COMP 472: Artificial Intelligence
Machine Learning part #2
Decision Trees video #4

- Russell & Norvig: Sections 19.3

Today

1. Introduction to ML
2. Naïve Bayes Classification
   a. Application to Spam Filtering
3. Decision Trees 

4. (Evaluation
5. Unsupervised Learning)
6. Neural Networks
   a. Perceptrons
   b. Multi Layered Neural Networks

Guess Who?

P1

P2

DT

yes
no
yes
no
class
class

Decision Trees

Ross Quinlan

- Simplest, but most successful form of learning algorithm
- Very well-know algorithm is ID3 (Quinlan, 1987) and its successor C4.5

3-48 . . .

1. Rank features based on how good they are to indicate the result
2. Put the most discriminating feature as a node (as a question) of a tree
3. Split the training examples so that those with different values for the chosen feature are in a different set
4. Repeat the same process with the nest most discriminating feature

F1=? yes no class1 F3=? yes no class2 class1

http://www.rulequest.com/Personal/

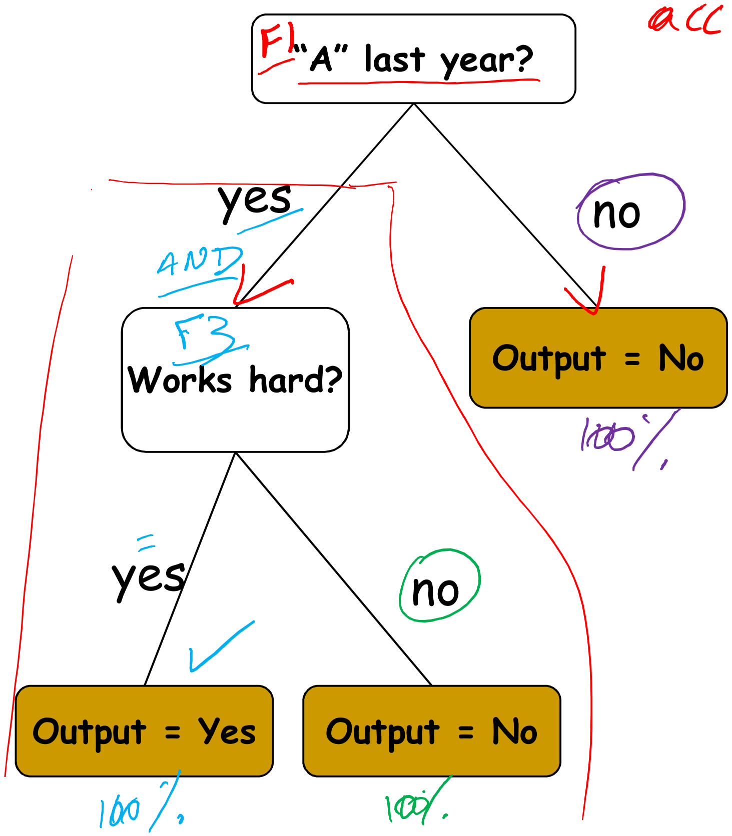
Example 1

Info on last year's students to determine if a student will get an 'A' this year

	Features (X)				Output f(X) class
Student	**'A' last year?** F1	**Black hair?** F2	**Works hard?** F3	**Drinks?** F4	**'A' this year?**
X1: Richard	Yes	Yes	No	Yes	**No**
X2: Alan	Yes	Yes	Yes	No	**Yes**
X3: Alison	No	No	Yes	No	**No**
X4: Jeff	No	Yes	No	Yes	**No**
X5: Gail	Yes	No	Yes	Yes	**Yes**
X6: Simon	No	Yes	Yes	Yes	**No**

Example 1

A random decision tree that fits the dataset with 100% accuracy



same data set as previous slide

	Features				Output f(X)
Student	'A' last year? F1	Black hair?	Works hard? F3	Drinks ?	'A' this year?
Richard	Yes	Yes	No	Yes	No
Alan	Yes	Yes	Yes	No	Yes
Alison	No	No	Yes	No	No
Jeff	No	Yes	No	Yes	No
Gail	Yes	No	Yes	Yes	Yes
Simon	No	Yes	Yes	Yes	No

- fits the training set with 100% accuracy
- but the features are chosen at random so there might be a shorter DT // better

Example 2: The Restaurant

- Goal: learn whether one should wait for a table
- Attributes

10 features

1. Alternate: another suitable restaurant nearby
2. Bar: comfortable bar for waiting
3. Fri/Sat: true on Fridays and Saturdays
4. Hungry: whether one is hungry
5. Patrons: how many people are present (none, some, full)
6. Price: price range ($, $$, $$$)
7. Raining: raining outside
8. Reservation: reservation made
9. Type: kind of restaurant (French, Italian, Thai, Burger)
10. WaitEstimate: estimated wait by host (0-10 mins, 10-30, 30-60, >60)

Example 2: The Restaurant

- Training data:

10 features.

