

# Regelungstechnik: *FAST Reference sheet*

Dino Colombo, Michael van Huffel

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## Testsignale 1. Ordnung

Name	Impulse	Step	Ramp	Harmonic
Picture				
Input	$\delta(t) = \lim_{\Delta \rightarrow \infty} \begin{cases} \Delta, & t < \frac{1}{\Delta} \\ 0, & t \geq \frac{1}{\Delta} \end{cases}$	$h(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$	$p(t) = h(t) \cdot t$	$c(t) = h(t) \cdot \cos(\omega t)$
Output	$y_\delta(t) = e^{-\frac{t}{\tau}} \cdot (x_0 + \frac{k}{\tau})$	$y_h(t) = e^{-\frac{t}{\tau}} \cdot x_0 + k \cdot (1 - e^{-\frac{t}{\tau}})$	$y_p(t) = e^{-\frac{t}{\tau}} x_0 + k(t + (e^{-\frac{t}{\tau}} - 1)\tau)$	$y_{c,\infty} = m(\omega) \cdot \cos(\omega t + \varphi(\omega))$

## Aussage über ein system

### Lyapunov Stabilität

1. **Asymptotisch stabil:**  $\lim_{t \rightarrow \infty} \|x(t)\| = 0$ , falls alle EW  $\text{Re}(\lambda_i) < 0$ .
2. **Stabil:** ( $\|x(t)\| < \infty \forall t \in [0, \infty]$ ), falls mehrere EW  $\text{Re}(\lambda_k) = 0$  und kein EW  $\text{Re}(\lambda_i) \neq 0$ .
3. **Instabil:**  $\lim_{t \rightarrow \infty} \|x(t)\| = \infty$  falls mindestens ein EW  $\text{Re}(\lambda_i) > 0$ .

### BIBO Stabilität

Ein System ist BIBO Stabil, falls für die Impulsantwort  $\delta(t)$  folgendes gilt:  $\int_0^\infty |\delta(t)| dt < \infty$

- $\text{Re}(\pi_i) < 0, \forall i \in \mathcal{N}$
- Nicht BIBO stabil in allen anderen Fällen.

### -barkeit

- **Steuer-/Erreich-:**  $\mathcal{R} = [b, \quad A \cdot b, \quad A^2 \cdot b, \quad \dots, \quad A^{n-1} \cdot b]$ : vollen Rang ( $\text{Det}(\mathcal{R}) \neq 0$ ).
- **Stabilisier-:** Ein (instabiles) System ist potentiell Stabilisierbar, falls alle Zustände, die nicht steuerbar sind asymptotisch stabil sind.

- **Beobachtbar-:**  $\mathcal{O} = \begin{bmatrix} c \\ c \cdot A \\ c \cdot A^2 \\ \vdots \\ c \cdot A^{n-1} \end{bmatrix}$ : vollen Rang ( $\text{Det}(\mathcal{O}) \neq 0$ )

- **Detektier-:** Ein System ist nur detektierbar, falls alle seine nicht-beobachtbaren Zustände asymptotisch stabil sind.

## Übertragungsfunktion

**Achtung:** #Pole = Ordnung des Systems

$$\Sigma(s) = \frac{Y(s)}{U(s)} = \frac{c \cdot \text{Adj}(s\mathbb{I} - A) \cdot b}{\det(s\mathbb{I} - A)} + d$$

Most common adj()

Adjunkte für eine  $n \times n$  Matrix:

$$adj(A) = C^T = ((-1)^{i+j} M_{ji})_{1 \leq i,j \leq n}$$

Adjunkte für eine  $2 \times 2$ -Matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \xrightarrow{\text{adjungieren}} \text{Adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Adjunkte für eine  $3 \times 3$ -Matrix:

$$adj \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = C^T = \begin{pmatrix} +\det \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix} & -\det \begin{pmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{pmatrix} & +\det \begin{pmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{pmatrix} \\ -\det \begin{pmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{pmatrix} & +\det \begin{pmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{pmatrix} & -\det \begin{pmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{pmatrix} \\ +\det \begin{pmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} & -\det \begin{pmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{pmatrix} & +\det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \end{pmatrix}$$

Laplace-Transformation

Wichtige Singaltransformationen:	
$x(t)$	$X(S)$
$\delta(t)$	1
$h(t) \quad (= 1)$	$\frac{1}{s}$
$p(t) \quad (= t)$	$\frac{1}{s^2}$
$h(t) \cdot t^n \cdot e^{\alpha \cdot t}$	$\frac{n!}{(s-\alpha)^{n+1}}$
$h(t) \cdot \sin(\omega \cdot t)$	$\frac{\omega}{s^2+\omega^2}$
$h(t) \cdot \cos(\omega \cdot t)$	$\frac{s}{s^2+\omega^2}$
$h(t) \cdot \sinh(\omega \cdot t)$	$\frac{\omega}{s^2-\omega^2}$
$h(t) \cdot \cosh(\omega \cdot t)$	$\frac{s}{s^2-\omega^2}$
$h(t) \cdot (e^{at} - 1)$	$\frac{a}{s(s-a)}$
$h(t) \cdot \frac{e^{at}-e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}$
$h(t) \cdot \frac{ae^{at}-be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}$

