COMP 472 Artificial Intelligence
State Space Search
Informed Search
More on Heuristics & Summary

part 3

video 6

- Russell & Norvig – Section 3.5.2

Today

1. State Space Representation
2. State Space Search
   a) Overview
   b) Uninformed search
      1. Breadth-first and Depth-first
      2. Depth-limited Search
      3. Iterative Deepening
      4. Uniform Cost
   c) Informed search
      1. Intro to Heuristics
      2. Hill climbing
      3. Greedy Best-First Search
      4. Algorithms A & A*
      5. More on Heuristics

   d) Summary

Evaluating Heuristics

$\forall\, h(n) = 0$

$\forall n\ h(n) \leq h^*(n)$

1. Admissibility:
   - "optimistic"
   - $h(n)$ never overestimates the actual cost of reaching the goal
   - guarantees to find the lowest cost solution path to the goal (if it exists) A*

2. Monotonicity:
   - "local admissibility"
   - guarantees to find the lowest cost path to each state n visited (i.e. popped from OPEN)

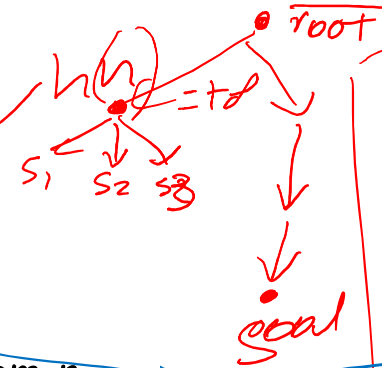
not informed

3. Informedness:
   - measure for the "quality" of a heuristic
   - the more informed, the less backtracking, the shorter the search path

Admissibility

actual

- A heuristic is admissible if it never overestimates the cost of reaching the goal
  - i.e.: guess
    - $h(n) \le h^*(n)$ for all n
      actual
      h^*
  - hence
    - h(goal) = h*(goal) = 0
    - h(n) = ∞ if we cannot reach the goal from *n*



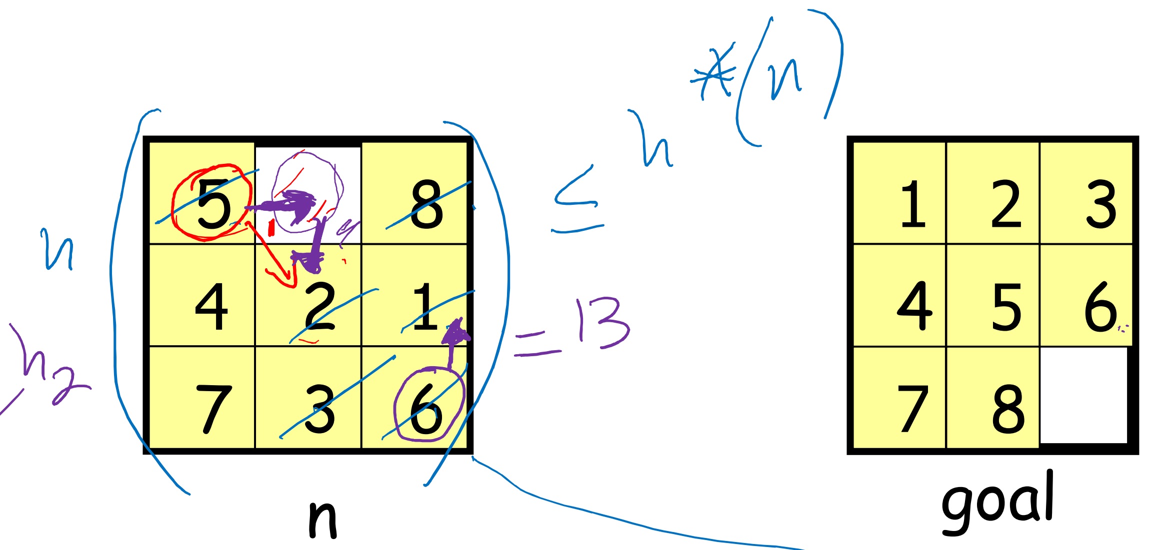
if $h(n) = h^*(n)$ $\forall n$
guess actual

