ETH zürich

RRE
Reliability and Risk Engineering

Introduction to Modeling and Optimization of Sustainable Energy Systems:

Optimal design of multi-energy systems (MES) and technology modeling

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Reliability and Risk Engineering

Since the last lecture, you are able to …

- ✓ Understand concept, motivation and main design questions of multi-energy systems (MES)
- ✓ Understand the different degrees of complexity when optimizing MES
- ✓ Formulate time-dependent optimization problem for MES optimal operation

After this lecture, you are able to …

- Formulate multi-objective optimization problem for MES optimal design
- Model energy conversion technologies within MES optimization
- Model energy storage technologies within MES optimization
- Understand the different degrees of complexity when optimizing MES
- Model energy conversion dynamics (***optional material, no exam***)

Modeling MES:
Multi-objective optimal design

The decision-making context determines the optimization problem

easier → more difficult

	easier	more difficult
Decision variables: Design vs Operation	Technology operation for given technology selection and size	Selection and size of technologies, coupled with their operation
Time dimension	Instantaneous	Time-dependent
Space dimension	Single node	Multi-node energy networks
Uncertainty	Deterministic optimization	Robust or stochastic optimization
Objective function	Single objective	Multiple objectives

The optimal operation problem is a special case of the optimal design problem (which requires resolving operation)

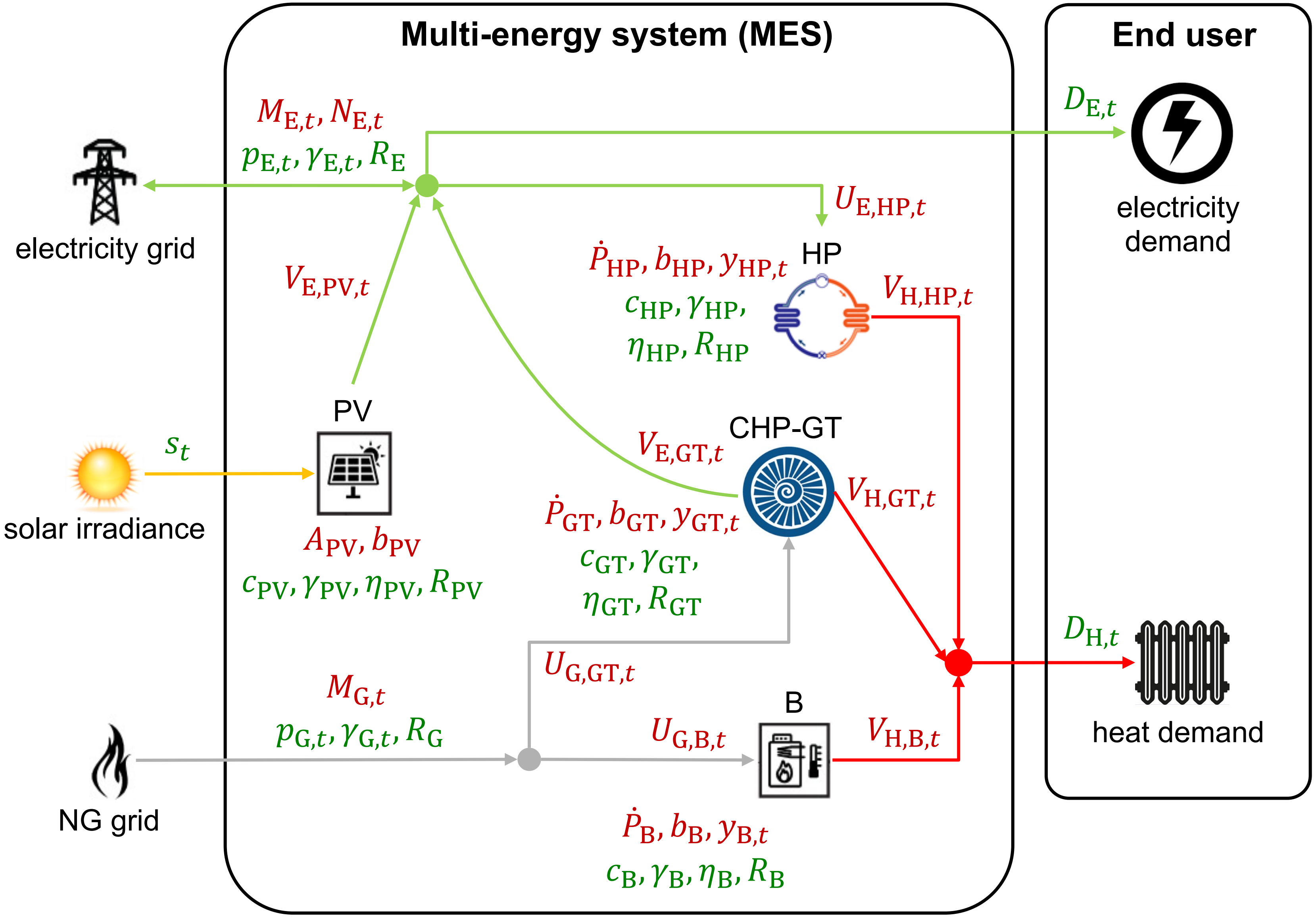
Formulation of the optimization problem

$$\min_{\boldsymbol{x}} \quad z = f(\boldsymbol{x})$$

$$\begin{aligned} \text{s.t.} \quad & g_j(\boldsymbol{x}) \leq 0, & j = 1, \dots, n \\ & h_i(\boldsymbol{x}) = 0, & i = 1, \dots, o \end{aligned}$$

objective function		z	$\mathbb{R}^{m(T)} \rightarrow \mathbb{R}$	System total annual cost, environmental impact, reliability and resilience
decision variables		$\boldsymbol{x}$	$\in \mathbb{R}^{m(T)}$	Technology installation and operation, energy exchanged with grids
constraints	equality	$h_i(\boldsymbol{x}) = 0$	$i = 1, \dots, o$	Energy conversion efficiency (typically nonlinear)
	(in)equality	$g_j(\boldsymbol{x}) \leq 0$	$j = 1, \dots, n$	Min/max-power constraints (typically nonlinear)

MES simple case-study: Optimal, multi-objective design



Decision variables	
Input energy	U
Output energy	V
Imported energy	M
Exported energy	N
Technology size	P
Technology selection	b
ON/OFF scheduling	y

Input data	
Weather conditions (solar)	s
Energy demand	D
Energy price	p
Energy grids and technology carbon footprint	γ
Technology cost	c
Technology performance	η
Component reliability	R

Design decision variables: Input and output energy, and technology size

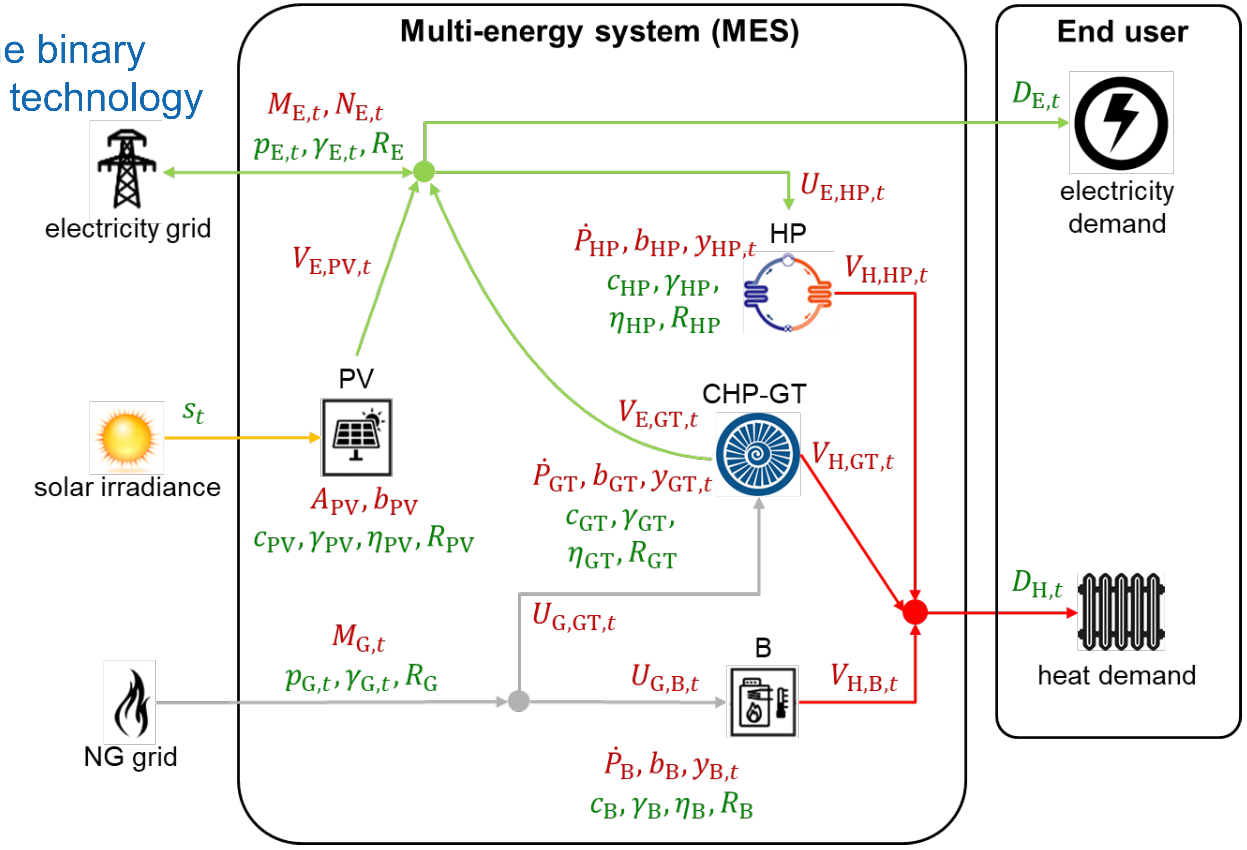
$$x = [M_{E,t}, M_{G,t}, N_{E,t}, U_{G,GT,t}, U_{E,HP,t}, U_{G,B,t}, V_{E,GT,t}, V_{H,GT,t}, V_{H,HP,t}, V_{H,B,t}, V_{E,PV,t}, y_{GT,t} y_{HP,t}, y_{B,t},$$ Operation variables

$$\dot{P}_{GT}, \dot{P}_{HP}, \dot{P}_{B}, \dot{P}_{PV}, b_{GT}, b_{PV}, b_{HP}, b_{B}]$$ Design variables

Considering one binary variable, b, per technology

$$x \in \{\mathbb{R} \text{ or } \{0,1\}\}^m, \qquad m(T) = 14T + 8$$

- Operation variables are vectors of dimension T, design variables are scalars
- Energy variables (operation), x, are connected to power variables (design), $\dot{x}$: $x = \dot{x}\Delta t$



Objective function: Total annual cost

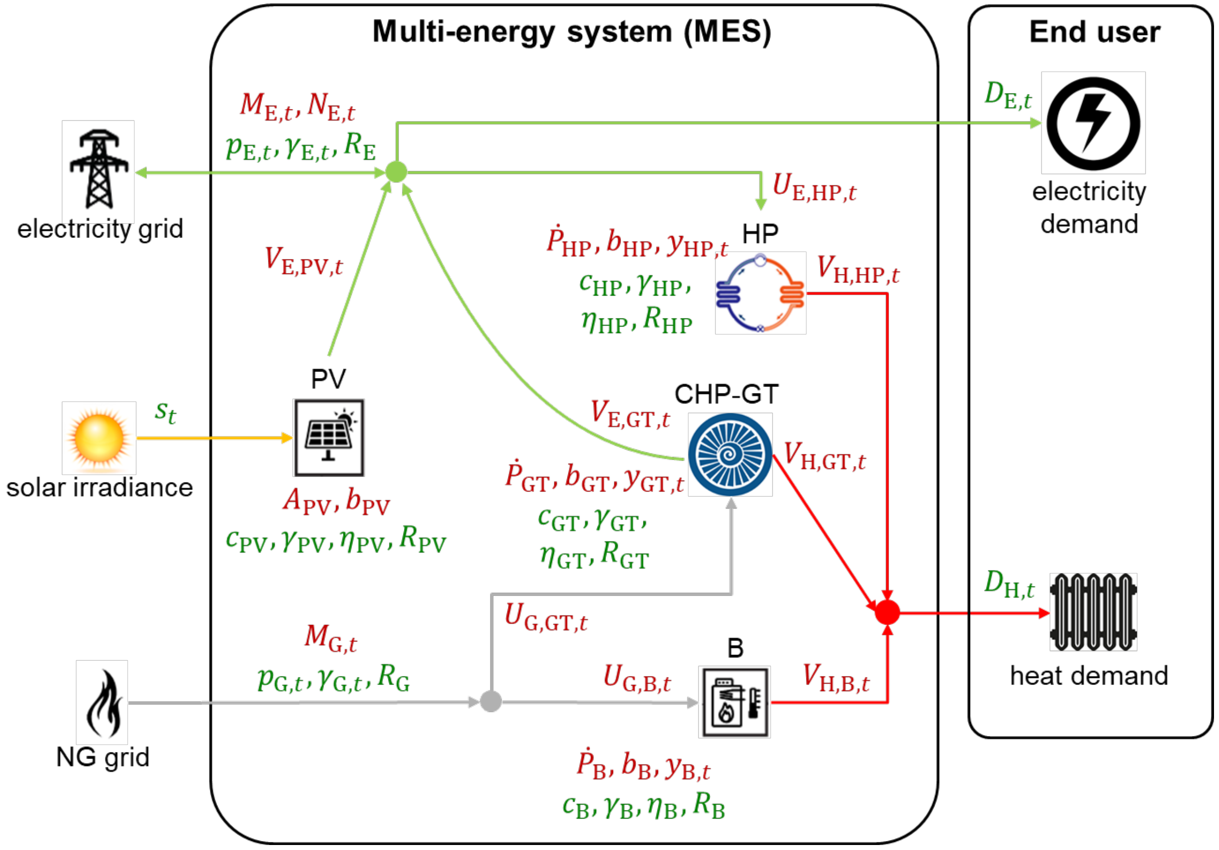
Installation costs

O&M costs

Neglect other costs

$$z_{\text{cost}} = I_{\text{AV}} + O + F + \cancel{X}$$

Energy costs



Objective function: Total annual cost – Installation costs

Installation costs

$$z_{\text{cost}} = I_{\text{AV}} + O + F$$

MINLP

where,

Installation cost of tech. k

$$I_{\text{AV}} = \sum_{k \in \mathcal{K}} I_k a_k$$

Annuity factor of tech. k

$$I_k = c(\dot{P}_k)$$

Nonlinear function of size

$$\dot{P}_{k,\min} b_k \leq \dot{P}_k \leq \dot{P}_{k,\max} b_k$$

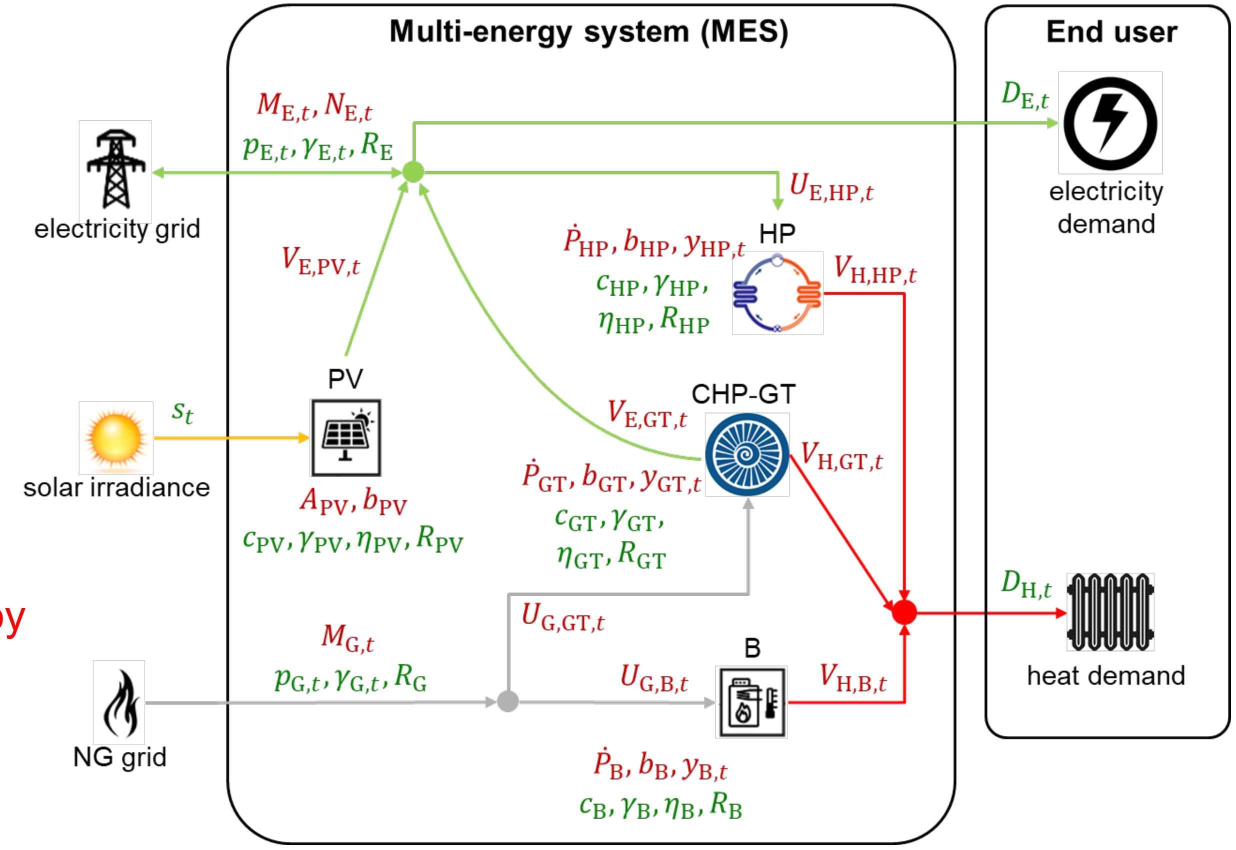
Size is constrained by installation variable

$$a_k = \frac{r}{1 - \frac{1}{(1+r)^{l_k}}}$$

Discount rate

lifetime

No time index: time horizon is fixed to one year by the annuity factor



Objective function: Total annual cost – O&M costs

O&M costs

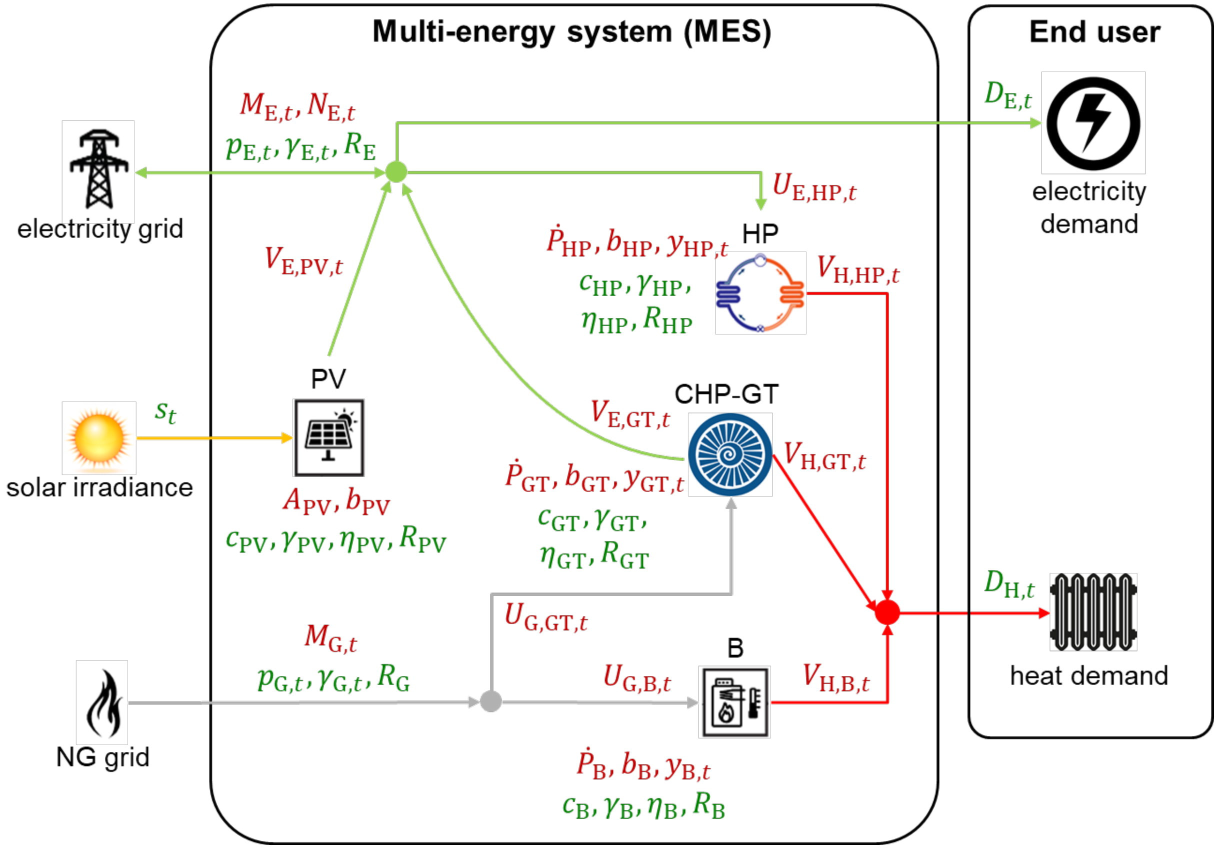
$$z_{\text{cost}} = I_{\text{AV}} + \underbrace{O}_{\text{O\&M costs}} + F$$

LP

where,

$$O = \sum_{k \in \mathcal{K}} v_k I_k a_k$$

- The O&M costs are often given as a fraction, v, of the installation costs
- They refer to the same time basis as the annualized investment cost
- Sometimes O&M expressed as a function of generated energy