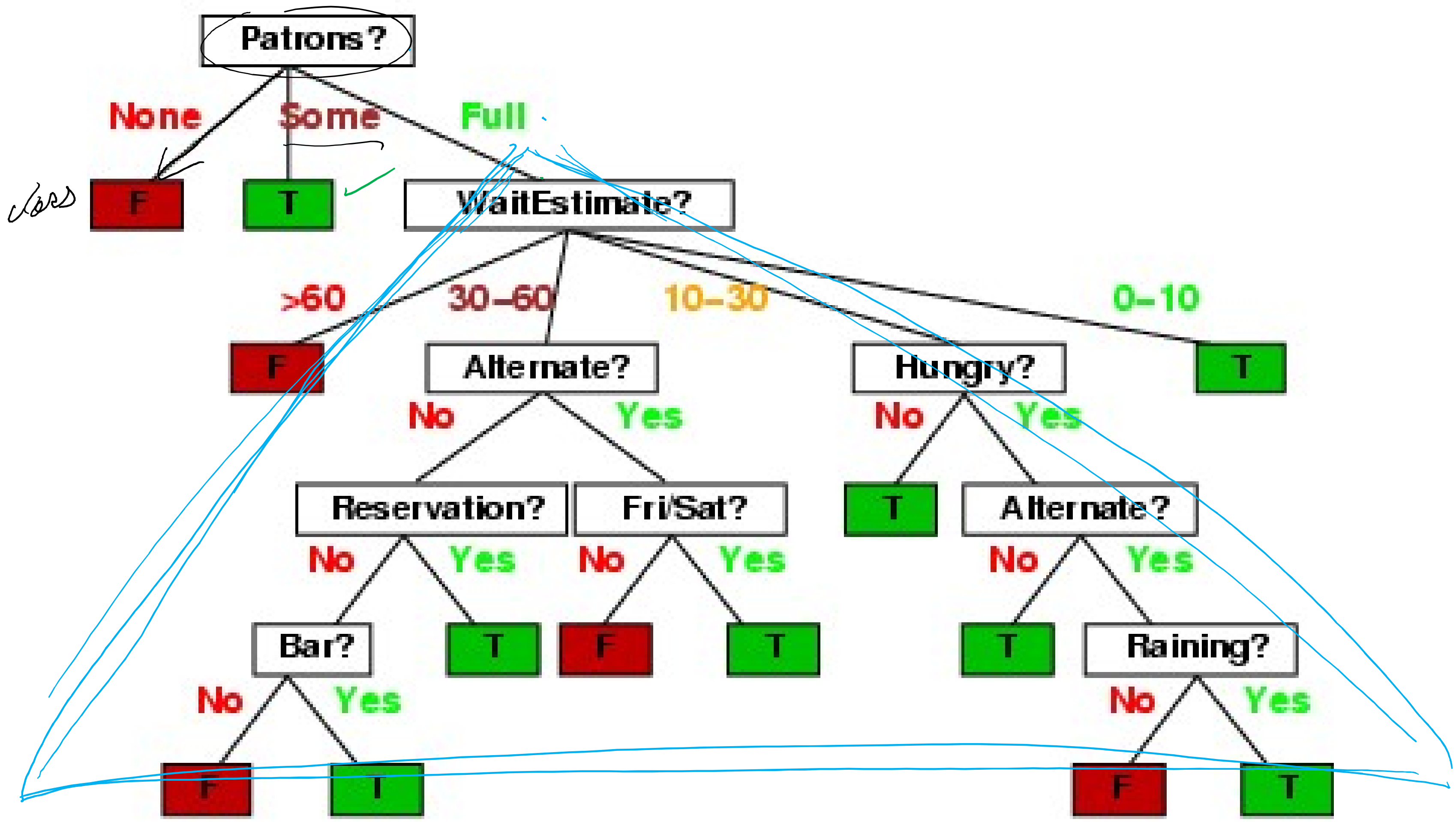
f(x) class

12 instances

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*Wait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

source: Norvig (2003)

A First Decision Tree



- But is it the best decision tree we can build?

source: Norvig (2003)

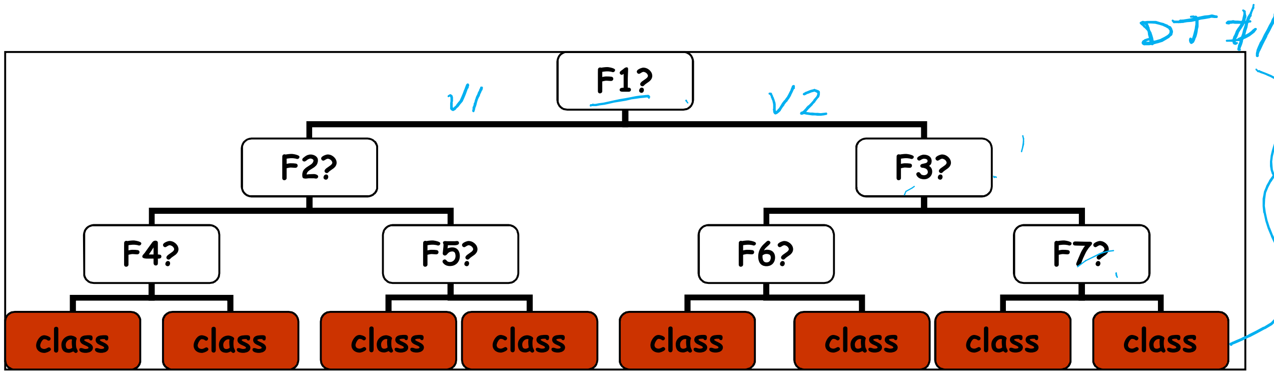
Ockham's Razor

It is vain to do more than can be done with less… Entities should not be multiplied beyond necessity. *[Ockham, 1324]*

