The Object-Z Specification Language

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Template for a class definition

ClassName

$< visibility\ list >$

$< parent\ class >$

$< state >$

$< initialization\ of\ state >$

$< list\ of\ operations >$

Example 1: Stack ADT – Visibility list and interface

$$\upharpoonright (Push, Pop, Top)$$

State schema

$$elements : seq\ T$$
$$count : \mathbb{N}$$

$$count >= 0$$

Initialization of state

$$
\begin{array}{|l}
\multicolumn{1}{l}{\textit{INIT}} \\
\hline
elements = \langle\rangle \\
count = 0 \\
\hline
\end{array}
$$

Operation Push

$$
\begin{array}{|l}
\multicolumn{1}{l}{\textit{Push}} \\
\hline
\Delta(elements, count) \\
el? : T \\
\hline
elements' = \langle el? \rangle \frown elements \\
count' = count + 1 \\
\hline
\end{array}
$$

Operation Pop

$$
\begin{array}{|l}
\hline
\textit{Pop} \\
\Delta(elements, count) \\
el! : T \\
\hline
count > 0 \\
el! = head(elements) \\
elements' = tail(elements) \\
count' = count - 1 \\
\hline
\end{array}
$$

Operation Top

$$
\begin{array}{|l}
\hline
\textit{Top} \\
el! : T \\
\hline
count > 0 \\
el! = head(elements) \\
elements' = elements \\
count' = count \\
\hline
\end{array}
$$

$Stack[T]$

$\upharpoonright (Push, Pop, Top)$

$$elements : seq\ T$$
$$count : \mathbb{N}$$

$$count >= 0$$

$INIT$

$$elements = \langle\rangle$$
$$count = 0$$

$Push$

$$\Delta(elements, count)$$
$$el? : T$$

$$elements' = \langle el? \rangle \frown elements$$
$$count' = count + 1$$

Pop

$$\Delta(elements, count)$$
$$el! : T$$

$$count > 0$$
$$el! = head(elements)$$
$$elements' = tail(elements)$$
$$count' = count - 1$$

Top

$$el! : T$$

$$count > 0$$
$$el! = head(elements)$$
$$elements' = elements$$
$$count' = count$$

Instantiating a stack of natural numbers

$$
\begin{array}{|l}
\hline
\textit{IntStack} \\
\hline
\quad items : Stack(\mathbb{N}) \\
\hline
Push \mathrel{\widehat{=}} items.Push \\
Pop \mathrel{\widehat{=}} items.Pop \\
Top \mathrel{\widehat{=}} items.Top \\
\hline
\end{array}
$$

Inheritance

- A class in Object-Z may be specified as a <u>specialization</u> or <u>extension</u> of another class using inheritance.

- A class S can inherit another class P by including the name of the parent class after the visibility list in S.

Inheritance for *specialization*

- The subclass is a specialized version of the parent class, and thus satisfies the specification (interface) of the parent class in all relevant aspects, adding any particular behavior through overriding.

Inheritance for *extension*

- A subclass merely adds new behavior and does not modify or alter any of the inherited features.

Inheritance /cont.

- The subclass inherits every feature (variables, constants, initial state schema and operations), except the visibility list.

- The subclass must define its own visibility list.

- This implies that a feature that is declared private in the parent class may now be declared as visible, and vice versa: A visible feature from the parent class can now be declared as private by not being included in the visibility list of the subclass.

State and behavior in the presence of inheritance

- State variables in the parent class are merged with those of the subclass.

- The subclass may redefine a state variable, but only in a compatible way, expanding or restricting the type of a variable with the same name, for example restricting an integer variable to one that can hold only positive integers.

State and behavior in the presence of inheritance /cont.

- If an operation is redefined in the subclass, the declaration of an operation in the parent class is merged with that of the same operation in the subclass.

- An operation's predicate part is conjoined with that of the same operation in the subclass.

