

Animation for Computer Games COMP 477/6311

Prof. Tiberiu Popa

Physics-based Animation

- We looked at two types of physics-based animation
 - Springs
 - Fluids
- Can you see a pattern that can serve as a template for other physics-based animations?
- For Lagrangian yes

Bender, J., Müller, M., Otaduy, M. A., & Teschner, M. (2013, May). Position-based Methods for the Simulation of Solid Objects in Computer Graphics. In *Eurographics (STARs)* (pp. 1-22).



Physics-based Animation

- Lagrangian
 - I. Develop the specific equations from F=ma
 - I. Derive internal and external forces
 - Derive acceleration
 - 2. Find a suitable representation/discretization
 - 3. Solve the resulting ODE using time integration techniques
 - 4. ?????



Physics-based Animation

- Lagrangian
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 - 2. Find a suitable representation/discretization
 - 3. Solve the resulting ODE using time integration techniques
 - 4. Contact and collision (coming soon to a lecture near you!!!)



Easier said then done

Cloth is an extremely complex object with very complex

behaviour





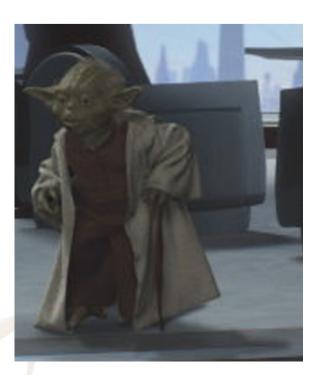


Bridson, R., Fedkiw, R., & Anderson, J. (2002, July). Robust treatment of collisions, contact and friction for cloth animation. In *Proceedings of the 29th annual conference on Computer graphics and interactive techniques* (pp. 594-603).



- Divide and conquer
 - I. Physical behavior
 - 2. Collisions and contact





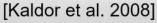




- I. Physical behavior
 - Many materials
 - Different physical properties









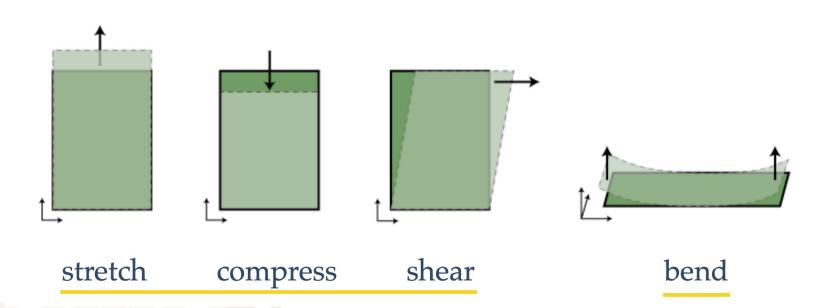
[Cirio et al. 2016]



[Sánchez-Banderas et al. 2020 💋]



Internal forces of cloth





Elastic objects \rightarrow tend to return to original shape when deformed Some internal forces push the object

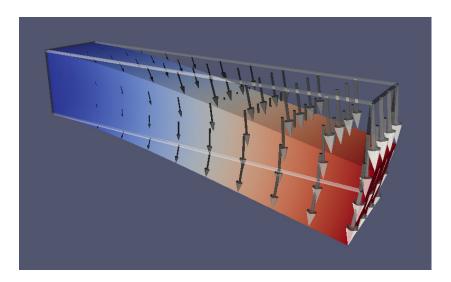
Where do those forces come from?