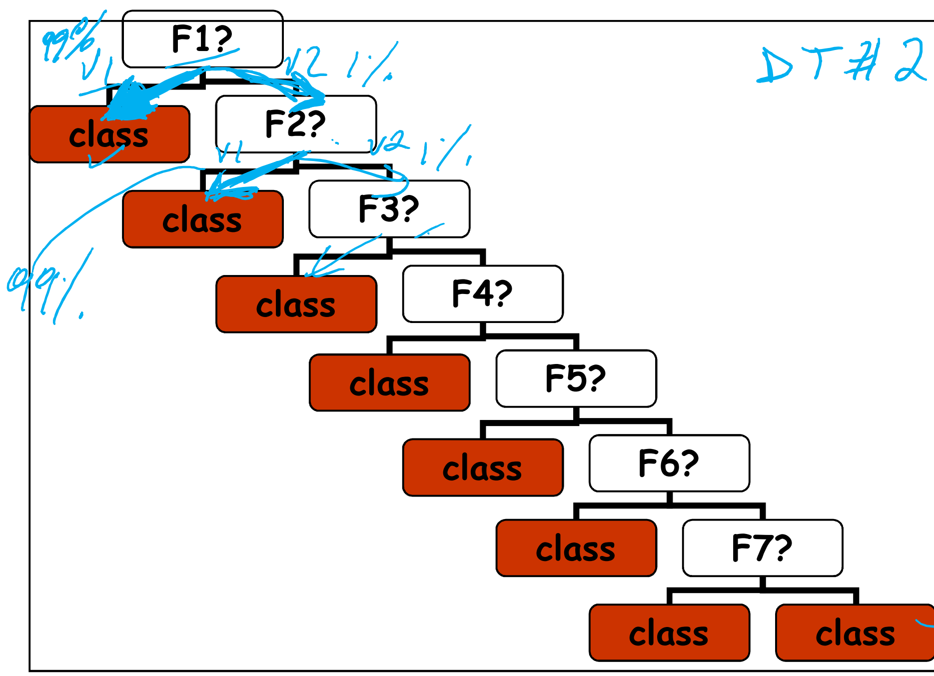
- In other words… always favor the simplest answer that correctly fits the training data
- i.e. the smallest tree on average
- This type of assumption is called inductive bias
  - inductive bias = making a choice beyond what the training instances contain

https://www.britannica.com/topic/Occams-razor

Which Tree is Best?



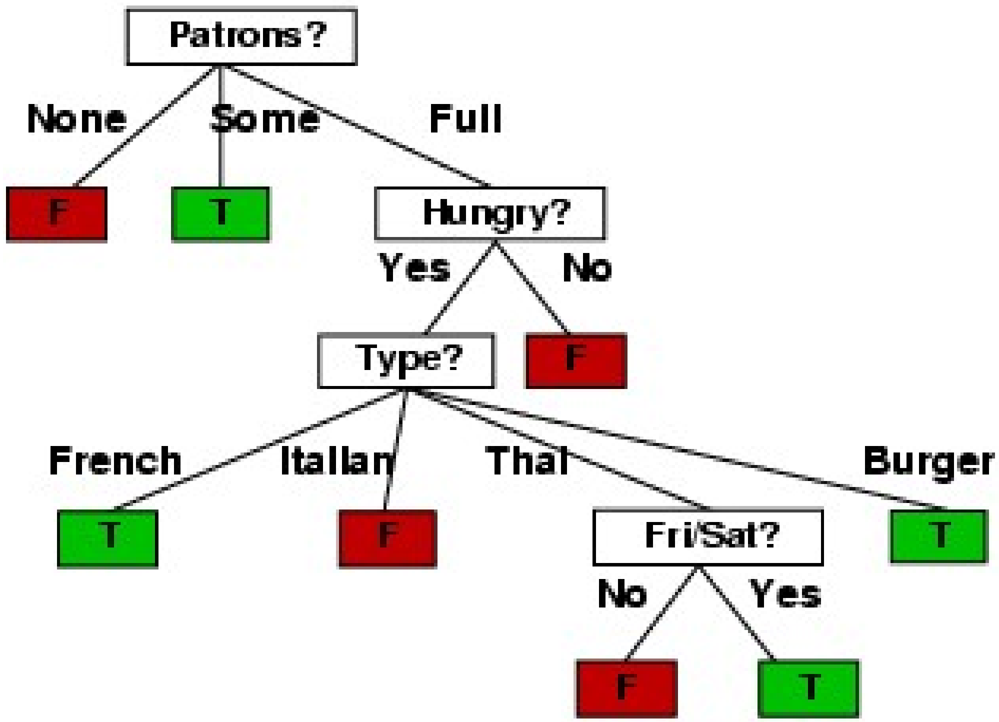
always asks 3 questions



may be the shortest tree on average depending on the probability of each branch

A Better Decision Tree

- 4 tests instead of 9
- 11 branches instead of 21



source: Norvig (2003)

