Problem 1:

Consider a one-dimensional particle size object that has the following initial conditions:

$$t_0 = 0, h = 1, v_0 = 1, x_0 = 2$$

The only force acting on this particle is a gravitational force with the constant -1. We consider no damping forces for this example.

Compute analytically $y(t)$, write both theEuler implicit and explicit integration scheme; Compute the three time-steps after 0 for both approximations as well as the analytical solution, look at the error and discuss it.

Problem 2:

Consider a one-dimensional spring as illustrated in class. The spring is fixed at the origin and at the other end of the spring there is a particle A whose motion in space we would like to compute. If there is no damping and the rest length of the spring is $L = 2$, the spring coefficient is $k = 3$,and mass is 1. The rest of the parameters are:

$$t_0 = 0, h = 1, v_0 = 1, x_0 = 3$$

Write both the Euler implicit and explicit integration scheme; Compute the two time-steps after 0 for explicit and one time-step after 0 for implicit.

Problem3: Inverse problems

You are working for a spring making factory and you are required to run an experiment to determine the stiffness coefficient k of a given spring.

The setup of the experiment, illustrated in the figure above, consists of placing the spring on a horizontal rail (i.e. no gravity forces) where one end of the spring is fixed while the position of the second one is recorded by a high-speed camera (i.e. the blue dot in the illustration above). The blue dot can only move in the x-axis and all the forces in the system are along x-axis. The rest length of the spring is $\mathrm{L} = 1$

The experiment consists of extending the blue dot to twice its rest length and releasing the spring. The position over time of the blue dot is recorded over time.

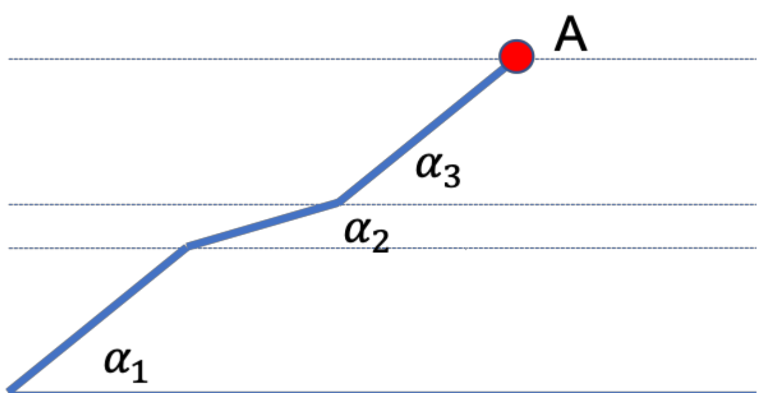
Assuming that:

- you release the spring at time $t_0 = 0$
- the camera starts recording at exactly t_0
- the camera records at a frequency of 1000 fps (unit of time we use are milliseconds (0.001 seconds)
- ignore damping forces

Compute the stiffness of the spring using both the explicit and implicit Euler integration schemes. For the explicit Euler integration use the position (in x direction) in the third frame which is 1.5. For the implicit Euler scheme use the position (in x direction) in the second frame which is $\frac{5}{3}$.

A variation of this problem can be knowing the stiffness and estimating the damping forces

Problem4: IK



Consider the 2D Kinematic link in the image above. The target in point A (in red) and the control angles are $\alpha_1, \alpha_2, \alpha_3$. The lengths of the kinematic arms are: 3, 2 and 3 respectively.

Assuming that we have the additional constraint $\alpha_1 = \alpha_3$, tailor the Jacobian method presented in class to solve the inverse kinematic problem for this chain and show the first iteration assuming $\Delta e = \binom{-1}{0}$. You can use the transpose of the Jacobian instead of the inverse.