- Algorithm A that uses an admissible heuristic
  - is called algorithm A*
  - guarantees to find the lowest cost solution path to the goal (if it exists)
  - note: does not guarantee to find the lowest cost search path
  - e.g.: uniform cost is admissible -- it uses f(n) = g(n) + 0

if $h(n) = 0$ $\forall n$
→ it is admissible but uninformed

→ i.e you can backtrack

Example: 8-Puzzle



numbered

- h1(n) = Hamming distance = number of misplaced tiles = 6
--> admissible

- h2(n) = Manhattan distance = 13
--> admissible

Problem with Admissibility

- Admissible heuristics may temporarily reach non-goal states along a suboptimal path
  - remember with uniform-cost… when we expanded a node, we had to check if it was already in the OPEN list with a higher path path, and if so, we would replace it with the current path cost/parent info

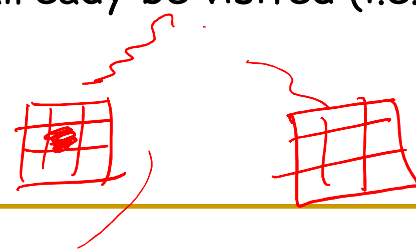
g(n)   < g(n)   g(n)

open   a-5   a 10 a-5

- With A*, if we have a node *n* in OPEN or even in CLOSED
  - We may later find *n* again, but with a lower *f(n)* (due to a lower *g(n)*… the *h(n)* will, by definition be the same).

g(n) + h(n)

  - So to ensure that the solution path has the lowest cost,
    - We may need to update the cost/parent info of node *n* in OPEN or even put *n* back in OPEN even if it has already be visited (i.e. in CLOSED)… expensive work…



Admissibility and A* Search

- Admissibility: $\forall n \ h(n) \le h^*(n)$
  - To guarantee to find the lowest cost solution path, when we generate a successors s: i.e. when node n is popped from OPEN

1. IF s is already in CLOSED
   IF s in CLOSED has a higher f-value due to a higher g-value i.e $g(s)$
   THEN place s and its new lower f-value in OPEN!
   // we found a lower cost path to s, but we had already expanded s…
   // to guarantee the lowest cost solution path, we need to put s back in OPEN and re-visit it again
   ELSE ignore s

expensive but necessary if you want to guarantee lowest cost solution

2. ELSE IF s is already in OPEN
   IF s in OPEN has a higher f-value
   THEN replace the old s in OPEN with the new lower f-value s
   // we found a lower cost path to s, and we had not expanded s yet
   // to guarantee the lowest cost solution path, we need to replace the old s in OPEN with the new
   // lower-cost s
   ELSE ignore s

3. ELSE insert s in OPEN
   // as usual

Monotonicity (aka consistent)

- Admissibility:
  - does <u>not</u> guarantee that <u>every</u> node *n* that is expanded (i.e. for which we generate the successors *s*) will have been found via the lowest cost. the first time we expand it
- Monotonicity
  - guarantees that!
  - Stronger property than admissibility
- If a heuristic is monotonic
  - We are guaranteed that once a node is popped from the OPEN list, we have found the lowest cost path to it

  - i.e we always find the lowest cost path to each node, the 1st time it is popped from OPEN!
  - So once a node is placed in the CLOSED list, if we encounter it again, we do not need to check that the 2nd encounter has a lower cost. We can just ignore it. (more efficient!)

