min}

total costs = investment costs + operation costs

total annualized costs = annualized investment costs + operation costs

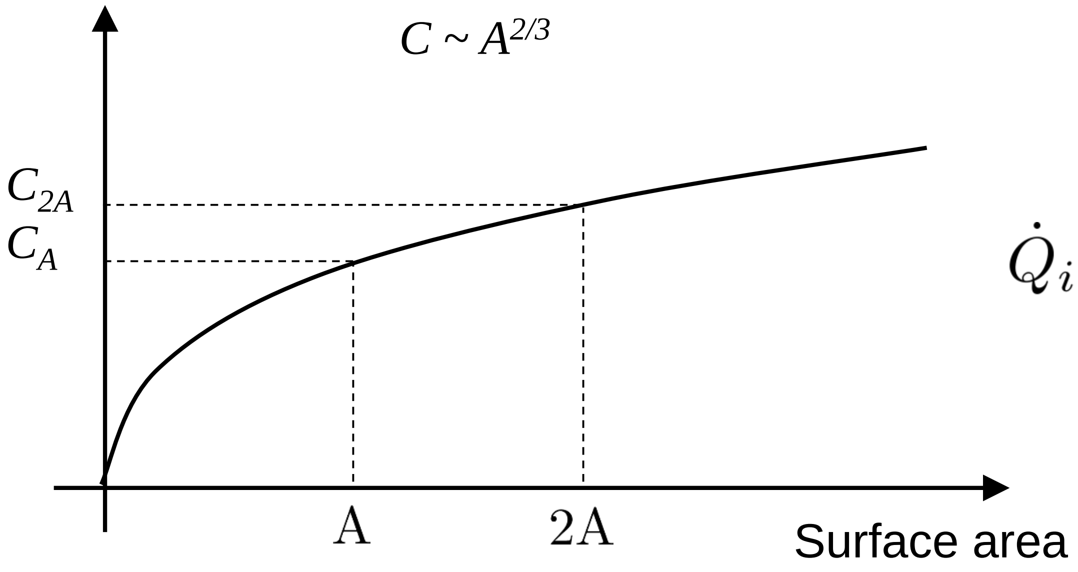


Cost-optimized heat exchanger networks

Total costs = operating costs + capital costs

→ minimum operating costs ≈ given by pinch methodology

→ minimum capital costs :



$$\dot{Q}_i = k_i A_i \Delta T_{\mathrm{ln},i}$$

„ better a few large than many small heat exchangers!"

Development of optimal HEN

Economic optimization:

min Total costs = Operating costs + Investment costs

Sequential approach :

2nd lecture

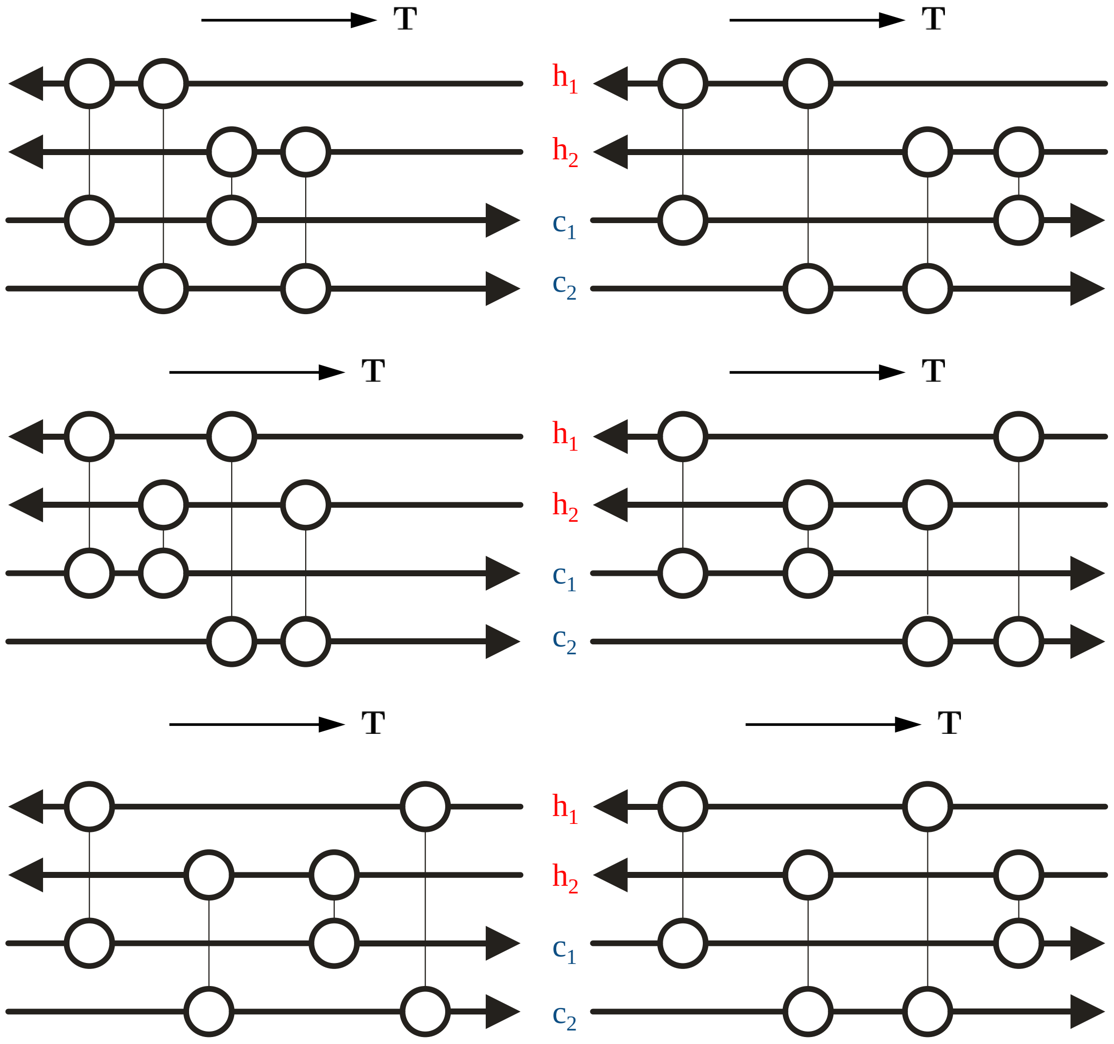
1) Targeting: Minimize utility costs (heat integration) $\Rightarrow \dot{Q}_{h,c}, \dot{Q}_{c,c}$

Now

2) Minimize number of heat exchangers (network search)
   s.t. Target (minimum utility costs)

Economic optimization vs. sequential approach
trade-off between utility costs and heat exchanger costs disregarded

Possible networks for 2 hot and 2 cold streams



Connection of h1-c1
at cold end (left)

connections = $(N_h \cdot N_c)!$

Euler theorem of graph theory (1)

upper limit for minimum number of heat exchangers for a fully heat integrated network

[MER = maximum heat recovery]

$$N_{min,MER} \leq (N_{hot} + N_{cold} + N_{utilities} - 1)_{above\ Pinch} + (N_{hot} + N_{cold} + N_{utilities} - 1)_{below\ Pinch}$$

BUT:

Limit does not take thermodynamics into account

→ cases with more heat exchangers are also possible

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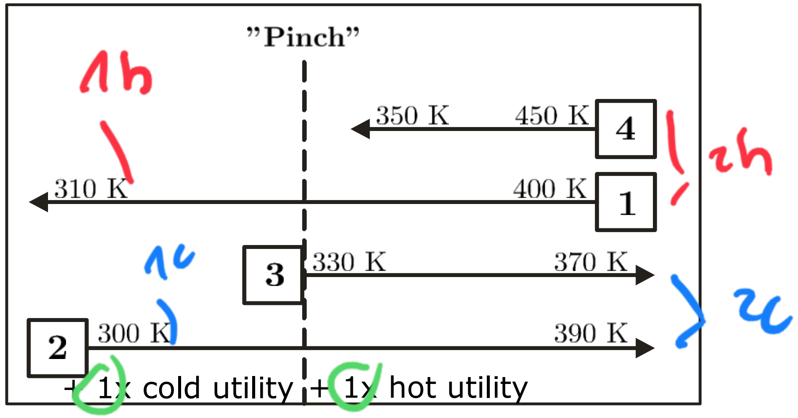
EDITOR'S CHOICE: PROCESS SYSTEMS ENGINEERING

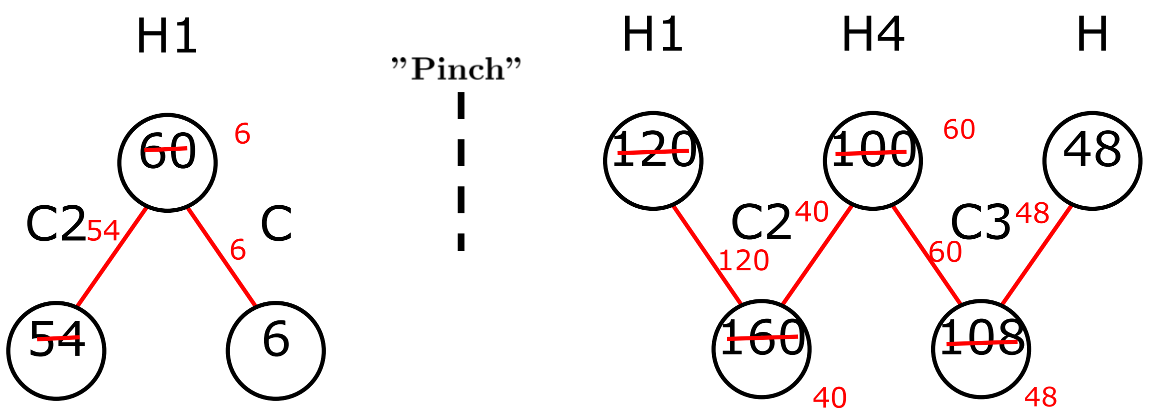
Globally optimal synthesis of heat exchanger networks. Part I: Minimal networks

Chenglin Chang, Alice Peccini, Yufei Wang, André L. H. Costa, Miguel J. Bagajewicz

First published: 10 May 2020 | https://doi.org/10.1002/aic.16267

Euler theorem of graph theory (2)



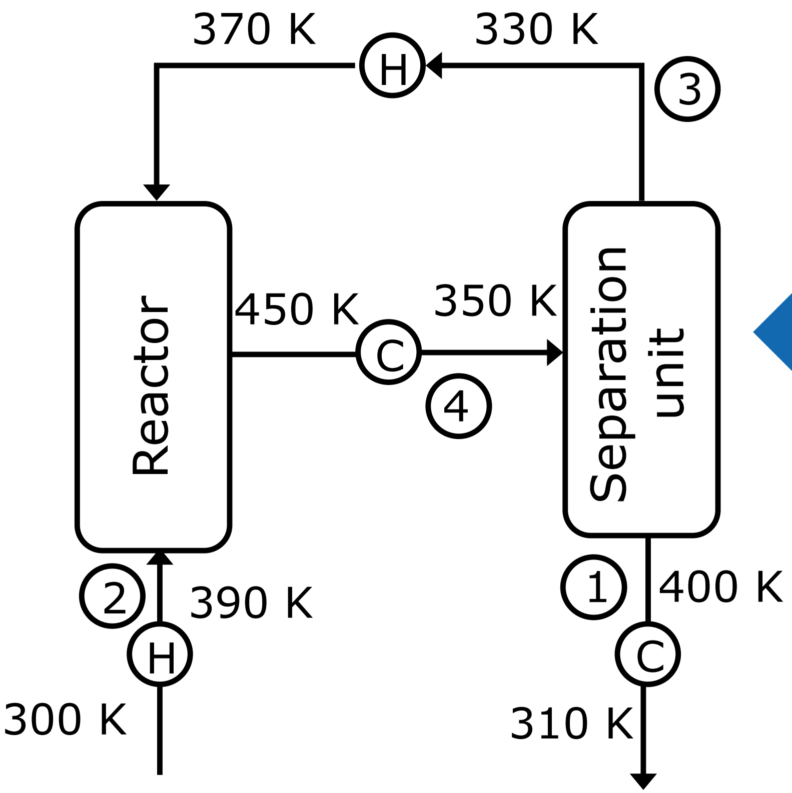


$$N_{min,MER} \le \underbrace{(1+1+1-1)}_{2} + \underbrace{(2+2+1-1)}_{4} = 6$$

Stream -No.	T_{in} [K]	T_{out} [K]	C_p [kW/K]
1(h)	400	310	2
2(c)	300	390	1,8
3(k)	330	370	4
4(c)	450	350	1

