Morphological processing

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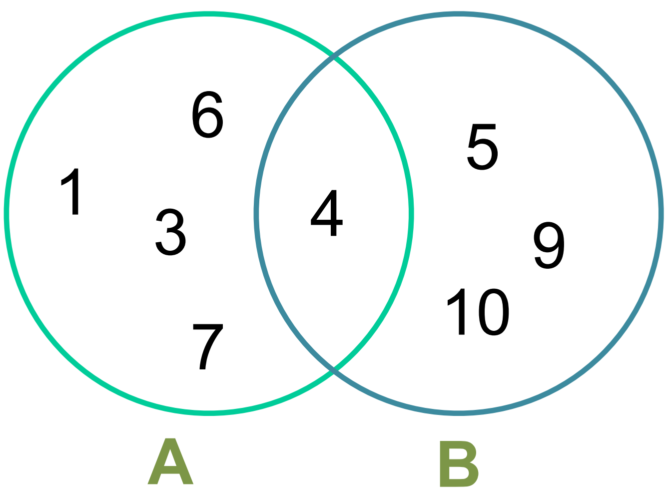
Morphology

The term ***morphology*** refers to the description of the properties of shape and structure of any objects.

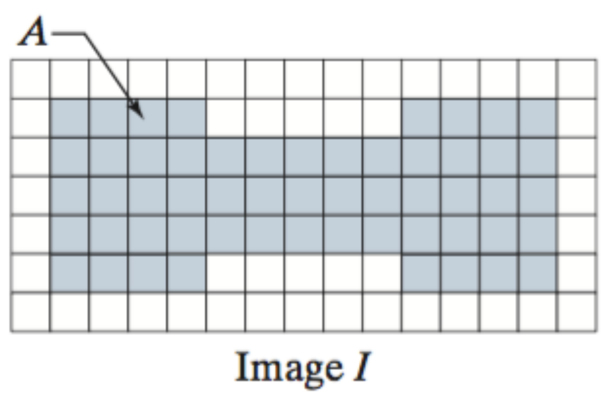
Operations of mathematical morphology were originally defined as operations on sets, but it soon became clear that they are also useful in the processing tasks of the set of points in the two-dimensional space, aka. images.

Sets



A = {1,3,4,6,7} B = {4,5,9,10}

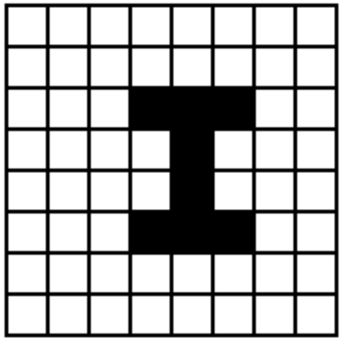
Defined in the Z^2 space



foreground pixels in an image

Morphological Processing

- A binary image can be considered as a set by considering “black” pixels (with image value “1”) as elements in the set and “white” pixels (with value “0”) as outside the set.

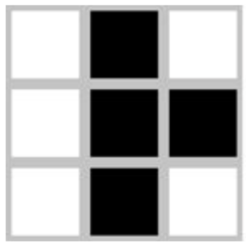


- Morphological filters are essentially set operations

Set Operations

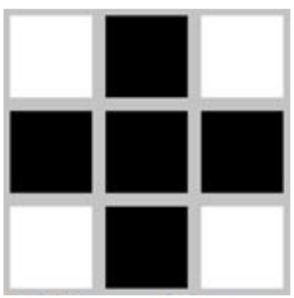
***Basic operations**: union, intersection, complement, difference*

Input Images



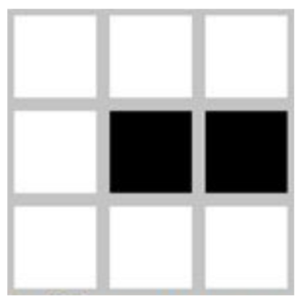
A

B



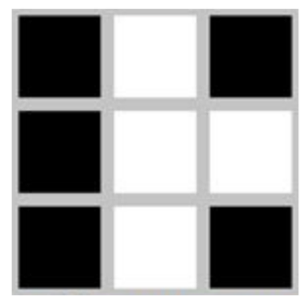
$C = A \cup B$

union (OR)



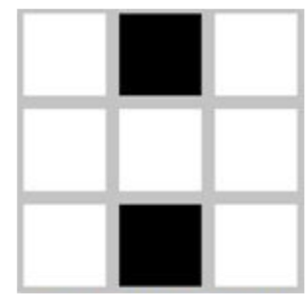
$C = A \cap B$

intersection (AND)



$C = A^c$

complement



$C = A \backslash B$

difference

Set Operations

Reflection $\hat{B}=\{w|w=-b, \text{ for } b \in B\}$

rotating by 180 degrees about the origin

Translation $(B)_z=\{c|c=b+z, \text{ for } b \in B\}$

moving the structuring elements by (z1,z2)

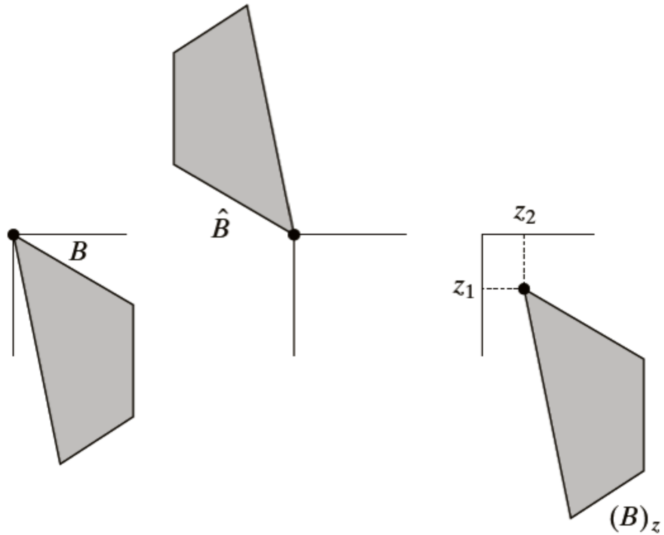
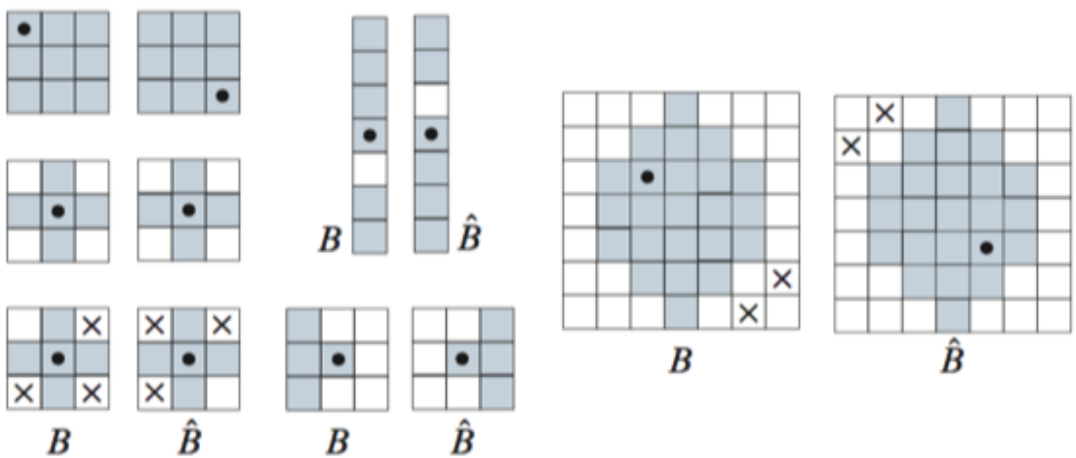
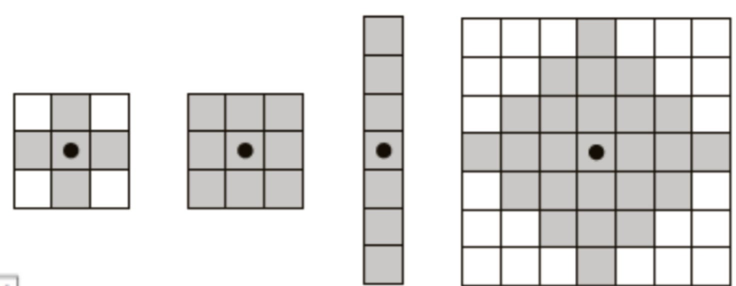


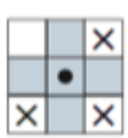
FIGURE 9.2 Structuring elements and their reflections about the origin (the ×'s are don't care elements, and the dots denote the origin). Reflection is rotation by 180° of an SE about its origin.



Structural elements

- The input data for the mathematical morphology are the two images: the image/**object** and a **structuring element (SE)**.
- Typically, a structuring element is much smaller than the **object** to be processed. The shape of the structured element can be arbitrary, as long as it can be represented as a binary image of a given size.





B

Structuring element

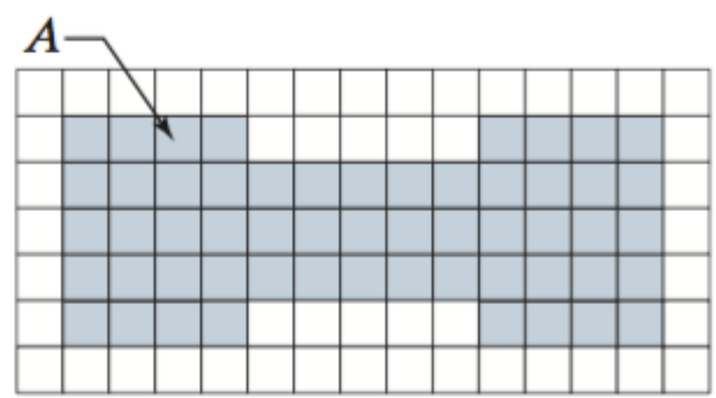


Image *I*

Object

Dilation

Dilation of set **A** by a structure element **B** is represented by $A \oplus B$

empty set

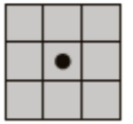
$$\boxed{A \oplus B} = \left\{ z \,\middle|\, (\hat{B})_z \cap A \neq \varnothing \right\}$$

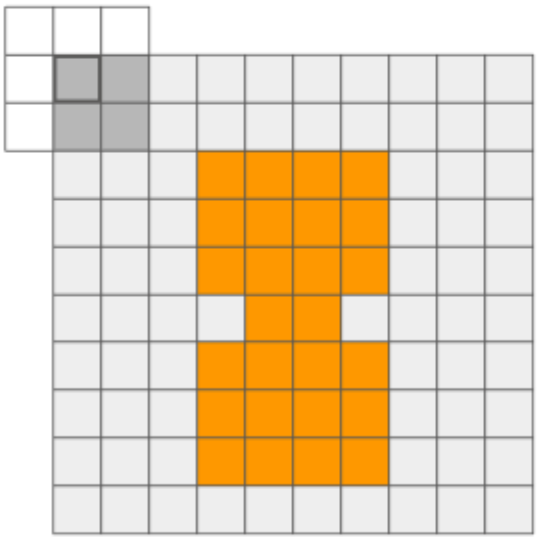
$C = A \oplus B$ is composed of all pixels that when $\hat{B}$ shifts its origin to these pixels, at least one pixel in $\hat{B}$ is in C

At the location of $A \subset A \oplus B$, the origin of $\hat{B}$ takes the value 1

