COMP 478/6771 Assignment 1 solutions –Fall 2022

Theoretical questions

Q1(a) **(5 points)**
All that histogram equalization does is re-map histogram components on the intensity scale. To obtain a uniform (flat) histogram would require in general that pixel intensities be re-distributed so that there are L groups of n/L pixels with the same intensity, where L is the number of allowed discrete intensity levels and n=MN is the total number of pixels in the input image. The histogram equalization method has no provisions for the type of (artificial) intensity redistribution process, considering the intensity values are discrete.

Q1(b) **(5 points)**
Let n=MN be the total number of pixels and let n_{r_j} be the number of pixels in the input image with intensity value r_j. Then the histogram equalization transformation is

$$s_k = T(r_k) = \frac{\sum_{j=0}^{k} n_{r_j}}{n}(\text{L-1})$$

Because every pixel (and no others) with value r_k is mapped to value s_k, it follows that $n_{s_k} = n_{r_k}$. A second pass of histogram equalization would produce value v_k according to the transformation

$$v_k = T(s_k) = \frac{\sum_{j=0}^{k} n_{s_j}}{n}(\text{L-1})$$

But $n_{s_j} = n_{r_j}$

$$v_k = T(s_k) = \frac{\sum_{j=0}^{k} n_{s_j}}{n}(L-1) = s_k$$

which shows that a second pass of histogram equalization would yield the same result as the first pass. Here, we assume negligible round-off errors.

***Note:** if a student demonstrates this with a concrete example without generalizing the demonstration to all cases, 3 points will be given. If a student mentions that in the ideal case, the histogram becomes a uniform distribution after histogram equalization, and thus the transformation function (i.e., the CDF of the new image) will remain the same, full mark will also be given.*

Q2 **(5 points)**
The median *m* of a set of numbers is such that half of the values in the set are below *m* and the other half are above it. A simple example will suffice to show that an operator that computes the median is nonlinear. Let $S_1 = \{1, -2, 3\}$ and $S_2 = \{4, 5, 6\}$ and a = b = 1, then $H(S_1)$ = median {1, -2, 3} = 1 and $H(S_2)$ = median {4, 5, 6} = 5. While $H(S_1 + S_2)$ = median{5, 3, 9} = 5 and $H(S_1 + S_2)$ is not equal to $H(S_1) + H(S_2)$ and the median operator is not linear.

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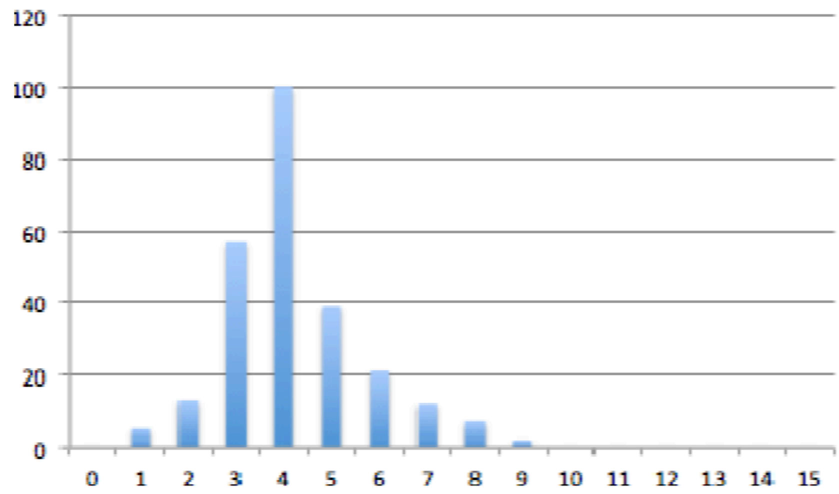
Q3 (a) **(5 points)**

a) Plot of image histogram



(b) **(10 points): (i) and (ii) are 5 points each.**

b)

