COMP 472: Artificial Intelligence
State Space Representation and Uninformed Search
Solutions

1 Question

Consider the following problem:

Once upon a time a farmer went to the market and purchased a fox, a goose, and a bag of beans. On his way home, the farmer came to the bank of a river and rented a boat. But in crossing the river by boat, the farmer could carry only himself and a single one of his purchases - the fox, the goose, or the bag of the beans.

If left alone, the fox would eat the goose, and the goose would eat the beans. The farmer's challenge was to carry himself and his purchases to the far bank of the river, leaving each purchase intact.

Represent this problem as a search problem. Choose a representation for the problem's states and:

(a) Write down the initial state.

Solution

Let `position(farmer, fox, goose, beans)` represent the position of the farmer, the fox, the goose and the beans with respect to the river. The possible positions are `o` for "original bank" and `f` for "far bank".

Initially, the state is: `position(o,o,o,o)`

(b) Write down the goal state.

Solution

`position(f,f,f,f)`

(c) Write down all illegal states.

Solution

```
 position(f,o,o,o)
position(f,o,o,f)
position(o,f,f,o)
position(o,f,f,f)
position(f,f,o,o)
position(o,o,f,f)
position(o,f,f,f)
```

(d) Write down the possible actions.

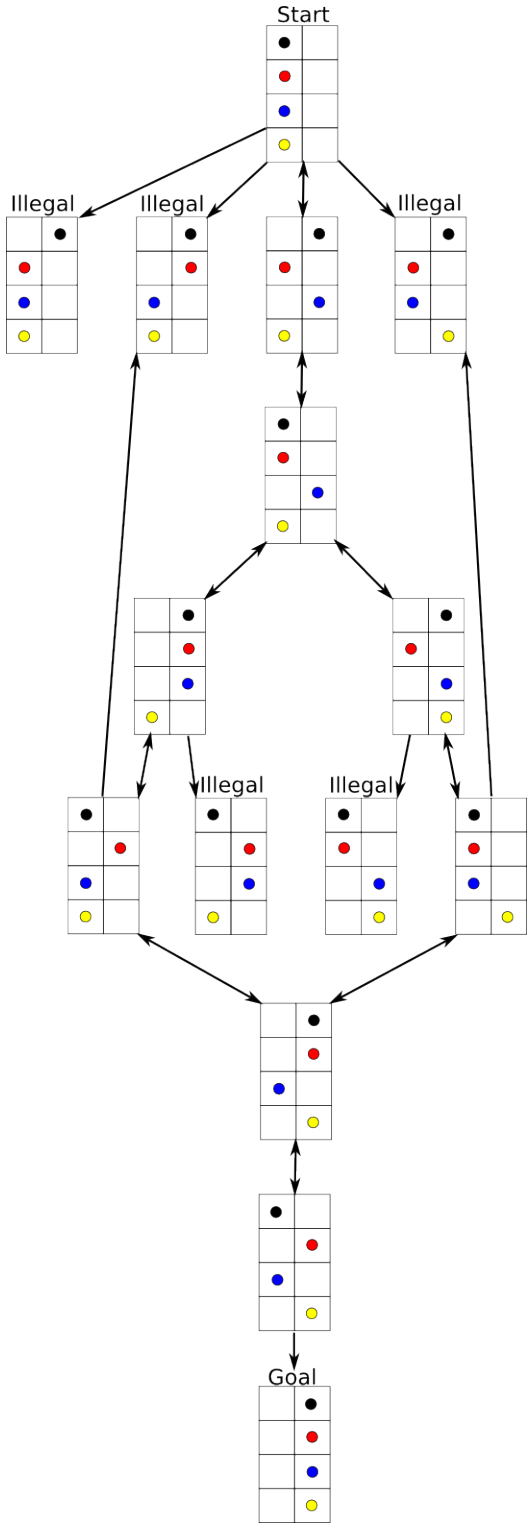
Solution

```
moveFar(farmer, empty)
moveFar(farmer, fox)
moveFar(farmer, goose)
moveFar(farmer, beans)
moveBack(farmer, empty)
moveBack(farmer, fox)
moveBack(farmer, goose)
moveBack(farmer, beans)
```

(e) Draw the state space for this problem.

