COMP 478/6771 Assignment 3 solutions – Fall 2022

Question 1. (10 points)

The spatial average (excluding the center term) is

$$g(x,y) = \frac{1}{4}[f(x,y+1) + f(x+1,y) + f(x-1,y) + f(x,y-1)]$$

From property 3 in Table 4.3,

$$\begin{aligned} G(u,v) &= \frac{1}{4}\left[e^{\frac{j2\pi v}{N}} + e^{\frac{j2\pi u}{M}} + e^{-\frac{j2\pi u}{M}} + e^{-\frac{j2\pi v}{N}}\right]F(u,v) \\ &= H(u,v)F(u,v) \end{aligned}$$

where

$$H(u,v) = \frac{1}{2}\left[\cos\left(\frac{2\pi u}{M}\right) + \cos\left(\frac{2\pi v}{N}\right)\right]$$

is the filter transfer function in the frequency domain.

Question 2. (16 points = 8 points for Part a and 8 points for Part b)

(a) From Section 2.6, we know that an operator, O, is linear if $O(af_1 + bf_2) = aO(f_1) + bO(f_2)$. From the definition of the Radon transform in Eq. (5.11-3),

$$\begin{aligned} O(af_1 + bf_2) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (af_1 + bf_2)\delta(x\cos\theta + y\sin\theta - \rho)dxdy \\ &= a\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_1\delta(x\cos\theta + y\sin\theta - \rho)dxdy \\ &\qquad + b\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_2\delta(x\cos\theta + y\sin\theta - \rho)dxdy \\ &= aO(f_1) + bO(f_2) \end{aligned}$$

thus showing that the Radon transform is a linear operation.

(b) Let $p = x - x_0$ and $q = y - y_0$. Then $dp = dx$ and $dq = dy$. From Eq. (5.11-3), the Radon transform of $f(x - x_0, y - y_0)$ is

$$\begin{aligned} g(\rho,\theta) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x - x_0, y - y_0)\delta(x\cos\theta + y\sin\theta - \rho)dxdy \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(p,q)\delta\left[(p + x_0)\cos\theta + (q + y_0)\sin\theta - \rho)\right]dpdq \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(p,q)\delta\left[p\cos\theta + q\sin\theta - (\rho - x_0\cos\theta - y_0\sin\theta)\right]dpdq \\ &= g(\rho - x_0\cos\theta - y_0\sin\theta, \theta). \end{aligned}$$

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Part II: Programming

Q1 (6 points)

I_1 : reconstructed image obtained by inverse Fourier Transform using magnitude $|F_A|$ of image A and phase $\Omega(F_B)$image B.

I_2 reconstructed image obtained by inverse Fourier transform using the magnitude $|F_B|$ of image B and phase $\Omega(F_A)$of image A. Among these two images, I_2 is the better reconstruction of the original image I_A .

If we compare the image I_2 reconstructed by using phase $\Omega(F_A)$ and magnitude $|F_B|$ with the image I_1 reconstructed by using phase $\Omega(F_B)$ and magnitude $|F_A|$, we can easily recognize that I_2 will have the structure of original image A while I_1 keeps the structure of image B.

Moreover, for the reconstructed image I_2 , all frequencies with their configurations of image A are still there except the magnitude is bigger or smaller. Of course, there will be some cases that the frequencies are absent since their magnitudes are 0 (corresponding to magnitudes of image B). However, the "locations" of all frequencies are still the same so all objects in image are kept. On the other hand, in case of I_1 , the structure will be changed corresponding to what we have in image B. Therefore, even we keep the magnitude $|F_A|$, the change of structural information results in making reconstructed image more different from original image A. As a result, I_2 is the better reconstruction of the original image I_A .

Q2 (18 points)

1) 4 points
2) 4 points
3) 4 points
4) 6 points: result comparison 4 points, appropriate comments 2 points