

Predicate logic

Dr. Constantinos Constantinides, P.Eng.

Department of Computer Science and Software Engineering
Concordia University Montreal, Canada

December 31, 2021

Introduction

- ▶ Much like propositional logic, predicate logic uses symbols to represent knowledge.
- ▶ Propositional logic does not allow us to say things like “*All objects of this class have this particular property*”, or “*Some objects of this class have this property*”, etc.

Introduction /cont.

- ▶ Let $P(x)$ denote the statement "*x is an odd number.*"
- ▶ The statement cannot be given a truth value since the value for x is not yet specified. We say that variable x is a *free variable* and it constitutes the *subject* of the statement, whereas "*is an odd number*" refers to a property of the subject and is called the *predicate*.
- ▶ $P(x)$ is called a *propositional function*, as each choice of x produces a proposition.

Introduction /cont.

- ▶ **Definition:** A *propositional function* is a statement containing one or more free variables. The statement becomes a proposition once values are assigned to all its free variables.
- ▶ In $P(x)$: “ x is an odd number.”, $P(5)$ becomes a proposition by setting $x = 5$ and its value is true, whereas the proposition $P(6)$ is false.
- ▶ Sometimes, the entire $P(x)$ is referred to as a predicate.

Using predicates to define sets: Untyped set comprehension

- ▶ $\{x \mid x < 5\}$ is the set of all values of x for which $x < 5$.
- ▶ This notation is an example of *untyped set comprehension*.
- ▶ We read “the set of values x such that $x < 5$.”

Using predicates to define sets: Untyped set comprehension /cont.

- ▶ A set comprehension can be modeled by a decision procedure where for an input variable it gives a result of true or false, e.g. $2 \in \{x \mid x < 5\}$ is true.

Using predicates to define sets: Typed set comprehension

- ▶ A *typed set comprehension* is of the form $\{x \in X \mid p(x)\}$ where $p(x)$ is a predicate with free variable x .
- ▶ For example, $\{x \in \mathbb{N} \mid x < 5\}$ represents the set of natural numbers less than five.
- ▶ Is $2.5 \in \{x \in \mathbb{N} \mid x < 5\}$ true? No, since $2.5 \notin \mathbb{N}$.

Set replacement

- ▶ An extension to set comprehension is to follow the declaration and predicate by a formula, e.g.
 $\{x \in \mathbb{N} \mid x < 5 \bullet x^2\}$ refers to a set of numbers which are squares of numbers less than 5, i.e. $\{0, 1, 4, 9, 16\}$.
- ▶ Each element in the set $\{x \in \mathbb{N} \mid x < 5\}$ is replaced by its square so that a new set is formed.

Quantifiers

- ▶ As its name suggests, a *quantifier* is an operator that states that a predicate is true for a given quantity of objects.
- ▶ The expression “for all”, denoted by the symbol \forall , is called the *universal quantifier*.
- ▶ The expression “there exists”, denoted by the symbol \exists is called the *existential quantifier*.

Universal quantification

- ▶ **Definition:** The *universal quantification* of $P(x)$ is the statement “ $P(x)$ is true for all values of x .” Symbolically this is expressed as $\forall x P(x)$.
- ▶ The statement $\forall x P(x)$ is true when $P(x)$ is true for all x . It is false when there is at least one x for which $P(x)$ is false.
- ▶ Its negation becomes $\neg \forall x P(x)$ and it is true when there is an x for which $P(x)$ is false, or $\exists x \neg P(x)$.

An initial example of a universal quantification

- ▶ In the domain of animals, how do we express “All cats are mammals”?
- ▶ We can rephrase the statement as “If x is a cat, then x is a mammal”:

$$\forall x (cat(x) \rightarrow mammal(x))$$

Example: Universal quantification

- ▶ How do we express “Only dogs bark”?
- ▶ We can rephrase the statement as “It barks only if it is a dog”, or “If it barks, then it is a dog”:

$$\forall x(barks(x) \rightarrow dog(x))$$

Existential quantification

- ▶ **Definition:** The *existential quantification* of $P(x)$ is the statement “There exists an element x such that $P(x)$ is true.” Symbolically this is expressed as $\exists xP(x)$.
- ▶ The statement $\exists xP(x)$ is true when there is at least one x for which $P(x)$ is true.
- ▶ It is false when $P(x)$ is false for all x . Its negation becomes $\neg\exists xP(x)$ and it is true when for every x , $P(x)$ is false, or $\forall x\neg P(x)$.
- ▶ In existential quantification we can make a distinction between “there exists at least one” (as defined above) and “there exists exactly one”, symbolically expressed as $\exists!$.

