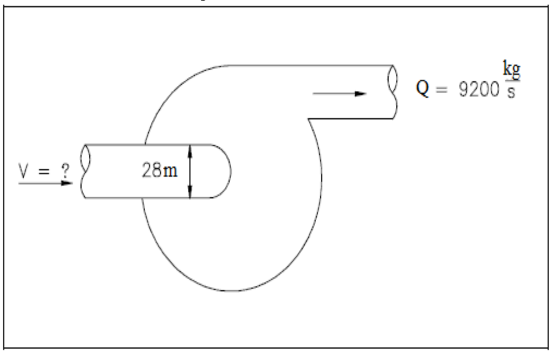
Problem Set 8
Physics, summer 2020/21

1) **(2p.)** A pipe with an inner diameter of 4 m contains water that flows at an average velocity of 14 m/s. The water in the pipe has a density of 62.44 kg/m^3. Calculate the volumetric flow rate of water in the pipe and the mass flow rate.

$$V = Av = (\pi r^2)v = \pi * 16m^2 * 14\frac{m}{s} = 703.72\frac{m^3}{s}$$
$$Q = \rho A v = 62.44\frac{kg}{m^3} * 703.72\frac{m^3}{s} = 43940.28\frac{kg}{s} \approx 44\frac{t}{s}$$

2) **(2p.)** Steady-state flow exists in a pipe that undergoes a gradual expansion from a diameter of 6 m to a diameter of 8 m. The density of the fluid in the pipe is constant at 60.8 kg/m^3. If the flow velocity is 22.4 m/s in the 6 m section, what is the flow velocity in the 8 m section?



$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$
$$v_2 = v_1 * \frac{\rho_1 A_1}{\rho_2 A_2} = v_1 \frac{\pi r_1^2}{\pi r_2^2}$$
$$= 22.4\frac{m}{s} * \left(\frac{3m}{4m}\right)^2$$
$$= 12.6\frac{m}{s}$$

3) **(2p.)** Suppose the flow rate of blood in a coronary artery has been reduced to half its normal value by plaque deposits. By what factor has the radius of the artery been reduced, assuming no turbulence occurs (laminar flow)?

Assuming laminar flow, Poiseuille's law states that

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\mu l}$$

We need to compare the artery radius before and after the flow rate reduction.

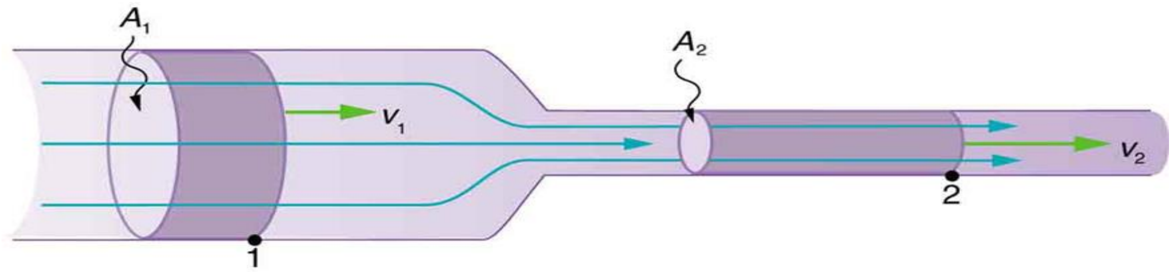
With a constant pressure difference assumed and the same length and viscosity, along the artery we have

$$\frac{Q_1}{r_1^4} = \frac{Q_2}{r_2^4}$$

So, given that $Q_2 = 0.5Q_1$, we find that $r_2^4 = 0.5\, r_1^4$.

Therefore, $r_2 = (0.5)^{0.25}\, r_1 = 0.841\, r_1$, a decrease in the artery radius of 16%.

4) **(3p)** Assume frictionless flow in a long, horizontal, conical pipe. The diameter is 2.0 m at one end and 4.0 m at the other. The pressure head at the smaller end is 16 m of water. If water flows through this cone at a rate of 125.6 m^3/s, find the velocities at the two ends and the pressure head at the larger end.



$$Q_1 = A_1 v_1 \rightarrow v_1 = \frac{Q_1}{A_1} = \frac{125.6\frac{m^3}{s}}{\pi * 4m^2} = 10\frac{m}{s}$$

$$Q_2 = A_2 v_2 \rightarrow v_2 = \frac{Q_2}{A_2} = \frac{125.6\frac{m^3}{s}}{\pi * 1m^2} = 40\frac{m}{s}$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\frac{P_1}{\rho g} = \frac{P_2 + \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)}{\rho g} = \frac{P_2}{\rho g} + 0.5 * \frac{(v_2^2 - v_1^2)}{g} + (h_2 - h_1)$$

$$\frac{P_1}{\rho g} = 16m + \frac{\left(1600\frac{m^2}{s^2} - 100\frac{m^2}{s^2}\right)}{20\frac{m}{s^2}} + 0m = 91m$$

5) **(1p.)** Daniel Bernoulli and his famous principle get a lot of attention when it comes to lift. A lot of aviation enthusiasts would argue, however, that Bernoulli is only part of the lift story though. A different, more modern view of airplane flight relies on another famous scientific principle. Which one?

a) Darwin’s theory of natural selection.

b) Newton’s third law of motion.

c) Einstein’s theory of relativity.

Move over Bernoulli, Newton wants to fly this plane. According to authors Anderson and Eberhardt, Newton's third law of motion is perfectly capable of explaining how a wing works: Grossly simplified, it says that the wing pushes the air down, so the air pushes the wing up.

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25.04.2021r.