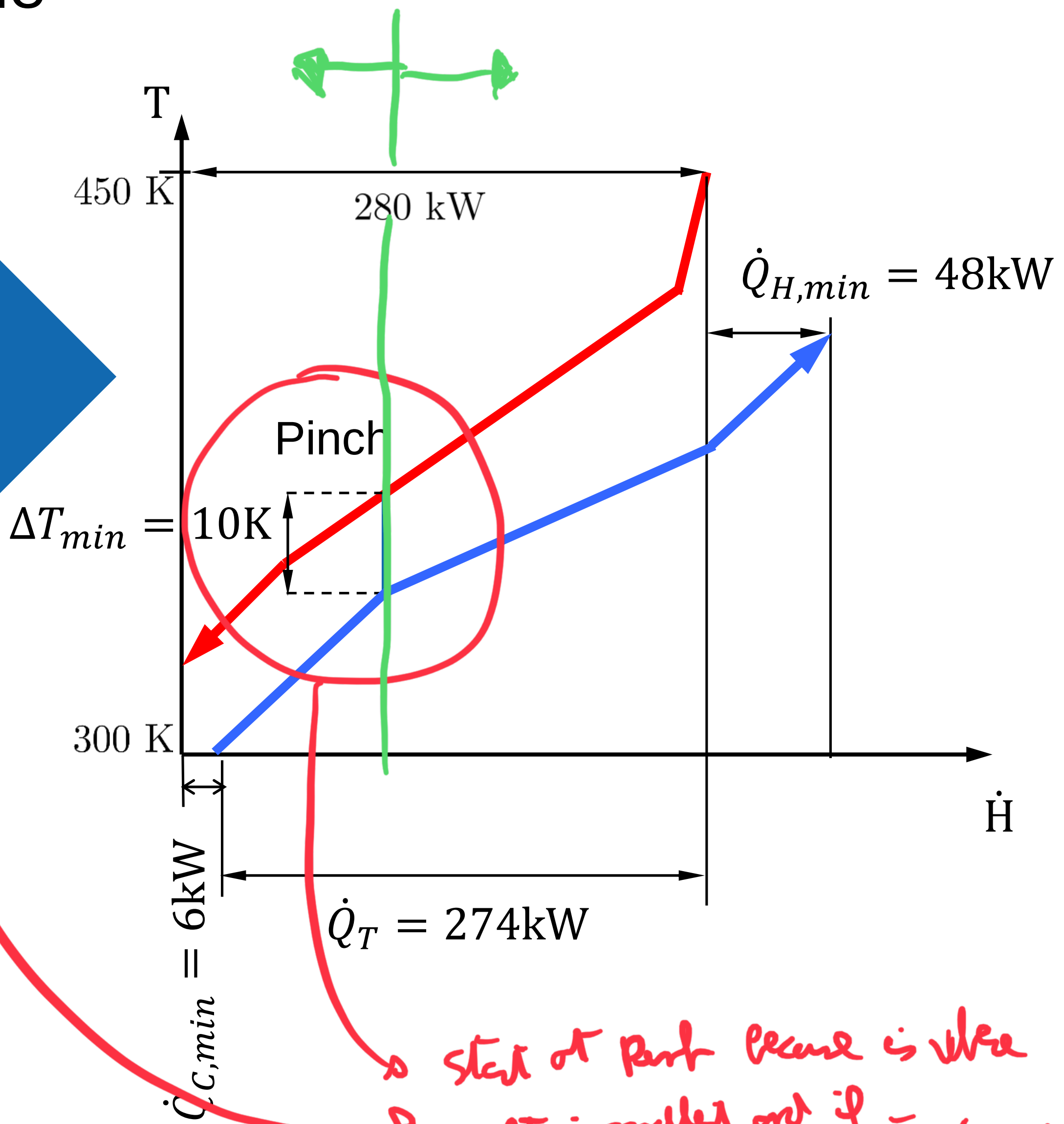
Each side: $N_{min,MER} \le \quad (N_{hot} + N_{cold} + N_{utilities} - 1)$

Hot and cold streams process example



Stream -No.	T_{in} [K]	T_{out} [K]	C_p [kW/K]
1(h)	400	310	2
2(c)	300	390	1,8
3(c)	330	370	4
4(h)	450	350	1

Heat Exchangers Where?

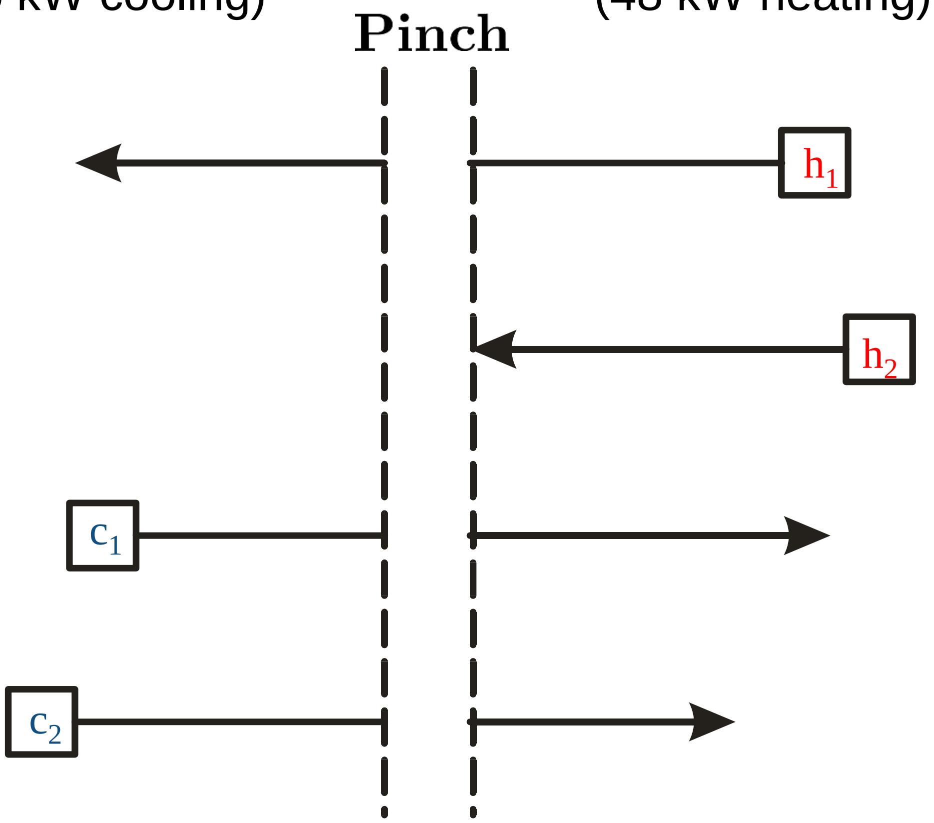


start at pinch because is where
ΔT is smallest and if
pinch doesn't work, nothing work

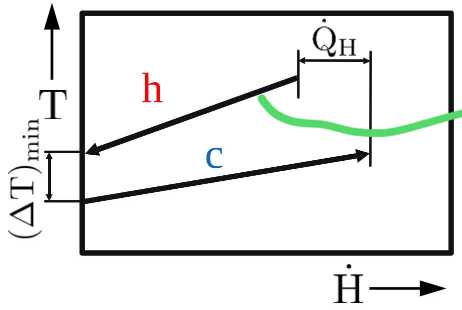
Subsystems and heat sinks divided @ pinch

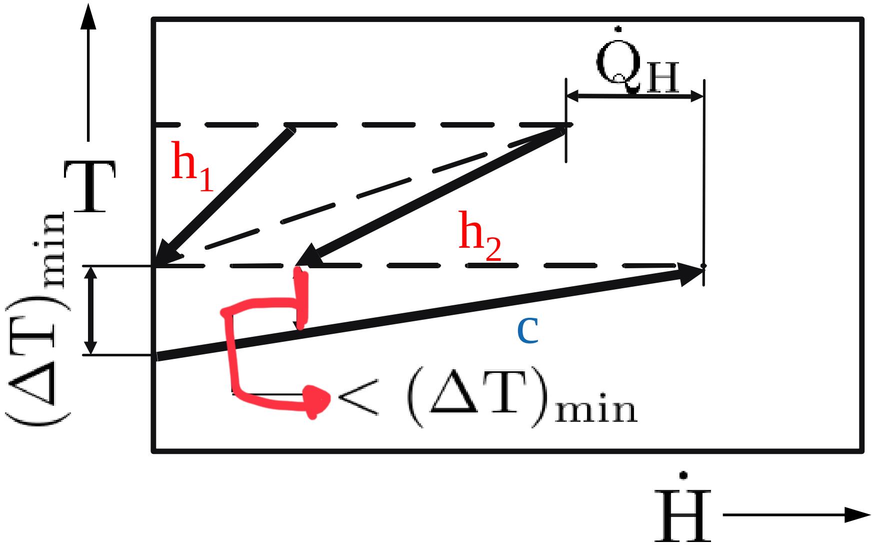
Heat source
(6 kW cooling)

Heat sink
(48 kW heating)

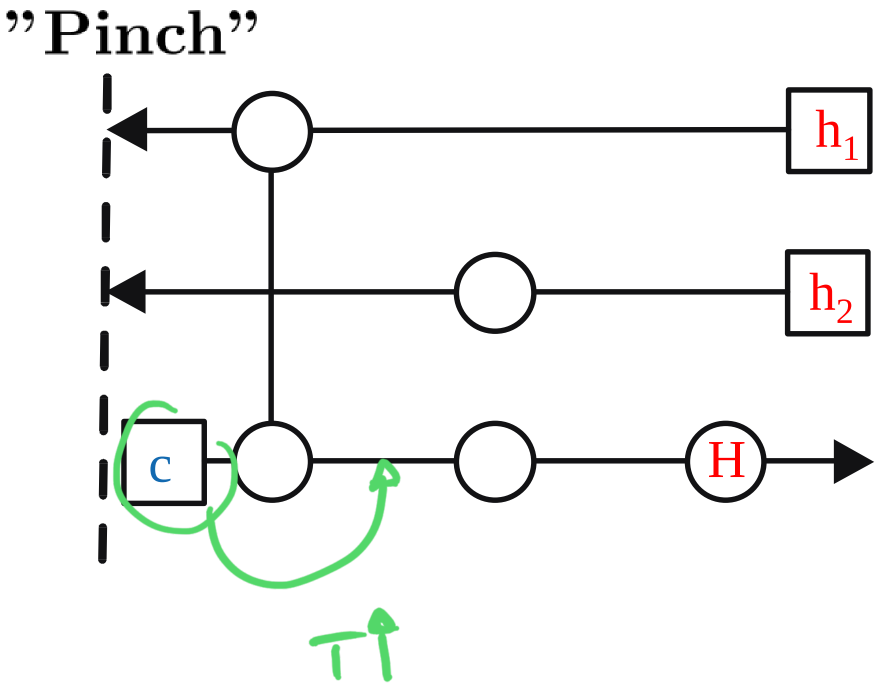


Connection of hot & cold streams above the "pinch" (1)