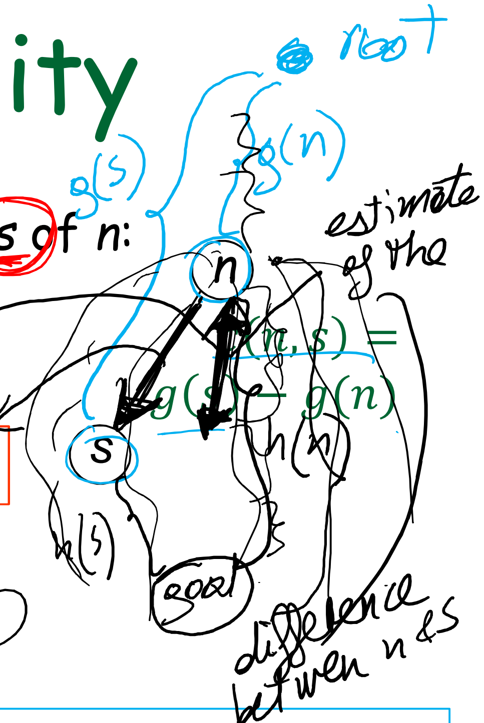
Monotonicity vs Admissibility

stronger

- *h* is monotonic if for every node *n* and every successor *s* of *n*:
  - $h(n) \leq c(n,s) + h(s)\ \forall n,s$
  - $h(n) - h(s) \leq c(n,s)$
  - $h(n) - h(s) \leq g(s) - g(n)$

Estimate of cost from n to s

Actual of cost from n to s



- -> monotonic = *h(n)* is optimistic for all transitions n-->s
- *f(n)* is non-decreasing along any path

Estimate of cost from n to goal

- admissibility = *h(n)* only needs to be optimistic for n-->goal
  - $h(n) \leq h^*(n)\ \forall n$
  - $h^*(n) = g(goal) - g(n) \ \rightarrow\ h(n) \leq g(goal) - g(n)$
  - $h(goal) = 0 \ \rightarrow\ \boxed{h(n) - h(goal)} \leq \boxed{g(goal) - g(n)}$

Actual of cost from n to goal

- Every monotonic *h(n)* is admissible (but not vice-versa)

Monotonicity and A* Search

- Monotonicity
  - Guarantees to find the lowest cost solution path
  - Guarantees to find the lowest cost path to every node, the first time we expand it.
  - --> no need to check the CLOSED list again!
  - So when we generate a successors s:

~~1. IF s is already in CLOSED~~

~~IF s in CLOSED has a higher f-value~~

~~THEN place s and its new lower f-value in OPEN !~~

~~*// we found a lower cost path to s, but we had already expanded s…*~~

~~*// to guarantee the lowest cost solution path, we need to put s back in OPEN and re-visit it again*~~

~~ELSE ignore s~~

2. ELSE IF s is already in OPEN

   IF s in OPEN has a higher f-value

   THEN replace the old s in OPEN with the new lower f-value s

   // we found a lower cost path to s, and we had not expanded s yet

   // to guarantee the lowest cost solution path, we need to replace the old s in OPEN with the new lower-
   // cost s

   ELSE ignore s

3. ELSE insert s in OPEN

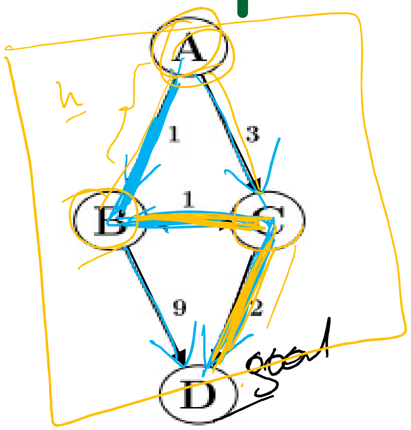
   // as usual

Example

ideal $h(n) = h^*(n)\ \forall n$

$h_1(n) = h^*(n)$

$\forall n \quad h(n) \le h^*(n)$



node	h_1	h_2	h^*
A	4	4	4
B	3	3	3
C	2	0	2
D	0	0	0

Solution paths

1. A B D -> cost of 10
2. A C D -> cost of 5
3. A B C D -> cost of 4
4. A C B D -> cost of 13

- Admissibility -- h*(A)=4 h*(B)=3 h*(C)=2 h*(D)=0
  - is h_1 admissible? Yes
  - is h_2 admissible? Yes