Subclassifying Stack to define BoundedStack
Inheritance for *specialization*

$BoundedStack[T]$

$\upharpoonright (Push, Pop, Top)$

$Stack[T]$

$capacity : \mathbb{N}$

$count <= capacity$

$\textsc{Init}$

$capacity = 10$

$Push$

$count < capacity$

Example 2: Queue ADT

Front of Queue

Rear of Queue

$$\Lambda = < \boxed{el_1}, el_2, \ldots, \boxed{el_n} >$$

Head of Λ

Queue ADT – State schema

$$elements : seq\ T$$
$$count = \mathbb{N}$$

$$count >= 0$$

Initialization of state

$$
\begin{array}{|l}
\hline
\textit{INIT} \\
\hline
elements = \langle\rangle \\
count = 0 \\
\hline
\end{array}
$$

Operation Enqueue

$$
\begin{array}{|l}
\hline
\textit{Enqueue} \\
\Delta(elements, count) \\
el? : T \\
\hline
elements' = elements \frown \langle el? \rangle \\
count' = count + 1 \\
\hline
\end{array}
$$

Operation Dequeue

Dequeue

$$\Delta(elements, count)$$
$$el! : T$$

$$count > 0$$
$$el! = head(elements)$$
$$elements' = tail(elements)$$
$$count' = count - 1$$

Instantiating a queue of natural numbers

$$
\begin{array}{|l}
\hline
\textbf{IntQueue} \\
\begin{array}{|l}
\hline
items : Queue(\mathbb{N}) \\
\hline
\end{array} \\
Enqueue \mathrel{\widehat{=}} items.Enqueue \\
Dequeue \mathrel{\widehat{=}} items.Dequeue \\
\hline
\end{array}
$$

Subclassifying Queue to define BoundedQueue
Inheritance for *specialization*

$BoundedQueue[T]$

$\upharpoonright (Enqueue, Dequeue)$

$Queue[T]$

$capacity : \mathbb{N}$

$count <= capacity$

$\textsc{Init}$

$capacity = 10$

$Enqueue$

$count < capacity$

Subclassifying Queue to define RessetableQueue Inheritance for *extension*

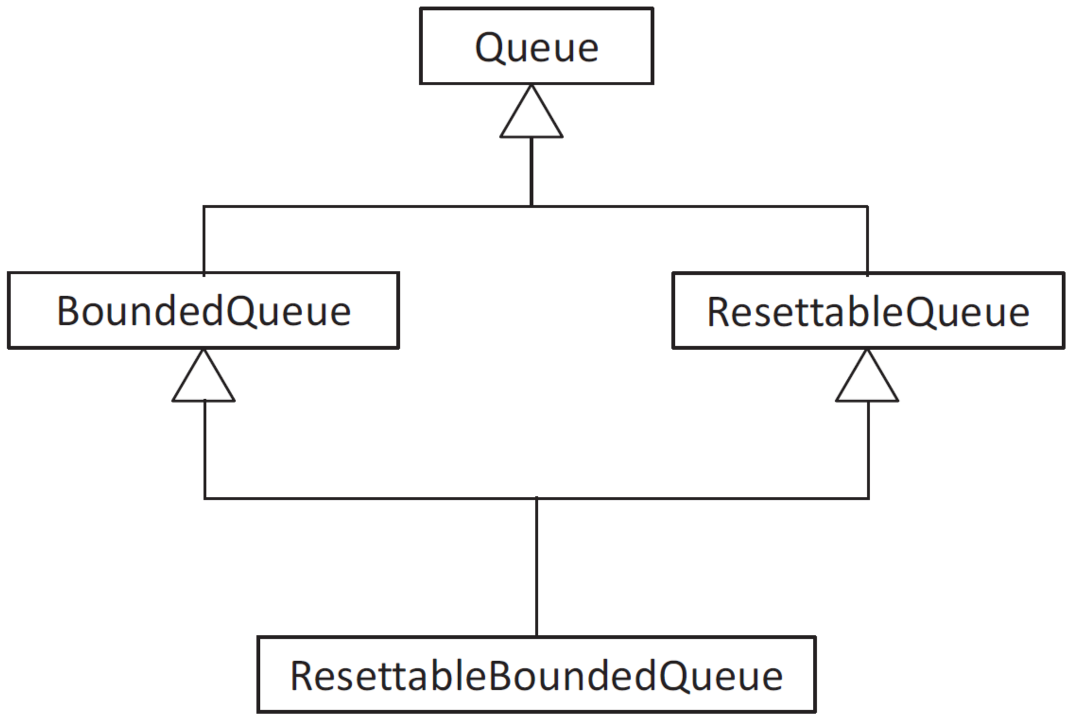
$$
\begin{array}{|l}
\hline
\textit{ResettableQueue}[T] \\
\upharpoonright (\textit{Enqueue}, \textit{Dequeue}, \textit{Reset}) \\
\textit{Queue}[T] \\
\quad \begin{array}{|l}
\hline
\textit{Reset} \\
\Delta(\textit{elements}, \textit{count}) \\
\hline
\textit{elements}' = \langle\rangle \\
\textit{count}' = 0 \\
\hline
\end{array} \\
\hline
\end{array}
$$

Inheritance for combination

- Object-Z supports multiple inheritance.