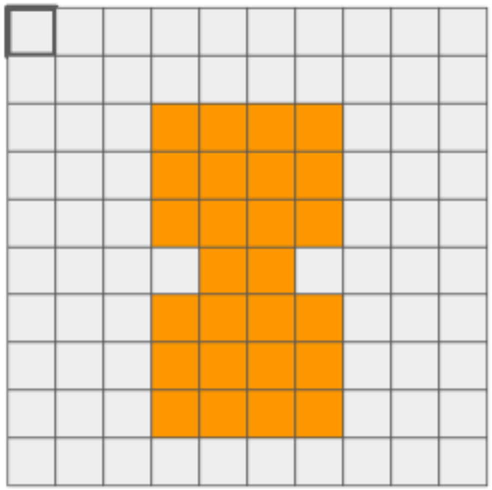
Dilation

SE





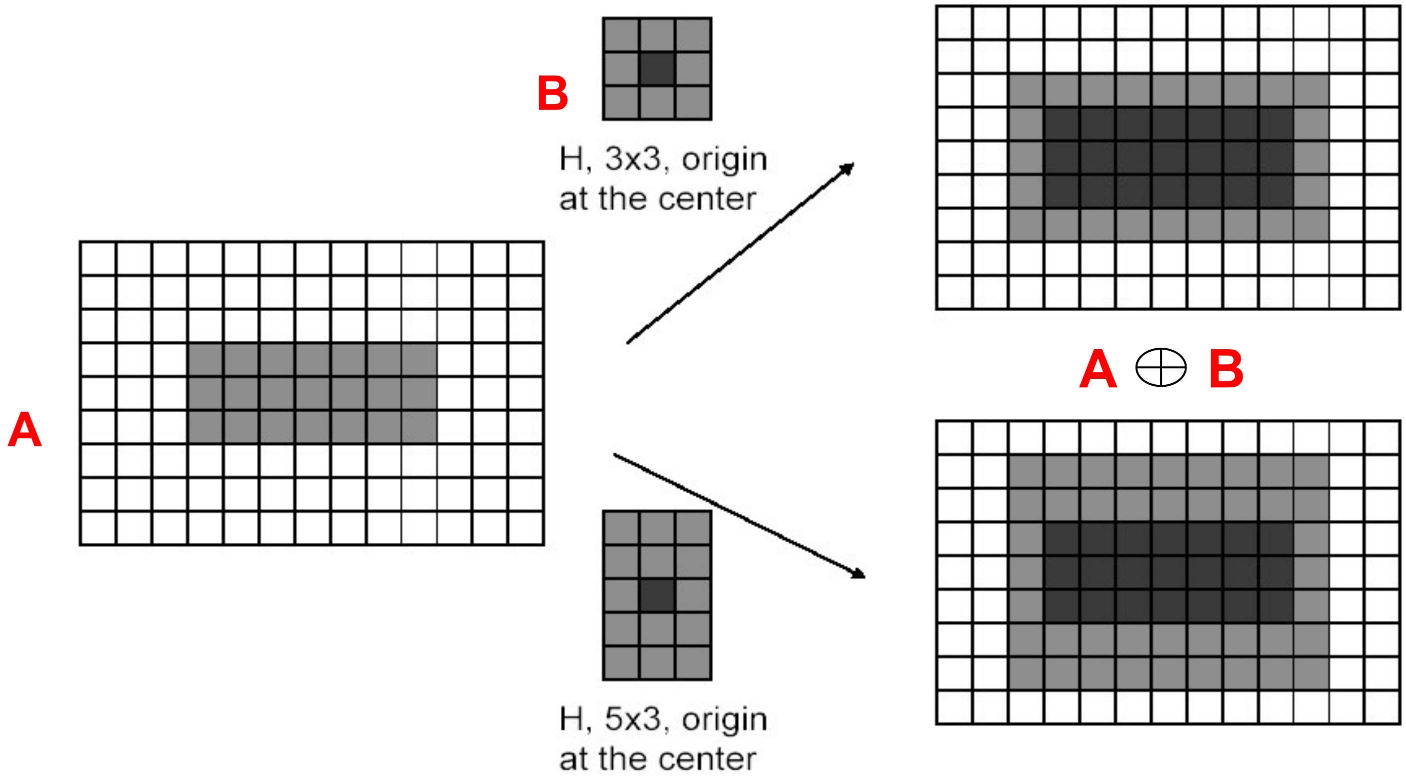
Object



Result

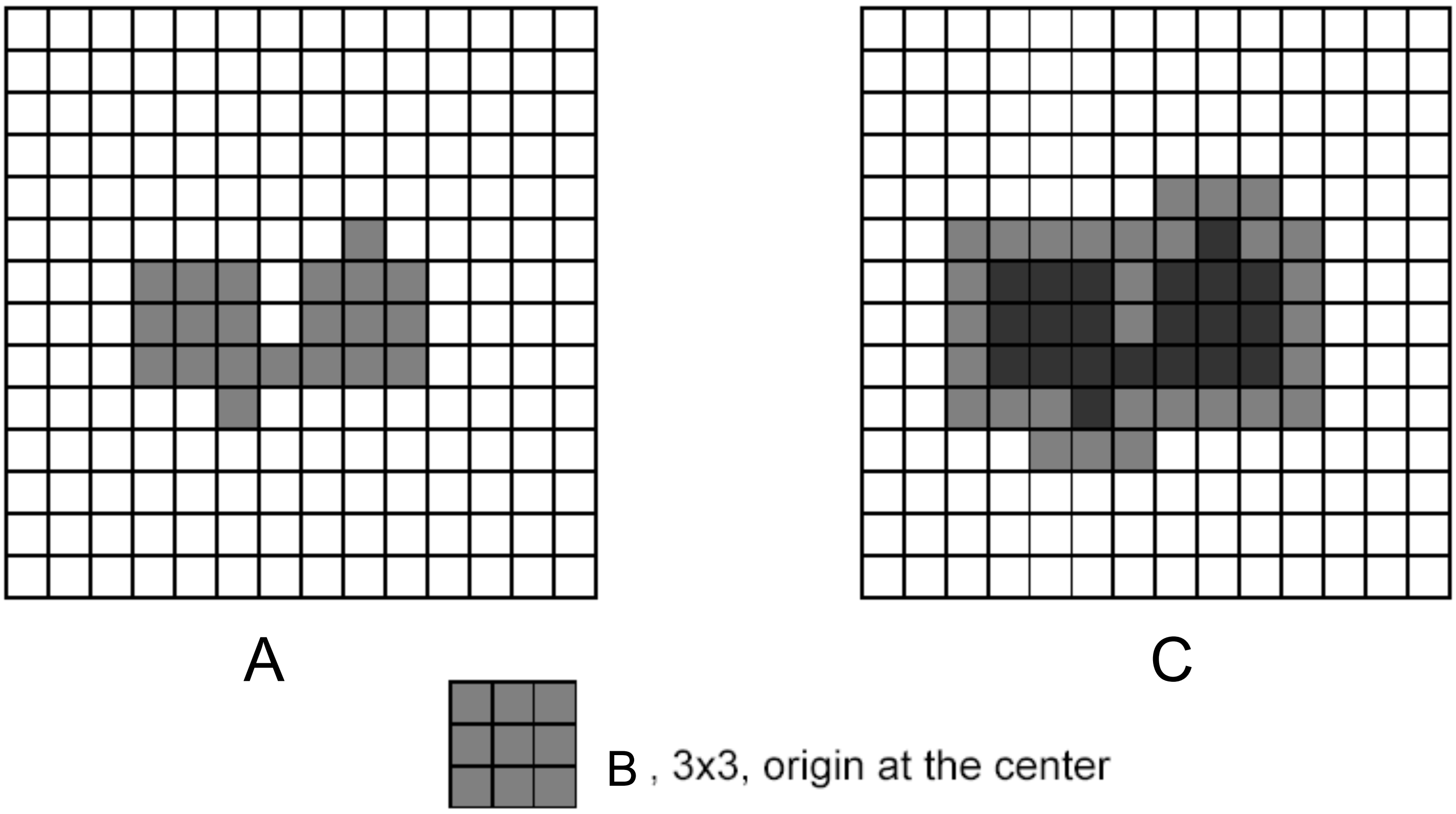
https://penny-xu.github.io/blog/mathematical-morphology

Dilation Example

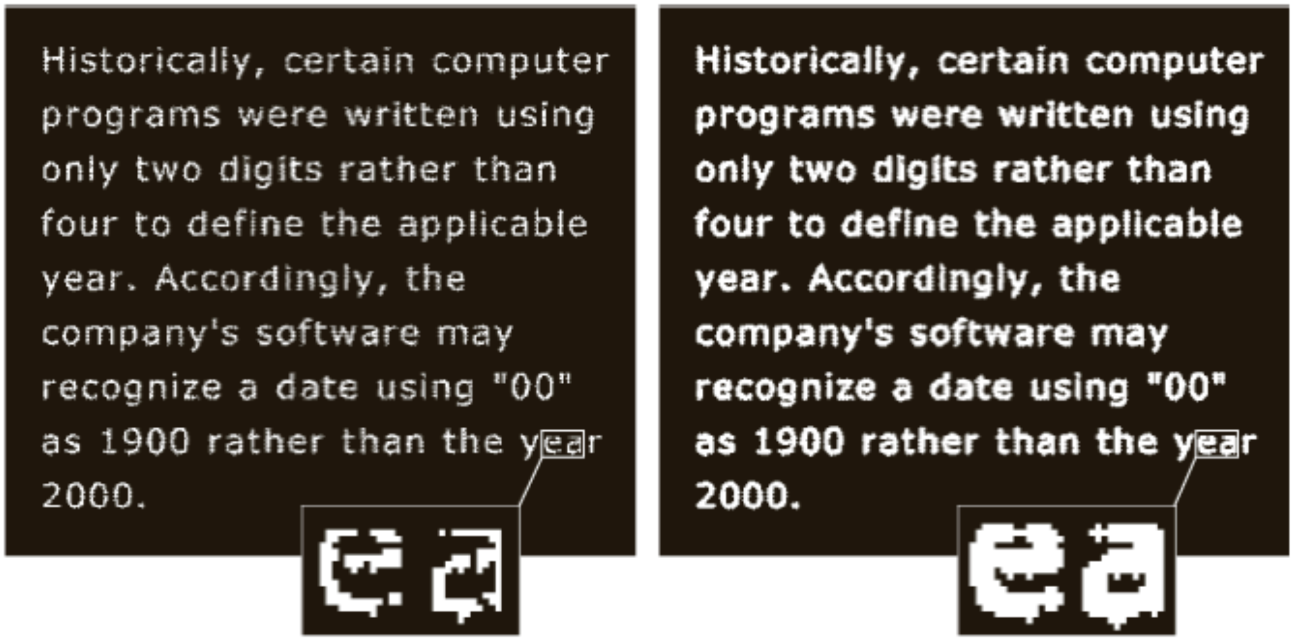


The dilation of A by B is the set of all displacements such that they overlap by at least one element.