Energy → ability of an object to do work

Elastic energy \rightarrow stored in the material



https://www.solarschools.net/knowledge-bank/energy/types/elastic



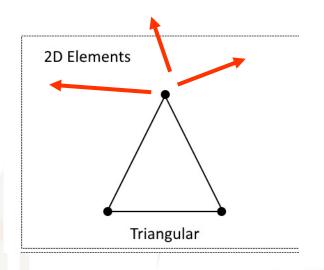
https://fenicsproject.org/pub/tutorial/html/. ftut1008.html

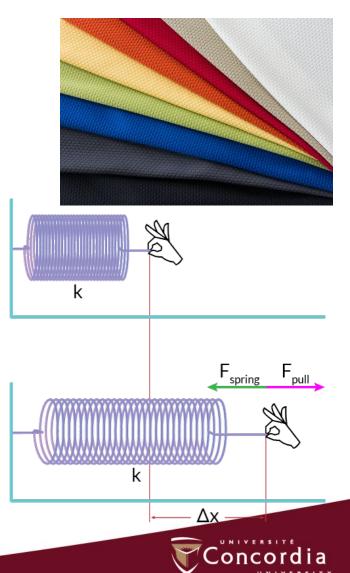


- I. Physical behavior
- Elastic behavior
 - Elastic potential energy (E or W)
 - Accumulates when deformed
 - ~sum of all forces at intermediate steps

$$-E = \int F dx$$

— QI) What kind of function is E?

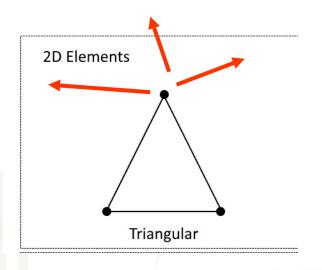


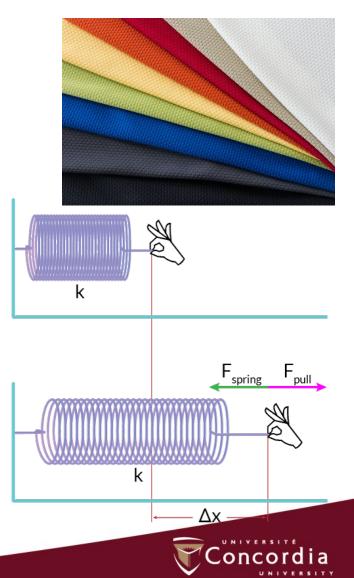


- I. Physical behavior
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$$-E = \int F dx$$

— Q2) What does E depend on?

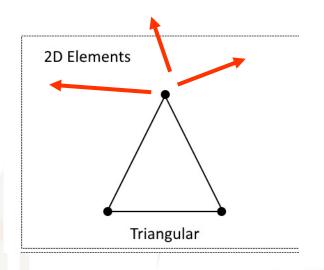


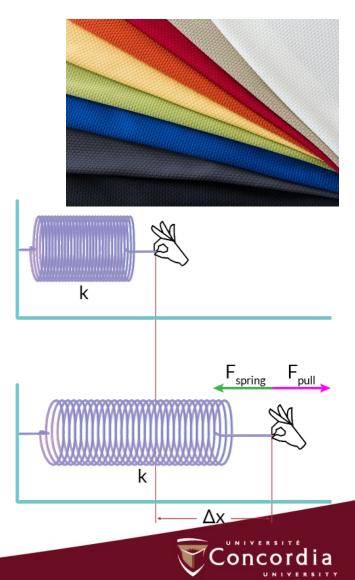


- I. Physical behavior
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$$-E = \int F dx$$

— Q3) What kind of function is E?

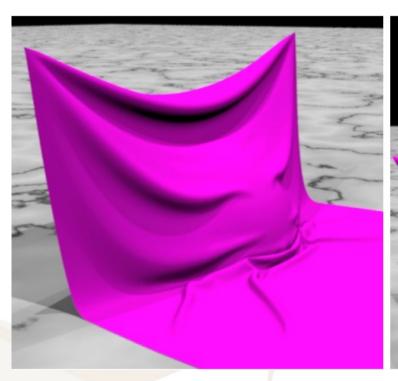


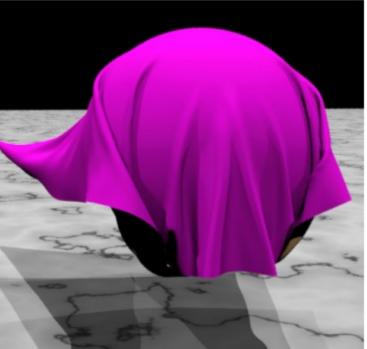


- Ingredients
 - Geometric and Physical model
 - How do I represent the geometry and forces?
 - What are the forces?
 - Time Integration scheme
 - Collision handling
 - Contact handling



- Triangular mesh
- How do I define the forces?