Example: Combining quantifiers

- ▶ Consider the following statement: “For every integer number x , there is a successor integer number y .”
- ▶ We can denote the successor property with the predicate $P(x, y) : y = x + 1$ and write the statement symbolically as

$$\forall x \exists y P(x, y)$$

Bound variables

- ▶ When we assign a value to a free variable, then the variable becomes a *bound variable*.
- ▶ Another way for a free variable to become bound is when we can apply a quantifier to it, such as “*There exists an x such that $x > 0$.*”
- ▶ We say that the variable x is bound by the quantifier “there exists.”

Precedence rules in the presence of quantifiers

- ▶ Both universal and existential quantifiers have a higher precedence than the rest of the connectives, but they have a lower precedence than the negation operator.
- ▶ Consider the predicates $p(x) = \text{"x is living."}$ and $q(x) = \text{"x is dead"}$:
 - 1 $\forall x(p(x) \vee q(x))$ is interpreted as everything is either living or dead.
 - 2 $\forall x p(x) \vee q(x)$ is interpreted as everything is living or x is dead.

Negation

- ▶ Consider the statement *“All birds are white”* or

$$\forall x P(x)$$

where the predicate $P(x)$ stands for *“bird is white.”*

- ▶ To negate the statement we can say *“It is not true (or: not the case) that all birds are white”* or *“Not all birds are white.”*

Negation /cont.

- ▶ Note that it would be incorrect to say “*All birds are not white*” as this would claim that there are no white birds which is clearly false.
- ▶ The negated statement can be written symbolically as

$$\neg[\forall xP(x)]$$

- ▶ An alternative way to express the negated statement is to say “*There is at least one bird that is not white*”, written symbolically as

$$\exists x[\neg P(x)]$$

Negation /cont.

- ▶ Note that the process of negating the statement corresponds to negating the predicate and changing the quantifier. In other words

$$\neg[\forall x P(x)] \equiv \exists x[\neg P(x)]$$

- ▶ Similarly, to negate a predicate with the existential quantifier, we can write

$$\neg[\exists x P(x)] \equiv \forall x[\neg P(x)]$$

Negation - Summary

Negated statement	Equivalent statement	When is it true?	When is it false?
$\neg[\forall x P(x)]$	$\exists x[\neg P(x)]$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x .
$\neg[\exists x P(x)]$	$\forall x[\neg P(x)]$	For every x , $P(x)$ is false.	There is an x for which $P(x)$ is true.

Nested quantifiers

- ▶ A quantifier is called *nested* if it occurs within the scope of another. For quantifiers of the same type, the order does not matter.
- ▶ However, the order in which quantifiers of different types are placed is important.
- ▶ $\forall x \exists y P(x, y)$ reads “**For every x there is a y for which $P(x, y)$ is true.**”
- ▶ In other words, no matter which x we choose, there must be a value of y (possibly depending on the choice of x) for which $P(x, y)$ is true.
- ▶ $\exists y \forall x P(x, y)$ reads “**There is an y that makes $P(x, y)$ true for every x .**”

Example 1: Nested quantifiers

- ▶ Consider the predicate $loves(x, y)$ denoting “ x loves y ”. We can express the following predicates using nested quantifiers:
- ▶ $\forall x \exists y loves(x, y)$ reads “Everyone loves someone”, i.e. no matter which x we choose, there must always be some y that makes $loves(x, y)$ true.
- ▶ $\exists y \forall x loves(x, y)$ reads “There is someone who is loved by everybody”, i.e. there is some particular y for which $loves(x, y)$ is true, regardless of the choice of x .

Example 1: Nested quantifiers /cont.

- ▶ If $\exists y \forall x P(x, y)$ is true, then $\forall x \exists y P(x, y)$ is also true, e.g. if “There is someone who is loved by everybody”, then we can safely conclude that “Everyone loves someone.”
- ▶ However, if $\forall x \exists y P(x, y)$ is true, e.g. “Everyone loves someone”, then it is not necessary that $\exists y \forall x \text{ loves}(x, y)$ is true, e.g. it is not necessary that “There is someone who is loved by everybody.”