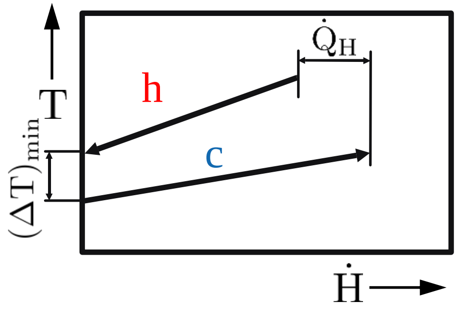


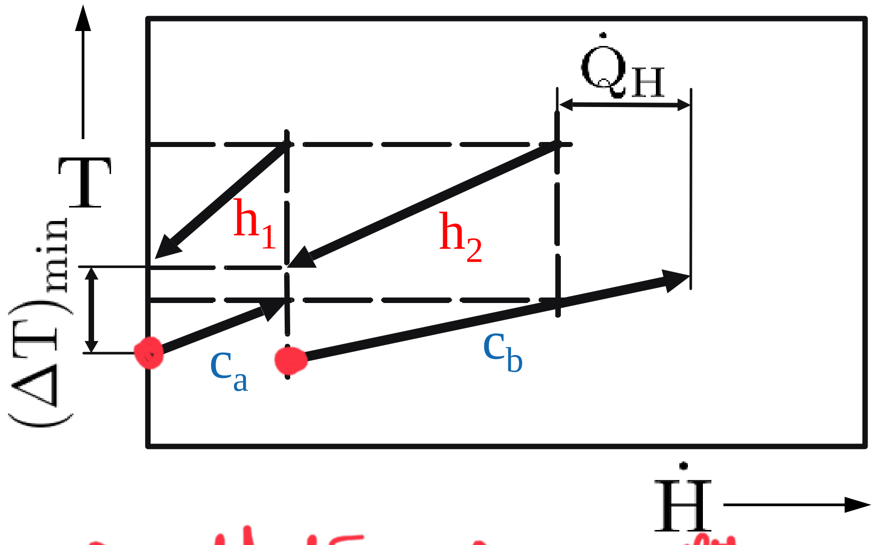


Heat sink subsystem 1



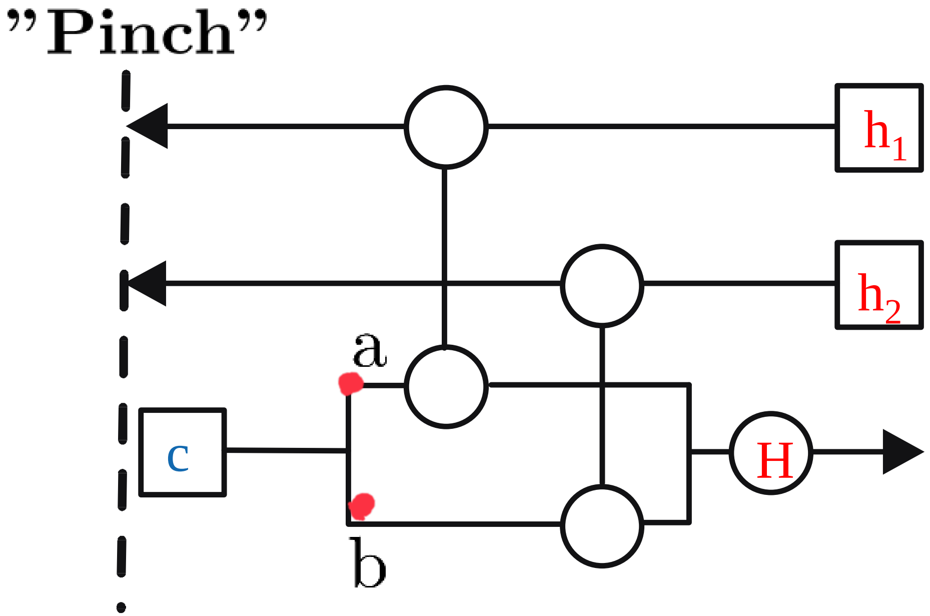
Connection of hot & cold streams above the “pinch” (1)





Heat sink subsystem 1

Rule: $N_c \geq N_h$ if not → split a cold stream

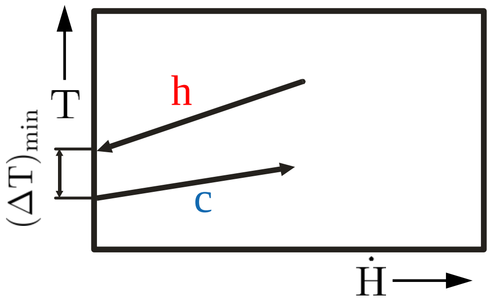


⇒ cold stream has to be flatter or $\dot{m}c_{p,c} > \dot{m}c_{p,h}$!

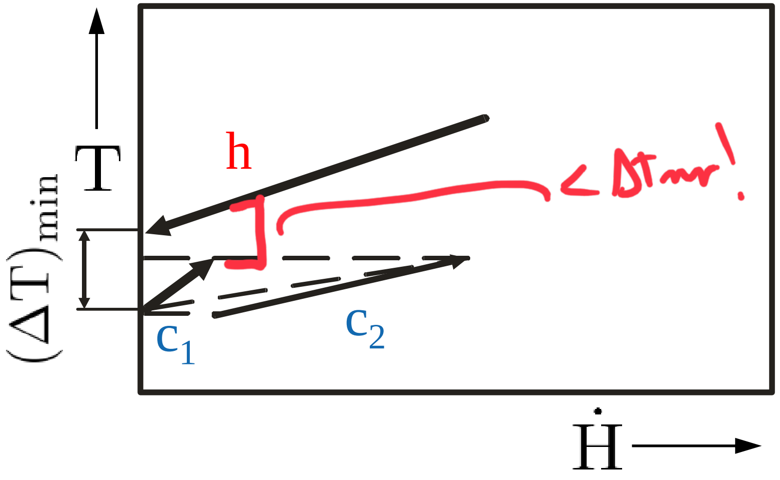
otherwise ΔT_{min} violated (see next slide) → if not possible ...

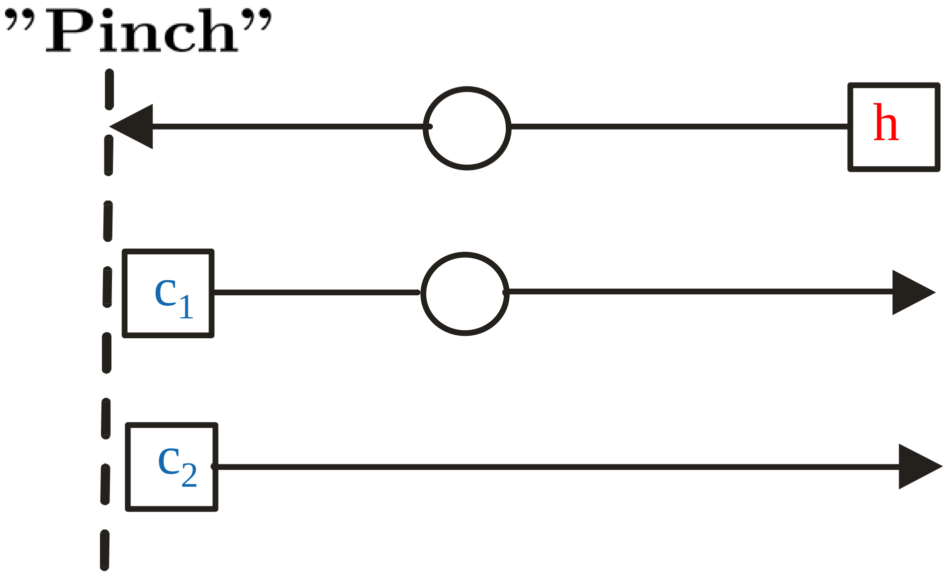
⇓

Connection of hot & cold streams above the “pinch” (2)

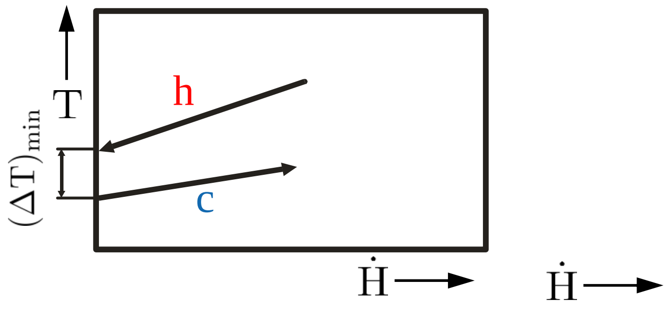


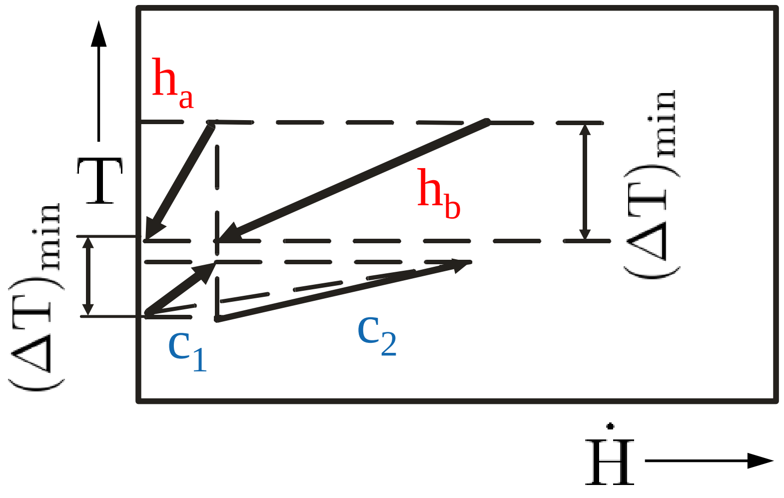
Heat sink subsystem 1





Connection of hot & cold streams above the “pinch” (2)