- A subclass is formed by combining features from more than one types.

Multiple inheritance



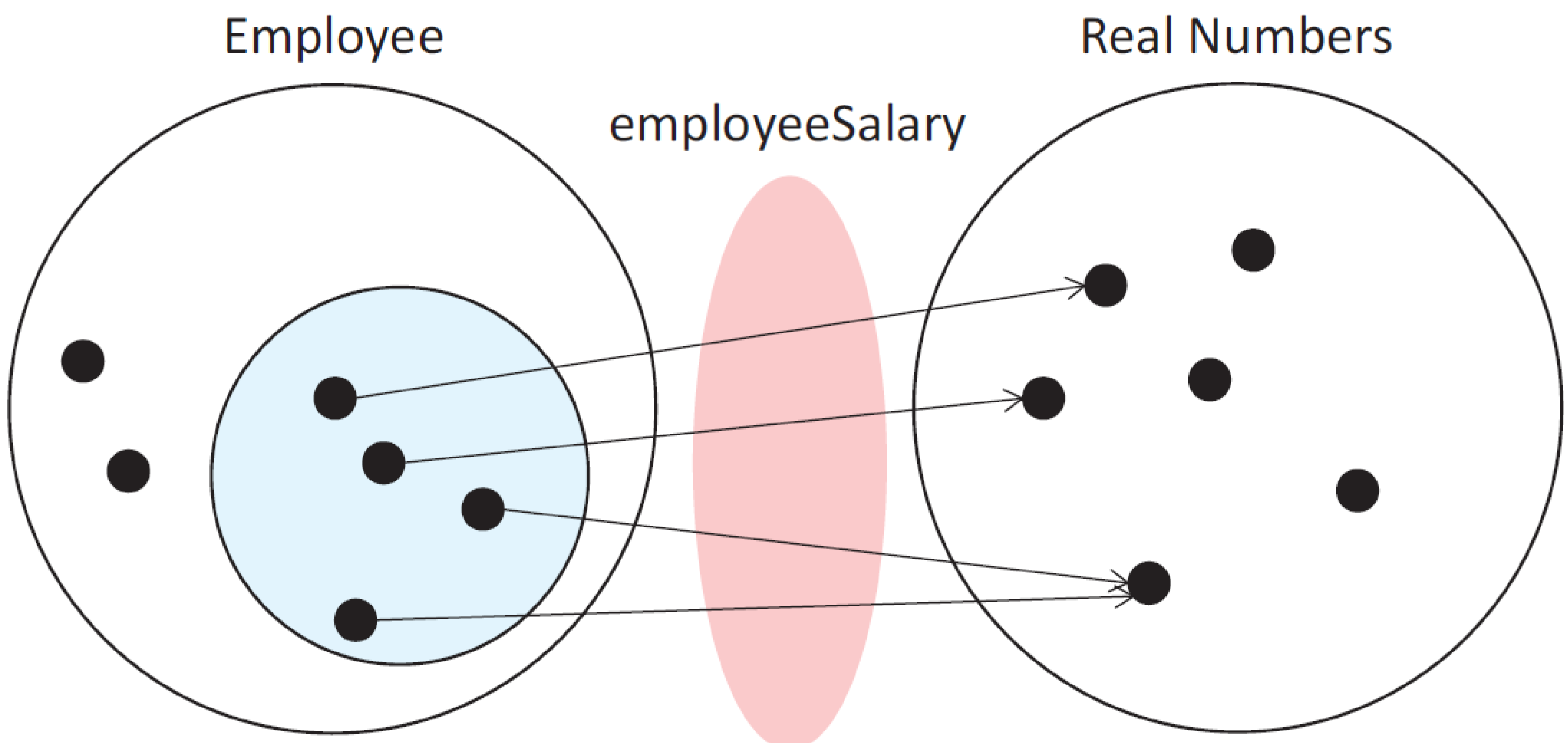
Class RessetableBoundedQueue Inheritance for *combination*

$$
\begin{array}{l}
\textit{ResettableBoundedQueue}[T] \\
\upharpoonright (\textit{Enqueue}, \textit{Dequeue}, \textit{Reset}) \\
\textit{BoundedQueue}[T] \\
\textit{ResettableQueue}[T]
\end{array}
$$

