Problem Set 7
Physics, summer 2020/21

1) **(1p.)** Figure shows a glass of ice water filled to the brim. Will the water overflow when the ice melts? Explain your answer.

Answer:
No. The mass of the ice cube is roughly the same as that of the liquid it pushes aside while floating in the glass. When the cube melts, the surrounding liquid can move into the space it filled. If the glass contains only ice and water, it will not overflow because the amount of extra water will be equal to that previously displaced by the cube(s). Diluted soda is slightly denser than water due to its sugar content. The water from the ice cube would therefore fill a larger volume than what it displaced and so, in this case, the melting ice might push some liquid out of the glass.

2) **(2p.)** Calculate the average density of the atmosphere, given that it extends to an altitude of 120 km. Compare this density with that of air equal 1.29 kg/m^3.

Answer:
If we solve p=hρg for density, we see that

$$\rho = \frac{P}{hg}$$

We then take p to be atmospheric pressure, h is given, and g is known, and so we can use this to calculate ρ.
Solution

$$\rho = \frac{P}{hg} = \frac{1.01 \times 10^5 \mathrm{Nm}^2}{120 \times 10^3 \mathrm{m} * 9.8 \frac{\mathrm{m}}{\mathrm{s}^2}} = 8.59 \times 10^{-2} \frac{\mathrm{kg}}{\mathrm{m}^3}$$

This result is the average density of air between the Earth's surface and the top of the Earth's atmosphere, which essentially ends at 120 km. The density of air at sea level is equal 1.29 kg/m^3 *— about 15 times its average value. Because air is so compressible, its density has its highest value near the Earth's surface and declines rapidly with altitude.*

3) **(2p.)** Calculate the depth below the surface of water at which the pressure due to the weight of the water equals 1.00 atm.

<u>Answer:</u>
If we solve p=hρg for density, we see that

$$h = \frac{P}{\rho g}$$

We then take p to be 1.00 atm and ρ to be the density of the water that creates the pressure.
<u>Solution:</u>

$$\rho = \frac{P}{hg} = \frac{1.01 \times 10^5 \mathrm{Nm}^2}{1.00 \times 10^3 \frac{\mathrm{kg}}{\mathrm{m}^3} * 9.8 \frac{\mathrm{m}}{\mathrm{s}^2}} = 10.3\mathrm{m}$$

Just 10.3 m of water creates the same pressure as 120 km of air. Since water is nearly incompressible, we can neglect any change in its density over this depth.

4) **(2p.)** What force must be exerted on the master cylinder of a hydraulic lift to support the weight of a 2000 kg car (a large car) resting on the slave cylinder? The master cylinder has a 2.00 cm diameter and the slave has a 24.0 cm diameter.

<u>Answer:</u>
We will use Pascal's Principle in Hydraulic System

$$\frac{F_{car}}{A_{slave}} = \frac{F_{lift}}{A_{cylinder}}$$

We should take $A = \pi \left(\frac{d}{2}\right)^2$ *for both surfaces, and then*

$$F_{lift} = \frac{F_{car} A_{cylinder}}{A_{slave}} = \frac{F_{car}\pi\left(\frac{d_{cylinder}}{2}\right)^2}{\pi\left(\frac{d_{slave}}{2}\right)^2} = \frac{m_{car} * g * \pi\left(\frac{d_{cylinder}}{2}\right)^2}{\pi\left(\frac{d_{slave}}{2}\right)^2}$$

<u>Solution:</u>

$$F_{lift} = \frac{m_{car} * g * \pi\left(\frac{d_{cylinder}}{2}\right)^2}{\pi\left(\frac{d_{slave}}{2}\right)^2} = \frac{2000\ kg * 9.8\frac{m^2}{s} * \pi\left(\frac{0.02\ m}{2}\right)^2}{\pi\left(\frac{0{,}24 m}{2}\right)^2} = 136{,}11\ N$$

5) **(3p.)** Calculate the contact angle θ for olive oil if capillary action raises it to a height of 7.07 cm in a glass tube with a radius of 0.100 mm. Is this value consistent with that for most organic liquids? (for olive oli take: γ = 0.032 N/m and ρ=0.92 kg/m^3)

Answer:

Here we will use equation $h = \frac{2\gamma cos\theta}{\rho g r}$ *and change it to find* θ

$cos\theta = \frac{h\rho g r}{2\gamma}$ *and then we have* $\theta = arccos\left(\frac{h\rho g r}{2\gamma}\right)$

Solution:

$$\theta = arccos\left(\frac{h\rho g r}{2\gamma}\right) =$$

$$= arccos\left(\frac{7.07 \times 10^{-2} m * 0.92 \frac{kg}{m^3} * 9.8 \frac{m}{s^2} * 0.1 \times 10^{-3} m}{2 * 0.032 \frac{N}{m}}\right) =$$

$$= arccos\left(\frac{6{,}37 \times 10^{-5} \frac{kg}{s^2}}{6{,}4 \times 10^{-5} \frac{kg^2}{s}}\right) = 1.571\, rad = 90°$$

This value is not consistent with that for most organic liquids

Interface	Contact angle Θ
Mercury–glass	140°
Water–glass	0°
Water–paraffin	107°
Water–silver	90°
Organic liquids (most)–glass	0°
Ethyl alcohol–glass	0°
Kerosene–glass	26°

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17.04.2021r.