Objective function: Total annual cost – O&M costs

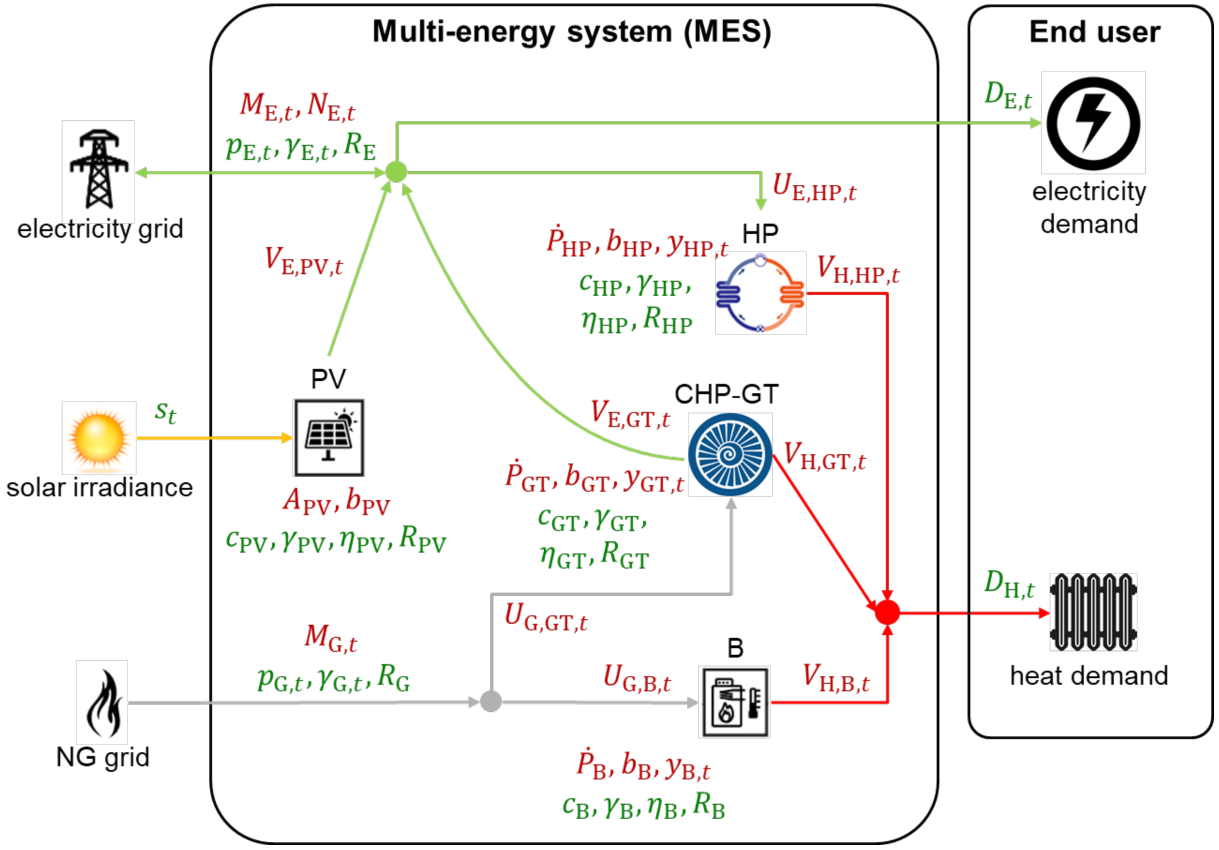
LP

$$z_{\text{cost}} = I_{\text{AV}} + O + \underset{\substack{\uparrow \\ \text{Energy costs}}}{F}$$

where,

$$F = \sum_{t=1}^{T} \sum_{j \in \mathcal{J}} p_{j,t} \left(M_{j,t} - N_{j,t} \right)$$

- Same as for optimal operation problem
- T refers to the same time basis as the annualized investment cost: one year used as time horizon
- Operation cost is given at the net of revenues coming from exported energy

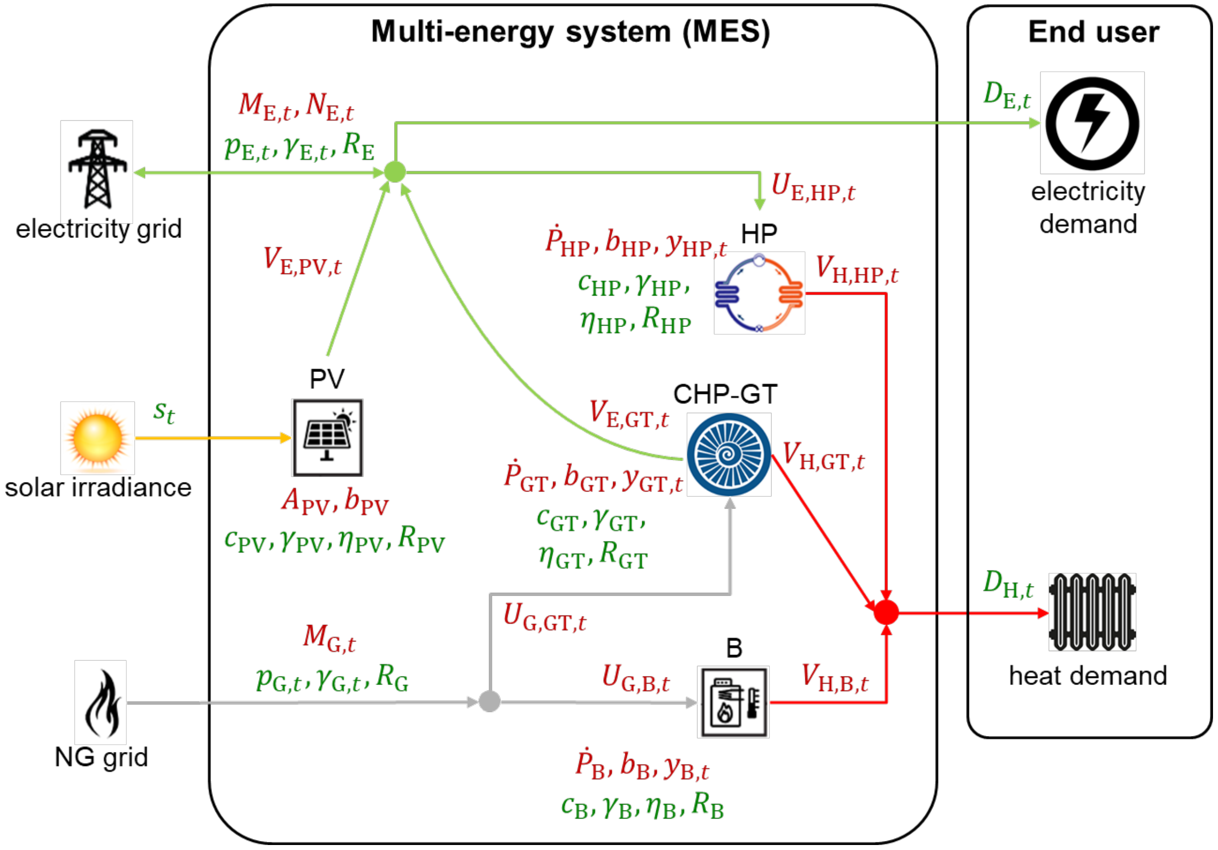


Operation objective function: Carbon emissions

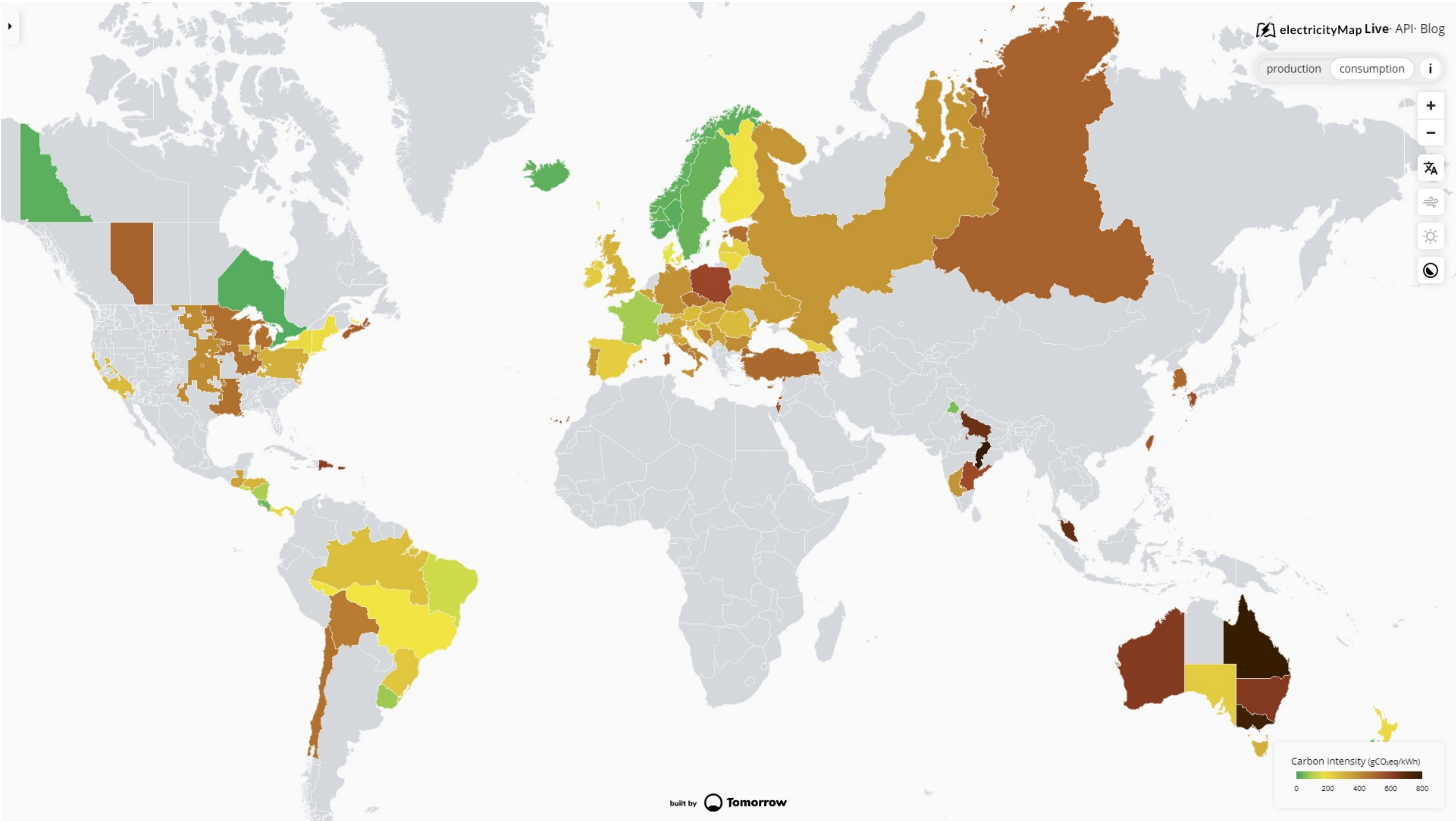
$$z_{\mathrm{CO2}} = \sum_{t=1}^{T} \left[\underbrace{\sum_{j \in \mathcal{J}} \gamma_{j,t} M_{j,t}}_{\text{Grids carbon emissions}} + \underbrace{\sum_{k \in \mathcal{K}} \gamma_k \overbrace{V_{\mathrm{FU},k,t}}^{\text{Technology functional unit}}}_{\text{Technologies carbon emissions}} \right]$$

LP

- The electricity grid carbon footprint depends on the grid production mix (typically at country level), and is time-dependent
- Carbon footprint of gas grid is generally constant, as are the technology carbon footprints
- From LCA (Lecture 7): “the functional unit defines the quantification of the identified functions of the product”. Here, it is the main energy output of a technology.

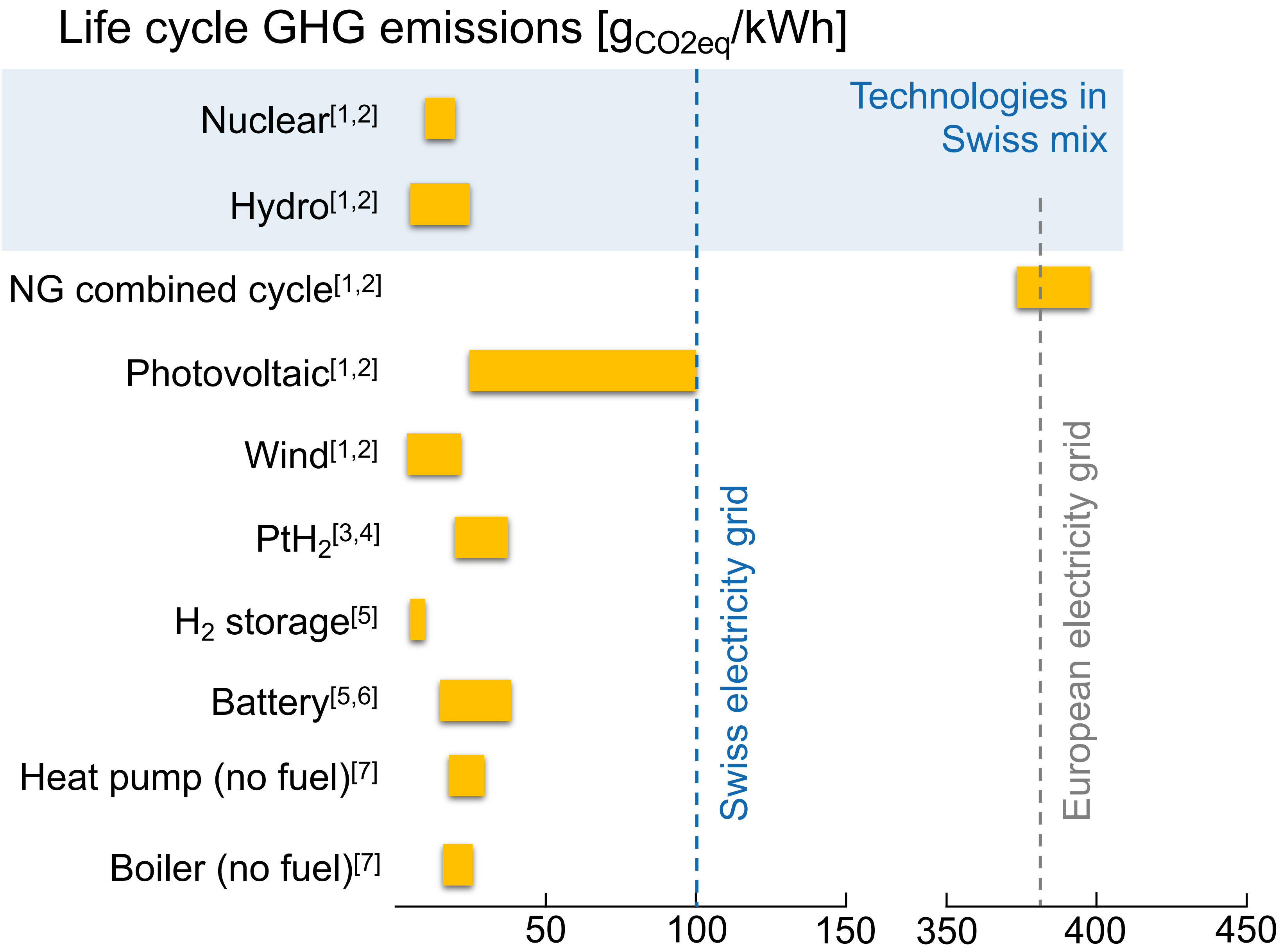


Operation objective function: Carbon emissions – Electricity grid



Real time carbon intensity of electricity grid (when available): Free API (non-commercial use) - https://www.electricitymap.org/map

Operation objective function: Carbon emissions – Technology LCA



Sources

1. PSI, *Potential, costs and environmental assessment of electricity generation technologies*, **2017**
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3. Simons and Bauer, *Appl. Energy*, **2015**, 157, 884-896
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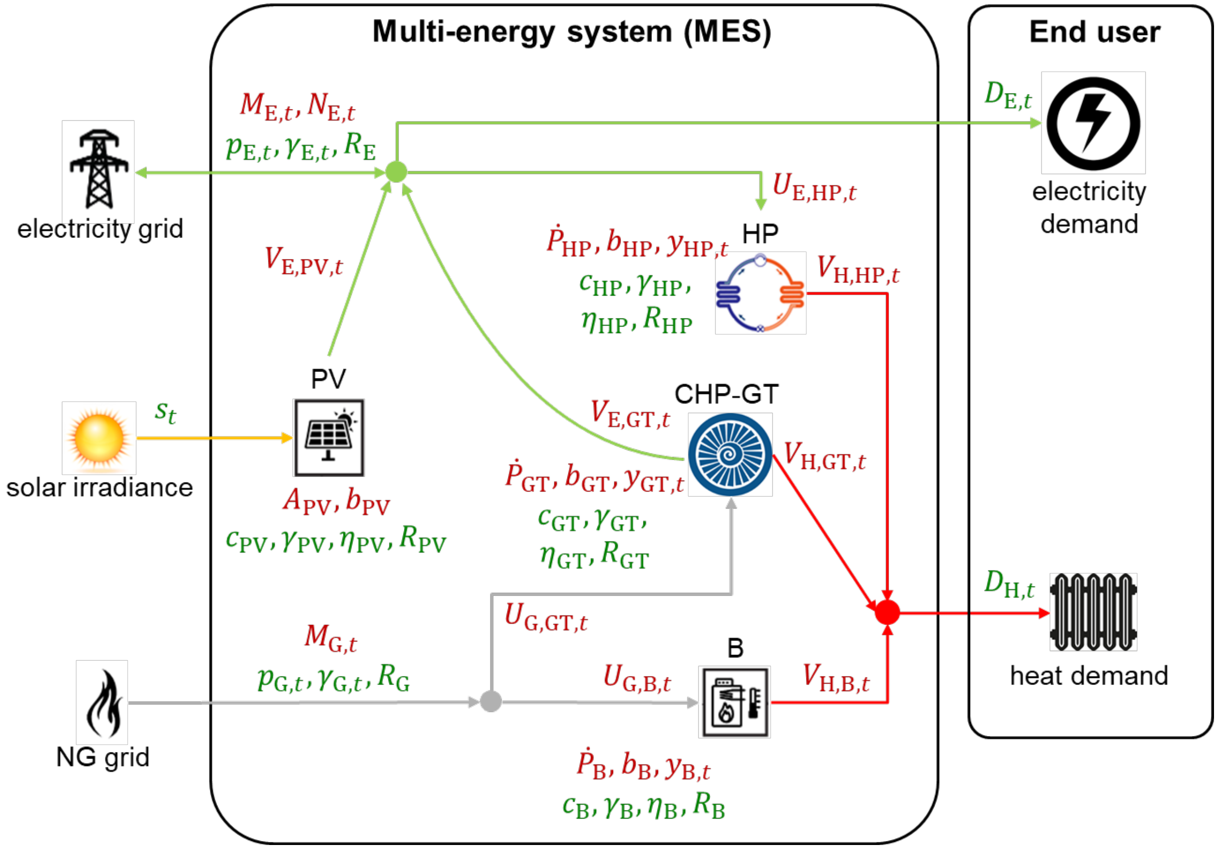
Operation objective function: Expected energy not supplied (EENS)

$$z_{\text{risk}} = \underbrace{\sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{E}} \pi_i \sum_{t=1}^{T} \text{DNS}_{j,t,i}}_{\text{Overall EENS}}$$

LP

- The overall expected energy not supplied (EENS) of energy carrier j depends on the **state probability**, π_i, which is a function of the reliability of energy grids (R_E and R_G) and technologies (R_{GT}, R_{HP}, R_{PV}, R_{B})
- Set of system states, $\mathcal{E}$, and their probabilities π_i are identified using an event tree
- $\text{DNS}_{j,t,i}$ is the **demand not supplied (DNS)** of carrier j at time t in system state i
- Assumption: The repair time is equal for all technologies, and all technologies can be repaired simultaneously (enough repair crews)

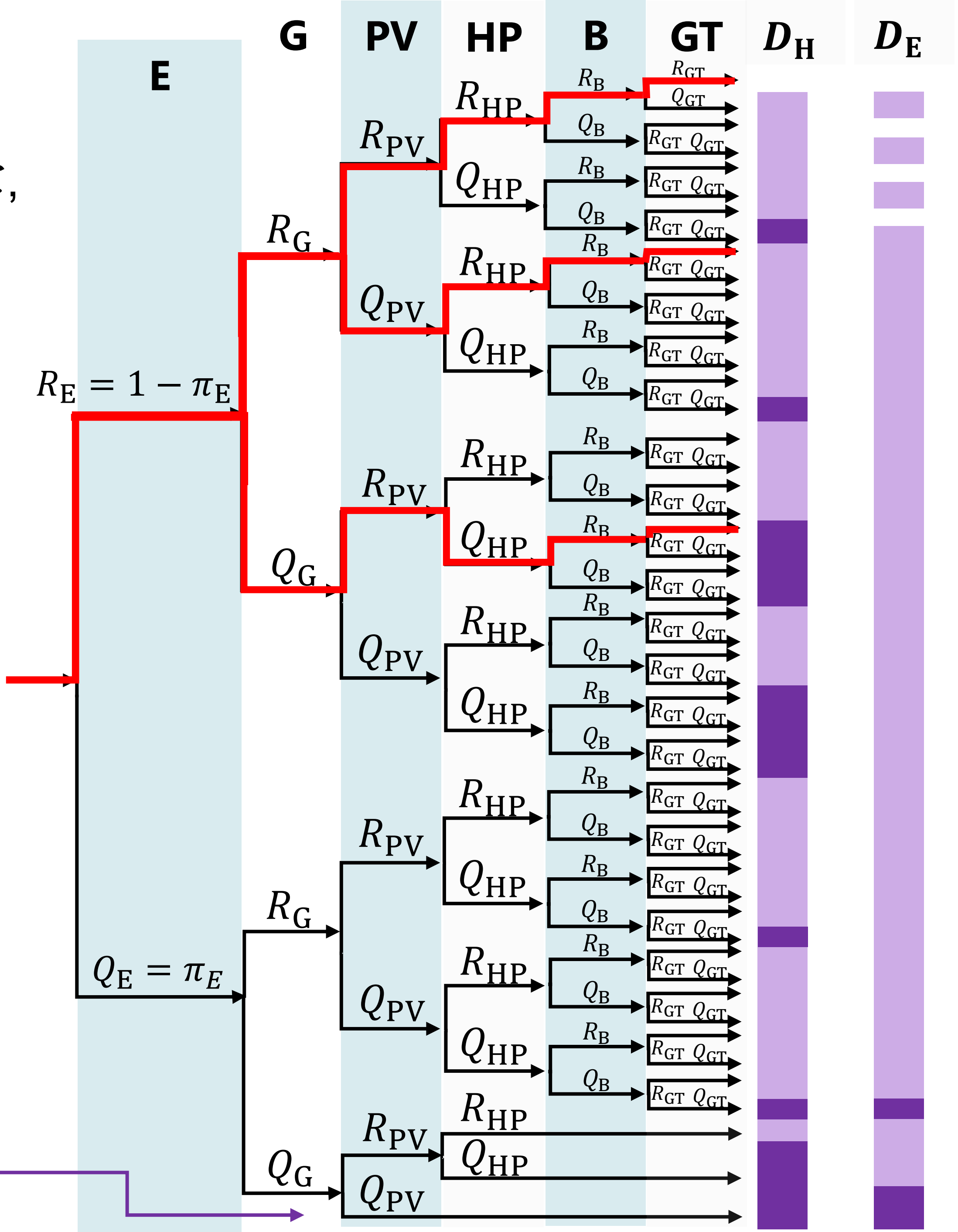


Recap: Event trees

- Visualization of the set of all possible system states, $\mathcal{E}$, using paths $i \in \mathcal{E}$ in a tree
- π_i is the product of all probabilities along path i
- Determine states for which the heat or electricity supply is completely impossible or possibly reduced

