FIL1006 - oblig 4

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1 Natural deduction with extensions

(a) Show that ”$\forall x(Fx \supset Gx)$” $\equiv$ ”$\forall x(\neg Gx \supset \neg Fx)$”

[1]	(1)	$\forall x(Fx \supset Gx)$	P
[2]	(2)	$\neg Gy$	P
[1]	(3)	$Fy \supset Gy$	(1) UI
[1][2]	(4)	$\neg Fy$	(2)(3) MT
[1]	(5)	$\neg Gy \supset \neg Fy$	[2](4) D
[1]	(6)	$\forall x(\neg Gx \supset \neg Fx)$	(5) UG

(b) Implication with quantifiers have the same rules as propositions. If p then q, and if not q then we know not p. With propositions we could reason: ”if I'd drop my phone then it would be broken” and ”if my phone is not broken then I have not dropped my phone”. But quantifiers with implication is a little different because they refer to things, example: ”if it's an apple it's delicious” and ”if it's not delicious, then it's not an apple”; both statements refer to the same objects, but with the negation and antecedent switch the statement refers inversely, but still captures the same objects as the first.

2 Abstract interpretation

(a) $\forall x \exists y(Fxy \wedge \neg Fyx)$

$$U = \{0, 1, 2\}$$
$$F = \{<0,1>, <1,2>, <2,1>\}$$
$$!F = \{\}$$

(b) $\forall x(Fxx \supset \exists y(Fyy))$

$$U = \{0\}$$
$$F = \{<0,0>\}$$

(c) $\exists x \exists y(Fxy \equiv Fyx)$

$$U = \{0, 1\}$$
$$F = \{<0,0>, <1,1>\}$$
$$!F = \{<0,1>\}$$

3 Formalize statements

(a) There is one and only one journalist who admires every politician.

J : journalist
P : politician
A : admires

$\exists x \forall y \forall z (Py \supset (Jx \wedge Axy \wedge ((Jz \wedge Azy) \supset x = y))$

(b) No journalist admires the Prime Minister

J : journalist
P : prime minister
A : admires

$\neg \exists x \exists y \forall z (Jx \wedge Py \wedge Axy \wedge (Pz \supset y = z))$

(c) Everyone gets something he/she wants and wants something he/she does not get

$U = \{\text{everything}\}$
P : person
W : want
G : get

$\forall x \exists y \exists z (Px \supset (Gxy \wedge Wxy \wedge \neg Gxz \wedge Wxz))$

4 Prove conclusion

C : composer
P : philosopher
L : logician
A : admires

(a) The conclusion is not valid. Here is a counterexample

$U = \{0, 1, 2, 3\}$
$C = \{0\}$
$P = \{1, 2\}$
$L = \{3\}$
$A = \{< 0, 1 >, < 2, 3 >\}$

The composer only admires 1, and the philosopher that admires the logician is not admired by anybody.

(b) Conclusion is valid, here is the deduction.

[1]	(1)	$\exists x(Cx \wedge \forall y(Py \supset Axy))$	P
[2]	(2)	$\exists x(Px \wedge \forall y(Ly \supset Axy))$	P
[1, 3]	(3)	$Cx \wedge \forall y(Py \supset Axy)$	(1) x EII
[2, 4]	(4)	$Px \wedge \forall y(Ly \supset Axy)$	(2) x EII
[1, 3]	(5)	$\forall y(Py \supset Axy)$	(3) CE
[1, 3]	(6)	$Py \supset Axy$	(5) y UI
[2, 4]	(7)	$\forall y(Ly \supset Axy)$	(4) CE
[2, 4]	(8)	$Ly \supset Axy$	(7) y UI
[9]	(9)	Ly	P
[2, 4, 9]	(10)	Axy	(8)(9) MP
[2, 4]	(11)	Px	(4) CE
[1, 2, 3, 4]	(12)	Axy	(6)(11) MP
[1, 2, 3, 4]	(13)	$\exists z(Azx)$	(12) EG
[1, 2, 4]	(14)	$\exists z(Azx)$	[3](13) EIE
[1, 2, 4, 9]	(15)	$Axy \wedge \exists z(Azx)$	(10)(14) CI
[1, 2, 4, 9]	(16)	$\exists y(Ayx \wedge \exists z(Azy))$	(15) EG
[1, 2, 9]	(17)	$\exists y(Ayx \wedge \exists z(Azy))$	[4](16) EIE
[1, 2]	(18)	$Lx \supset \exists y(Ayx \wedge \exists z(Azy))$	[9](17) D
[1, 2]	(19)	$\forall x(Lx \supset \exists y(Ayx \wedge \exists z(Azy)))$	(18) UG