- Monotonic
  - is h_1 monotonic? Yes
    - $h_1(A) - h_1(B) \le g(B) - g(A) \quad 4-3 \le 1-0 \quad 1 \le 1$
    - $h_1(A) - h_1(C) \le g(C) - g(A) \quad 4-2 \le 2-0 \quad 2 \le 2$
    - $h_1(A) - h_1(D) \le g(D) - g(A) \quad 4-0 \le 4-0 \quad 4 \le 4$
    - ...
  - is h_2 monotonic? No

    $h_2(A) - h_2(C) \nleq g(C) - g(A) \quad 4-0 \nleq 2-0 \quad 3 \nleq 2$

    $h_2(B) - h_2(C) \nleq g(C) - g(B) \quad 3-0 \nleq 2-1 \quad 3 \nleq 1$

$\forall n\ s$

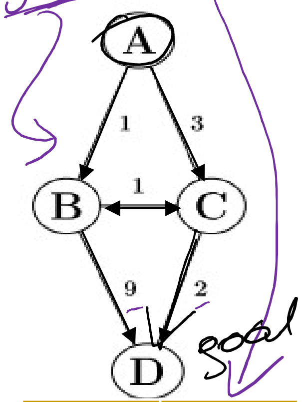
Example - h_2 admissible + not monotonic

node n

parent of n

$f(n) = g(n) + h(n)$

	OPEN (unsorted... work in progress)	OPEN	CLOSED
1	• $A_{null}4_{g=0+h=4}$	1. $A_{null}4_{g=0+h=4}$	
2	• $B_A4_{g=1+h=3}$ • $C_A3_{g=3+h=0}$ sort	1. $C_A3_{g=3+h=0}$ // we will explore C directly from A, but there is a lower cost path to C (ABC). h1 found it because it is monotonic, but h2 is not monotonic, so it could not guarantee that when we expand a node, we have found the lowest cost path to it… 2. $B_A4_{g=1+h=3}$	$A_{null}4_{g=0+h=4}$
3	• $B_C7_{g=3+1+h=3}$ // B is already in OPEN (see below) but with a lower cost path. We do not replace the old, and ignore this version • $D_C5_{g=3+2+h=0}$ • $B_A4_{g=1+h=3}$	1. $B_A4_{g=1+h=3}$ 2. $D_C5_{g=3+2+h=0}$	$A_{null}4_{g=0+h=4}$ $C_A3_{g=3+h=0}$
4	• $C_B2_{g=1+1+h=0}$ // C was already in CLOSED but with a higher f-value, we just found a lower cost path to C… we need to put this version back in OPEN ;-(• $D_B10_{g=1+9+h=0}$ // D is already in OPEN (see below) but with a lower cost path. We do not replace the old, and ignore this version • $D_C5_{g=2+2+h=0}$	1. $C_B2_{g=1+1+h=0}$ 2. $D_C5_{g=3+2+h=0}$	$A_{null}4_{g=0+h=4}$ ~~$C_A3_{g=3+h=0}$~~ $B_A4_{g=1+h=3}$
5	• $B_C6_{g=1+1+1+h=3}$ • $D_C4_{g=1+1+2+h=0}$ // D is already in OPEN (see below) but with a higher cost path. We replace the old version with version • $D_C5_{g=1+9+h=0}$	1. $D_C4_{g=1+1+2+h=0}$ 2. $B_C6_{g=1+1+1+h=3}$	$A_{null}4_{g=0+h=4}$ $B_A4_{g=1+h=3}$ $C_B2_{g=1+1+h=0}$
		goal(D_C4) = true! Solution path = $D_C\,C_B\,B_A A_{null}$ cost = 2+1+1 = 4 // lowest cost path found in 5 steps	