State set	Impact on Supply	DNS	Example (D_H)
$\mathcal{E}_N$	No impact on supply	$\mathrm{DNS}_{j,t,i} = 0$	$R_\mathrm{E} R_\mathrm{G} R_\mathrm{PV} R_\mathrm{HP} R_\mathrm{B} R_\mathrm{GT}$ Everything operational
$\mathcal{E}_I$	Supply is impossible	$\mathrm{DNS}_{j,t,i} = D_{j,t}$	$R_\mathrm{E} Q_\mathrm{G} R_\mathrm{PV} Q_\mathrm{HP} R_\mathrm{B} R_\mathrm{GT}$ No gas, no heat pump
$\mathcal{E}_R$	Supply may be reduced	$0 \le \mathrm{DNS}_{j,t,i} \le D_{j,t}$	$R_\mathrm{E} Q_\mathrm{G} Q_\mathrm{PV} R_\mathrm{HP} R_\mathrm{B} R_\mathrm{GT}$ PV or gas grid failed. May lead to DNS if grid electricity is insufficient to operate heat pump

Determining DNS: See Constraints (3)



Tree reduction: Compound paths $\mathcal{E}_I$ without inputs, because no energy can be supplied for all of them

Constraints (1): Energy balances for all energy carriers

LP

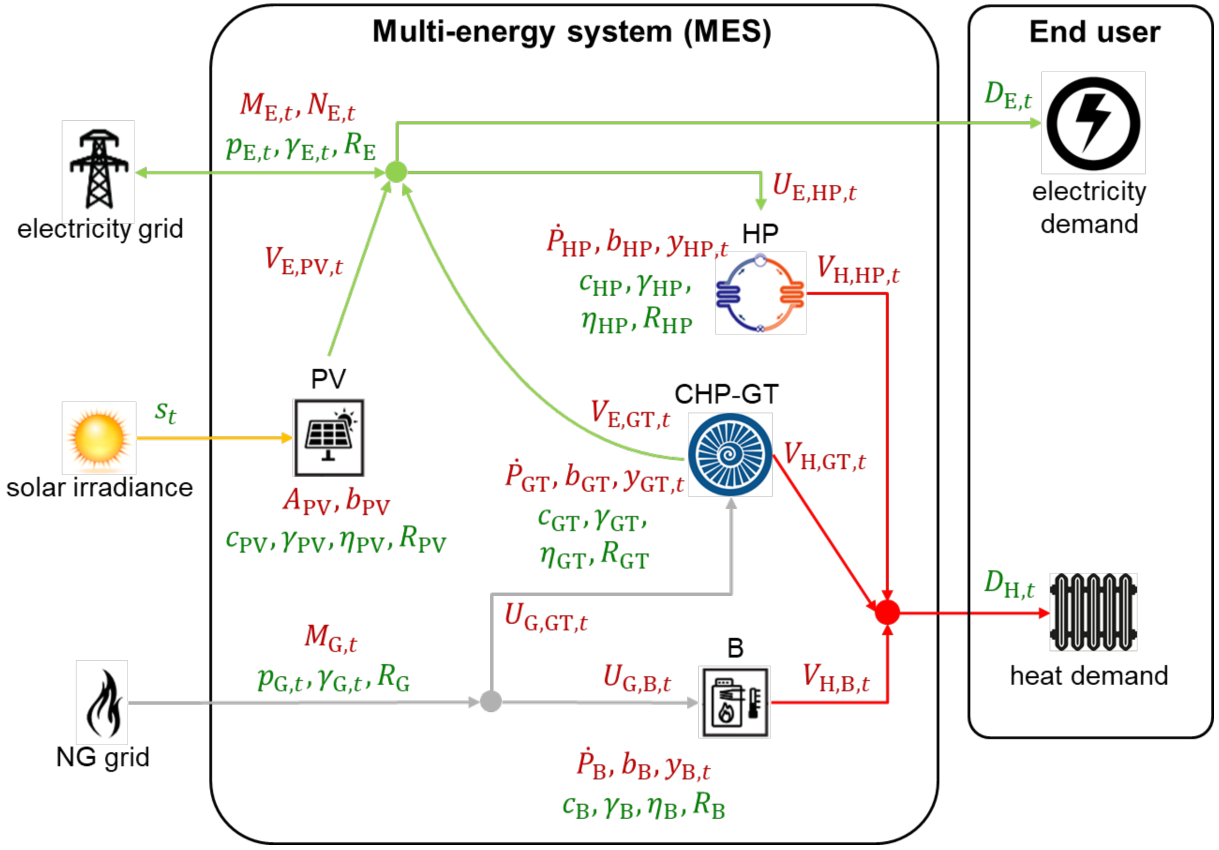
Generic energy carrier, *j*

$$\sum_{k \in \mathcal{K}} \left(V_{j,k,t} - U_{j,k,t}\right) + M_{j,t} - N_{j,t} = D_{j,t} \ ,$$

$$0 \leq M_{j,t} \leq M_{j,\max}, \ \ 0 \leq N_{j,t} \leq N_{j,\max} \ ,$$

$$\forall j \in \mathcal{J}, t \in \{1, \ldots, T\}$$

- Same as for optimal operation problem
- The maximum constraint on imported/exported energy can be zero if energy grid is not present (e.g. heat)



Constraints (2): Performance of conversion technology – Dispatchable

MINLP

Generic dispatchable technology, k

$$V_{\bar{j},k,t} = \underbrace{\eta_{\underline{j}\bar{j},k}}_{}(U_{\underline{j},k,t}, \boxed{P_k})$$

conversion efficiency: not a function of time

Design optimization: performance is a function of energy input and technology size

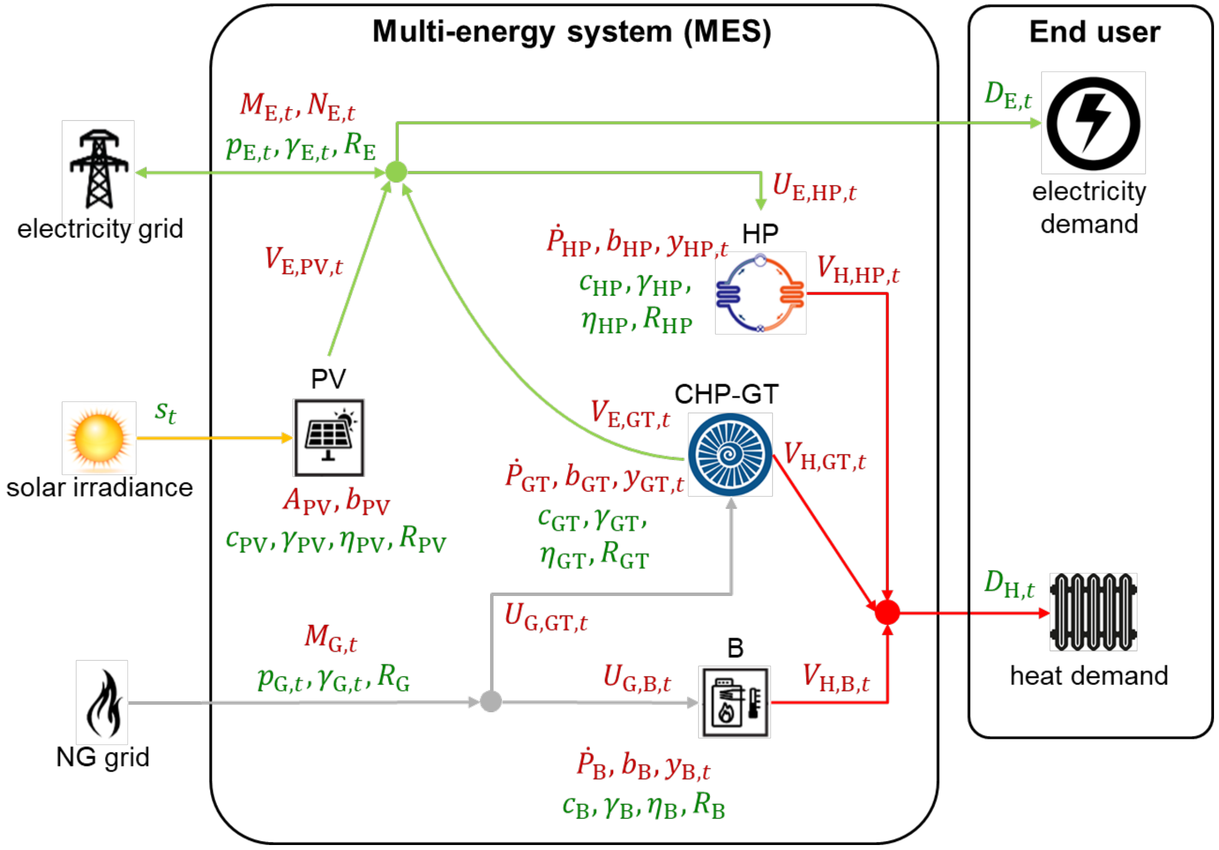
$$y_{k,t}\underbrace{\delta_k}_{}P_k \leq U_{\underline{j},k,t} \leq y_{k,t}P_k$$

Minimum power as fraction of size

$$b_k\dot{P}_{k,\min} \leq \dot{P}_k \leq b_k\dot{P}_{k,\max}$$

$$\forall \bar{j} \in \bar{\mathcal{J}}_k, \underline{j} \in \underline{\mathcal{J}}_k, k \in \mathcal{K}, t \in \{1, \dots, T\}$$

- For conventional technologies the input energy is a decision variable, constrained by the size
- Size ranging from minimum to maximum value



Constraints (2): Performance of conversion technology – RE

LP

Generic RE technology, k

$$V_{\bar{J},k,t} = \underbrace{\eta_k(w_t)}\,A_k,$$

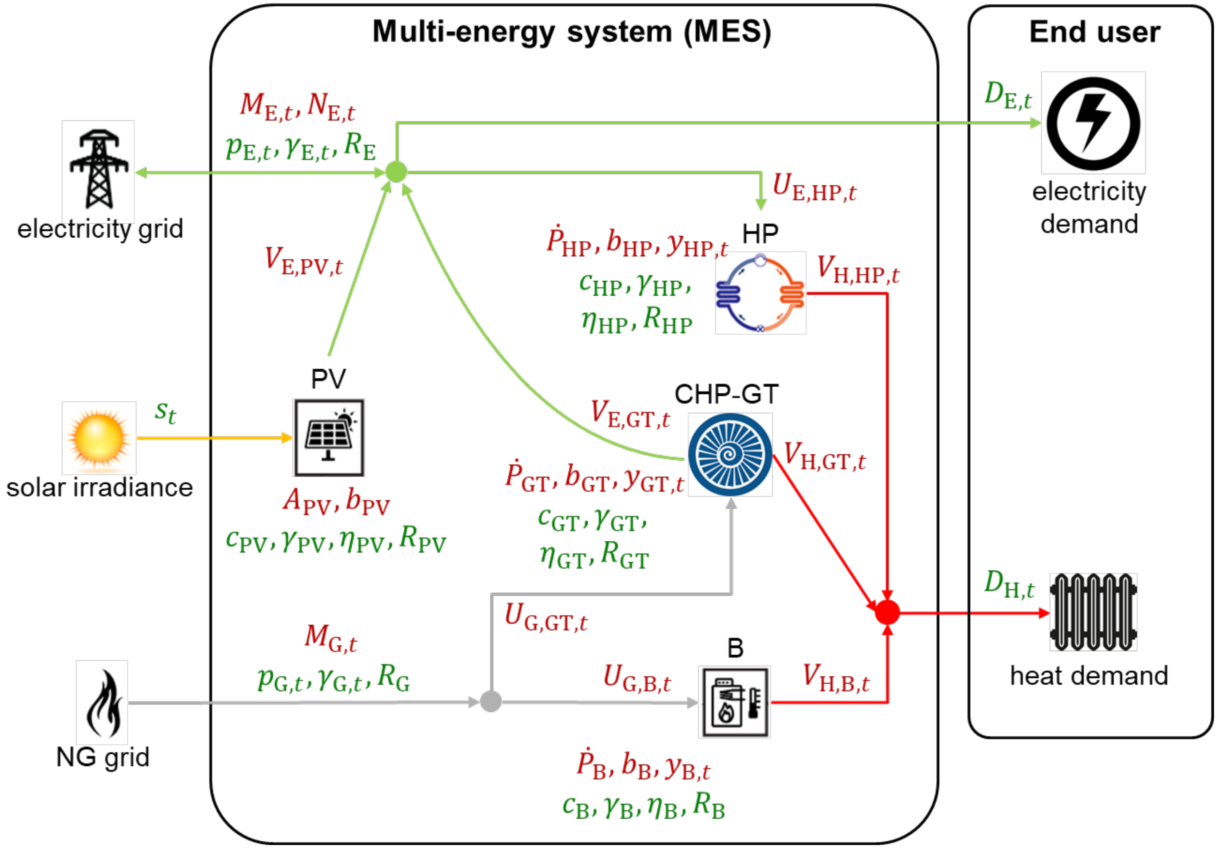
conversion efficiency function of time-dependent weather conditions, w (e.g. solar irradiance, wind speed)

$$b_k A_{k,\min} \leq A_k \leq b_k A_{k,\max}\,,$$

Minimum installed area   Maximum installed area

$$\forall \bar{J} \in \bar{\mathcal{J}}_k, k \in \mathcal{K}, t \in \{1, \dots, T\}$$

- For RE technologies the input energy (i.e. weather conditions) is an input data
- It scales with the size, which can range from a minimum to a maximum value



Constraints (3): Determining demand not supplied (DNS)

1. Consider all operational variables and technology constraints (2) for each system state i where supply may be reduced
2. Set the inputs and outputs of all failed (F) technologies $\mathcal{K}_{F,i}$ to zero
3. Set grid imports and exports of failed carriers $\mathcal{J}_{F,i}$ to zero
4. Introduce demand not supplied (DNS) in energy balances constraints (1)

$$U_{\underline{j},k,t,i} = V_{\bar{j},k,t,i} = 0$$

$$\forall \bar{j} \in \bar{\mathcal{J}}_k, \underline{j} \in \underline{\mathcal{J}}_k, k \in \mathcal{K}_{F,i}, t \in \{1, \dots, T\}$$

$$N_{j,t,i} = M_{j,t,i} = 0$$

$$\forall \ j \in \mathcal{J}_{F,i}, t \in \{1, \dots, T\}$$

$$\sum_{k \in \mathcal{K}} \left(V_{j,k,t,i} - U_{j,k,t,i}\right) + M_{j,t,i} - N_{j,t,i} = D_{j,t} - DNS_{j,t,i} \ ,$$

$$0 \leq DNS_{j,t,i} \leq D_{j,t},$$

$$\forall j \in \mathcal{J}, t \in \{1, \dots, T\}$$

General MILP formulation: Optimal design (no storage)

Objective function

$$\min_{x}\; z_{\text{cost}} = \sum_{k\in\mathcal{K}} (1+\nu_k) I_k a_k + \sum_{t=1}^{T}\sum_{j\in\mathcal{J}} p_{j,t}\left(M_j - N_j\right)$$

$$\min_{x}\; z_{\text{CO2}} = \sum_{t=1}^{T}\left[\sum_{j\in\mathcal{J}} \gamma_{j,t} M_{j,t} + \sum_{k\in\mathcal{K}} \gamma_k V_t^*\right]$$

$$\min_{x}\; z_{\text{risk}} = \sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{E}} \pi_i \sum_{t=1}^{T} \text{DNS}_{j,t,i}$$

s. t.

Energy balances

$$\sum_{k\in\mathcal{K}} \left(V_{j,k,t} - U_{j,k,t}\right) + M_{j,t} - N_{j,t} = D_{j,t}$$

$$0 \le M_{j,t} \le M_{j,\max},\;\; 0 \le N_{j,t} \le N_{j,\max} \qquad \forall j \in \mathcal{J}, t \in \{1,\dots,T\}$$

⋮

MES structure

$\mathcal{J}$ = Set of energy carriers, {E, G, H}

$\mathcal{K}$ = Set of technologies, {PV, HP, B}

$\mathcal{K}_{\text{R}}$ = Set of renewable energy technologies, {PV}

$\mathcal{K}_{\text{C}}$ = Set of dispatchable technologies, {HP, B, GT}

$\bar{\mathcal{J}}_k$ = Set of output carriers for technology k

$\underline{\mathcal{J}}_k$ = Set of input carriers for technology k

T = Length of the time horizon

General MILP formulation: Optimal design (no storage)

⋮

RE (non-dispatchable) technologies

$$V_{\bar{j},k,t} = \eta_k(w_t)\, A_k,$$

$$b_k A_{k,\min} \leq A_k \leq b_k A_{k,\max} \qquad \forall \bar{j} \in \bar{\mathcal{J}}_k, k \in \mathcal{K}_\mathrm{R}, t \in \{1, \dots, T\}$$

Dispatchable technologies

$$V_{\bar{j},k,t} = \eta_{\underline{j}\bar{j},k}(U_{\underline{j},k,t}, P_k)$$

$$y_{k,t}\delta_k P_k \leq U_{\underline{j},k,t} \leq y_{k,t} P_k$$

$$b_k \dot{P}_{k,\min} \leq \dot{P}_k \leq b_k \dot{P}_{k,\max} \qquad \forall \bar{j} \in \bar{\mathcal{J}}_k, \underline{j} \in \underline{\mathcal{J}}_k, k \in \mathcal{K}_\mathrm{C}, t \in \{1, \dots, T\}$$

⋮

MES structure

$\mathcal{J}$ = Set of energy carriers, $\{\mathrm{E}, \mathrm{G}, \mathrm{H}\}$

$\mathcal{K}$ = Set of technologies, $\{\mathrm{PV}, \mathrm{HP}, \mathrm{B}\}$

$\mathcal{K}_\mathrm{R}$ = Set of renewable energy technologies, $\{\mathrm{PV}\}$

$\mathcal{K}_\mathrm{C}$ = Set of dispatchable technologies, $\{\mathrm{HP}, \mathrm{B}, \mathrm{GT}\}$

$\bar{\mathcal{J}}_k$ = Set of output carriers for technology k

$\underline{\mathcal{J}}_k$ = Set of input carriers for technology k

T = Length of the time horizon

General MILP formulation: Optimal design (no storage)

⋮

Demand not supplied

$$DNS_{j,t,i} = 0 \;\; \forall i \in \mathcal{E}_N$$

$$DNS_{j,t,i} = D_{j,t} \;\; \forall i \in \mathcal{E}_I$$

Technology operation constraints for scenario i

$$U_{\underline{j},k,t,i} = V_{\bar{j},k,t,i} = 0 \;\; \forall \bar{j} \in \bar{\mathcal{J}}_k, \underline{j} \in \underline{\mathcal{J}}_k, k \in \mathcal{K}_{F,i}$$

$$N_{j,t,i} = M_{j,t,i} = 0 \;\; \forall \; j \in \mathcal{J}_{F,i}$$

$$\sum_{k \in \mathcal{K}} \left(V_{j,k,t,i} - U_{j,k,t,i}\right) + M_{j,t,i} - N_{j,t,i} = D_{j,t} - DNS_{j,t,i}$$

$$0 \leq DNS_{j,t,i} \leq D_{j,t} \qquad \forall i \in \mathcal{E}_R, j \in \mathcal{J}, t \in \{1, \ldots, T\}$$

MES structure

$\mathcal{J}$ = Set of energy carriers, {E, G, H}

$\mathcal{K}$ = Set of technologies, {PV, HP, B}

$\mathcal{K}_\mathrm{R}$ = Set of renewable energy technologies, {PV}

$\mathcal{K}_\mathrm{C}$ = Set of dispatchable technologies, {HP, B, GT}

$\bar{\mathcal{J}}_k$ = Set of output carriers for technology k

$\underline{\mathcal{J}}_k$ = Set of input carriers for technology k

T = Length of the time horizon

$\mathcal{E}_N, \mathcal{E}_I, \mathcal{E}_R$ = Set of system states where supply is not impacted (N), impossible (I) or reduced (R)

$\mathcal{K}_{F,i}$ = Set of failed technologies in state i

$\mathcal{J}_{F,i}$ = Set of failed carrier grid imports in state i

After this lecture, you are able to …

✓ Formulate multi-objective optimization problem for MES optimal design (this can be solved through the methods introduced in Lecture 8)