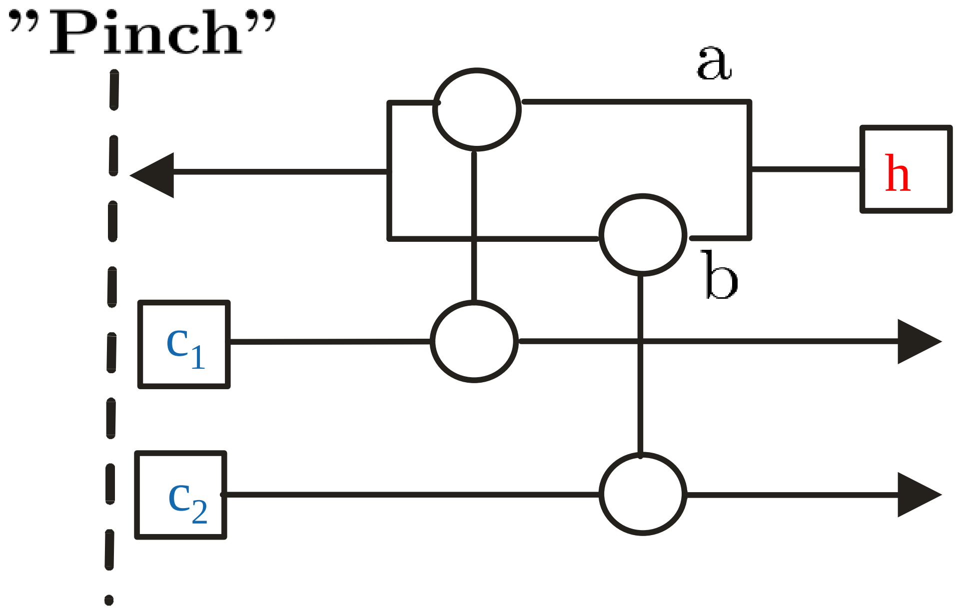




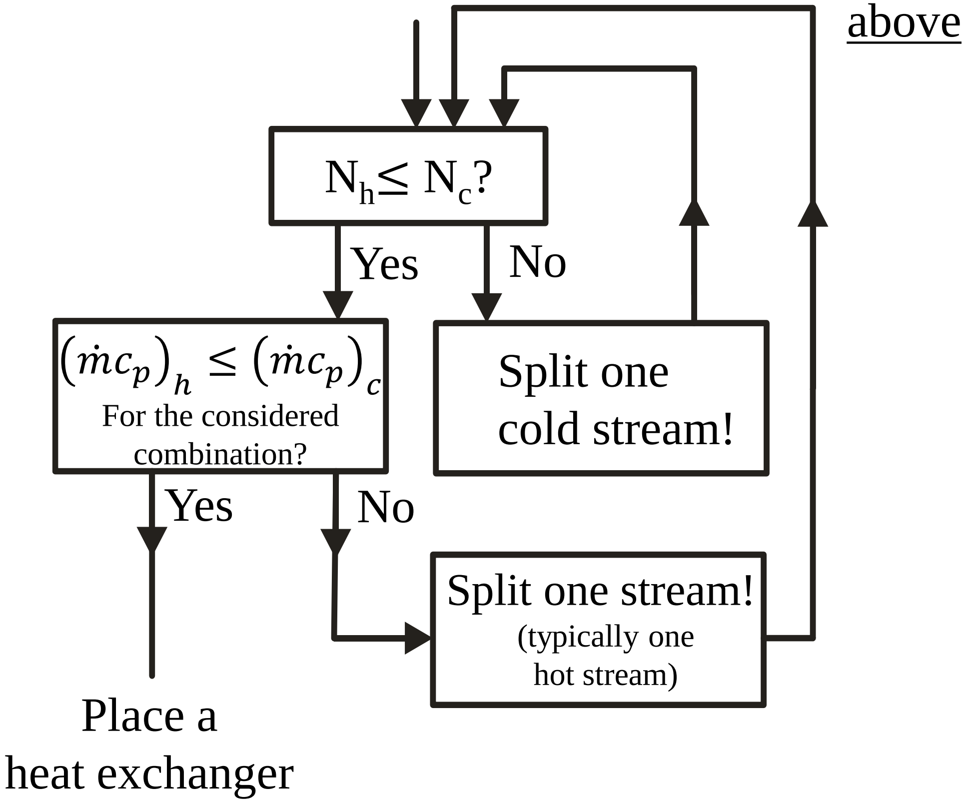
Heat sink subsystem 1

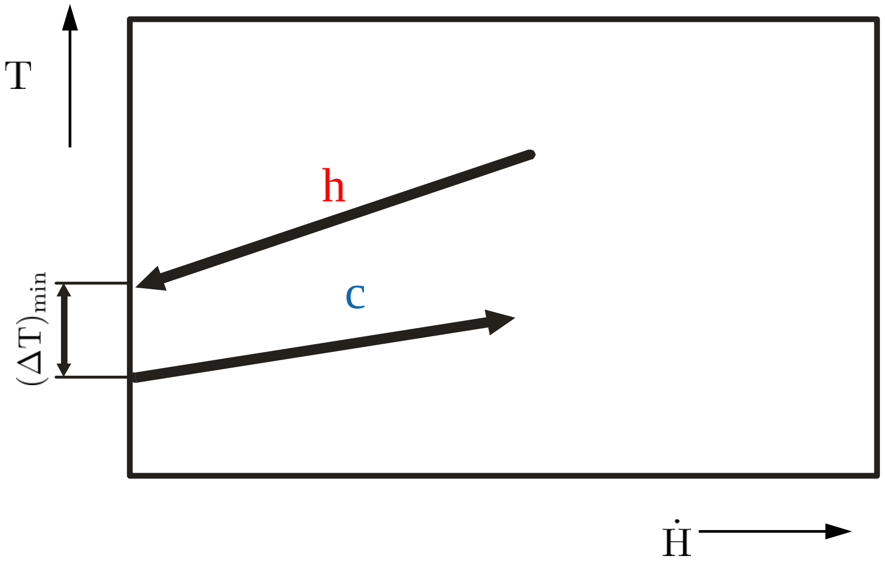
Rule: $(mc_p)_c \geq (mc_p)_h$

↳ split hot stream!

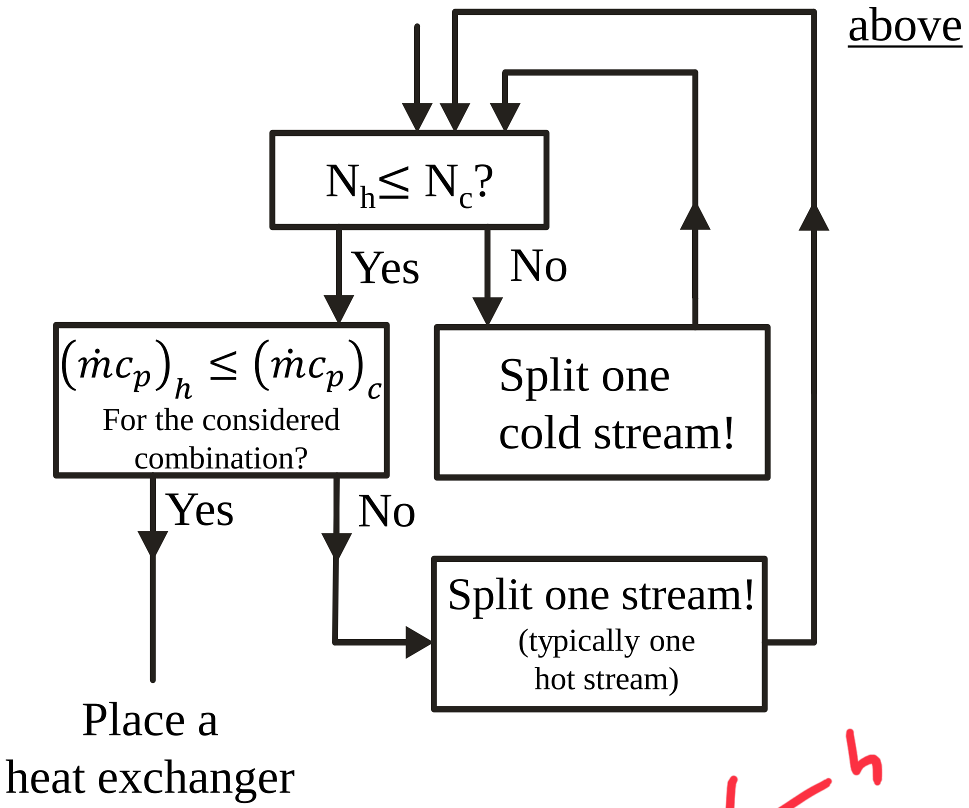


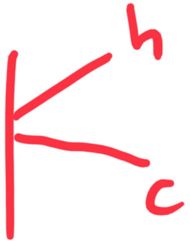
Algorithm for interconnection of Streams (above)

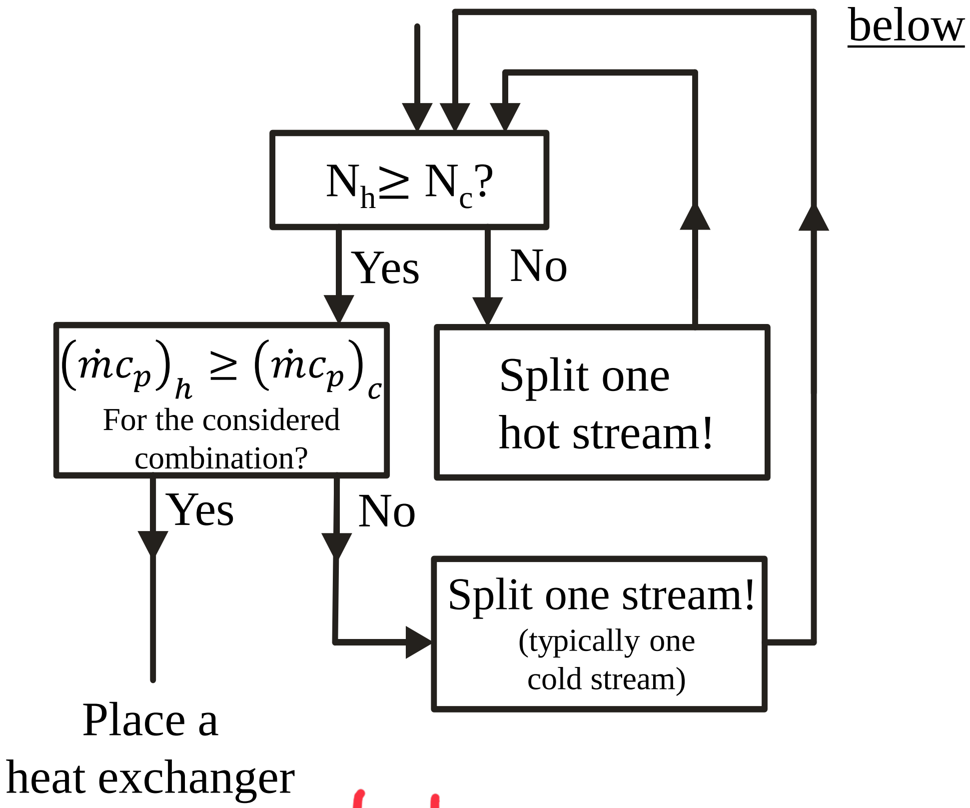




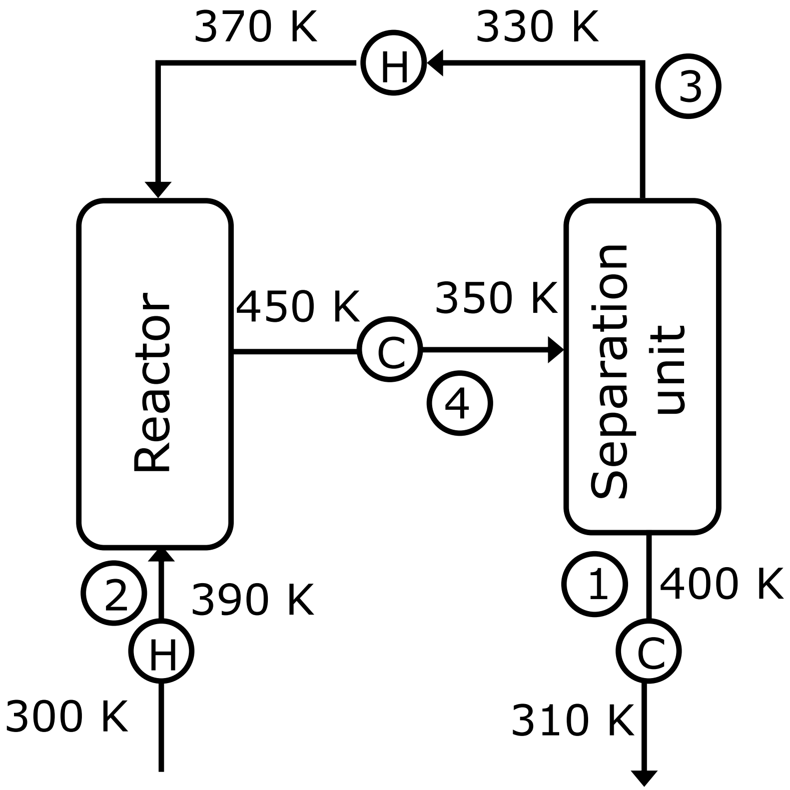
Algorithm for interconnection of Streams





