ETH zürich

RRE
Reliability and Risk Engineering

Introduction to Modeling and Optimization of Sustainable Energy Systems:
Exam Recap

Niklas Nolzen & David Shu
Energy and Process Systems Engineering
Dr. Paolo Gabrielli & Alissa Ganter
Reliability and Risk Engineering

Agenda

- ✓ General information
- ✓ EPSE topics
- ✓ RRE topics
- ✓ Questions

Whenever you have questions, please ask!!!

General information

Time & Place

- place: HIL D 15
- starting time: 9:00 a.m.
- 120 minutes

Allowed Auxiliaries

- non-programmable calculator
- pen
- ruler

Not allowed auxiliaries

-

Reminder Office Hour: Thursday, 27.01. from 1:15 pm - 2:45 pm via Zoom

Exam structure

- Similar to mock exam
- Three tasks

Task 1	epse ENERGY & PROCESS SYSTEMS ENGINEERING | RRE Reliability and Risk Engineering
Task 2	epse ENERGY & PROCESS SYSTEMS ENGINEERING
Task 3	RRE Reliability and Risk Engineering

- Grading is based on your calculation steps and the final result.

Lecture plan

No.	Date	Content	
1	29.09.	Introduction & Models	
2	06.10.	Heat integration	Applications
3	13.10.	Continuous Optimization	Methods
4	20.10.	Heat exchanger networks	Applications
5	27.10.	Discrete Optimization	Methods
6	03.11.	Life Cycle Assessment (LCA)	Metrics
7	10.11.	Thermoeconomics	Metrics
8	17.11.	Risk Key Performance Indicators for Security	Metrics
9	24.11.	Multi-energy dimension: introduction	Methods & Applications
10	01.12.	Design dimensions: technology modelling	
11	08.12.	Space dimensions: energy networks	
12	15.12.	Uncertainty in energy systems	
13	22.12.	Recap (online)	

Optimization problem

General optimization problem

$$\min_{x,y} \quad z = f(\boldsymbol{x}, \boldsymbol{y})$$

Objective function

$$\text{s.t.} \quad g_j(\boldsymbol{x}, \boldsymbol{y}) \le 0, \; j = 1, \dots, n$$

$$h_i(\boldsymbol{x}, \boldsymbol{y}) = 0, \; i = 1, \dots, o$$

Inequality constraints

equality constraints

$$\boldsymbol{x} \in \mathbb{R}^m, \boldsymbol{y} \in \mathbb{Z}^t$$

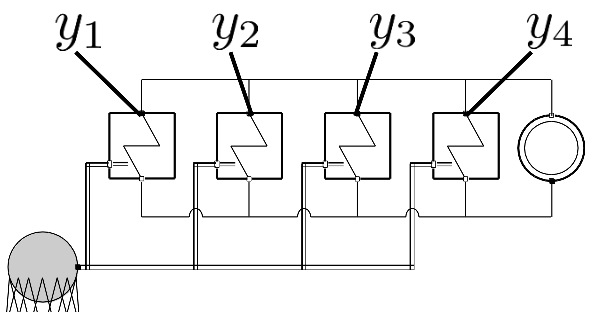
variables

Solve with
Branch&Bound algorithm

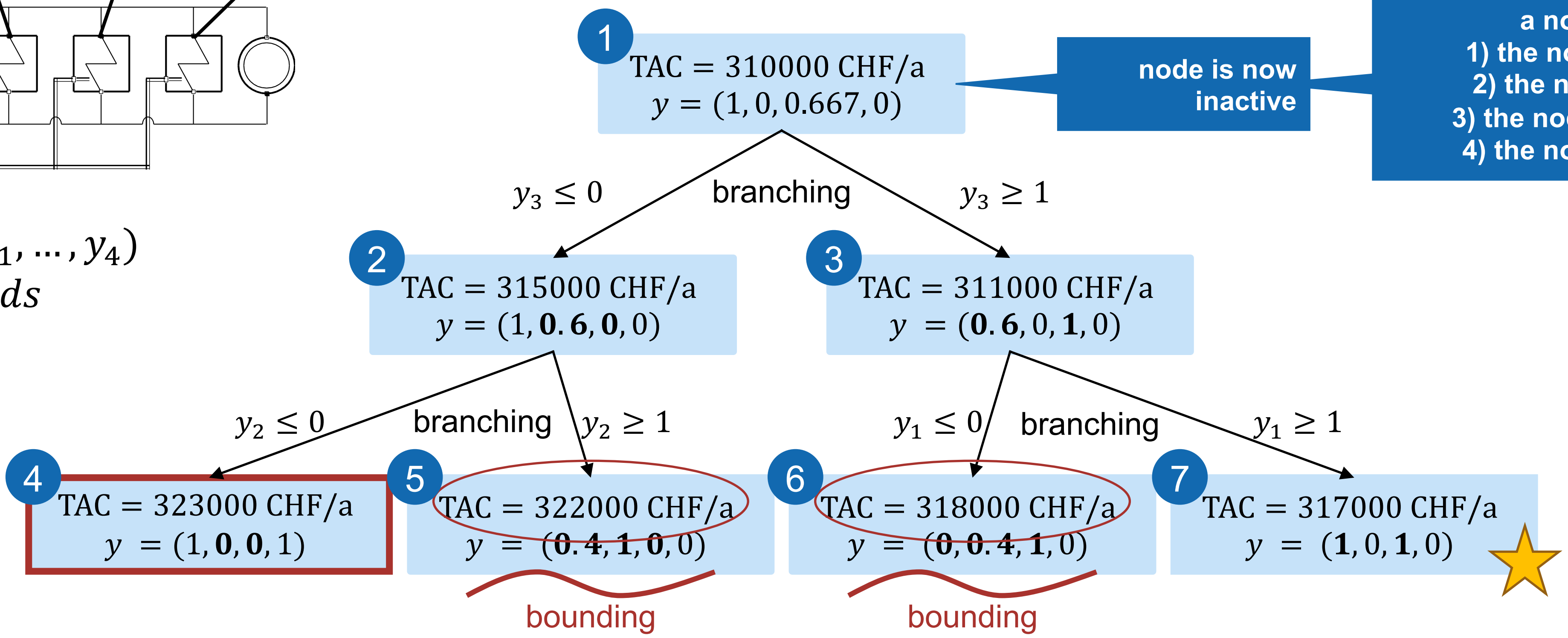
equations \ variables	continuous	discrete (& continuous)
linear	linear program (LP)	mixed-integer linear program (MILP)
nonlinear	nonlinear program (NLP)	mixed-integer nonlinear program (MINLP)

Branch-&-Bound: Lower & upper bound to assess solution quality

See Lecture 5!



$\min TAC(y_1, \dots, y_4)$
$s.t. \; demands$
...



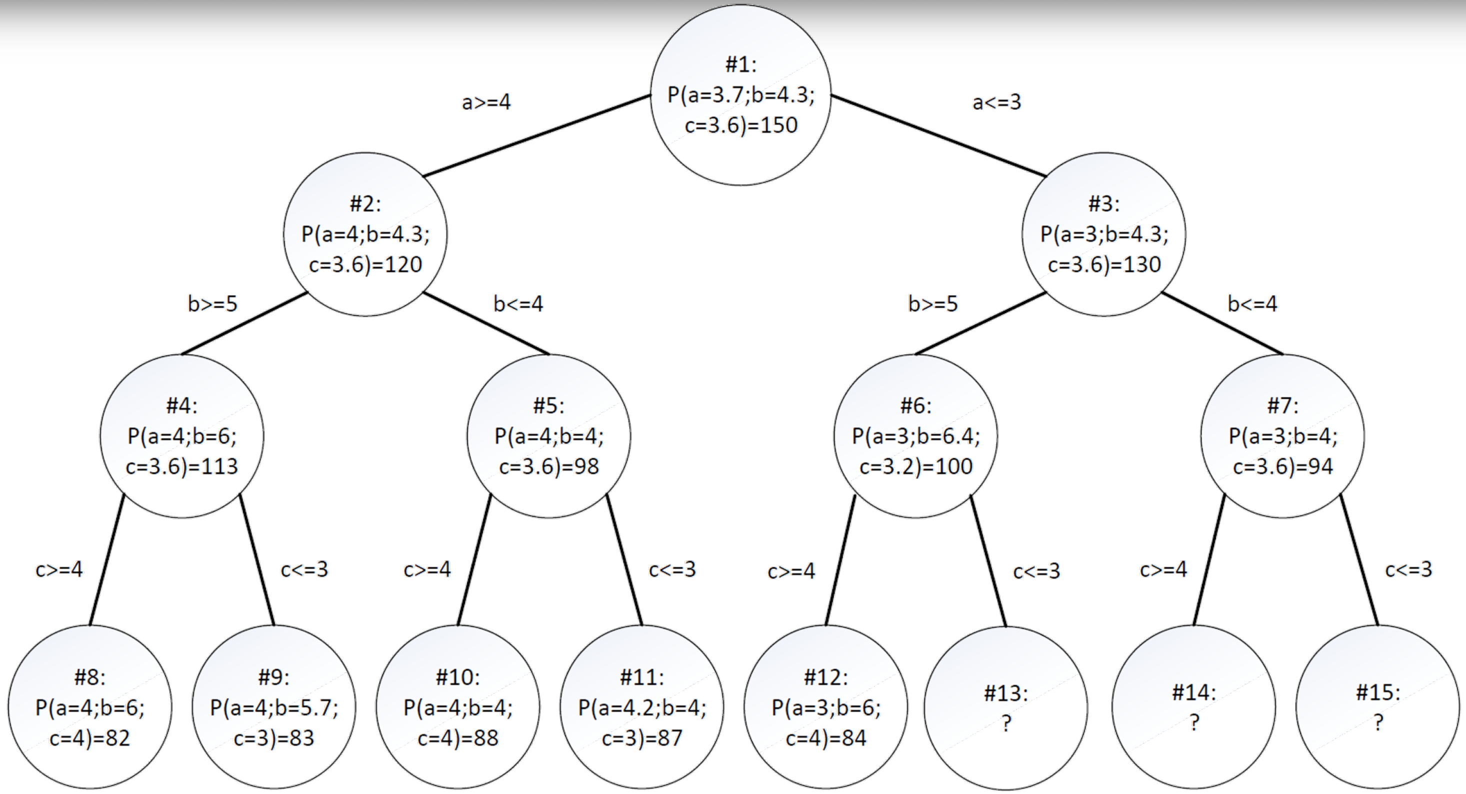
node is now inactive

a node is inactive, if
1) the node is branched,
2) the node is bounded,
3) the node is feasible, or
4) the node is infeasible.

node	1	2	3	4	5	6	7
lower bound	310000 CHF/a	310000 CHF/a	311000 CHF/a	311000 CHF/a	311000 CHF/a	311000 CHF/a	317000 CHF/a
upper bound	-	-	-	323000 CHF/a	323000 CHF/a	323000 CHF/a	317000 CHF/a
gap	-	-	-	12000 CHF/a	12000 CHF/a	12000 CHF/a	0 CHF/a

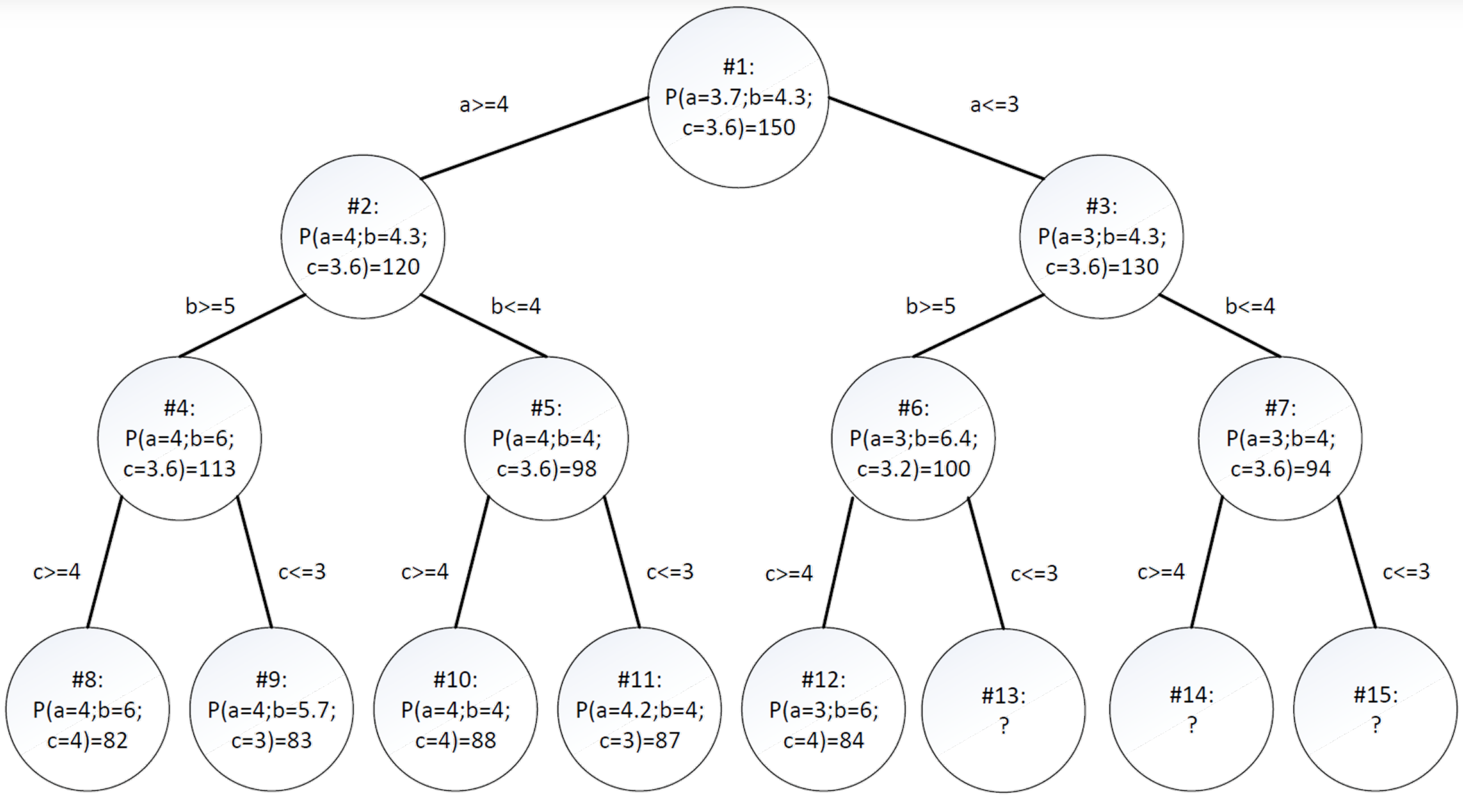
Exemplary task: Branch & Bound

The incomplete Branch-and-Bound Search Tree to solve a mixed-integer linear optimization problem is given in Figure 2. In each node, the number of the node #, the values of the relaxed integer decision variables (a,b,c) and the value of the objective function P(a,b,c) are given. The search algorithm runs through the Branch-and-Bound Search Tree in ascending order. At the time of consideration, the relaxed problems in the nodes #1–#12 are solved.