Choosing the Next Feature

- The key problem is choosing which feature to split a given set of examples
- Different measures have been proposed
- ID3 uses Maximum Information-Gain
  - i.e. we choose the feature that has the largest information gain
  - we expect this feature to result in the smallest tree on average
  - based on information theory

Essential Information Theory

- Developed by claude Shannon in the 40s
- Shannon developed the notion of entropy (aka information content) of a random variable (RV)

- Entropy measures how "predictable" a RV is
  - If you already have a good idea about the answer (e.g. 90/10 split)
    → low entropy // sure thing
  - If you have no idea about the answer (e.g. 50/50 split)
    → high entropy // total chaos

Entropy

x_i = 'e' …

x_n n …

z

x

hello

- Let X be a discrete RV with *i* possible outcomes x_i
- Entropy (or information content) of X

$$H(X) = -\sum_{i=1}^{n} p(x_i)\log_2 p(x_i)$$

base 2 always

- measures:
  - the *amount of information* in a RV
  - *average uncertainty* of a RV
  - *average length of a message* needed to transmit an outcome x_i of that RV when encoded optimally over a binary channel
- measured in bits

Entropy of a Coin Toss

$$H(X) = -\sum_{x_i \in X} p(x_i)\log_2 p(x_i)$$

Entropy (or information content)

prob x_1 = "head"

prob x_2 = "tail"

$$H(\text{fair coin toss}) = -\sum_{x_i \in X} p(x_i)\log_2 p(x_i) = H\left(\frac{1}{2}, \frac{1}{2}\right)$$

$$= -\left(\frac{1}{2}\log_2\frac{1}{2} + \frac{1}{2}\log_2\frac{1}{2}\right) = 1 \text{ bit}$$

head

tail

entropy of a fair coin toss (the RV) with 2 possible outcomes, each with a probability of 1/2

a RV with only 2 outcomes x_1 and x_2 will have $1 \geq H(X) \geq 0$

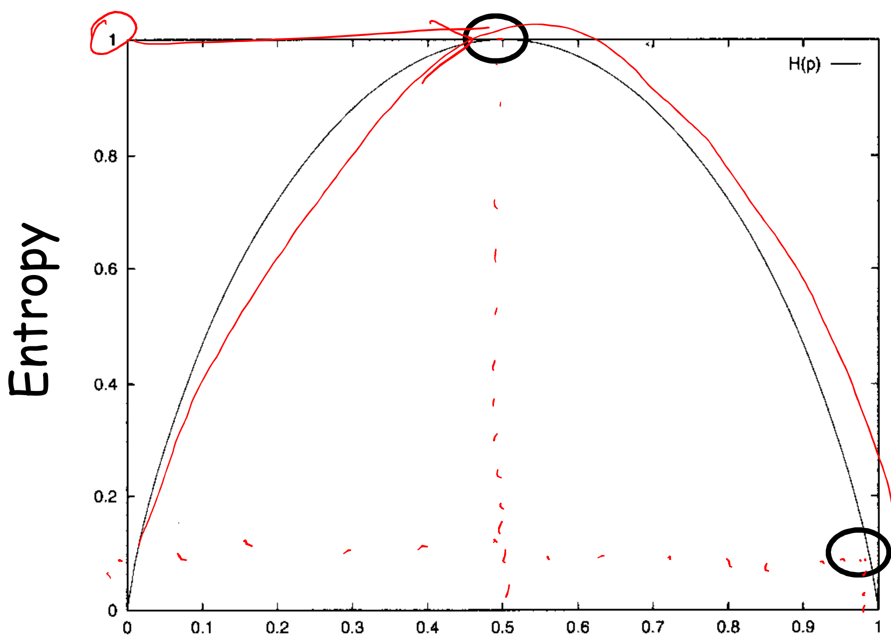
// sure thing

// total chaos

(unpredictable)

Example: The Coin Toss

- Fair coin: $H(X) = -\sum_{i=1}^{n} p(x_i)\log_2 p(x_i) = -\left(\overbrace{\frac{1}{2}\log_2\frac{1}{2}}^{\text{head}} + \overbrace{\frac{1}{2}\log_2\frac{1}{2}}^{\text{tail}}\right) = 1\,\text{bit}$
- Rigged coin: $H(X) = -\sum_{i=1}^{n} p(x_i)\log_2 p(x_i) = -\left(\underbrace{\frac{99}{100}\log_2\frac{99}{100}}_{\text{head}} + \frac{1}{100}\log_2\frac{1}{100}\right) = 0.08\,\text{bits}$



fair coin -> high entropy

rigged coin -> low entropy

P(head)

So what?

for each attribute/feature I will compute how much knowing its values will reduce the entropy of the class RV

- Training data:

10 features

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*Wait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

x_1

x_2

entropy(Target Wait) = 1

source: Norvig (2003)

Information Gain

- information gain
  - measure the entropy reduction of a RV, once a piece of information is known
  - used to measure the "discriminating power" of an attribute A given a data set S