Wichtige Eigenschaften:

Linearität	$\mathcal{L}\{ax_1(t) + bx_2(T)\} = aX_1(s) + bX_2(s)$
Ähnlichkeit	$\mathcal{L}\{\frac{1}{a} \cdot x(\frac{t}{a})\} = X(s \cdot a)$
Verschiebung	$\mathcal{L}\{x(t - T)\} = e^{-T \cdot s} \cdot X(S)$
Dämpfung	$\mathcal{L}\{(x(t) \cdot e^{a \cdot t})\} = X(s - a)$
Ableitung t	$\mathcal{L}\{\frac{d}{dt}x(t)\} = s \cdot X(s) - x(0)$
2. Ableitung t	$\mathcal{L}\{\frac{d^2}{dt^2}x(t)\} = s^2 \cdot X(s) - s \cdot x(0) - \frac{d}{dt}x(0)$
n-te Abl. t	$s^n \cdot X(s) - s^{n-1}x(0) - \dots - s^0 \frac{d^{n-1}}{dt^{n-1}}$
Ableitung s	$\mathcal{L}\{t \cdot x(t)\} = -\frac{d}{ds}X(s)$
Integration t	$\mathcal{L}\{\int_0^t x(\tau)d\tau\} = \frac{1}{s} \cdot X(s)$
Integration s	$\mathcal{L}\{\frac{1}{t} \cdot x(t)\} = \int_s^\infty X(\sigma)d\sigma$
Convolution t	$\mathcal{L}\{x_1(t) * x_2(t)\} = X_1(s) \cdot X_2(s)$
Convolution s	$\mathcal{L}\{x_1(t) \cdot x_2(t)\} = X_1(s) * X_2(s)$
Anfangswert	$\lim_{t \rightarrow 0^+} x(t) = \lim_{s \rightarrow \infty} s \cdot X(s)$
Endwert	$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} s \cdot X(s)$

## dB Skala

Dec	dB	dB	Dec
$\infty$	$\infty$	$\infty$	$\infty$
1000	60	1000	$1 \cdot 10^{50}$
100	40	100	$100000 = 10^5$
50	33.98	80	$10000 = 10^4$
20	26.02	60	$1000 = 10^3$
10	20	40	100
9	19.08	30	31.62
8	18.06	20	10
7	16.90	15	5.62
6	15.56	10	$3.16 = \sqrt{10}$
5	13.98	9	2.82
4	12.04	8	2.51
3	9.54	7	2.24
2	6.02	6	$\approx 2$
1	0	5	$1.78 = \sqrt[4]{10}$
$\frac{1}{2} = 0.5$	-6.02	4	1.58
$\frac{1}{3} \approx 0.33$	-9.54	3	$1.41 \approx \sqrt{2}$
$\frac{1}{4} = 0.25$	-12.04	2	$1.26 = \sqrt[10]{10}$
$\frac{1}{5} = 0.2$	-13.98	1	$1.12 = \sqrt[20]{10}$
$\frac{1}{6} \approx 0.17$	-15.56	0.1	$\approx 1.01$
$\frac{1}{7} \approx 0.14$	-16.90	0.01	$\approx 1.001$
0.1	-20.00	0	1
0.01	-40.00	$x_{dB} < 0$	$-\frac{1}{x_{dec}}$
0	$-\infty$	$-\infty$	0

$$\frac{1}{x_{dB}} = -x_{dB} \leftrightarrow \frac{1}{5dB} = -5dB$$

## Trigonometric function

$\alpha$ deg	$\alpha$ rad	$\cos(\alpha)$	$\sin(\alpha)$	$\tan(\alpha)$	$\cot(\alpha)$
0°	0	1	0	0	–
30°	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
60°	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	0	1	–	0

