INF2080
Oblig 1

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Problem 1: Regular languages

Problem 1a

$A \cap B = n_{\mathcal{A}} \cdot n_{\mathcal{B}}$
$A^* = n_{\mathcal{A}} \bigvee n_{\mathcal{A}}{+}1$

Problem 1b

$A \cap B = n_{\mathcal{A}} \cdot n_{\mathcal{B}}$
$AB = n_{\mathcal{A}} + n_{\mathcal{B}}$
$A^* = n_{\mathcal{A}}{+}1$

Problem 1c

Regex that define the same language as the NFA:

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^(aa+)?b(cc+)?$
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Problem 1d

Please refere to the DFA Figure 1.12 p.38 Sipser and replace a and b with 0 and 1.

Problem 2: all-NFAs

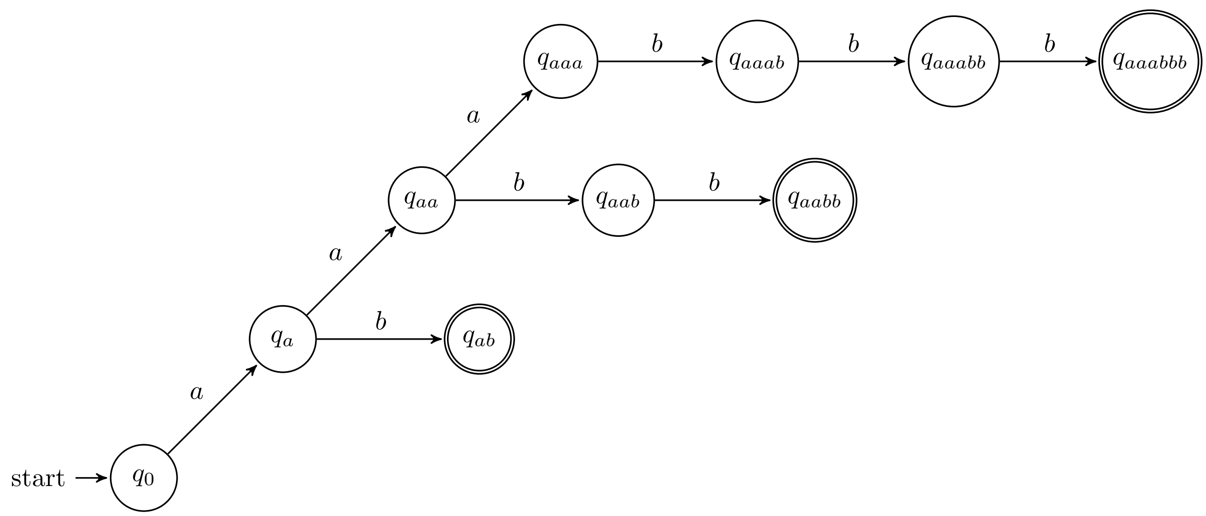
We start with the all-NFA $N = (Q, \Sigma, \delta, q_0, F)$ and construct a DFA $M = (Q', \Sigma', \delta', q_0', F')$ that recognize the same language as the NFA. We do the same approach as Sipser Theorem 1.39.

$Q' = P(Q)$
$\delta'(R, a) = U\delta(r, a)$
$q_0' = \{q_0\}$
$F' = \{R \subseteq Q' \mid R \subseteq F\}$

Note that the F' definition is altered, this is because needed the subsets of all the accepted states in Q.

Problem 3: Non-regular languages

Problem 3a



DFA for the language $\{ab, aabb, aaabbb\}$.

Problem 3b

Please refere to the PDA Figure 2.15 p.115 Sipser and replace 0 and 1 with a and 1.

Problem 3c

Using the pumping lemma, give a detailed proof that $\{a^n b^n \mid n \in \mathbb{N}\}$ is not a regular language; that is, no deterministic *finite* automaton may define it.
We assume the language $A = \{a^n b^n \mid n \in \mathbb{N}\}$ is regular. p is the pumping length

as defined by the pumping lemma. Our string $s \in A$ and $|s| \geq p$ which means $s = a^p b^p$. We split s into three pieces, $s = xy^iz$ where $i \geq 0$ and $xy^iz \in A$. Given that the alphabet is only two two symbols, and the entire language of strings has only one of the symbols on half of the string and the other symbol on the other side of the string $(ab, aabb, aaabbb, aaaabbbb, ...)$, that gives us three possible states the y can be in:

1. only a's
2. a's and b's
3. only b's

In the first and third case we have an unequal distribution of a's and b's, that are not elements of A. The second case gives us equally many a's and b's but they are not in the right order, A a has only a's on one side and b's on the other, eg. $aaabababbbb \notin A$. This gives us a contradiction. A can not be a regular language.

Problem 3d

To show that $\{a^n b^n \mid n \in \mathbb{N}\}$ is a context free language we design a CFG $G = (\{S\}, \{a, b\}, R, S)$, where $R : S \longrightarrow aSb|ab$.