  - Let Values(A) = the set of values that attribute A can take
  - Let S_v = the set of examples in the data set which have value v for attribute A (for each value v from Values(A))

// see slide 15

information gain (or entropy reduction)

$$gain(S, A) = H(S) - H(S|A)$$

$$= H(S) - \sum_{v \in values(A)} \frac{|S_v|}{|S|} \times H(S_v)$$

Some Intuition

H(output) = 1 // total chaos

3 features

Size	Color	Shape	Output
Big	Red	Circle	+
Small	Red	Circle	+
Small	Red	Square	-
Big	Blue	Circle	-

- *Size* is the least discriminating attribute (i.e. smallest information gain)

- *Shape* and *color* are the most discriminating attributes (i.e. highest information gain)

shape
color } more discriminating than size

A Small Example (1)

let's try F2 = color

Size	Color (F2)	Shape	Output
Big	Red	Circle	+
Small	Red	Circle	+
Small	Red	Square	-
Big	Blue	Circle	-

x_1

x_2

Values(Color) = {red,blue}

Color

red: 2+ 1-

blue: 0+ 1-

$$gain(S, Color) = H(S) - \sum_{v \in values(Color)} \frac{|S_v|}{|S|} \times H(S_v)$$

entropy of output

$$H(S) = -\left(\frac{2}{4}\log_2\frac{2}{4} + \frac{2}{4}\log_2\frac{2}{4}\right) = 1$$ // total chaos, not knowing anything

for each v of Values(Color)

- $H(S|Color = red) = H\left(\frac{2}{3}, \frac{1}{3}\right) = -\left(\frac{2}{3}\log_2\frac{2}{3} + \frac{1}{3}\log_2\frac{1}{3}\right) = 0.918$
- $H(S|Color = blue) = H(1, 0) = -\left(\frac{1}{1}\log_2\frac{1}{1}\right) = 0$ // sure thing

$$H(S|Color) = \frac{3}{4}(0.918) + \frac{1}{4}(0) = 0.6885$$ knowing the color

Red + Blue

$$gain(Color) = H(S) - H(S|Color) = 1 - 0.6885 = 0.3115$$

Note: by definition,

- $Log_2 0 = -\infty$
- $0\log_2 0$ is 0

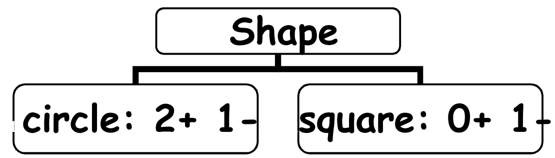
1 time out of 4 we have a blue

3 times out of 4 we have a red

A Small Example (2)

F3

Size	Color	Shape	Output
Big	Red	Circle	+
Small	Red	Circle	+
Small	Red	Square	-
Big	Blue	Circle	-



$$H(S) = -\left(\frac{2}{4}\log_2\frac{2}{4} + \frac{2}{4}\log_2\frac{2}{4}\right) = 1$$

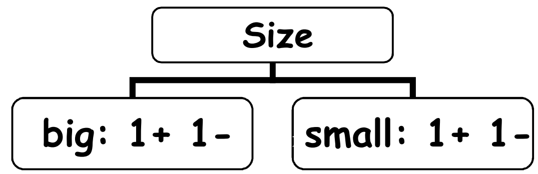
$$H(S \mid Shape) = \frac{3}{4}(0.918) + \frac{1}{4}(0) = 0.6885$$

$$gain(Shape) = H(S) - H(S \mid Shape) = 1 - 0.6885 = 0.3115$$

A Small Example (3)

F1

Size	Color	Shape	Output
Big	Red	Circle	+
Small	Red	Circle	+
Small	Red	Square	-
Big	Blue	Circle	-



$$H(S) = -\left(\frac{2}{4}\log_2\frac{2}{4} + \frac{2}{4}\log_2\frac{2}{4}\right) = 1$$

H(Output) without knowing any feature

$H(S|Size) = \frac{2}{4}(1) + \frac{2}{4}(1)$

gain(Size) = H(S) – H(S|Size) = 1-1=0

H(output/Size) knowing the size

A Small Example (4)

Size	Color	Shape	Output
Big	Red	Circle	+
Small	Red	Circle	+
Small	Red	Square	-
Big	Blue	Circle	-

$$\boxed{\begin{aligned} gain(Shape) &= 0.3115 \\ gain(Color) &= 0.3115 \\ gain(Size) &= 0 \end{aligned}}$$

- So first separate according to either *color* or *shape* (root of the tree)

A Small Example (4)

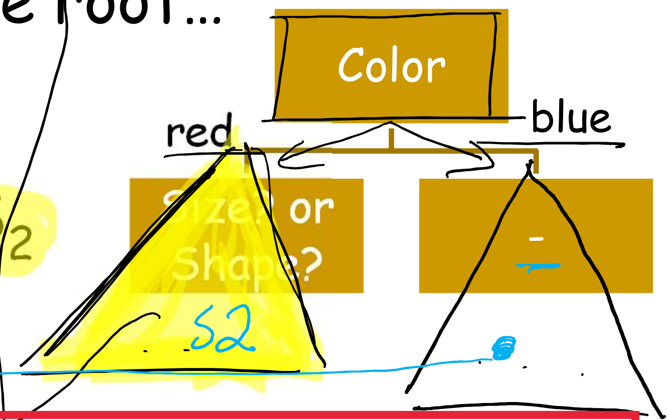
all instances in the data set

- Let's assume we pick *Color* for the root…

Size	Color	Shape	Output
Big	Red	Circle	+
Small	Red	Circle	+
Small	Red	Square	-
Big	Blue	Circle	-