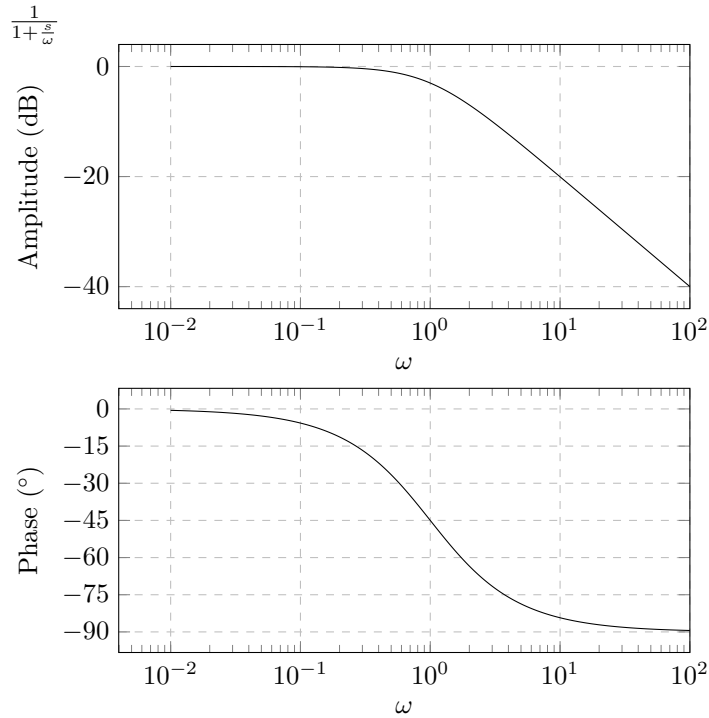
## Einfluss von Nullstellen

Standardelemente	Verstärkung $[\frac{dB}{dec}]$	Phase
Stabiler Pol	−20 bei $\omega_g$	−90° bei $\omega_g$
Instabiler Pol	−20 bei $\omega_g$	+90° bei $\omega_g$
Minimalphasige NullST	+20 bei $\omega_g$	+90° bei $\omega_g$
Nichtminimalphasige NullST	+20 bei $\omega_g$	−90° bei $\omega_g$
Delay um $\tau(\forall \omega)$	0	$-\frac{180}{\pi} \cdot \omega \cdot \tau^\circ$

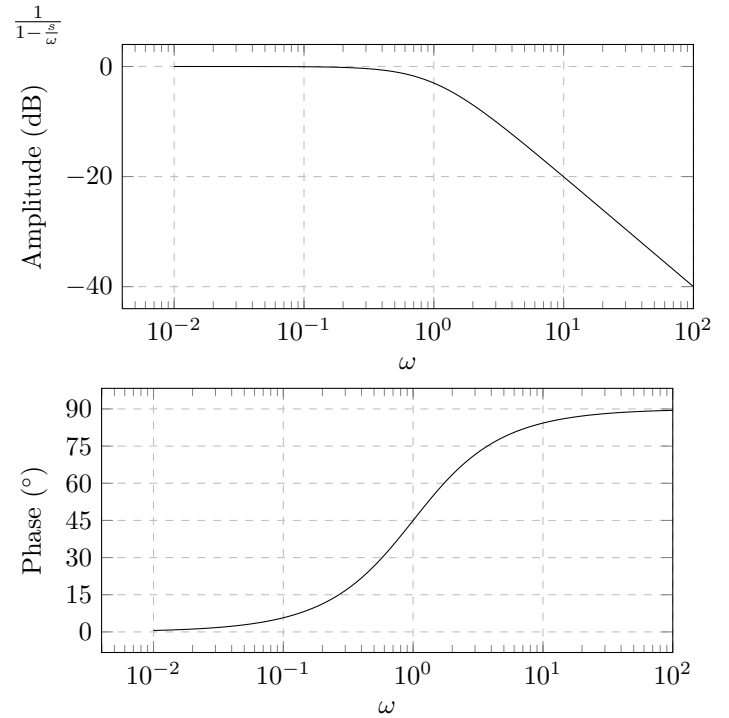
$\omega_g = \frac{1}{\tau}$ : Cutoff-frequency (Eckfrequenz) immer bei  $-3dB$