Another Example of Dilation



Example of Dilation



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Erosion

For ***erosion***, the structural element passes through all pixels of the image. The pixel corresponds to the reference of the structure element is turned on if the entire structure element falls with foreground area (pixels).

$$A \ominus B = \{z | (B)_z \subseteq A\}$$
$$A \ominus B = \{z | (B)_z \cap A^c = \emptyset\}$$

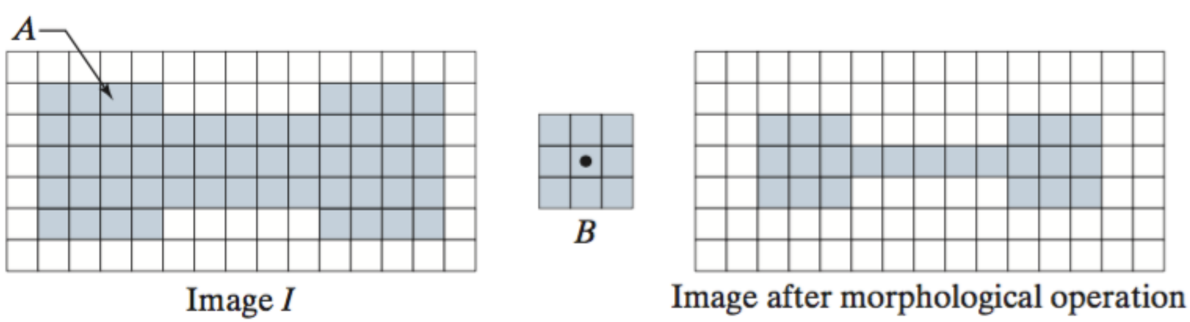
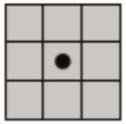
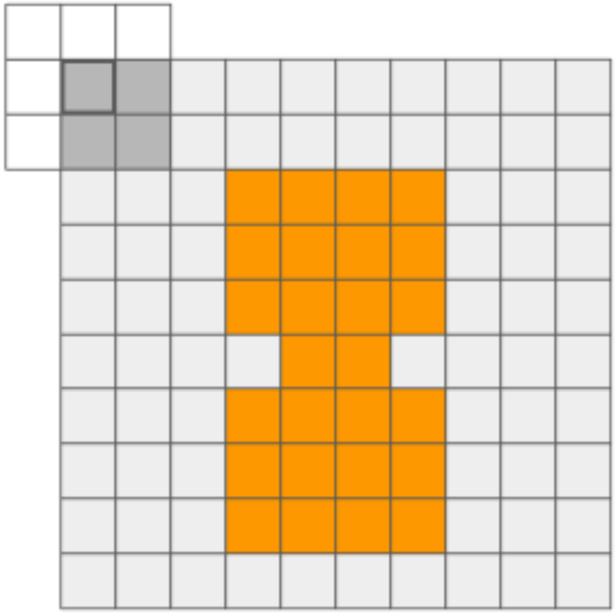


FIGURE 9.3 (a) A binary image containing one object (set), A. (b) A structuring element, B. (c) Image resulting from a morphological operation (see text).

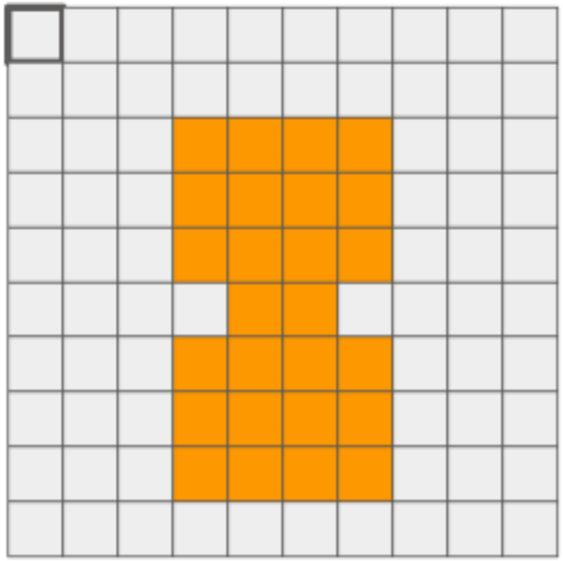
Erosion

SE





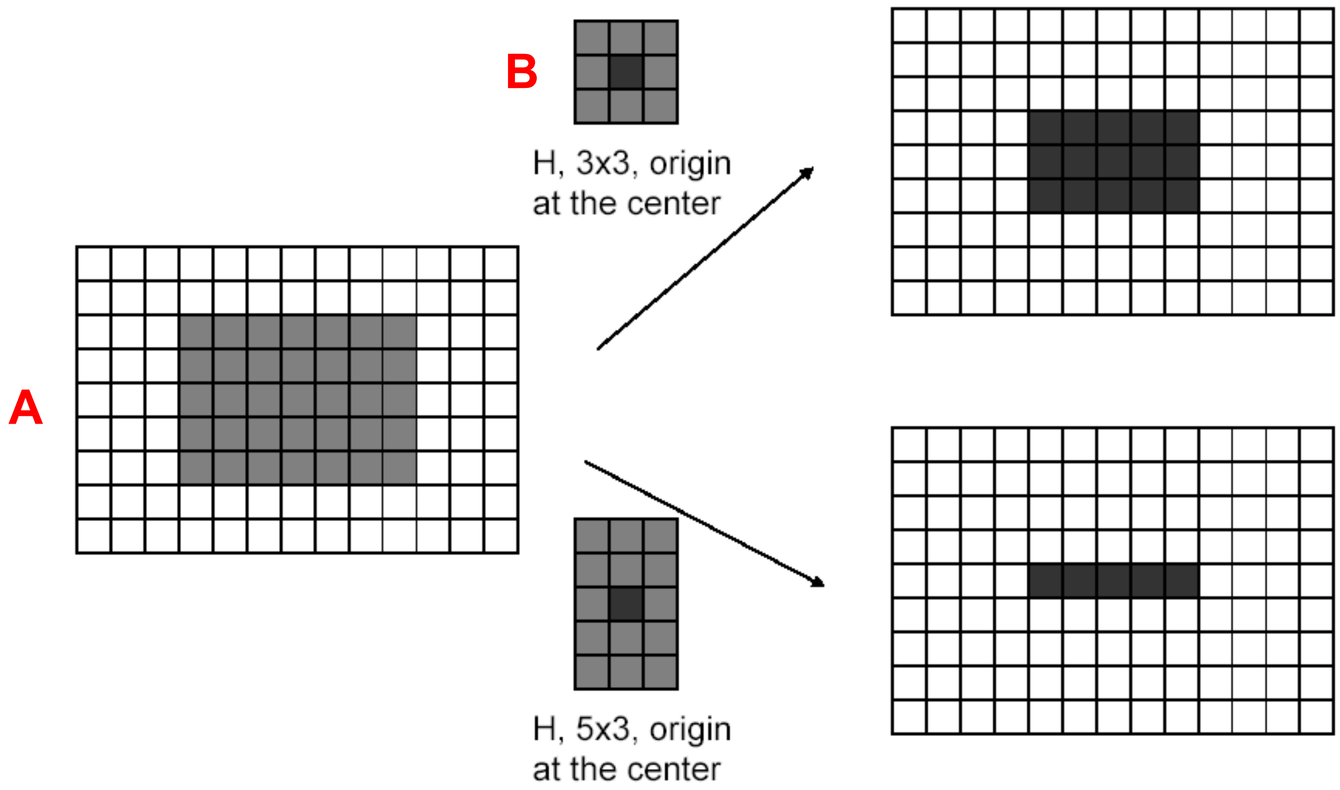
Object



Result

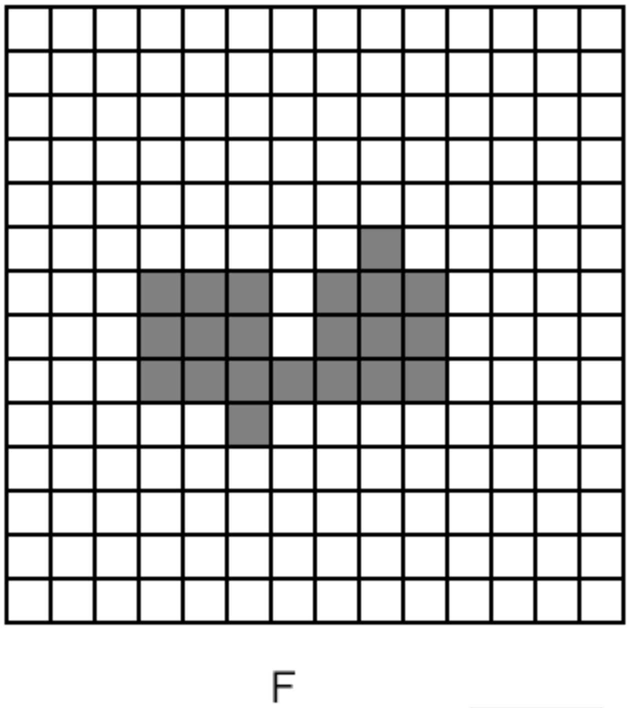
https://penny-xu.github.io/blog/mathematical-morphology

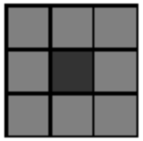
Erosion (Opposite of Dilation)



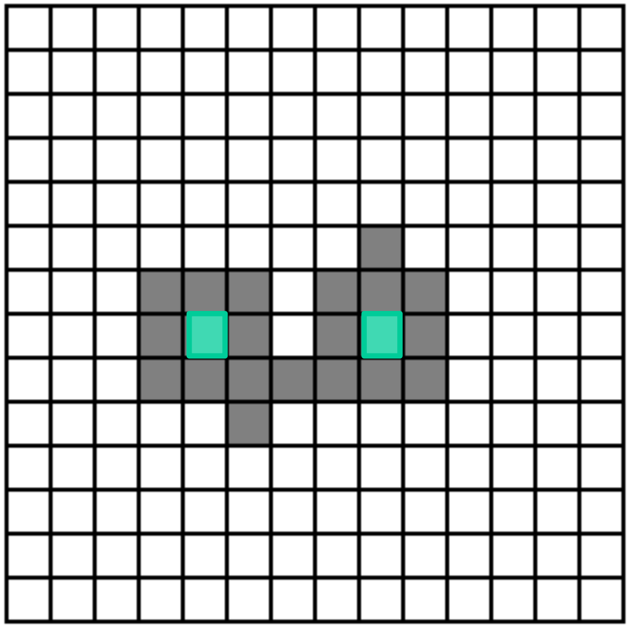
Passing B through A: at each location of origin of B, if B is completely contained in A, mark that location as a member of the new set (erosion set).