Exemplary task: Branch & Bound

a) Identify and explain why the optimization problem in the Branch-and-Bound Search Tree is a minimization or a maximization problem. Determine the value of the lower and the upper bound after the search algorithm processed node #12.



Objective function decreases

LP subproblem are upper bounds

Maximization problem

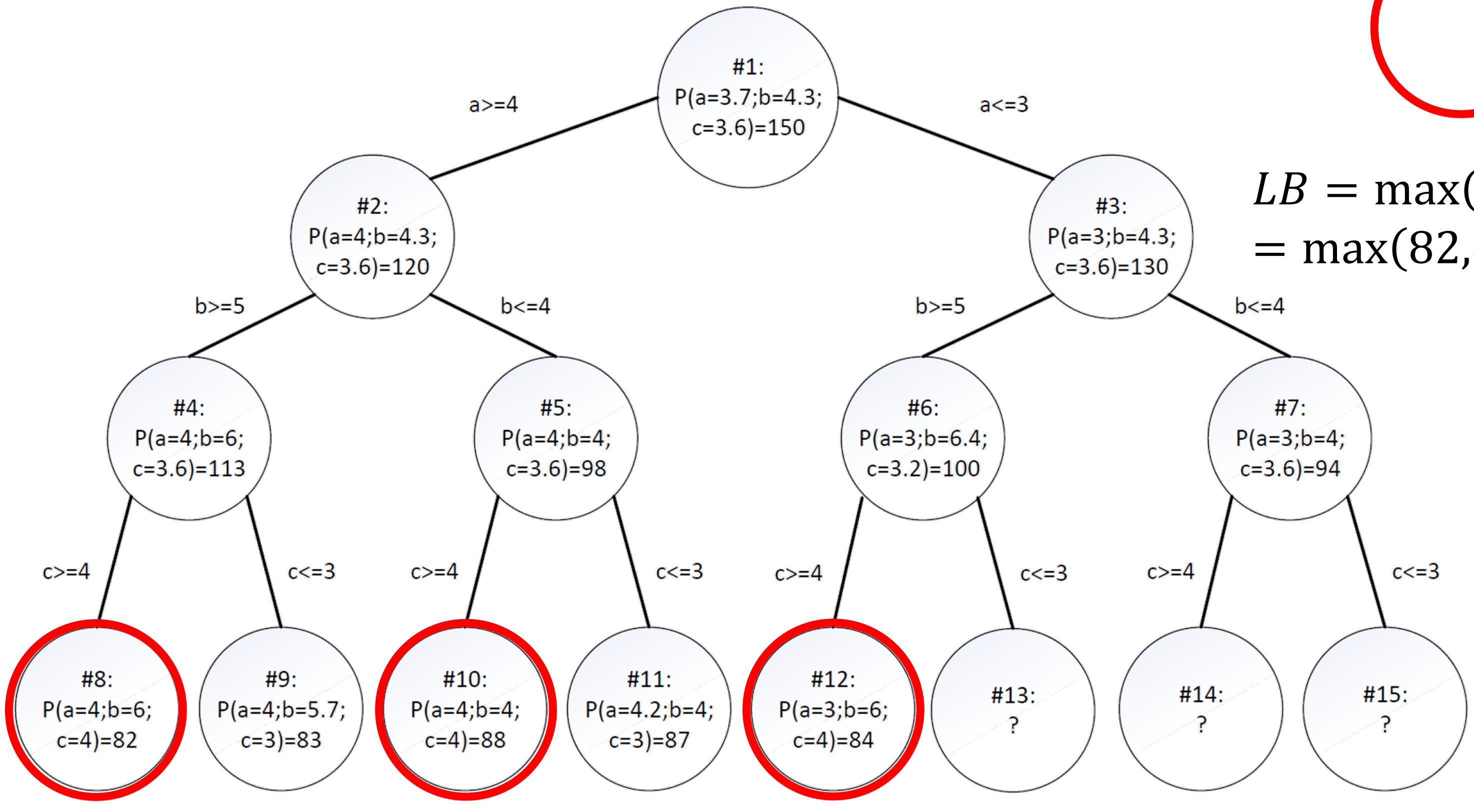
Exemplary task: Branch & Bound

a) Identify and explain why the optimization problem in the Branch-and-Bound Search Tree is a minimization or a maximization problem. Determine the value of the lower and the upper bound after the search algorithm processed node #12.

LB (in maximization): currently best feasible solution

Integer feasible solutions

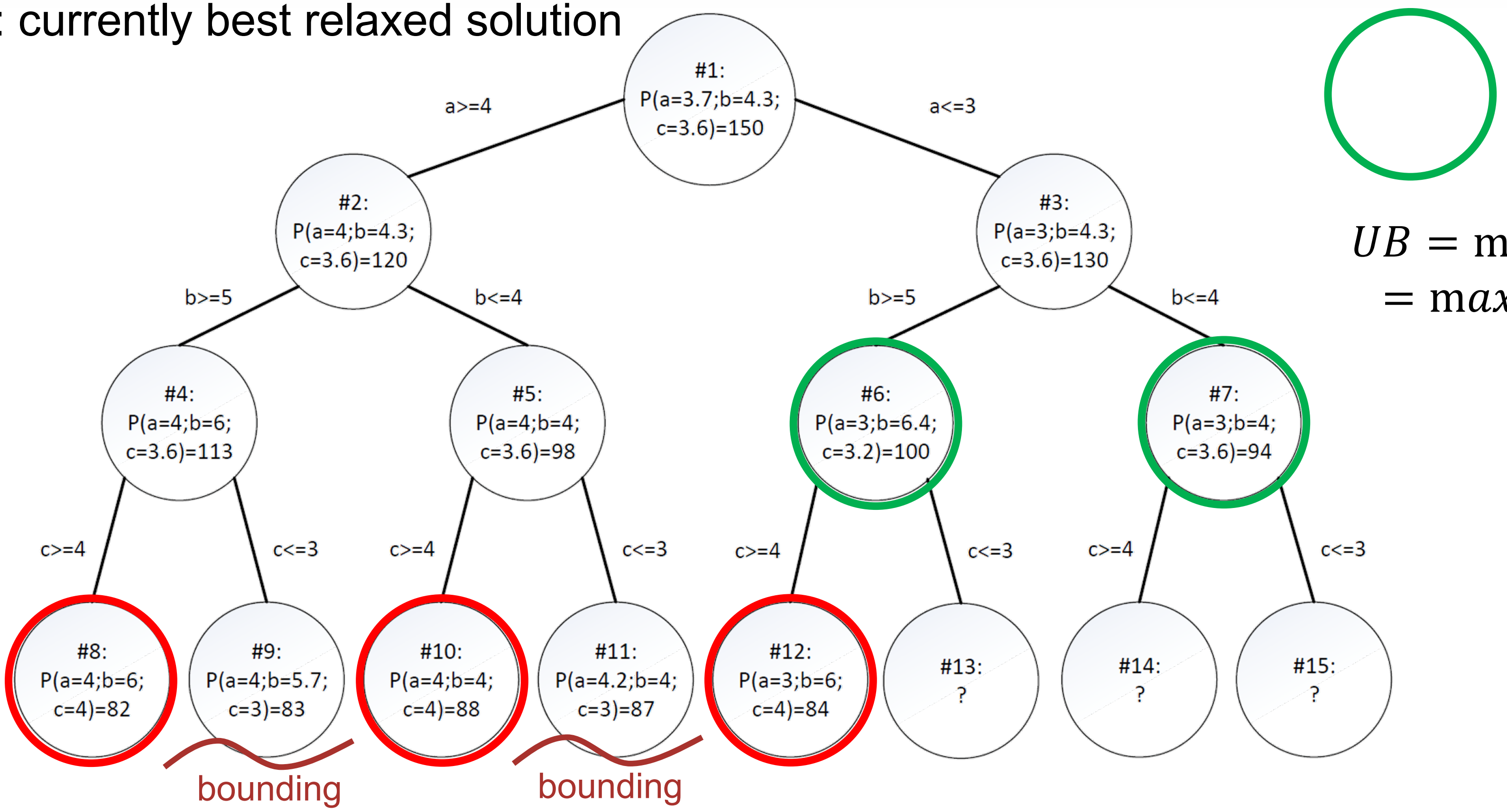
$$LB = \max(integer\ feasible\ solutions)$$
$$= \max(82,84,88) = 88$$



Exemplary task: Branch & Bound

a) Identify and explain why the optimization problem in the Branch-and-Bound Search Tree is a minimization or a maximization problem. Determine the value of the lower and the upper bound after the search algorithm processed node #12.

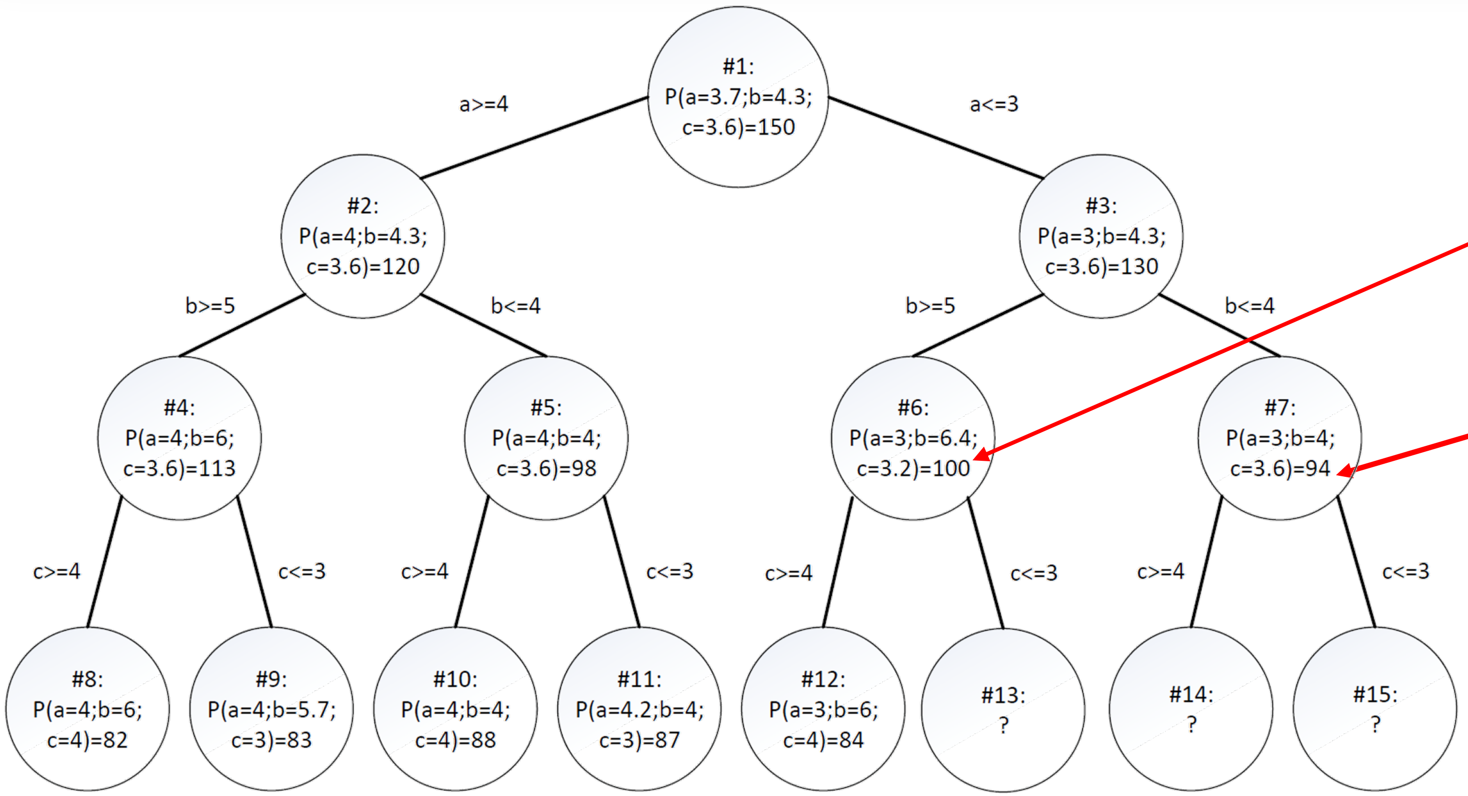
UB (in maximization): currently best relaxed solution



$$UB = \max(active\ nodes)$$
$$= \max(94{,}100) = 100$$

Exemplary task: Branch & Bound

b) Consider node #13: Assume the integer decision variables (a,b,c) reach integer values. Determine the maximum value for the objective function. Furthermore, determine the minimum value for the objective function such that the search algorithm would terminate with an optimal solution.



