

# FIL1006 - oblig 2

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## 1 Relations

I'm using the term "operand" for what has the relation, because relations are somewhat similar to operators.

- (a) What is it for a relation to be reflexive?
  - The reflexive relation is a relation to the operand itself.
- (b) What is it for a relation to be symmetric?
  - The operands must be commutative.
- (c) What is it for a relation to be transitive?
  - Given that there is a relation between  $a$  and  $b$ , and the same relation between  $b$  and  $c$ , if the relation is transitive there must also be a relation between  $a$  and  $c$ .
- (d) Is the relation 'x is related as a family member to y' an equivalence relation? Discuss.
  - No, it's not. First think intuitively; are everyone that are related to you as a family member also you? Thought not. The "family member" relation is both symmetric and transitive, but not reflexive.

## 2 Formalize the statement

- (a) Formalize the following statement using truth-functional connectives and sentence variables.

"Either the President is not a genius, or, if there is a conspiracy, then neither the Senator nor the judge is right."

  - $p$  = the president is not a genius
  - $q$  = the senator is right
  - $r$  = the judge is right
  - $s$  = there is a conspiracy
  - $t = s \supset \neg(q \vee r)$
  - $(p \wedge \neg t) \vee (\neg p \wedge t)$
- (b) Does the statement in (a) imply that the Senator is right?
  - No, it does not imply the Senator to be right.  
 $(p \wedge \neg t) \vee (\neg p \wedge t) \equiv (p \wedge s \wedge (q \vee r)) \vee (p \vee (s \wedge (q \vee r)))$   
We can see that in the equivalent statement the " $q$ " variable is connected to an "or" operator, therefore we cannot say the statement implies anything from the variable.

### 3 Insert quotation marks

- (a) Bernhard Borge is a pseudonym for the author André Bjerke.
- "Bernhard Borge" is a pseudonym for the author André Bjerke.
- (b) Be not afraid of greatness is a quote from a play called The Twelfth Night.
- "Be not afraid of greatness" is a quote from a play called The Twelfth Night.
- (c) One of the De Morgan equivalences is  $\neg p \cdot \neg q$  is equivalent to  $\neg(p \vee q)$ .
- One of the De Morgan equivalences is "  $\neg p \cdot \neg q$  " is equivalent to "  $\neg(p \vee q)$  ".
- (d) River Thames consists of 10 letters and rhymes with burning flames.
- "River Thames" consists of 10 letters and rhymes with "burning flames".

### 4 Natural deduction

- (a) " $p \supset q \vee r$ " and " $\neg r$ " implies " $p \supset q$ "

|           |     |                      |           |
|-----------|-----|----------------------|-----------|
| [1]       | (1) | $p \supset q \vee r$ | P         |
| [2]       | (2) | $\neg r$             | P         |
| [3]       | (3) | $p$                  | P         |
| [1][3]    | (4) | $q \vee r$           | (1)(3) MP |
| [1][2][3] | (5) | $q$                  | (2)(4) CE |
| [1][2]    | (6) | $p \supset q$        | [3](5) D  |

- (b) " $p \wedge q$ " and " $q \supset r$ " implies " $p \wedge (r \vee q)$ "

|        |     |                       |           |
|--------|-----|-----------------------|-----------|
| [1]    | (1) | $p \wedge q$          | P         |
| [2]    | (2) | $q \supset r$         | P         |
| [1]    | (3) | $p$                   | (1) CE    |
| [1]    | (4) | $q$                   | (1) CE    |
| [4]    | (5) | $r \vee q$            | (4) DI    |
| [1][4] | (6) | $p \wedge (r \vee q)$ | (3)(5) CI |

- (c) " $p \supset \neg q$ " and " $r \vee s \supset q$ " implies " $r \supset \neg p$ "

|           |     |                      |           |
|-----------|-----|----------------------|-----------|
| [1]       | (1) | $p \supset \neg q$   | P         |
| [2]       | (2) | $r \vee s \supset q$ | P         |
| [3]       | (3) | $r$                  | P         |
| [2][3]    | (4) | $q$                  | (2)(3) MP |
| [2][3]    | (5) | $\neg \neg q$        | (4) DN    |
| [1][2][3] | (6) | $\neg p$             | (1)(5) MT |
| [1][2]    | (7) | $r \supset \neg p$   | [3](6) D  |

(d) " $p \supset \neg q$ " implies " $p \wedge r \supset (q \vee r \supset r)$ "

|     |     |   |          |
|-----|-----|---|----------|
| [1] | (1) | $p \supset \neg q$                        | P        |
| [2] | (2) | $p \wedge r$                              | P        |
| [2] | (3) | $r$                                       | (2) CE   |
| [2] | (4) | $q \vee r$                                | (3) DI   |
| [2] | (5) | $q \vee r \supset r$                      | (3)(4) D |
| []  | (6) | $p \wedge r \supset (q \vee r \supset r)$ | [2](5) D |

## 5 Natural deduction II

By using natural deduction, show that " $p \supset (q \supset r)$ " is equivalent to " $p \wedge q \supset r$ ".

|           |     |                           |           |
|-----------|-----|---------------------------|-----------|
| [1]       | (1) | $p \wedge q \supset r$    | P         |
| [2]       | (2) | $p$                       | P         |
| [3]       | (3) | $q$                       | P         |
| [2][3]    | (4) | $p \wedge q$              | (2)(3) CI |
| [1][2][3] | (5) | $r$                       | (1)(4) MP |
| [1][2]    | (6) | $q \supset r$             | [3](5) D  |
| [1]       | (7) | $p \supset (q \supset r)$ | [2](6) D  |

|        |     |                           |           |
|--------|-----|---------------------------|-----------|
| [1]    | (1) | $p \supset (q \supset r)$ | P         |
| [2]    | (2) | $p \wedge q$              | P         |
| [2]    | (3) | $p$                       | (2) CE    |
| [2]    | (4) | $q$                       | (2) CE    |
| [1][2] | (5) | $q \supset r$             | (1)(3) MP |
| [1][2] | (6) | $r$                       | (4)(5) MP |
| [1]    | (7) | $p \wedge q \supset r$    | [2](6) D  |