Question #1 Time Integration

Assume a point-sized body (i.e. no angular velocity) in a one-dimensional space has the following forces applied to it:

$$F(t) = m \cdot g \cdot sin(t)$$

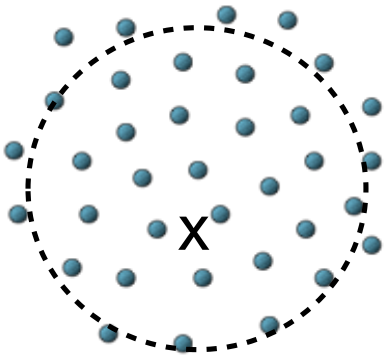
where F(t) is the sum of al forces exerted on the object at time t, m is the mass of the object and g is a constant. Let x(t) be the position of the object at time t and v(t) be the velocity of the object at time t.

1. Derive the explicit Euler time integration scheme as shown in class to estimate x and v.
2. Derive the implicit Euler time integration scheme as shown in class to estimate x and v.
3. Derive a closed form (or analytical) solution for x(t) and v(t)
4. Does the explicit Euler integration scheme give an identical solution as the closed form solution? Justify your answer? (**HINT: you can justify this question in two ways. The first way is to compute the difference between the result from the explicit Euler and the closed for solution. However this is tedious. The second and better way is to show it using Taylor series. Full marks will only be awarded for the second solution.**
5. Given the assumptions above (i.e. point sized object and one dimensional space) can you provide a necessary and sufficient condition for F such that the explicit Euler scheme yields the same results as the analytical solution? (**HINT: use the Taylor series**).

Question #2 Fluid Simulation and SPH

$$m\frac{\partial u}{\partial t} = -m \cdot u \cdot \nabla u + m \cdot g - V \cdot \nabla \mathrm{p} + \mathrm{V} \cdot \mu \cdot \nabla \cdot \nabla \mathrm{u}$$

a) The equation above is one of the Navier-Stokes (NS) equations as shown in class. This equation is derived from Newton's second law of motion: F = ma. Identify all terms of this equation and explain concisely what they are and what they represent. Furthermore, identify each term in the NS equation above with one of the terms in Newton's second law of motion (i.e. what terms correspond to which forces and what terms correspond to the mass and acceleration).



b) Smooth Particle Hydrodynamics (SPH) methods use particles to approximate a certain volume of fluid and use a kernel to spatially compute certain quantities. For instance, computing the temperature at a location x at a given time t is done using the following formula:

$$T(x) = \sum_j \frac{m_j}{\rho_j} T_J W(x, x_j, h)$$

Explain all the terms in this formula. Be precise but no verbose.

Question #3 Time Integration returns (Déjà vu)

Assume a point-sized object (i.e. no angular velocity) in a one-dimensional space has the following forces applied to it:

$$F(t) = m \cdot (-12t + 6)$$

where F(t) is the sum of all forces exerted on the object at time t and m is the mass of the object. Let x(t) be the position of the object at time t and v(t) be the velocity of the object at time t.

a) Derive the explicit Euler time integration scheme as shown in class to estimate x and v.
b) Derive the implicit Euler time integration scheme as shown in class to estimate x and v.
c) Derive a closed form (or analytical) solution for x(t) and v(t)
d) What maximum order scheme would you need in this case in order to have the most accurate result? Justify your answer?