Handling errors and providing robust specifications

$$
\begin{array}{|l}
\hline
\textit{Name} \\
\upharpoonright (Op_1) \\
\ldots \\
\begin{array}{|l}
\hline
Op_1OK \\
\ldots \\
\hline
\ldots \\
\hline
\end{array} \\
\begin{array}{|l}
\hline
Success \\
result! : Report \\
\hline
result! = ok \\
\hline
\end{array} \\
\begin{array}{|l}
\hline
Error \\
result! : Report \\
\hline
\ldots \\
result! = error \\
\hline
\end{array} \\
Op_1 \mathrel{\widehat{=}} (Op_1OK \land Success) \oplus Error \\
\hline
\end{array}
$$

Example: Managing employees



Interface, state schema and initialization

$$\upharpoonright (AddEmployee, DeleteEmployee, ModifySalary)$$

$$employeeSalary : Employee \pfun \mathbb{R}$$

$$\forall\, d : \mathrm{dom}\ employeeSalary \bullet employeeSalary(d) > 0.0$$

$$\textit{INIT}$$

$$employeeSalary = \varnothing$$

Operation AddEmployee

$$
\begin{array}{|l}
\hline
\textit{AddEmployee} \\
\Delta(\mathit{employeeSalary}) \\
\mathit{newEmployee?} : \mathit{Employee} \\
\mathit{salary?} : \mathbb{R} \\
\hline
\mathit{salary?} > 0.0 \\
\mathit{newEmployee?} \notin \operatorname{dom}\ \mathit{employeeSalary} \\
\mathit{employeeSalary}' = \mathit{employeeSalary} \cup \{\mathit{newEmployee?} \mapsto \mathit{salary?}\} \\
\hline
\end{array}
$$

Operation DeleteEmployee

$$
\begin{array}{|l}
\hline
\textit{DeleteEmployee} \\
\Delta(employeeSalary) \\
who? : Employee \\
\hline
who? \in \mathrm{dom}\ \ employeeSalary \\
employeeSalary' = \{who?\} \lhd employeeSalary \\
\hline
\end{array}
$$

Operation ModifySalary

$$
\begin{array}{|l}
\hline \textit{ModifySalary} \\
\Delta(employeeSalary) \\
employee? : Employee \\
newSalary? : \mathbb{R} \\
\hline
newSalary? > 0.0 \\
employee? \in \mathrm{dom}\ \ employeeSalary \\
employeeSalary' = employeeSalary \oplus \{employee? \mapsto newSalary?\} \\
\hline
\end{array}
$$

Examining the specification: Initial state

employeeSalary	**dom employeeSalary**	**ran employeeSalary**
{ }	{ }	{ }

$$\forall\, d : \mathrm{dom}\ employeeSalary \bullet employeeSalary(d) > 0.0$$ ✓ **Invariant**

$$\begin{array}{|l}\hline \textit{INIT} \\ employeeSalary = \varnothing \\ \hline\end{array}$$

AddEmployee(Syd, 90)

employeeSalary	**dom employeeSalary**	**ran employeeSalary**
{ }	{ }	{ }

$$\forall d : \mathrm{dom}\ employeeSalary \bullet employeeSalary(d) > 0.0$$ ✓ **Invariant**

$$
\begin{array}{l}
\textbf{AddEmployee} \\
\hline
\Delta(employeeSalary) \\
newEmployee? : Employee \\
salary? : \mathbb{R} \\
\hline
\boxed{\begin{array}{l} salary? > 0.0 \ \checkmark \\ newEmployee? \notin \mathrm{dom}\ employeeSalary \ \checkmark \end{array}} \\
employeeSalary' = employeeSalary \cup \{newEmployee? \mapsto salary?\} \\
\hline
\end{array}
$$