Another Erosion Example

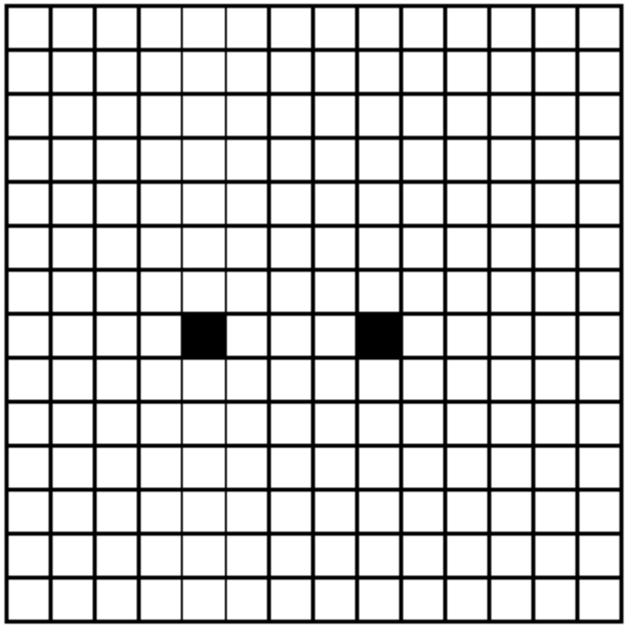




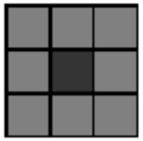
Another Erosion Example



F



G



H, 3x3, origin at the center

Erosion Example

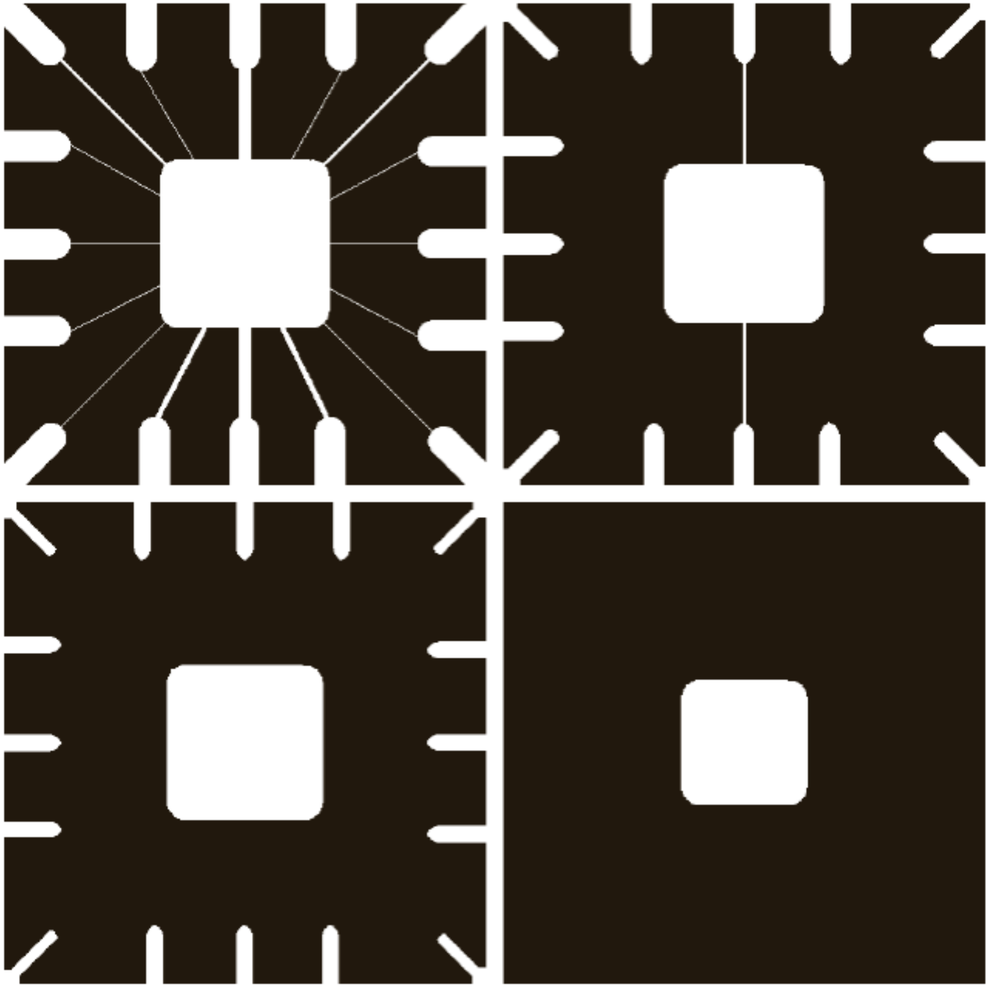


FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

Closing and Opening

- Closing

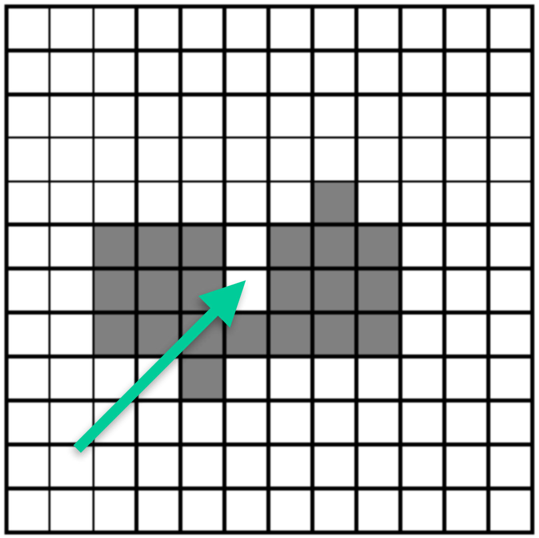
$$A \bullet B = (A \oplus B) \ominus B$$

  - Smooth the contour of an image
  - Fill small gaps and holes
- Opening

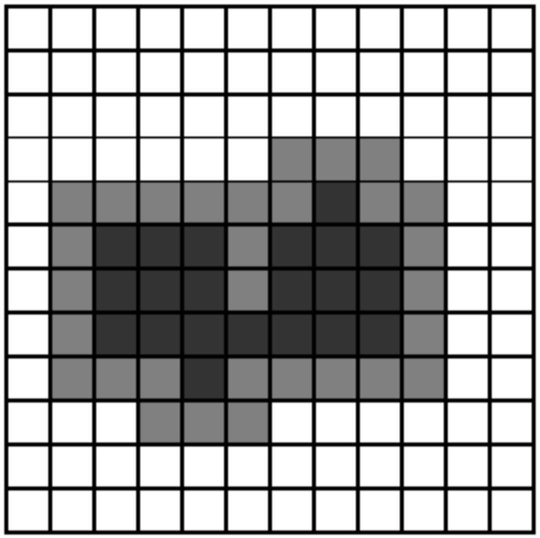
$$A \circ B = (A \ominus B) \oplus B$$

  - Smooth the contour of an image
  - Eliminate false touching, thin ridges and branches