After #12: $LB = 88$
$UB = 100$

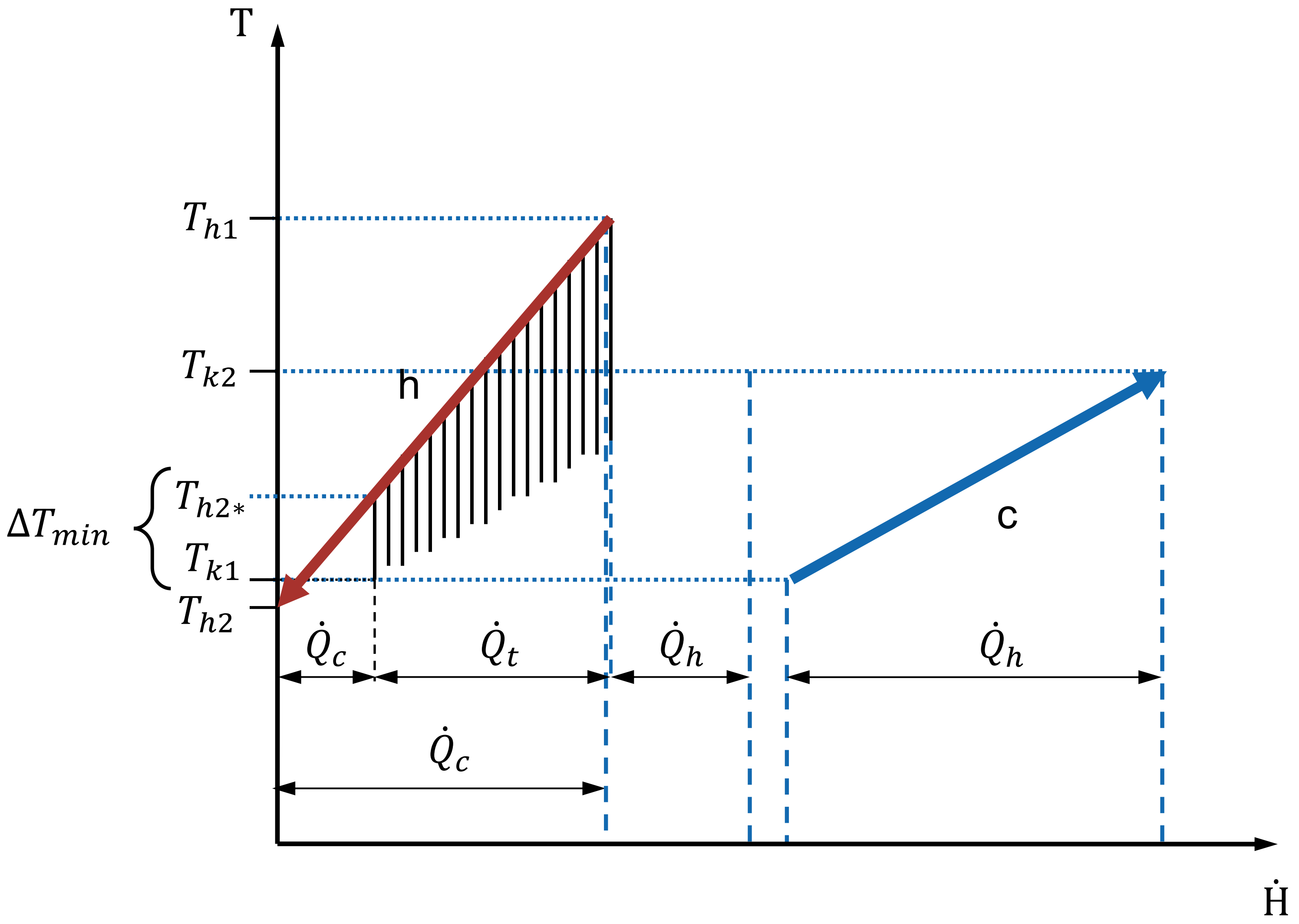
Maximum possible value of objective function is 100:
$P(\#13) \leq 100$

For termination, #7 needs to be bounded:
Bounding, if $P(\#13) > 94$

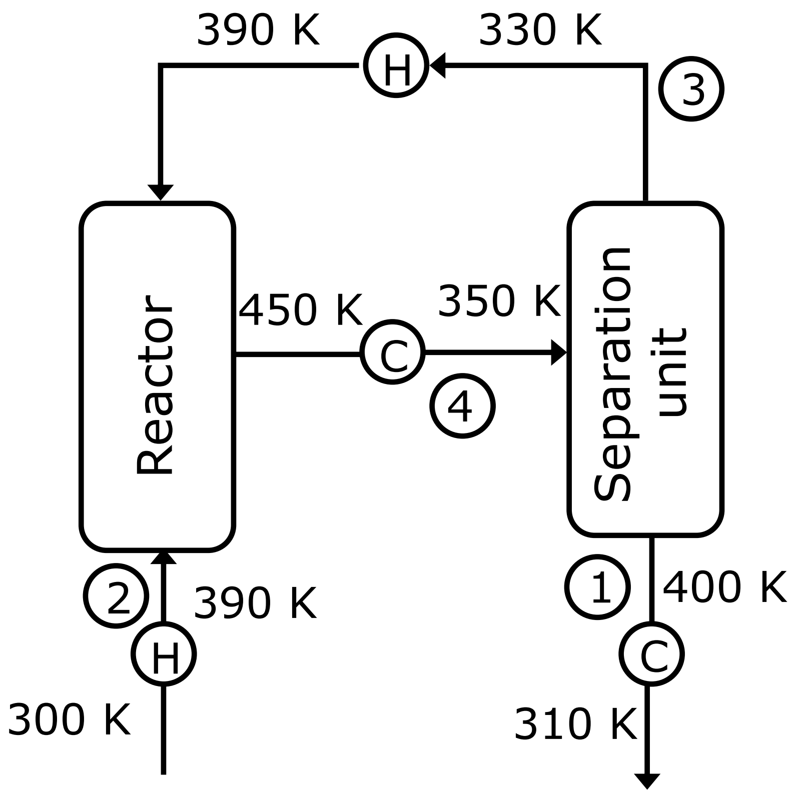
Termination after #13, if
$94 < P(\#13) \leq 100$

Changes in state in the t,H-diagram

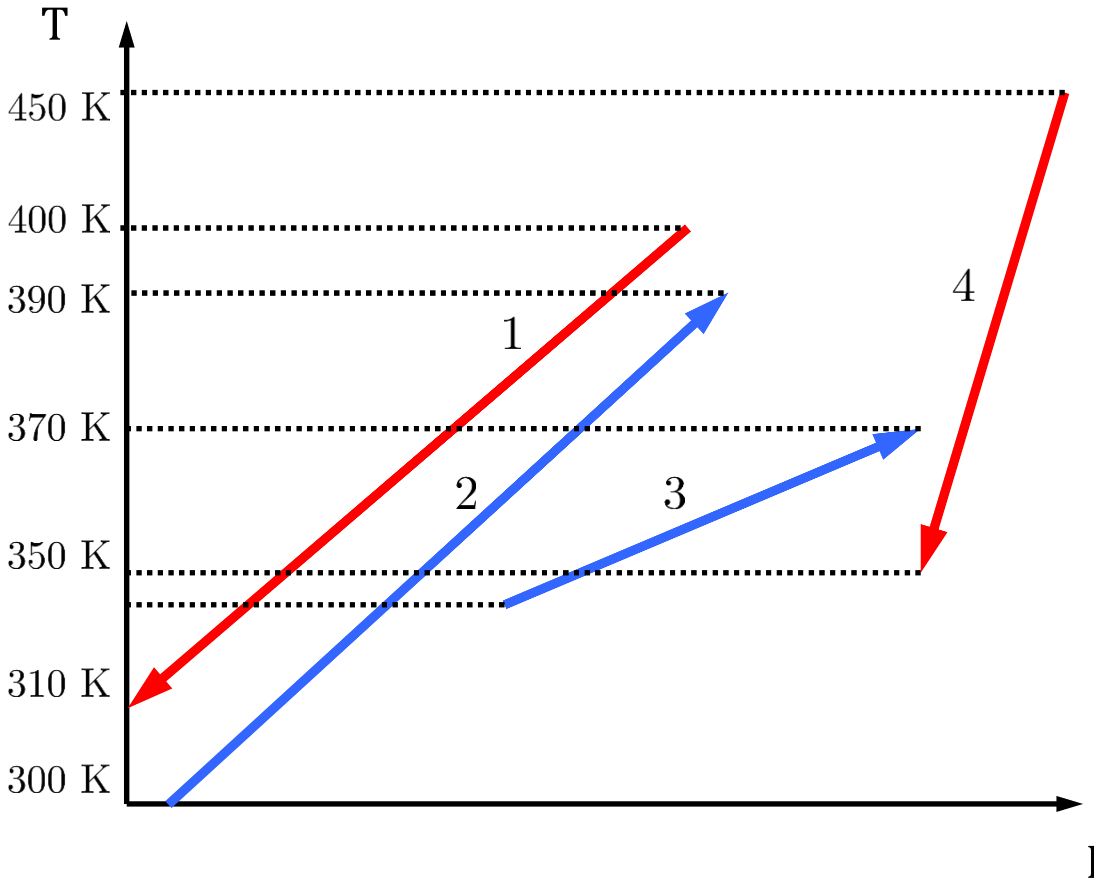
See Lecture 2!



Hot and cold streams process example



Stream	T_{in} [K]	T_{out} [K]	$\dot{C}_p$ [kW/K]
1(h)	400	310	2
2(c)	300	390	1,8
3(c)	330	370	4
4(h)	450	350	1



See Lecture 2!

Mock Exam Task 1.2

Mock Exam Introduction to Modeling and Optimization
of Sustainable Energy Systems — Fall Semester 2020/2021

Name: ______________________ Student ID: ______________

2. As an energy manager of MOE Inc., your task is the heat integration of a new process P1. The streams of process P1 are given in Table 1. Determine the heat cascade for the process data of process P1 given in Table 1 and draw the grand composite curve in Figure 2. Assume a minimal allowed approach temperature of $\Delta T_{\text{min}} = 20\,\text{K}$. Determine the pinch temperatures of the hot and cold streams ($t_{\text{Pinch,h}}$ and $t_{\text{Pinch,c}}$). Hatch the parts where internal heat integration is possible.

Table 1: Process data of the considered process P1.

Stream i	$(\dot{m} \cdot c_\text{p})_i$ / kW/K	$t_{i,\text{in}}$ / °C	$t_{i,\text{out}}$ / °C
1	25	60	320
2	20	180	340
3	10	400	200
4	15	260	80
5	20	200	120

What is the minimum amount of utilities requried?
From streams to grand composite curve

See Lecture 2!

1. adjust temperatures of streams to consider **minimum approach temperature**

$$T_h^* = T_h - \frac{\Delta T_{min}}{2}$$
$$T_c^* = T_c + \frac{\Delta T_{min}}{2}$$

2. fill **heat cascade:**
   1. determine the heat capacity flows in each temperature intervals
   2. determine the amount of heat surplus/deficit in each temperature interval
   3. cascade the surplus/deficits through temperature intervals
   4. shift the sum of surplus/deficits

Table 2: Heat cascade

t^* / °C	Δt^* / K	$\sum(\dot{m}\cdot c_p)_h$ / kW/K	$\sum(\dot{m}\cdot c_p)_c$ / kW/K	$\dot{Q}_i$ / kW	$\Delta\dot{Q}_i$ / kW	$\Delta\dot{Q}_i^*$ / kW
390					0	3800
	40	10	-	400		
350					400	4200
	20	10	20	-200		
330					200	4000
	80	10	45	-2800		
250					-2600	1200
	60	25	45	-1200		
190					-3800	0
	80	35	25	800		
110					-3000	800
	40	15	25	-400		
70					-3400	400

3. draw & interpret the **grand composite curve**
   1. where should we apply heat integration
   2. minimum utility demands & temperature requirements
   3. pinch temperatures

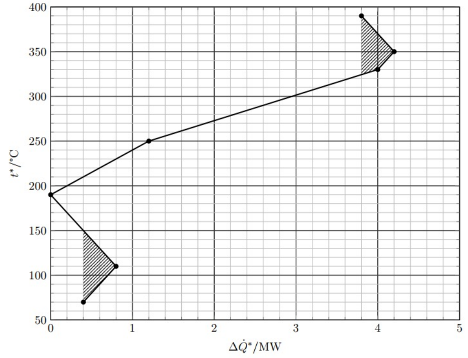


Figure 15: Grand composite curve

Mock Exam Task 1.2
Minimum approach temperature

$$T_h^* = T_h - \frac{\Delta T_{min}}{2}$$
$$T_c^* = T_c + \frac{\Delta T_{min}}{2}$$

$\boxed{\Delta T_{min} = 20 \text{ K}}$

Table 1: Process data of the considered process P1.

	Stream i	$(\dot{m} \cdot c_{\mathrm{p}})_i$ / kW/K	$t_{i,\mathrm{in}}$ / °C	$t_{i,\mathrm{out}}$ / °C
c	1	25	~~60~~ 70	~~320~~ 330
c	2	20	~~180~~ 190	~~340~~ 350
h	3	10	~~400~~ 390	~~200~~ 190
h	4	15	~~260~~ 250	~~80~~ 70
h	5	20	~~200~~ 190	~~120~~ 110

What is the minimum amount of utilities requried?
From streams to grand composite curve

See Lecture 2!

1. adjust temperatures of streams to consider **minimum approach temperature**

$$T_h^* = T_h - \frac{\Delta T_{min}}{2}$$

$$T_c^* = T_c + \frac{\Delta T_{min}}{2}$$

2. fill **heat cascade:**
    1. determine the heat capacity flows in each temperature intervals
    2. determine the amount of heat surplus/deficit in each temperature interval
    3. cascade the surplus/deficits through temperature intervals
    4. shift the sum of surplus/deficits

Table 2: Heat cascade

t^* / °C	Δt^* / K	$\sum(\dot{m} \cdot c_p)_h$ / kW/K	$\sum(\dot{m} \cdot c_p)_c$ / kW/K	$\dot{Q}_i$ / kW	$\Delta\dot{Q}_i$ / kW	$\Delta\dot{Q}_i^*$ / kW
390					0	3800
	40	10	-	400		
350					400	4200
	20	10	20	-200		
330					200	4000
	80	10	45	-2800		
250					-2600	1200
	60	25	45	-1200		
190					-3800	0
	80	35	25	800		
110					-3000	800
	40	15	25	-400		
70					-3400	400

3. draw & interpret the **grand composite curve**
    1. where should we apply heat integration
    2. minimum utility demands & temperature requirements
    3. pinch temperatures

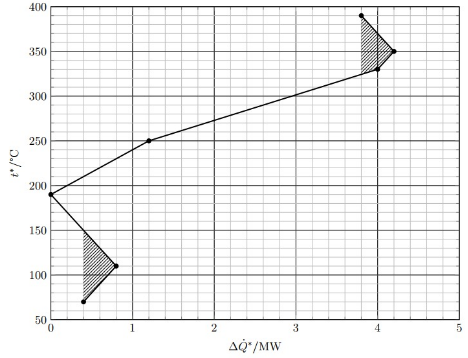


Figure 15: Grand composite curve

Mock Exam Task 1.2
Heat cascade

See Lecture 2!