Solution

To make it easier, the figure below represents each state graphically. The first column is the original bank `o`, the second column is the far back `f`; and the colored dots represent an entity (black → farmer, red → fox, blue → goose, and yellow → beans). However, we could have used the predicate `position` instead. For example, the initial state is equivalent to `position(o,o,o,o)` and the goal state is equivalent to `position(f,f,f,f)`.


Start
Illegal
Illegal
Illegal
Illegal
Illegal
Goal

(f) Find a sequence of moves to solve this problem.

Solution

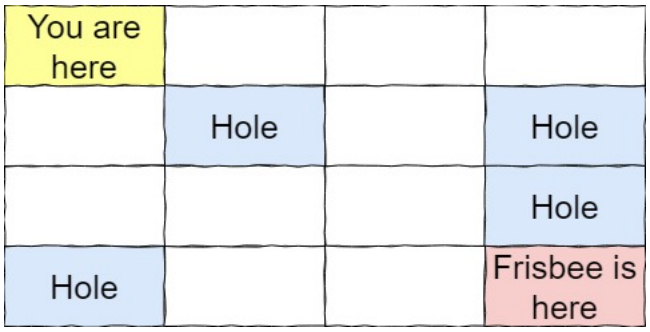
```
moveFar(farmer, goose),
moveBack(farmer, empty),
moveFar(farmer, fox),
moveBack(farmer, goose),
moveFar(farmer, beans),
moveBack(farmer, empty),
moveBack(farmer, goose)
```

OR

```
moveFar(farmer, goose),
moveBack(farmer, empty),
moveFar(farmer, beans),
moveBack(farmer, goose),
moveFar(farmer, fox),
moveBack(farmer, empty),
moveBack(farmer, goose)
```

2 Question

Exercise from OpenAI[1]*:* Winter is here. You and your friends were tossing around a Frisbee at the park when you made a wild throw that left the Frisbee out in the middle of the lake. The water is mostly frozen, but there are a few holes where the ice has melted. If you step into one of those holes you'll fall into the freezing water. At this time, there's an international Frisbee shortage, so it's absolutely imperative that you navigate across the lake and retrieve the disc as soon as you can. The surface is described using a rectangular grid like the figure below:



(a) Let a 4×4 matrix represent the above grid. The position of each cell in the grid can then be represented by the indices of the elements of the matrix. Given this representation, write down the initial state and the goal states.

Solution

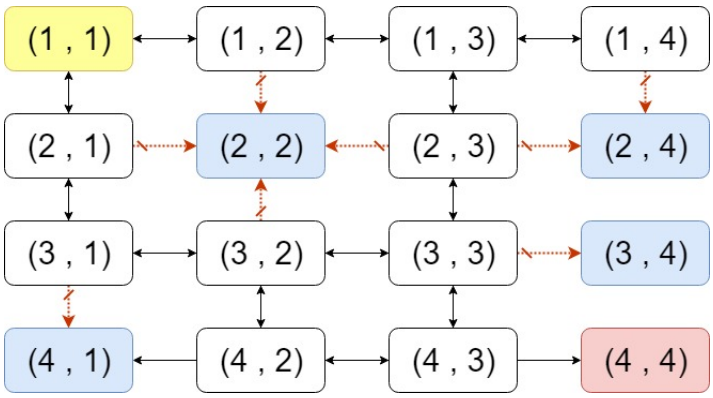
- Initial state: $(1, 1)$
- Goal state: $(4, 4)$

(b) Assume that the possible actions in a grid world are moving left, right, up, and down. Since we would like to reach the goal state as soon as possible (i.e. minimizing the number of actions), then we can assign a constant uniform cost for each action, for example a cost of 1.
Draw the state space for this problem.

Solution

The state space is shown in the figure below, where the initial state is colored yellow, the goal state is colored red and illegal states are colored blue.