## Graph bei $\omega = 1$

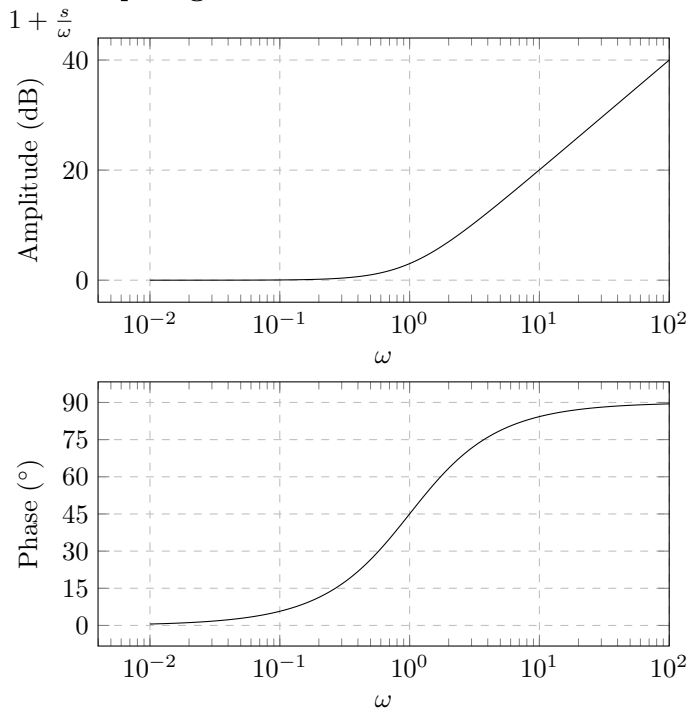
### Stabiler Pol



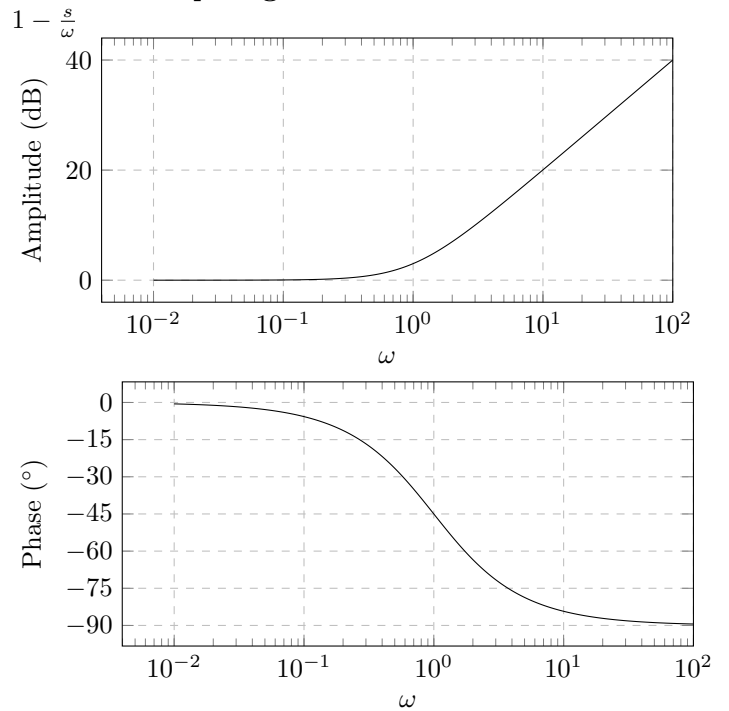
### Instabiler Pol



### Minimalphasige Nullstelle



### Nichtminimalphasige Nullstelle



## Systemtyp $k$

Der **Systemtyp**  $k$  = Vielfachheit offener Integratoren ( $\frac{1}{s^k}$ )

$$\angle \Sigma(0) = \begin{cases} -k \cdot \frac{\pi}{2}, & \text{sgn}(\frac{b_0}{a_0}) > 0 \\ -\pi - k \cdot \frac{\pi}{2}, & \text{sgn}(\frac{b_0}{a_0}) < 0 \text{ (neg. stat. Gain)} \end{cases}$$

## Relativer Grad $r = n - m$

Die Steigung des Magnitudenverlauf im Bode-Diagramm konvergiert asymptotisch zu:

$$\frac{\partial |\Sigma(j\omega)|_{dB}}{\partial \log(\omega)} = -r \cdot 20 \frac{dB}{\text{decade}}$$

## Nyquist

*Asymptotisch – stabil*  $\leftrightarrow n_c \stackrel{!}{=} \frac{n_0}{2} + n_+$

$n_c$ : Anzahl Umrundungen um den kritischen Punkt (-1,0)  
Positiv falls Umrundung gegen Uhrzeigersinn.

$n_0$ : Anzahl Pole von  $L(s)$  mit Realteil = 0

$n_+$ : Anzahl Pole von  $L(s)$  mit Realteil > 0

## Frequenzbedingung des geschlossenen Regelkreises

$$\omega_c = \text{Durchtrittsfrequenz } \omega_c = \begin{cases} \omega_c > \max\{10 \cdot \omega_d, 2 \cdot \omega_{\pi+}\} \\ \omega_c < \min\{\frac{1}{10} \cdot \omega_n, \frac{1}{10} \cdot \omega_2, \frac{1}{2} \cdot \omega_\tau, \frac{1}{2} \cdot \omega_{\zeta+}\} \end{cases}$$

1.  $\omega_2 \leftrightarrow |W_2(j\omega_2)| = 1$
2.  $\omega_\tau = \frac{1}{\tau}$ , mit  $L_\tau(s) = C(s) \cdot P(s)e^{-\tau \cdot s}$ . Wenn konservativ Faktor  $\frac{1}{5}$
3.  $\omega_{\zeta+}$  kleinster positiver Nullstelle. Wenn konservativ Faktor  $\frac{1}{5}$
4.  $\omega_n \leftrightarrow |N(j\omega_n)| = 0$
5.  $\omega_{\pi+}$  grössten positiver Pol. Wenn konservativ Faktor 5
6.  $\omega_d \leftrightarrow |D(j\omega_d)| = 0$

## PID-Regler

$$C_{\text{PID}}(s) = k_p \cdot \overbrace{\left( \frac{T_d \cdot T_i \cdot s^2 + T_i \cdot s + 1}{T_i \cdot s} \right)}^{\text{kausal}} \cdot \frac{1}{(\tau \cdot s + 1)^2}$$

nicht kausal

## Ziegler-Nicholas Parameter

$$|k_p^* \cdot P(j\omega^*)| \stackrel{!}{=} 1 \quad \angle k_p^* \cdot P(j\omega^*) \stackrel{!}{=} -\pi \quad T^* = \frac{2\pi}{\omega^*}$$