Table 2: Heat cascade

t^* / °C	Δt^* / K	$\sum(\dot{m}\cdot c_\mathrm{p})_\mathrm{h}$ / kW/K	$\sum(\dot{m}\cdot c_\mathrm{p})_\mathrm{c}$ / kW/K	$\dot{Q}_i$ / kW	$\Delta\dot{Q}_i$ / kW	$\Delta\dot{Q}_i^*$ / kW

Table 1: Process data of the considered process P1.

	Stream i	$(\dot{m}\cdot c_\mathrm{p})_i$ / kW/K	$t_{i,\mathrm{in}}$ / °C	$t_{i,\mathrm{out}}$ / °C
c	1	25	~~60~~ 70	~~320~~ 330
c	2	20	~~180~~ 190	~~340~~ 350
h	3	10	~~400~~ 390	~~200~~ 190
h	4	15	~~260~~ 250	~~80~~ 70
h	5	20	~~200~~ 190	~~120~~ 110

Mock Exam Task 1.2
Heat cascade

Table 1: Process data of the considered

	Stream i	$(\dot{m} \cdot c_p)_i$ / kW/K	$t_{i,\mathrm{in}}$ / °C	$t_{i,\ldots}$
c	1	25	~~60~~ 70	
c	2	20	~~180~~ 190	~~340~~
h	3	10	~~400~~ 390	~~200~~ 190
h	4	15	~~260~~ 250	~~80~~ 70
h	5	20	~~200~~ 190	~~120~~ 110

See Lecture 2!

Table 2: Heat cascade

t^* / °C	Δt^* / K	$+\sum(\dot{m} \cdot c_p)_h$ / kW/K	$-\sum(\dot{m} \cdot c_p)_c$ / kW/K	$\dot{Q}_i$ / kW	$\Delta\dot{Q}_i$ / kW	$\Delta\dot{Q}_i^*$ / kW
390						
	40	· (10	-) =			
350						
	20	· (10	- 20) =			
330						
	80	· (10	- 45) =			
250						
	60	· (25	- 45) =			
190						
	80	· (35	- 25) =			
110						
	40	· (15	- 25) =			
70						

What is the minimum amount of utilities requried?
From streams to grand composite curve

See Lecture 2!

1. adjust temperatures of streams to consider **minimum approach temperature**

$$T_h^* = T_h - \frac{\Delta T_{min}}{2}$$
$$T_c^* = T_c + \frac{\Delta T_{min}}{2}$$

2. fill **heat cascade:**
   1. determine the heat capacity flows in each temperature intervals
   2. determine the amount of heat surplus/deficit in each temperature interval
   3. cascade the surplus/deficits through temperature intervals
   4. shift the sum of surplus/deficits

Table 2: Heat cascade

t^* / °C	Δt^* / K	$\sum(\dot{m} \cdot c_p)_h$ / kW/K	$\sum(\dot{m} \cdot c_p)_c$ / kW/K	$\dot{Q}_i$ / kW	$\Delta\dot{Q}_i$ / kW	$\Delta\dot{Q}_i^*$ / kW
390					0	3800
	40	10	-	400		
350					400	4200
	20	10	20	-200		
330					200	4000
	80	10	45	-2800		
250					-2600	1200
	60	25	45	-1200		
190					-3800	0
	80	35	25	800		
110					-3000	800
	40	15	25	-400		
70					-3400	400

3. draw & interpret the **grand composite curve**
   1. where should we apply heat integration
   2. minimum utility demands & temperature requirements
   3. pinch temperatures

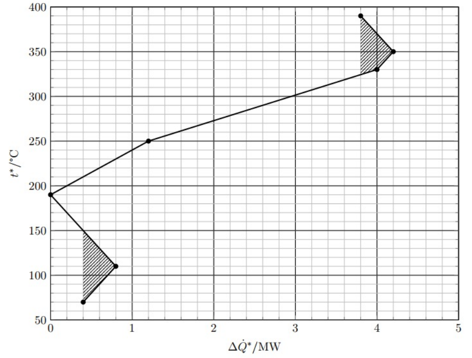


Figure 15: Grand composite curve

Mock Exam Task 1.2
Grand Composite Curve

See Lecture 2!

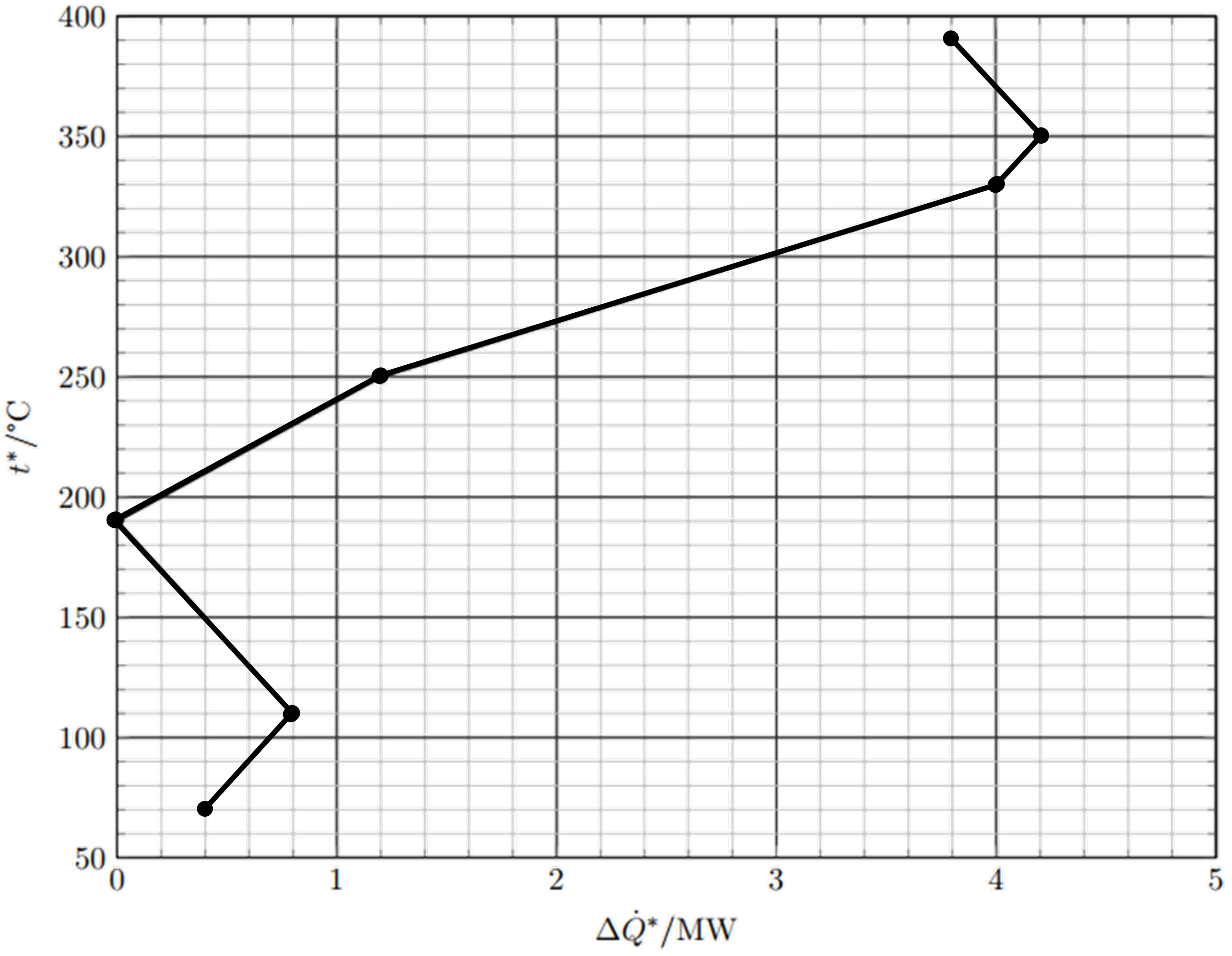


Figure 15: Grand composite curve

Table 2: Heat cascade

t^* / °C	Δt^* / K	$\sum(\dot{m}\cdot c_p)_h$ / kW/K	$\sum(\dot{m}\cdot c_p)_c$ / kW/K	$\dot{Q}_i$ / kW	$\Delta\dot{Q}_i$ / kW	$\Delta\dot{Q}_i^*$ / kW
390					0	3800
	40	10	-	400		
350					400	4200
	20	10	20	-200		
330					200	4000
	80	10	45	-2800		
250					-2600	1200
	60	25	45	-1200		
190					-3800	0
	80	35	25	800		
110					-3000	800
	40	15	25	-400		
70					-3400	400

Mock Exam Task 1.2

Mock Exam Introduction to Modeling and Optimization of Sustainable Energy Systems

Fall Semester 2020/2021

Name: ______________________ Student ID: ______________

2. As an energy manager of MOE Inc., your task is the heat integration of a new process P1. The streams of process P1 are given in Table 1. Determine the heat cascade for the process data of process P1 given in Table 1 and draw the grand composite curve in Figure 2. Assume a minimal allowed approach temperature of $\Delta T_{\text{min}} = 20\,\text{K}$. Determine the pinch temperatures of the hot and cold streams ($t_{\text{Pinch,h}}$ and $t_{\text{Pinch,c}}$). Hatch the parts where internal heat integration is possible.

Table 1: Process data of the considered process P1.

Stream i	$(\dot{m} \cdot c_\text{p})_i$ / kW/K	$t_{i,\text{in}}$ / °C	$t_{i,\text{out}}$ / °C
1	25	60	320
2	20	180	340
3	10	400	200
4	15	260	80
5	20	200	120

Mock Exam Task 1.2, Grand Composite Curve

See Lecture 2!

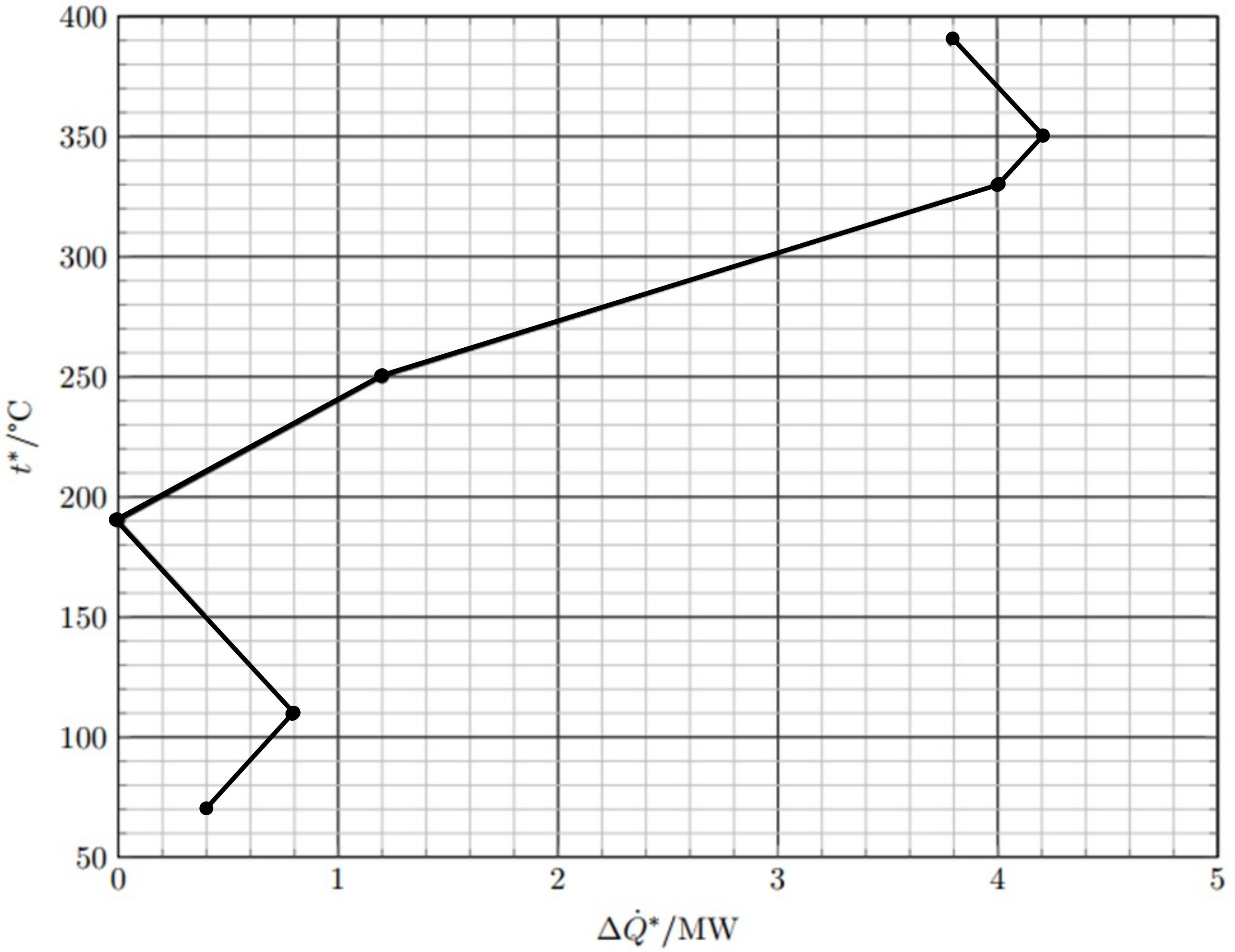


Figure 15: Grand composite curve

$$T_h^* = T_h - \frac{\Delta T_{min}}{2}$$

$$T_c^* = T_c + \frac{\Delta T_{min}}{2}$$

$$T^{h,pinch} = 200\ °C$$

$$T^{c,pinch} = 180\ °C$$

Bonus ☺:

$\dot{Q}^{c,min} = 0.4\ \text{MW},\ T^{c,max} = 140\ °C$

$\dot{Q}^{h,min} = 3.8\ \text{MW},\ T^{h,min} \approx 324\ °C$

What is the minimum amount of utilities requried?
From streams to grand composite curve

See Lecture 2!