Example 1: Nested quantifiers

- ▶ We can also translate the following sentences into formal statements:
- ▶ “Someone loves someone.” $\exists x \exists y \text{ loves}(x, y)$, or $\exists y \exists x \text{ loves}(x, y)$
- ▶ “Everyone loves someone.” $\forall x \exists y \text{ loves}(x, y)$
- ▶ “Everyone is loved by someone.” $\forall y \exists x \text{ loves}(x, y)$
- ▶ “There is someone who loves everyone.” $\exists x \forall y \text{ loves}(x, y)$
- ▶ “There is someone who is loved by everybody.” $\exists y \forall x \text{ loves}(x, y)$
- ▶ “Everyone loves everyone.” $\forall x \forall y \text{ loves}(x, y)$, or $\forall y \forall x \text{ loves}(x, y)$

Example 2: Nested quantifiers

- ▶ Consider the predicate $B(p, r)$ denoting the predicate “*Person p has booked room r .*” and the sentence “*No room is booked by more than one person.*”
- ▶ If no room is booked by more than one person, then the predicates $B(p, r)$ and $B(q, r)$ cannot both be true unless p and q denote the same person.
- ▶ Symbolically this can be expressed as

$$\forall p \forall q \forall r [(B(p, r) \wedge B(q, r)) \rightarrow (p = q)]$$

Example 3: Nested quantifiers

- Consider the following: “Every airline x flies to exactly one city y .” This can be formulated as:

$$\forall x \exists! y (airline(x) \wedge city(y) \wedge flies(x, y))$$

Example 4: Nested quantifiers

- ▶ Let $sendMessage(x, y)$ be the statement “ x has sent a message to y ” where the domain is all students in class. Note that x and y can be the same person.
- ▶ $\exists x \exists y P(x, y)$: There is some student who has sent a message to some student.
- ▶ $\forall x \forall y P(x, y)$: Every student in the class has sent a message to every student in the class.

Example 4: Nested quantifiers /cont.

- ▶ $\exists y \forall x P(x, y)$: Recall that this reads "**There in a y that makes $P(x, y)$ true for every x .**" There is a student in class who has been sent a message by every student in class.
- ▶ $\exists x \forall y P(x, y)$: This is similar to the above, only the roles have been changed: There is some student who has sent a message to every student in the class.

Example 4: Nested quantifiers /cont.

- ▶ $\forall x \exists y P(x, y)$: Recall that this reads “**For every** x **there is a** y **for which** $P(x, y)$ **is true.**” Every student in class has sent a message to some (at least one) student in class.
- ▶ $\forall y \exists x P(x, y)$: This is similar to the above, only the roles have been changed: Every student in class has been sent a message from some (at least one) student in class.

Example 5: Nested quantifiers

- ▶ Consider the statement “Everyone has exactly one movie that he or she likes.”
- ▶ Let us express the statement in formal logic by introducing the predicate $likes(x, y)$ to represent the statement “ x likes movie y .”
- ▶ Let us start with $\forall x \exists y likes(x, y)$ which reads “**For every x there is a y for which $likes(x, y)$ is true**”, or “Everyone likes some movie.”
- ▶ This does not fully capture the original statement because it does not exclude the possibility that there may be more than one movie that a person likes.

Example 5: Nested quantifiers /cont.

- ▶ To complete the statement we need to add that no other movie (i.e. no movie which is not y) is liked by x , or $\forall z((z \neq y) \rightarrow \neg \text{likes}(x, z))$.
- ▶ Putting everything together we have

$$\forall x \exists y (\text{likes}(x, y) \wedge \forall z ((z \neq y) \rightarrow \neg \text{likes}(x, z)))$$

Negating nested quantifiers

The negation of nested quantifiers follows the same rules as with single quantifiers. The statement

$$\forall x \exists y P(x, y)$$

reads “For every x , there is a y for which $P(x, y)$ is true.” The negated statement would read “There is an x such that $P(x, y)$ is false for every y ”, or

$$\exists x \forall y \neg P(x, y)$$

The statement

$$\exists x \forall y P(x, y)$$

reads “There is an x for which $P(x, y)$ is true for every y .” The negated statement would read “For every x there is a y for which $P(x, y)$ is false”, or

$$\forall x \exists y \neg P(x, y)$$

Summary of quantifications of two variables

Statement	When is it true	When is it false
$\forall x \forall y P(x, y)$ $\forall y \forall x P(x, y)$	$P(x, y)$ is true for every pair (x, y) .	There is a pair (x, y) for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every x there is a y for which $P(x, y)$ is true.	There is an x such that $P(x, y)$ is false for every y .
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y .	For every x there is a y for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$ $\exists y \exists x P(x, y)$	There is a pair (x, y) , for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair (x, y) .