Regler	$k_p$	$T_i$	$T_d$
P	$0.5 \cdot k_p^*$	$\infty \cdot T^*$	$0 \cdot T^*$
PI	$0.45 \cdot k_p^*$	$0.85 \cdot T^*$	$0 \cdot T^*$
PD	$0.55 \cdot k_p^*$	$\infty \cdot T^*$	$0.15 \cdot T^*$
PID	$0.6 \cdot k_p^*$	$0.5 \cdot T^*$	$0.125 \cdot T^*$

## Åström-Hägglund Verfahren

$$\{k_p^*, T^*, |P(0)|, \mu_{\min}\} \rightarrow \{k_p, T_i, T_d\}$$

PI	$\mu_{\min} = 0.7$			$\mu_{\min} = 0.5$		
x	$\alpha_{0,x}$	$\alpha_{1,x}$	$\alpha_{2,x}$	$\alpha_{0,x}$	$\alpha_{1,x}$	$\alpha_{2,x}$
$k_p/k_p^*$	0.053	2.90	-2.60	0.13	1.90	-1.30
$T_i/T^*$	0.90	-4.40	2.70	0.90	-4.40	2.70
$a$	1.10	-0.0061	1.8	0.48	0.40	-0.17
PID	$\mu_{\min} = 0.7$			$\mu_{\min} = 0.5$		
x	$\alpha_{0,x}$	$\alpha_{1,x}$	$\alpha_{2,x}$	$\alpha_{0,x}$	$\alpha_{1,x}$	$\alpha_{2,x}$
$k_p/k_p^*$	0.33	-0.31	-1.00	0.72	-1.60	1.20
$T_i/T^*$	0.76	-1.60	-0.36	0.59	-1.30	0.38
$T_d/T^*$	0.17	-0.46	-2.10	0.15	-1.40	0.56
$a$	0.58	-1.3000	3.5	0.25	0.56	-1.20

$$\square = \square^* \cdot \alpha_{0,x} \cdot e^{\alpha_{1,x} \cdot \kappa + \alpha_{2,x} \cdot \kappa^2} \quad \text{mit } \kappa = \frac{1}{|P(0)| \cdot k_p^*}$$

Phasenreserve

$\gamma$	Verstärkungsreserve	Verstärkungsreserve zu $(-1 + 0j)$ bei $\angle L(j\omega) = -\pi$
$\varphi$	Phasenreserve	Phasenabstand zu $-\pi$ bei der Durchtrittsfrequenz $\omega_c$
$\mu$	kritische Abstand	Kleinste Distanz zwischen $(-1 + 0j)$ und $L(j\omega)$

Spezifikation der Sensitivität

Nominelle Regelgüte:  $\|S(s) \cdot W_1(s)\|_\infty < 1 \Rightarrow |S(j\omega)| < |W_1^{-1}(j\omega)| \leftrightarrow |W_1(j\omega)| < |1 + L(j\omega)|$

Robuste Regelgüte:  $|W_1(j\omega) \cdot S(j\omega)| + |W_2(j\omega) \cdot T(j\omega)| < 1 \leftrightarrow |W_1(j\omega)| + |W_2(j\omega) \cdot L(j\omega)| < |1 + L(j\omega)|$

Root Locus

Vorgehen

**Ursprung der Asymptoten berechnen:**  $\sigma_a = \frac{1}{n-m} (\sum_{i=1}^n \text{Re}(\pi_i) - \sum_{i=1}^m \text{Re}(\zeta_i))$ , die Asymptoten verlassen den Punkt  $(\sigma_a + j \cdot 0)$ .  
**Winkel der Asymptoten bestimmen**  $\delta_i = \frac{\pi}{n-m} \cdot (2 \cdot (i - 1) + 1)[\text{rad}]$ ,  $i = 1, \dots, n - m$

Zugehörigkeitstest

Man kann testen ob ein Punkt  $z \in \mathbb{C}$  auf der Wurzelortskurven liegt, indem man ihn in diese Gleichung einsetzt ( $k_p > 0$ )

$$\sum_{i=1}^m \angle(z - \zeta_i) - \sum_{i=1}^n \angle(z - \pi_i) \stackrel{!}{=} \begin{cases} -\pi \pm 2\pi \cdot k, k \in \mathbb{N} & k_p > 0 \\ 0 \mod 2\pi & k_p < 0 \end{cases}$$

Skizzierhilfen

- Root Locus ist Symmetrisch zur Re-Achse.
- Except the constant transfer function  $P(s) = 0$  there is no other rational transfer function for which the positive and the negative root locus curve is identical.
- Treffen sich 2 Pole, dann drehen beide sich um  $90^\circ$  in der komplexen Ebene.
- A Pkt auf der Re-Achse kann sein Teil der RL nur wenn das # reelle Pole und Nullstelle auf sein rechts ist ungerade.
- $r = n - m$  Pole divergen entlang gerader Asymptoten  $\rightarrow \infty$ .
- Consider an open loop function  $L(s) = k \cdot \frac{N(s)}{D(s)}$  with  $k > 0$ . If the degree of  $N(s)$  and  $D(S)$  are at most 1, then the root locus curve lies only on the real axis.