1. adjust temperatures of streams to consider **minimum approach temperature**

$$T_h^* = T_h - \frac{\Delta T_{min}}{2}$$
$$T_c^* = T_c + \frac{\Delta T_{min}}{2}$$

2. fill **heat cascade:**
   1. determine the heat capacity flows in each temperature intervals
   2. determine the amount of heat surplus/deficit in each temperature interval
   3. cascade the surplus/deficits through temperature intervals
   4. shift the sum of surplus/deficits

Table 2: Heat cascade

t^* / °C	Δt^* / K	$\sum(\dot m \cdot c_p)_h$ / kW/K	$\sum(\dot m \cdot c_p)_c$ / kW/K	$\dot Q_i$ / kW	$\Delta \dot Q_i$ / kW	$\Delta \dot Q_i^*$ / kW
390					0	3800
	40	10	-	400		
350					400	4200
	20	10	20	-200		
330					200	4000
	80	10	45	-2800		
250					-2600	1200
	60	25	45	-1200		
190					-3800	0
	80	35	25	800		
110					-3000	800
	40	15	25	-400		
70					-3400	400

3. draw & interpret the **grand composite curve**
   1. where should we apply heat integration
   2. minimum utility demands & temperature requirements
   3. pinch temperatures

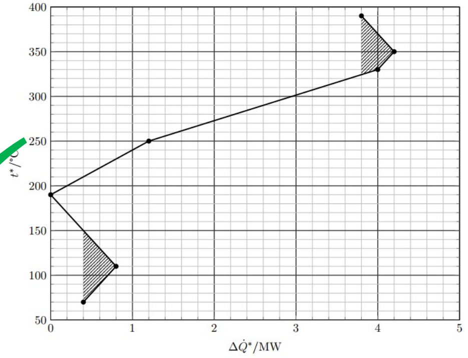


Figure 15: Grand composite curve

LP problem for finding the minimum amount of utilities
From lecture 4

See Lecture 4!

objective function

$$s.t. \qquad 0 = \sum_h \Delta\dot{H}_h^{(z)} + \sum_H \dot{Q}_{U,H}^{(z)} - \sum_c \Delta\dot{H}_c^{(z)} - \sum_C \dot{Q}_{U,C}^{(z)} + \Delta\dot{Q}_{z-1}^* - \Delta\dot{Q}_z^* \quad, \forall z$$

$$\Delta\dot{Q}_0^* = 0,$$
$$\Delta\dot{Q}_{z_{max}}^* = 0,$$
$$\Delta\dot{Q}_z^* \geq 0, \forall z$$
$$\dot{Q}_{U,H}^{(z)} \geq 0, \forall z$$
$$\dot{Q}_{U,C}^{(z)} \geq 0, \forall z$$

What is the minimum amount of utilities requried?
We can also formulate an optimization problem!

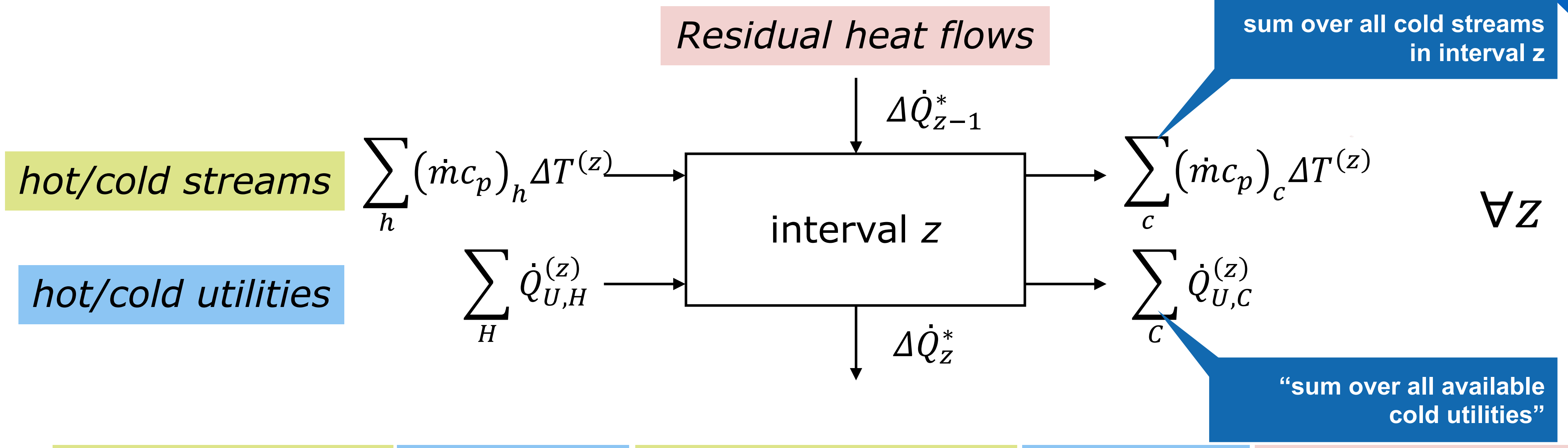
See Lecture 4!

Table 2: Heat cascade

temperature interval z	t^* / °C	Δt^* / K	$\sum(\dot{m}\cdot c_\mathrm{p})_\mathrm{h}$ / kW/K	$\sum(\dot{m}\cdot c_\mathrm{p})_\mathrm{c}$ / kW/K	$\dot{Q}_i$ / kW	$\Delta\dot{Q}_i$ / kW	$\Delta\dot{Q}_i^*$ / kW
	390					0	3800
1		40	10	-	400		
	350					400	4200
2		20	10	20	-200		
	330					200	4000
3		80	10	45	-2800		
	250					-2600	1200
4		60	25	45	-1200		
	190					-3800	0
5		80	35	25	800		
	110					-3000	800
6		40	15	25	-400		
	70					-3400	400

Energy balances
From lecture 4

See Lecture 4!



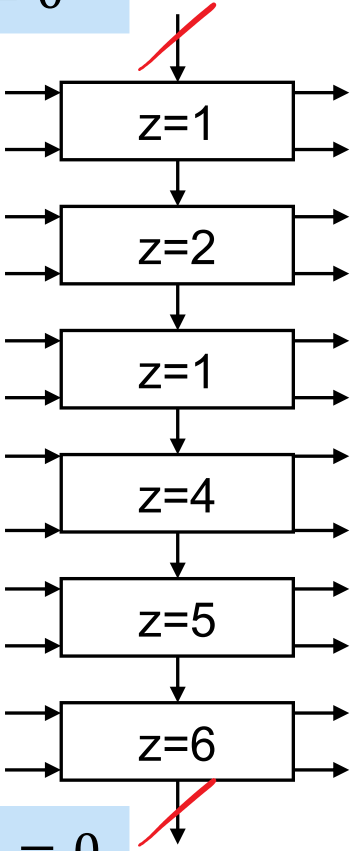
$$0 = \sum_h (\dot{m}c_p)_h \Delta T^{(z)} + \sum_H \dot{Q}^{(z)}_{U,H} - \sum_c (\dot{m}c_p)_c \Delta T^{(z)} - \sum_C \dot{Q}^{(z)}_{U,C} + \Delta\dot{Q}^*_{z-1} - \Delta\dot{Q}^*_z,$$

$$\Delta\dot{Q}^*_z \geq 0, \forall z$$
$$\dot{Q}^{(z)}_{U,H} \geq 0, \forall z$$
$$\dot{Q}^{(z)}_{U,C} \geq 0, \forall z$$

We can also formulate an optimization problem

See Lecture 4!

$$\Delta\dot{Q}_0^* = 0$$



$$\Delta\dot{Q}_{z_{max}}^* = 0$$

Table 2: Heat cascade

temperature interval	t^* / °C	Δt^* / K	$\sum(\dot{m}\cdot c_\mathrm{p})_\mathrm{h}$ / kW/K	$\sum(\dot{m}\cdot c_\mathrm{p})_\mathrm{c}$ / kW/K	$\dot{Q}_i$ / kW	$\Delta\dot{Q}_i$ / kW	$\Delta\dot{Q}_i^*$ / kW
	390					0	3800
1		40	10	-	400		
	350					400	4200
2		20	10	20	-200		
	330					200	4000
3		80	10	45	-2800		
	250					-2600	1200
4		60	25	45	-1200		
	190					-3800	0
5		80	35	25	800		
	110					-3000	800
6		40	15	25	-400		
	70					-3400	400

LP problem for finding the minimum amount of utilities
From lecture 4

See Lecture 4!

objective function

$$s.t. \qquad 0 = \sum_h \Delta \dot{H}_h^{(z)} + \sum_H \dot{Q}_{U,H}^{(z)} - \sum_c \Delta \dot{H}_c^{(z)} - \sum_C \dot{Q}_{U,C}^{(z)} + \Delta \dot{Q}_{z-1}^{*} - \Delta \dot{Q}_z^{*} \quad , \forall z$$

$$\Delta \dot{Q}_0^{*} = 0,$$
$$\Delta \dot{Q}_{z_{max}}^{*} = 0,$$
$$\Delta \dot{Q}_z^{*} \geq 0, \forall z$$
$$\dot{Q}_{U,H}^{(z)} \geq 0, \forall z$$
$$\dot{Q}_{U,C}^{(z)} \geq 0, \forall z$$

Heuristic & Optimization Problem

	heuristic	optimization problem
heat integration	Heat cascade & grand composite curve in **L2**	LP formulation in **L4**
heat exchanger network design	pinch method in **L4**	MILP formulation in **L4/L5**

Overview of Lectures 7 – 12

- **Lecture 7/8:** Key-Performance Indicators
- **Lecture 9/10:** Optimal design and operation of multi-energy systems
- **Lecture 11:** Energy networks
- **Lecture 12:** Uncertainty in energy networks

Lecture 7/8: Key-Performance Indicators (KPIs)

- Thermodynamic KPIs
  - Cumulative energy demand (CED)*
  - Energy conversion efficiency*
  - Energy storage efficiency*

- Economic KPIs
  - Net present value* (NPV)
  - Internal rate of return* (IRR)
  - Levelized cost of energy* (LCOE)
  - Total annual cost* (TAC)

- Security KPIs
  - Reliability
  - Total risk
  - Uncertainty
  - Expected energy not supplied (EENS)
  - Expected power not supplied (EPNS)
  - System average interruption frequency index (SAIFI)
  - System average duration index (SAIDI)

***You are expected to remember the formula**

Lecture 9/10: Optimal design and operation of multi-energy systems

- Understand concept, motivation and main design questions of multi-energy systems (MES)

- Understand the different degrees of complexity when optimizing MES
  - Single objective vs. multi-objective optimization problems
  - Optimal design vs. optimal operation
  - Time and space dimension
  - Uncertainty (deterministic vs. sensitivity and uncertainty analysis)

- Formulation of the optimization problem
  - Model energy conversion technologies within MES optimization
    - Approximation* for modelling the efficiency of energy conversion technologies
  - Model energy storage technologies within MES optimization

General MILP formulation: Optimal design and operation

Objective function

$$\min_{\boldsymbol{x}} \; z_{\text{cost}} = \sum_{k\in\mathcal{K}} (1+v_k) I_k a_k + \sum_{t=1}^{T} \sum_{j\in\mathcal{J}} p_{j,t}(M_j - N_j)$$

Minimize cost, emissions, or risk

s. t.

Energy balances

$$\sum_{k\in\mathcal{K}} (V_{j,k,t} - U_{j,k,t}) + M_{j,t} - N_{j,t} = D_{j,t}$$

$$0 \le M_{j,t} \le M_{j,\max}, \;\; 0 \le N_{j,t} \le N_{j,\max} \qquad \forall j \in \mathcal{J}, t \in \{1,\dots,T\}$$

RE (non-dispatchable) technologies

$$V_{\bar{j},k,t} = \eta_k(w_t)\, A_k,$$

$$b_k A_{k,\min} \le A_k \le b_k A_{k,\max} \qquad \forall \bar{j} \in \bar{\mathcal{J}}_k, k \in \mathcal{K}_{\text{R}}, t \in \{1,\dots,T\}$$

Dispatchable technologies

$$V_{\bar{j},k,t} = \eta_{\underline{j}\bar{j},k}(U_{\underline{j},k,t}, P_k)$$

$$y_{k,t}\delta_k P_k \le U_{\underline{j},k,t} \le y_{k,t} P_k$$

$$b_k \dot{P}_{k,\min} \le \dot{P}_k \le b_k \dot{P}_{k,\max} \qquad \forall \bar{j} \in \bar{\mathcal{J}}_k, \underline{j} \in \underline{\mathcal{J}}_k, k \in \mathcal{K}_{\text{C}}, t \in \{1,\dots,T\}$$

MES structure

$\mathcal{J}$ = Set of energy carriers, {E, G, H}

$\mathcal{K}$ = Set of technologies, {PV, HP, B}

$\mathcal{K}_{\text{R}}$ = Set of renewable energy technologies, {PV}

$\mathcal{K}_{\text{C}}$ = Set of dispatchable technologies, {HP, B, GT}

$\bar{\mathcal{J}}_k$ = Set of output carriers for technology k

$\underline{\mathcal{J}}_k$ = Set of input carriers for technology k

T = Length of the time horizon

General MILP formulation: Optimal design and operation

⋮

Storage technologies

$$S_{j,k,t} = (1-\lambda_k \Delta t) S_{j,k,t-1} + \eta_k^{\mathrm{c}} U_{j,k,t} - \frac{V_{j,k,t}}{\eta_k^{\mathrm{d}}},$$

$$0 \le S_{j,k,t} \le P_k\,, \qquad 0 \le U_{j,k,t} \le \frac{P_k}{\tau^{\mathrm{c}}}, \qquad 0 \le V_{j,k,t} \le \frac{P_k}{\tau^{\mathrm{d}}}\,,$$

$$S_{j,k,0} = S_{j,k,T}$$

$$b_k P_{k,\min} \le P_k \le b_k P_{k,\max} \qquad \forall j \in \mathcal{J}_k, k \in \mathcal{K}_{\mathrm{S}}, t \in \{1,\dots,T\}$$

Demand not supplied

$$DNS_{j,t,i} = 0 \;\; \forall i \in \mathcal{E}_N$$

$$DNS_{j,t,i} = D_{j,t} \;\; \forall i \in \mathcal{E}_I$$

Technology operation constraints for scenario i

$$U_{\underline{j},k,t,i} = V_{\bar{j},k,t,i} = 0 \;\; \forall \bar{j} \in \bar{\mathcal{J}}_k, \underline{j} \in \underline{\mathcal{J}}_k, k \in \mathcal{K}_{F,i}$$

$$N_{j,t,i} = M_{j,t,i} = 0 \;\; \forall \; j \in \mathcal{J}_{F,i}$$

$$\sum_{k\in\mathcal{K}} \left(V_{j,k,t,i} - U_{j,k,t,i}\right) + M_{j,t,i} - N_{j,t,i} = D_{j,t} - DNS_{j,t,i}$$

$$0 \le DNS_{j,t,i} \le D_{j,t} \qquad \forall i \in \mathcal{E}_R, j \in \mathcal{J}, t \in \{1,\dots,T\}$$

MES structure

$\mathcal{J}$ = Set of energy carriers, {E, G, H}

$\mathcal{K}$ = Set of technologies, {PV, HP, B}

$\mathcal{K}_{\mathrm{R}}$ = Set of renewable energy technologies, {PV}

$\mathcal{K}_{\mathrm{C}}$ = Set of dispatchable technologies, {HP, B, GT}

$\mathcal{K}_{\mathrm{S}}$ = Set of storage technologies, {BS, TS}

$\bar{\mathcal{J}}_k$ = Set of output carriers for technology k

$\underline{\mathcal{J}}_k$ = Set of input carriers for technology k

T = Length of the time horizon

$\mathcal{E}_N, \mathcal{E}_I, \mathcal{E}_R$ = Set of system states where supply is not impacted (N), impossible (I) or reduced (R)

$\mathcal{K}_{F,i}$ = Set of failed technologies in state i

$\mathcal{J}_{F,i}$ = Set of failed carrier grid imports in state i

Lecture 11: Energy networks

- Describe the energy network modeling process
- Define energy networks i.e. gas, electricity and thermal networks with
  - a graph with node and link quantities
  - an optimization problem with decision variables and constraints
- Solve network equations with backward-forward sweep
- Describe the process of linearizing physical laws
- Model non-unique flow directions and topology changes

Note: Flow equations will be provided in the exam.