- Model energy conversion technologies within MES optimization

- Model energy storage technologies within MES optimization

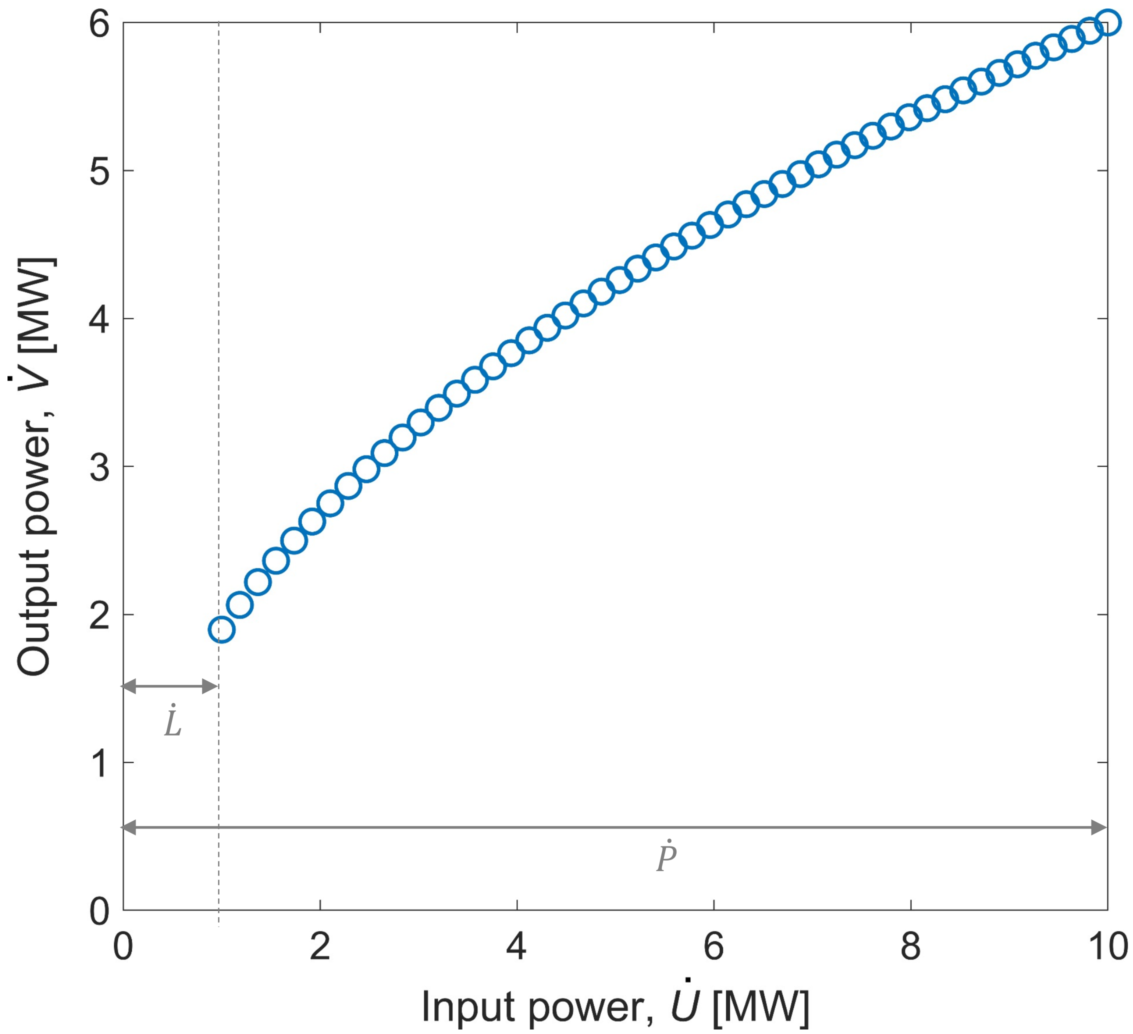
✓ Understand the different degrees of complexity when optimizing MES

- Model energy conversion dynamics (***optional material, no exam***)

Modeling MES:
Linearization of constraints for LP and MILP formulations

Linearizing conversion performance: From non-linear models …

Consider a conversion technology, e.g. HP, and its heat generation. Most often, the off-design behavior results in a nonlinear correlation between input, $\dot{U}$, and output power, $\dot{V}$



- Technology performance are given in terms of power
- We translate this to energy by implicitly consider a consistent time interval, Δt:

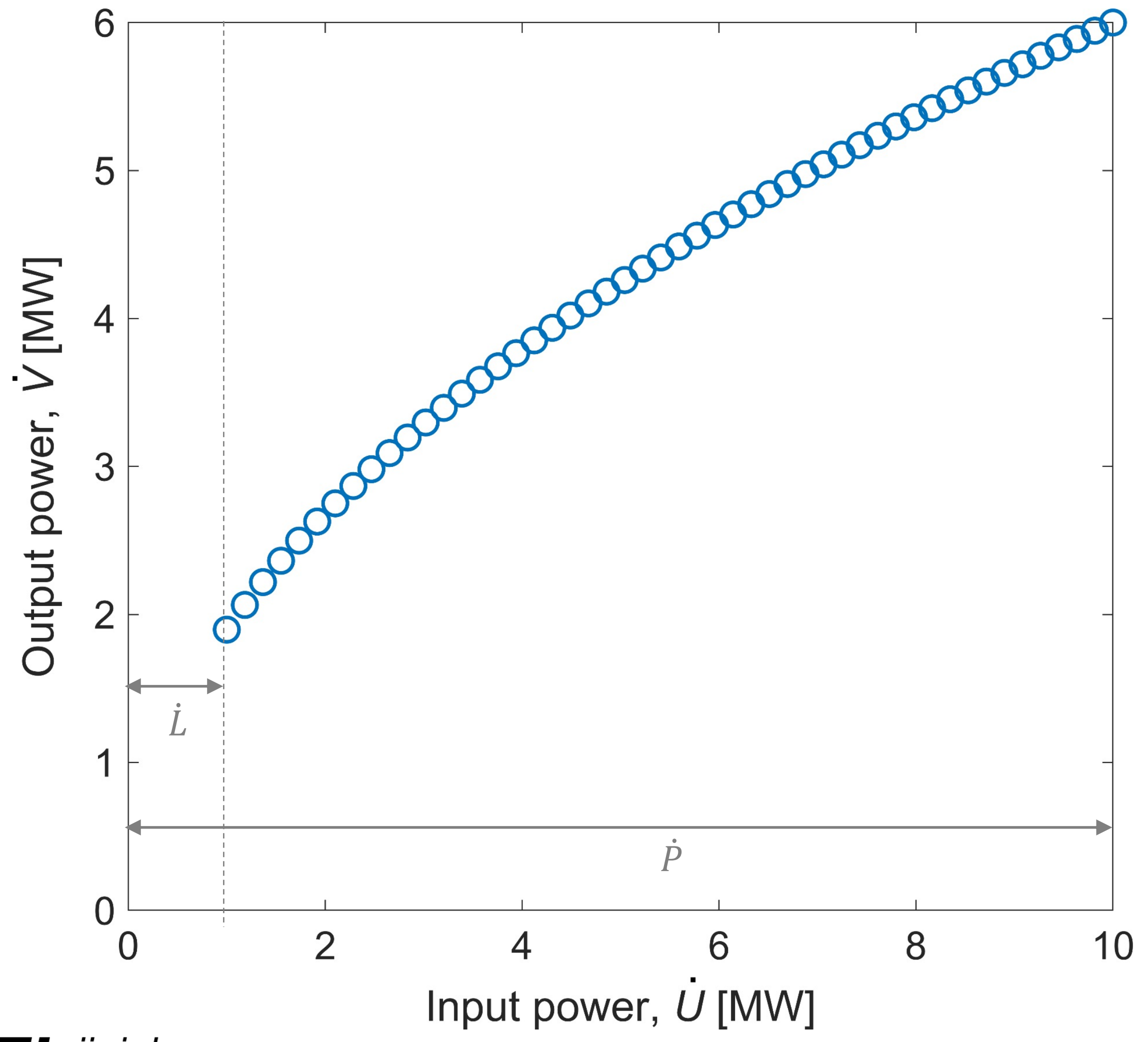
$$U = \dot{U}\Delta t, \; V = \dot{V}\Delta t, \; L = \dot{L}\Delta t, \; P = \dot{P}\Delta t$$

- We do not report the time index for simplicity
- We consider constant size at first

$\dot{L} \in \mathbb{R}$, minimum power (input)
$\dot{P} \in \mathbb{R}$, rated power (input)

Linearizing conversion performance: From non-linear models …

Consider a conversion technology, e.g. HP, and its heat generation. Most often, the off-design behavior results in a nonlinear correlation between input, U, and output energy, V



Actual behavior (MINLP)

$$V_{\mathrm{H,HP}} = \eta_{\mathrm{HP}}(U_{\mathrm{E,HP}})$$

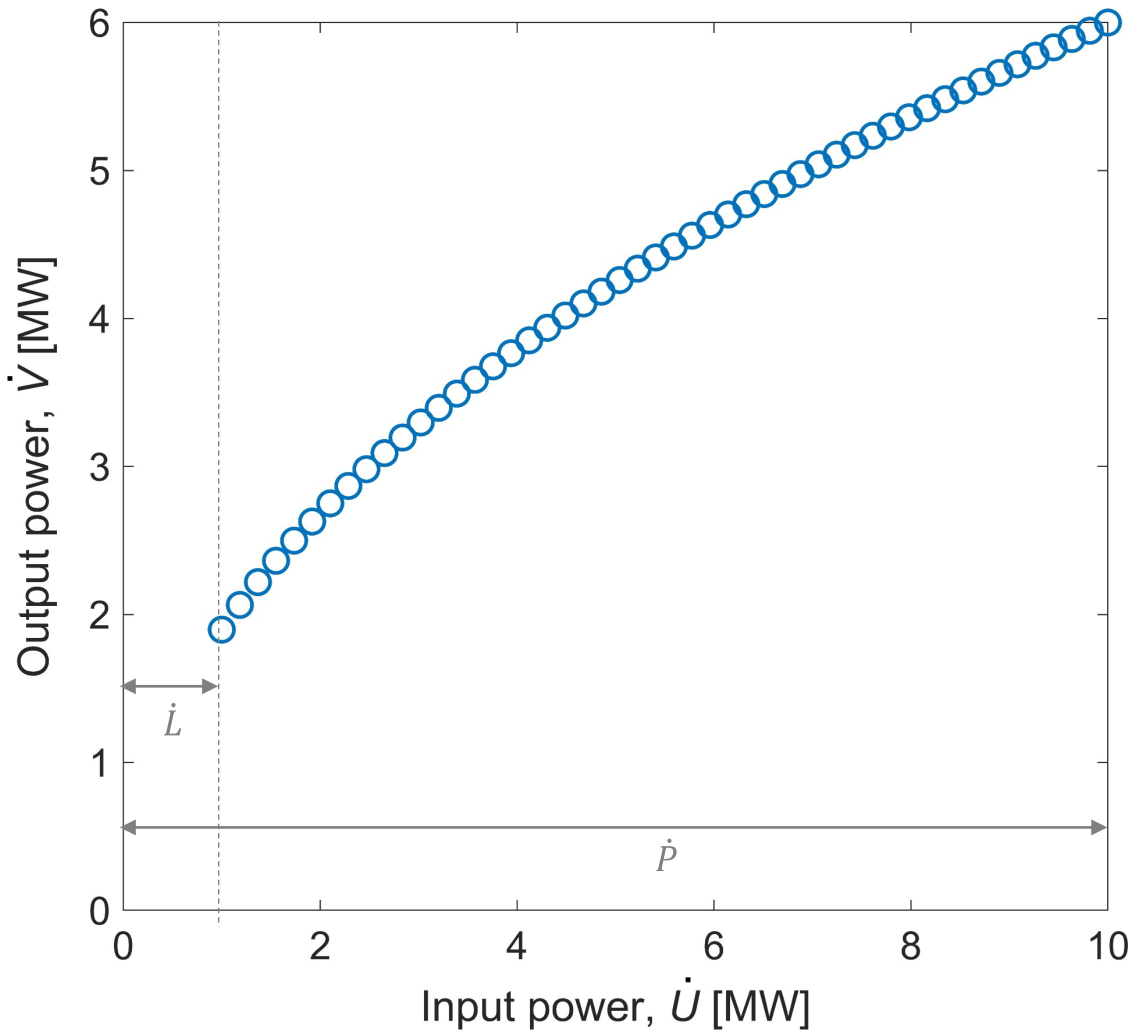
$$y_{\mathrm{HP}} L_{\mathrm{HP}} \leq U_{\mathrm{E,HP}} \leq y_{\mathrm{HP}} P_{\mathrm{HP}}$$

Non-linear function, e.g.
$V_{\mathrm{H,HP}} = \alpha\sqrt{U_{E,HP}}$

$\dot{L} \in \mathbb{R}$, minimum power (input)
$\dot{P} \in \mathbb{R}$, rated power (input)
$y \in \{0,1\}$, ON/OFF operation

Linearizing conversion performance: From non-linear models …

Consider a conversion technology, e.g. HP, and its heat generation. Most often, the off-design behavior results in a nonlinear correlation between input, U, and output energy, V



Actual behavior (MINLP)

$$V = \eta(U)$$

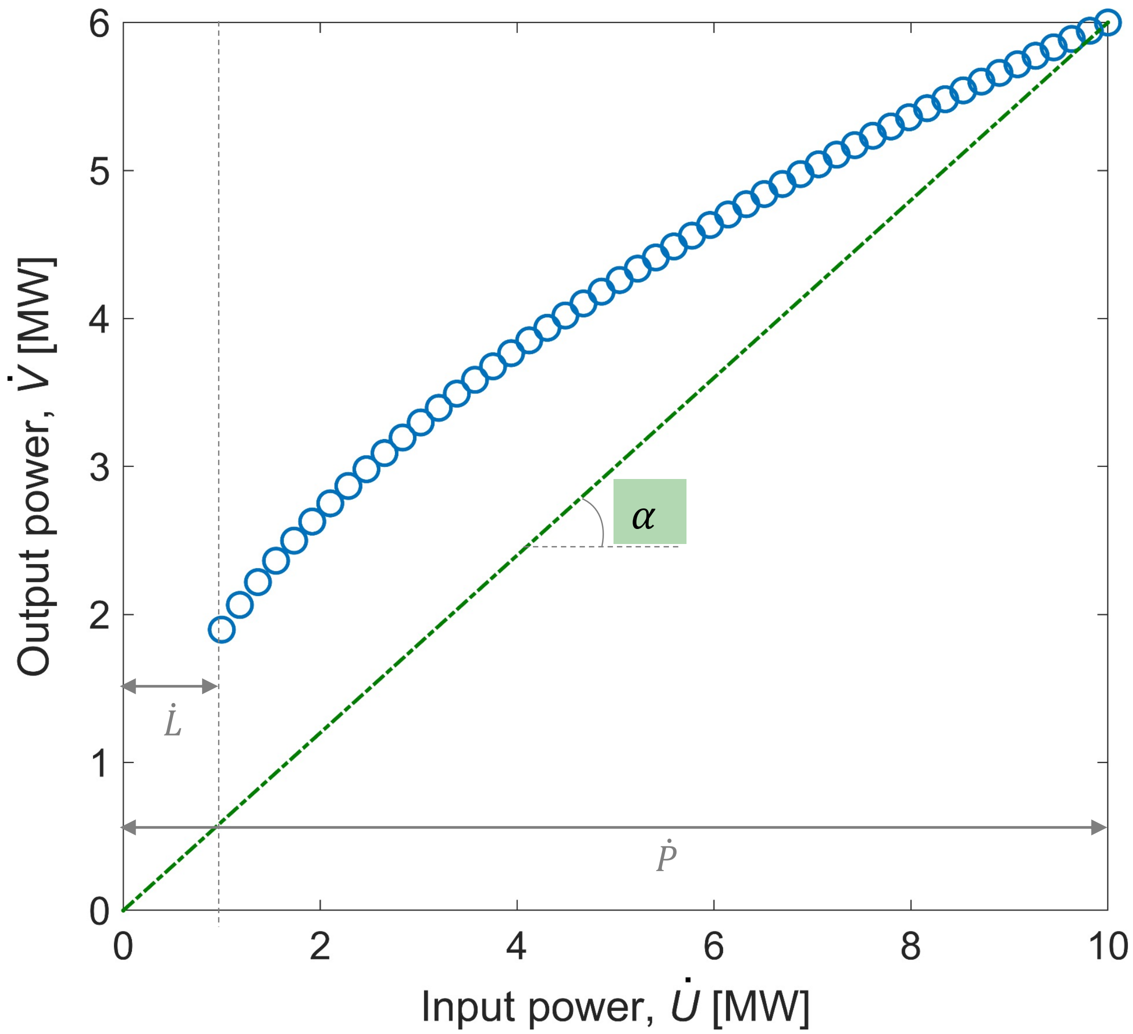
$$yL \leq U \leq yP$$

Simplified notation for a generic technology:
$V = \alpha\sqrt{U}$

$\dot{L} \in \mathbb{R}$, minimum power (input)
$\dot{P} \in \mathbb{R}$, rated power (input)
$y \in \{0,1\}$, ON/OFF operation

Linearizing conversion performance: … to linear models …

Consider a conversion technology, e.g. HP, and its heat generation. Most often, the off-design behavior results in a nonlinear correlation between input, U, and output energy, V



Actual behavior (MINLP)

$$V = \eta(U)$$

$$yL \leq U \leq yP$$

Linear approximation (LP)

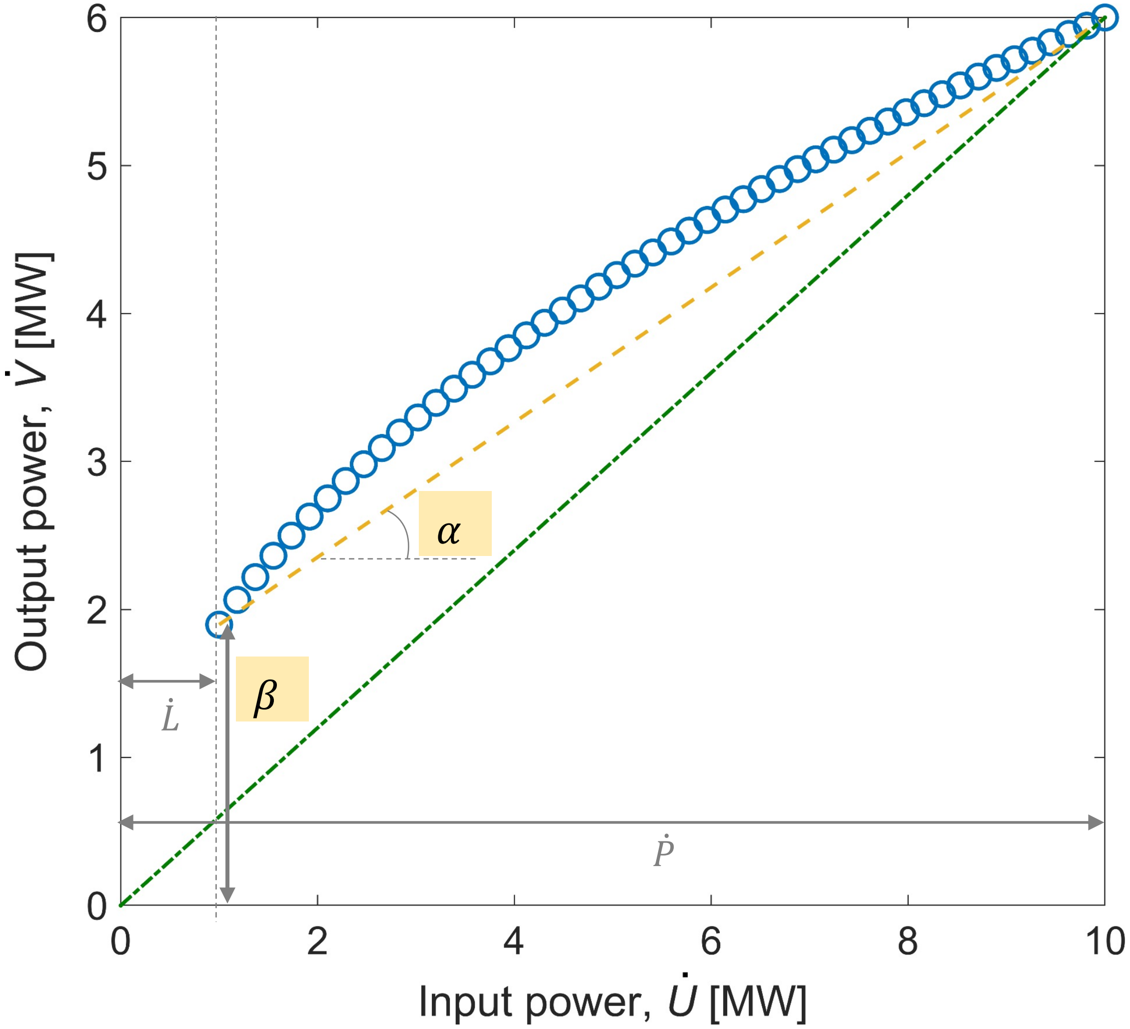
$$V = \alpha U$$

$$0 \leq U \leq P$$

$\dot{L} \in \mathbb{R}$, minimum power (input)
$\dot{P} \in \mathbb{R}$, rated power (input)
$y \in \{0,1\}$, ON/OFF operation

Linearizing conversion performance: … to mixed-integer linear models

Consider a conversion technology, e.g. HP, and its heat generation. Most often, the off-design behavior results in a nonlinear correlation between input, U, and output energy, V