S2

S_2



only use the instances with color = red

$$H(S_2) = -\left(\frac{2}{3}\log_2\frac{2}{3} + \frac{1}{3}\log_2\frac{1}{3}\right)$$

only use the instances with color = blue

for each v of Values(Size)

$$H(S_2 \mid Size = big) = H\left(\frac{1}{1}, \frac{0}{1}\right) = 0$$

$$H(S_2 \mid Size = small) = H\left(\frac{1}{2}, \frac{1}{2}\right) = 1$$

$$H(S_2 \mid Size) = \frac{1}{3}(0) + \frac{2}{3}(1)$$

$$gain(Size) = H(S_2) - H(S_2 \mid Size)$$

for each v of Values(Shape)

$$H(S_2 \mid Shape = circle) = H\left(\frac{2}{2}, \frac{0}{2}\right) = 0$$

$$H(S_2 \mid Shape = square) = H\left(\frac{0}{1}, \frac{1}{1}\right) = 0$$

$H(S_2 \mid Shape)$

$$gain(Shape) = H(S_2) - H(S_2 \mid Shape)$$

Highest gain is the root of the subtree

Back to the Restaurant

pick max info gain

- Training data:

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*Wait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

source: Norvig (2003)

The Restaurant Example

$gain(alt) = ...$ $gain(bar) = ...$ $gain(fri) = ...$ $gain(hun) = ...$

∀ features

$$gain(pat) = 1 - \left(\frac{2}{12} \times H\left(\frac{0}{2}, \frac{2}{2}\right) + \frac{4}{12} \times H\left(\frac{0}{4}, \frac{4}{4}\right) + \frac{6}{12} \times H\left(\frac{2}{6}, \frac{4}{6}\right) \right)$$

$$= 1 - \left(\frac{2}{12} \times -\left(\frac{0}{2} \log_2 \frac{0}{2} + \frac{2}{2} \log_2 \frac{2}{2} \right) + \frac{4}{12} \times -\left(\frac{0}{4} \log_2 \frac{0}{4} + \frac{4}{4} \log_2 \frac{4}{4} \right) + \ldots \right) \approx 0.541 \text{bits}$$

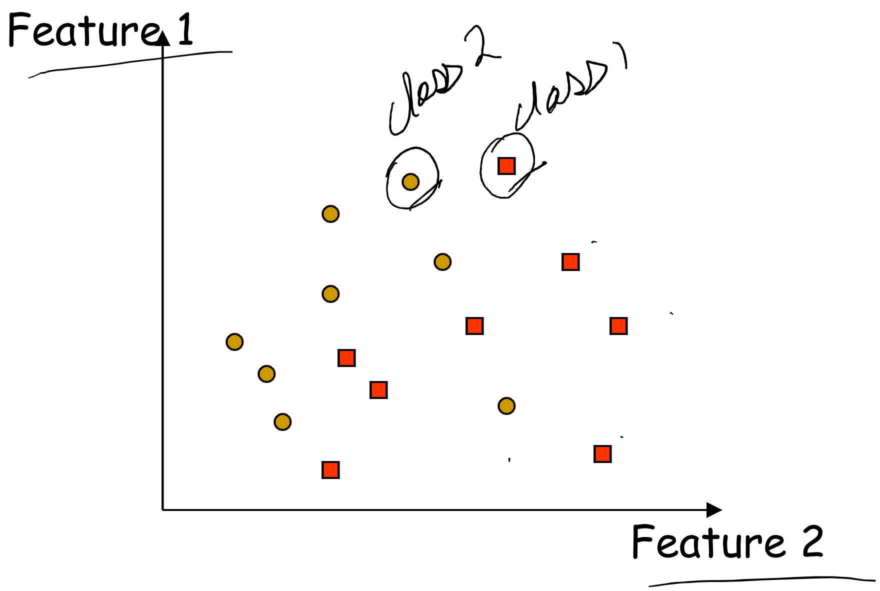
$gain(price) = ...$ $gain(rain) = ...$ $gain(res) = ...$

$$gain(type) = 1 - \left(\frac{2}{12} \times H\left(\frac{1}{2}, \frac{1}{2}\right) + \frac{2}{12} \times H\left(\frac{1}{2}, \frac{1}{2}\right) + \frac{4}{12} \times H\left(\frac{2}{4}, \frac{2}{4}\right) + \frac{4}{12} \times H\left(\frac{2}{4}, \frac{2}{4}\right) \right) = 0 \text{ bits}$$