Combined MES technology and network optimization

Lecture 11

$$\min_{\boldsymbol{x}} \; z$$

Objective function

$\mathrm{s.t.}$

Lectures 9, 10

RE (non-dispatchable) technologies

Lectures 9, 10

Dispatchable technologies

Lecture 10

Storage technologies

Lecture 10

Demand not supplied

⋮

MES structure

$\mathcal{J}$ = Set of energy carriers, $\{\mathrm{E, G, H}\}$

$\mathcal{N}$ = Set of nodes

$\mathcal{L}$ = Set of links

$\mathcal{K}_{\mathrm{a}}$ = Set of technologies, installed at node a

T = Length of the time horizon

Combined MES technology and network optimization

⋮

Energy balances

$$\sum_{k\in\mathcal{K}_a}\left(V_{G,k,t}-U_{G,k,t}\right)-D_{G,a,t}=\mathrm{LHV}\dot{m}_{a,t},$$

$$\sum_{k\in\mathcal{K}_a}\left(V_{H,k,t}-U_{H,k,t}\right)-D_{H,a,t}=\dot{Q}_{a,t},$$

$$\sum_{k\in\mathcal{K}_a}\left(V_{E,k,t}-U_{E,k,t}\right)-D_{E,a,t}=P_{a,t},$$

$$D_{E,a,t}f_Q=Q_{a,t},$$

$$M_{Q,t}-N_{Q,t}=Q_{1,t}$$

$$M_{E,t}-N_{E,t}=P_{1,t}$$

$$M_{G,t}=\mathrm{LHV}\dot{m}_{1,t},$$

$$0\le M_{j,t}\le M_{j,\max},\;\; 0\le N_{j,t}\le N_{j,\max} \qquad \forall a\in\mathcal{N}\backslash\{1\}, t\in\{1,\dots,T\}$$

⋮

MES structure

$\mathcal{J}$ = Set of energy carriers, {E, G, H}

$\mathcal{N}$ = Set of nodes

$\mathcal{L}$ = Set of links

$\mathcal{K}_\mathrm{a}$ = Set of technologies, installed at node a

T = Length of the time horizon

Combined MES technology and network optimization

⋮

Gas network

$$-\dot{m}_{a,\max} \leq \dot{m}_{a,t} \leq \dot{m}_{a,\max}$$

$$p_{a,\min} \leq p_{a,t} \leq p_{a,\max}$$

$$p_{1,t} = p_{\text{ref}}$$

$$\dot{m}_{a,t} + \sum_b \dot{m}_{b,a,t} - \sum_b \dot{m}_{a,b,t} = 0$$

$$p_{a,t}^2 - p_{b,t}^2 = k\, \dot{m}_{a,b,t}^{1.848} \qquad \forall a \in \mathcal{N}, (a,b) \in \mathcal{L}, t \in \{1, \dots, T\}$$

⋮

MES structure

$\mathcal{J}$ = Set of energy carriers, {E, G, H}

$\mathcal{N}$ = Set of nodes

$\mathcal{L}$ = Set of links

$\mathcal{K}_\mathrm{a}$ = Set of technologies, installed at node a

T = Length of the time horizon

Combined MES technology and network optimization

⋮

Thermal network

$$\dot{Q}_{a,t} = c_p \dot{m}_{a,t}(T^{\mathrm{S}}_{a,t} - T^{\mathrm{R}}_{a,t})$$

$$-\dot{m}_{a,\max} \leq \dot{m}_{a,t} \leq \dot{m}_{a,\max}$$

$$p_{a,\min} \leq p_{a,t} \leq p_{a,\max}$$

$$p_{1,t} = p_{\mathrm{ref}}$$

$$\dot{m}_{a,t} + \sum_b \dot{m}_{b,a,t} - \sum_b \dot{m}_{a,b,t} = 0$$

$$\left(\dot{m}^{\mathrm{E}}_{a,t} + \sum_b \dot{m}_{a,b,t}\right) T^{\mathrm{S/R}}_{a,t} = \left(\dot{m}^{\mathrm{I}}_{a,t} T^{I}_{a,t} + \sum_b \dot{m}_{b,a,t}\, T^{\mathrm{P}}_{a,t}\right)$$

$$0 \leq \dot{m}_{a,b,t} \leq \dot{m}_{a,b,\max}$$

$$\dot{Q}_{\mathrm{L},a,b,t} = c_{\mathrm{p}} \dot{m}_{a,b,t}(T^{\mathrm{P}}_{a,t} - T^{\mathrm{P}}_{b,t})$$

$$T^{\mathrm{P}}_{b,t} = \left(T^{\mathrm{P}}_{a,t} - T_{\mathrm{amb},t}\right) e^{-\frac{\lambda l}{c_{\mathrm{p}} \dot{m}_{a,b,t}}} + T_{\mathrm{amb},t}$$

$$p_{a,t} - p_{b,t} = 0.01 \frac{8lf}{\pi^2 \rho d^5} \dot{m}^2_{a,b,t}$$

$$\forall a \in \mathcal{N}, (a,b) \in \mathcal{L}, t \in \{1, \dots, T\}$$

⋮

MES structure

$\mathcal{J}$ = Set of energy carriers, {E, G, H}

$\mathcal{N}$ = Set of nodes

$\mathcal{L}$ = Set of links

$\mathcal{K}_{\mathrm{a}}$ = Set of technologies, installed at node a

T = Length of the time horizon

Combined MES technology and network optimization

⋮

Electricity network (DistFlow)

$$-P_{a,\max} \le P_{a,t} \le P_{a,\max}$$

$$-Q_{a,\max} \le Q_{a,t} \le Q_{a,\max}$$

$$u_{a,\min}^2 \le \tilde{u}_{a,t} \le u_{a,\max}^2$$

$$\tilde{u}_{1,t} = u_{\mathrm{ref}}^2$$

$$P_{a,t} + \sum_b P_{b,a,t} - \sum_b P_{a,b,t} = 0$$

$$Q_{a,t} + \sum_b Q_{b,a,t} - \sum_b Q_{a,b,t} = 0$$

$$P_{a,b,t}^2 + Q_{a,b,t}^2 \le \left(u_{a,\min} i_{a,b,\max}\right)^2$$

$$\tilde{u}_{a,t} - \tilde{u}_{b,t} = 2\left(r_{a,b} P_{a,b,t} + x_{a,b} Q_{a,b,t}\right)$$

$$\forall a \in \mathcal{N}, (a,b) \in \mathcal{L}, t \in \{1, \dots, T\}$$

MES structure

$\mathcal{J}$ = Set of energy carriers, {E, G, H}

$\mathcal{N}$ = Set of nodes

$\mathcal{L}$ = Set of links

$\mathcal{K}_\mathrm{a}$ = Set of technologies, installed at node a

T = Length of the time horizon

Lecture 12: Uncertainty in energy systems

- Understand how uncertainty affects energy systems, their modeling and optimization
- Describe the uncertainty occurring within modeling and optimization of energy systems:
  - Aleatory and epistemic uncertainty
  - Probability distributions and uncertainty ranges
- Apply sensitivity analysis and uncertainty analysis within modeling and optimization of energy systems
- Interpret the results and make decisions under uncertainty

Understand the different degrees of complexity when optimizing MES

easier → more difficult

Decision variables: Design vs Operation	Technology operation for given technology selection and size	Selection and size of technologies, coupled with their operation
Time dimension	Instantaneous	Time-dependent
Space dimension	Single node	Multi-node energy networks
Uncertainty	Deterministic optimization	Robust or stochastic optimization
Objective function	Single objective	Multiple objectives

The optimal operation problem is a special case of the optimal design problem (which requires resolving operation)

Exam problems

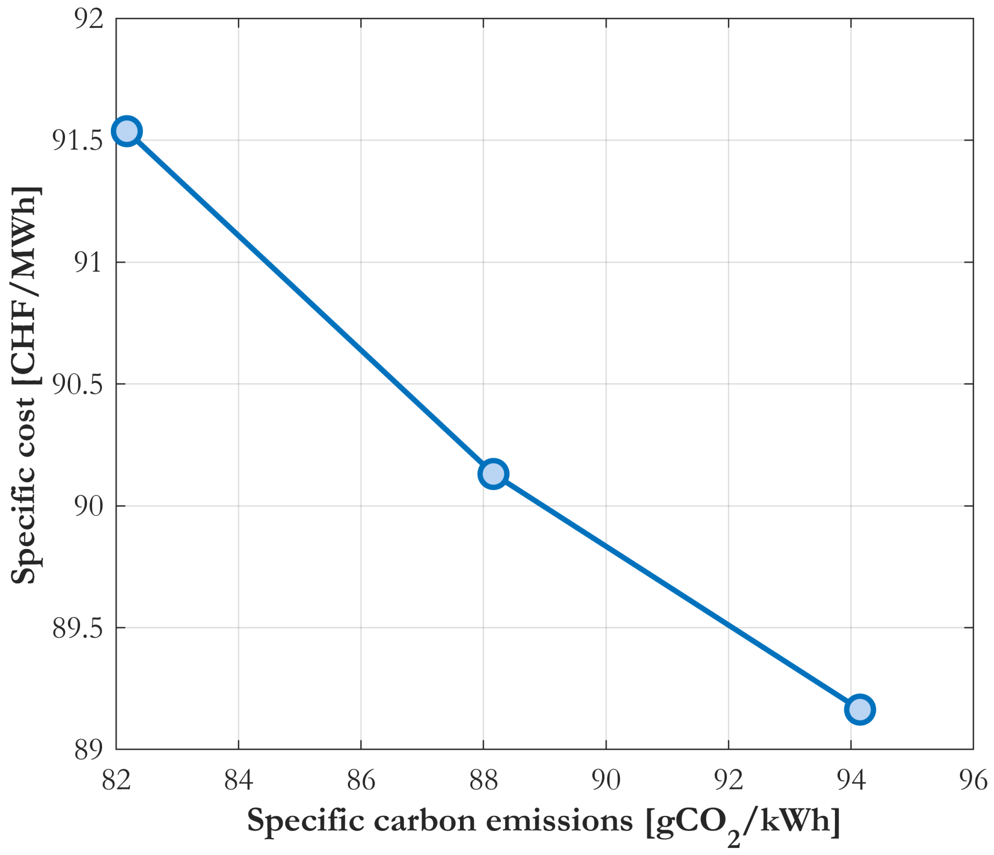
- Multi-objective optimization (Task 3.5 from the Mock exam)

- Thermal networks (Task 1.4 d-g from the Mock exam)

Multi-objective optimization

1. Describe the algorithm to determine the cost-emissions Pareto front through the ϵ-constraint method

2. Report the following quantities:
   - The minimum-cost value $c_{\min}$
   - The minimum-emissions value $e_{\min}$
   - The minimum-emissions value subject to the minimum-cost constraint, $e_{\max}$
   - The minimum-cost value subject to the minimum-emissions constraint, $c_{\max}$



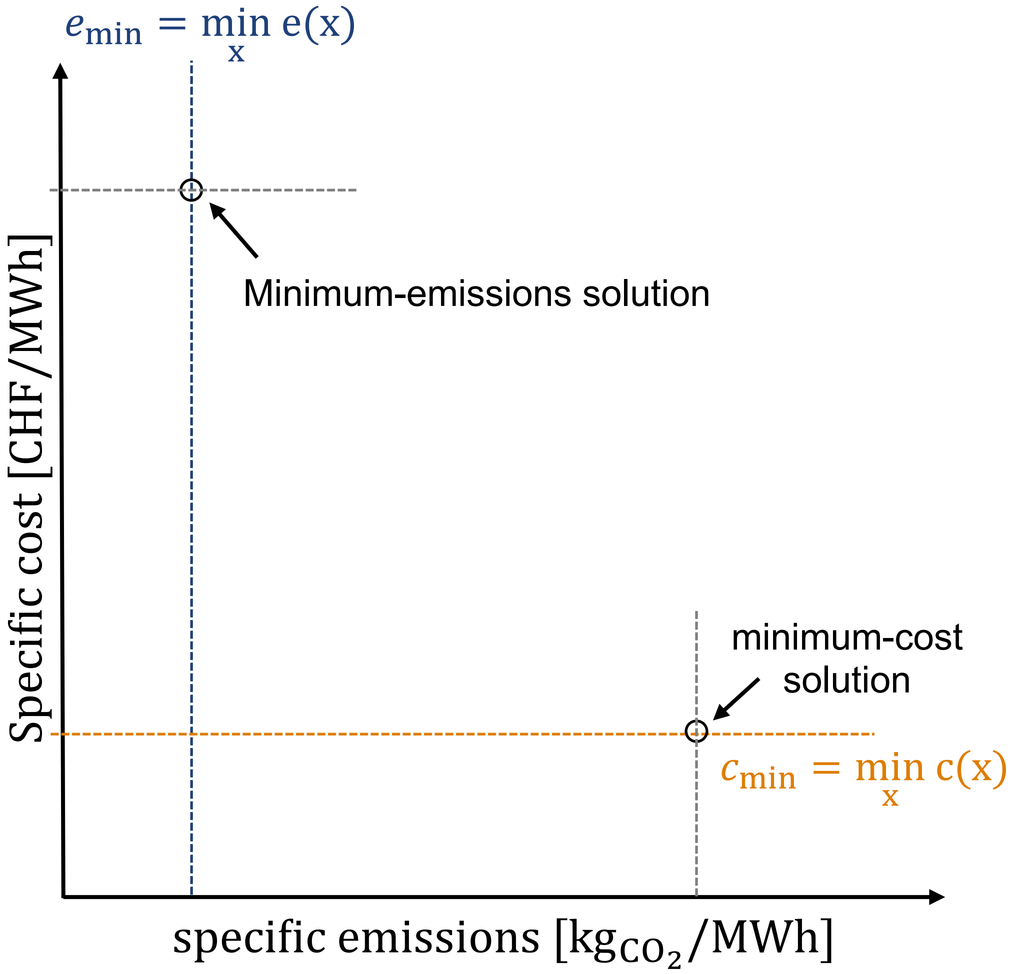
Multi-objective optimization: The ϵ-constraint method