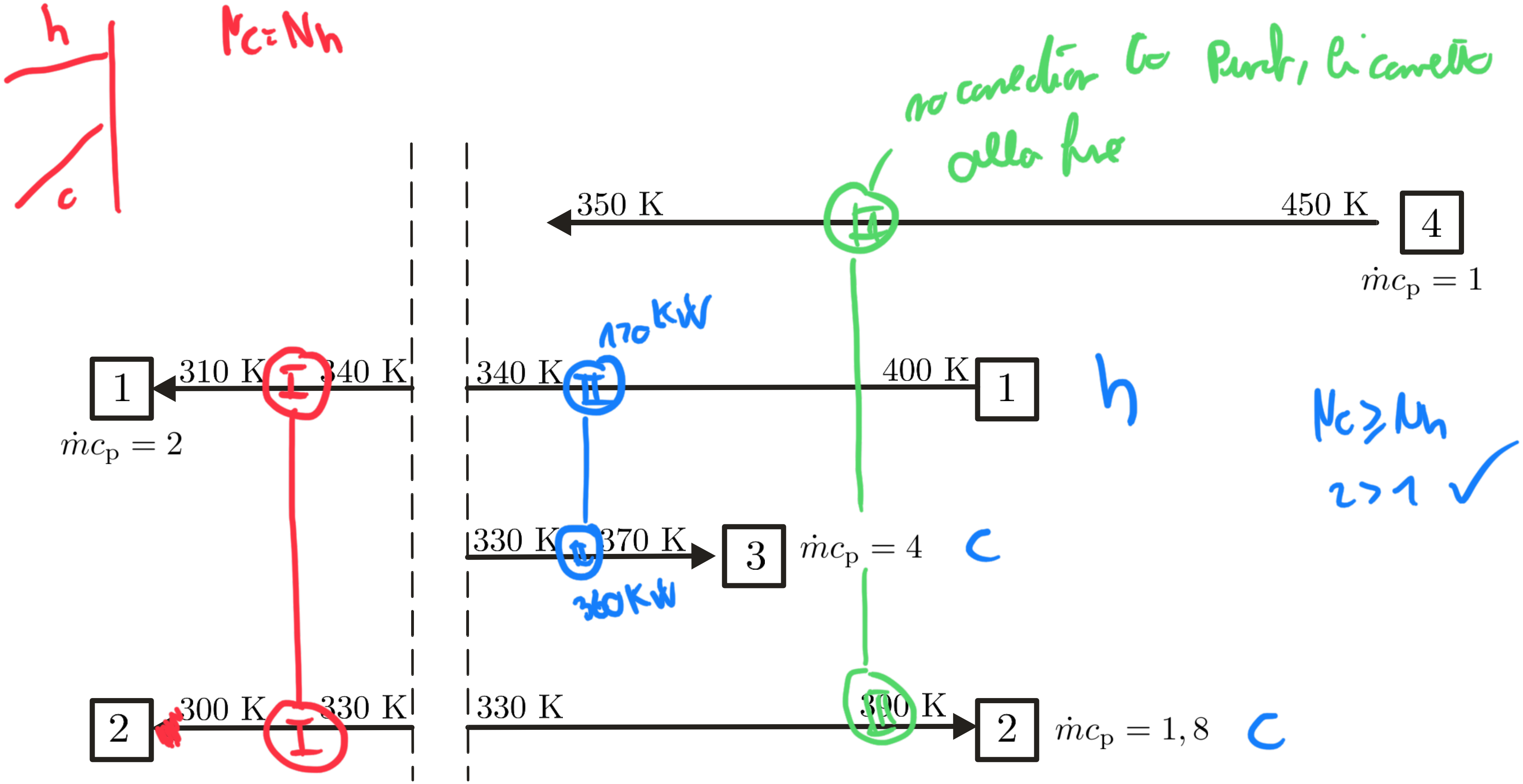


Hot and cold streams of the example process

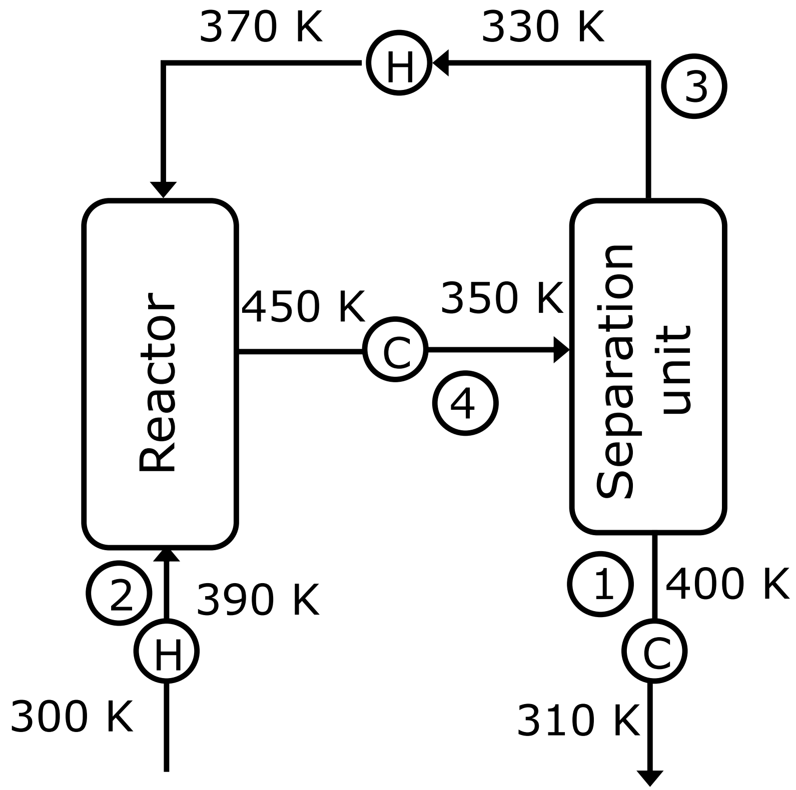


Stream -No.	T_{in} [K]	T_{out} [K]	C_p [kW/K]
1(h)	400	310	2
2(c)	300	390	1,8
3(c)	330	370	4
4(h)	450	350	1

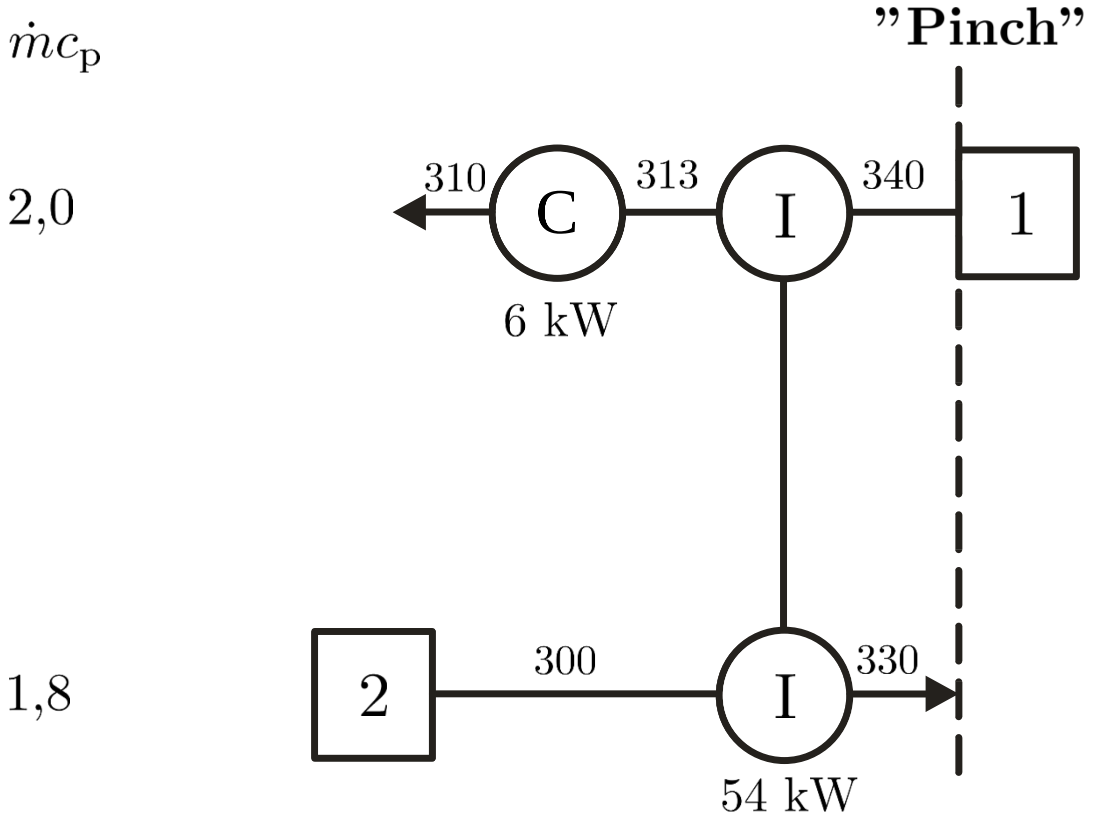
Grid representation of streams with "pinch" separation



Connection of streams below the "pinch"



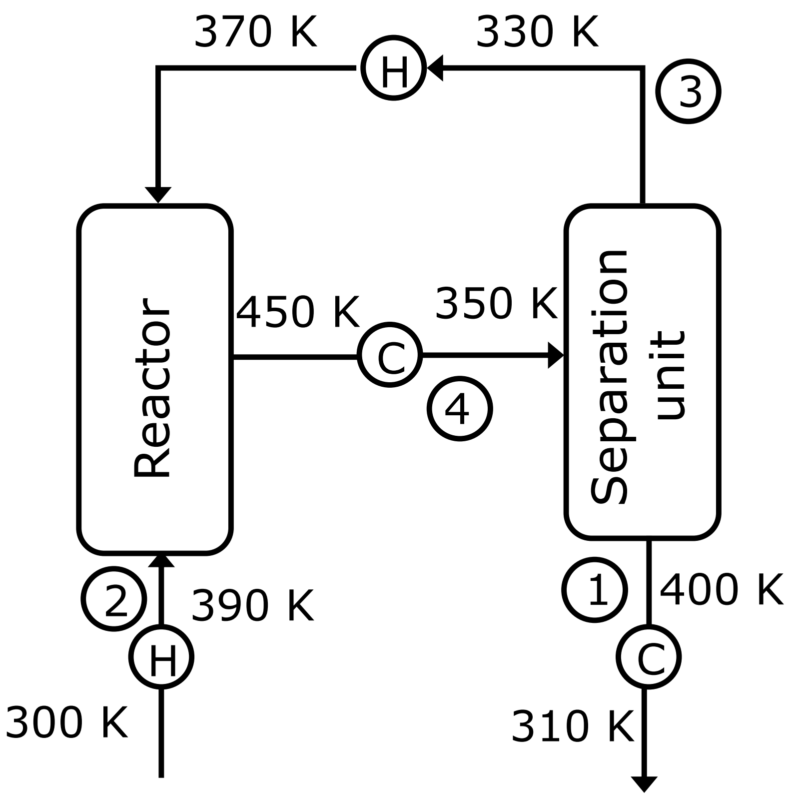
Stream -No.	T_{in} [K]	T_{out} [K]	C_p [kW/K]
1(h)	400	310	2
2(c)	300	390	1,8
3(c)	330	370	4
4(h)	450	350	1



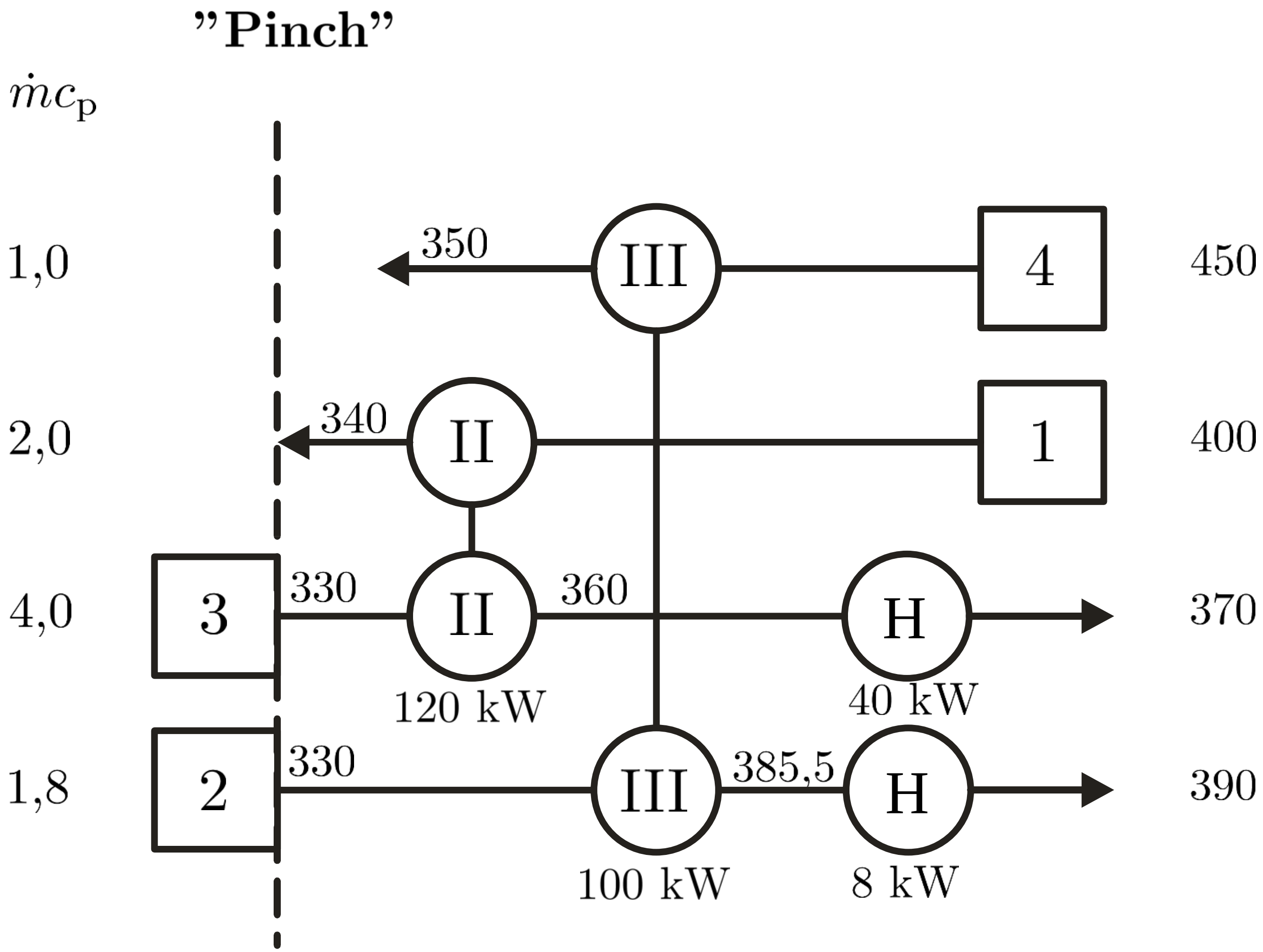
[1]

