Problem Set 5
Physics, summer 2020/21

1) **(4p.)** Suppose a cosmic ray colliding with a nucleus in the Earth's upper atmosphere produces a muon that has a velocity v=0.950c . The muon then travels at constant velocity and lives 1.52μs as measured in the muon's frame of reference. (You can imagine this as the muon's internal clock.) How long does the muon live as measured by an Earth-bound observer?

Answer:
A clock moving with the system being measured observes the proper time, so the time we are given is $\Delta t0=1.52\mu s$. The Earth-bound observer measures Δt as given by the equation $\Delta t=\gamma\Delta t0$. Since we know the velocity, the calculation is straightforward.

Solution

1) Identify the knowns: $v=0.95c$, $\Delta t_0=1.52\mu s$
2) Identify the unknown. Δt
3) Choose the appropriate equation.

Use, $\Delta t=\gamma\Delta t_0$, where

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

4) Plug the knowns into the equation.

First find γ .

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.95*c)^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{0.95^2*c^2}{c^2}}} = \frac{1}{\sqrt{1-0.95^2}} = 3.2.$$

Now we use the calculated value of γ to determine Δt .

$\Delta t = \gamma\Delta t_0 = (3.20)*(1.52\mu s) = 4.87\mu s$

One implication of this example is that since $\gamma=3.20$ at 95.0% of the speed of light ($v=0.95*c$), the relativistic effects are significant. The two time intervals differ by this factor of 3.20, where classically they would be the same. Something moving at 0.950c is said to be highly relativistic.

2) **(3p.)** A particle is traveling through the Earth's atmosphere at a speed of 0.750c . To an Earth-bound observer, the distance it travels is 2.50 km. How far does the particle travel in the particle's frame of reference?

Answer:

1) Identify the knowns: $v=0.75c$, $L_0=2.50km$
2) Identify the unknown. L
3) Choose the appropriate equation.

Use, $L = L_0/\gamma = L_0\sqrt{1-\frac{v^2}{c^2}}$, *becouse*

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

4) Plug the knowns into the equation.

First find γ .

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.75*c)^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{0.75^2*c^2}{c^2}}} = \frac{1}{\sqrt{1-0.75^2}} = 1.51$$

Now we use the calculated value of γ *to determine* L .

$L = L_0/\gamma = (2.5\ km)/(1.51) = 1.65\ km$

3) **(3p.)** What is the momentum of an electron traveling at a speed 0.985c? The rest mass of the electron is 9.11×10^{-31}kg.

Answer:

$$p = \gamma mu = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{(9.11\times10^{-31}\ \mathrm{kg})(0.985)(3.00\times10^8\ \mathrm{m/s})}{\sqrt{1-\frac{(0.985c)^2}{c^2}}} = 1.56\times10^{-21}\ \mathrm{kg\cdot m/s}$$

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