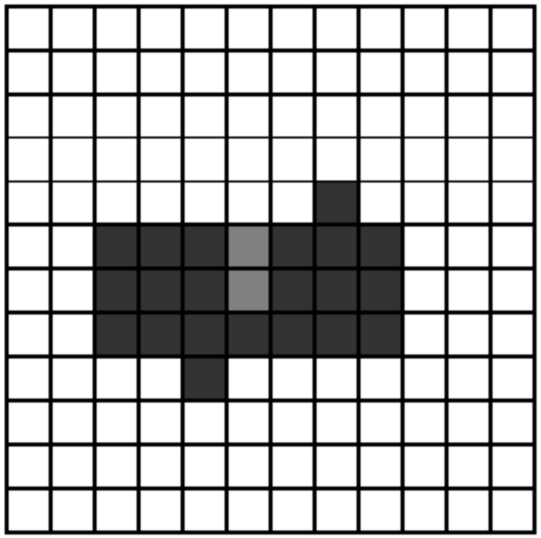
Closing Example



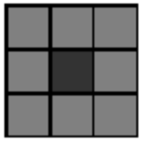
F



$F \oplus H$

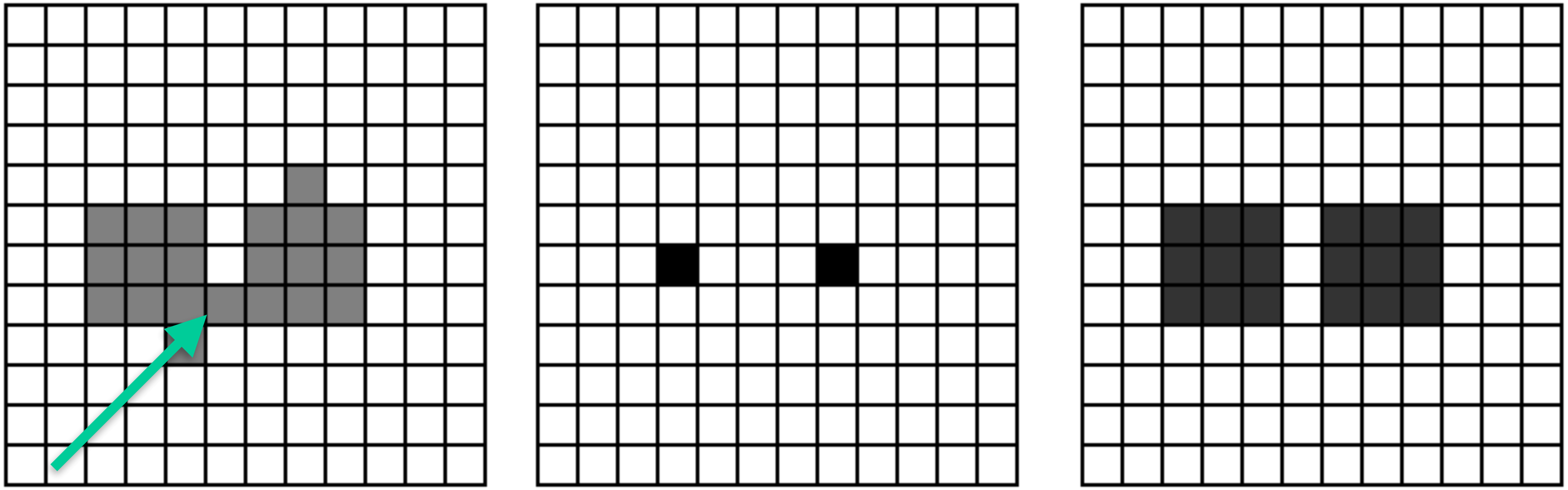


$(F \oplus H)\ominus H$

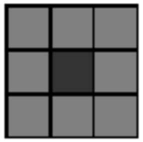


H, 3x3, origin at the center

Opening Example



F $\quad\quad F \Theta H \quad\quad (F \Theta H) \oplus H$



H, 3x3, origin at the center

Opening & Closing Example

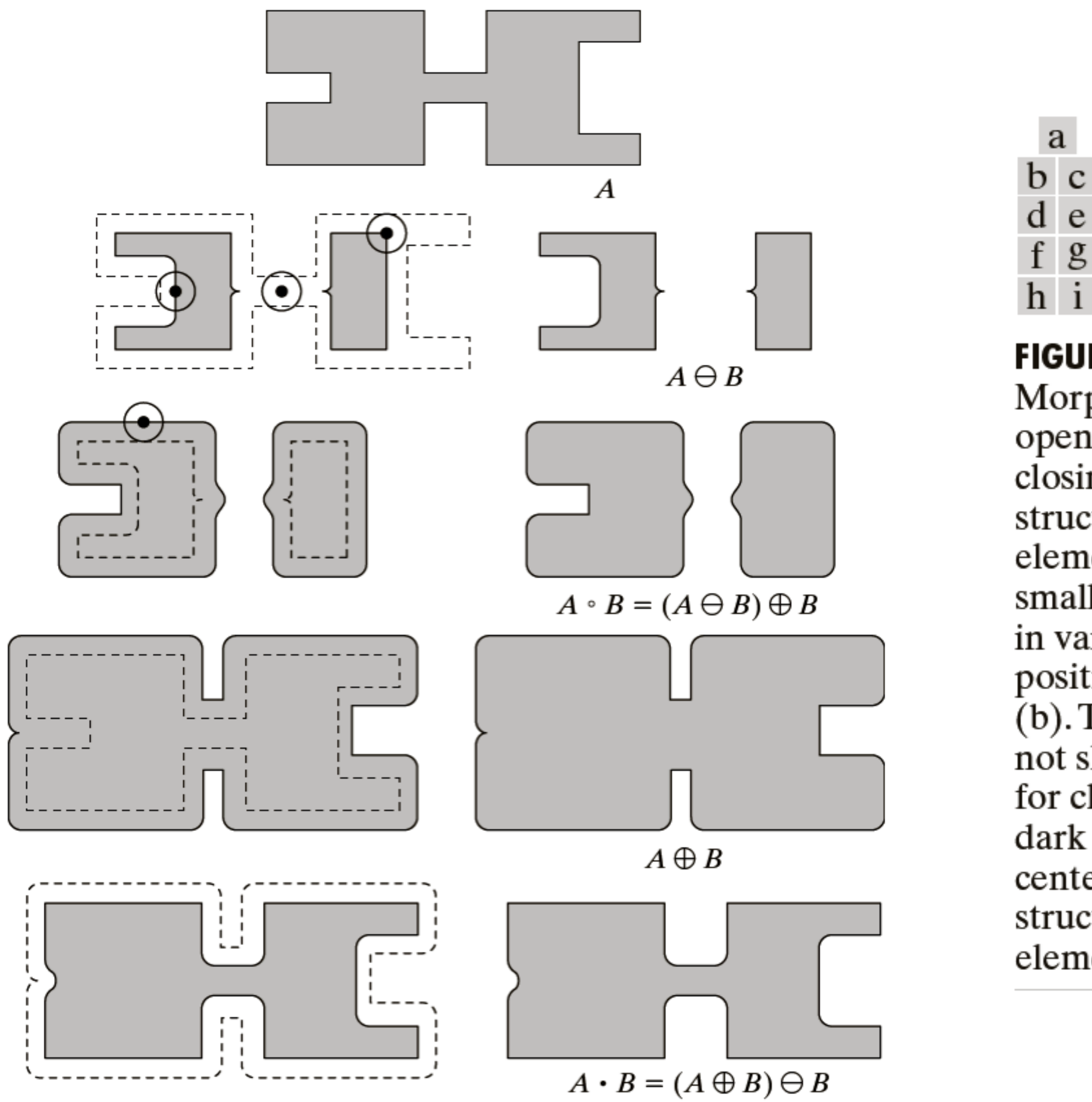
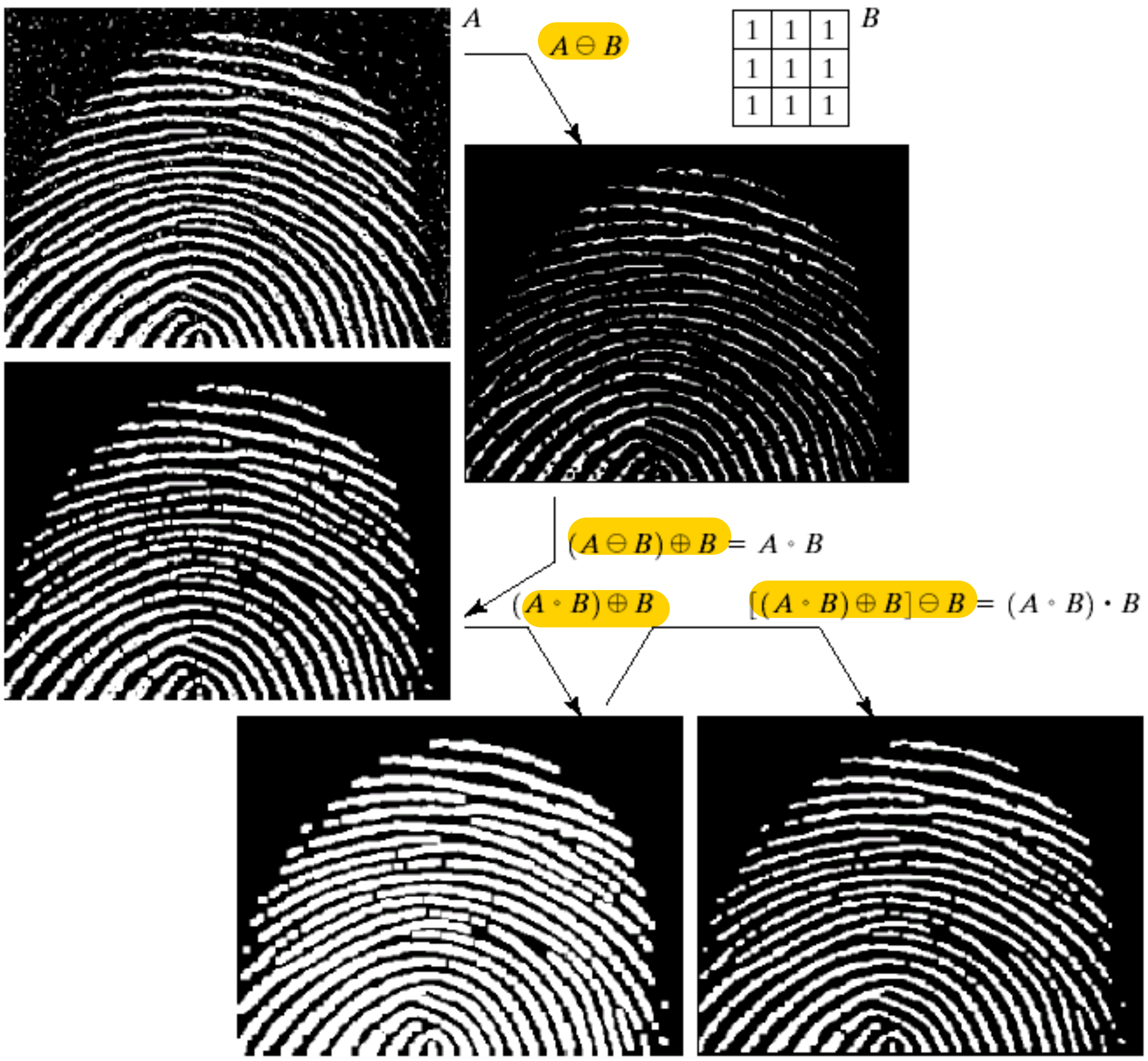


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Morphological Filtering Example: Opening followed by Closing

open



a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(c) Eroded image.
(d) Opening of A.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

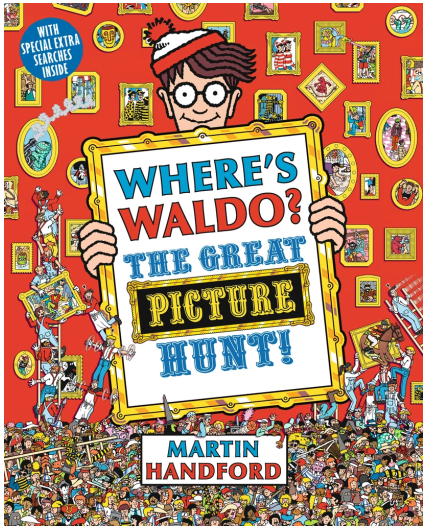
closing

Take a break!

Hit-or-Miss Transform

- ***Hit-or-Miss transform (HMT)*** is a basic tool for shape detection
- HMT uses two structuring elements: $\boldsymbol{B_1}$ for detecting shapes in the foreground, and $\boldsymbol{B_2}$ for detecting shapes in the background
- The word "miss" in the HMT arises from the fact that $\boldsymbol{B_2}$ finding a match in $\boldsymbol{A^c}$ is the same as $\boldsymbol{B_2}$ missing a match in $\boldsymbol{A}$.

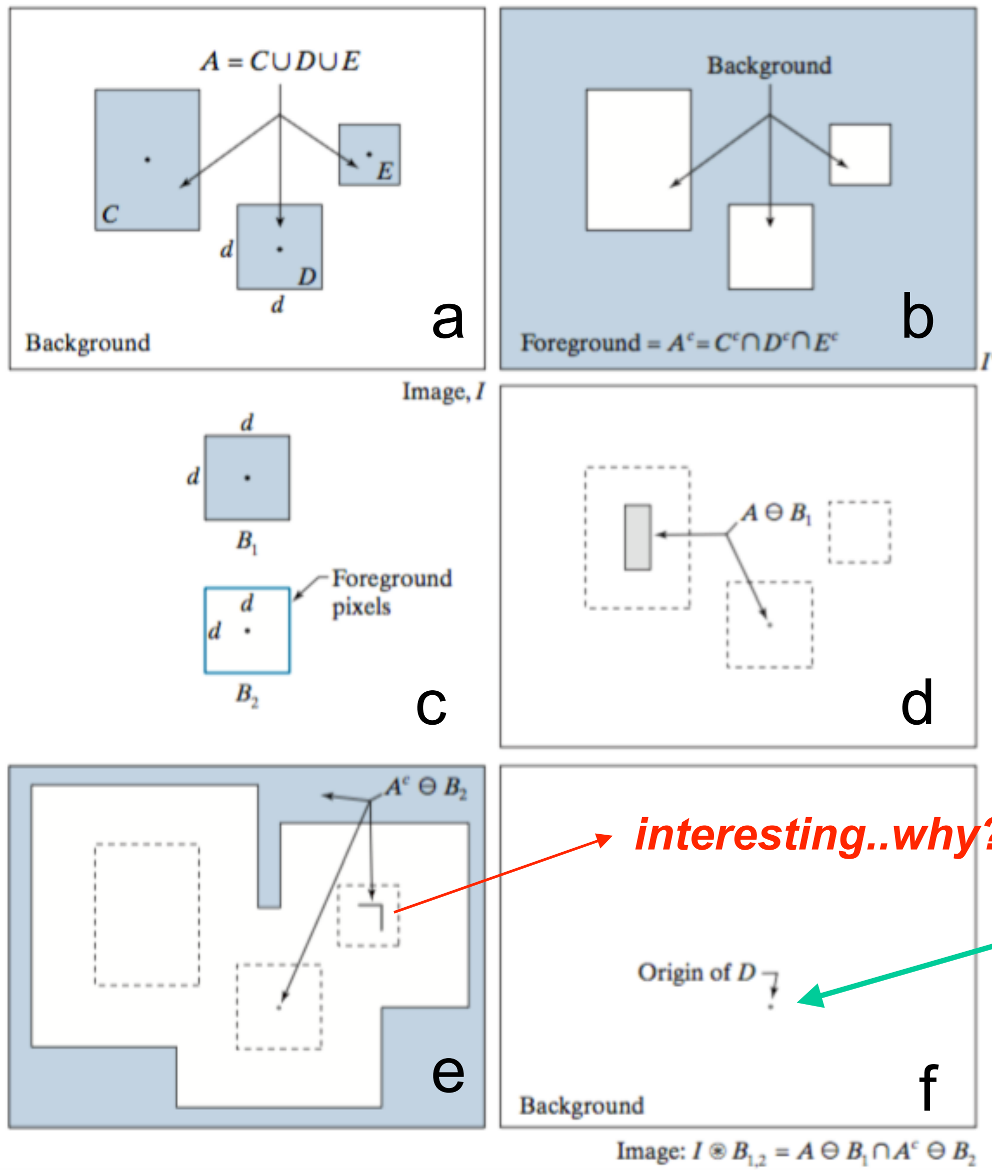
$$
\begin{aligned}
I \circledast B_{1,2} &= \left\{ z \,\middle|\, (B_1)_z \subseteq A \text{ and } (B_2)_z \subseteq A^c \right\} \\
&= \left(\underline{A \ominus B_1} \right) \cap \left(\underline{A^c \ominus B_2} \right)
\end{aligned}
$$



Hit-or-Miss Transform

a b
c d
e f

FIGURE 9.12
(a) Image consisting of a foreground (1's) equal to the union, A, of set of objects, and a background of 0's.
(b) Image with its foreground defined as A^c.
(c) Structuring elements designed to detect object D.
(d) Erosion of A by B_1.
(e) Erosion of A^c by B_2.
(f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origin of their respective components. Each dot is a single pixel.

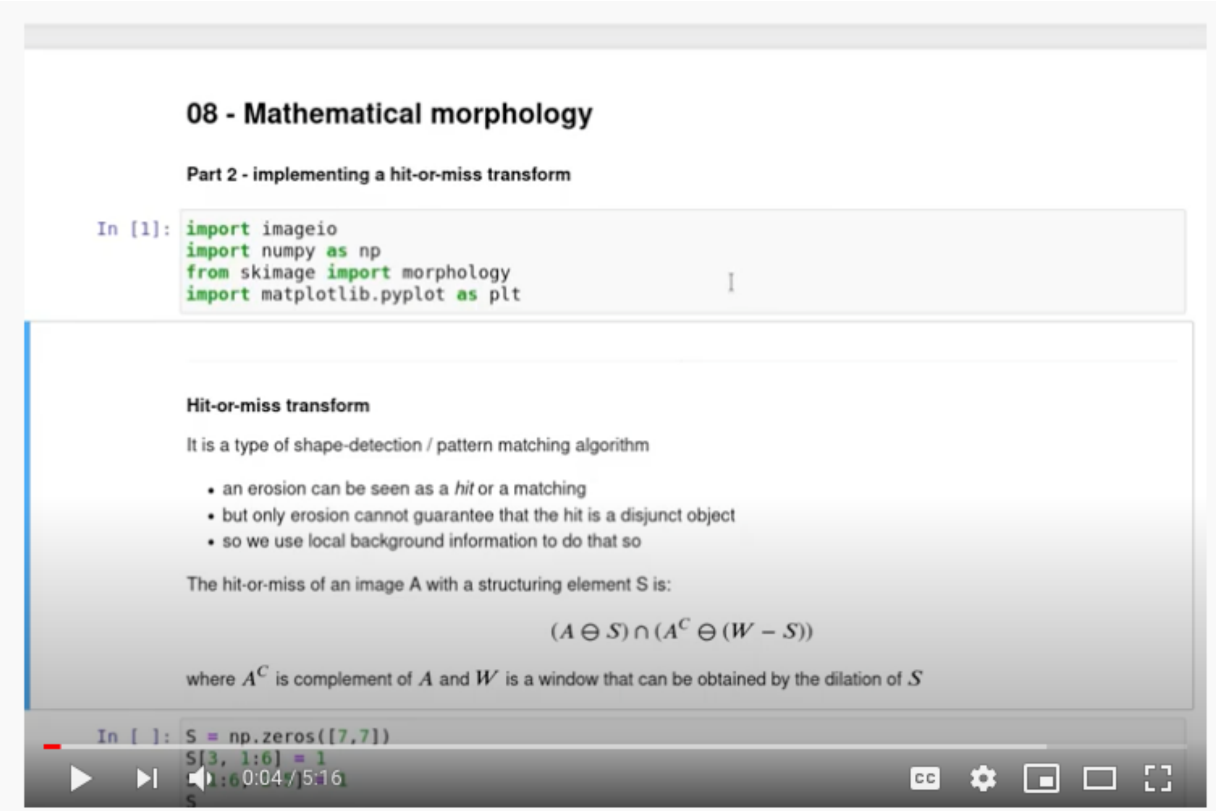


Several shapes

interesting..why?