Z-Transform

$X(z) = \mathbb{Z}\{x(k)\} = \sum_{k=0}^\infty z^{-k} \cdot x(k) \leftrightarrow z = e^{sT_s}$

Emulation

$z = e^{sT_s}$

$\approx 1 + sT_s$

$\Rightarrow s \approx \frac{z - 1}{T_s}$

Euler Forward

$z = \frac{1}{e^{-sT_s}}$

$\approx \frac{1}{1 - sT_s}$

$\Rightarrow s \approx \frac{z - 1}{zT_s}$

Euler Backward

$z = \frac{e^{sT_s/2}}{e^{-sT_s/2}}$

$\approx \frac{1 + sT_s/2}{1 - sT_s/2}$

$\Rightarrow s \approx \frac{2(z - 1)}{T_s(z + 1)}$

Tustin Emulation

Hurwitz Stabilitätskriterium

$a_n \cdot s^n + a_{n-1} \cdot s^{n-1} + \dots + a_1 \cdot s + a_0,$      $a_n > 0$  Stabil wenn Koeffizienten  $a_i$  bekannt. Falls alle führenden Hauptminoren der Hurwitzmatrix

$$H_1 = a_{n-1}, H_2 = \begin{vmatrix} a_{n-1} & a_n \\ a_{n-3} & a_{n-2} \end{vmatrix}, H_3 = \begin{vmatrix} a_{n-1} & a_n & 0 \\ a_{n-3} & a_{n-2} & a_{n-1} \\ a_{n-5} & a_{n-4} & a_{n-3} \end{vmatrix}$$

usw  
positive sind, dann sind alle NST in der linken komplexen Halbebene.

MIMO/RGA

$$\mathbf{RGA} \begin{bmatrix} \frac{P_{11}P_{22}}{P_{11}P_{22}-P_{12}P_{21}} & \frac{-P_{12}P_{21}}{P_{22}P_{11}-P_{21}P_{12}} \\ \frac{-P_{12}P_{21}}{P_{22}P_{11}-P_{21}P_{12}} & \frac{P_{11}P_{22}}{P_{11}P_{22}-P_{12}P_{21}} \end{bmatrix}$$

Singulärwerte

Lineare Abbildung mit:  $\sigma_{\min}(M) \leq \frac{\|y\|}{\|u\|} \leq \sigma_{\max}(M)$  wobei  $\|.\|$  die euklische Norm ist und

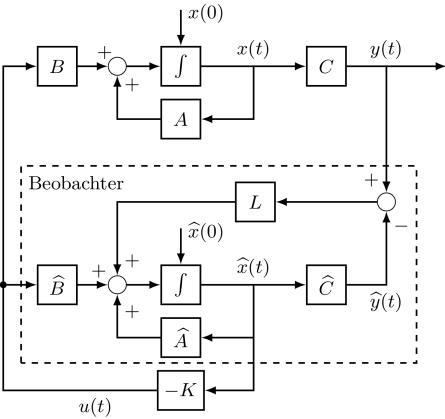
$$\sigma_i(M) = \sqrt{\lambda_i(\overline{M}^T \cdot M)} > 0,$$

**Wichtig:** die Eigenwert von  $\overline{M}^T \cdot M$ .  
 $\overline{M}^T$  bedeutet komplexe conj. des Elementes der Matrix.

LQR

**Linear:** da das System linear ist:  $\frac{d}{dt}x(t) = A \cdot x(t) + B \cdot u(t), \ x(t) \in \mathbb{R}^n, \ u(t) \in \mathbb{R}^m$     **Quadratic:** Definition einer *quadratischen* Kostenfunktion J:  $J(u(t)) = \int_0^\infty \left( x(u(t))^T \cdot Q \cdot x(u(t)) + u(t)^T \cdot R \cdot u(t) \right) dt$     **Lsg:**  $u^*(t) = -K \cdot x(t)$ , wobei  $K = R^{-1} \cdot B^T \cdot \Phi$     **Ricatti Gleichung:**  $\Phi \cdot B \cdot R^{-1} \cdot B^T \cdot \Phi - \Phi \cdot A - A^T \cdot \Phi - Q = 0$