Precondition

AddEmployee(Syd, 90)

employeeSalary

{
(Syd, 90)
}

dom employeeSalary

{ Syd }

ran employeeSalary

{ 90 }

$$\forall\, d : \mathrm{dom}\ \ employeeSalary \bullet employeeSalary(d) > 0.0$$ ✓ **Invariant**

$AddEmployee$

$\Delta(employeeSalary)$
$newEmployee? : Employee$
$salary? : \mathbb{R}$

$salary? > 0.0$
$newEmployee? \notin \mathrm{dom}\ \ employeeSalary$

Postcondition

$\boxed{employeeSalary' = employeeSalary \cup \{newEmployee? \mapsto salary?\}}$

AddEmployee(David, 100)

employeeSalary

{
  (Syd, 90)
}

dom employeeSalary

{ Syd }

ran employeeSalary

{ 90 }

$$\forall\, d : \mathrm{dom}\ \ employeeSalary \bullet employeeSalary(d) > 0.0$$ ✓ **Invariant**

$$
\begin{array}{|l}
\hline
\multicolumn{1}{l}{\textit{AddEmployee}} \\
\Delta(employeeSalary) \\
newEmployee? : Employee \\
salary? : \mathbb{R} \\
\hline
\boxed{\begin{array}{l} salary? > 0.0 \ \checkmark \\ newEmployee? \notin \mathrm{dom}\ \ employeeSalary \ \checkmark \end{array}} \ \textbf{Precondition} \\
employeeSalary' = employeeSalary \cup \{newEmployee? \mapsto salary?\} \\
\hline
\end{array}
$$

AddEmployee(David, 100)

employeeSalary

{
(Syd, 90) ,
(David, 100)
}

dom employeeSalary

{ Syd, David }

ran employeeSalary

{ 90, 100 }

$$\forall\, d : \mathrm{dom}\ \ employeeSalary \bullet employeeSalary(d) > 0.0 \checkmark$$

Invariant

$$
\begin{array}{|l}
\multicolumn{1}{l}{\_\ AddEmployee\ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \\
\Delta(employeeSalary) \\
newEmployee? : Employee \\
salary? : \mathbb{R} \\
\hline
salary? > 0.0 \\
newEmployee? \notin \mathrm{dom}\ \ employeeSalary \\
\boxed{employeeSalary' = employeeSalary \cup \{newEmployee? \mapsto salary?\}} \\
\hline
\end{array}
$$

Postcondition

AddEmployee(Roger, 100)

employeeSalary

{
(Syd, 90) ,
(David, 100)
}

dom employeeSalary

{ Syd, David }

ran employeeSalary

{ 90, 100 }

$$\forall\, d : \mathrm{dom}\ \ employeeSalary \bullet employeeSalary(d) > 0.0$$ ✓ **Invariant**

AddEmployee

$$\Delta(employeeSalary)$$
$$newEmployee? : Employee$$
$$salary? : \mathbb{R}$$

$$\boxed{\begin{array}{l} salary? > 0.0 \ \checkmark \\ newEmployee? \notin \mathrm{dom}\ \ employeeSalary \ \checkmark \end{array}}$$ **Precondition**

$$employeeSalary' = employeeSalary \cup \{newEmployee? \mapsto salary?\}$$

AddEmployee(Roger, 100)

employeeSalary

{
(Syd, 90) ,
(David, 100),
(Roger, 100)
}

dom employeeSalary

{ Syd, David, Roger }

ran employeeSalary

{ 90, 100 }

$$\forall\, d : \mathrm{dom}\ \ employeeSalary \bullet employeeSalary(d) > 0.0$$ ✓ **Invariant**

$$
\begin{array}{|l}
\hline
\textit{AddEmployee} \\
\Delta(employeeSalary) \\
newEmployee? : Employee \\
salary? : \mathbb{R} \\
\hline
salary? > 0.0 \\
newEmployee? \notin \mathrm{dom}\ \ employeeSalary \\
\boxed{employeeSalary' = employeeSalary \cup \{newEmployee? \mapsto salary?\}} \\
\hline
\end{array}
$$