i) Calculating s_k and histogram equalization

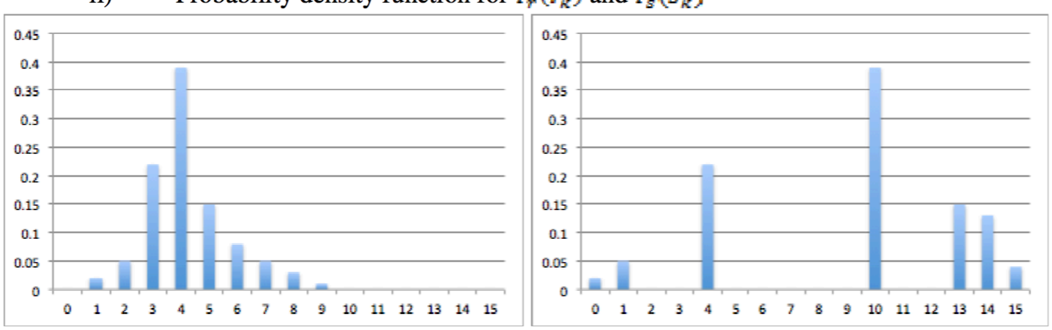
Gray levels r_k	Number of pixels n_k	$p_r(r_k) = \frac{n_k}{n}$	$s_k = \frac{L-1}{n}\sum_{j=0}^{k} n_j$	Discrete values
0	0	0	0	0
1	5	0.02	0.293	0
2	13	0.05	1.055	1
3	57	0.22	4.395	4
4	100	0.39	10.25	10
5	39	0.15	12.54	13
6	21	0.08	13.77	14
7	12	0.05	14.47	14
8	7	0.03	14.88	15
9	2	0.01	15	15
10	0	0	15	15
11	0	0	15	15
12	0	0	15	15
13	0	0	15	15
14	0	0	15	15
15	0	0	15	15

Which $L = 16$ and $n = 256$.

To do histogram equalization we map the gray level values by $S = T(r)$ transformation which yield the following table.

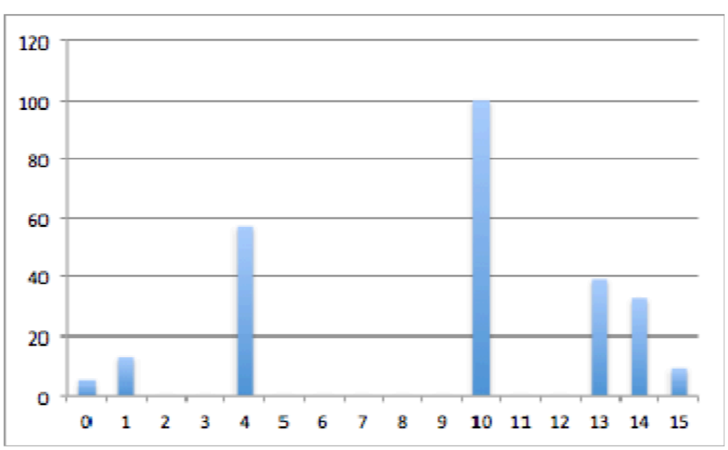
Gray levels s_k	Number of pixels n_{s_k}	$p_s(s_k) = \frac{n_{s_k}}{n}$
0	5	0.02
1	13	0.05
2	0	0
3	0	0
4	57	0.22
5	0	0
6	0	0
7	0	0
8	0	0
9	0	0
10	100	0.39
11	0	0
12	0	0
13	39	0.15
14	33	0.13
15	9	0.04

ii) Probability density function for $P_r(r_k)$ and $P_s(s_k)$



$P_r(r_k)$ $P_s(s_k)$

(c) (**5 points**) Histogram of image after histogram equalization



Q4 **(5 points)**

The question is meant to help the students understand that histogram carry no information about spatial information of the image content. For the case of $f(x,y) + g(x,y)$, the height of each bin for the histogram remains the same, but the entire histogram will be shifted to the right (**2 points**). For the case of $f(x,y)*g(x,y)$, the height of each bin of the histogram will still remain the same. However, their spacing between each other will increase, and thus the new histogram will be more spread out than the original (**3 points**).

Part II: programming questions

1- 1 point
2- 3 points
3- 2 points
4- 3 points
5- 1 point