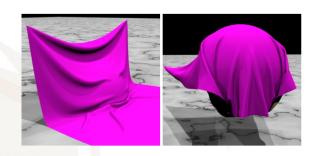




- Forces vs. Energy
- Energy
 scalar field (potential at every point in space)

$$-\frac{\partial E}{\partial X} = -F_{int}$$

- New equation: $\ddot{X} = M^{-1}(-\frac{\partial E}{\partial X} + F_{ext})$
- Why bother?

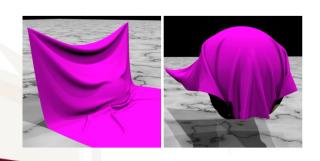




$$-\ddot{X} = M^{-1}(-\frac{\partial E}{\partial X} + F_e)$$

- Acceleration Internal forces External forces
- We can compute/estimate internal energy of the cloth:

$$-E = E_{stretch} + E_{shear} + E_{bending}$$

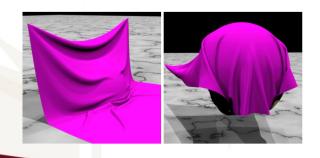




 We can compute/estimate internal energy of the cloth:

$$-E = E_{stretch} + E_{shear} + E_{bending}$$

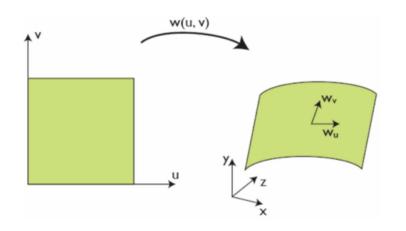
- Properties of energy:
 - Non-negative
 - 0 only in the rest pose

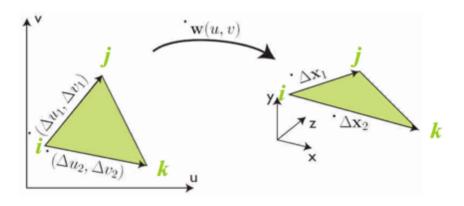




Baraff-Witkin model

Stretch





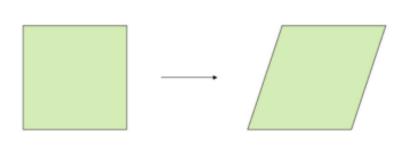
$$\mathbf{w}_u = \partial \mathbf{w}/\partial u$$
 and $\mathbf{w}_v = \partial \mathbf{w}/\partial v$

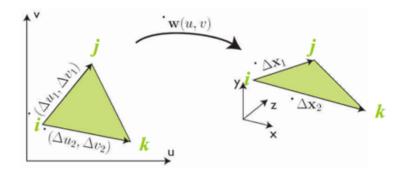
$$\mathbf{C}(\mathbf{x}) = a \begin{pmatrix} \|\mathbf{w}_{u}(\mathbf{x})\| - 1 \\ \|\mathbf{w}_{v}(\mathbf{x})\| - 1 \end{pmatrix}$$



Baraff-Witkin model

Shear





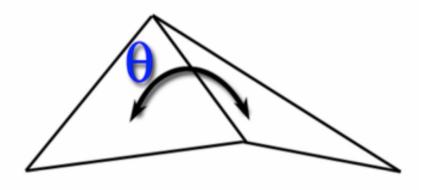
$$\mathbf{w}_u = \partial \mathbf{w}/\partial u$$
 and $\mathbf{w}_v = \partial \mathbf{w}/\partial v$