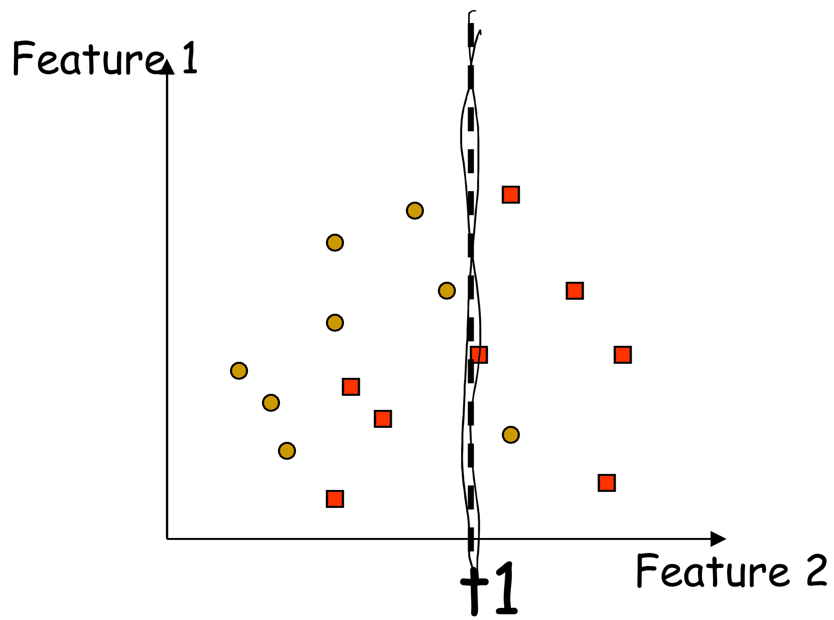
$gain(est) = ...$

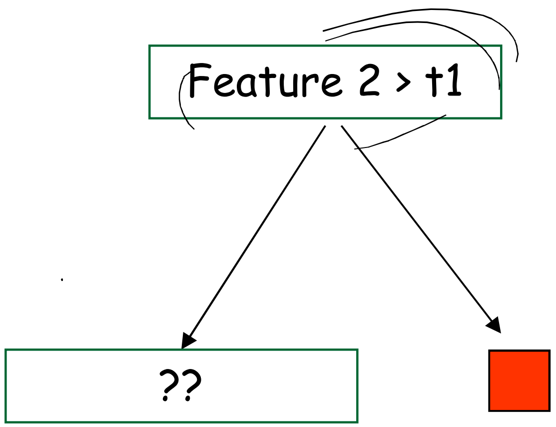
- Attribute pat (Patron) has the highest gain, so root of the tree should be attribute *Patrons*
- do recursively for subtrees