Actual behavior (MINLP)

$$V = \eta(U)$$

$$yL \leq U \leq yP$$

Linear approximation (LP)

$$V = \alpha U$$

$$0 \leq U \leq P$$

Affine approximation

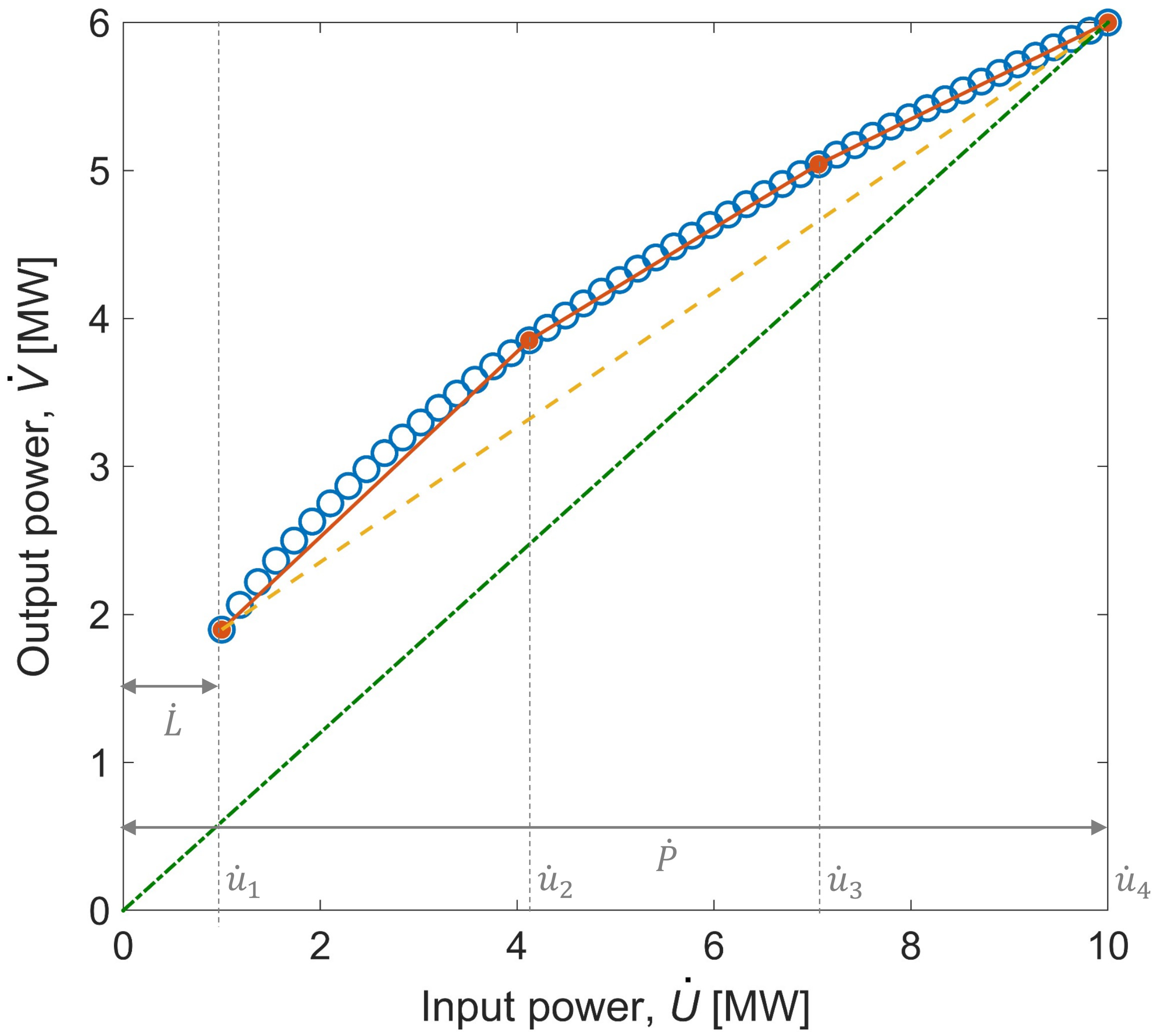
$$V = \alpha U + \beta y$$

$$yL \leq U \leq yP$$

$\dot{L} \in \mathbb{R}$, minimum power (input)
$\dot{P} \in \mathbb{R}$, rated power (input)
$y \in \{0,1\}$, ON/OFF operation

Linearizing conversion performance: … to mixed-integer linear models

Consider a conversion technology, e.g. HP, and its heat generation. Most often, the off-design behavior results in a nonlinear correlation between input, U, and output energy, V



Actual behavior (MINLP)

$$V = \eta(U)$$

$$yL \leq U \leq yP$$

Linear approximation (LP)

$$V = \alpha U$$

$$0 \leq U \leq P$$

Affine approximation

$$V = \alpha U + \beta y$$

$$yL \leq U \leq yP$$

PWA approximation (MILP)

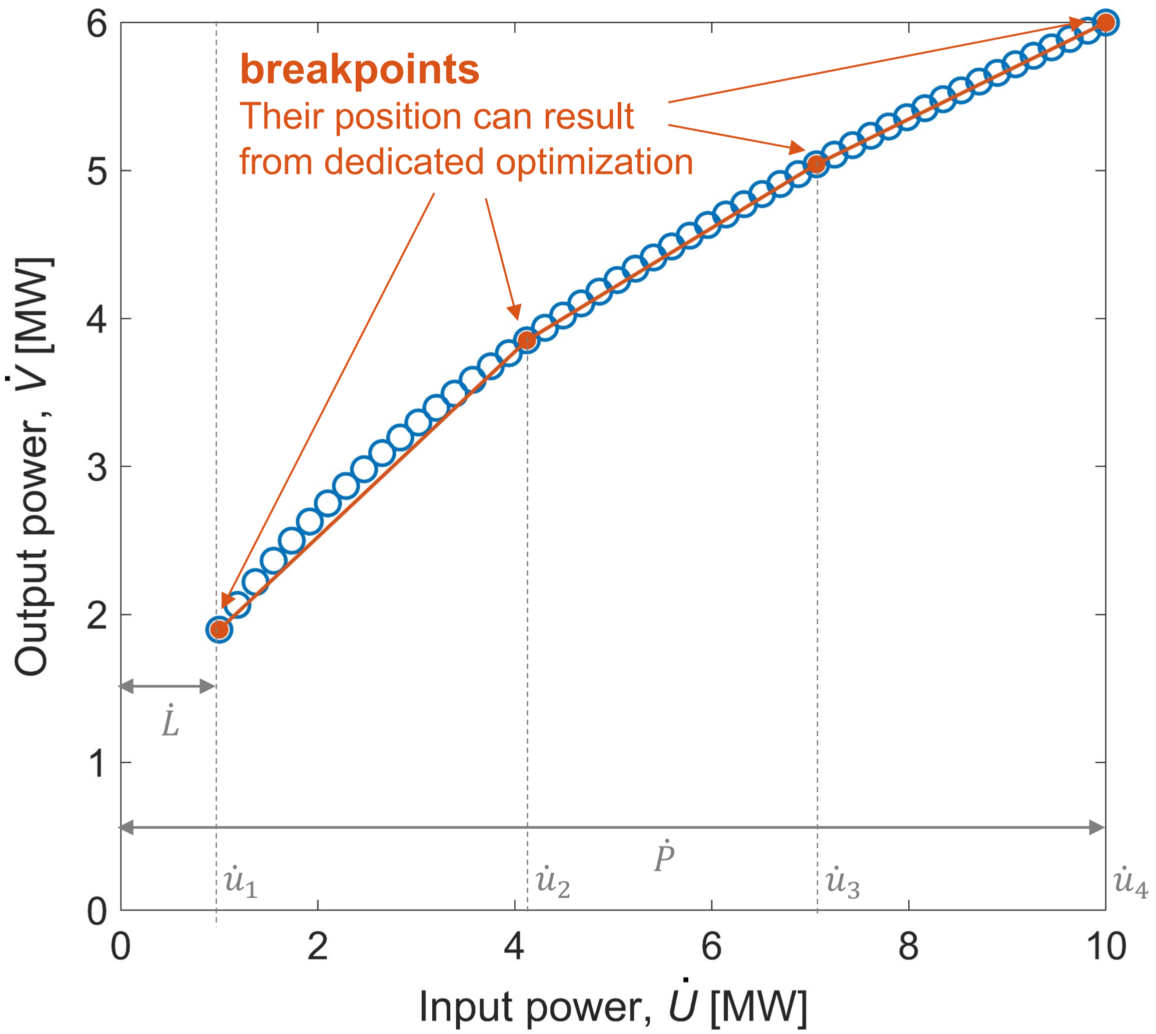
$$V = \sum_{i=1}^{n} (\alpha_i U + \beta_i) y_i$$

$$\sum_{i=1}^{n} y_i \leq 1$$

$$\sum_{i=1}^{n} (y_i u_i) \leq U \leq \sum_{i=1}^{n} (y_i u_{i+1})$$

$\dot{L} \in \mathbb{R}$, minimum power (input)
$\dot{P} \in \mathbb{R}$, rated power (input)
$y_i \in \{0,1\}$, operation point

Piecewise affine (PWA) approximation



$$V = \alpha_1 U y_1 + \alpha_2 U y_2 + \alpha_3 U y_3 + \beta_1 y_1 + \beta_2 y_2 + \beta_3 y_3$$

$$y_1 + y_2 + y_3 \leq 1$$

$$y_1 u_1 + y_2 u_2 + y_3 u_3 \leq U \leq y_1 u_2 + y_2 u_3 + y_3 u_4$$

$n = 3$

PWA approximation (MILP)

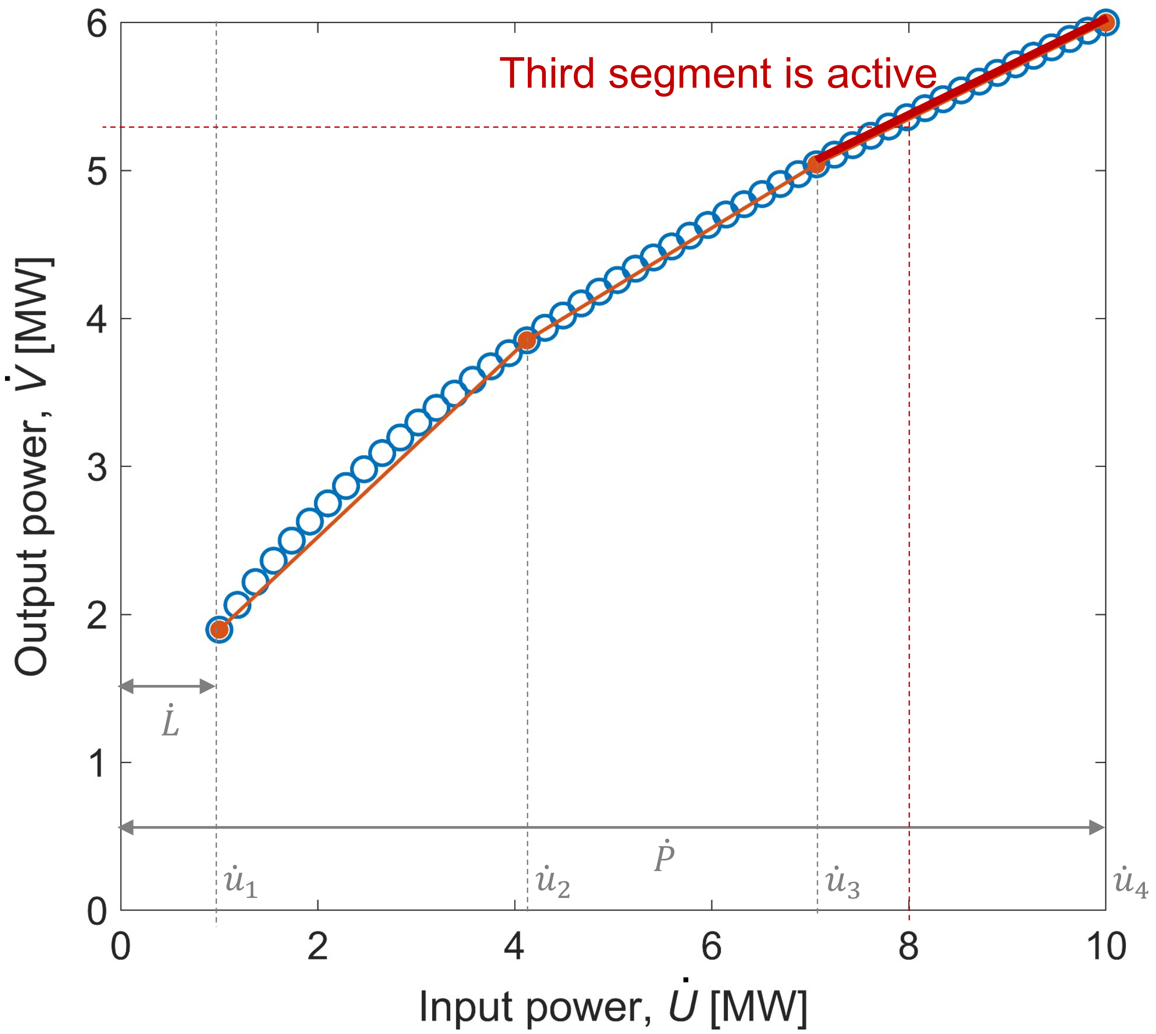
$$V = \sum_{i=1}^{n} (\alpha_i U + \beta_i) y_i$$

$$\sum_{i=1}^{n} y_i \leq 1$$

$$\sum_{i=1}^{n} (y_i u_i) \leq U \leq \sum_{i=1}^{n} (y_i u_{i+1})$$

$\dot{L} \in \mathbb{R}$, minimum power (input)
$\dot{P} \in \mathbb{R}$, rated power (input)
$y_i \in \{0,1\}$, operation point

Piecewise affine (PWA) approximation: Example



$$V = \alpha_1 \cancel{U y_1} + \alpha_2 \cancel{U y_2} + \alpha_3 U y_3 + \cancel{\beta_1 y_1} + \cancel{\beta_2 y_2} + \beta_3 y_3$$

$$y_1 + y_2 + y_3 \leq 1 \rightarrow y_1 = 0, y_2 = 0, y_3 = 1$$

$$y_1 u_1 + y_2 u_2 + y_3 u_3 \leq U \leq y_1 u_2 + y_2 u_3 + y_3 u_4 \rightarrow$$

$$u_3 \leq U \leq u_4$$

$n = 3$

PWA approximation (MILP)

$$V = \sum_{i=1}^{n} (\alpha_i U + \beta_i) y_i$$

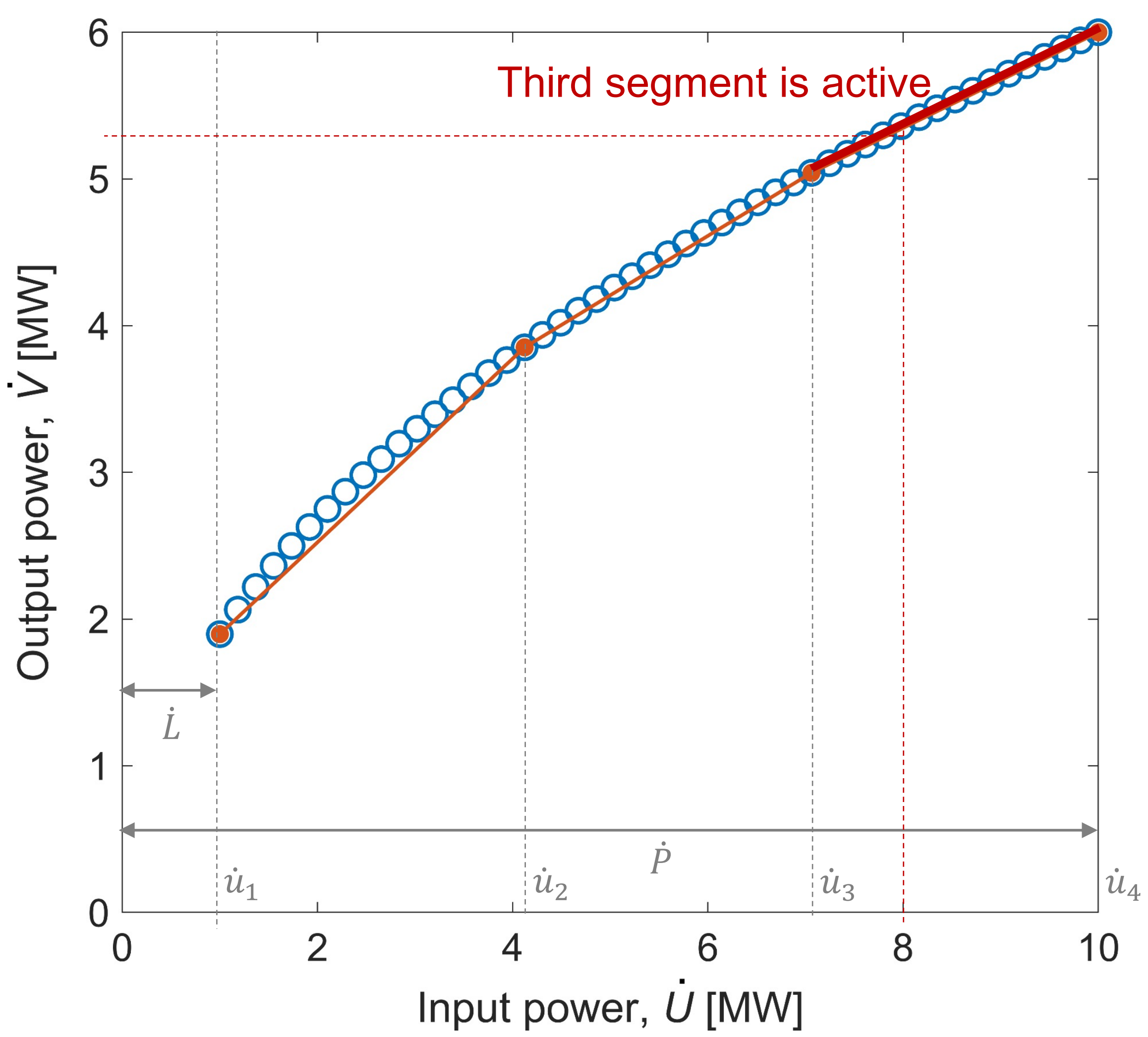
$$\sum_{i=1}^{n} y_i \leq 1$$

$$\sum_{i=1}^{n} (y_i u_i) \leq U \leq \sum_{i=1}^{n} (y_i u_{i+1})$$

$\dot{L} \in \mathbb{R}$, minimum power (input)
$\dot{P} \in \mathbb{R}$, rated power (input)
$y_i \in \{0,1\}$, operation point

Piecewise affine (PWA) approximation: Example

The product between a binary and a continuous variable can be written in linear form: when we have this product we still have a MILP (coming soon)



$$V = \cancel{\alpha_1 U y_1} + \cancel{\alpha_2 U y_2} + \alpha_3 \boxed{U y_3} + \cancel{\beta_1 y_1} + \cancel{\beta_2 y_2} + \beta_3 y_3$$

$$y_1 + y_2 + y_3 \leq 1 \rightarrow y_1 = 0, y_2 = 0, y_3 = 1$$

$$y_1 u_1 + y_2 u_2 + y_3 u_3 \leq U \leq y_1 u_2 + y_2 u_3 + y_3 u_4 \rightarrow$$
$$u_3 \leq U \leq u_4$$

$n = 3$

PWA approximation (MILP)

$$V = \sum_{i=1}^{n} (\alpha_i U + \beta_i) y_i$$

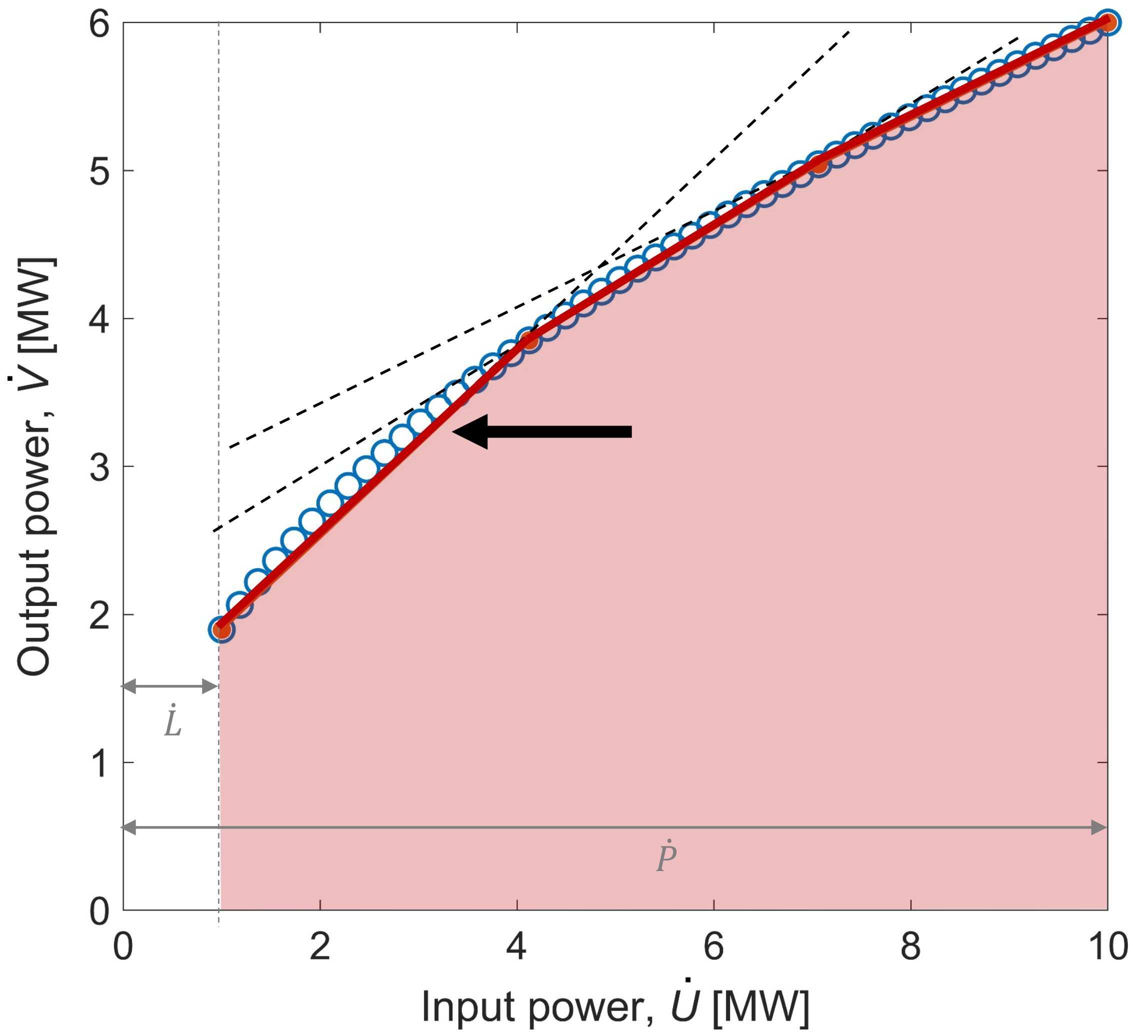
$$\sum_{i=1}^{n} y_i \leq 1$$

$$\sum_{i=1}^{n} (y_i u_i) \leq U \leq \sum_{i=1}^{n} (y_i u_{i+1})$$

$\dot{L} \in \mathbb{R}$, minimum power (input)
$\dot{P} \in \mathbb{R}$, rated power (input)
$y_i \in \{0,1\}$, operation point

Piecewise affine (PWA) approximation: Formulation with inequality

A similar constraint (≥) can be used to minimize a convex function (i.e. input energy, cost, emission,…)



- When we have a **concave function** to be **maximized** (often the case with energy efficiency), we can write the PWA constraint as:

$$V \leq \alpha_i U + \beta_i y, \qquad i = 1, \dots, n$$

concavity $\qquad \alpha_i \geq \alpha_{i+1}, \qquad i = 1, \dots, n-1$

- This allows reducing the number of binary variables from n to 1, simplifying the optimization problem
- Several other formulations possible (e.g. AIMMS Modeling Guide - Integer Programming Tricks: www.aimms.com)

$n = 3$

Breakpoint position not needed anymore

PWA approximation (MILP)

$$V \leq \alpha_i U + \beta_i y, \;\; i = 1, \dots, n$$

$$yL \leq U \leq yP$$

$\dot{L} \in \mathbb{R}$, minimum power (input)
$\dot{P} \in \mathbb{R}$, rated power (input)
$y \in \{0,1\}$, ON/OFF operation

Linearization of product between continuous and binary variable

- Goal: Linearize the product Uy, where $y \in \{0,1\}$ is a binary variable and U is a continuous variable bounded between $U_{\min}$ and $U_{\max}$
- Basic idea: Introduce an auxiliary continuous variable $\tilde{U} = Uy$ and constrain it such that:

$$\begin{cases} \tilde{U} = U, & \text{if } y = 1 \\ \tilde{U} = 0, & \text{if } y = 0 \end{cases}$$

- This can be done as:

$$U_{\min} y \leq \tilde{U} \leq U_{\max} y$$

$$U - U_{\max}(1 - y) \leq \tilde{U} \leq U$$

$\Rightarrow$

$y = 0 \Rightarrow \tilde{U} = 0$
$0 \leq \tilde{U} \leq 0$
$U - U_{\max} \leq \tilde{U} \leq U$

$y = 1 \Rightarrow \tilde{U} = U$
$U_{\min} \leq \tilde{U} \leq U_{\max}$
$U \leq \tilde{U} \leq U$

- How to choose $U_{\min}$ and $U_{\max}$ if there are no obvious bounds (e.g. zero and maximum size):
  - Set them big enough such that they are not tight at the optimum, but not too big (solver numerics)
  - If the optimum value of U after solving is either $U_{\min}$ or $U_{\max}$, increase bounds and solve again

General MILP formulation with affine approximations

Objective function

$$\min_{\boldsymbol{x}} \ z_{\text{cost}} = \sum_{k\in\mathcal{K}} (1+\nu_k)\left[\left(\iota_k \dot{P}_k + \zeta_k\right) b_k\right] a_k + \sum_{t=1}^{T} \sum_{j\in\mathcal{J}} p_{j,t}\left(M_j - N_j\right)$$

...

s. t.