2 HEX

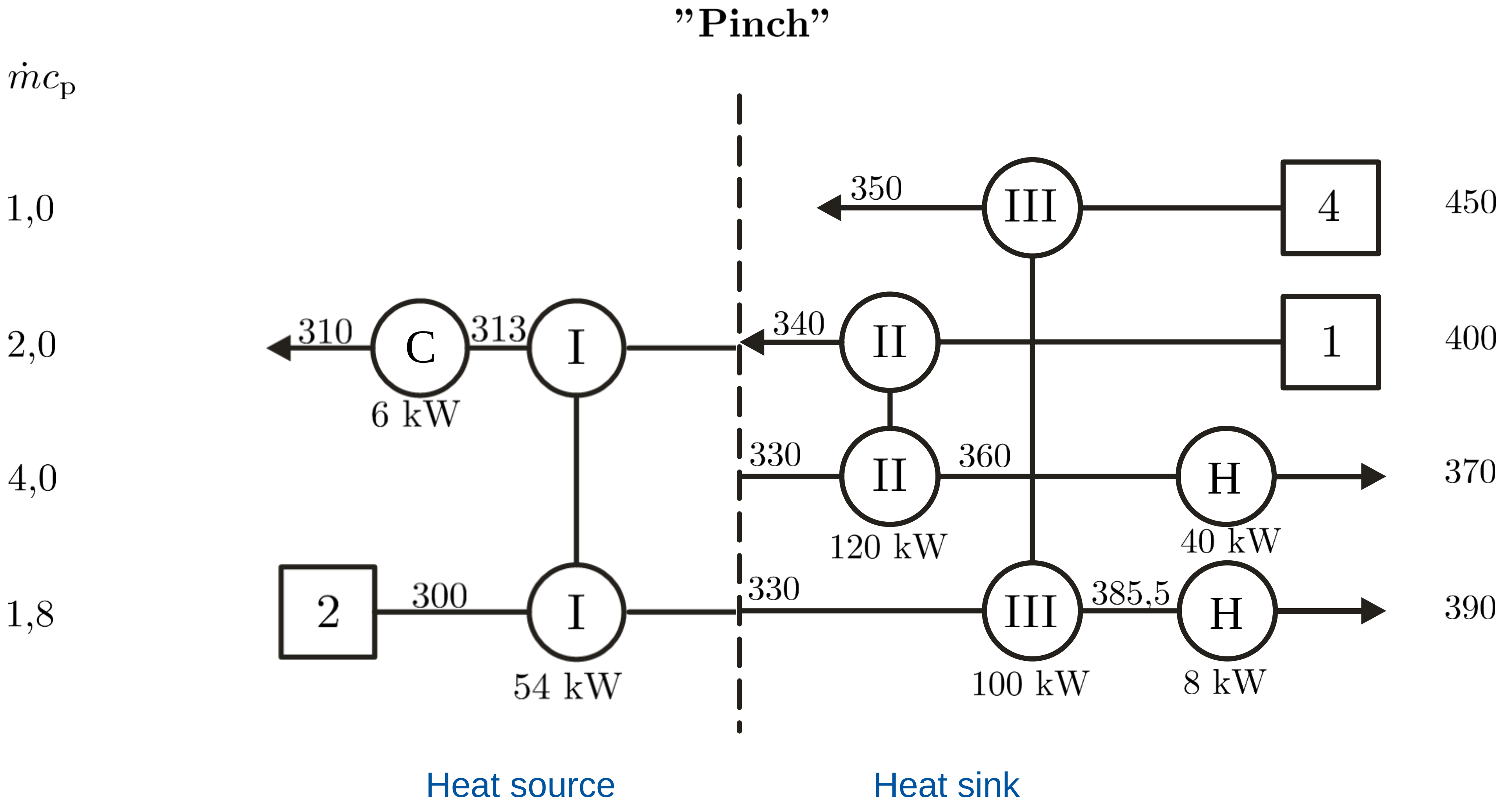
Connection of streams above the "pinch"



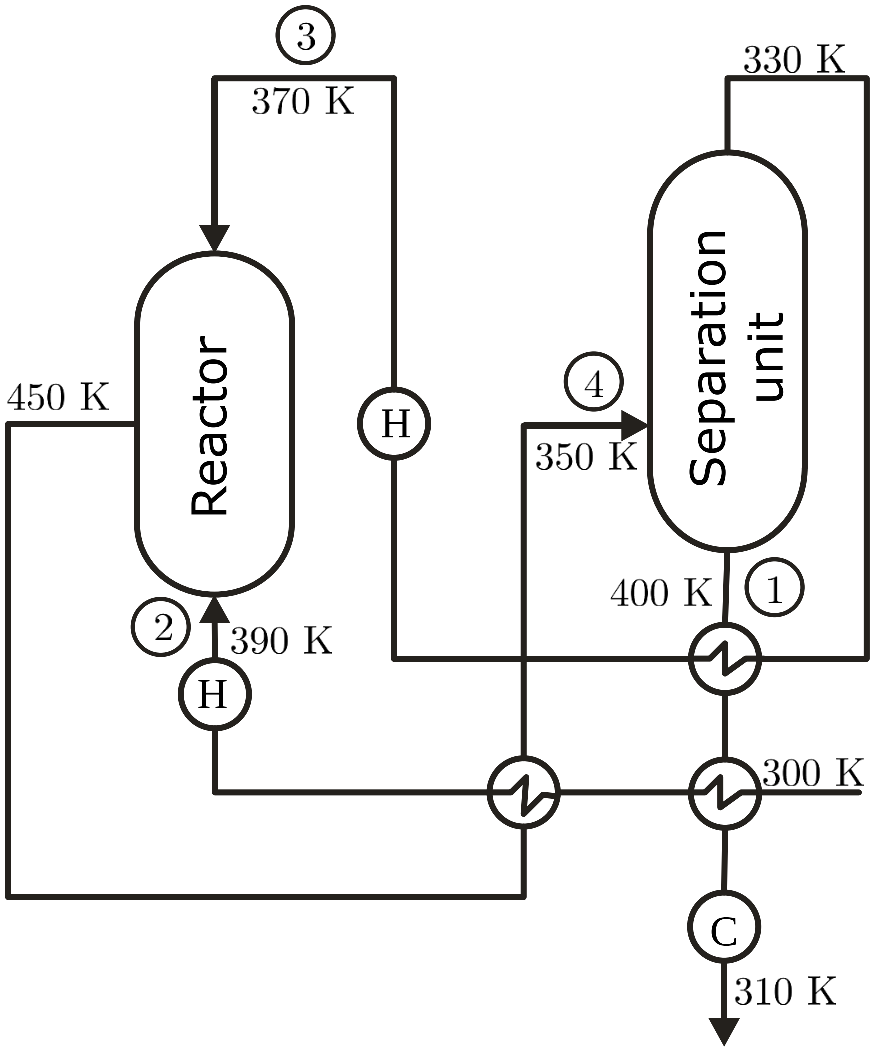
Stream -No.	T_{in} [K]	T_{out} [K]	C_p [kW/K]
1(h)	400	310	2
2(c)	300	390	1,8
3(c)	330	370	4
4(h)	450	350	1



