COMP 478/6771 Assignment 2 solutions – Fall 2022

Theoretical questions

Q1 (**7 points**)

The most extreme case is when the mask is positioned on the center pixel of a 5-pixel gap, along a thin segment, in which case a 5×5 mask would encompass a completely blank field. Since this is known to be the largest gap, the next (odd) mask size up is guaranteed to encompass some of the pixels in the segment. Thus, the smallest mask that will do the job is a 7 × 7 averaging mask.

The smallest average value produced by the mask is when it encompasses only **two pixels** of the segment. This average value is a gray-scale value, not binary, like the rest of the segment pixels. Denote the smallest average value by $A_{\min}$, and the binary values of pixels in the thin segment by B. Clearly, $A_{\min}$ is less than B . Then, setting the binarizing threshold slightly smaller than $A_{\min}$ will create one binary pixel of value B in the center of the mask.
For the case of 7 × 7 averaging mask, $A_{\min} = \frac{2}{7\times7}$.

Q2 (**7 points**)

In general, the sum of all elements in the filter should be zero, with the center element being a negative number. In this way, no additional information is introduced through the filtering operation. (**3 points**)

In general, increasing the size of the "Laplacian-like" mask produces blurring. This is because a larger "Laplacian-like" mask does not follow the definition of a second derivative. Consider an image consisting of two vertical bands, a black band (intensity =0) on the left and a white band (intensity = 255) on the right, with the transition between the bands occurring through the center of the image. That is, the image has a sharp vertical edge through its center. We know that a second derivative should produce a double edge in the region of the vertical edge when a 3× 3 Laplacian mask is centered on the edge. As the center of the mask moves more than two pixels on either side of the edge the entire mask will encompass a constant area and its response would be zero. However, suppose that the mask is much larger. As its center moves through, say, the black area, the mask can still be contained in the white area. The sum of the products will therefore be different from 0, making the region of filtered edges more blurry. (**4 points**)

COMP 478/6771 Assignment 2 solutions – Fall 2022

Q2a (**7 points**)

a)

$$
\begin{aligned}
F(\mu) &= \int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu t}dt \\
&= \int_{0}^{W} Ae^{-j2\pi\mu t}dt \\
&= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]_{0}^{W} \\
&= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu W} - 1\right] \\
&= \frac{A}{j2\pi\mu}\left[e^{j\pi\mu W} - e^{-j\pi\mu W}\right]e^{-j\pi\mu W} \\
&= \frac{A}{\pi\mu}\sin(\pi\mu W)\, e^{-j\pi\mu W} \\
&= AW\left[\frac{\sin(\pi\mu W)}{\pi\mu W}\right]e^{-j\pi\mu W}
\end{aligned}
$$

The only difference between this result and the result in Example 4.1 is the exponential term. It is a phase term that accounts for the shift in the function. The magnitude of the Fourier transform is the same in both cases, as expected.

In case $A = W = 1$, we have $F(\mu) = \left[\frac{\sin(\pi\mu)}{\pi\mu}\right]e^{-j\pi\mu}$.

Q2b (**7 points**)

b) Since the tent function is the convolution of two equal box functions, by the convolution

theorem, the Fourier transform of the spatial convolution of two functions is the product their transforms. Recall that the trans- form of a box is a sinc function. Therefore, the Fourier transform of a tent function is a sinc function squared.
From Example 4.1, we have

$$F(\mu) = AW\frac{\sin(\pi\mu W)}{\pi\mu W}$$

The Fourier transform of the tent function is

$$F(\mu)F(\mu) = (AW)^2\frac{\sin^2(\pi\mu W)}{(\pi\mu W)^2}$$

Part II: Programming (22 points)

8 points: The program functions correctly
6 points: Demonstrate of results
4 points: Discussion about the choice of parameters and filters
4 points: Compare the results with ***adaptthresh()*** function in MATLAB