Equivalences

$$\forall x \forall y P(x, y) \equiv \forall y \forall x P(x, y)$$

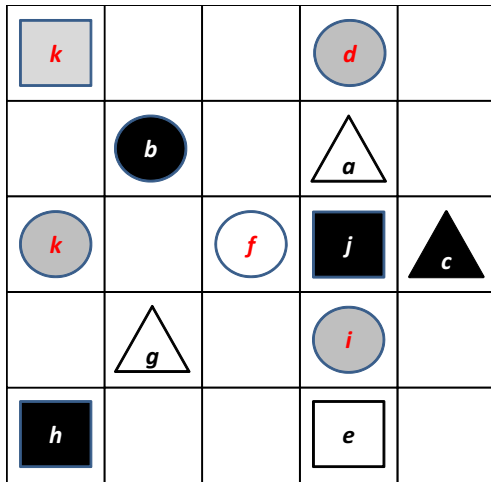
$$\exists x \exists y P(x, y) \equiv \exists y \exists x P(x, y)$$

$$\exists x (P(x) \rightarrow Q(x)) \equiv \forall x P(x) \rightarrow \exists x Q(x)$$

$$\exists x (P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$$

$$\forall x (P(x) \wedge Q(x)) \equiv \forall x P(x) \wedge \forall x Q(x)$$

Example: Formalizing sentences from a natural language with Tarski's world

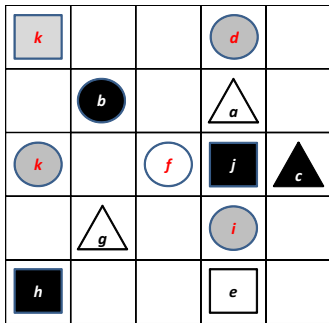


Example: Formalizing sentences from a natural language with Tarski's world /cont.

We adopt the following predicates:

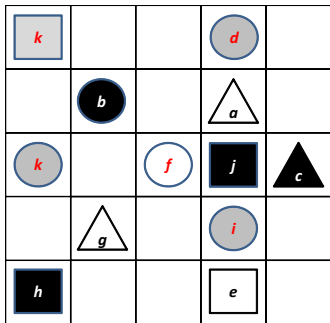
- ▶ *square* (x) indicates that x is a square.
- ▶ *circle* (x) indicates that x is a circle.
- ▶ *triangle* (x) indicates that x is a triangle.
- ▶ *black* (x) indicates that x is a black.
- ▶ *gray* (x) indicates that x is a gray.
- ▶ *white* (x) indicates that x is a white.
- ▶ *aboveOf* (x, y) indicates that x is above y , perhaps in a different column.
- ▶ *rightOf* (x, y) indicates that x is to the right of y , perhaps in a different row.
- ▶ *sameColor* (x, y) indicates that x and y have the same color.

Example: Formalizing sentences from a natural language with Tarski's world /cont.



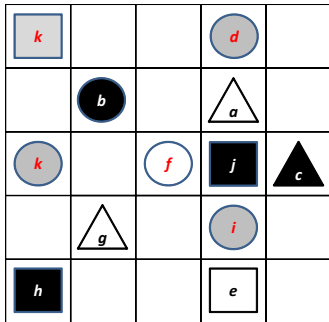
- ▶ All squares are black: $\forall x (square(x) \rightarrow black(x))$.
False, since e and k are both squares and non-black. Note that to prove false one counter-example would be enough.
- ▶ Everything white is a triangle. $\forall x (white(x) \rightarrow triangle(x))$.
False, since e and f are both white and not triangles.

Example: Formalizing sentences from a natural language with Tarski's world /cont.



- ▶ There is a square that lies to the left of d .
 $\exists x (square(x) \wedge rightOf(d, x))$.
True: h and k both lie to the left of d .
- ▶ There is a black circle. $\exists x (circle(x) \wedge black(x))$. True: b .

