

# COMP 472 - Artificial Intelligence

## Word Embeddings

### *Solutions*

**Question 1** Consider the following sentence:

“the cat drinks the milk”

We will use this sentence to train a CBOW Word2Vec model. Assume that:

- you want to produce word embeddings of dimension 2,
- you use a context window of size 2 (1 word before and 1 word after the target word), and
- your vocabulary only contains the words in the sentence above

(a) Using only the sentence above, how many instances will be generated as training set?  
3 instances

| Instance | Context Word-1 | Context Word+1 | To Predict |
|----------|----------------|----------------|------------|
| 1        | the            | drinks         | cat        |
| 2        | cat            | the            | drinks     |
| 3        | drinks         | milk           | the        |

(b) List the one-hot vectors that correspond to each word in the vocabulary. (Assume alphabetical ordering)

| Word   | Hot Vector |   |   |   |
|--------|------------|---|---|---|
| cat    | 1          | 0 | 0 | 0 |
| drinks | 0          | 1 | 0 | 0 |
| milk   | 0          | 0 | 1 | 0 |
| the    | 0          | 0 | 0 | 1 |

(c) List the one-hot vectors that correspond to each training instance in the input layer.

| Instance | Context        | Word   | Hot Vector |   |   |   |
|----------|----------------|--------|------------|---|---|---|
| 1        | Context Word-1 | the    | 0          | 0 | 0 | 1 |
|          | Context Word+1 | drinks | 0          | 1 | 0 | 0 |
| 2        | Context Word-1 | cat    | 1          | 0 | 0 | 0 |
|          | Context Word+1 | the    | 0          | 0 | 0 | 1 |
| 3        | Context Word-1 | drinks | 0          | 1 | 0 | 0 |
|          | Context Word+1 | milk   | 0          | 0 | 1 | 0 |

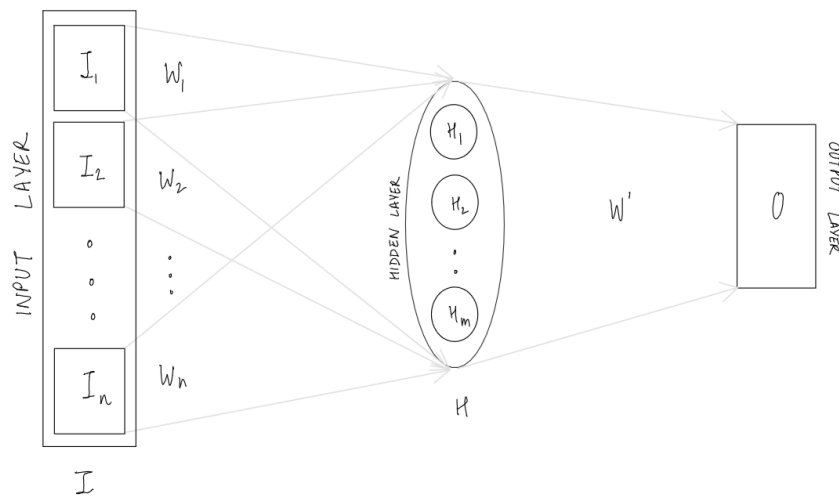
(d) How many nodes will the hidden layer contain?

*Number of nodes in hidden layer = 2*

(e) What is the target hot vector for each training instance?

| Instance | To Predict | Hot Vector |   |   |   |
|----------|------------|------------|---|---|---|
| 1        | cat        | 1          | 0 | 0 | 0 |
| 2        | drinks     | 0          | 1 | 0 | 0 |
| 3        | the        | 0          | 0 | 0 | 1 |

(f) Assume that the Word2Vec model is trained with the standard network depicted below:



i. What will be the values of  $n$  and  $m$ ?

$n = 2, m = 2$

ii. What will be the sizes of  $I, W_i$  (for each  $1 \leq i \leq n$ ),  $W'$  and  $O$ ?

Size of  $I_1 = I_2 = 1 \times 4, I = 2 \times 4$

Size of  $W_1 = W_2 = 4 \times 2$

Size of  $W' = 2 \times 4$

Size of  $O = 1 \times 4$

(g) Assume that we have these weight vectors:

$$W = \begin{bmatrix} 2 & 6 \\ 4 & 3 \\ 1 & 4 \\ 5 & 2 \end{bmatrix}$$

$$W' = \begin{bmatrix} 6 & 2 & 8 & 3 \\ 4 & 5 & 9 & 7 \end{bmatrix}$$

To compute the final probabilities at the output layer, we use the softmax function as shown in class. Recall that for a given vector of size  $k$ , the softmax function is defined as:

$$p_i = \frac{e^{x_i}}{\sum_{i=1}^k e^{x_i}}, \text{ where } 1 \leq i \leq k$$

- i. Trace the first feed forward pass in the network and show the values propagated all the way to the output layer.

Instance 1 -

| Instance | Context        | Word   | Hot Vector |   |   |   | To Predict | Target |   |   |   |
|----------|----------------|--------|------------|---|---|---|------------|--------|---|---|---|
| 1        | Context Word-1 | the    | 0          | 0 | 0 | 1 | cat        | 1      | 0 | 0 | 0 |
|          | Context Word+1 | drinks | 0          | 1 | 0 | 0 |            |        |   |   |   |

Calculate the output of each hidden node for each context word

$$H = I \times W = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 6 \\ 4 & 3 \\ 1 & 4 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}$$

Take the average

$$H_{AVG} = [4.5 \quad 2.5]$$

Calculate output

$$O = H_{AVG} \times W' = [4.5 \quad 2.5] \times \begin{bmatrix} 6 & 2 & 8 & 3 \\ 4 & 5 & 9 & 7 \end{bmatrix} = [37 \quad 21.5 \quad 58.5 \quad 31]$$

Calculate softmax probabilities for the output

$$\begin{aligned} softmax(O) &= softmax([37 \quad 21.5 \quad 58.5 \quad 31]) \\ &= [4.6 \times 10^{-10} \quad 8.53 \times 10^{-17} \quad 0.99 \quad 1.14 \times 10^{-12}] \end{aligned}$$

- ii. What is the error after the first pass?

Calculate error

$$E = O - T = [4.6 \times 10^{-10} \quad 8.53 \times 10^{-17} \quad 0.99 \quad 1.14 \times 10^{-12}] - [1 \quad 0 \quad 0 \quad 0]$$

$$= [\approx -1 \quad 8.53 \times 10^{-17} \quad 0.99 \quad 1.14 \times 10^{-12}]$$