LQG



$$u(t) = -K \cdot \hat{x}(t)$$

## More Laplace transform

$\{f(t)\}$	$f(t)$	$\{f(t)\}$	$f(t)$
$1/s$	1	$e^{-as}/s$	$u(t-a)$
$1/s^2$	$t$	$e^{-as}$	$\delta(t-a)$
$1/s^n$	$t^{n-1}/(n-1)!$	$\frac{1}{\sqrt{s}}e^{-\omega/s}$	$\frac{1}{\sqrt{\pi t}}\cos 2\sqrt{\omega t}$
$1/\sqrt{s}$	$1/\sqrt{\pi t}$	$e^{-k\sqrt{s}}$	$\frac{k}{2\sqrt{\pi t^3}}e^{-k^2/4t}$
$1/s^{3/2}$	$2\sqrt{t/\pi}$	$\frac{1}{(s-a)(s-b)}$	$\frac{1}{a-b}(e^{at}-e^{bt})$
$1/s^k$	$t^{k-1}/\Gamma(k)$	$\frac{s}{(s-a)(s-b)}$	$\frac{1}{a-b}(ae^{at}-be^{bt})$
$\frac{1}{s-a}$	$e^{at}$	$\frac{2\omega^3}{(s^2+\omega^2)^2}$	$\sin \omega t - \omega t \cos \omega t$
$\frac{1}{(s-a)^2}$	$te^{at}$	$\frac{2\omega s}{(s^2+\omega^2)^2}$	$t \sin \omega t$
$\frac{1}{(s-a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{at}$	$\frac{2\omega s^2}{(s^2+\omega^2)^2}$	$\sin \omega t + \omega t \cos \omega t$
$\frac{1}{(s-a)^k}$	$\frac{1}{\Gamma(k)}t^{k-1}e^{at}$	$\frac{s^2}{(s^2+a^2)(s^2+b^2)}$	$\frac{1}{b^2-a^2}(\cos at - \cos bt)$
$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\frac{4\omega^3}{s^4+4\omega^4}$	$\sin \omega t \cos \omega t - \cos \omega t \sinh \omega t$
$\frac{a}{s^2-a^2}$	$\sinh at$	$\frac{2\omega^2 s}{s^4+4\omega^4}$	$\sin \omega t \sinh \omega t$
$\frac{\omega}{(s-a)^2+\omega^2}$	$e^{at} \sin \omega t$	$\frac{2\omega^3}{s^4-\omega^4}$	$\sinh \omega t - \sin \omega t$
$\frac{\omega}{(s-a)^2-\omega^2}$	$e^{at} \sinh \omega t$	$\frac{2\omega^2 s}{s^4-\omega^4}$	$\cosh \omega t - \cos \omega t$
$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\ln \frac{s-a}{s-b}$	$\frac{1}{t}(e^{bt}-e^{at})$
$\frac{s}{s^2-a^2}$	$\cosh at$	$\ln \frac{s^2+\omega^2}{s^2}$	$\frac{2}{t}(1-\cos \omega t)$
$\frac{s-a}{(s-a)^2+\omega^2}$	$e^{at} \cos \omega t$	$\ln \frac{s^2-\omega^2}{s^2}$	$\frac{2}{r}(1-\cosh \omega t)$
$\frac{s-a}{(s-a)^2-\omega^2}$	$e^{at} \cosh \omega t$		
$\frac{\omega^2}{s(s^2+\omega^2)}$	$1 - \cos \omega t$		
$\frac{\omega^3}{s^2(s^2+\omega^2)}$	$\omega t - \sin \omega t$		