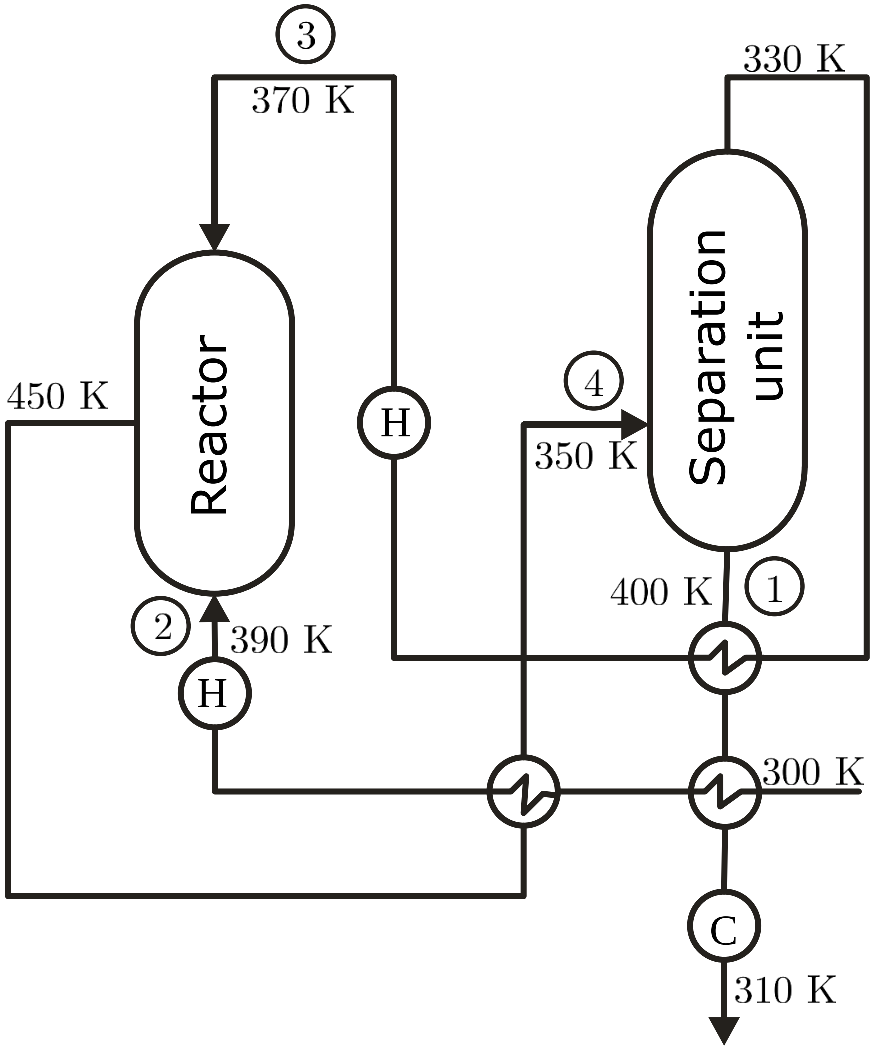
Complete heat exchanger network

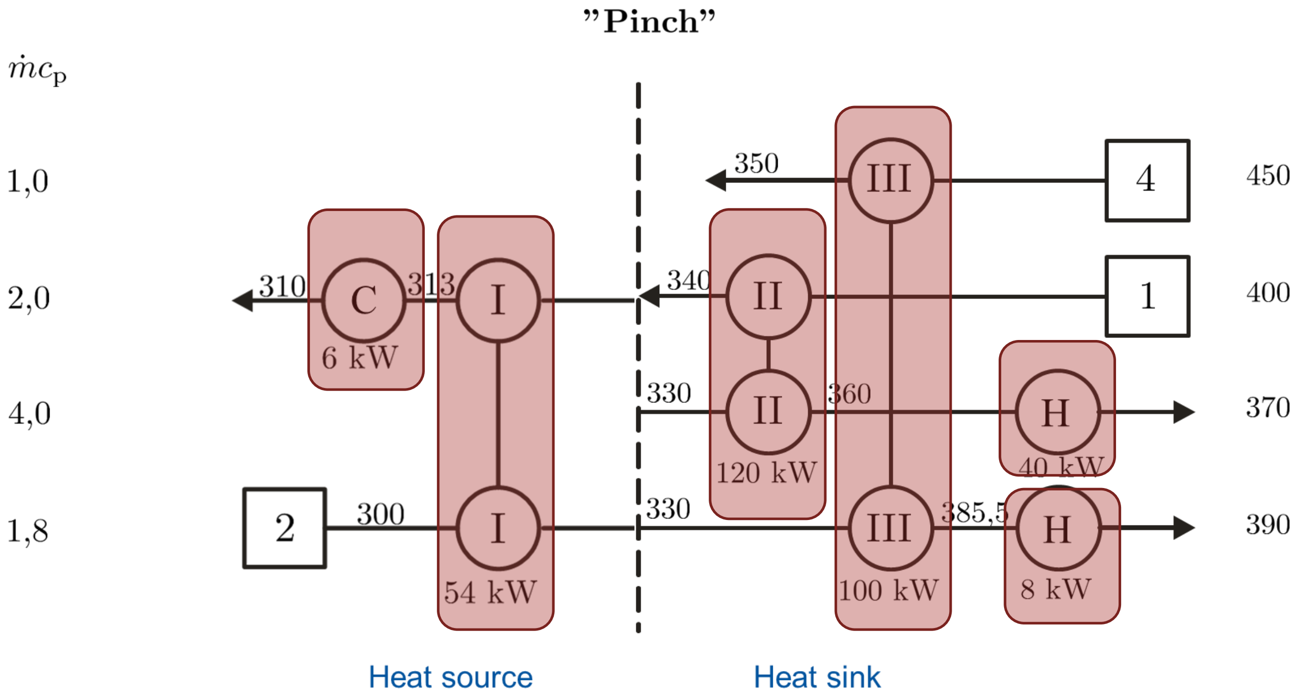


Connection of streams in the flow diagram



Heat exchanger network





6 heat exchanger

optimal network?

After this lecture, you are able to…

- ✓ formulate **heat integration as optimization problem**
- ✓ **design networks** for heat integration of hot and cold streams **using pinch technology.**
- ✓ find a **feasible solution** for the heat exchanger design problem using **heuristics.**
- ▪ formulate the **modelling equations** for designing **heat exchanger networks.**

Development of optimal HEN

Economic optimization:

min Total costs = Operating costs + Investment costs

Sequential approach :

2nd lecture → 1) Targeting: Minimize utility costs (heat integration)

Now → 2) Minimize number of heat exchangers (network search) s.t. Target (minimum utility costs)

Economic optimization vs. sequential approach
trade-off between utility costs and heat exchanger costs disregarded

Operating costs minimization (LP)

$$\min_{\dot{Q}_{U,H},\dot{Q}_{U,C},\Delta\dot{Q}_z^*} \sum_H c_{U,H} \cdot \dot{Q}_{U,H} + \sum_C c_{U,C} \cdot \dot{Q}_{U,C}$$

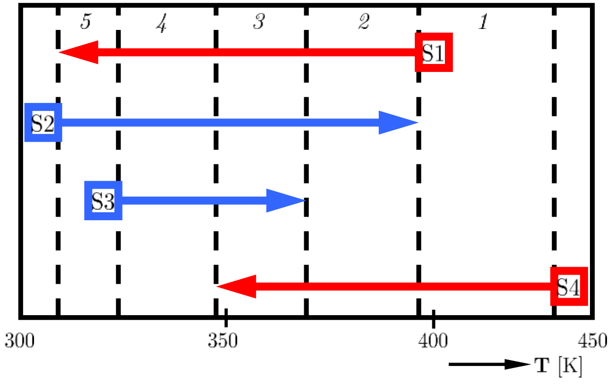
$s.t.$

$$\forall H \quad \dot{Q}_{U,H} = \sum_{z=1}^{z_{max}=5} \dot{Q}_{U,H}^{(z)},$$

$$\forall C \quad \dot{Q}_{U,C} = \sum_{z=1}^{z_{max}=5} \dot{Q}_{U,C}^{(z)},$$

$$\Delta\dot{H}_h^{(z)} = (\dot{m}c_p)_h \cdot \Delta T_h^{(z)}$$

$$\Delta\dot{H}_c^{(z)} = (\dot{m}c_p)_c \cdot \Delta T_c^{(z)}$$



$$\forall z \quad 0 = \sum_h \Delta\dot{H}_h^{(z)} + \sum_H \dot{Q}_{U,H}^{(z)} - \sum_c \Delta\dot{H}_c^{(z)} - \sum_C \dot{Q}_{U,C}^{(z)} + \Delta\dot{Q}_{z-1}^* - \Delta\dot{Q}_z^*,$$

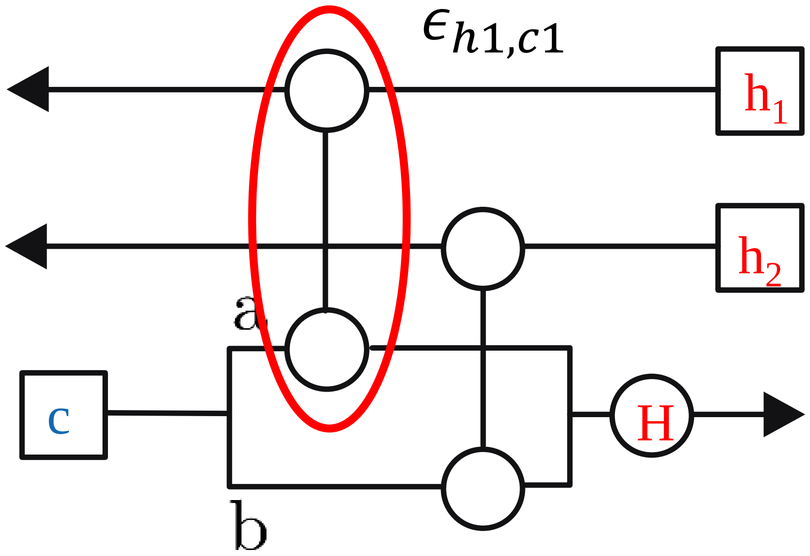
$$\Delta\dot{Q}_0^* = 0, \Delta\dot{Q}_{z_{max}}^* = 0,$$

$$\Delta\dot{Q}_z^* \geq 0, \dot{Q}_{U,H}^{(z)} \geq 0, \dot{Q}_{U,C}^{(z)} \geq 0,$$

unknown

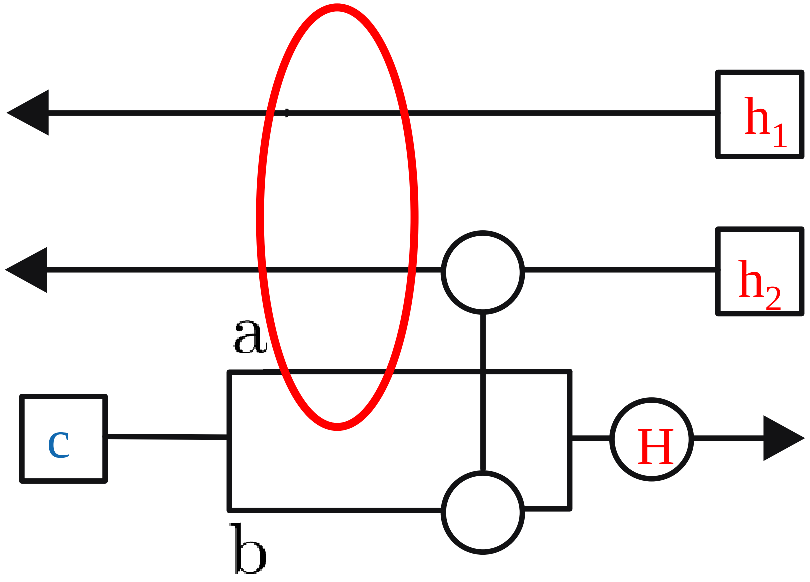
known

Binary decision variables



Decision to match h1 and c1

$$\epsilon_{h1,c1} = 1$$



Decision to **not** match h1 and c1

$$\epsilon_{h1,c1} = 0$$

Optimization: Number of heat exchangers

Sequential approach:

2) Minimize number of heat exchangers (network search)
s.t. Target (minimum utility costs)

Necessary condition:

At least one heat exchanger per stream coupling

$$\min_{\Delta \dot{Q}_z^*, \dot{Q}_{h,c}^{(z)}} \sum_h \sum_c \epsilon_{h,c}$$

Number of stream couplings
≤ Number of heat exchangers

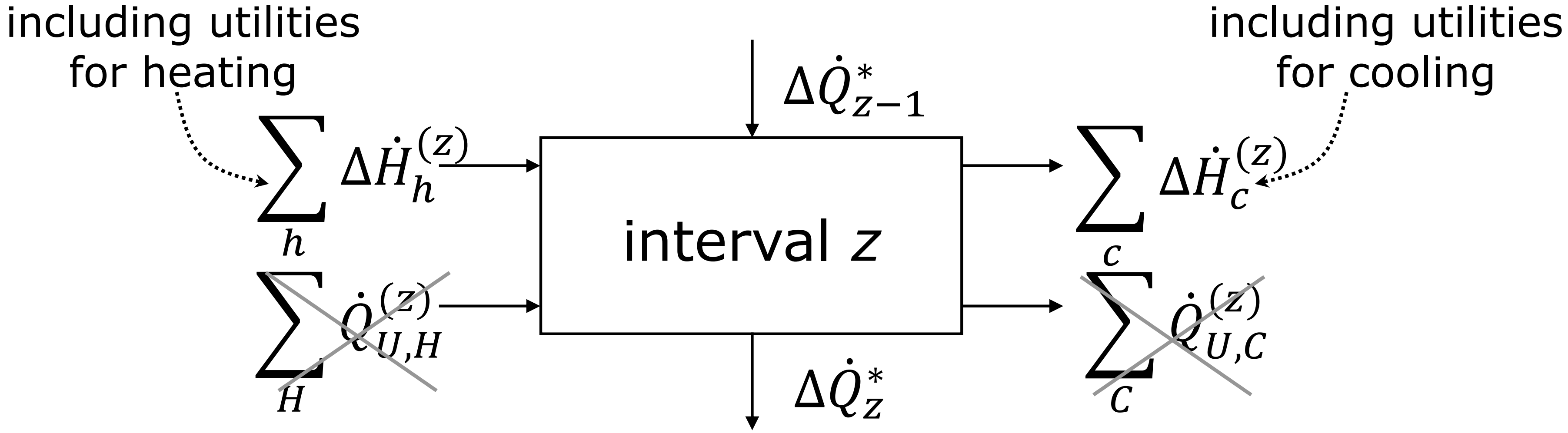
$s.t.$ **Energy balances**

stream coupling

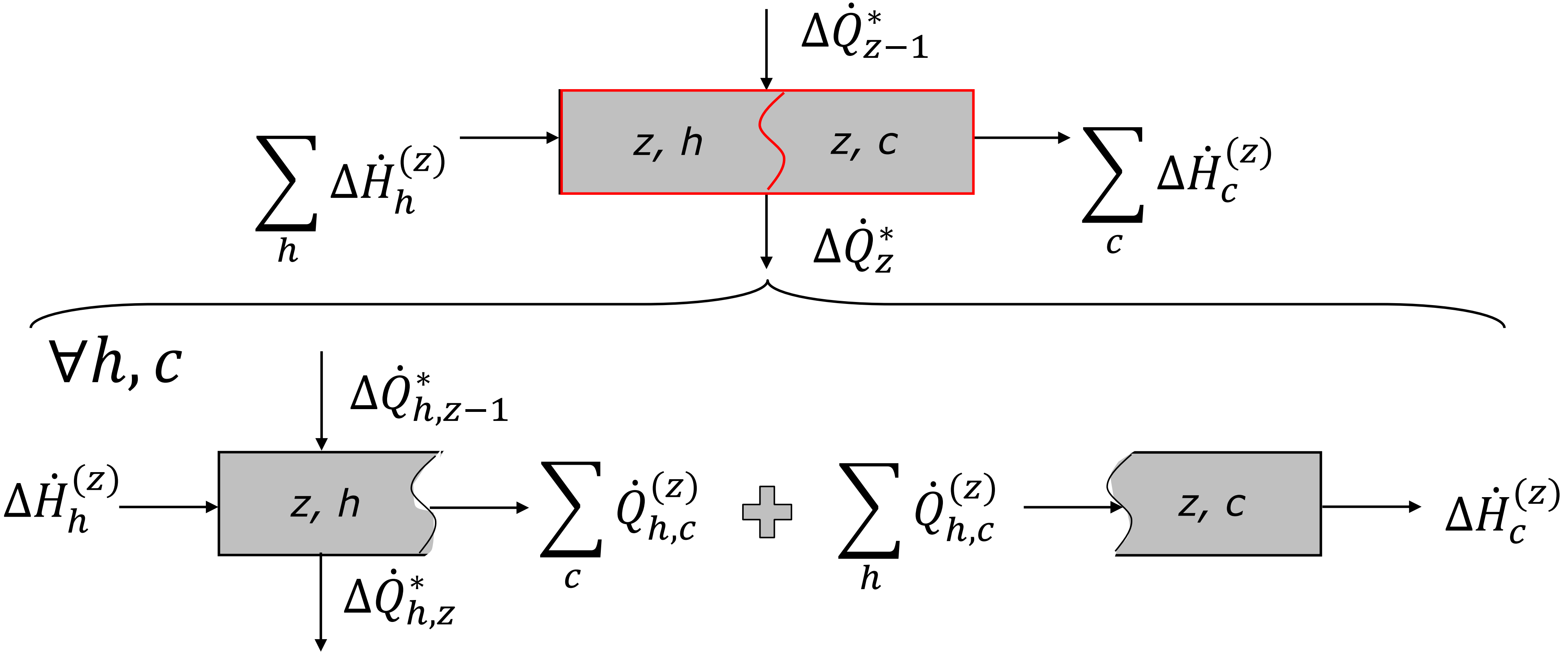
Papoulias, Grossmann. *A structural optimization approach in process synthesis II: Heat recovery networks.* Comput Chem Eng, *7:*707-721, 1983.

Problem Formulation

- utilities = hot/cold streams
- Input data:
  - Same as for operating cost-LP $\Delta\dot{H}_h^{(z)}, \Delta\dot{H}_c^{(z)}$
  - Required utilities (LP results) $\dot{Q}_{U,H}, \dot{Q}_{U,C}$



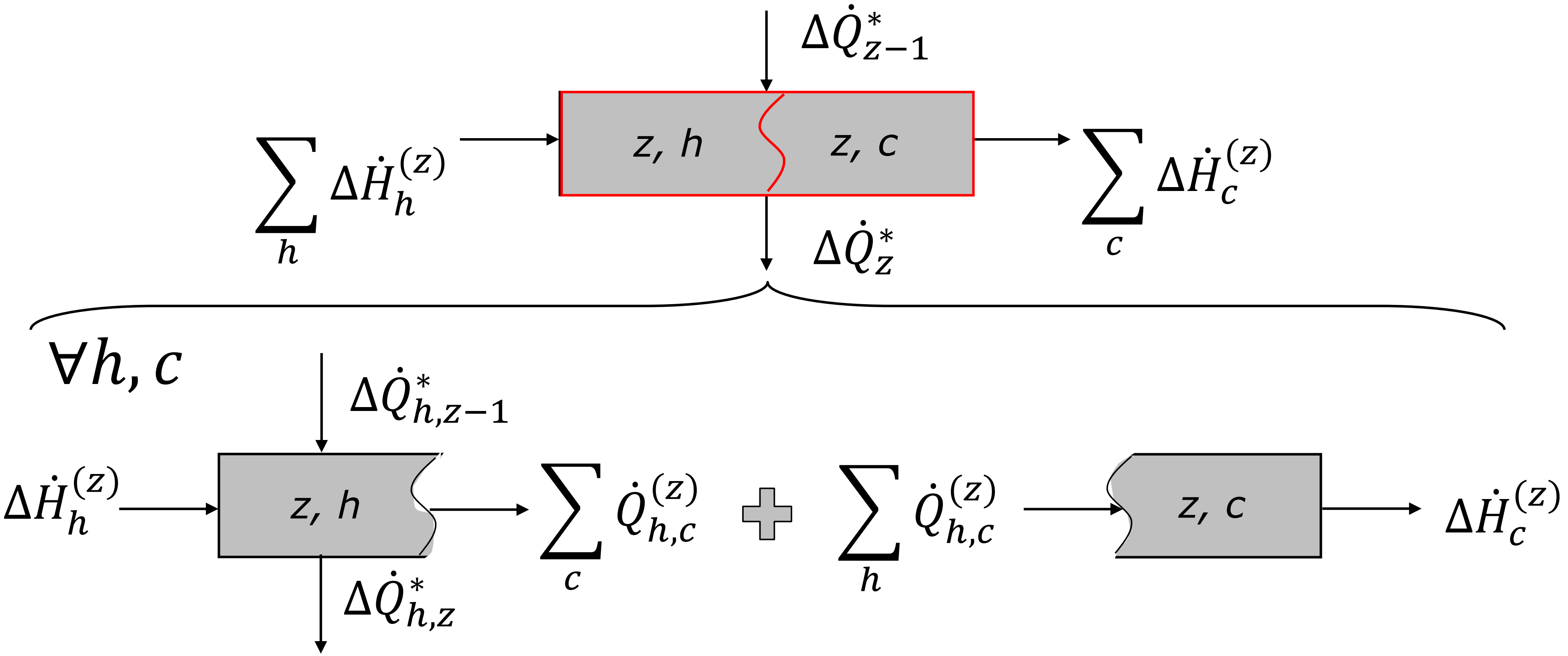
Energy balances around all streams



$$\forall h, z \quad 0 = -\Delta\dot{Q}^*_{h,z} + \Delta\dot{Q}^*_{h,z-1} + \Delta\dot{H}_h^{(z)} - \sum_c \dot{Q}_{h,c}^{(z)} \cdot \varepsilon_{hc}$$

$$\forall c, z \quad 0 = -\Delta\dot{H}_c^{(z)} + \sum_h \dot{Q}_{h,c}^{(z)}$$

Energy balances around all streams + stream couplings

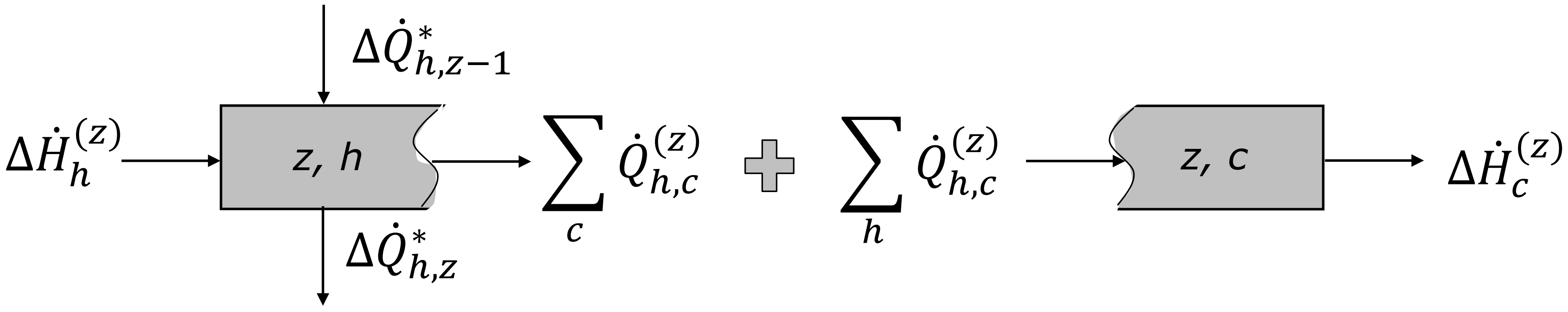


$$\forall h, z \quad 0 = -\Delta\dot{Q}^*_{h,z} + \Delta\dot{Q}^*_{h,z-1} + \Delta\dot{H}_h^{(z)} - \sum_c \epsilon_{h,c}\dot{Q}_{h,c}^{(z)}$$

$$\forall c, z \quad 0 = -\Delta\dot{H}_c^{(z)} + \sum_h \epsilon_{h,c}\dot{Q}_{h,c}^{(z)}$$

unknown

Coupling of two streams h and c



$$\forall h,c \quad \sum_z \dot{Q}_{h,c}^{(z)} \le \varepsilon_{h,c}\cdot M\cdot \underbrace{\min\left\{\sum_z \Delta\dot{H}_h^{(z)}, \sum_z \Delta\dot{H}_c^{(z)}\right\}}_{\Delta\dot{Q}_{h,c}^{max}}$$

with $\epsilon_{h,c} \in \{0,1\}$,

$$\epsilon_{h,c} = \begin{Bmatrix} 1, \text{if } h \text{ and } c \text{ connected} \\ 0, \text{ else} \end{Bmatrix}$$

Optimization: Number of heat exchangers

$$\min_{\Delta \dot{Q}^*_z, \dot{Q}^{(z)}_{h,c}} \sum_h \sum_c \epsilon_{h,c}$$

Number of stream couplings
≤ Number of heat exchangers

$s.t.$ **Energy balances:**

$$\forall h, z \quad 0 = -\Delta \dot{Q}^*_{h,z} + \Delta \dot{Q}^*_{h,z-1} + \Delta \dot{H}^{(z)}_h - \textstyle\sum_c \dot{Q}^{(z)}_{h,c},$$

$$0 = \Delta \dot{Q}^*_{h,1},$$

$$0 = \Delta \dot{Q}^*_{h,z_{max}},$$

$$\forall c, z \quad 0 = -\Delta \dot{H}^{(z)}_c + \sum_h \dot{Q}^{(z)}_{h,c}$$

Stream couplings:

$$\forall h, c \quad \sum_z \dot{Q}^{(z)}_{h,c} \leq \epsilon_{h,c} \cdot \Delta \dot{Q}^{max}_{h,c}$$

"cont M" ($= \min(\Delta\dot{H}_h, \Delta\dot{H}_c)$

Papoulias, Grossmann. *A structural optimization approach in process synthesis II: Heat recovery networks.* Comput Chem Eng, *7:*707-721, 1983.

Optimization: Number of heat exchangers

unknown
known

$$\min_{\Delta\dot{Q}^*_z,\dot{Q}^{(z)}_{h,c}} \sum_h \sum_c \epsilon_{h,c}$$

Number of stream couplings
≤ Number of heat exchangers

$s.t.$ **Energy balances:**

$$\forall h,z \quad 0 = -\Delta\dot{Q}^*_{h,z} + \Delta\dot{Q}^*_{h,z-1} + \Delta\dot{H}^{(z)}_h - \textstyle\sum_c \dot{Q}^{(z)}_{h,c},$$

$$0 = \Delta\dot{Q}^*_{h,1},$$

$$0 = \Delta\dot{Q}^*_{h,z_{max}},$$

$$\forall c,z \quad 0 = -\Delta\dot{H}^{(z)}_c + \sum_h \dot{Q}^{(z)}_{h,c}$$

Stream couplings:

$$\forall h,c \quad \sum_z \dot{Q}^{(z)}_{h,c} \leq \epsilon_{h,c} \cdot \Delta\dot{Q}^{max}_{h,c}$$

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