[1] `https://gym.openai.com/envs/FrozenLake-v0/`



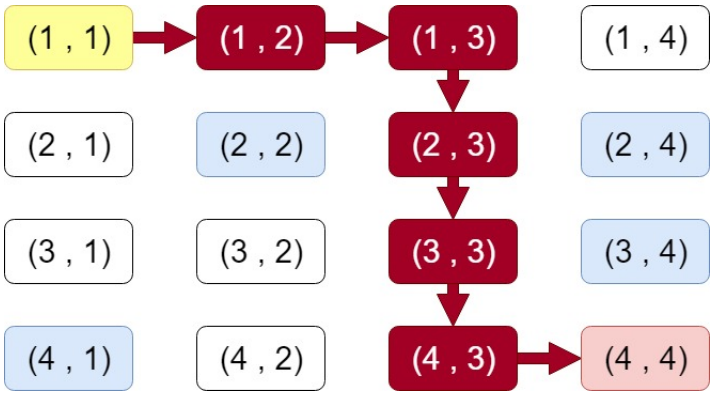
(c) Find a sequence of moves to solve this problem.

Solution

Any of the following sequence of moves constitute a possible solution for this problem:

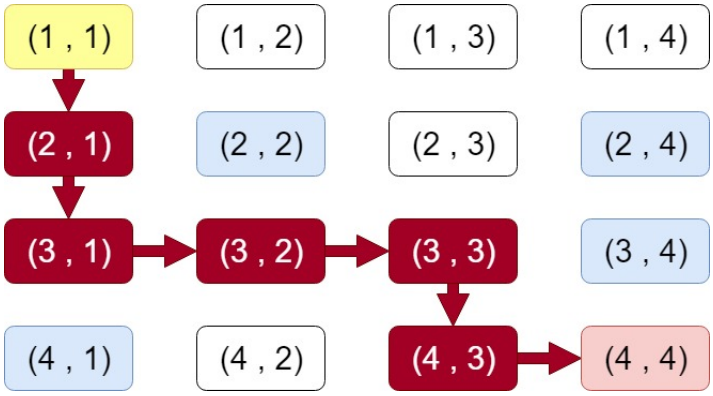
Solution Path 1

Solution Path 1: $(1,1) \rightarrow (1,2) \rightarrow (1,3) \rightarrow (2,3) \rightarrow (3,3) \rightarrow (4,3) \rightarrow (4,4)$
Cost of solution path: 6 (since each move has a cost of 1)



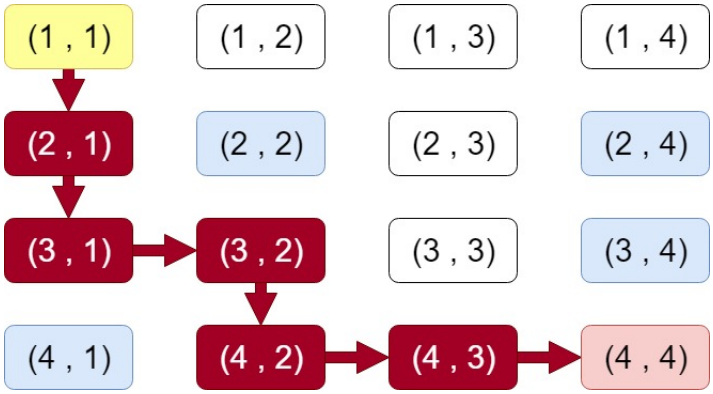
Solution Path 2

Solution Path 2: $(1,1) \rightarrow (2,1) \rightarrow (3,1) \rightarrow (3,2) \rightarrow (3,3) \rightarrow (4,3) \rightarrow (4,4)$
Cost of solution path: 6 (since each move has a cost of 1)



Solution Path 3

Solution Path 3: $(1,1) \rightarrow (2,1) \rightarrow (3,1) \rightarrow (3,2) \rightarrow (4,2) \rightarrow (4,3) \rightarrow (4,4)$
Cost of solution path: 6 (since each move has a cost of 1)