Location of the target shape

Hit-or-Miss Transform

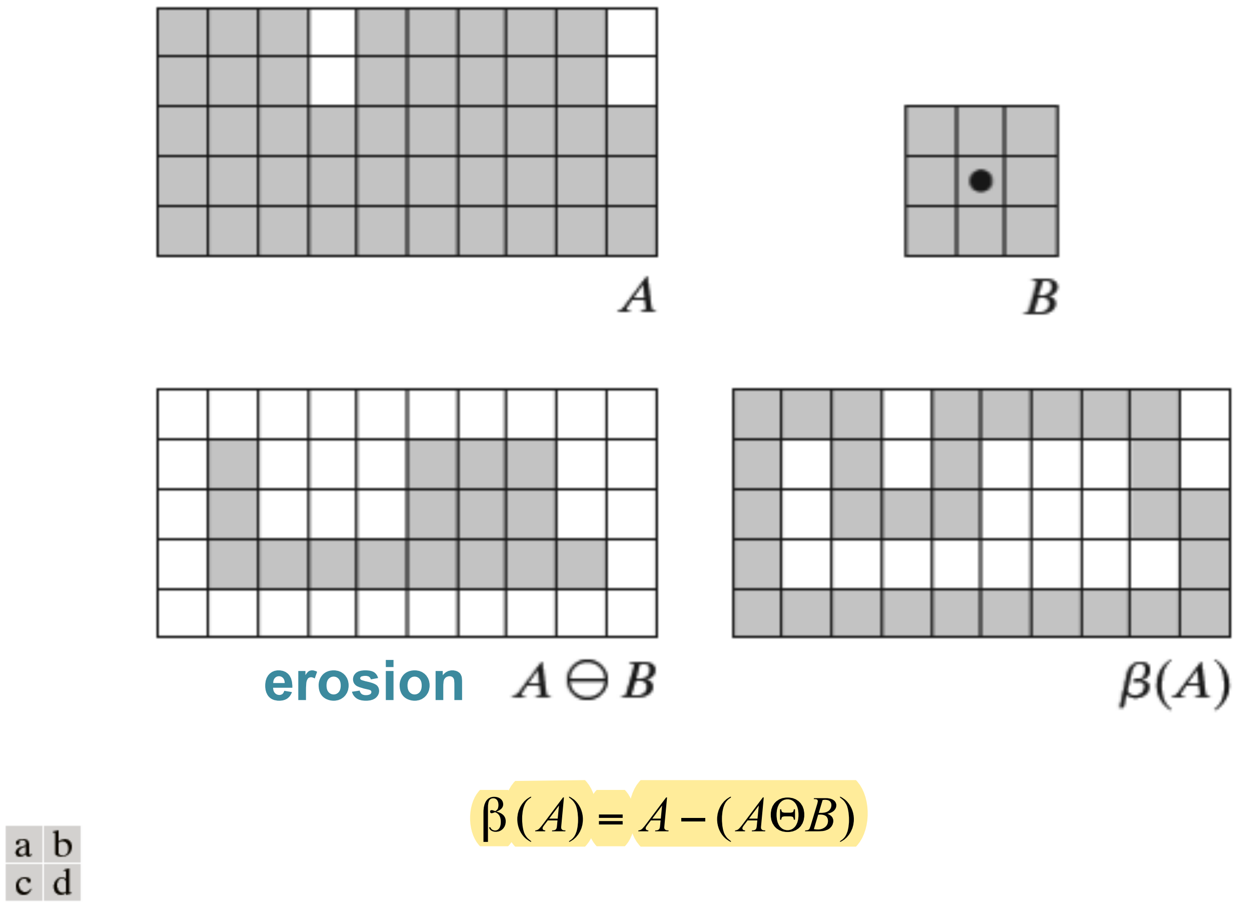


DIP 08 - Mathematical Morphology (4) - implementing the hit-or-miss in python

https://www.youtube.com/watch?v=P8NwA_p07a8

Question: How is hit-or-miss transform differ from the template matching using cross-correlation ?

Boundary Extraction



$$\beta(A) = A - (A \Theta B)$$

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

Boundary Extraction

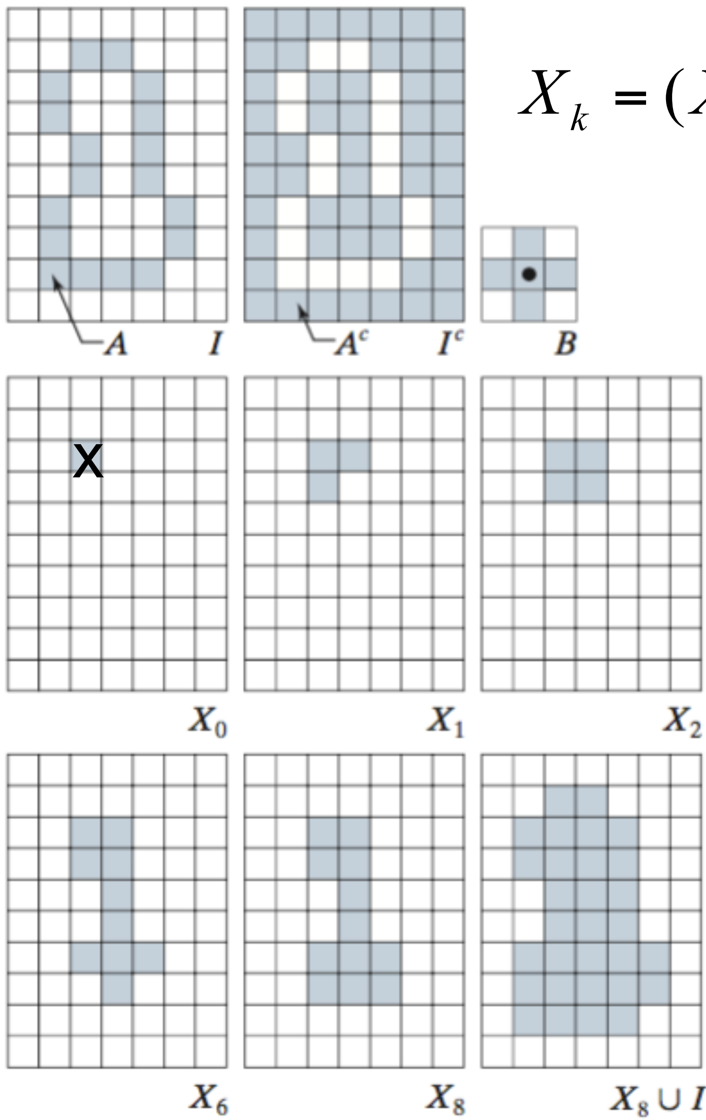
a b

FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Hole Filling

a b c
d e f
g h i

FIGURE 9.17 Hole filling. (a) Set A (shown shaded) contained in image I. (b) Complement of I. (c) Structuring element B. *Only* the foreground elements are used in computations (d) Initial point inside hole, set to 1. (e)–(h) Various steps of Eq. (9-19). (i) Final result [union of (a) and (h)].

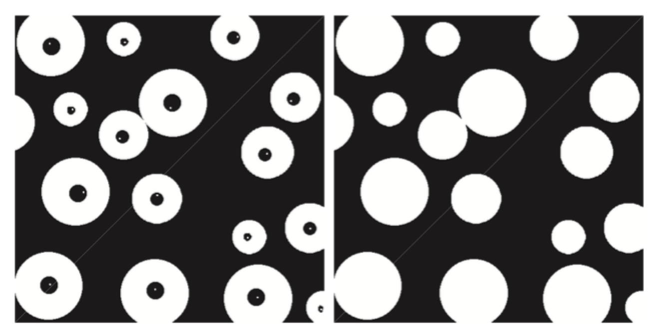


$$X_k = (X_{k-1} \oplus B) \cap A^c \qquad k = 1,2,3,\ldots$$

dilation

A ***hole*** may be defined as a background region surrounded by a connected border of foreground pixels.

Conditional dilation: *constrain the dilation within the hole by checking with the complement of A at each operation*



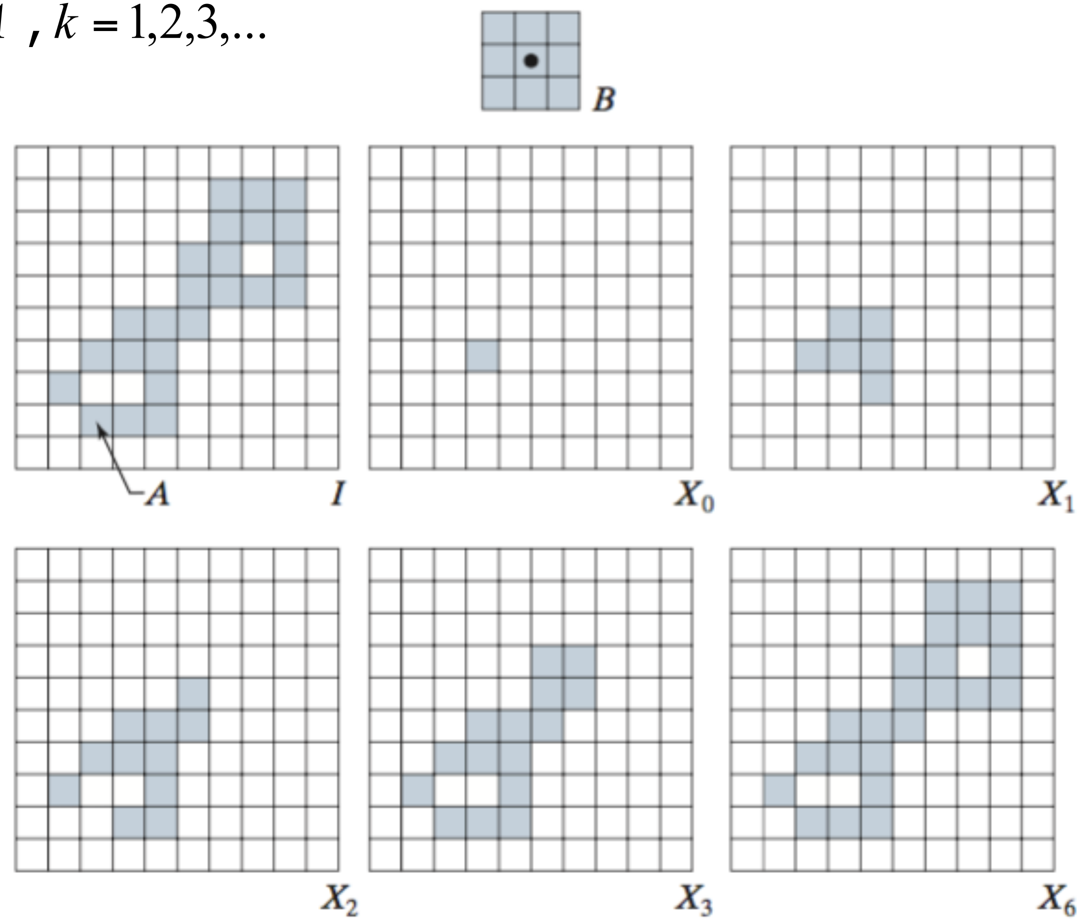
Extraction of Connected Components

Finding pixels that are connected: similar to hole-filling, but checking with A itself at each step, instead of the complement of A

