FIL1006 - oblig 3

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03.04.18

1 Natural deduction

(a) Show that ”$p \vee \neg p$” is valid

[1]	(1)	$p \vee \neg p$	P
[1]	(2)	p	(1) DE
[1]	(3)	$\neg p$	(1) DE
[1]	(4)	$\neg p$	(2) NI
[1]	(5)	$\neg\neg p$	(3) NI
[1]	(6)	p	(5) DN
[1]	(7)	$\neg p \vee p$	(4)(6) DI

(b) ”$\neg p \equiv \neg q$” implies ”$p \equiv q$”

[1]	(1)	$(\neg p \supset \neg q) \wedge (\neg q \supset \neg p)$	P
[2]	(2)	p	P
[3]	(3)	q	P
[1]	(4)	$\neg p \supset \neg q$	(1) CE
[1]	(5)	$\neg q \supset \neg p$	(1) CE
[1][3]	(6)	$\neg\neg p$	(3)(4) MT
[1][3]	(7)	p	(6) DN
[1][2]	(8)	$\neg\neg q$	(2)(5) MT
[1][2]	(9)	q	(8) DN
[1]	(10)	$p \supset q$	[2](9) D
[1]	(11)	$q \supset p$	[3](7) D
[1]	(12)	$(p \supset q) \wedge (q \supset p)$	(10)(11) CI

(c) ”$p \wedge (q \vee r)$” is equivalent to ”$(p \wedge q) \vee (p \wedge r)$”

[1]	(1)	$p \wedge (q \vee r)$	P
[1]	(2)	p	(1) CE
[1]	(3)	$q \vee r$	(1) CE
[1]	(4)	q	(3) DE
[1]	(5)	r	(3) DE
[1]	(6)	$p \wedge q$	(2)(4) CI
[1]	(7)	$p \wedge r$	(2)(5) CI
[1]	(8)	$(p \wedge q) \vee (p \wedge r)$	(6)(7) DI

(d) ”$\neg(p \wedge \neg q)$” is equivalent to ”$p \supset q$”

[1]	(1)	$\neg(p \wedge \neg q)$	P
[2]	(2)	p	P
[1][2]	(3)	$\neg(\neg q)$	(1)(2) CE
[1][2]	(4)	q	(3) DN
[1]	(5)	$p \supset q$	[2](4) D

2 Formalize predicate logic I

(a) Nothing is solid and everything is energy

S : is solid
E : is energy

$\underline{\forall x(\neg Sx \wedge Ex)}$

(b) No elephants are pink, but some of the elephants that Nina draws are pink.

P : pink elephant
D : drawn by Nina

$\underline{\forall x \neg(Rx) \wedge \exists x(Dx \supset Rx)}$

(c) Some thinkers are creative and others are skilled

T : a thinker
C : is creative
S : is skilled

$\underline{\exists x(Tx \wedge Cx) \wedge \exists x(Tx \wedge Sx)}$

(d) Only beavers build dams.

B : is a beaver
D : builds dams

$\underline{\exists x(Dx \supset Bx)}$

(e) The giraffes at the zoo do not eat any fruit given to them.

G : giraffe at the zoo
F : eats fruit given

$\underline{\forall x(Gx \supset \neg Fx)}$

3 Formalize predicate logic II

(a) All animals have rights.

R : have rights

$\underline{\forall x(Rx)}$

(b) Some small animals work hard.

S : are small
W : work hard

$\underline{\exists x(Sx \wedge Wx)}$

(c) If any animal is suffering, a human animal should intervene.

S : is suffering
H : is human
I : should intervene

$\underline{\forall x \exists y(Sx \supset (Hy \wedge Iy))}$

4 Interpret schema

(a) $\exists x(Fx \wedge \neg Gx) \wedge \exists x(Gx \wedge \neg Fx)$

F : is X
G : is Y

Some X aren't Y, and some Y aren't X

(b) $\neg \exists x(Fx \wedge Gx) \wedge \forall x(Fx \equiv \neg Gx)$

F : is X
G : is Y

Nothing is both X and Y, being X is the same as not being Y.

(c) $\forall x \neg(Fx \wedge Gx)$

F : is X
G : is Y

Nothing is both X and Y

5 Axiom of extensionality

(a) Explain why "$\exists x(Fx \vee Gx)$" is equivalent to "$\exists x(Fx) \vee \exists x(Gx)$"

The existence quantifier is running over the same set in both instances on the last schema. As long as either of the predicates is true the whole expression is true. It's the same if we run through the entire set once and check if either predicate is true, or if we run it once per predicate.

(b) Show that "$\exists x(Fx \vee Gx)$" implies "$\exists x(Fx) \vee \exists x(Gx)$"

Fx	Gx	$Fx \vee Gx$	$(Fx \vee Gx)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

(c) Show that "$\exists x(Fx) \lor \exists x(Gx)$" implies "$\exists x(Fx \lor Gx)$"

I showed that they were identical in (b). If this is wrong then I'll take the retake oblig. Either way I'll get to see how to properly use the instances in the answer sheet.