$$C(\mathbf{x}) = a\mathbf{w}_u(\mathbf{x})^T \mathbf{w}_v(\mathbf{x})$$



Baraff-Witkin model

- Bending
- Constraint the angle



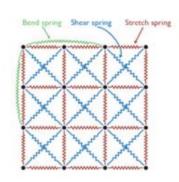


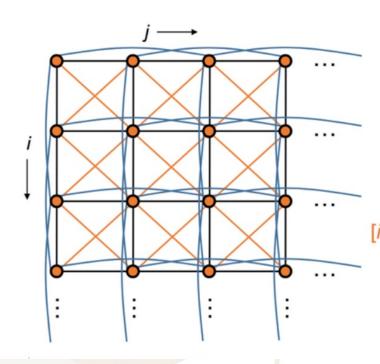
- From constraints to energy
- Take the L2 norm: $\|C(x)\|_2$
- Is it physically accurate?
- Not always, but often is plausible

Baraff, D., & Witkin, A. (1998, July). Large steps in cloth simulation. In *Proceedings of the 25th annual conference on Computer graphics and interactive techniques* (pp. 43-54).



Spring systems and spring forces





Types of springs

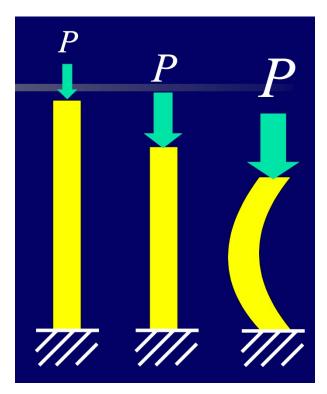
Structural ——
$$[i, j] - [i, j + 1]; [i, j] - [i + 1, j]$$
Shear ——
$$[i, j] - [i + 1, j + 1]; [i + 1, j] - [i, j + 1]$$
Flexion (bend) ——

[i, j]—[i, i + 2]; [i, j]—[i + 2, j]



- Explicit Euler does not work
- Implicit methods such as Implicit Euler
- Other problems:
 - Cloth not-really hookian



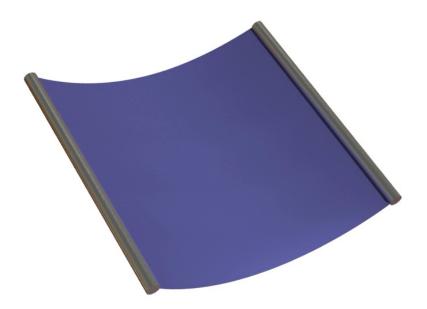


Choi, K. J., & Ko, H. S. (2005). Stable but responsive cloth. In ACM SIGGRAPH 2005 Courses (pp. 1-es).



- Baraff-Witkin (hookian)
 - Shearing term helps achieving wrinkling
 - Numerical problems under compression
 - Hessian of the energies is indefinite under compression
- As Rigid as Possible (ARAP)
 - Energy that explicitly avoids compression
 - Numerically stable
 - Results in stiff clothing
- Superelastic material
 - St. Venant Kirchhoff (StVK)
 - Also has some (other) numerical problems

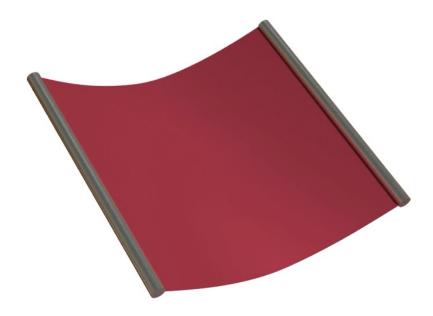
Concordia







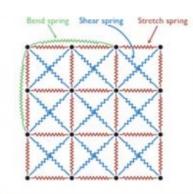






Collision Handling

- Collision detection and handling
 - Penalty method
 - Add "fake" forces
 - Impulse method
 - Change velocities





Time Integration

Bend spring Shear spring Stretch spri

- Collision detection and handling
 - Penalty method
 - Cons: does not prevent interpenetration
 - Impulse method
 - Numerical instabilities as velocity field is not continuous anymore
 - Repulsion forces
 - Main Idea: slow down particles before they get to the surface
 - Tangential (sliding) motion
 - Add friction forces