Postcondition

DeleteEmployee(Syd)

employeeSalary	**dom employeeSalary**	**ran employeeSalary**
{ (Syd, 90) , (David, 100), (Roger, 100) }	{ Syd, David, Roger }	{ 90, 100 }

$$\forall\, d : \mathrm{dom}\ \ employeeSalary \bullet employeeSalary(d) > 0.0$$ **Invariant**

$$
\begin{array}{|l}
\hline
\textit{DeleteEmployee} \\
\Delta(employeeSalary) \\
who? : Employee \\
\hline
\boxed{who? \in \mathrm{dom}\ \ employeeSalary} \\
employeeSalary' = \{who?\} \mathbin{\lhd\mkern-14mu-} employeeSalary \\
\hline
\end{array}
$$

Precondition

DeleteEmployee(Syd)

employeeSalary

{
~~(Syd, 90)~~ ,
(David, 100),
(Roger, 100)
}

dom employeeSalary

{ ~~Syd,~~ David, Roger }

ran employeeSalary

~~{ 90,~~ 100 }

$$\forall\, d : \mathrm{dom}\ \ employeeSalary \bullet employeeSalary(d) > 0.0$$ ✓ **Invariant**

$$\begin{array}{|l}
\multicolumn{1}{l}{\underline{\ DeleteEmployee\ \hspace{20em}}}\\
\Delta(employeeSalary)\\
who? : Employee\\
\hline
who? \in \mathrm{dom}\ \ employeeSalary\\
\boxed{employeeSalary' = \{who?\} \lhd employeeSalary}\\
\hline
\end{array}$$

Postcondition

ModifySalary(David, 110)

employeeSalary

{
(David, 100),
(Roger, 100)
}

dom employeeSalary

{ David, Roger }

ran employeeSalary

{ 100 }

$$\forall\, d : \mathrm{dom}\ \ employeeSalary \bullet employeeSalary(d) > 0.0$$ ✓ **Invariant**

ModifySalary

$$\Delta(employeeSalary)$$
$$employee? : Employee$$
$$newSalary? : \mathbb{R}$$

$$newSalary? > 0.0$$ ✓
$$employee? \in \mathrm{dom}\ \ employeeSalary$$ ✓

Precondition

$$employeeSalary' = employeeSalary \oplus \{employee? \mapsto newSalary?\}$$

ModifySalary(David, 110)

employeeSalary

{
(David, 110),
(Roger, 100)
}

dom employeeSalary

{ David, Roger }

ran employeeSalary

{ 100, 110 }

$$\forall\, d : \mathrm{dom}\ employeeSalary \bullet employeeSalary(d) > 0.0$$ ✓ **Invariant**

ModifySalary

$$\Delta(employeeSalary)$$
$$employee? : Employee$$
$$newSalary? : \mathbb{R}$$

$$newSalary? > 0.0$$
$$employee? \in \mathrm{dom}\ employeeSalary$$
Postcondition
$$\boxed{employeeSalary' = employeeSalary \oplus \{employee? \mapsto newSalary?\}}$$

Example: CreditCard
Visibility list, constants, state and initialization

CreditCard

$$\upharpoonright (\mathit{Withdraw}, \mathit{Deposit}, \mathit{GetAvailableFunds})$$

$$\mathit{number} : \mathbb{N}$$
$$\mathit{limit} : \mathbb{R}$$

$$\mathit{limit} \in \{1000, 5000, 10000\}$$

$$\mathit{balance} : \mathbb{R}$$

$$\mathit{balance} + \mathit{limit} \geq 0$$

INIT

$$\mathit{balance} = 0$$

Operation Withdraw

$$
\begin{array}{|l}
\hline
\textit{Withdraw} \\
\Delta(balance) \\
amount? : \mathbb{R} \\
\hline
amount? > 0 \\
amount? \leq balance + limit \\
balance' = balance - amount? \\
\hline
\end{array}
$$

Operation Deposit

$$
\begin{array}{|l}
\hline
\textit{Deposit} \\
\Delta(balance) \\
amount? : \mathbb{R} \\
\hline
amount? > 0 \\
balance' = balance + amount? \\
\hline
\end{array}
$$

Operation GetAvailableFunds

$$\begin{array}{|l}
\multicolumn{1}{l}{\underline{\quad GetAvailableFunds \quad}} \\
amount! : \mathbb{R} \\
\hline
amount! = balance + limit \\
\hline
\end{array}$$

