Cable crossing developement

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To help with the development, it helps to look at figure 1. The values of

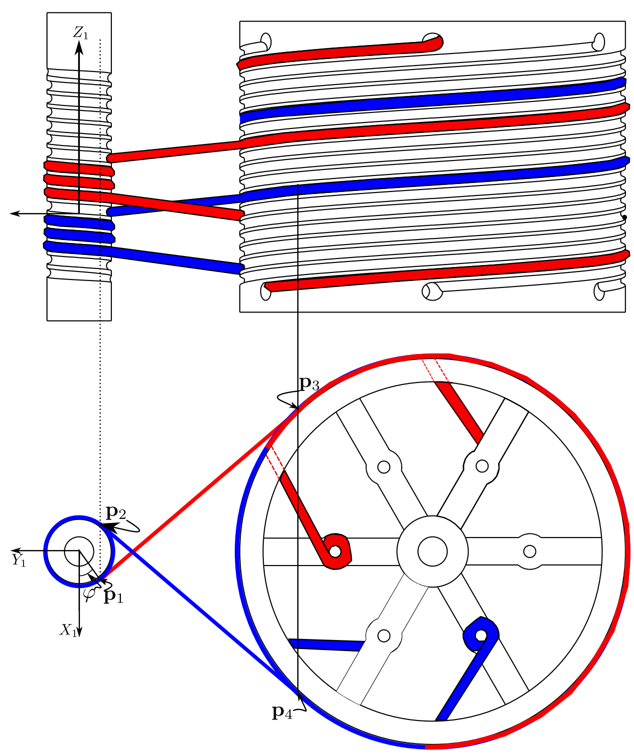

FIGURE 1 – cable crossing condition.

$\mathbf{p}_1$ and $\mathbf{p}_2$ are given by

$$\mathbf{p}_1 = \begin{bmatrix} r_1 c\varphi \\ -r_1 s\varphi \\ \frac{H_1\varphi}{2\pi} \end{bmatrix}, \quad \mathbf{p}_2 = \begin{bmatrix} -r_1 c\varphi \\ -r_1 s\varphi \\ \frac{H_1}{2\pi}(3\pi - \varphi). \end{bmatrix} \tag{1}$$

Also, we know that at points $\mathbf{p}_1$ and $\mathbf{p}_2$ the unit vectors that are tangeant and that go in the same direction as the cables (towards the large pulley) are expressed respectively as

$$\mathbf{u}_1 = \frac{\begin{bmatrix} -r_1 s\varphi \\ -r_1 c\varphi \\ \frac{H_1}{2\pi} \end{bmatrix}}{\rho_1}, \quad \mathbf{u}_2 = \frac{\begin{bmatrix} r_1 s(3\pi - \varphi) \\ r_1 c(3\pi - \varphi) \\ \frac{-H_1}{2\pi} \end{bmatrix}}{\rho_1} = \frac{\begin{bmatrix} r_1 s\varphi \\ -r_1 c\varphi \\ \frac{-H_1}{2\pi} \end{bmatrix}}{\rho_1}. \tag{2}$$

With the expressions in (1) and (2), we can express the positions of $\mathbf{p}_3$ and $\mathbf{p}_4$ respectively as

$$\mathbf{p}_3 = \mathbf{p}_1 + L_f \mathbf{u}_1, \tag{3}$$

$$\mathbf{p}_4 = \mathbf{p}_2 + L_f \mathbf{u}_2. \tag{4}$$

The line l_1 that passes through $\mathbf{p}_1$ and $\mathbf{p}_2$ can be expressed with the form

$$l_1 : \mathbf{p}(s_1) = \mathbf{p}_1 + (\mathbf{p}_3 - \mathbf{p}_1)s_1, \quad s_1 \in \mathbb{R}, \tag{5}$$

where s_1 is an arbitrary parameter. In the same manner, the line l_2 that passes through points $\mathbf{p}_3$ and $\mathbf{p}_4$ can be expressed with the form

$$l_2 : \mathbf{p}(s_2) = \mathbf{p}_2 + (\mathbf{p}_4 - \mathbf{p}_2)s_2, \quad s_2 \in \mathbb{R}, \tag{6}$$

where s_2 is also an arbitrary parameter. The expressions in equations (5) and (6) can be further simplified to

$$l_1 : \mathbf{p}(s_1) = \mathbf{p}_1 + L_f s_1 \mathbf{u}_1, \quad l_2 : \mathbf{p}(s_2) = \mathbf{p}_2 + L_f s_2 \mathbf{u}_2. \tag{7}$$

The shortest distance d between l_1 and l_2 is given by the following equation

$$d = L_f^2 \frac{\mathbf{u}_1 \times \mathbf{u}_2}{\|\mathbf{u}_1 \times \mathbf{u}_2\|} \cdot (\mathbf{p}_2 - \mathbf{p}_1). \tag{8}$$

$$L_f^2 = \frac{\rho_1^2 \left(D^2 - (r_2 + r_1)^2\right)}{r_1^2} \tag{9}$$

$$\mathbf{u}_1 \times \mathbf{u}_2 = \frac{1}{\rho_1^2} \begin{bmatrix} -r_1 s\varphi \\ -r_1 c\varphi \\ \frac{H_1}{2\pi} \end{bmatrix} \times \begin{bmatrix} r_1 s\varphi \\ -r_1 c\varphi \\ \frac{-H_1}{2\pi} \end{bmatrix} = \frac{2 r_1 c\varphi}{\rho_1^2} \begin{bmatrix} \frac{H_1}{2\pi} \\ 0 \\ r_1 s\varphi \end{bmatrix} \tag{10}$$

$$\|\mathbf{u}_1 \times \mathbf{u}_2\| = \tag{11}$$