Example: Formalizing sentences from a natural language with Tarski's world /cont.



- ▶ All circles are above g . $\forall x (circle(x) \rightarrow aboveOf(x, g))$.
False, since i is a circle and it is not above g .
- ▶ For every square, there exists a circle of the same color.

$$\forall x (square(x) \rightarrow \exists y (circle(y) \wedge sameColour(x, y)))$$

This is true.

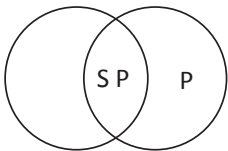
Categorical propositions

- ▶ A *categorical proposition* (or *categorical statement*) is a proposition that asserts or denies that all or some of the members of one category (the *subject term*) are included in another (the *predicate term*).
- ▶ For example, the categorical proposition “*All birds are white*” asserts that all members of category *birds* are included in category *being white*.
- ▶ The category of the subject (*birds*) and refers to what the proposition is about, whereas the category of the predicate (*being white*) refers to what the proposition affirms (or denies) about the subject.

Standard forms of categorical propositions

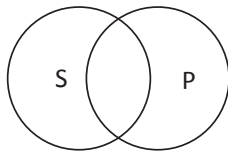
- ▶ Aristotle identified four primary distinct types of categorical proposition and gave them standard forms (referred to as A, E, I, and O).
- ▶ For the subject category S, and the predicate category P, the four standard forms are:
 - ▶ All S are P. (A form)
 - ▶ No S are P. (E form)
 - ▶ Some S are P. (I form)
 - ▶ Some S are not P. (O form)

Visualizing categorical propositions



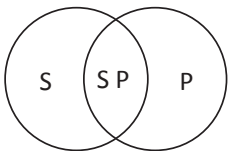
All S are P

A



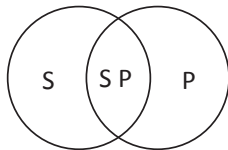
No S are P

E



Some S are P

I



Some S are not P

O

Properties of categorical propositions: Quantity

- ▶ *Quantity* refers to the number of members of the subject class that are used in the proposition.
- ▶ If the proposition refers to all members of the subject class, it is *universal*.
- ▶ If the proposition does not employ all members of the subject class, it is *particular*.
- ▶ Categorical propositions A and E have a *universal quantity* as they make a claim about all members of the subject class, whereas propositions I and O have a *particular quantity* as they make a claim about some (at least one) member of the subject class.

Properties of categorical propositions: Quality

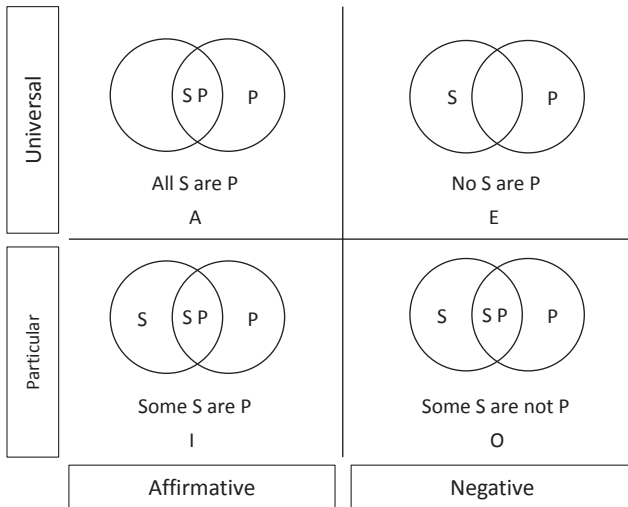
- ▶ *Quality* describes whether the proposition affirms or denies the inclusion of a subject within the class of the predicate.
- ▶ The two possible qualities are called *affirmative* and *negative*.
- ▶ Propositions A and I have an *affirmative quality* as they affirm class membership, whereas propositions E and O have *negative quality* as they deny class membership.