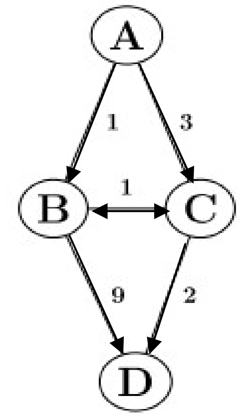


node	h_2
A	4
h(B)	3
h(C)	0
D	0

lowest cost path found in 5 steps

Example – h_1 admissible + monotonic

	OPEN (unsorted… work in progress)	OPEN	CLOSED
1	• $A_{null}4_{g=0+h=4}$	1. $A_{null}4_{g=0+h=4}$	
2	• $B_A4_{g=1+h=3}$ • $C_A5_{g=3+h=2}$	1. $B_A4_{g=1+h=3}$ 2. $C_A5_{g=3+h=2}$	$A_{null}4_{g=0+h=4}$
3	• $C_B4_{g=1+1+h=2}$ *// C already in OPEN with a higher f-value, replace old version with this one* • $D_B10_{g=1+9+h=0}$ • $C_A5_{g=3+h=2}$	1. $C_B4_{g=1+1+h=2}$ 2. $D_B10_{g=1+9+h=0}$	$A_{null}4_{g=0+h=4}$ $B_A4_{g=1+h=3}$
4	• $D_C4_{g=2+2+h=0}$ *// D already in OPEN with a higher f-value, replace old version with this one* • $B_C6_{g=2+1+h=3}$ *// B already in CLOSED but since h1 is monotonic, we do not need to check the f-value of the version in CLOSED because we know that the version in CLOSED will have a lower f-value, so can ignore this version* • $D_B10_{g=1+9+h=0}$	1. $D_C4_{g=2+2+h=0}$ 2. $B_C6_{g=2+1+h=3}$	$A_{null}4_{g=0+h=4}$ $B_A4_{g=1+h=3}$ $C_B4_{g=1+1+h=2}$
		goal(D_C4) = true! Solution path = $D_C\, C_B\, B_A\, A_{null}$ cost = 2+1+1 = 4 *// lowest cost path found in 4 steps*	



node	h_1
A	4
B	3
C	2
D	0

Admissible + Monotonic

lowest cost path found in 4 steps

Informedness

rank the nodes

most promising — least promising

- Intuition:
  - $h(n) = 0$ for all nodes is less informed
  - number of misplaced tiles is less informed than Manhattan distance
- Formally:
  - given 2 admissible heuristics h_1 and h_2 *// ie. $h_1(n) \leq h^*(n)$ and $h_2(n) \leq h^*(n)$* $\forall n$
    - if $h_1(n) \leq h_2(n)$, for all states n
    - then h_2 is more informed than h_1
    - aka h_2 dominates h_1
- So?
  - a more informed heuristic expands fewer nodes
  - aka the search path is shorter
  - however, you need to consider the computational cost of evaluating the heuristic… $h(n)$
  - the time spent computing heuristics must be recovered by a better search

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Summary

Search	Uses h(n)?	Uses g(n)?	OPEN list
Breadth-first	No	No	Priority queue sorted by level
Depth-first	No	No	Stack
Depth-limited	No	No	Stack
Iterative Deepening	No	No	Stack
Uniform Cost *- guarantees to find the lowest cost solution path*	No	Yes	Priority queue sorted by g(n) h(n) When generating successors: - If successor s already in OPEN with higher g(n), replace old version with new s - If successor s already in CLOSED, ignore s
Hill Climbing	Yes	No	N/A
Greedy Best-First *- no constraints on h(n)* *- no guarantee to find lowest cost solution path*	Yes	No	Priority queue sorted by h(n)
Algorithm A *- no constraints on h(n)* *- no guarantee to find lowest cost solution path*	Yes	Yes	Priority queue sorted by f(n) identical
Algorithm A* *- h(n) must be admissible* $\forall n \; h(n) \leq h^*(n)$ *- guarantees to find the lowest cost solution path* provided	Yes	Yes	Priority queue sorted by f(n) If h(n) is NOT monotonic When generating successors: - If successor s already in OPEN with higher f(n), replace old version with new s - If successor s already in CLOSED with higher f(n), replace old version with new s If h(n) IS monotonic When generating successors: - If successor s already in OPEN with higher f(n), replace old version with new s - If successor s already in CLOSED, ignore it.

un informed

informed

Today

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Up Next

1. Part 4: Adversarial Search