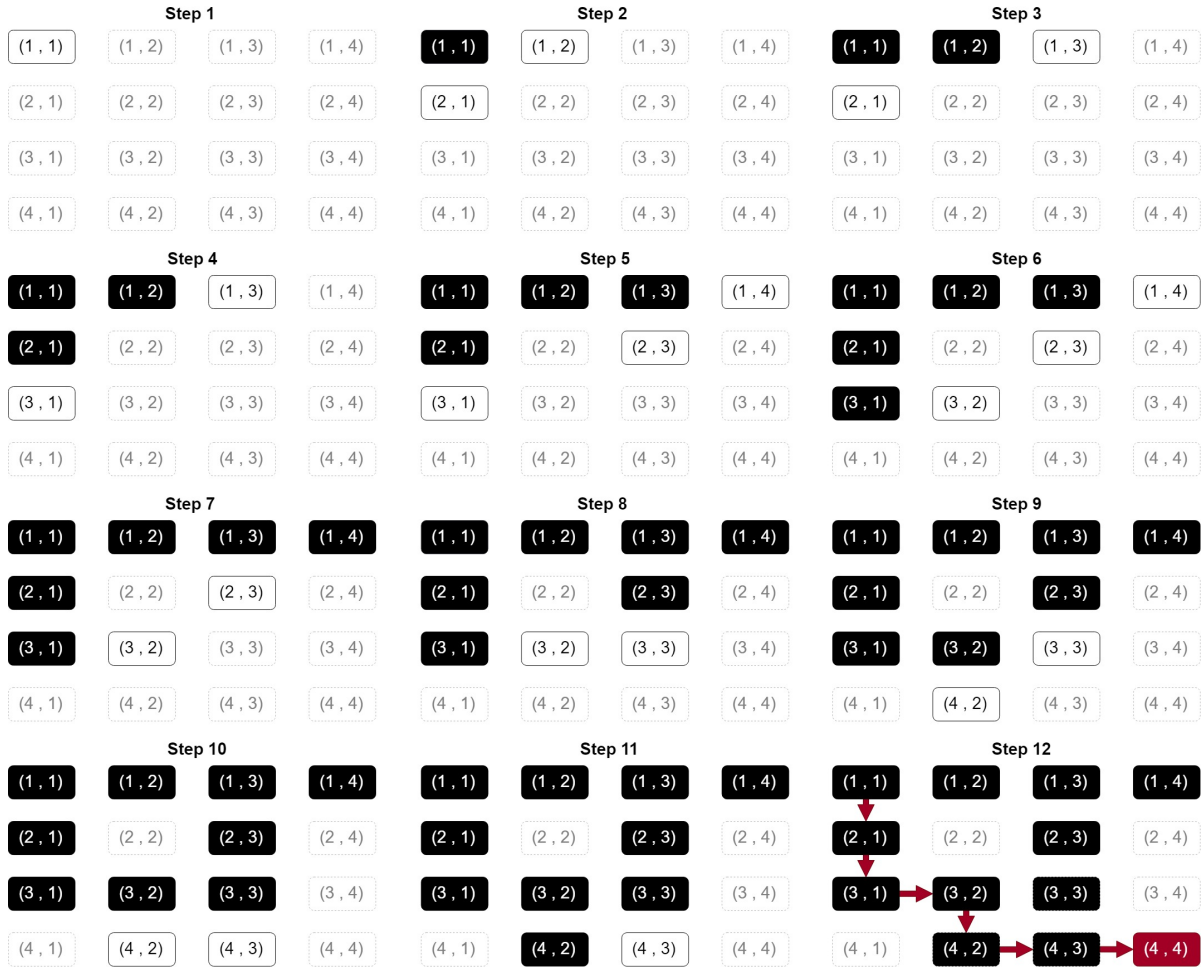
(d) Perform a Breadth-First Search on this state space. At each step, indicate the content of the OPEN and CLOSED lists.

Solution

The trace of Breadth-First Search is listed below.