Summary of categorical propositions

Letting S and P stand for subject and predicate respectively, we have four forms of categorical propositions as shown below:

NAME	FORM	TITLE	
A	All S are P	Universal affirmative	$\forall xP(x)$
E	No S are P	Universal negative	$\forall x[\neg P(x)]$
I	Some S are P	Particular affirmative	$\exists xP(x)$
O	Some S are not P	Particular negative	$\exists x[\neg P(x)]$

Visualizing categorical propositions with their properties



Formalizing sentences from a natural language

- ▶ In the episode “Fidelity” of House M. D., Dr. Gregory House says: *“I don’t ask why patients lie, I just assume they all do.”*
- ▶ Consider the following list of categorical propositions where $P(x)$ denotes the subject “ x is a patient” and $Q(x)$ denotes the predicate “ x lies”:
 - ▶ “Every patient lies.” $\forall x (P(x) \rightarrow Q(x))$ (All P are Q: Type A)
 - ▶ “No patient lies.” $\forall x (P(x) \rightarrow \neg Q(x))$ (No P are Q: Type E)
 - ▶ “Some patient lies.” $\exists x (P(x) \wedge Q(x))$ (Some P are Q: Type I)
 - ▶ “Not all patients lie.” $\exists x (P(x) \wedge \neg Q(x))$ (Some P are not Q: Type O)

Formalizing sentences from a natural language /cont.

- ▶ Consider the statement “**All** patients **are** honest.” What type is it? Is it A?
- ▶ No. Given the subject and predicate, the statement is of type E: “No patient lies.”
- ▶ Similarly, the statement “**No** patient **is** honest” is of type A because, given the subject and predicate it is interpreted as “All patients lie.”
- ▶ To determine the proper type, a statement should be interpreted w.r.t. the subject and predicate.

Formalizing categorical propositions

- ▶ “Some patient lies.” $\exists x (P(x) \wedge Q(x))$
- ▶ “No patient lies.” $\forall x (P(x) \rightarrow \neg Q(x))$
- ▶ “All patients lie.” $\forall x (P(x) \rightarrow Q(x))$
- ▶ “Not all patients lie.” $\exists x (P(x) \wedge \neg Q(x))$
- ▶ “Every patient lies.” $\forall x (P(x) \rightarrow Q(x))$
- ▶ “There is an honest patient.” $\exists x (P(x) \wedge \neg Q(x))$
- ▶ “No patient is honest.” $\forall x (P(x) \rightarrow Q(x))$
- ▶ “All patients are honest.” $\forall x (P(x) \rightarrow \neg Q(x))$

Formalizing sentences from a natural language /cont.

- ▶ We notice that each formalization satisfies one of the following two properties:
- ▶ The universal quantifier $\forall x$ quantifies an implication.
- ▶ The existential quantifier $\exists x$ quantifies a conjunction.

Formalizing sentences from a natural language /cont.

- ▶ Consider the statement “Some patient lies” which was formalized as $\exists x (P(x) \wedge Q(x))$.
- ▶ Can we argue that the sentence can also be formalized as $\exists x (P(x) \rightarrow Q(x))$?
- ▶ This would be equivalent to $\forall x P(x) \rightarrow \exists x Q(x)$ which means “If everyone is a patient then someone lies” which does not convey the meaning of the original sentence.

Contradictory categorical propositions

- ▶ **Definition:** A pair of categorical propositions are called *contradictories* if they have opposite truth values: they cannot both be true and cannot both be false.

Contradictory categorical propositions: Example 1

- ▶ Consider the statement “Every person owns a house.” Its contradictory statement is “Not every person owns a house.”
- ▶ Given a subject category “x is a person” and predicate category “x owns a house”, then “Every person owns a house” is of type A.
- ▶ Its contradictory statement “Not every person owns a house” can be rephrased as “Some people do not own a house” which is of type O.
- ▶ Universal affirmations and particular denials are contradictory statements.

Contradictory categorical propositions: Example 2

- ▶ Consider the statement “No people suffer from hunger.” Its contradictory statement is “Some people suffer from hunger.”
- ▶ Given a subject category “x is a person” and predicate category “x suffers from hunger”, then “No people suffer from hunger.” is of type E.
- ▶ Its contradictory statement “Some people suffer from hunger” is of type I.
- ▶ Universal denials and particular affirmations are contradictory statements.

Contradictory categorical propositions: Example 3

- ▶ let $P(x)$ to denote “ x is a patient” and $Q(x)$ to denote “ x lies”
- ▶ “All patients lie” ($\forall x (P(x) \rightarrow Q(x))$), and “There is an honest patient” (which can be interpreted as There is some patient who does not lie) ($\exists x (P(x) \wedge \neg Q(x))$) is a pair of contradictory propositions.
- ▶ “No patient lies” ($\forall x (P(x) \rightarrow \neg Q(x))$) and “Some patient lies” ($\exists x (P(x) \wedge Q(x))$) is a pair of contradictory categorical propositions.