Installation cost, I_k, can also be expressed through an affine approximation: here written as an affine approximation of size

⋮

Dispatchable technologies

$$V_{\bar{j},k,t} = \alpha_{\underline{j}\bar{j},k} U_{\underline{j},k,t} + \beta_{\underline{j}\bar{j},k} P_k y_{k,t}$$

$$y_{k,t}\delta_k P_k \leq U_{\underline{j},k,t} \leq y_{k,t} P_k$$

$$b_k \dot{P}_{k,\min} \leq \dot{P}_k \leq b_k \dot{P}_{k,\max} \qquad \forall \bar{j} \in \bar{\mathcal{J}}_k, \underline{j} \in \underline{\mathcal{J}}_k, k \in \mathcal{K}_\text{C}, t \in \{1, \dots, T\}$$

⋮

MES structure

$\mathcal{J}$ = Set of energy carriers, {E, G, H}

$\mathcal{K}$ = Set of technologies, {PV, HP, B}

$\mathcal{K}_\text{R}$ = Set of renewable energy technologies, {PV}

$\mathcal{K}_\text{C}$ = Set of dispatchable technologies, {HP, B, GT}

$\bar{\mathcal{J}}_k$ = Set of output carriers for technology k

$\underline{\mathcal{J}}_k$ = Set of input carriers for technology k

T = Length of the time horizon

Conversion performance, η_k, written as affine approximation of input energy and size (the constant term of the affine approximation scales with size in a design problem)

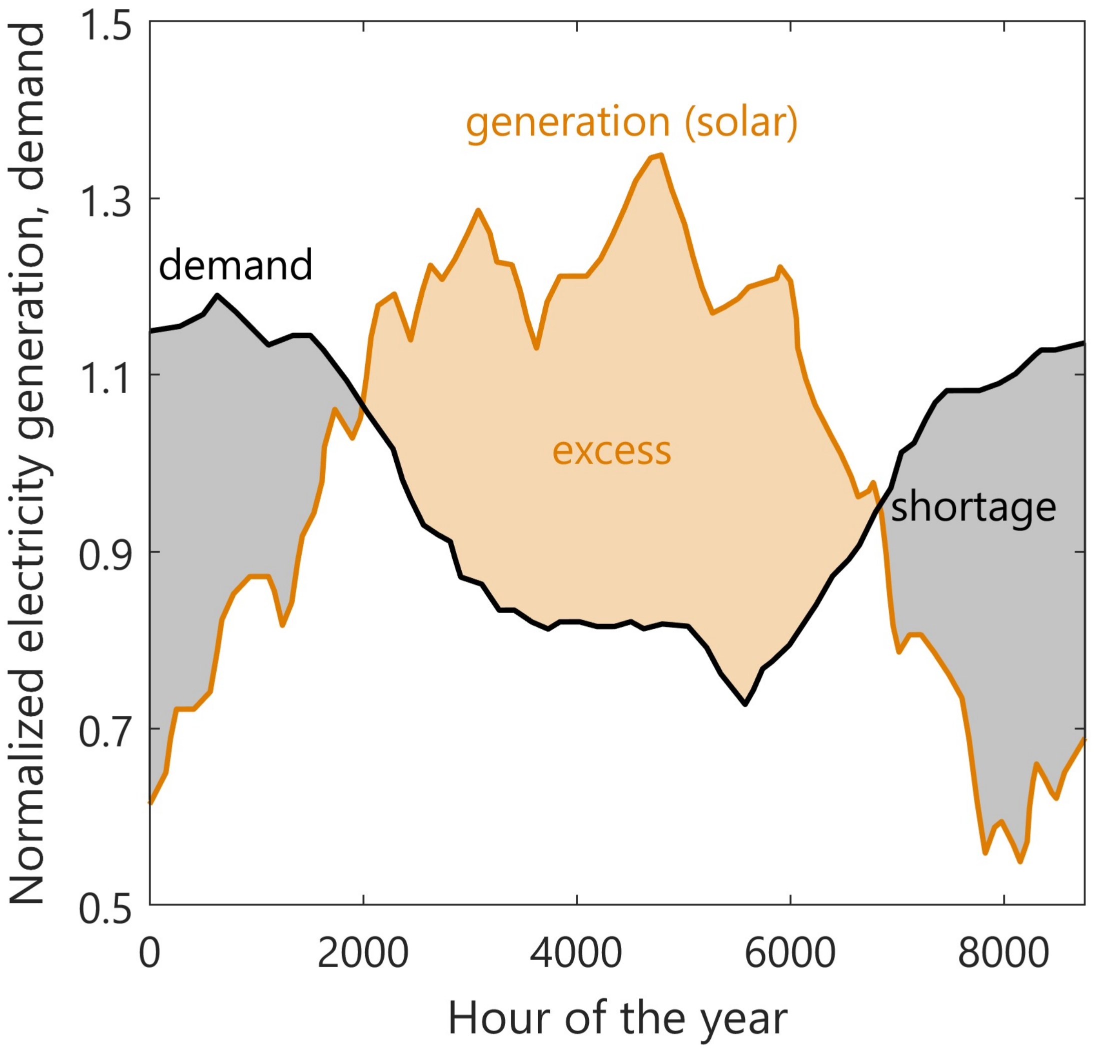
After this lecture, you are able to …

- ✓ Formulate multi-objective optimization problem for MES optimal design (this can be solved through the methods introduced in Lecture 8)
- ✓ Model energy conversion technologies within MES optimization
- Model energy storage technologies within MES optimization
- ✓ Understand the different degrees of complexity when optimizing MES
- Model energy conversion dynamics (***optional material, no exam***)

Modeling MES:
Energy storage technologies

Time dimension is key for sustainable MES

- MES are most often used in combination with renewable energy sources to satisfy a variable energy demand
- Energy storage is crucial in moments of excess energy production for later use in moments of shortage energy production
- Several forms of energy storage are possible, such as electricity storage (batteries), thermal storage (hot water tanks or latent thermal storage), power to gas (e.g. hydrogen storage)



P. Gabrielli et al., Seasonal energy storage for zero-emissions multi-energy systems via underground hydrogen storage. *Renewable and Sustainable Energy Reviews* 2020, **121**, 109629

Constraints: Modeling energy storage

Multi-energy system (MES)

The technology size is the maximum stored **energy** (not maximum rated power)

Input energy: $U_{j,k,t}$

Technology size: P_k

Energy storage technology, *k*

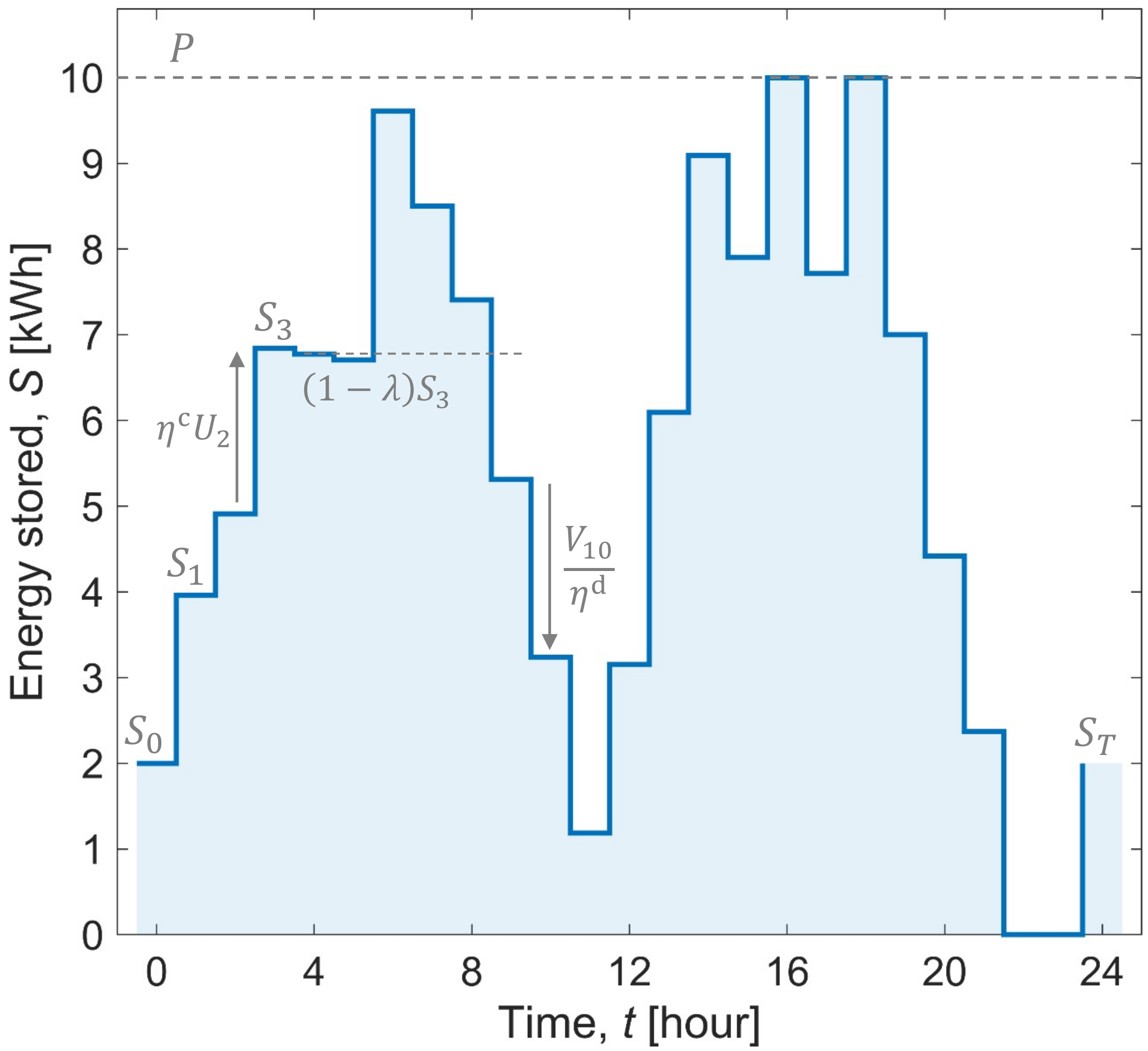
Stored Energy: $S_{j,k,t}$

Charging/discharging efficiency: $\eta_k^{\mathrm{c/d}}$

Self-discharge: λ_k

One additional decision variable: stored energy of type j, at time instant t

Output energy: $V_{j,k,t}$



Constraints: Modeling energy storage

LP

Generic storage technology, *k*

Energy stored at time t

Energy stored at time $t-1$

$$S_{j,k,t} = (1-\lambda_k \Delta t) S_{j,k,t-1} + \eta_k^{\mathrm{c}} U_{j,k,t} - \frac{V_{j,k,t}}{\eta_k^{\mathrm{d}}},$$

self-discharge coefficient

charging efficiency

discharging efficiency

$$0 \le S_{j,k,t} \le P_k \,,$$

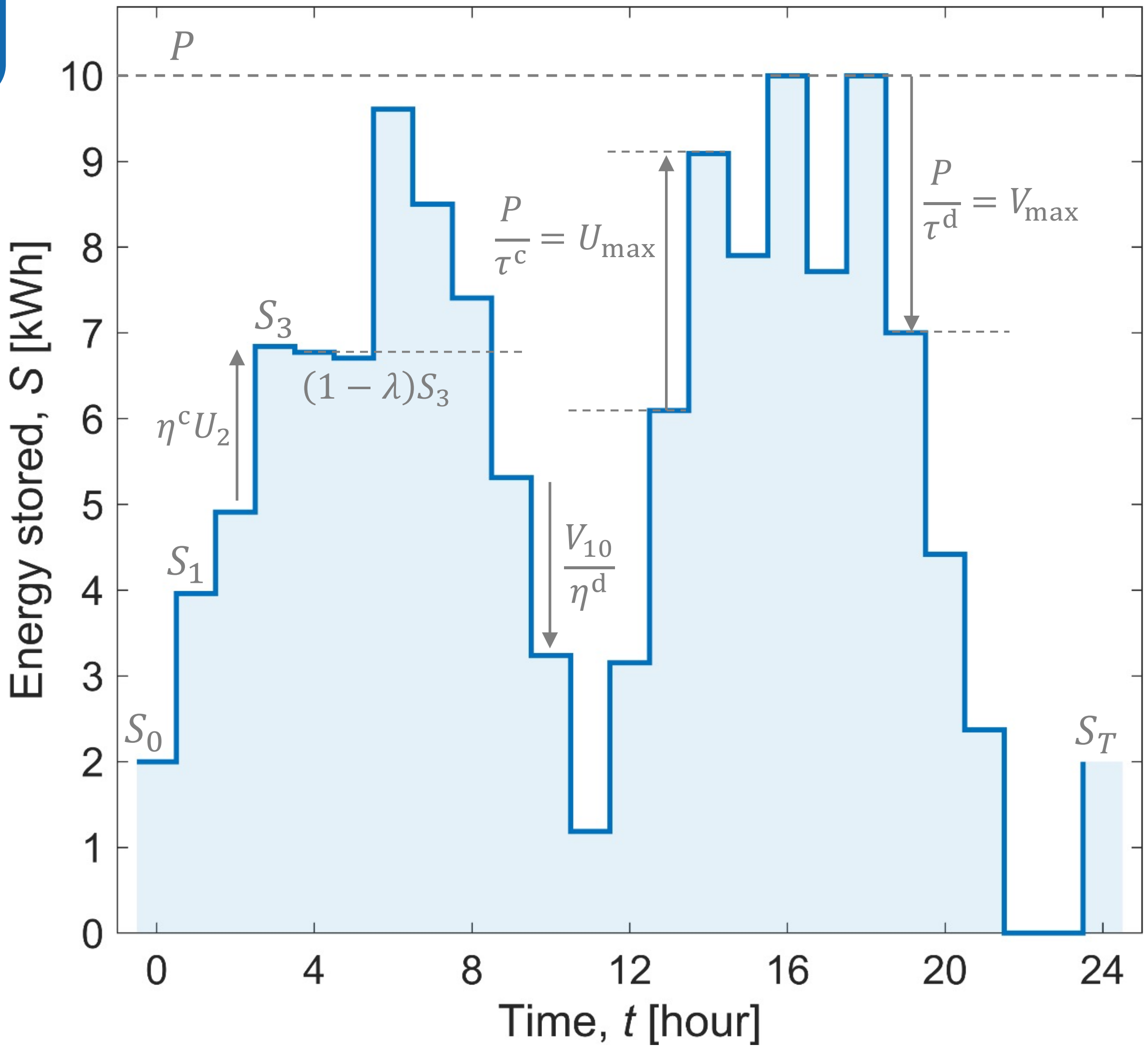
Minimum number of time intervals for full charge (τ^{c}) / discharge (τ^{d})

$$0 \le U_{j,k,t} \le \frac{P_k}{\tau^{\mathrm{c}}}, \qquad 0 \le V_{j,k,t} \le \frac{P_k}{\tau^{\mathrm{d}}} \,,$$

$$S_{j,k,0} = S_{j,k,T}$$

Periodicity constraint: stored energy at beginning and end of time horizon must be equal to ensure consistent storage operation

$$\forall j \in \mathcal{J}_k, k \in \mathcal{K}_S, t \in \{1, \dots, T\}$$



Constraints: Modeling energy storage – alternative with binary variables

Generic storage technology, *k*

MILP

$$S_{j,k,t} = (1 - \lambda_k \Delta t) S_{j,k,t-1} + \eta_k^{\mathrm{c}} U_{j,k,t} - \frac{V_{j,k,t}}{\eta_k^{\mathrm{d}}},$$

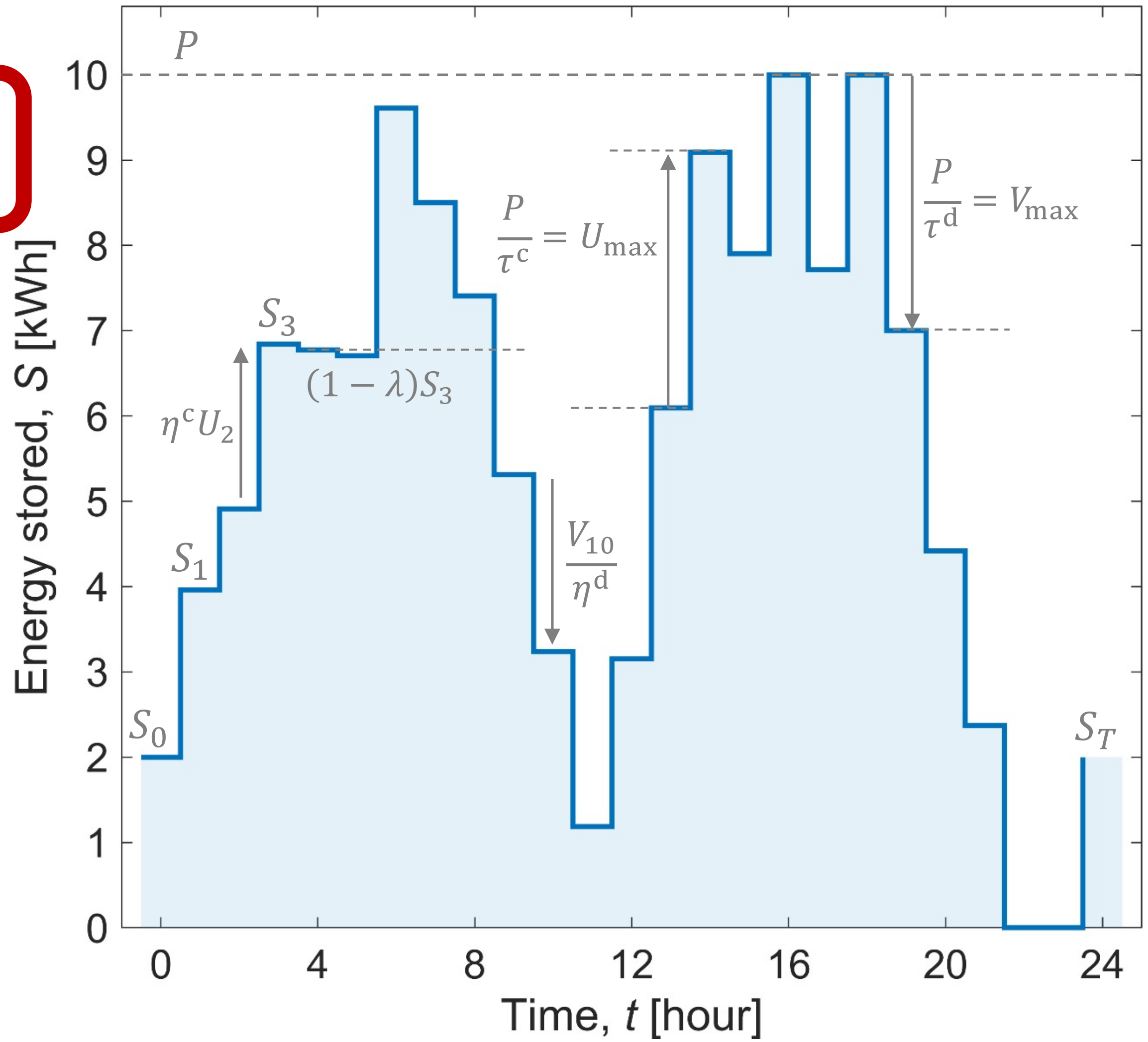
$$0 \leq S_{j,k,t} \leq P_k \,,$$

$$0 \leq U_{j,k,t} \leq \frac{P_k}{\tau^{\mathrm{c}}} c_{k,t}, \qquad 0 \leq V_{j,k,t} \leq \frac{P_k}{\tau^{\mathrm{d}}} (1 - c_{k,t}) \,,$$

$$S_{j,k,0} = S_{j,k,T}$$

Additional binary decision variable, $c_{k,t}$

$$\forall j \in \mathcal{J}_k, k \in \mathcal{K}_{\mathrm{S}}, t \in \{1, \dots, T\}$$



$c_{k,t} = 1$ when charging at time t; $c_{k,t} = 0$ when discharging at time t

$c_{k,t}$ is not strictly necessary: the optimizer avoids charging and discharging during the same time interval t due to the energy loss given by round-trip efficiency, $\eta_k^{\mathrm{c}} \eta_k^{\mathrm{d}}$

