Theory questions

Question 1. (8 points)

1. In Otsu's method for thresholding in Chapter 10, derive Eq. (10.3.15) by using Eqs. (10.3.10), (10.3.11) and (10.3.14). You should give all steps in your derivation.

We have the following equations:

$$P_1 m_1 + P_2 m_2 = m_G \qquad (10.3.10)$$

$$P_1 + P_2 = 1 \qquad (10.3.11)$$

$$\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2 \qquad (10.3.14)$$

We will derive Eq. (10.3.15) from Eqs. (10.3.10), (10.3.11) and (10.3.14)

$$\sigma_B^2 = P_1 P_2 (m_1 - m_2)^2 \qquad (10.3.15)$$

Combining Eq. (10.3.14) and (10.3.10), we have:

$$\begin{aligned}
\sigma_B^2 &= P_1(m_1 - P_1 m_1 - P_2 m_2)^2 + P_2(m_2 - P_1 m_1 - P_2 m_2)^2 \\
&= P_1\big((1 - P_1)m_1 - P_2 m_2\big)^2 + P_2\big((1 - P_2)m_2 - P_1 m_1\big)^2 \quad (\text{since } P_1 + P_2 = 1) \\
&= P_1(P_2 m_1 - P_2 m_2)^2 + P_2(P_1 m_2 - P_1 m_1)^2 \\
&= P_1 P_2^2 (m_1 - m_2)^2 + P_1^2 P_2 (m_1 - m_2)^2 \\
&= P_1 P_2 (m_1 - m_2)^2 \qquad (\text{since } P_1 + P_2 = 1)
\end{aligned}$$

Question 2. (8 points: First question=4 points; second question: 4 points)

Students can find the answer of this question in Chapter 10, section 2 of the textbook.

If Hough transform is carried out in the Cartesian (x, y) coordinate system, we use the slope-intercept form of the equation of a straight line:

$$\boldsymbol{y = ax + b}$$

However, it does not represent vertical lines (i.e. m → ∞) well. The general form of equation of a line (i.e. $ax + by + c = 0$ may represent vertical lines but it requires a Cartesian (x, y, z) 3D coordinate system which is a more complex space.

For the question, it is OK to just mention the form y=ax+b. For the second half of question, the student should summarize the steps for Hough transform in the textbook.

Question 3. (8 points: for a selected method, clear elaboration of the steps for shape identification=7 points, mentioning of how the number of shapes is counted = 1 point)

Several methods can work. The student may choose to use "hit-or-miss transform", "Hough transform for circles", or even image erosion with a kernel same to the bigger polka dot.

Part II: Programming questions

Question 1. (10 points: part a=5 points, part b = 5 points)

Part (a): correct implementation of the algorithm = 3 points; demonstration of the result = 2 points;

Part (b): Demonstration of the results =3 points; comments on the differences before and after averaging filtering = 2 points.

After applying the averaging filtering, the segmentation result should be improved. Specifically, the "speckles" within each segmented region are reduced.

Question 2. (16 points)

Part(a) Correct demonstration of the results = 5 points
Part(b) Correct demonstration of the results = 5 points
Part(c) Appropriate comments = 6 points.

Suggested comments for Part c: The Haar wavelet is supposed to produce less smooth approximation images as its power to represent higher order polynomials is weaker than the Daubechies-4 wavelet.