Contrary categorical propositions

- ▶ **Definition:** A pair of categorical propositions are called *contraries* if they cannot both be true, but could both be false.

Contrary categorical propositions: Example 1

- ▶ Consider the statement “All people are rich.” Its contrary statement is “No people are rich.”
- ▶ Given a subject category “x is a person” and predicate category “x is rich”, then “All people are rich” is of type A and “No people are rich” is of type E.
- ▶ A pair of universal statements are contraries.

Contrary categorical propositions: Example 2

- ▶ “All patients lie” ($\forall x (P(x) \rightarrow Q(x))$) and “No patient lies” ($\forall x (P(x) \rightarrow \neg Q(x))$) are contrary categorical propositions.

Subcontrary categorical propositions

- ▶ **Definition:** A pair of categorical propositions are called *subcontraries* if they cannot both be false but could both be true.

Subcontrary categorical propositions: Example

- ▶ “Some patient lies” ($\exists x (P(x) \wedge Q(x))$) and “There is an honest patient” (or There is some patient who does not lie) ($\exists x (P(x) \wedge \neg Q(x))$) are subcontrary categorical propositions.

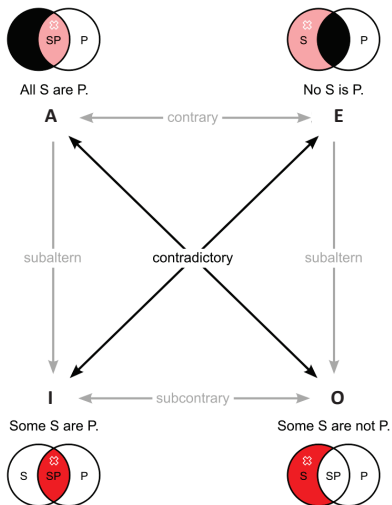
Subaltern and superaltern categorical propositions

- ▶ **Definition:** Two categorical propositions are called *superaltern* and *subaltern* if the subaltern must be true if its *superaltern* is true (and subsequently the superaltern must be false if the subaltern is false).
- ▶ I is a subaltern of A (Some S are P, if All S are P).
- ▶ O is a subaltern of E (Some S are not P, if No S are P).

Subaltern and superaltern categorical propositions: Example

- ▶ “Some patient lies” ($\exists x (P(x) \wedge Q(x))$) is a subaltern of “All patients lie” ($\forall x (P(x) \rightarrow Q(x))$).
- ▶ “There is an honest patient” ($\exists x (P(x) \wedge \neg Q(x))$) is a subaltern of “No patient lies” ($\forall x (P(x) \rightarrow \neg Q(x))$).

Summary: The square of opposition



Categorical syllogisms

- ▶ Recall from propositional logic that a syllogism is an inference in which one proposition necessarily follows from two others.
- ▶ A *categorical syllogism* consists of three parts (major premise, minor premise, and conclusion) all of which are *categorical propositions*.

Categorical syllogisms /cont.

Some valid forms of categorical syllogisms are shown below:

All M are P. All S are M. Therefore, All S are P.

All P are M. Some S are not M. Therefore, Some S are not P.

Some M are not P. All M are S. Therefore, Some S are not P.

All P are M. No M are S. Therefore, No S are P.

All M are P. Some S are M. Therefore, Some S are P.

No M are P. Some S are M. Therefore, Some S are not P.

Universal conditional statements

- ▶ A universal conditional statement has the form $\forall x$ if $P(x)$ then $Q(x)$, or $\forall x(P(x) \rightarrow Q(x))$.
- ▶ It can be proven that its negation has the form $\exists x(P(x) \wedge \neg Q(x))$.
- ▶ For the universal conditional statement above,
- ▶ its contrapositive statement is

$$\forall x(\neg Q(x) \rightarrow \neg P(x))$$

- ▶ its converse statement is

$$\forall x(Q(x) \rightarrow P(x))$$

- ▶ and its inverse is

$$\forall x(\neg P(x) \rightarrow \neg Q(x))$$