Step	Visited	OPEN	CLOSED
1		$(1,1)$	$\emptyset$
2	$(1,1)$	$(1,2),(2,1)$	$(1,1)$
3	$(1,2)$	$(2,1),(1,3)$	$(1,2),(1,1)$
4	$(2,1)$	$(1,3),(3,1)$	$(2,1),(1,2),(1,1)$
5	$(1,3)$	$(3,1),(1,4),(2,3)$	$(1,3),(2,1),(1,2),(1,1)$
6	$(3,1)$	$(1,4),(2,3),(3,2)$	$(3,1),(1,3),(2,1),(1,2),(1,1)$
7	$(1,4)$	$(2,3),(3,2)$	$(1,4),(3,1),(1,3),(2,1),(1,2),(1,1)$
8	$(2,3)$	$(3,2),(3,3)$	$(2,3),(1,4),(3,1),(1,3),(2,1),(1,2),(1,1)$
9	$(3,2)$	$(3,3),(4,2)$	$(3,2),(2,3),(1,4),(3,1),(1,3),(2,1),(1,2),(1,1)$
10	$(3,3)$	$(4,2),(4,3)$	$(3,3),(3,2),(2,3),(1,4),(3,1),(1,3),(2,1),(1,2),(1,1)$
11	$(4,2)$	$(4,3)$	$(4,2),(3,3),(3,2),(2,3),(1,4),(3,1),(1,3),(2,1),(1,2),(1,1)$
12	$(4,3)$	$(4,4)$	$(4,3),(4,2),(3,3),(3,2),(2,3),(1,4),(3,1),(1,3),(2,1),(1,2),(1,1)$
13	$(4,4)$		$(4,4),(4,3),(4,2),(3,3),(3,2),(2,3),(1,4),(3,1),(1,3),(2,1),(1,2),(1,1)$

The goal state (4,4) is visited, so the search terminates.



Progress of Breadth-First Search. The cells with solid white background represent the open set. The cells with solid black background represent the closed set and the cell with solid red background is the goal state.

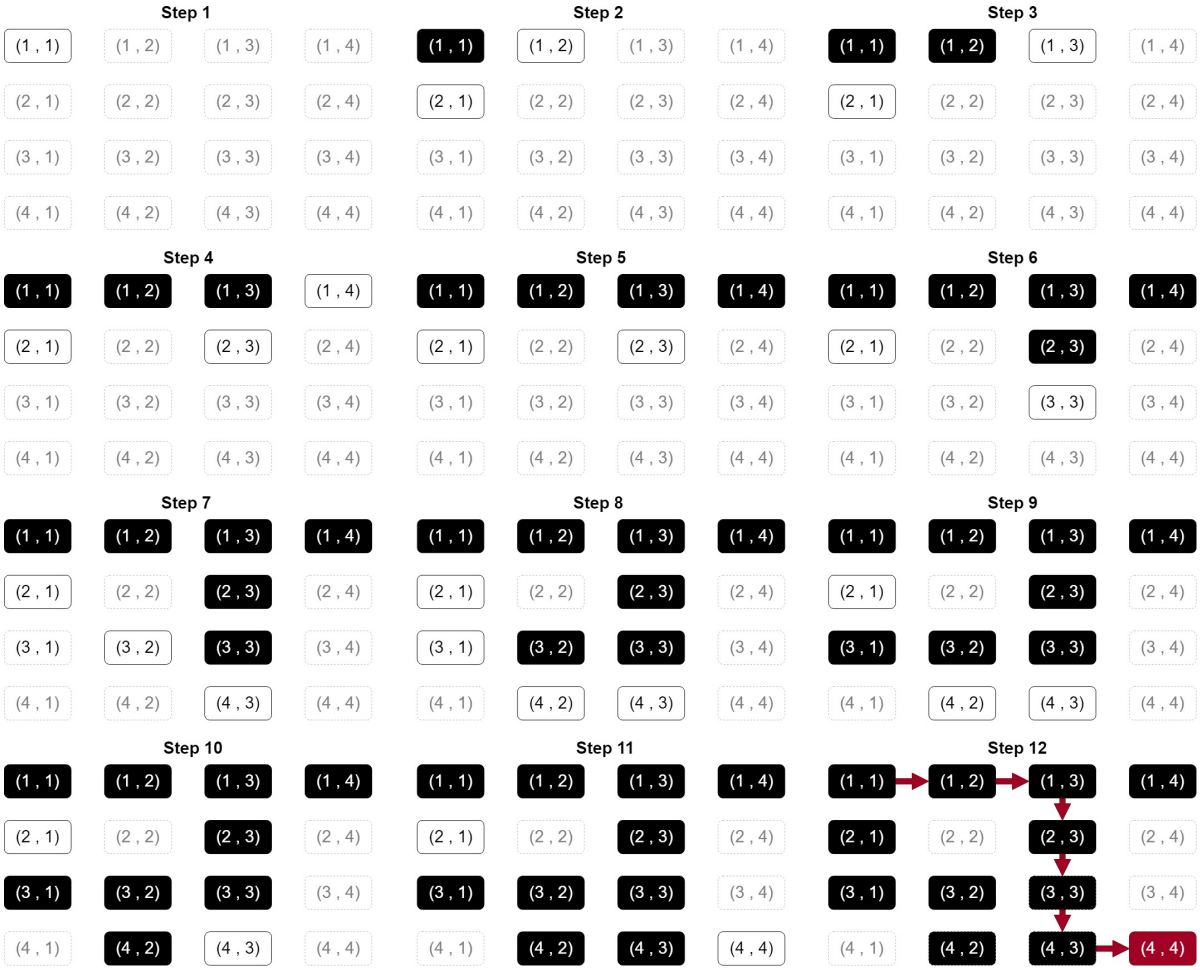
(e) Perform a Depth-First Search on this state space. At each step, indicate the content of the OPEN and CLOSED lists.