Decision Boundaries of Decision Trees

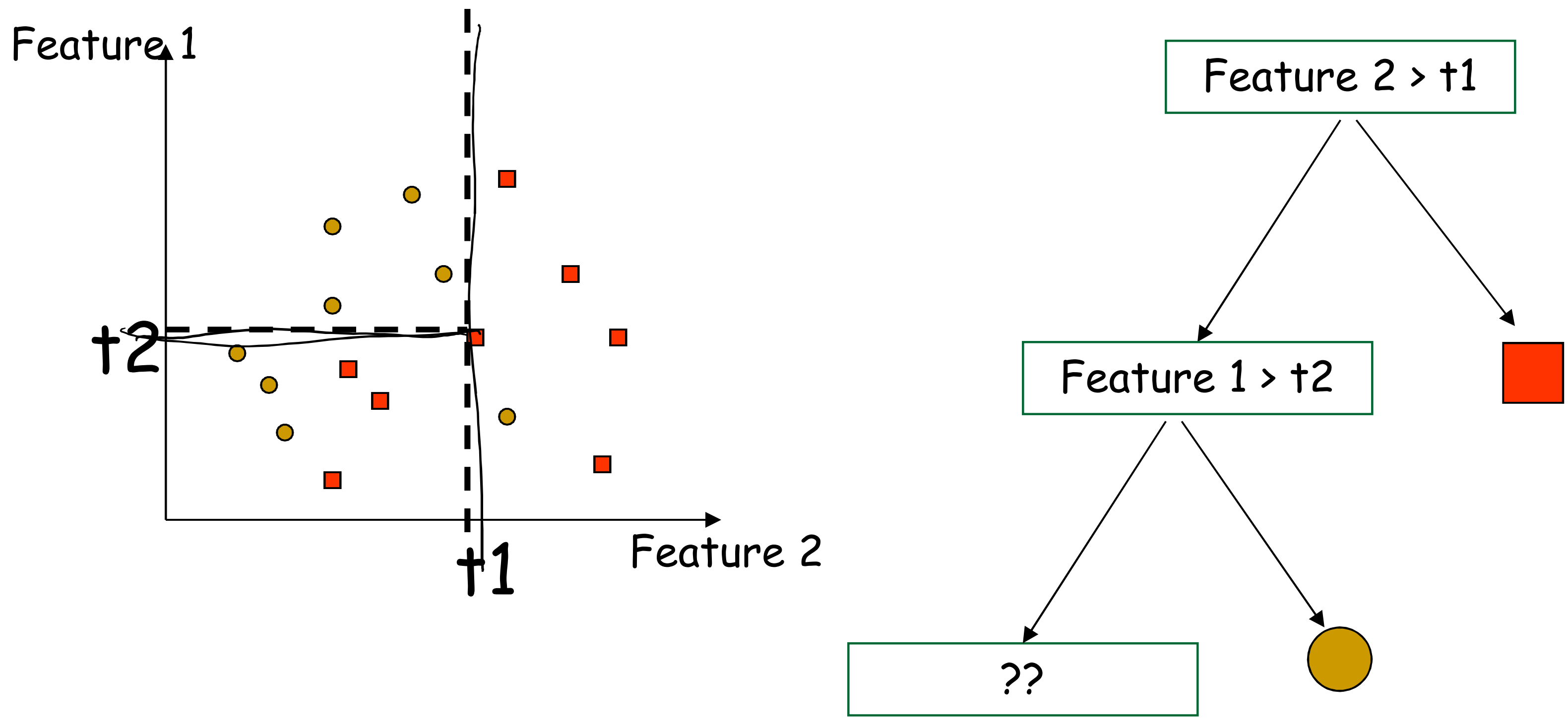


Decision Boundaries of Decision Trees

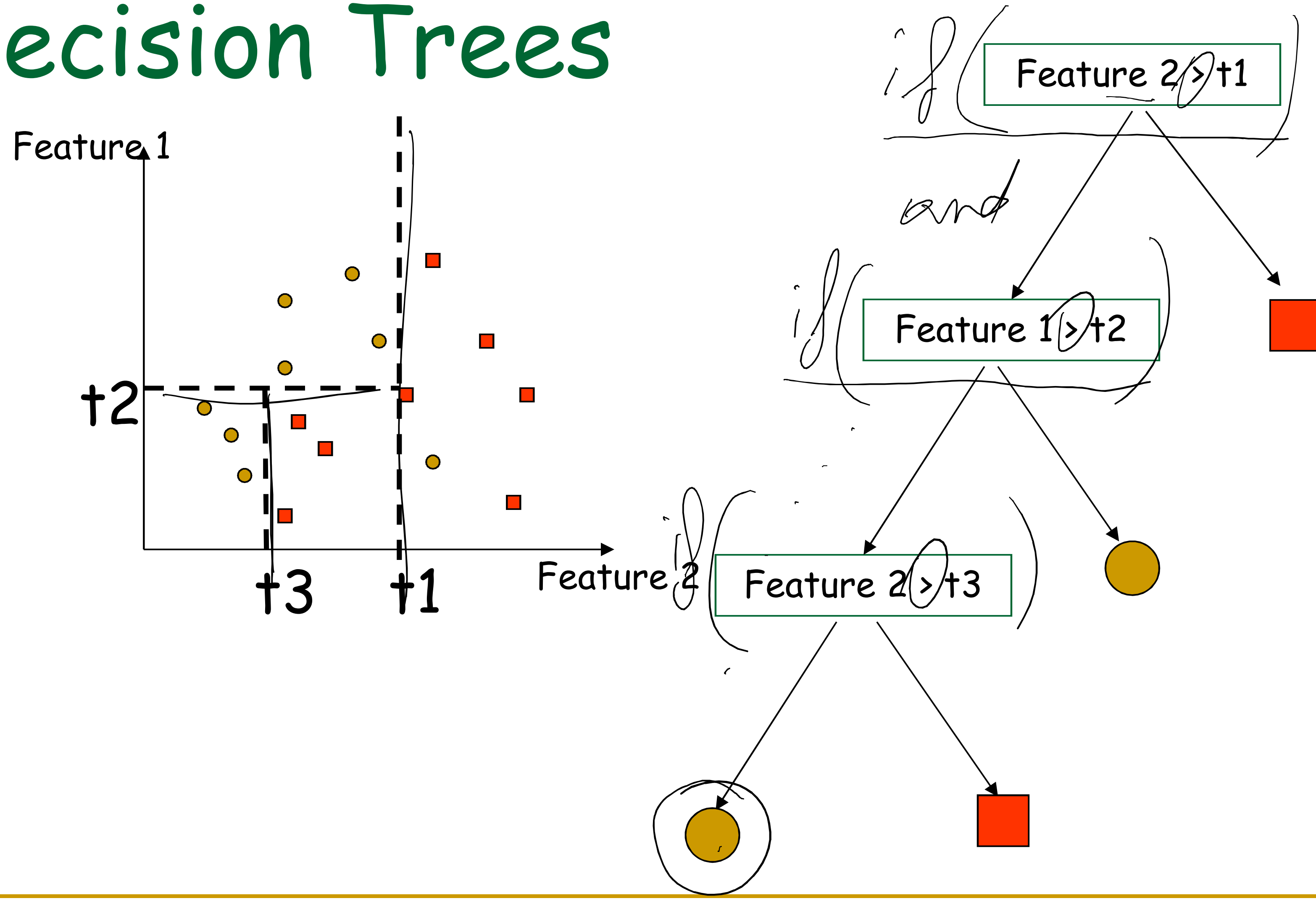




Decision Boundaries of Decision Trees



Decision Boundaries of Decision Trees



Applications of Decision Trees

- One of the most widely used learning methods in practice
  - Fast
  - Simple
  - Traceable (<-- very important!)

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    a. Application to Spam Filtering
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5. Unsupervised Learning)
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    a. Perceptrons
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supervised learning