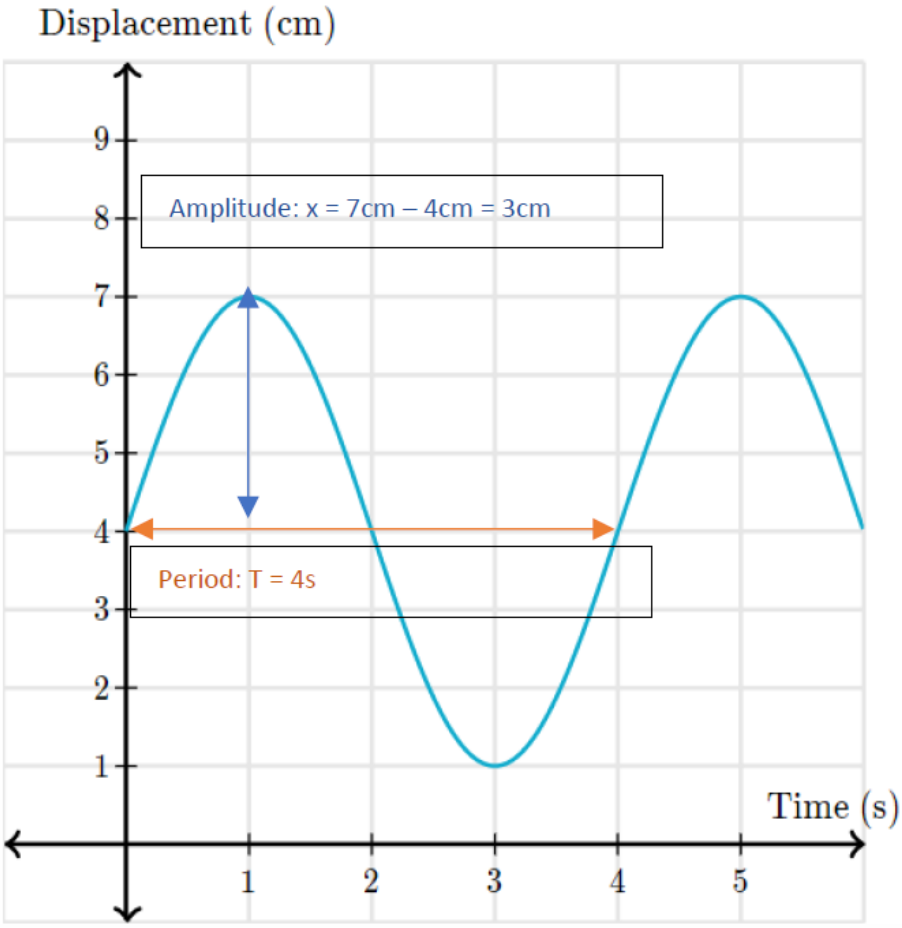
Problem Set 4
Physics, summer 2020/21

1) **(4p.)** A student extends then releases a mass attached to a spring. A graph of the mass's displacement over time is shown below.



a) What is the period, frequency and amplitude of the oscillation?
b) What is the average speed and velocity between 2s and 5s?

Answer:
*a) Amplitude is equal **3cm**. Period is equal **4s**, so frequency **f = 1/T = 1/4s = 0.25s**.*
*b) Distance d =9cm, V_{avg}=d/Δt=9/(5-2)=**3cm/s**, S_{avg}=(7-4)/(5-2)=**1cm/s**.*

2) **(2p.)** Find the acceleration due to Earth's gravity at the distance of the Moon. The radius of the Moon's nearly circular orbit is 3.84×10^8m, and mass of Earth is equal 5.98×10^{24}kg.

Answer:
Substituting known values into the expression for g found above, remembering that M is the mass of Earth not the Moon, yields

$$g == \frac{GM}{r^2} = (6.67\times10^{-11} N\cdot m^2 kg^2)\times\frac{5.98\times10^{24} kg}{(3.84\times10^8 m)^2} = 2.70\times10^{-3} m/s^2.$$

3) **(3p.)** Calculate the centripetal acceleration needed to keep the Moon in its orbit (assuming a circular orbit about a fixed Earth), and compare it with the value of the acceleration due to Earth's gravity that you have just found. Assume that the period (the time it takes to make one complete rotation) of the Moon's orbit is 27.3 days. Remember formulas from previous classes (rotational motion).

<u>*Answer:*</u>

Centripetal acceleration can be calculated using either form of

$$\begin{cases} a_c = \dfrac{v^2}{r} \\ a_c = r\omega^2 \end{cases}$$

We choose to use the second form:
$a_c = r\omega^2$
where ω *is the angular velocity of the Moon about Earth.*
Given that the period (the time it takes to make one complete rotation) of the Moon's orbit is 27.3 days, and using

$1\ d \times 24 hr/d \times 60 min/hr \times 60 s/min = 86{,}400\ s$

we see that

$\omega = \Delta\theta/\Delta t = 2\pi rad/((27.3\ d)(86{,}400\ s/d)) = 2.66 \times 10^{-6} rad/s.$

The centripetal acceleration is

$$a_c = r\omega^2 = (3.84 \times 10^8 m)\left(2.66 \times \frac{10^{-6} rad}{s}\right)^2 = 2.72 \times 10^{-3} m/s^2.$$

The direction of the acceleration is toward the center of the Earth.
Comment: The centripetal acceleration of the Moon found in differs by less than 1% from the acceleration due to Earth's gravity found in previous exercise. This agreement is approximate because the Moon's orbit is slightly elliptical, and Earth is not stationary (rather the Earth-Moon system rotates about its center of mass, which is located some 1700 km below Earth's surface). The clear implication is that Earth's gravitational force causes the Moon to orbit Earth.

4) **(1p.)** A planet moves around the sun in an elliptical orbit with the sun at one of its foci. The physical quantity associated with the motion of the planet that remains constant with time is? Choose an answer and comment that choice.
   a) velocity
   b) centripetal force
   c) linear momentum
   d) <u>angular momentum</u>

Answer: D.
Angular momentum is conserved (constant) because of the force of gravitational attraction between the planets and the sun exerts zero torque on the planet.

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