Example: CreditCard2
Subclassifying CreditCard

$CreditCard2$

$\upharpoonright (Withdraw, Deposit, GetAvailableFunds)$

$CreditCard$

$withdrawals : \mathbb{N}$

$INIT$

$withdrawals = 0$

$Withdraw$

$\Delta(withdrawals)$

$withdrawals' = withdrawals + 1$

Example: CreditCompany Visibility list, state and initialization

$$
\begin{array}{l}
\textit{CreditCompany} \\
\hline
\upharpoonright (\mathit{AddAccount}, \mathit{DeleteAccount}) \\
\hline
\mathit{accounts} : \mathbb{P}\, \mathit{CreditCard} \\
\mathit{count} : \mathbb{N} \\
\hline
\forall\, a_i, a_j : \mathit{accounts} \bullet a_i.\mathit{number} \neq a_j.\mathit{number} \\
\mathit{count} = \#\mathit{accounts} \\
\hline
\textit{INIT} \\
\hline
\mathit{accounts} = \{\} \\
\hline
\end{array}
$$

Operation AddAccount

$$
\begin{array}{|l}
\hline
\textit{AddAccount} \\
\Delta(accounts) \\
account? : CreditCard \\
\hline
account? \notin accounts \\
accounts' = accounts \cup \{account?\} \\
count' = count + 1 \\
\hline
\end{array}
$$

Operation DeleteAccount

$$
\begin{array}{|l}
\multicolumn{1}{l}{\textit{DeleteAccount}} \\
\hline
\Delta(accounts) \\
account? : CreditCard \\
\hline
account? \in accounts \\
accounts' = accounts \setminus \{account?\} \\
count' = count - 1 \\
\hline
\end{array}
$$

Inheritance and subtyping

- Each class defines a type and all instances of the class constitute legitimate values of that type.
- Every instance of a subclass is also an instance of a superclass, but not vice-versa.
- The type defined by the subclass is a subset of the type defined by its superclasses as the set of all instances of a subclass is included in the set of all instances of its superclass.

Polymorphism

- In

  $$account : \downarrow Account$$

  variable *account* can hold an instance of *Account* or any of its subclasses.

- The declaration

  $$accounts : \mathbb{P} \downarrow Account$$

  indicates that *account* is a set of elements from *Account* as well as from any of its subclasses.

Example: Bank
[To be covered in tutorials this week]

Account

$\upharpoonright (accountNumber, Deposit, Withdraw)$

SavingsAccount

$\upharpoonright (accountNumber, balance, Deposit, Withdraw)$
$Account$

Bank

$accounts : \mathbb{P} \downarrow Account$

$\forall a_1, a_2 : accounts \bullet a_1.accountNumber = a_2.accountNumber \Leftrightarrow a_1 = a_2$

Cancellation and redefinition of features through renaming

$A[T]$

$x : T$

$y : \mathbb{P}\, T$

$x \in y$

Op

$\Delta(x)$

$x? : T$

$x? \in y$

$x' = x?$

Cancellation and redefinition of features through renaming /cont.

$B[T]$

$\upharpoonright (Op)$

$A[y1/y, Op1/Op]$

$$
\begin{array}{|l}
y : \text{bag } T \\
\hline
x \in y
\end{array}
$$

Op

$$
\begin{array}{|l}
\Delta(x) \\
x? : T \\
\hline
x? \in y \\
x' = x?
\end{array}
$$

Explicit redefinition and removal of operations

$$
\begin{array}{|l}
\multicolumn{1}{l}{\textit{DoublePushStack}[T]} \\
\hline
\upharpoonright (Push, Pop, Top) \\
BoundedStack[T][\mathbf{redef}\ Push] \\
\\
\begin{array}{|l}
\multicolumn{1}{l}{Push} \\
\hline
\Delta(elements, count) \\
item? : T \\
\hline
count < capacity - 1 \\
elements' = \langle item?, item? \rangle \frown elements \\
count' = count + 2 \\
\hline
\end{array} \\
\hline
\end{array}
$$

Explicit redefinition and removal of operations /cont.

$$\begin{array}{|l}
\hline
\textit{OnlyPushStack}[T] \\
\hline
\upharpoonright (\textit{Push}) \\
\textit{Stack}[T][\textbf{remove } \textit{Pop}] \\
\hline
\end{array}$$