# Problem description

Let us consider an annulus A. An annulus is a ring-shaped object, bounded by two concentric circles C and C'. Let us assume that C' is the smaller one. Inside the annulus, there is a set of n points, named P\_n, that are uniformly randomly distributed. For these points, we have a convex hull CH(P\_n).

A convex hull of a set of points X in an Euclidean space (or affine space over the reals) is the smalled convex set that contains X. If we were to have a bounded set X of points on a plane, we could visialize the convex hull as a rubber band streched arround the set X.

The objective of our assignement is to experimentaly analyze the length and the area of CH(P\_n). We also have to experimentaly analyze the probability, that the convex hull CH(P\_n) contains the inner border of the annulus, C'.

The results will depend on the relative size of the smaller annulus and the number of points inside the annulus.

# Work plan

As we have to analyse the data with respect to radius r of the smaller circle C’ and number of points n, we decided it would be best if we measure the radius relatively to the radius of the bigger circle.

We set the bigger radius to 1 and took 101 smaller r-s, from 0 to 1. This represents 1% increase of the smaller radius, relative to the bigger one, for which we calculate all of the required properties.

For n, we took 16 different values: 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 30, 40, 50, 100, 200, 500

At smaller values, the differences in length, area and probability should be more noticeable with small increases in n than at higher values.

We have two method for point generation. One distributes the points uniformly over the area of the annulus, using a probability transformation of uniformly distributed polar coordinates parameter.

The other also uses uniformly distributed parameters of polar coordinates, but without the transformation, so the points are not uniformly distributed over the annulus.

For each combination of r, n and generating method, we did 4000 repeats, each time with a new set of randomly generated points. For every combination, we calculated mean length, mean area and the number of times the smaller circle C’ was included inside the convex hull. From this, we calculated the experimental probability of the convex hull containing the inner circle.

Most of the programing work will be done in Python, with some graphics and analysis, done in R. Most of the project specific code will be written by us, with the help of numpy and scipy packages and predefined functions.

# Hypothesis

Our hypothesis is, that the length of the convex hull will increase significantly with the number of points till a certain point, and then only marginally with further increase. The size of the radius r will also be a significant factor in the length of the hull, presumably even more so than n, as generated points will be squeezed to the outskirts of the annulus with the radius increase.

Similar applies to the area.

In both cases, the “true” uniform distribution should produce longer length and bigger area, as it generates points uniformly throughout the annulus, in comparison to the other method, in which the point density is greater near the smaller annulus.

The probability of C’ inclusion should increase with the number of points and decrease with the radius. The method of point generation should be of some importance. Presumably, if points are generated “truly” uniformly, the probability of inclusion should be higher as with the other method, because there will again be more points on the outskirts of the annulus, meaning a bigger hull.