$$X_k = (X_{k-1} \oplus B) \cap A \text{ , } k = 1,2,3,\ldots$$

a
b c d
e f g

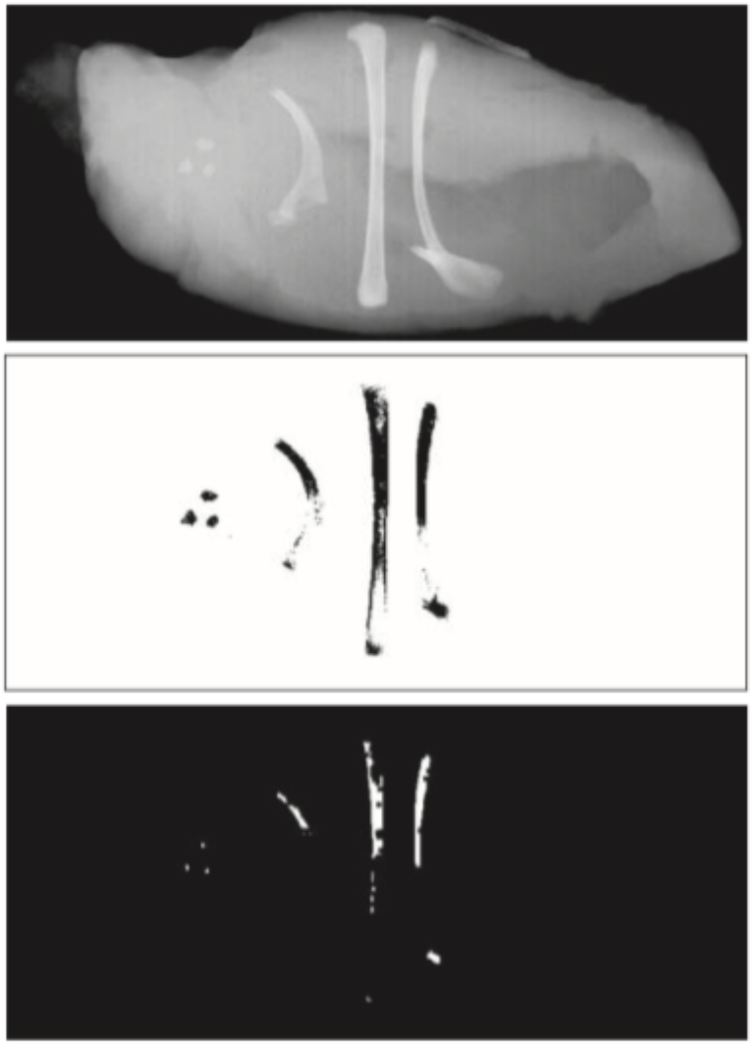
FIGURE 9.19
(a) Structuring element.
(b) Image containing a set with one connecte component.
(c) Initial array containing a 1 in the region of the connected component.
(d)–(g) Various steps in the iteration of Eq. (9-20)



Extraction of Connected Components

a
b
c d

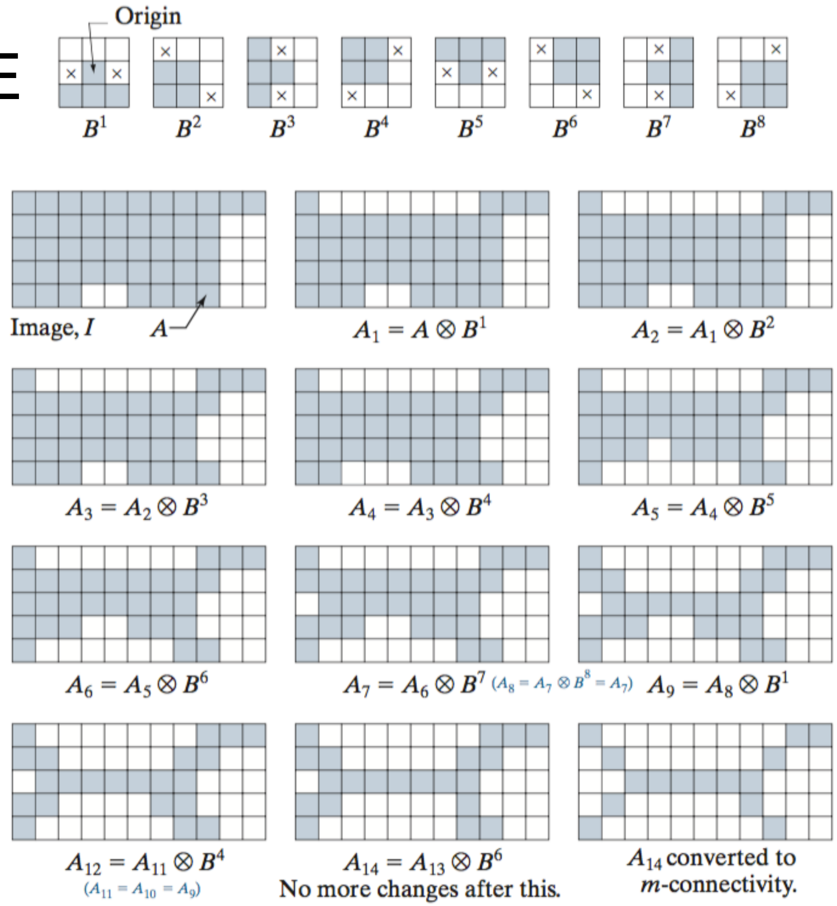
FIGURE 9.20
(a) X-ray image of a chicken filet with bone fragments.
(b) Thresholded image (shown as the negative for clarity).
(c) Image eroded with a 5×5 SE of 1's.
(d) Number of pixels in the connected components of (c).
(Image (a) courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Thinning

SE



	a	
b	c	d
e	f	g
h	i	j
k	l	m

FIGURE 9.23
(a) Structuring elements.
(b) Set A.
(c) Result of thinning A with B^1 (shaded).
(d) Result of thinning A_1 with B_2.
(e)–(i) Results of thinning with the next six SEs. (There was no change between A_7 and A_8.)
(j)–(k) Result of using the first four elements again.
(l) Result after convergence.
(m) Result converted to m-connectivity.

$$\begin{aligned} A \otimes B &= A - (A \circledast B) \\ &= A \cap (A \circledast B)^c \end{aligned}$$

$$A \otimes \{B\} = \left(\left(\ldots\left(\left(A \otimes B^1\right) \otimes B^2\right)\ldots\right) \otimes B^n\right)$$

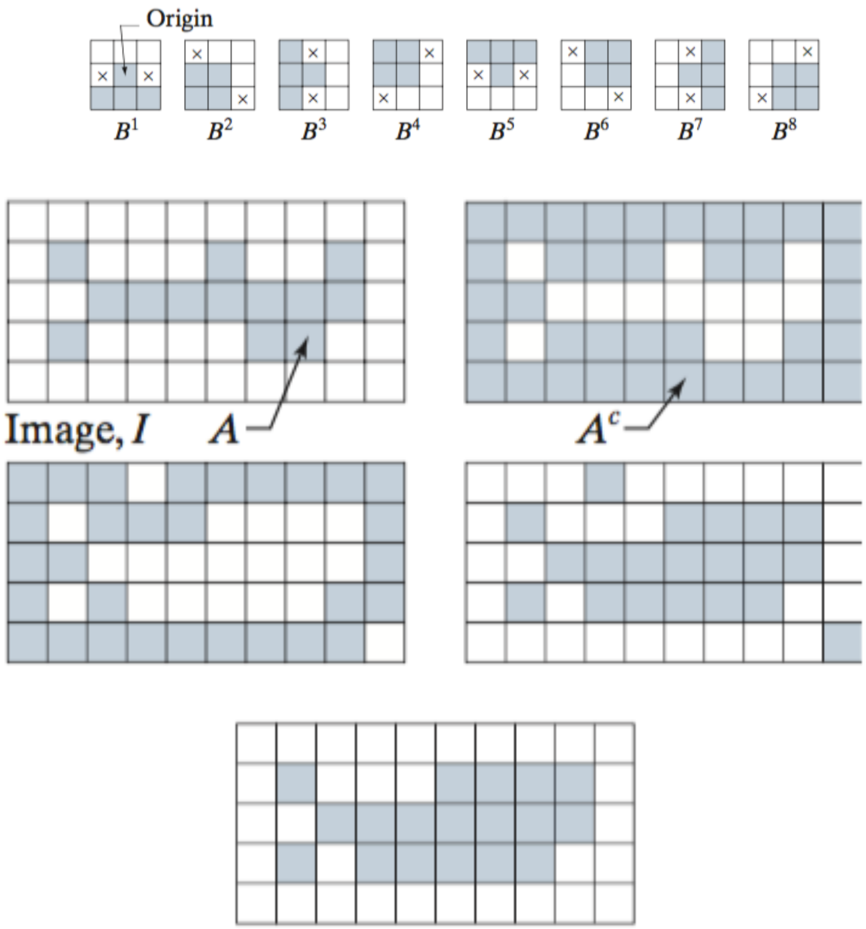
$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

Shrink objects

	a	
b	c	d
e	f	g
h	i	j
k	l	m

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m-connectivity.

Thickening



$$\boxed{A \odot B = A \cup (A \circledast B)}$$

$$A \odot \{B\} = \left(\left(\ldots\left(\left(A \odot B^1\right) \odot B^2\right)\ldots\right) \odot B^n\right)$$

a b
c d
e

FIGURE 9.24
(a) Set A.
(b) Complement of A.
(c) Result of thinning the complement.
(d) Thickened set obtained by complementing (c).
(e) Final result, with no disconnected points.

same kernels as thinning but with 1’s and 0’s switched

Differences? thinning vs. erosion ; thickening vs. dilation

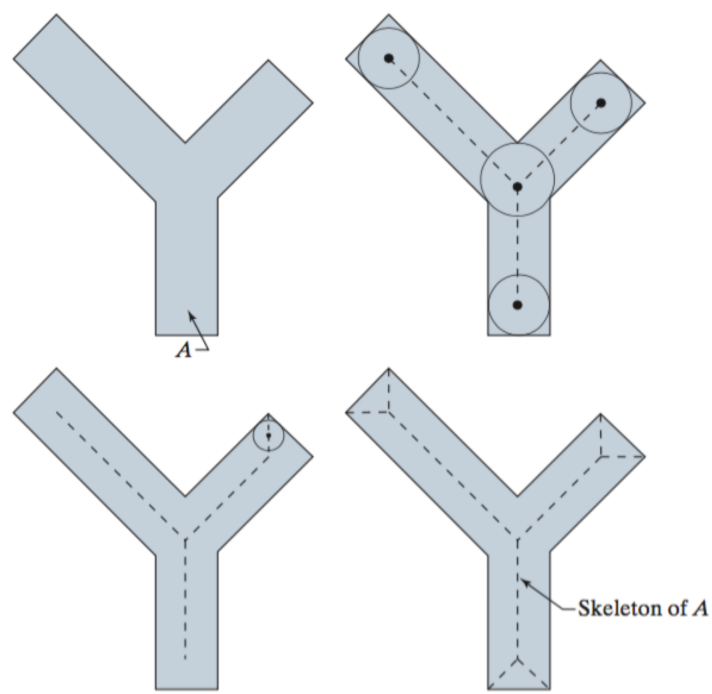
Skeletonization

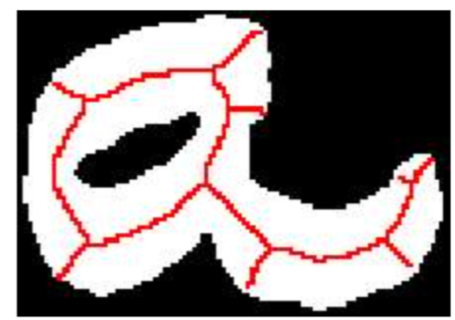
- Compact or minimal representation of objects in an image while retaining homotopy of the image (the object can be reconstructed from the skeleton)
- Is the best of all points that are equally distanced from two closest points of the object boundary
- Equivalently, the union of all maximal disk centres that are contained in the object

Analogy: staring a fire at the boundary of an object, letting it burn inward

a b
c d

FIGURE 9.25
(a) Set A. (b) Various positions of maximum disks whose centers partially define the skeleton of A. (c) Another maximum disk, whose center defines a different segment of the skeleton of A. (d) Complete skeleton (dashed).