General MILP formulation: Optimal design and operation

⋮

RE (non-dispatchable) technologies

$$V_{\bar{j},k,t} = \eta_k(w_t)\, A_k,$$

$$b_k A_{k,\min} \leq A_k \leq b_k A_{k,\max}\,, \qquad \forall \bar{j} \in \bar{\mathcal{J}}_k, k \in \mathcal{K}_\mathrm{R}, t \in \{1, \dots, T\}$$

Dispatchable technologies

$$V_{\bar{j},k,t} = \alpha_{\underline{j}\bar{j},k} U_{\underline{j},k,t} + \beta_{\underline{j}\bar{j},k} P_k y_{k,t}\,,$$

$$y_{k,t}\delta_k P_k \leq U_{\underline{j},k,t} \leq y_{k,t} P_k\,,$$

$$b_k \dot{P}_{k,\min} \leq \dot{P}_k \leq b_k \dot{P}_{k,\max}\,, \qquad \forall \bar{j} \in \bar{\mathcal{J}}_k, \underline{j} \in \underline{\mathcal{J}}_k, k \in \mathcal{K}_\mathrm{C}, t \in \{1, \dots, T\}$$

Storage technologies

$$S_{j,k,t} = (1 - \lambda_k \Delta t) S_{j,k,t-1} + \eta_k^\mathrm{c} U_{j,k,t} - \frac{V_{j,k,t}}{\eta_k^\mathrm{d}},$$

$$0 \leq S_{j,k,t} \leq P_k\,, \qquad 0 \leq U_{j,k,t} \leq \frac{P_k}{\tau^\mathrm{c}}, \qquad 0 \leq V_{j,k,t} \leq \frac{P_k}{\tau^\mathrm{d}}\,,$$

$$S_{j,k,0} = S_{j,k,T}$$

$$b_k P_{k,\min} \leq P_k \leq b_k P_{k,\max} \qquad \forall j \in \mathcal{J}_k, k \in \mathcal{K}_\mathrm{S}, t \in \{1, \dots, T\}$$

⋮

MES structure

$\mathcal{J}$ = Set of energy carriers, {E, G, H}

$\mathcal{K}$ = Set of technologies, {PV, HP, B}

$\mathcal{K}_\mathrm{R}$ = Set of renewable energy technologies, {PV}

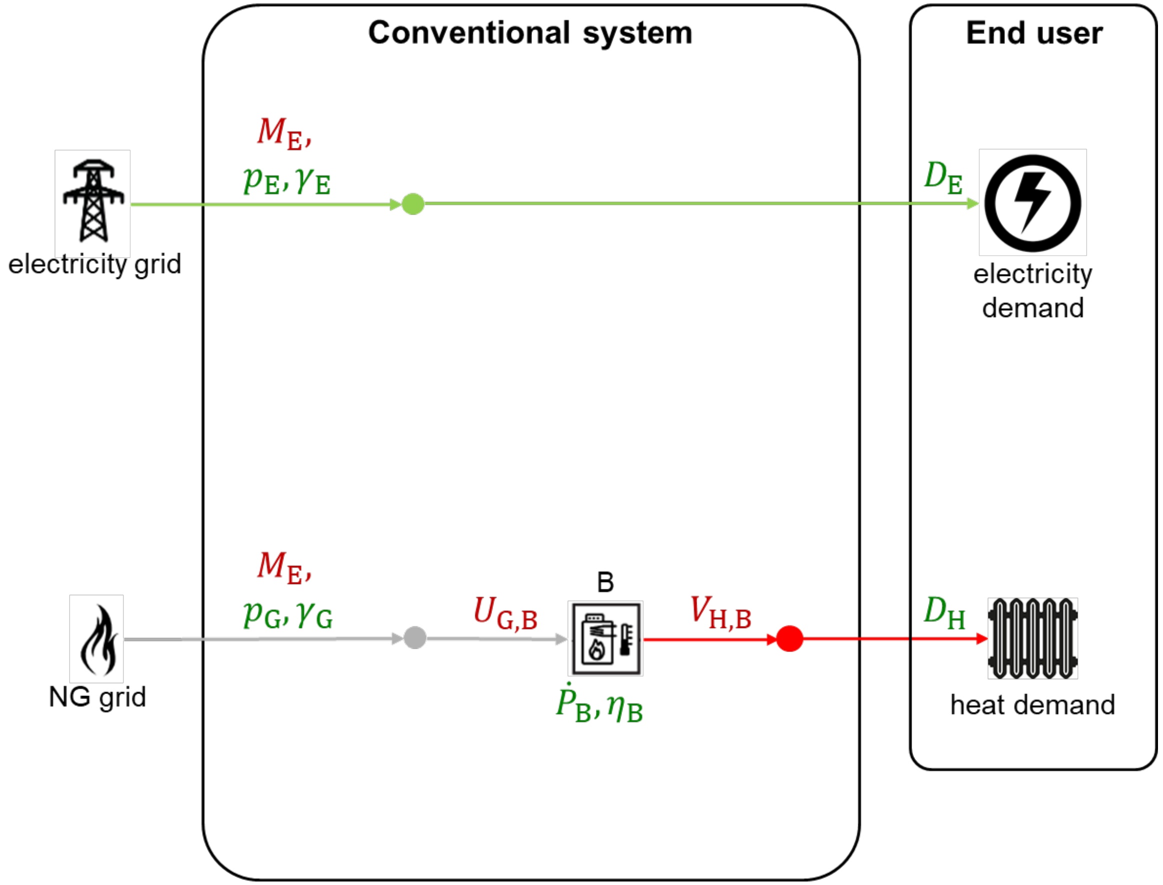
$\mathcal{K}_\mathrm{C}$ = Set of conventional technologies, {HP, B}

$\bar{\mathcal{J}}_k$ = Set of output carriers for technology k

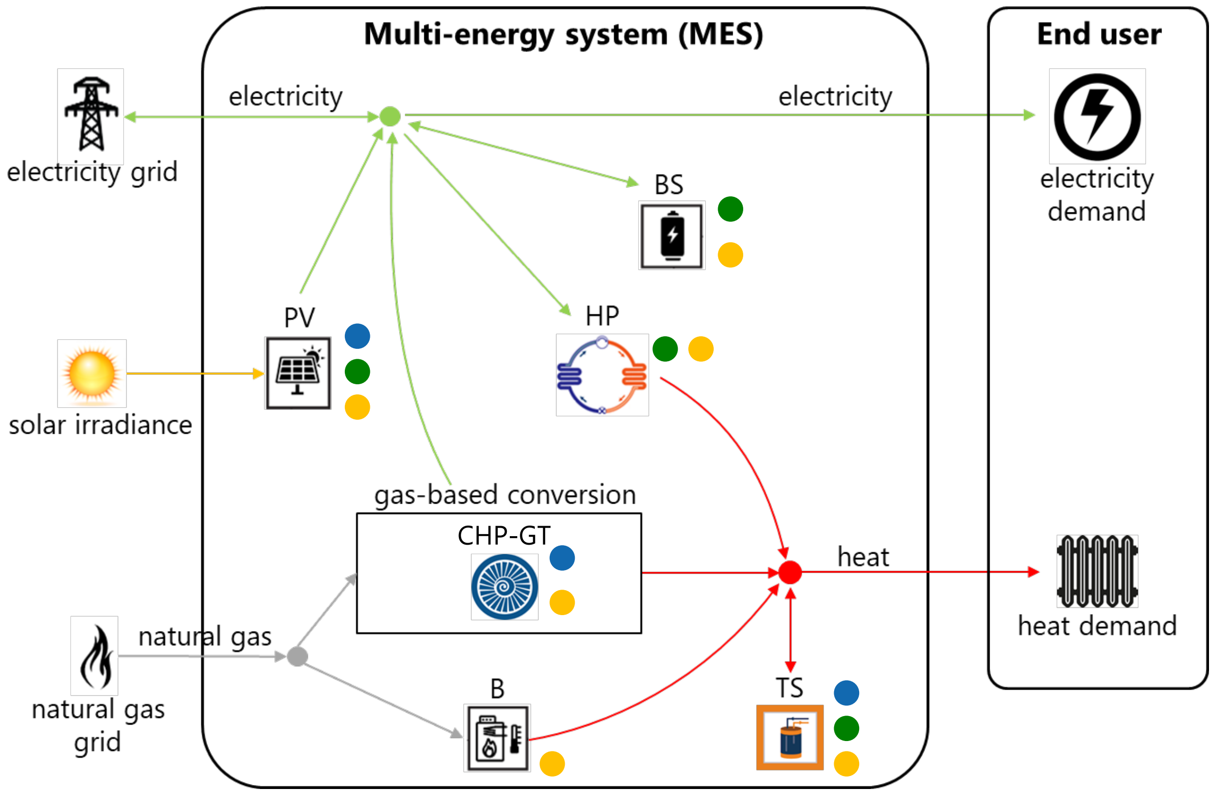
$\underline{\mathcal{J}}_k$ = Set of input carriers for technology k

T = Length of the time horizon

MES optimal design example: Comparison with conventional system



Objective function			
Minimum-cost optimization	z_{cost}	2.01	MCHF/a
Minimum-emissions optimization	z_{CO2}	4091	tCO_2/a
Minimum-risk optimization	z_{risk}	$1.78 \cdot 10^5$	kWh/a



Objective function			
Minimum-cost optimization	z_{cost}	**1.48**	MCHF/a
Minimum-emissions optimization	z_{CO2}	**1222**	tCO_2/a
Minimum-risk optimization	z_{risk}	**0.09**	kWh/a

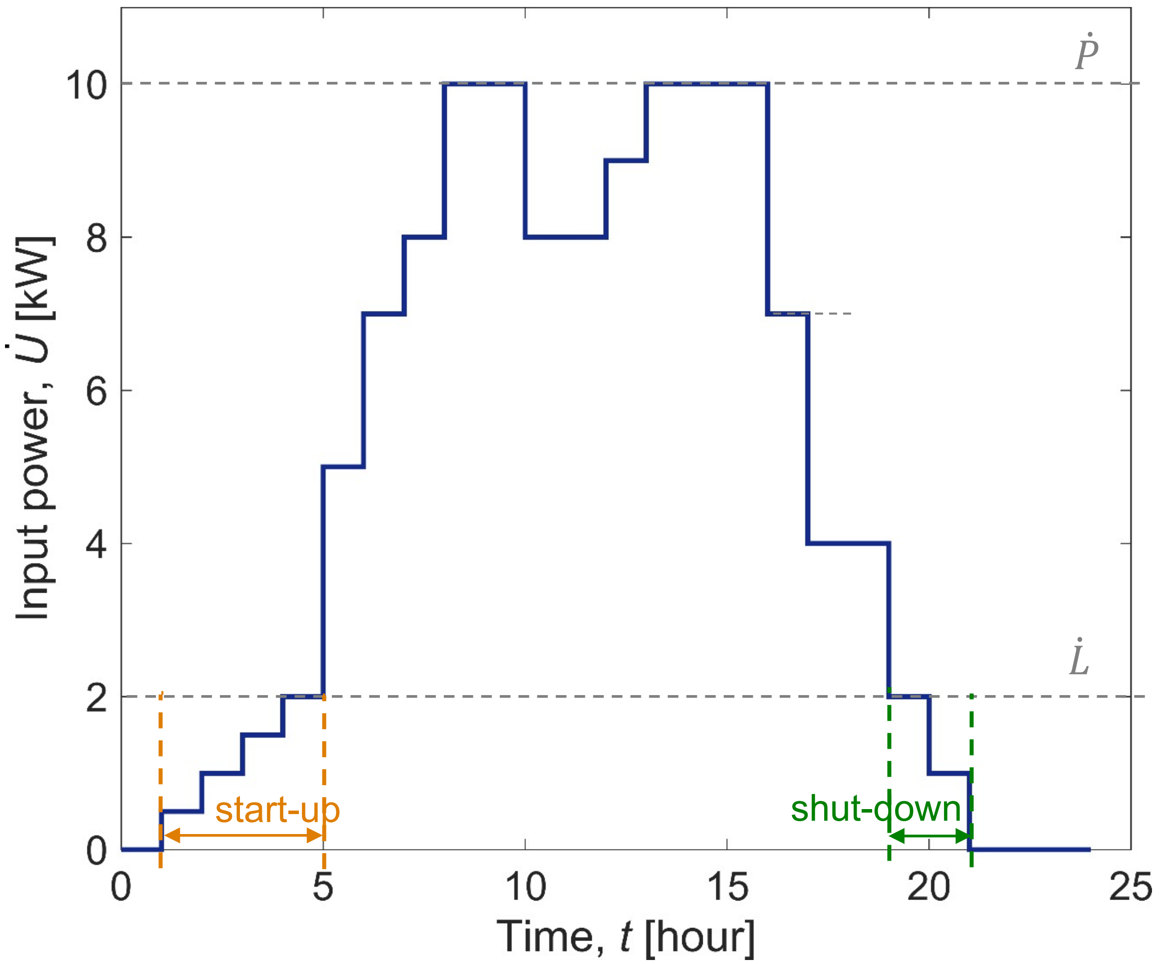
After this lecture, you are able to …

- ✓ Formulate multi-objective optimization problem for MES optimal design (this can be solved through the methods introduced in Lecture 8)
- ✓ Model energy conversion technologies within MES optimization
- ✓ Model energy storage technologies within MES optimization
- ✓ Understand the different degrees of complexity when optimizing MES
- Model energy conversion dynamics (***optional material, no exam***)

Advanced modeling topic ***(no exam)***: Energy conversion dynamics

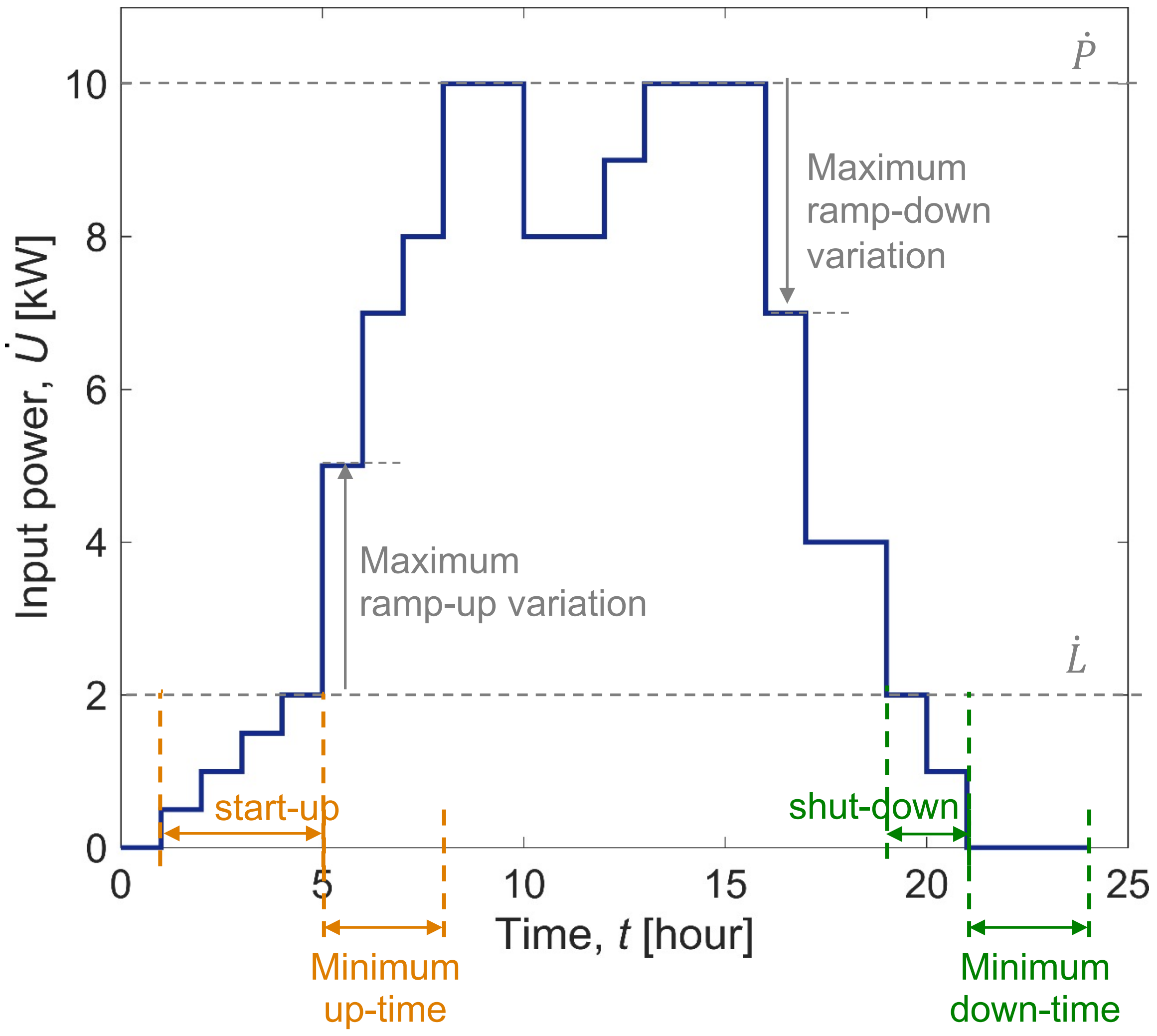
Constraints: Conversion dynamics – start-up / shut-down

- Energy conversion technologies are subject to various constraints limiting the dynamic evolution of input/output energy between following time intervals
- Such constraints are formulated by limiting the variation of input/output energy or ON/OFF status between time intervals
- Conversion dynamics constraints model:
  - *Start-up phase*: the energy technology is turned ON and is forced to generate energy with a controlled profile until to the minimum value
  - *Shut-down phase*: the energy technology is turned OFF and is forced to generate energy with a controlled profile until zero



Constraints: Conversion dynamics – Ramp-up/down and up/down-times

- Conversion dynamics constraints model:
  - *Ramp-up / -down constraints*: the variation in generated energy between two following time intervals is limited by a maximum value, which reflects technological limitations (e.g. variation in turbine rotational speed)
  - *Minimum up- / down-times*: once turned ON or OFF, the energy technology is forced to stay in this status for a minimum amount of time, which reflect technology limitations (e.g. temperature variation across a fuel cell stack)



Constraints: Conversion dynamics – Generation limits

Generic conversion technology, *k*, and carrier, *j*

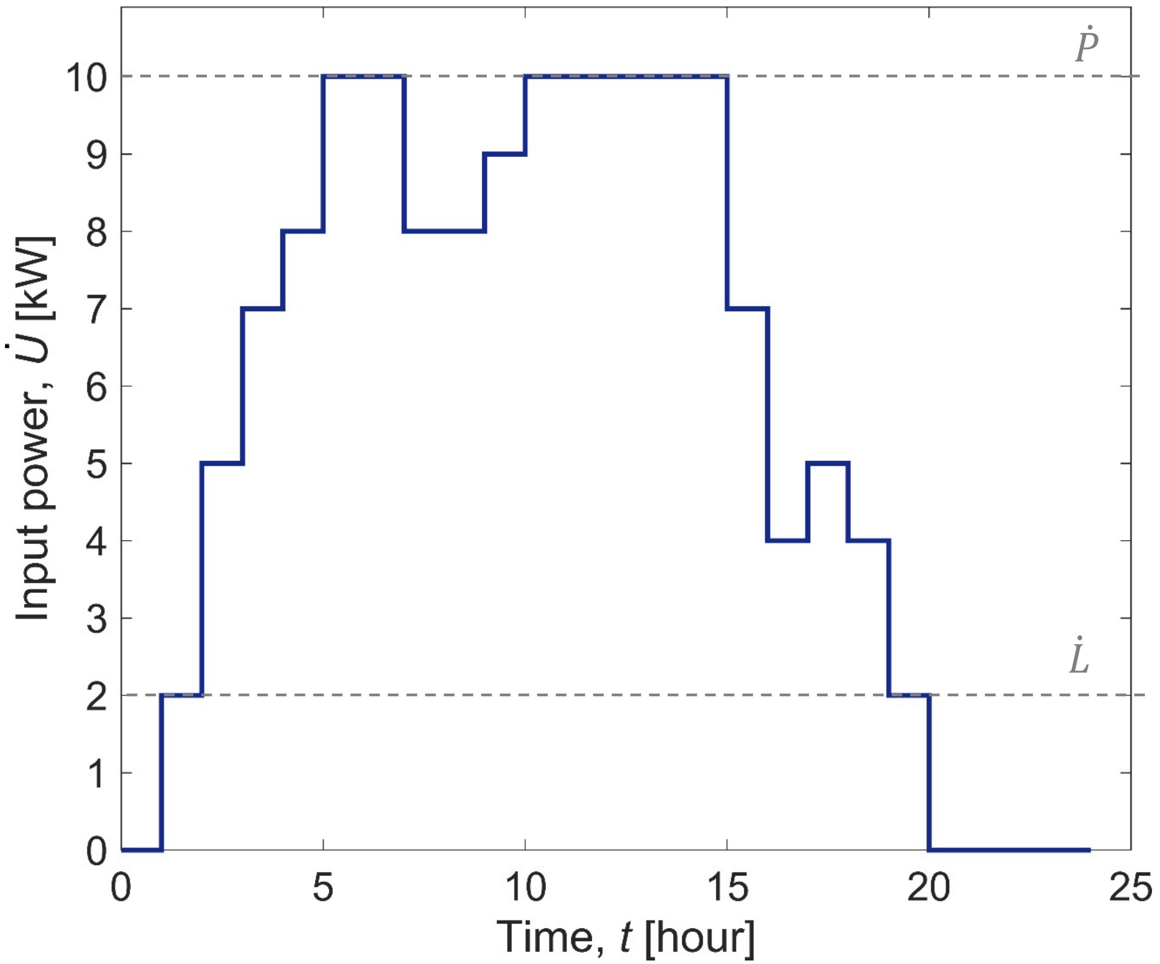
$$y_{k,t}\delta_k P_k \leq U_{j,k,t} \leq y_{k,t}P_k \,, \qquad t \in \{1, \dots, T\}$$

$$y_{k,t}L_k \leq U_{j,k,t} \leq \overline{U}_{j,k,t} \,,$$

Introduction of an auxiliary variable, $\overline{U}_{j,k,t}$, for modeling generation limits and ramping constraints

$$0 \leq \overline{U}_{j,k,t} \leq y_{k,t}P_k \,,$$

$$\forall t \in \{1, \dots, T\}$$



M. Carrion, J. M. Arroyo, A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem, *IEEE Transactions on Power Systems*, 2006, **21**(3), 1371-1378