Solution

The trace of Depth-First Search is listed below.

Step	Visited	OPEN	CLOSED
1		(1,1)	$\emptyset$
2	$(1,1)$	$(1,2),(2,1)$	$(1,1)$
3	$(1,2)$	$(1,3),(2,1)$	$(1,2),(1,1)$
4	$(1,3)$	$(1,4),(2,3),(2,1)$	$(1,3),(1,2),(1,1)$
5	$(1,4)$	$(2,3),(2,1)$	$(1,4),(1,3),(1,2),(1,1)$
6	$(2,3)$	$(3,3),(2,1)$	(2,3), (1,4), (1,3), (1,2), (1,1)
7	$(3,3)$	$(3,2),(4,3),(2,1)$	$(3,3),(2,3),(1,4),(1,3),(1,2),(1,1)$
8	$(3,2)$	$(3,1),(4,2),(4,3),(2,1)$	(3,2), (3,3), (2,3), (1,4), (1,3), (1,2), (1,1)
9	$(3,1)$	$(4,2),(4,3),(2,1)$	(3,1), (3,2), (3,3), (2,3), (1,4), (1,3), (1,2), (1,1)
10	$(4,2)$	$(4,3),(2,1)$	$(4,2),(3,1),(3,2),(3,3),(2,3),(1,4),(1,3),(1,2),(1,1)$
11	$(4,3)$	$(4,4),(2,1)$	$(4,3),(4,2),(3,1),(3,2),(3,3),(2,3),(1,4),(1,3),(1,2),(1,1)$
12	$(4,4)$	$(2,1)$	$(4,4),(4,3),(4,2),(3,1),(3,2),(3,3),(2,3),(1,4),(1,3),(1,2),(1,1)$

The goal state (4,4) is visited, so the search terminates.



Progress of Depth-First Search. The cells with solid white background represent the open set. The cells with solid black background represent the closed set and the cell

with solid red background is the goal state.

(f) Perform a Uniform Cost Search on this state space. At each step, indicate the content of the OPEN and CLOSED lists.

Solution

Since we have assumed that all move costs are equal, then Uniform-Cost Search and Breadth-First Search will have the same result.