Skeletonization

Skeletonization algorithm

$Input(A, B);\ \ A: the\ given\ set, B: structuring\ element\ for\ S(A)$

$for\ \ k = 0,1,2,....$

$do\ \ S_k(A) = (A \Theta kB) - (A \Theta kB) \circ B$; $\boxed{\Theta = erosion,\ \ \circ = opening}$

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

Reconstruction algorithm

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)\ \ ;\qquad \oplus = dilation$$

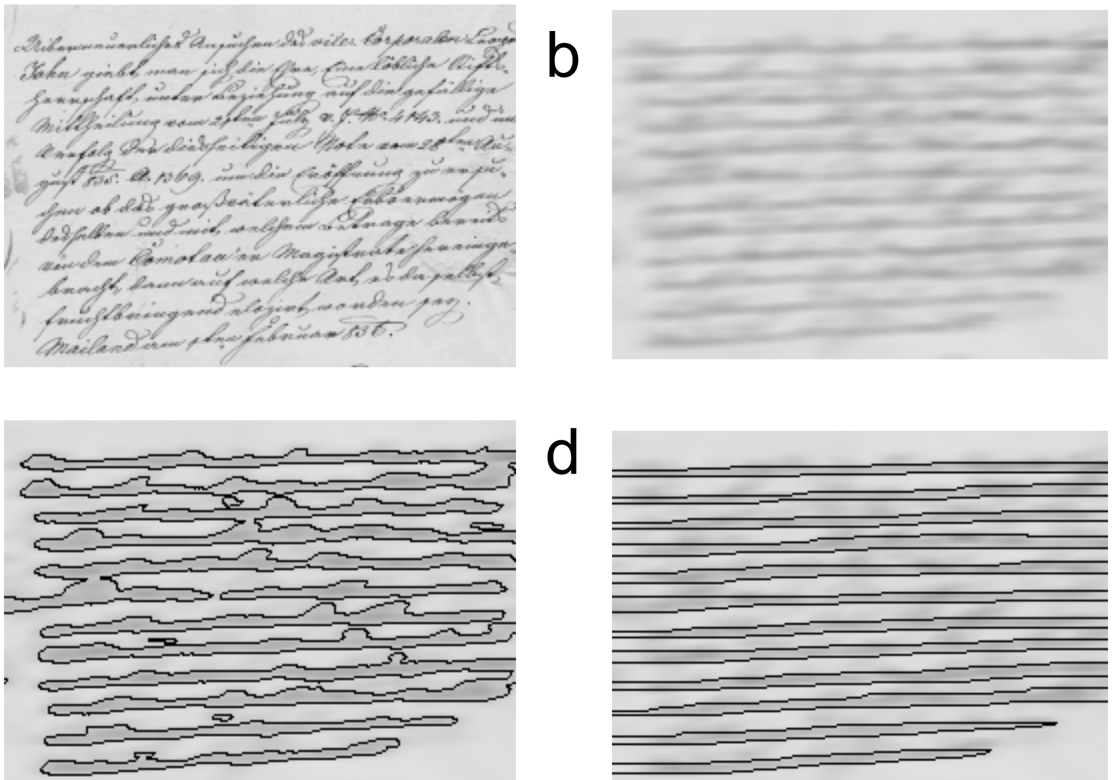
A for loop is used

$$A \circ B = (A \Theta B) \oplus B$$

k = k successive operations

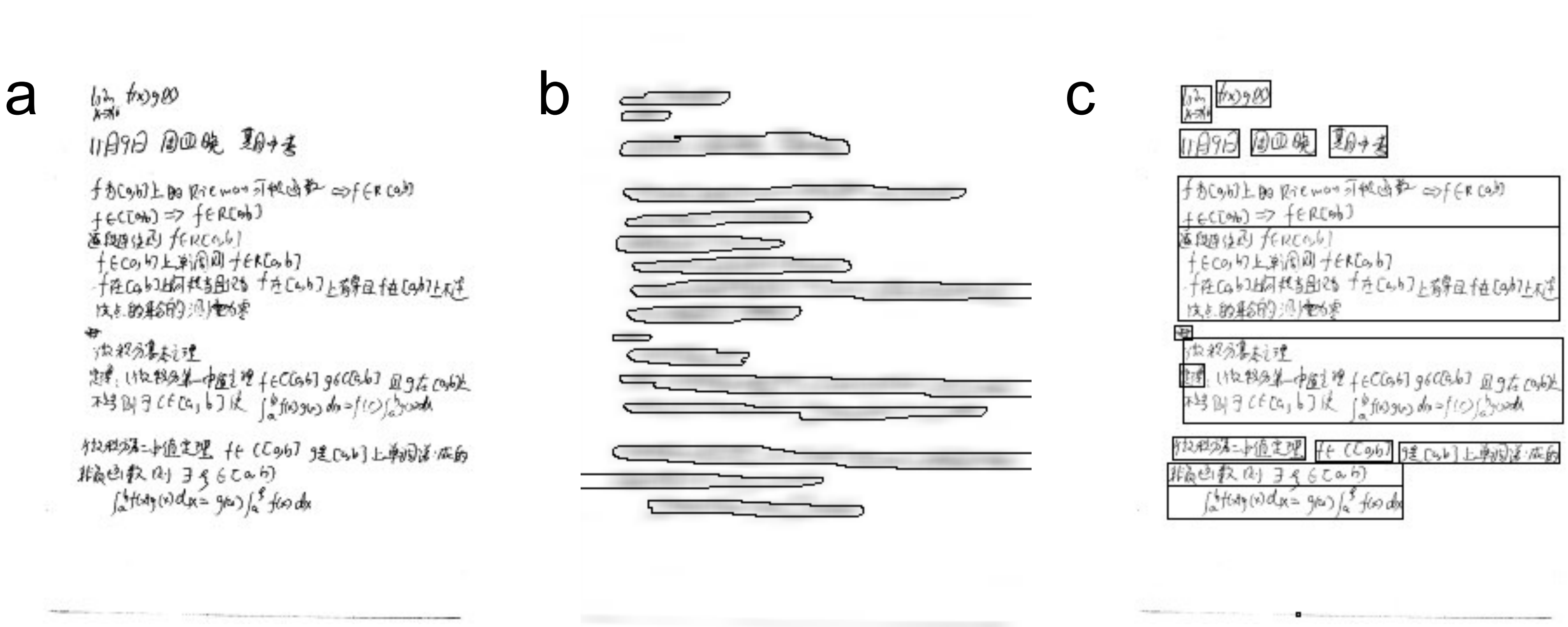
FIGURE 9.26 Implementation of Eqs. (9-28) through (9-33). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

Morphological Filtering Example: Handwritten Document Processing



Process of text line segmentation. a: original image; b: blurred image; c: segmented image; d: final segmentation result after morphological processing.

Morphological Filtering Example: Handwritten Document Processing



Process of text line segmentation. a: original image;
b: segmentation result based on morphological processing;
c: segmentation result based on connected component analysis.

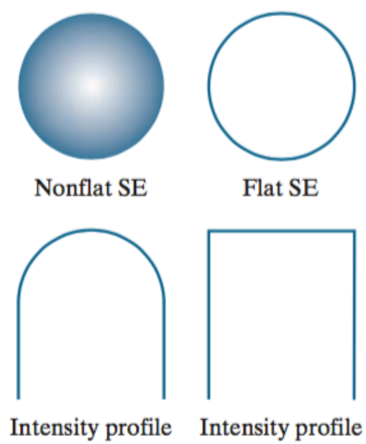
Grayscale Morphology

Structuring elements in grayscale morphology perform the same basic functions as their binary counterparts

Structuring elements in grayscale morphology belong to one of two categories: **non-flat** and **flat**

a b
c d

FIGURE 9.36 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their centers. All examples in this section are based on flat SEs.



not often used

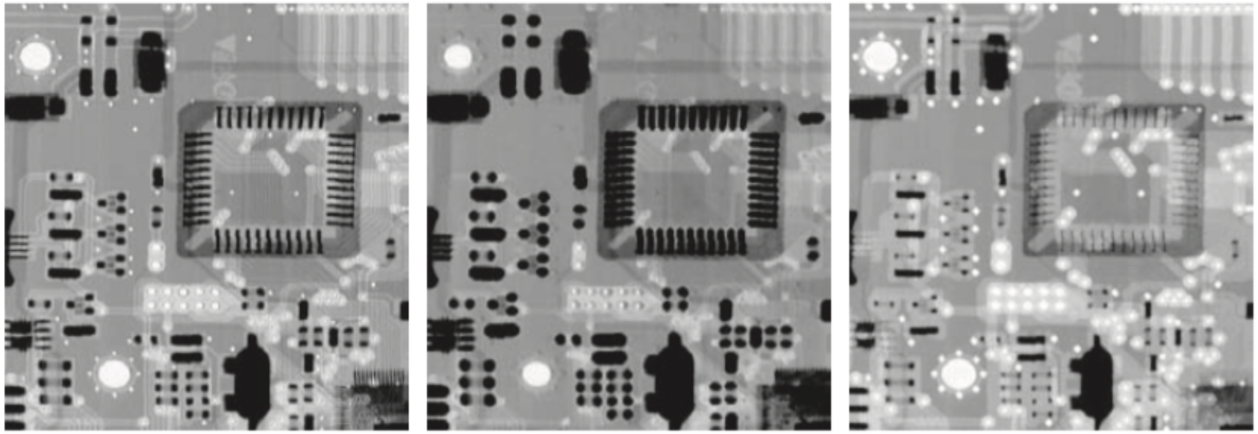
Grayscale Morphology

The *grayscale erosion* of ***f*** by a flat structuring element ***b*** at location ***(x, y)*** is defined as the *minimum* value of the image in the region coincident with ***b(x, y)*** when the origin of ***b*** is at (*x*, *y*). In equation form, the **erosion** at (*x*, *y*) of an image *f* by a structuring element *b* is given as

$$\boxed{[f \ominus b](x,y) = \min_{(s,t)\in b}\{f(x+s, y+t)\}}$$

The **dilation** at (*x*, *y*) of an image *f* by a structuring element *b* is given as

$$\boxed{[f \oplus b](x,y) = \max_{(s,t)\in \hat{b}}\{f(x-s, y-t)\}}$$



a b c

FIGURE 9.37 (a) Gray-scale X-ray image of size 448×425 pixels. (b) Erosion using a flat disk SE with a radius of 2 pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

Summary

- Set operations, structuring elements, objects
- Dilation, erosion -> closing, opening
- Hit-or-miss transform, boundary extraction, hole filling, connected component extraction
- Thinning, thickening, skeletonization
- Simple grayscale morphology: flat SE erosion and dilation

Reading materials

- Chapter 9
- Page 635-663, 674-677

Questions

- How does "hit-or-miss" transform differ from cross-correlation in terms of locating an object in an image using image kernels?
- What are the differences between thinning and erosion?
- What are the differences between thickening and dilation?
- To smooth out a foreground object, what type of morphological operation can be used?
- Page 25 of the slide: explain why there is a "corner" shape left in the image after erosion?
- What are the ways to spot a disk shape of known size in an image?