1. The minimum-cost value $c_{\min}$ is determined with the following optimization problem.

$$\begin{aligned} \min_{x} \quad & c(x) \\ s.t. \quad & \boldsymbol{H}x = 0 \\ & \boldsymbol{G}x \leq 0 \end{aligned}$$

2. The minimum-emissions value, $e_{\min}$, is determined with the following optimization problem.

$$\begin{aligned} \min_{x} \quad & e(x) \\ s.t. \quad & \boldsymbol{H}x = 0 \\ & \boldsymbol{G}x \leq 0 \end{aligned}$$

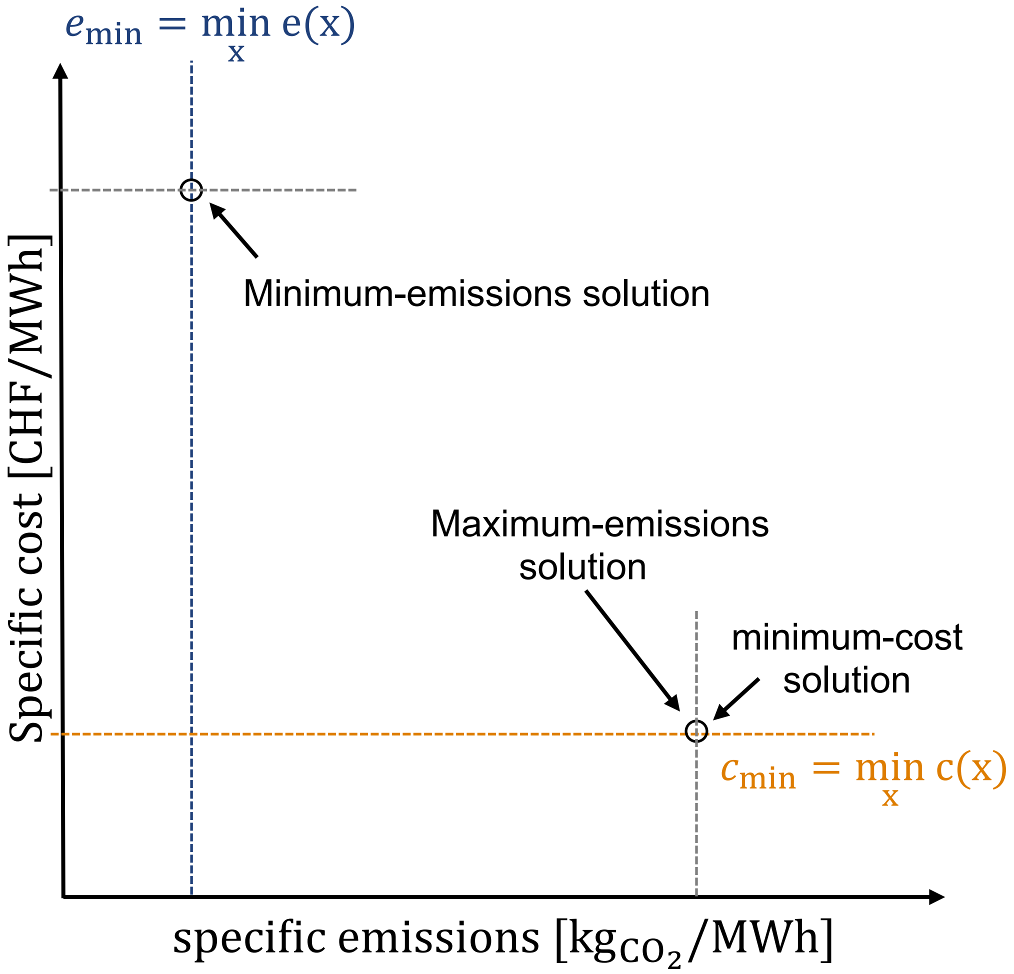


Multi-objective optimization: The ϵ-constraint method

3. The minimum-emissions value subject to the minimum-cost constraint, $e_{\max}$, is determined with the following optimization problem.

$$
\begin{aligned}
\min_{x} \quad & e(x) \\
s.t. \quad & \boldsymbol{H}x = 0 \\
& \boldsymbol{G}x \leq 0 \\
& \mathrm{c}(x) \leq \mathrm{c}_{\min}
\end{aligned}
$$

Note: If $e_{\min} = e_{\max}$, then the minimum-cost and minimum-emissions solution coincide and the algorithm stops. If $e_{\min} < e_{\max}$, the algorithm proceeds.

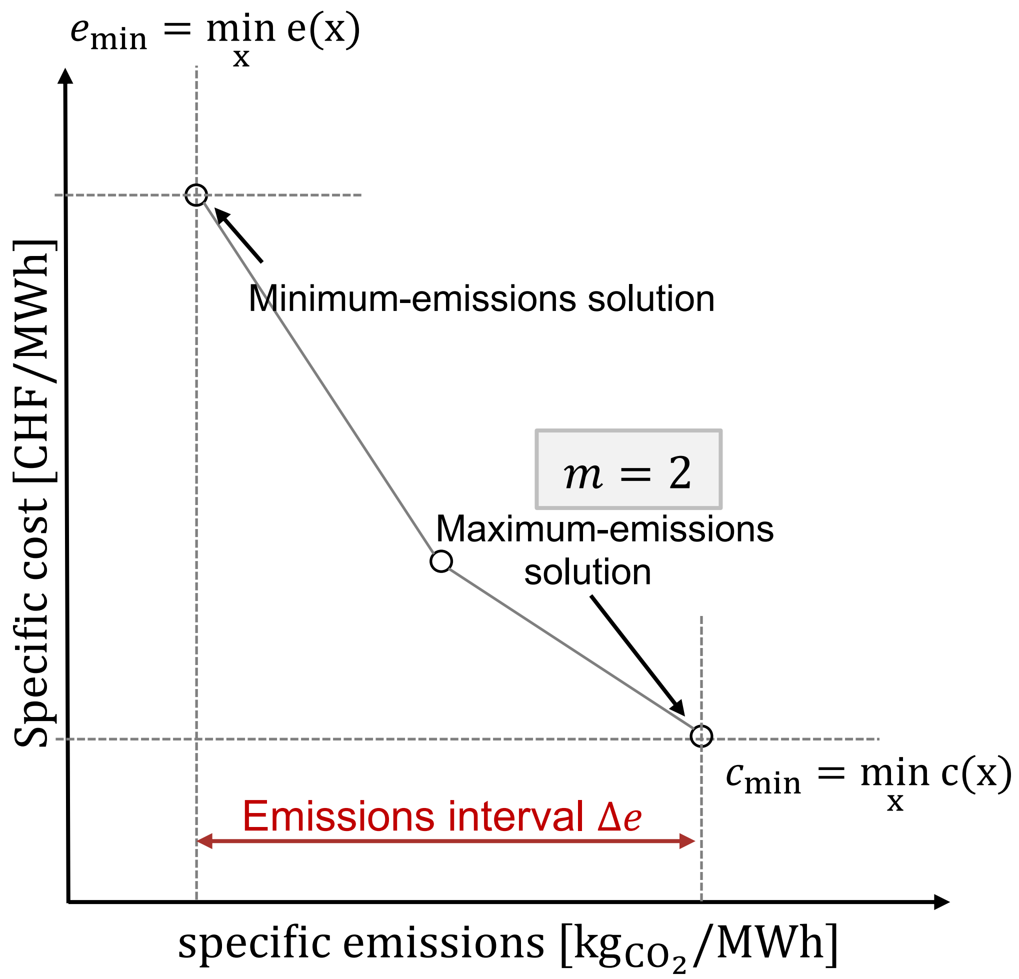


Multi-objective optimization: The ϵ-constraint method

4. The emissions interval, $\Delta e = e_{\max} - e_{\min}$, is divided into m sections.

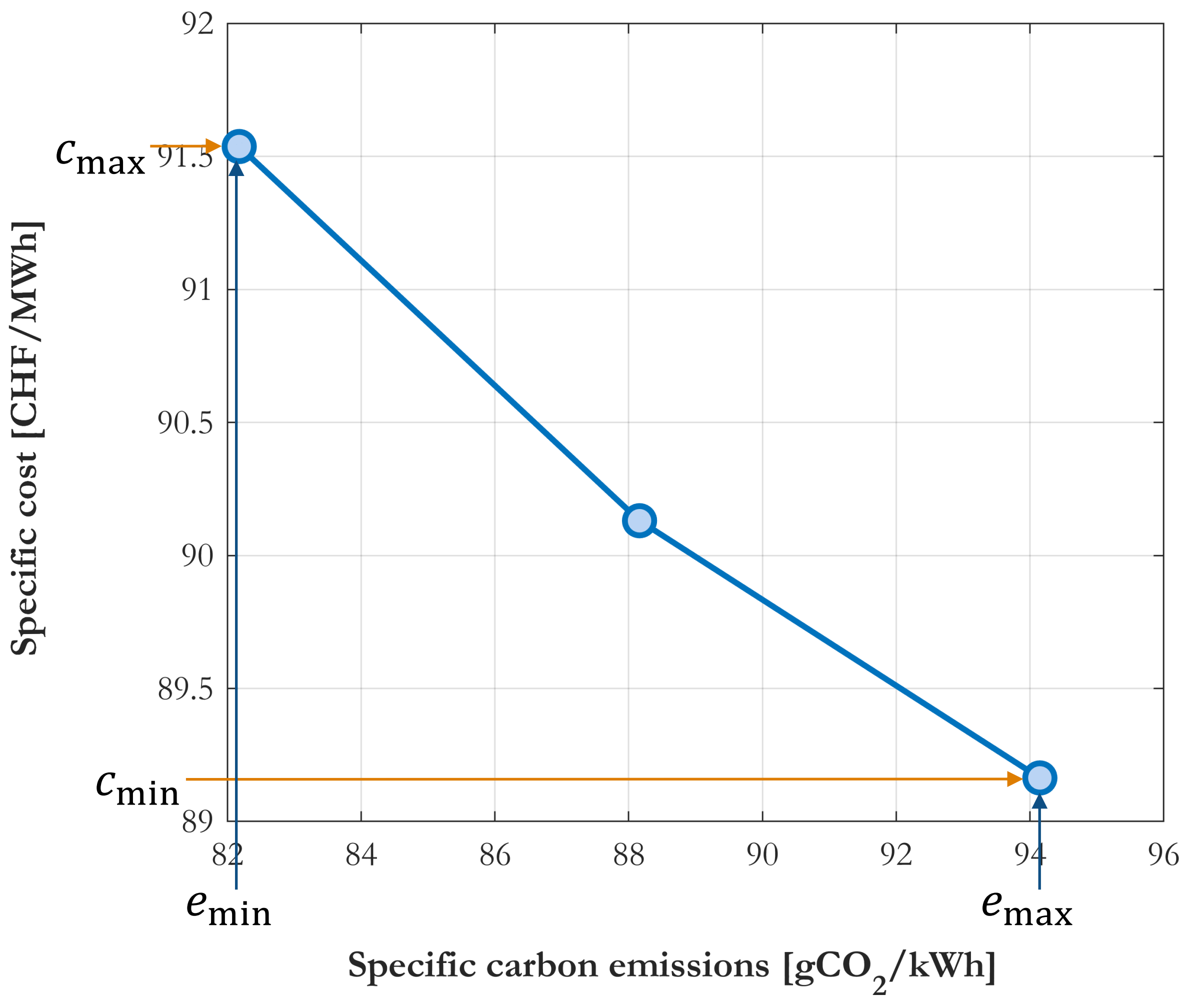
5. For all $i \in \{1, \dots, m\}$, the following optimization problems are solved:

$$\begin{aligned} \min_{x} \quad & c(x) \\ s.t. \quad & \boldsymbol{H}x = 0 \\ & \boldsymbol{G}x \leq 0 \\ & e(x) \leq e_{\max} - \frac{i\Delta e}{m} \end{aligned}$$



Multi-objective optimization: Pareto front

- The minimum-cost value $c_{\min}$
- The minimum-emissions value $e_{\min}$
- The minimum-emissions value subject to the minimum-cost constraint, $e_{\max}$
- The minimum-cost value subject to the minimum-emissions constraint, $c_{\max}$



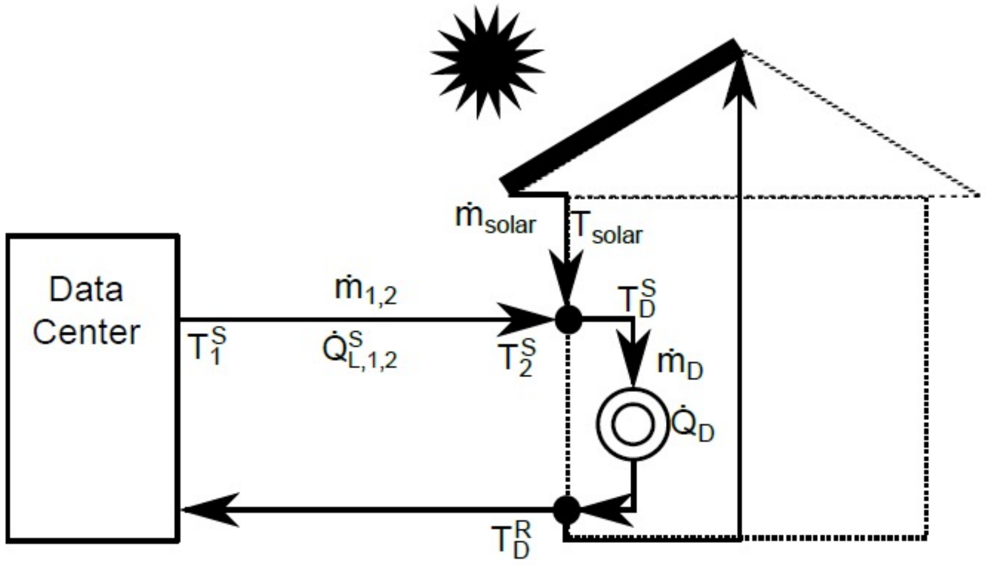
Exam problems

- Multi-objective optimization (Task 3.5 from the Mock exam)

- **Thermal networks (Task 1.4 d-g from the Mock exam)**

Thermal networks

Consider a data center that uses water for cooling and plans to use the waste energy to heat a nearby building. Losses might result in an insufficient supply temperature; therefore, solar thermal panels are considered to provide additional heat.