$k > 0, n \in \mathbb{N}, a \neq b, \gamma \approx 0.5772$



# MATLAB Commands

Command	Description
bode(SYS)	Draws the Bode plot of the dynamic system SYS.
[MAG,PHASE] = bode(SYS,W)	Return the response magnitudes and phases in degrees (along with the frequency vector W if unspecified).
[MAG,PHASE,W] = bode(SYS)	No plot is drawn on the screen.
W = logspace(-2,3,1e3)	$10^{-2} < w < 10^3$ mit 1000 log-Werten
nyquist(SYS)	Draws the Nyquist plot of the dynamic system SYS.
[RE,IM] = nyquist(SYS,W)	Return the real parts RE and imaginary parts IM of the frequency response (along with the frequency vector W if unspecified).
[RE,IM,W] = nyquist(SYS)	
sys = ss(A,B,C,D)	Creates an object SYS representing the continuous-time state-space model
(E =) [V,D] = eig(A)	Produces a diag matrix D of eigenvalues and a matrix V whose columns are eigenvectors so that $A \cdot V = V \cdot D$ . (Column vector E containing the Eigval of a matrix A.)
co = ctrb(A,B) (= ctrb(SYS))	Returns the controllability matrix $[BABA^2B\dots]$ .
ob = obsv(A,C) (= obsv(SYS))	Returns the observability matrix $[C; CA; CA^2\dots]$
s = tf('s')	Specifies the transfer function $H(s) = s$ (Laplace variable).
SYS = tf(NUM,DEN, Ts, 'InputDelay', T)	Creates a continuous-time transfer function SYS with numerator NUM and denominator DEN and optimal time delay $T$ . (For discrete-time models add a sample time $T_s$ )
P = tf(sys)	Converts any dynamic system SYS to the transfer function representation.
MSYS = minreal(SYS)	Produces, for a given LTI model SYS, an equivalent model MSYS where all cancelling pole/zero pairs or non minimal state dynamics are eliminated. For state-space models, minreal produces a minimal realization MSYS of SYS where all uncontrollable or unobservable modes have been removed.
X = fminsearch(FUN,X0)	Starts at X0 and attempts to find a local minimizer X of the function FUN. FUN is a function handle. FUN accepts input X and returns a scalar function value F evaluated at X. X0 can be a scalar, vector or matrix.
P = pole(SYS)	Returns poles P of the dynamic system SYS as a column vector. For state-space models, the poles are the eigenvalues of the A matrix. (Bei MIMO System die Pole der SISO-Elemente)
[Z,G] = zero(SYS)	Computes the zeros Z and gain G of the single-input, single-output dynamic system SYS.
Z = tzero(SYS,TOL)	Computes invariant zeros of the dynamic system SYS. For state-space models with matrices A,B,C,D,E ( $= I$ ), the invariant zeros are the complex values s for which the rank of the matrix $\begin{bmatrix} A - sE & B \\ C & D \end{bmatrix}$ drops below its normal value. For minimal realizations, this coincides with the transmission zeros of SYS (values of s for which its transfer function drops rank).
Ydb = mag2db(Y)	Converts magnitude data Y into dB values. (db2mag analog)
[NUM,DEN] = tfdata(SYS)	Returns the num. and denom. of the transfer function SYS.
[Z,P,K] = tf2zp(NUM,DEN)	Finds the zeros, poles, and gains from a transferfunction in the form of $H(s) = K \cdot \frac{(s - z1)(s - z2) \dots (s - zn)}{(s - p1)(s - p2) \dots (s - pn)}$
[Z,P,K] = zpndata(SYS)	Returns the zeros, poles, and gain for each I/O channel of the dynamic system SYS.
K = dcgain(SYS)	Computes the steady-state (D.C. or low frequency) gain of the dynamic system SYS ( $P(j \cdot 0)$ )
eye(M)/ eye(M,N)/ eye(size(A))	M-by-M/M-by-N/siz(A) matrix with 1's on the diagonal and zeros elsewhere.
zeros(M)/ zeros(M,N)/ zeros(size(A))	M-by-M/M-by-N/size(A) matrix of zeros.
rga = P.*inv(P')	MATLAB code to compute the RGA matrix of P
bodemag(SYS)	Plots the magnitude of the frequency response of the linear system SYS (useful for RGA/ <b>Bode plot without the phase diagram</b> ).
sigma(SYS)	Produces a singular value (SV) plot of the frequency response of the dynamic system SYS.
[U,S,V] = svd(X)	Produces a diagonal matrix S, of the same dimension as X and with nonnegative diagonal elements in decreasing order, and unitary matrices U and V so that $X = U \cdot S \cdot V^T$ .
FRESP = evalfr(SYS,X)	Evaluates the transfer function of the continuous- or discrete-time linear model SYS at the complex number S=X or Z=X. <b>X = j.omega</b> (Eval System, then plug it into svd)
SimOut = sim('MODEL', PARAMETERS)	Simulates your Simulink model, where 'PARAMETERS' represents a list of parameter name-value pairs.
L = series(C,P)	Connects the input/ouput models C and P in series.
M = feedback(M1,M2)	$(M1, M2) = (1, L) \Rightarrow S(s)$ and $(M1, M2) = (L, 1) \Rightarrow T(s)$ . Computes a closed-loop model M (assumes negative feedback: for positive add "+1")
$\mu_{\min} = \min(\text{abs}(\text{bode}(1 + L)))$	Computes the minimum return difference of a given open loop gain L
[Gm,Pm,Wcg,Wcp] = margin(SYS)	Computes the gain margin Gm, the phase margin Pm, and the associated frequencies Wcg and Wcp, for the SISO open-loop model SYS. The gain margin Gm is defined as $1/G$ where G is the gain at the -180 phase crossing. The phase margin Pm is in degrees. $\mathbf{k_p^* = Gm}$ , $\mathbf{T^* = \frac{2\pi}{W_{cg}}}$
margin(SYS)	Creates a Bode plot of the open loop and marks the gain and phase margins in the plot.
B = squeeze(A)	Returns an array B with the same elements as A but with all the singleton dimensions removed.
SYSD = c2d(SYSC,TS,METHOD)	Computes a discrete-time model SYSD with sample time TS that approximates the continuous-time model SYSC (method = zoh, foh, impulse, tustin,...)
SYSC = d2c(SYSD,METHOD)	Computes a continuous-time model SYSC that approximates the discrete-time model SYSD. (method = as above)
[K,S,CLP] = lqr(SYS,Q,R) = lqr(A,B,Q,R)	Calculates the optimal gain matrix K for the continuous or discrete state-space model SYS. lqr also returns the solution S of the associated algebraic Riccati equation and the closed-loop poles $\text{CLP} = \text{eig}(A - B \cdot K)$ .
Ki_lqri = -K_tilde(:,n+1:end)	Extraction of $K_I$ from $\tilde{K}$