Constraints: Conversion dynamics – Start-up / ramp-up

Generic conversion technology, *k,* and carrier, *j*

$$\bar{U}_{j,k,t} - U_{j,k,t-1} \leq y_{k,t-1}\Delta U^{\mathrm{RU}} +$$

$$+ \left(y_{k,t} - y_{k,t-1}\right)\Delta U^{\mathrm{SU}} +$$

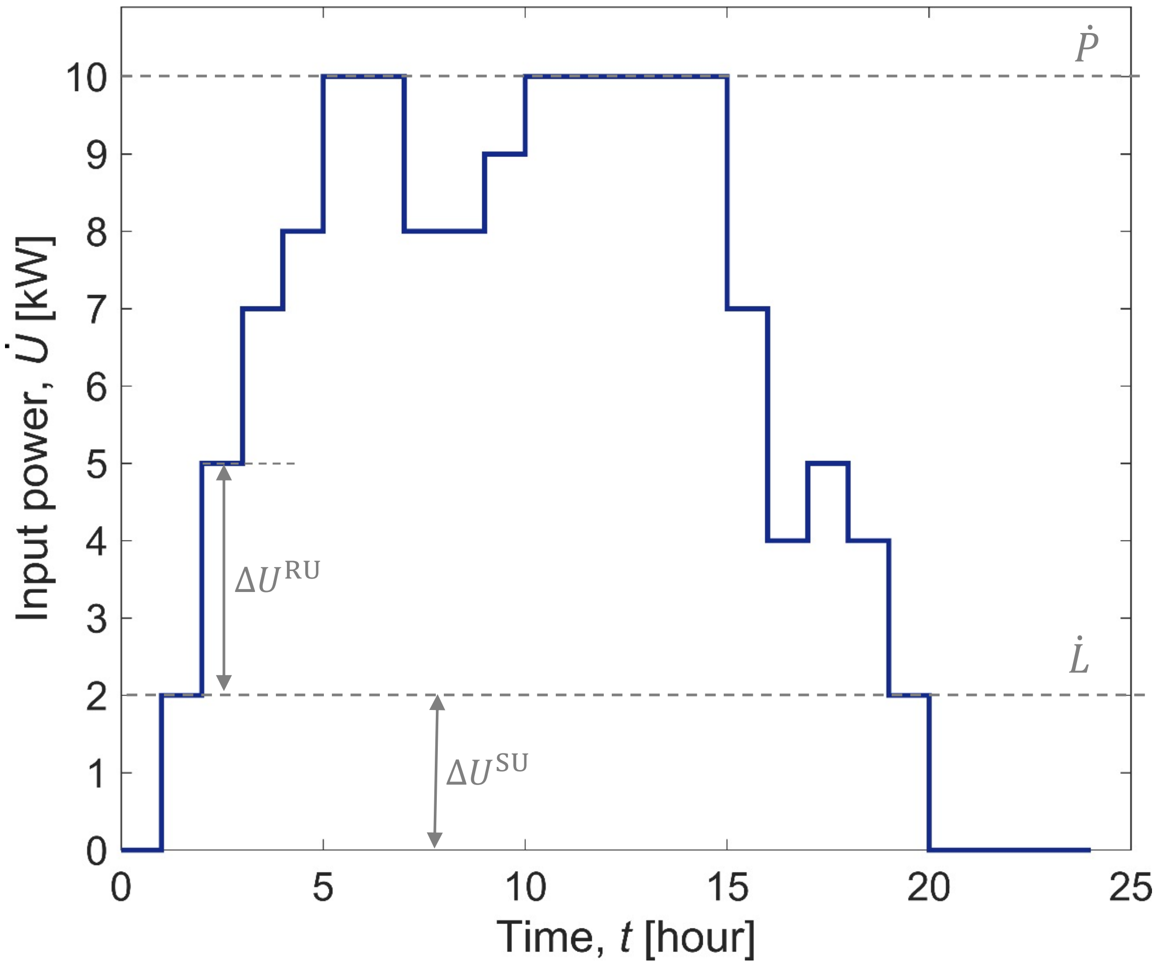
$$+\left(1 - y_{k,t}\right)\dot{P}_k$$

ramp-up limitation

start-up limitation

ensuring constraint is non-limiting if not in start-up/ramp-up mode

$$\forall t \in \{1, \dots, T\}$$



M. Carrion, J. M. Arroyo, A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem, *IEEE Transactions on Power Systems*, 2006, **21**(3), 1371-1378

Constraints: Conversion dynamics – Shut-down / ramp-down

Generic conversion technology, *k,* and carrier, *j*

$$U_{j,k,t-1} - \overline{U}_{j,k,t} \leq y_{k,t}\Delta U^{\mathrm{RD}} +$$

ramp-down limitation

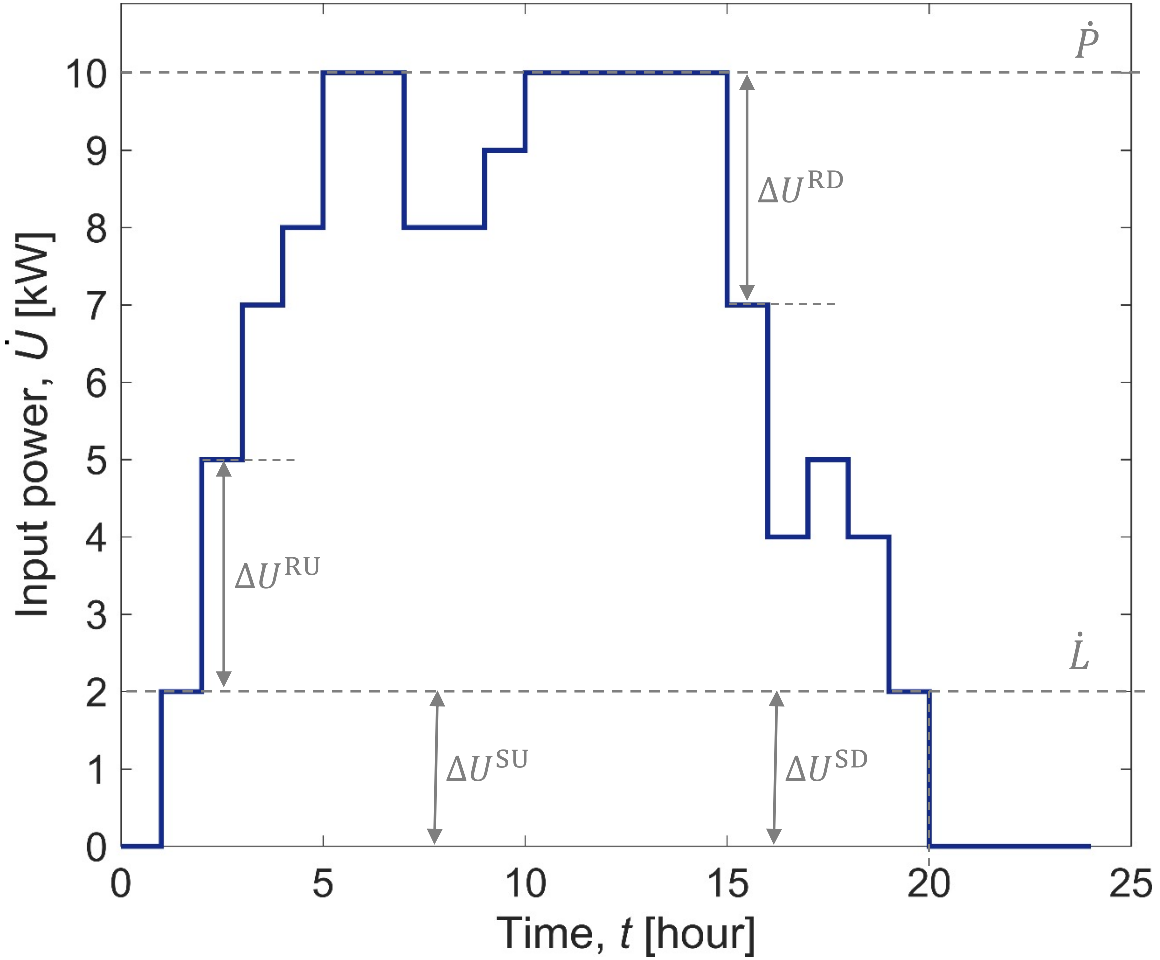
$$+ \left(y_{k,t-1} - y_{k,t}\right)\Delta U^{\mathrm{SD}} +$$

shut-down limitation

$$+\left(1 - y_{k,t-1}\right)\dot{P}_k$$

ensuring constraint is non-limiting if not in shut-down/ramp-down mode

$$\forall t \in \{1, \ldots, T\}$$



M. Carrion, J. M. Arroyo, A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem, *IEEE Transactions on Power Systems*, 2006, **21**(3), 1371-1378

Constraints: Conversion dynamics – Minimum up-time

Generic conversion technology, *k*

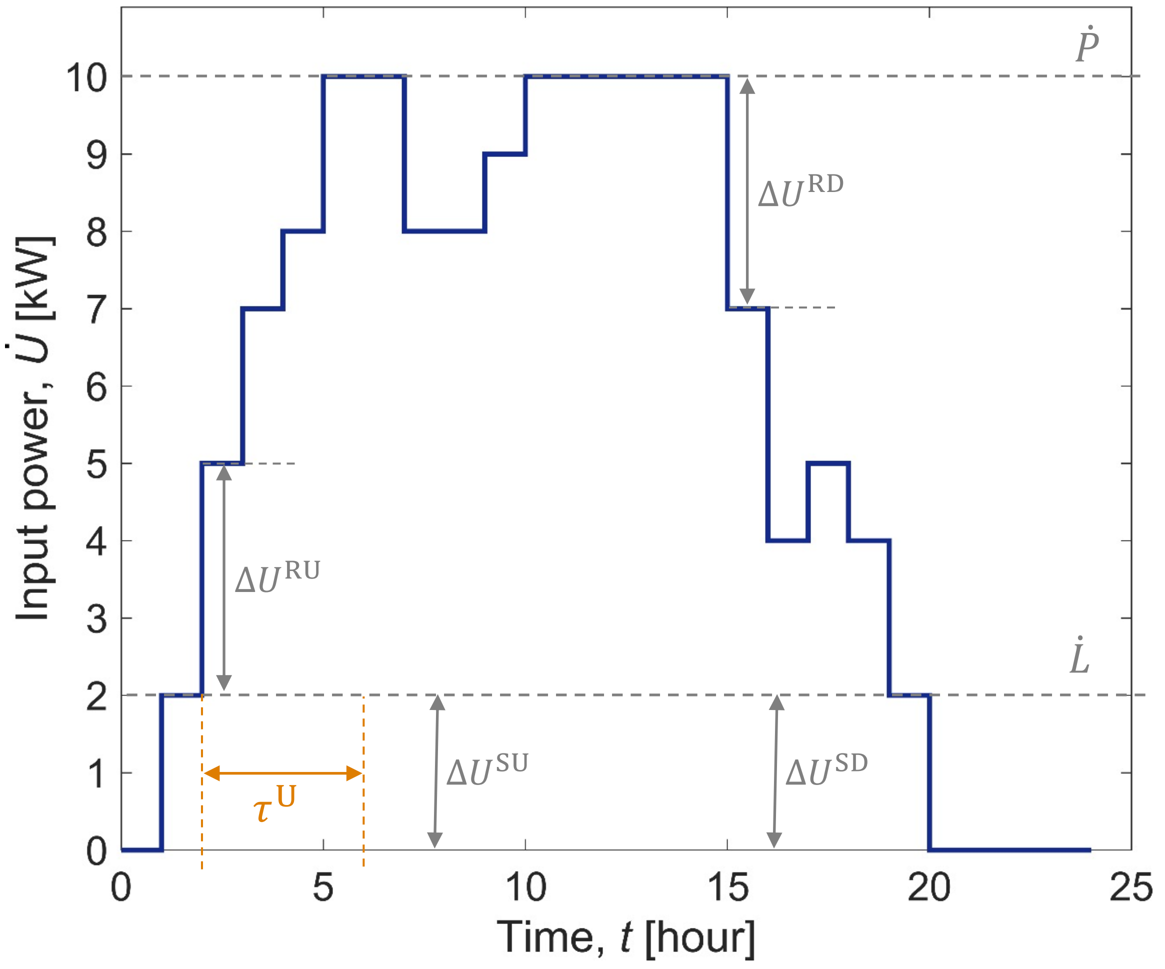
$$\sum_{t=1}^{\tau_0^{\mathrm{U}}} \left(1 - y_{k,t}\right) = 0$$

τ_0^{U}: initial time during which the technology should be ON

$$\sum_{u=t}^{t+\tau^{\mathrm{U}}-1} y_u \geq \left(y_{k,t} - y_{k,t-1}\right)\tau^{\mathrm{U}}, \quad \forall t \in \{\tau_0^{\mathrm{U}} + 1, \dots, T - \tau^{\mathrm{U}} + 1\}$$

τ^{U}: minimum up-time (not limiting in the figure)

$$\sum_{u=t}^{T} y_u \geq \left(y_{k,t} - y_{k,t-1}\right), \qquad \forall t \in \{T - \tau^{\mathrm{U}} + 2, \dots, T\}$$



M. Carrion, J. M. Arroyo, A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem, *IEEE Transactions on Power Systems*, 2006, **21**(3), 1371-1378

Constraints: Conversion dynamics – Minimum down-time

Generic conversion technology, *k*

$$\sum_{t=1}^{\tau_0^{\mathrm{D}}} y_{k,t} = 0$$

τ_0^{D}: initial time during which the technology should be OFF

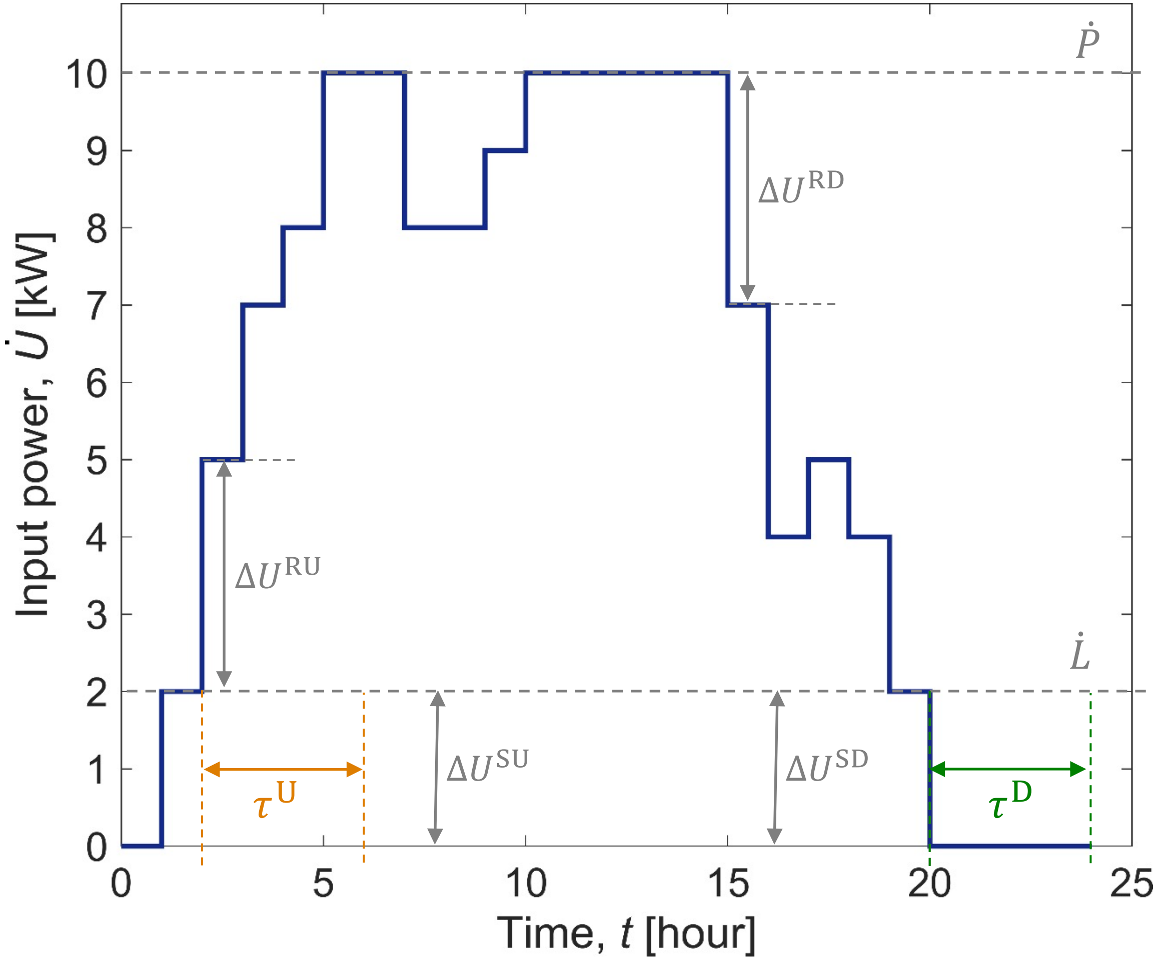
$$\sum_{u=t}^{t+\tau^{\mathrm{D}}-1} (1 - y_u) \geq \left(y_{k,t-1} - y_{k,t}\right)\tau^{\mathrm{D}}\,,$$

$$\forall t \in \{\tau_0^{\mathrm{D}} + 1, \dots, T - \tau^{\mathrm{D}} + 1\}$$

τ^{D}: minimum down-time (limiting in the figure)

$$\sum_{u=t}^{T} (1 - y_u) \geq \left(y_{k,t-1} - y_{k,t}\right),$$

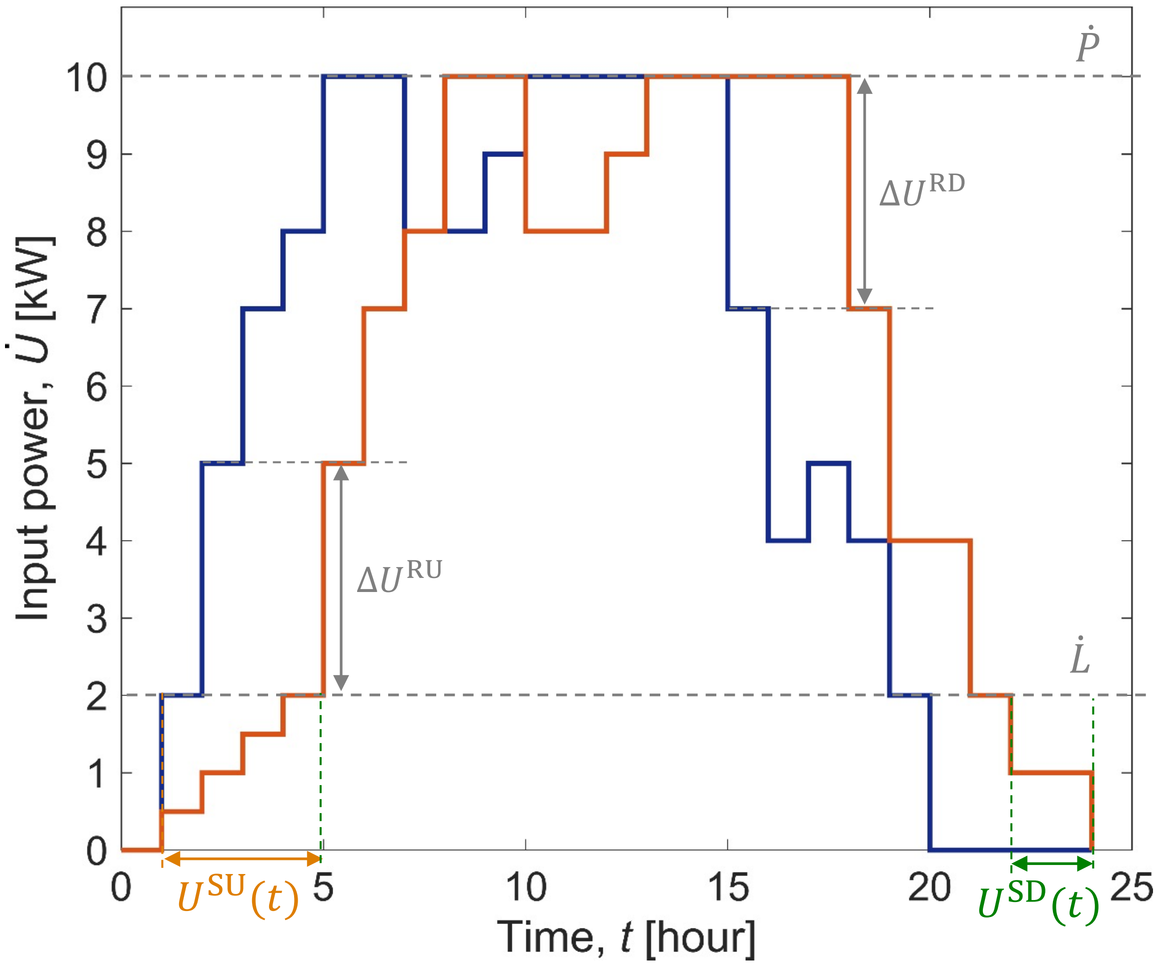
$$\forall t \in \{T - \tau^{\mathrm{D}} + 2, \dots, T\}$$



M. Carrion, J. M. Arroyo, A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem, *IEEE Transactions on Power Systems*, 2006, **21**(3), 1371-1378

Constraints: Conversion dynamics – Detailed start-up/shut-down modeling

- Detailed start-up and shut-down trajectories can be modeled
- This requires the introduction of two additional binary variables:
  - $y_{k,t}^{\mathrm{U}}$, equal to 1 if technology k is started up at time interval t
  - $y_{k,t}^{\mathrm{D}}$, equal to 1 if technology k is shut down at time interval t
- More complex constraints are required, as presented by Arroyo and Conejo (reference below – optional reading)



J. M. Arroyo, A. J. Conejo, Modeling of start-up and shut-down power trajectories of thermal units, *IEEE Transactions on Power Systems*, 2004, **19**(3), 1562-1568

After this lecture, you are able to …

- ✓ Formulate multi-objective optimization problem for MES optimal design (this can be solved through the methods introduced in Lecture 8)
- ✓ Model energy conversion technologies within MES optimization
- ✓ Model energy storage technologies within MES optimization
- ✓ Understand the different degrees of complexity when optimizing MES
- ✓ Model energy conversion dynamics (***optional material, no exam***)