A mass flow of $\dot{m}_\mathrm{D}$ at a temperature T_D^S is required to satisfy the heat demand $\dot{Q}_D$ of the building. The solar panels provide a mass flow of $\dot{m}_\mathrm{solar}$ at temperature T_solar.

$\dot{m}_\mathrm{solar} = 0.02\,\frac{kg}{s}$ $\quad T_\mathrm{solar} = 50°C$ $\quad \lambda = 0.1\,\frac{W}{m\,K}$

$\dot{m}_\mathrm{D} = 0.1\,\frac{kg}{s}$ $\quad T_\mathrm{D}^S = 40°C$ $\quad L = 200m$

$\dot{Q}_\mathrm{D} = 4000W$ $\quad T_\mathrm{amb} = 15°C$ $\quad c_\mathrm{p} = \frac{4180J}{kg\,K}$

Thermal networks

Calculate the additional mass flow $\dot{m}_{1,2}$ and temperature T_2^S required to satisfy the demand.

- Mass balance:

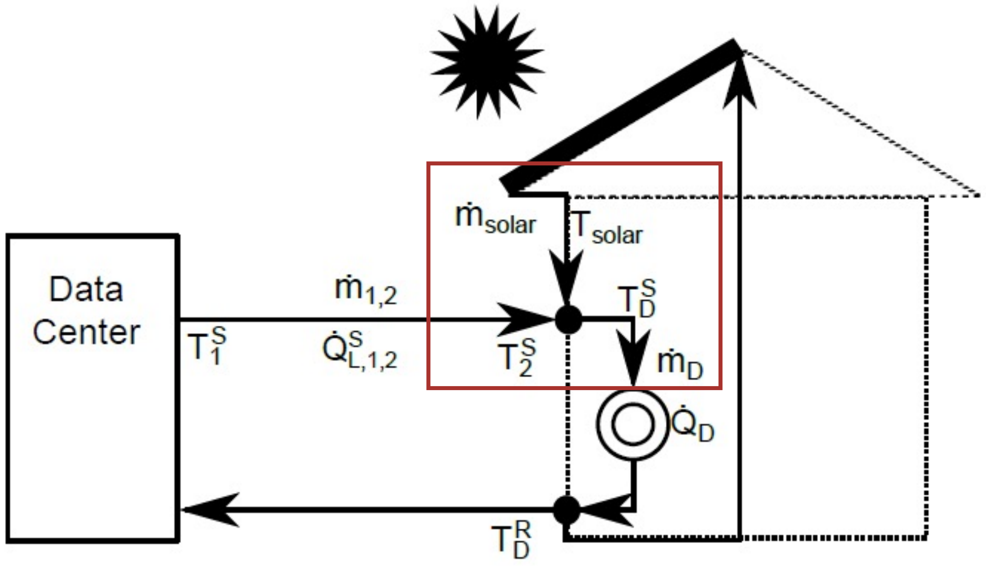
$$\dot{m}_{1,2} = \dot{m}_\mathrm{D} - \dot{m}_\mathrm{solar} = 0.08\,\frac{kg}{s}$$

- Energy balance:

$$\dot{m}_{1,2}\,T_2^\mathrm{S} + \dot{m}_\mathrm{solar}T_\mathrm{solar} = \dot{m}_\mathrm{D}T_\mathrm{D}$$

- Solving for T_2^S :

$$T_2^\mathrm{S} = \frac{\dot{m}_\mathrm{D}T_\mathrm{D} - \dot{m}_\mathrm{solar}T_\mathrm{solar}}{\dot{m}_{1,2}} = 37.5\ °C$$



$$\dot{m}_\mathrm{solar} = 0.02\,\frac{kg}{s} \qquad T_\mathrm{solar} = 50°C \qquad \lambda = 0.1\,\frac{W}{m\,K}$$

$$\dot{m}_\mathrm{D} = 0.1\,\frac{kg}{s} \qquad T_\mathrm{D}^\mathrm{S} = 40°C \qquad L = 200m$$

$$\dot{Q}_\mathrm{D} = 4000W \qquad T_\mathrm{amb} = 15°C \qquad c_\mathrm{p} = \frac{4180J}{kg\,K}$$

Thermal networks

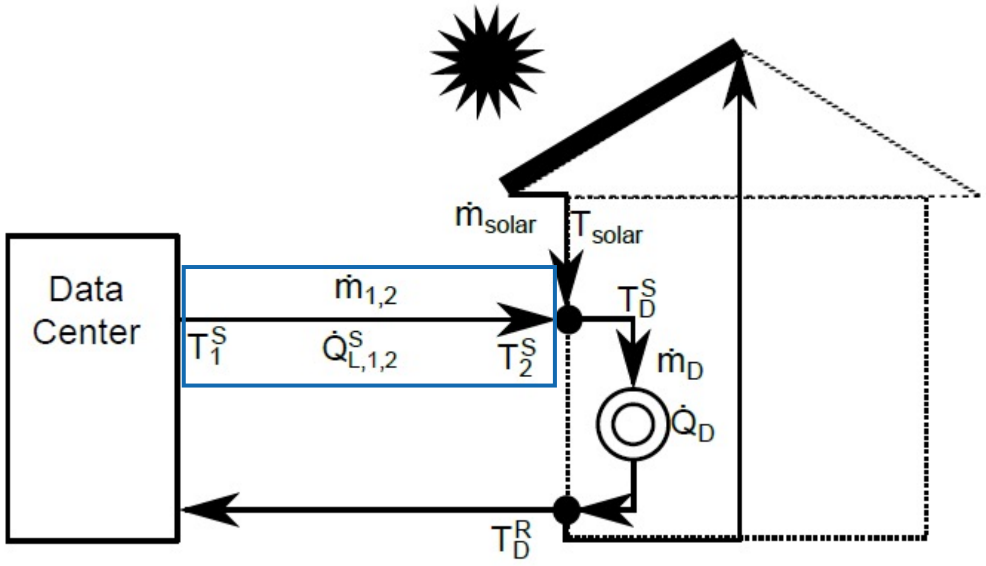
Calculate the required supply temperature T_1^S of waste heat at node 1. The pipe between nodes 1 and 2 has a length L and a heat transfer coefficient.

- The pipe equation between the data center and the building is applied:

$$T_2^S = \left(T_1^S - T_{\text{amb}}\right) e^{-\frac{\lambda L}{c_\text{p} \dot{m}_{1,2}}} + T_{amb}$$

- Solving for T_1^S:

$$T_1^S = (T_2^S - T_{amb}) e^{\frac{\lambda L}{c_\text{p} \dot{m}_{1,2}}} = 38.6\ °C$$



$\dot{m}_\text{solar} = 0.02 \frac{kg}{s}$ $\quad T_\text{solar} = 50°C$ $\quad \lambda = 0.1 \frac{W}{m\ K}$

$\dot{m}_\text{D} = 0.1 \frac{kg}{s}$ $\quad T_\text{D}^S = 40°C$ $\quad L = 200m$

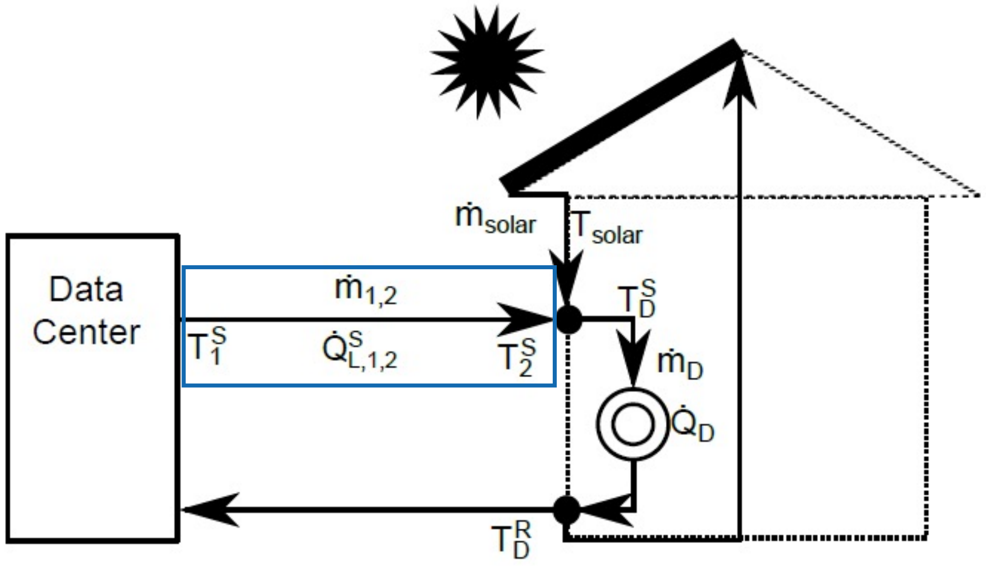
$\dot{Q}_\text{D} = 4000W$ $\quad T_\text{amb} = 15°C$ $\quad c_\text{p} = \frac{4180J}{kg\ K}$

Thermal networks

Calculate the losses $\dot{Q}^S_{\mathrm{L},1,2}$ between the data center and the building.

- Energy balance:

$$Q^S_{L,1,2} = c_\mathrm{p}\dot{m}_{1,2}\left(\mathrm{T}^S_1 - \mathrm{T}^S_2\right) = 460.9\ \mathrm{W}$$



$$\dot{m}_{\mathrm{solar}} = 0.02\,\frac{kg}{s} \qquad T_{\mathrm{solar}} = 50°C \qquad \lambda = 0.1\,\frac{W}{m\ K}$$

$$\dot{m}_{\mathrm{D}} = 0.1\,\frac{kg}{s} \qquad T^S_{\mathrm{D}} = 40°C \qquad L = 200m$$

$$\dot{Q}_{\mathrm{D}} = 4000W \qquad T_{\mathrm{amb}} = 15°C \qquad c_{\mathrm{p}} = \frac{4180J}{kg\ K}$$

Thermal networks

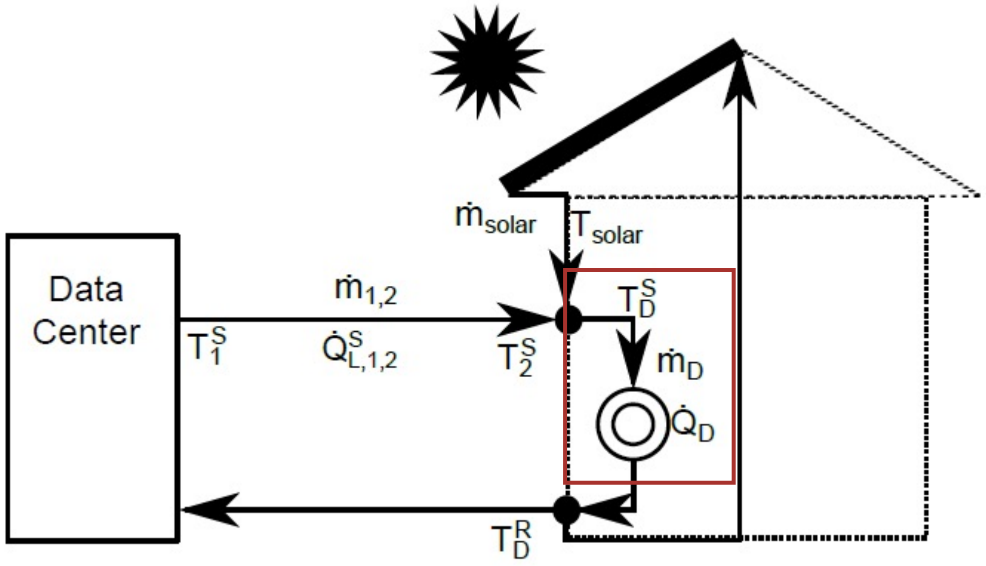
Calculate the return temperature T_D^R which results from supplying a demand of $\dot{Q}_\mathrm{D}$.

- Energy balance:

$$\dot{Q}_\mathrm{D} = c_\mathrm{p}\dot{m}_\mathrm{D}(T_\mathrm{D}^\mathrm{R} - T_\mathrm{D}^\mathrm{S})$$

- Solving for T_D^R:

$$T_\mathrm{D}^\mathrm{R} = T_\mathrm{D}^\mathrm{S} - \frac{\dot{Q}_\mathrm{D}}{c_\mathrm{p}\dot{m}_\mathrm{D}} = 30.4\,°C$$



$\dot{m}_\mathrm{solar} = 0.02\,\frac{kg}{s}$ $\quad T_\mathrm{solar} = 50°C$ $\quad \lambda = 0.1\,\frac{W}{m\,K}$

$\dot{m}_\mathrm{D} = 0.1\,\frac{kg}{s}$ $\quad T_\mathrm{D}^\mathrm{S} = 40°C$ $\quad L = 200m$

$\dot{Q}_\mathrm{D} = 4000W$ $\quad T_\mathrm{amb} = 15°C$ $\quad c_\mathrm{p} = \frac{4180J}{kg\,K}$