Work plan – 4th group

Laura Carvajal, Andraž Pirnovar, Jaša Štefan

Problem description

Let us consider an annulus *A*. An annulus is a ring-shaped object, bounded by two concentric circles *C* and *C'*. Let us assume that *C'* is the smaller one. Inside the annulus, there is a set of *n* points, named P_n, that are uniformly randomly distributed. For these points, we find a convex hull $CH(P_n)$.
A convex hull of a set of points *X* in a Euclidean space (or affine space over the reals) is the smaller convex set that contains *X*. If we were to have a bounded set *X* of points on a plane, we could visualize the convex hull as a rubber band stretched around the set *X*.

The objective of our assignment is to experimentally analyse the length and the area of $CH(P_n)$. We also have to experimentally analyse the probability that the convex hull $CH(P_n)$ contains the inner border (smaller circumference) of the annulus, *C'*.

Work plan

As we have to analyse the data with respect to radius *r* of the smaller circle *C'* and number of points *n*, we decided it would be best if we measure the radius relatively to the radius of the bigger circle.

We set the bigger radius to 1 and took 101 smaller radiuses, from 0 to 1. This represents 1% increase of the smaller radius, relative to the bigger one, for which we calculate all of the required properties.

For *n*, we took 16 different values: 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 30, 40, 50, 100, 200, 500.
At smaller values, the differences in length, area and probability should be more noticeable with small increases in *n* than at higher values.

We use two methods for point generation. One distributes the points uniformly over the area of the annulus, using a probability transformation of uniformly distributed polar coordinates parameter.
The other also uses uniformly distributed parameters of polar coordinates, but without the transformation, so the points are not uniformly distributed over the annulus.

For each combination of *r*, *n* and generating method, we did 4000 repetitions, each time with a new set of randomly generated points. For every combination, we calculated *mean length*, *mean area* and the number of times the smaller circle *C'* was included inside the convex hull. From this, we calculated the experimental probability of the convex hull containing the inner circle.

Data is saved to a *csv*, so that it is easy to import into *R*. There we plan to plot graphs that show how do the ratio between the bigger and smaller radius, and the number of points affect the probability of smaller circle inclusion.

Most of the programing work will be done in *Python* and *Matlab*, with some graphics and analysis done in *R*. Most of the project specific code will be written by us, with the help of *numpy* and *scipy* packages and predefined functions.

Hypothesis

The results should depend on the proportion between the smaller and the bigger circle and also on the number of points inside the annulus.

Our hypothesis is that the length of the convex hull will increase significantly with the number of points till a certain point and then only marginally with further increase. The size of the radius *r* will also be a significant factor in the length of the hull, presumably even more so than *n*, as generated points will be squeezed to the outskirts of the annulus with the radius increase. Similar applies to the area.

The probability of the convex hull including the smaller circle, *C'*, should increase with *n* and decrease with *r*.

The method of point generation should be of some importance. Presumably, if points are generated "truly" uniformly, the probability of inclusion should be higher than with the other method, as there will be more points on the outskirts of the annulus. Therefore, the convex hull from the "true" uniform distribution should be longer and should cover a larger area. Because of a bigger hull, we expect the probability of a smaller circle inclusion to be higher when points are distributed "truly" uniformly.